AP

If $S = S_1 \cup S_2 \ldots S_n$ and $S_i \cap S_j = \emptyset$ for $i \neq j$

Then $|S| = \sum_{i \in [n]} |S_i|$

MP

- If a length n sequence of choices bijectively maps to elements of S
- # of choices at each step does not depend on previous step, s.t. step i has c_i choices

$$\Rightarrow |S| = \prod_{i \in [n]} c_i$$

Permutations

Count no. of bijections from $[n]$ to $[n]$

n Step Process

1. Choose x s.t. $f(x) = 1$ n choices
2. Choose x s.t. $f(x) = 2$ $n-1$ choices

⋮

n. Choose x s.t. $f(x) = n$ 1 choice

by MP: $(n)(n-1)(n-2)\ldots(1) = n!$

* Permuting n things

Arrangements

Count no. injections from $[k]$ to $[n]$ $(k \leq n)$

1. choose $f(1)$ — n choices
2. choose $f(2)$ — $n-1$ choices
3. choose $f(3)$ — $n-2$ choices

$\vdots$

k. choose $f(k)$ — $n-k+1$ choices

$\therefore \; {}^{n}P_{r}$ $\dfrac{n!}{(n-k)!}$

* arranging an ordered list of length n from k

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1. choose $f[[k]]$ $\binom{n}{k}$
2. permute them to construct f $k!$

Selections

Count no. subsets of $[n]$ of size k: defined to be $\binom{n}{k}$

$$\frac{k!}{(n-k)!} = \binom{n}{k} k!$$

$$\Rightarrow \binom{n}{k} = \frac{k!}{(n-k)!\, k!}$$

No. solutions to $x_1 + x_2 + \dots x_k = n$

for $x_i \geq 0$

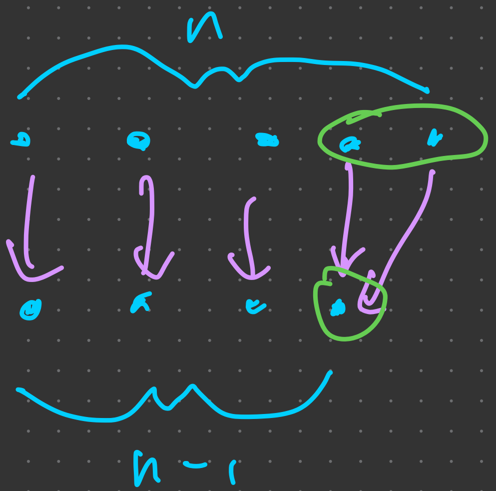
$\star \mid \star\star \mid\mid \star\star \mid \star$

- $k-1$ bars placed among n stars divides n into k pieces
- To specify such a string we need to place the $k-1$ bars among $n+k-1$ total symbols

So there are $\binom{n+k-1}{k-1}$ total solutions

Count no. of surjections

from $[n]$ to $[n-1]$



① Choose $y \in [n-1]$ s.t. y is mapped

to twice $\binom{n-1}{1}$

② Choose 2 values from $[n]$ $\{x_1, x_2\}$

such that $f(x_1) = y$ and $f(x_2) = y$

$\binom{n}{2}$

③ permute the remaining $n-2$ elements

in $[n]$ $(n-2)!$

By MP, $(n-1)\binom{n}{2}(n-2)!$ such functions

Count no. of 5 card hands with no Jacks OR no Queens, OR no Kings, OR no Aces.

Let S = set of 5 card hands with at least one Jack, Queen, King, and Ace

$S = S_{dup} \cup S_{nodup}$

* there could be two of the same face card

S_{dup}

1. choose rank to be dupped $\binom{4}{1}$
2. choose two of this card $\binom{4}{2}$
3. choose one of the other 3 ranks $\binom{4}{1}^3$

S_{nodup}

1. Choose one for each rank $\binom{4}{1}^4$
2. Choose the last card $\binom{52-16}{1}$

By MP & AP,

$$|S| = \binom{4}{1}\binom{4}{2}\binom{4}{1}^3 + \binom{4}{1}^4\binom{36}{1}$$

Finally, no. cards originally ~~used~~ for it

$$\binom{52}{5} - \binom{4}{1}\binom{4}{2}\binom{4}{1}^3 - \binom{4}{1}^4\binom{36}{1}$$

a)

count no. relations on $[2n]$

$$2^{(2n)^2}$$

b) count no. relations on $[2n]$

such that $\forall x \in [2n]$, $x \equiv y \mod 2$

$\implies xRy$

Lets count the set $S = \{ (x,y) \in [2n]$

$x \equiv y \mod 2$

$n=1$

1 2

$n=2$

1 2 3 4

$n=3$

1 2 3 4 5 6

★ n even numbers

★ n odd numbers

$\boxed{2n^2}$

$$2^{\left((2n)^2 - 2n^2\right)}$$