Discrete Probability Space:

Sample Space, probability measure

$(\Omega, \mathbb{P})$

Ω — countable set — $\Omega = \{$ ⚀, ⚁, ⚂, ⚃, ⚄, ⚅ $\}$

$\mathbb{P}$ — function $\underline{P(\Omega)} \to [0,1]$ — note power set! — $\mathbb{P}(\{$ ⚁, ⚃, ⚅ $\}) = \frac{1}{2}$

outcome — element of Ω — outcome of rolling a four = ⚃

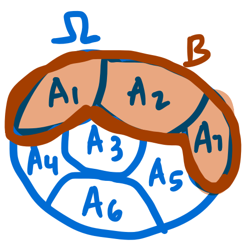
event — subset of Ω — event odd rolled = $\{$ ⚀, ⚂, ⚄ $\}$

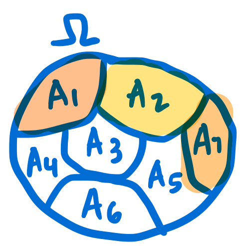
aka a set of outcomes
aka an element of $P(\Omega)$
aka in the domain of $\mathbb{P}$

$\mathbb{P}$ can't just be any ol' function — to be a legit *probability measure*:

i) $\mathbb{P}(\Omega) = 1$

ii) $\mathbb{P}\left(\bigcup_{i \in I} A_i\right) = \sum_{i \in I} \mathbb{P}(A_i)$] Countable Additivity





"you can add partitions of events the way you'd expect and nothing weird happens"

$\mathbb{P}(B) = \mathbb{P}(A_1) + \mathbb{P}(A_2) + \mathbb{P}(A_7)$

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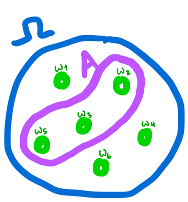
sample space — $\Omega = \{$ ⚀, ⚁, ⚂, ⚃, ⚄, ⚅ $\}$ — outcome

probability measure — $\mathbb{P}(\{$ ⚁ or ⚃ or ⚅ $\}) = \frac{1}{2}$ — event

($\mathbb{P}$ is a probability measure)

$\Longleftrightarrow$

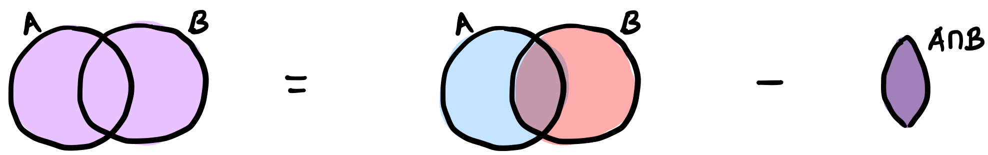
$$\left(\sum_{\omega \in A} \mathbb{P}(\{\omega\}) = \mathbb{P}(A) \quad \text{for all } A \subseteq \Omega \right)$$



$\mathbb{P}(\{\omega_5\}) + \mathbb{P}(\{\omega_3\}) + \mathbb{P}(\{\omega_2\}) = \mathbb{P}(A)$

Theorem 7.1.18: $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

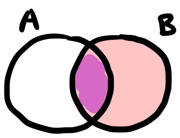
we want this — oops! we double-counted — so we compensate here



$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

Conditional Probability:

$$\underbrace{\mathbb{P}(A|B)}_{\text{"probability of A given B"}} = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$



$\mathbb{P}(A|B) = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$

"out of all the ways B can happen,
only consider the ones where A also happens"

Independent:

$$\left[\underbrace{\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)}\right] \Leftrightarrow \left[\text{A and B are independent}\right]$$

where does this come from?
intuitively we want $\mathbb{P}(A|B) = \mathbb{P}(A)$
plug this into the conditional probability eq.

mutually independent: $\mathbb{P}(A_1 \cap A_2 \cap \ldots A_n) = \mathbb{P}(A_1)\,\mathbb{P}(A_2) \ldots \mathbb{P}(A_n)$

Proposition 7.1.29:

$$\underbrace{\mathbb{P}(A)}_{\text{probability of A}} = \underbrace{\mathbb{P}(A|B)\,\mathbb{P}(B)}_{\text{probability of A if B happens}} + \underbrace{\mathbb{P}(A|B^c)\,\mathbb{P}(B^c)}_{\text{probability of A if B doesn't happen}}$$

these are the only 2 options — they partition the sample space
You could also partition into > 2 partions

Bayes Theorem:

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(A|B)\,\mathbb{P}(B)}{\mathbb{P}(A)}$$

try proving these two yourself!
(Use the definition of conditional probability, the fact that $A \cap B = B \cap A$, and your beautiful smart brain algebraic manipulation skillz to prove these two) I believe in you !☆

Corollary 7.1.34:

Combine Baye's Thm. and proposition 7.1.29 to get this super useful result!

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(A|B)\,\mathbb{P}(B)}{\mathbb{P}(A|B)\,\mathbb{P}(B) + \mathbb{P}(A|B^c)\,\mathbb{P}(B^c)}$$