

- You have  $n$  pieces of candy
- There are  $k$  distinguishable kids

$n=5$  

$k=3$



- Introduce  $k-1$  dividing bars

$k-1=2$  

These bars let you visualize different ways of allotting candy — ex:



Each "super arrangement" corresponds to a candy distribution.

- Awesome now we've set up a bijection between super arrangements and candy distributions

- Since we have a bijection, this means that we can count the candy distributions by just counting the super arrangements

...so now, let's just figure out how to count super arrangements

We have  $(n + k - 1)$  objects (object := ~~xxx~~ or  $|$ )

So how do we arrange the  $n + k - 1$  objects?  
It matters whether the object is candy or a bar...

► One way of thinking about each arrangement is like:

- Place  $n + k - 1$  objects in a line

$(?) \quad (?) \quad (?) \quad (?) \quad (?) \quad (?) \quad (?)$

- Pick  $k - 1$  of them to be the bars

$(?) \quad | \quad (?) \quad (?) \quad | \quad (?) \quad (?)$

- So that the rest  $((n + k - 1) - (k - 1) = n)$  are candy

~~xxx~~  $|$  ~~xxx~~ ~~xxx~~  $|$  ~~xxx~~ ~~xxx~~

► This leads to the formula:

$$\binom{n + k - 1}{k - 1}$$

► Also note that you could have picked the candy first so:

$$\binom{n + k - 1}{n} \text{ also works the same.}$$