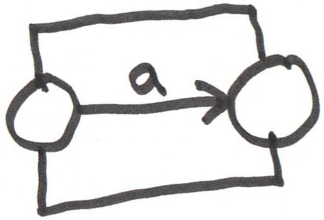
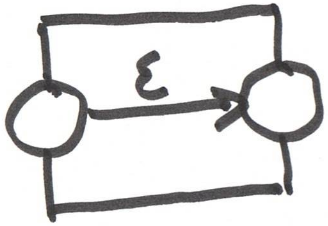
THOMPSON'S CONSTRUCTION

$a \in \Sigma$

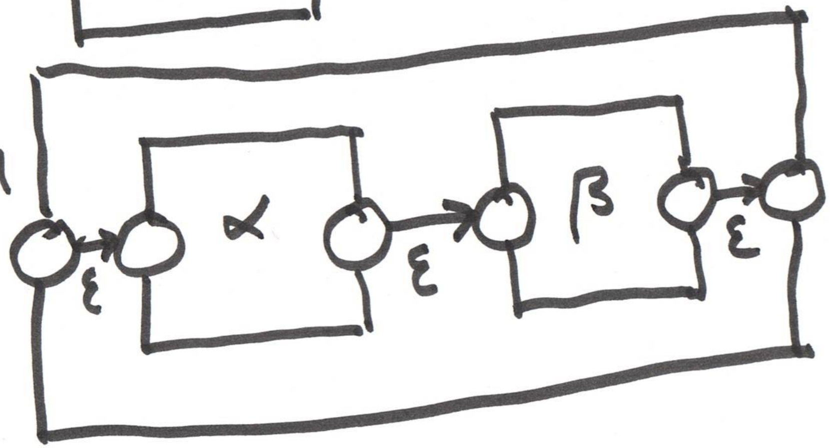


ε



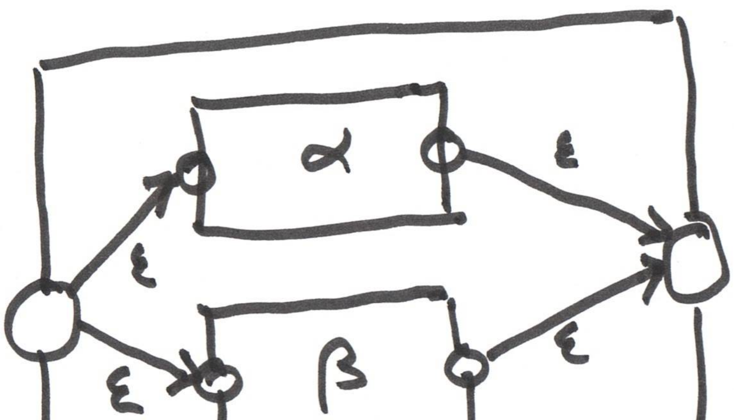
Concatenation

$\alpha\beta$



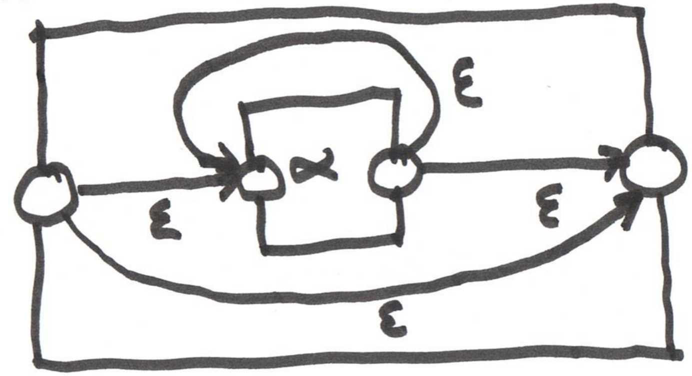
alternation

$\alpha | \beta$



Kleene closure

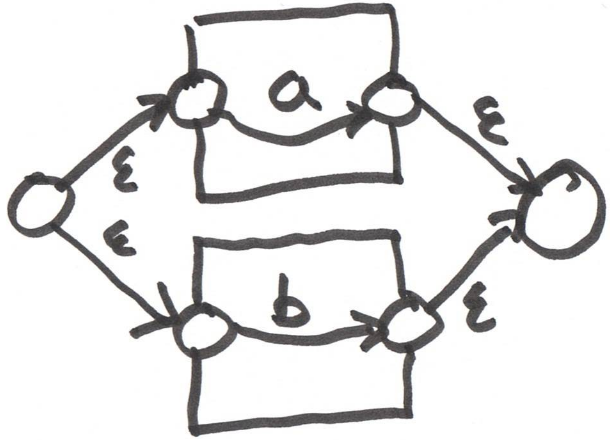
α^*



