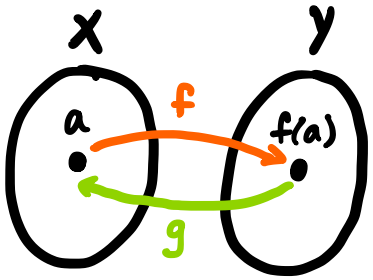
Left-inverse:

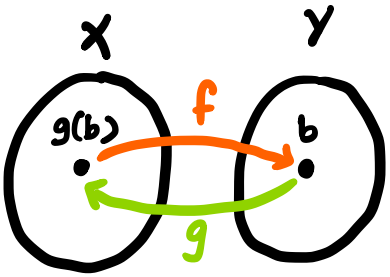
$g(f(a)) = a$



(f has a left inverse) $\Leftrightarrow$ (f is injective)

Right-inverse:

$f(g(b)) = b$



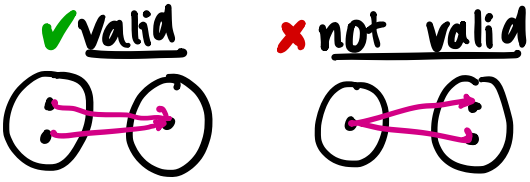
(f has a right inverse) $\Leftrightarrow$ (f is surjective)

Two-Sided inverse:

(f has a two-sided inverse) $\Leftrightarrow$ (f is bijective)

✱ Note the subtle BUT IMPORTANT difference:

- Remember that for $f: X \to Y$ to be a valid function
  - ✔ we can have multiple x's go to the same y's but
  - ✘ we cannot have the same x go to multiple y's

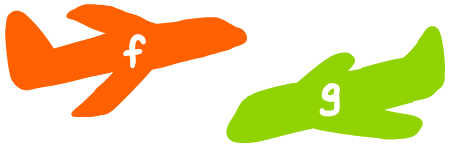


Let's explore how this works intuitively!

⊛ The subtle BUT IMPORTANT difference helps you understand why left inverse ⇔ inj. and right inverse ⇔ surj.

Analogy: f: Cities → Cities (Destinations)

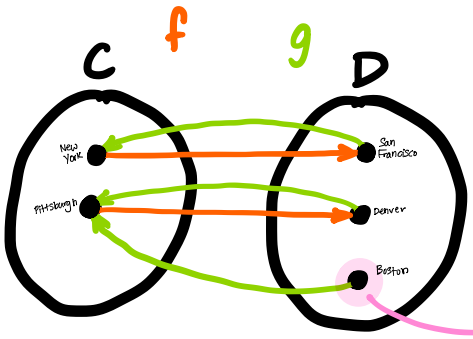
g: inverse of f (let's explore whether it's left or right)



Left Inverse:

Tells you **exactly** where you came from

"Some destinations might be unreachable, but each reachable destination has a return flight"



unreachable destination] that's why there is NOT necessarily a right inverse

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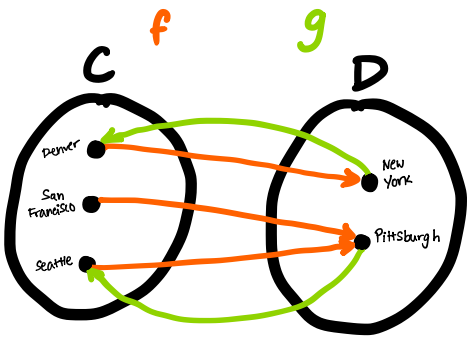
Right Inverse:

Tells you where you **might** have come from

"every destination is reachable, but I can't give an exact return flight since I don't know where you came from"



even if you flew:
San Francisco → Pittsburgh

the "return flight" would take you:
Pittsburgh → Seattle

] that's why there is NOT necessarily a left inverse

(it might go to the wrong city)