1. Let $d = \gcd(252, 198)$. Use the Euclidean Algorithm to compute d, and then find all integer solutions x, y to the equation $252x + 198y = d$.
Show your work.

2. (i) Prove, by considering the remainders that x and y can leave when divided by 3, that for all $x, y \in \mathbb{Z} \quad 3 \mid (x^2 + y^2) \implies (3 \mid x \text{ and } 3 \mid y)$.

(ii) Prove that for all prime numbers $p \geq 3$ and integers x,
$p^2 \mid (x^2 - 1) \implies (p^2 \mid x + 1 \quad \text{or} \quad p^2 \mid x - 1)$.

3. Note that modular arithmetic is not on exam 2 in fall 2022, but you can work this problem for practice prior to exam 3. Provide brief justification for your answers.

(i) Find the remainder when 3^{103} is divided by 53.

(ii) Let p be a positive prime number. Determine $\gcd((p-1)! + 1, p!)$.

(iii) Determine the digits a and b given that $72 \mid 437213ab$. Hint: $72 = 8 \cdot 9$.

4. Show that for every positive integer n, $\displaystyle\sum_{i=1}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$.

5. Suppose that $\sim$ is an equivalence relation on $\mathbb{N}$ that satisfies the following condition:

$$\forall n \geq 1, \exists k \in \mathbb{N} \text{ s.t. } 2k < n \text{ and } 2k \sim n$$

In other words, every natural is related to some even natural smaller than itself. Show, by induction, that $\forall n \in \mathbb{N},\ 0 \sim n$.

Bonus. Show that every prime number is irreducible.