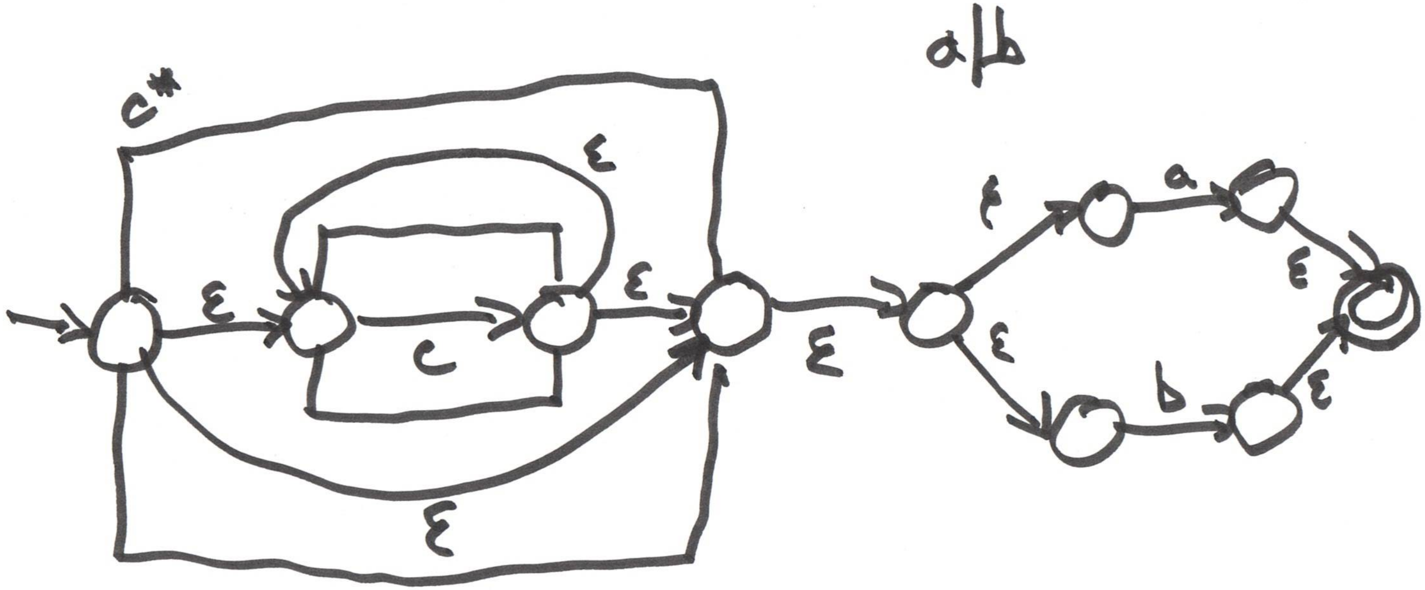
$c^*(a|b)$

Reg. Expr. $\longrightarrow$ NFA $\longrightarrow$ DFA $\longrightarrow$ Min. state DFA

"compiling" a regular expression



Q \ Σ	a	b	c
q_0			
q_1			
q_2			



1 table lookup / character.

Kleene's construction

states $0 \ldots n-1$

R^k_{ij} = regular expression for paths from state i to state j that have intermediate nodes at most k.

