21-128 Lecture

Wednesday, August 31, 2022 11:04 AM

1.2, 1.3 Friday
DH2315 Saturday 1-4

$\forall x \in \mathbb{R} \quad |x^2| = |x|^2$

$\forall x, y \in \mathbb{R} \quad |(x+iy)^2| = |x+iy|^2$

$(x^2+y^2)^2 = (2xy)^2 + (x^2-y^2)^2$

Propositional variable $\rightarrow$ $P_{\sim}$ T or F

False if p is true and q is false

Logical connectives $\rightarrow$ $p \wedge q$, $p \vee q$, $p \Rightarrow q$, $p \Leftrightarrow q$, $\neg p$

Propositional formula $\rightarrow$ $P \wedge (R \vee Q) \equiv (P \wedge R) \vee (P \wedge Q)$

Predicate $P(x_1, x_2, \ldots, x_N)$

Logical formula

$\forall, \exists, !$, s.t.

$\exists y \in \mathbb{N}\ \forall x \in \mathbb{N}$ "$x+y$ is Even" for y if $x = y+1$

$x+y = 2y+1$ which is odd

THM The product of two consecutive positive integers is never a perfect square

$n(n+1) = n^2 + n$

assume that $m \in \mathbb{N}^+$ and $m(m+1) = N^2$ for some $N \in \mathbb{N}$

observe that $m^2 < m^2 + m = N^2 < (m+1)^2$

$\Rightarrow m < N < m+1$

THM $\sqrt{2} \notin \mathbb{Q}$

PF Assume $\exists\, M, N \in \mathbb{N}$

s.t. $\sqrt{2} = \frac{M}{N}$ and are not both even

$\Rightarrow \sqrt{2}N = M \Rightarrow 2N^2 = M^2 \Rightarrow M^2$ is even $\Rightarrow M$ is even

$\Rightarrow m = 2k$ for some $k \in \mathbb{N}$

$\Rightarrow 2N^2 = (2k)^2 \Rightarrow N^2 = 2k^2 \Rightarrow N^2$ is even $\Rightarrow N$ is even

A contradiction