Relations Review

151/128 Staff

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1. For each example below, determine whether the given relation R is an equivalence relation on the given set S:

   (a) (MIT 6.042) $S = P$, where P is the set of all people in the world today; $(x, y) \in R$ if and only if x is at least as tall as y.

   (b) (BYU) $S = \mathbb{Z}$; $(x, y) \in R$ if and only if $2x + 5y \equiv 0 \pmod 7$.

   (c) (For those of you in Matrices) $S = \mathbb{R}^n$ for some $2 \leq n \in \mathbb{N}$; $(\vec{x}, \vec{y}) \in R$ if and only if $\vec{x}$ and $\vec{y}$ are linearly independent.

2. Suppose R is a relation on the set X that is both an equivalence relation and a partial order relation. Prove that R is the equality relation on X.

3. Let R be a relation on $\mathcal{P}([2n])$ defined by $(A, B) \in R$ if and only if $A \cap [n] = B \cap [n]$. Is R an equivalence relation? What happens if instead of using $[n], [2n]$, we generalize to C, S where $C \subseteq S$?

4. (Tripos 2011) Write down an equivalence relation on the positive integers that has exactly four equivalence classes, of which two are infinite and two are finite.