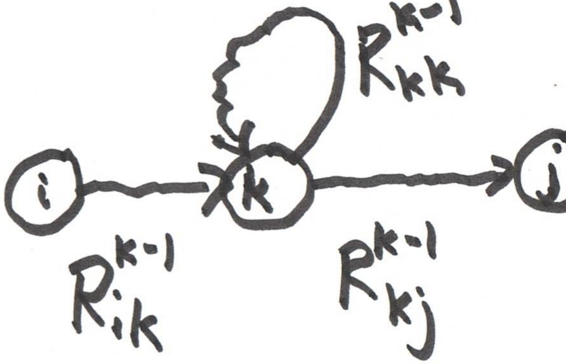
start with $k = -1$ (directly from i to j)

$a|e$

$b|g|\varepsilon$

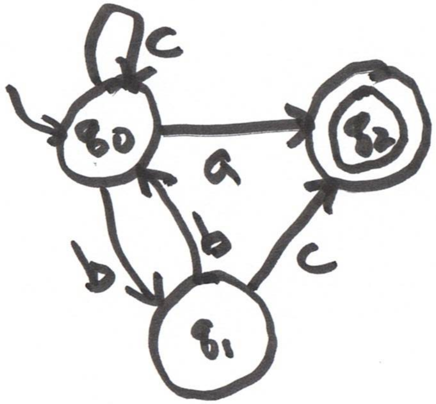
R_{ij}^{k}

don't have state k as intermediate state R_{ij}^{k-1}



$$R_{ik}^{k-1}\left(R_{kk}^{k-1}\right)^{*} R_{kj}^{k-1}$$

$$= R_{ij}^{k-1} \mid R_{ik}^{k-1}\left(R_{kk}^{k-1}\right)^{*} R_{kj}^{k-1}$$



k=-1

to →

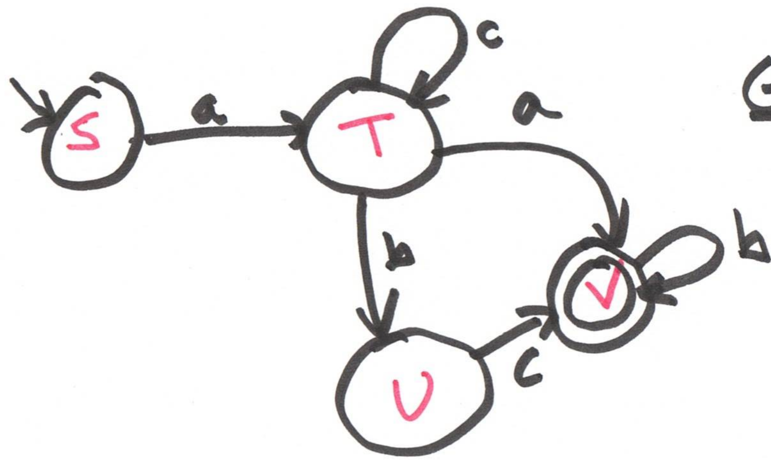
from ↓	0	1	2
0	$c\|\varepsilon$	b	a
1	b	ε	c
2	X	X	ε

$(c|\varepsilon)|((c|\varepsilon(c|\varepsilon)^* c|\varepsilon)$

k=0

	0	1	2
0	$\cancel{c\varepsilon}\|c^*$	$b\|c^*b$	c^*a
1	$b\|bc^*$	$\varepsilon\|bc^*b$	
2			

DFA



Grammar

$S \to aT$

$T \to aV$

$T \to bU$

$T \to cT$

$U \to cV$

$V \to \varepsilon$

$V \to bV$

Right-regular grammar

productions are of this form. or $A \to cB$ or $A \to \varepsilon$

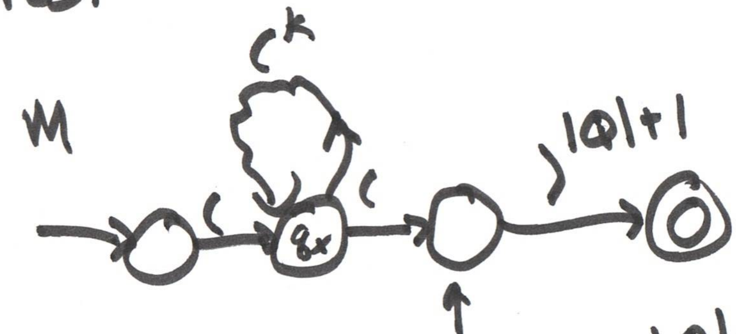
$S \to (S)$
$\;\;\to \varepsilon$

$\varepsilon \mid () \mid (()) \mid ((())) \mid \dots$

$\{ (^k)^k \mid k \geq 0 \}$

Suppose $\exists$ DFA M that recognizes this.
M has $|Q|$ of states.

$(^{|Q|+1}$



$(^{|Q|+1-k} \,)^{|Q|+1}$