

Cantor-Schroeder-Bernstein Theorem (CSB)

A and B are two sets

You have two injective functions $f: A \rightarrow B$ and $g: B \rightarrow A$

CSB says you can find a bijective function $h: A \leftrightarrow B$

For the finite case this is easy:

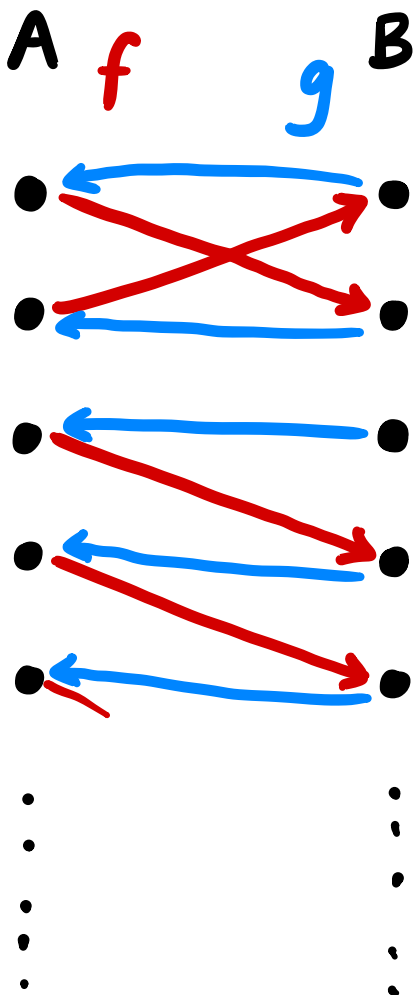
$$\left. \begin{array}{l} \text{injection } f: A \rightarrow B \Rightarrow |A| \leq |B| \\ \text{injection } g: B \rightarrow A \Rightarrow |B| \leq |A| \end{array} \right\} \Rightarrow |A| = |B| \Rightarrow \text{bijection } h: A \leftrightarrow B$$

But for infinite case we don't get that because

"injection from X to Y implies $|X| \leq |Y|$ " was proven by induction.

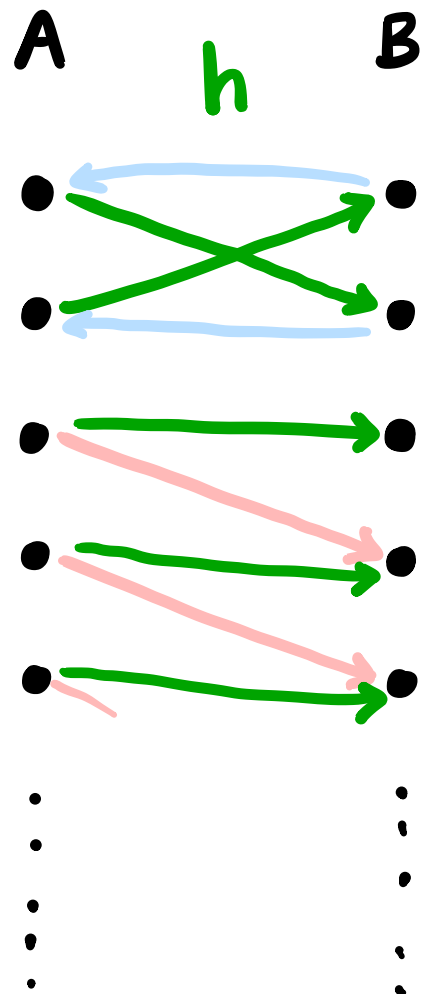
You cannot induct all the way to infinity

So we need to be a bit more clever for the infinite case....



"good case"
follow f
when you can

"bad case"
follow g
(but reverse
the arrows)
when you
cannot



☆ AoPS has a really nice formal proof, but you really just need the intuition which is the picture