# INDUCTIONFEST

**Inductionfest** was first held in 2004. It is always so successful that it is held the following year.

**Instructions:** Work the first problem. After working a problem, work the problem that follows it.

**Example 1.** Show that for every positive integer $n$, $\quad 1 \cdot 1! + 2 \cdot 2! + \cdots n \cdot n! = (n+1)! - 1$.

**Example 2.** Let $h$ be a real number, $h \geq -1$. Show that for every positive integer $n$,

$$1 + nh \leq (1+h)^n.$$

**Example 3.** Show that for every positive integer $n$, $\quad 21 \mid (4^{n+1} + 5^{2n-1})$.

**Example 4.** Consider the sequence $\{a_n\}$ defined by $a_0 = 2$, $a_1 = 1$, and $a_n = 3a_{n-1} + a_{n-2}$ when $n \geq 2$. First show that for all natural numbers $n$, $\quad 3 \nmid a_n$. As a second problem, show that $a_n$ is even if and only if $n$ is a multiple of 3.

**Example 5.** Show that every positive integer $n$ can be written as a sum of Fibonacci numbers, such that no number is repeated and no pair of adjacent Fibonacci numbers are used (for example, 1 can't be repeated and 5 and 8 can't both be used).