21-128 Lecture

Friday, September 2, 2022 11:07 AM

Read 2.1, bring laptops to LaTex

Prop vars, connectives, quantifiers
Predicate
Logical Formula

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$p \Rightarrow q \equiv \neg p \vee q$

$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$

$p \Leftrightarrow q \equiv p \Rightarrow q \wedge q \Rightarrow p$

$\neg(\exists z \in S\ p(z))$
$\equiv \forall z \in S\ \neg p(z)$

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$\equiv \exists z \in S\ \neg p(z)$

THM $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ [(y<x) \wedge (|x-y|=1)]$

PF Let $x \in \mathbb{R}$

Consider $y = x-1$

1st $y<x$ since $x-1<x$

2nd $|x-y| = |x-(x-1)| = |1| = 1$

Q.E.D

must prove the negation true

THM

$\neg(\exists y \in \mathbb{R}\ \forall x \in \mathbb{R}\ [(y<x) \wedge (|x-y|=1)])$

$\equiv \forall y \in \mathbb{R}\ \exists x \in \mathbb{R}\ \neg((y<x) \wedge (|x-y|=1))$

$\equiv \forall y \in \mathbb{R}\ \exists x \in \mathbb{R}\ ((y \geq x) \vee (|x-y| \neq 1))$

PF Let $y \in \mathbb{R}$

Consider $y = x+2$

Note that $y = x+2 > x$ since $2>0$. Q.E.D

$\neg(\exists!\, x \in S\ p(x))$

$\equiv \neg([\exists x \in S\ p(x)] \wedge [\forall x, y \in S\ p(x) \wedge p(y) \Rightarrow x=y])$

THM $\forall N \in \mathbb{N}\ \underbrace{(N \text{ is prime} \Rightarrow ((N \text{ is odd}) \vee (N=2)))}$

PF Let $N \in \mathbb{I}$

$\equiv \neg(N \text{ is odd}) \vee (N=2) \Rightarrow N \text{ is composite}$

$\equiv (N \text{ is even}) \wedge (N \neq 2) \Rightarrow N \text{ is composite}$