Relations
↳ def for if two elements in a set are related
↳ ex: $\equiv_3$ mod 3 ⇒ $3 R 6$

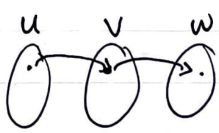
Relation R on set S,
① reflexive: $aRa \quad \forall a \in S$
② symmetric: $\forall a,b \in S \quad aRb \Rightarrow bRa$
③ transitive: $\forall a,b,c \in S \quad aRb \wedge bRc \Rightarrow aRc$
④ antisymmetric: $\forall a,b \in S \quad aRb \wedge bRa \Rightarrow a=b$

possible to be both

equivalence relations: ① ② ③
↳ equivalence classes: ~~partition of S st $\forall x \in S$ xRy~~ $\forall x,y$
equiv class on an elem x is the set $[x]_\sim = \{y \in S \mid xRy\}$
↳ in this equiv class $\forall a,b \in [x]_\sim \quad aRb$

① relation $\cong$ on $P(\mathbb{N})$ via $U \cong V \Leftrightarrow \exists$ bij $f: U \to V$
★ Note U, V are sets
Show $\cong$ is an equiv relation

① reflexive: Let $U \in P(\mathbb{N})$, WTS $U \cong U \Leftrightarrow \exists$ bij $f: U \to U$
consider $f: U \to U$ via $f(x) = x$ ✓ identity is bijective ✓
② symmetric: Let $U, V \in P(\mathbb{N})$, $U \cong V$, WTS $V \cong U \Leftrightarrow \exists$ bij $g: V \to U$
$U \cong V \Rightarrow \exists$ bij $f: U \to V$ def of $\cong$
$\Rightarrow \exists$ inverse $f^{-1}: V \to U$ def of bij & f^{-1} is bijective ✓
③ transitive: Let $U, V, W \in P(\mathbb{N})$, $U \cong V \wedge V \cong W$, WTS $U \cong W \Leftrightarrow \exists$ bij $h: U \to W$
we know $\exists$ bij $f: U \to V$ & $g: V \to W$



Consider $h: U \to W$ via $h(u) = g \circ f(u)$ $u \in U$
show well defined

pf of bij: Let $h^{-1}: W \to U$ via $h^{-1}(w) = f^{-1} \circ g^{-1}(w)$
↳ pf of R inv: $h \circ h^{-1}(w) = g(f(f^{-1}(g^{-1}(w))))$
$= g(g^{-1}(w))$
$= w$ ✓
↳ pf of L inv: $h^{-1} \circ h(u) = f^{-1}(g^{-1}(g(f(u))))$
$= f^{-1}(f(u))$
$= u$ ✓

- equivalence classes? $[U]_\sim = \{V \mid |U| = |V|\}$

② Relation R on $P(\mathbb{N})$ via $URV \Leftrightarrow \exists$ inj $f: U \to V$

Equivalence relation?

① transitive ✓

② symmetric X counterexample $U = \{1\}$ $V = \{1,2\}$

URV but $\overline{VRU}$

③ transitive? Assume for $U, V, W \in P(\mathbb{N})$ $URV \wedge VRW$ ✓

③ Relation R on $\mathbb{Z}$ via $aRb \Leftrightarrow$ ~~$ab \equiv 1 \bmod 5$~~ $a | b$

Equivalence relation?

NOT symmetric

④ Relation R on set of ppl via $aRb \Leftrightarrow$ a & b are emotionally close / friends

vs a & b are married