Note how countable and uncountable have no overlap

Countable · infinite · uncountable

finite   Countably
Infinite
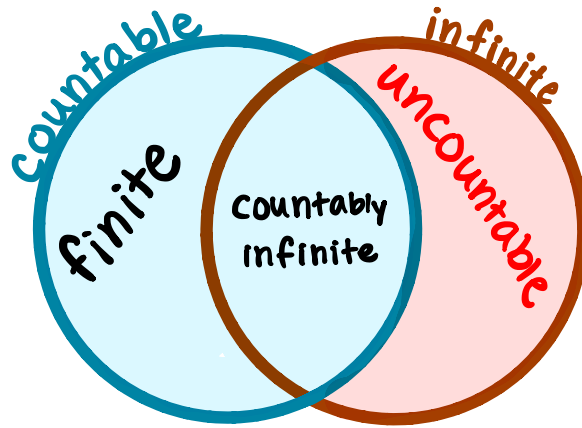
Countable

Uncountable

Use functions

want: A is countable
know: C is countable

| Injection | $f: A \to C$ |

| Surjection | $f: C \to A$ |

Good ideas for C

$$\mathbb{N} \quad \mathbb{Z} \quad \mathbb{Q}$$
$$\mathbb{N} \times \mathbb{N}$$

Other good tools

Countable Union of countable sets

Finite Cartesian Product of countable sets

☆ See other Sheet for reminders about these two!

Use functions

want: A is uncountable
know: U is uncountable

| Injection | $f: U \to A$ |

| Surjection | $f: A \to U$ |

Good ideas for U

$$\mathbb{R} \qquad \mathcal{P}(\mathbb{N})$$
$$\{0, 1\}^{\mathbb{N}}$$

Other good tools

Diagonalization