

# INDUCTION DON'TS

- ④ DON'T jump straight into the BC without first stating  $P(n)$ . (It's a sad way to lose easy points :))

$$P(n) := \sum_{i=1}^n 2^i = 2^{n+1} - 2$$

- ⑤ DON'T accidentally quantify  $n$  inside the proposition

~~$P(n) := \sum_{i=1}^n 2^i = 2^{n+1} - 2 \quad n \in \mathbb{N}, n \geq 1$~~

This proposition doesn't make sense!  $P(n)$  implies you can "plug in" any  $n$  you want, but  $n \in \mathbb{N}, n \geq 1$  means you've already fixed  $n$  to those values!

$P(n) := \sum_{i=1}^n 2^i = 2^{n+1} - 2$  ✓

WTS that  $P(n)$  is true for all  $n \in \mathbb{N}, n \geq 1$

Note how the quantification is separate from the proposition statement. You can use your own format (more like Clive's, or Mackey's, or your own personal way), but this key separation should still be there

- ⑥ DON'T assume more than you can in your IH

~~Let  $k \in \mathbb{N}, k \geq 1$~~

~~Assume  $P(k)$  is true for all  $k$ . WTS  $P(k+1)$~~

You already assumed what you wanted to show!

Fix some  $k \in \mathbb{N}, k \geq 1$  ✓

Assume  $P(k)$  is true (for this specific  $k$ ) WTS  $P(k+1)$

Assume that  $P(k)$  is true for some  $k$ .

Use that to show it works for the next number

## ⊛ DON'T prove the opposite direction

$$~~P(k+1) \Rightarrow P(k)~~$$

$$[P(k) \Rightarrow P(k+1)]$$

\*note that technically, if you are proving something like "P(n) holds for all negative integers" you might have to go backwards, but don't worry about that\*

This may seem obvious at first, but once you are in the midst of a big, complicated algebraic induction proof, it may be easy to accidentally go backwards!

## ⊛ DON'T get confused with strong induction IH

IH: Fix  $x \in \mathbb{N}$  Assume  $P(y)$  for all  $y \leq x$

This is one way to phrase a strong induction IH

~~WIS:  $P(y+1)$~~

Remember IH is assuming  $P(0)$  and  $P(1)$  and....  $P(x-1)$  and  $P(x)$   
Showing that  $P(y+1)$  is true doesn't add to this

✓ WIS:  $P(x+1)$

Since we know  $P(0) \wedge P(1) \wedge \dots \wedge P(x)$ , it makes sense to show  $P(x+1)$

## ⊛ DON'T forget extra BC's!

Yeah, a majority of the induction proofs we do have just one (1) base case, but don't fall into a habit of this! Always check that you're done as many as you need (sometimes you need 2, maybe even 3 base cases!

How do you know you need more BC's?: (but I don't think it gets much more than that)—also yikes nested parens)

- Sometimes a proof will follow the classic domino analogy

"if one domino falls, then the next one will"  $\rightarrow$  need one BC

- Other times a proof relies on something with like special dominoes where

"if two consecutive dominoes fall, then the next one will"  $\rightarrow$  need 2 BCs

This happens a lot with Fibonacci-like questions