Example 7.1.35

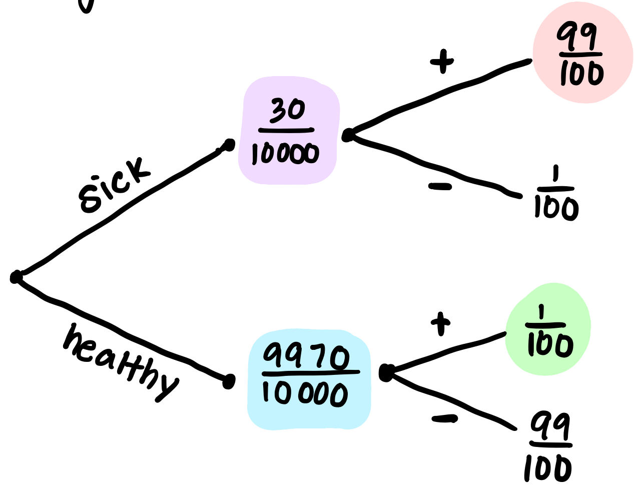
10000 people
30 sick
9970 healthy

Test is 99% accurate]

this means given that you're sick it will correctly predict + (positive) 99% of the time

And given that you're healthy it will correctly predict − (negative) 99% of the time

If a person takes the test and comes back positive (+), what is the probability they are actually sick?



- notice we have info for test result given sickness
- but we want info for sickness given test result
- This switcheroo requires Bayes Thm.

$$\mathbb{P}(\text{sick} \mid +) = \frac{\mathbb{P}(+\mid \text{sick})\,\mathbb{P}(\text{sick})}{\mathbb{P}(+\mid\text{sick})\,\mathbb{P}(\text{sick}) + \mathbb{P}(+\mid\text{healthy})\,\mathbb{P}(\text{healthy})}$$

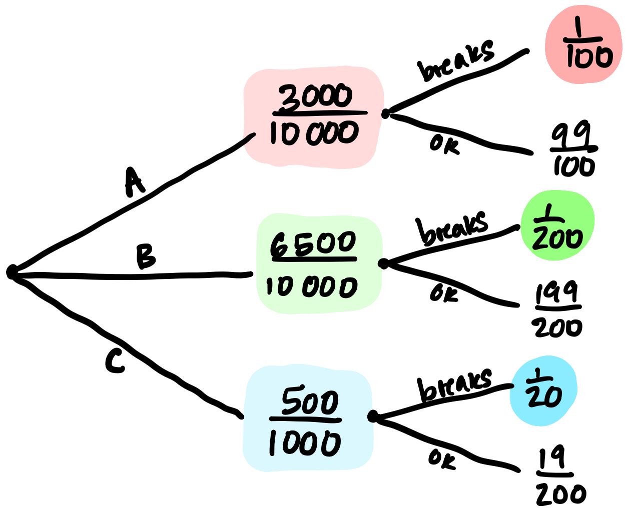
$$= \frac{\left(\frac{99}{100}\right)\left(\frac{30}{10000}\right)}{\left(\frac{99}{100}\right)\left(\frac{30}{10000}\right) + \left(\frac{1}{100}\right)\left(\frac{9970}{10000}\right)} = \frac{297}{1294} \approx 0.23$$

Wow look! Even though the test was 99% accurate, because the probability that you're even sick to begin with is so low ($\frac{30}{10000} = 0.3\%$), even if the test is positive, the probability that you're actually sick is $\approx$ 23%.

Example 7.1.37:

3000 Model A breaks down $\frac{1}{100}$
6500 Model B breaks down $\frac{1}{200}$
500 Model C breaks down $\frac{1}{20}$

notice we have: break given Model
but we want Model given break



$$P(C \mid \text{breaks}) = \frac{\mathbb{P}(\text{breaks} \mid C)\,\mathbb{P}(C)}{\mathbb{P}(\text{breaks} \mid C)\,\mathbb{P}(C) + \mathbb{P}(\text{breaks} \mid B)\,\mathbb{P}(B) + \mathbb{P}(\text{breaks} \mid A)\,\mathbb{P}(A)}$$

$$= \frac{\left(\frac{1}{20}\right)\left(\frac{50}{1000}\right)}{\left(\frac{1}{20}\right)\left(\frac{50}{1000}\right) + \left(\frac{1}{200}\right)\left(\frac{6\,500}{10\,000}\right) + \left(\frac{1}{100}\right)\left(\frac{3000}{10\,000}\right)}$$

$$= \frac{2}{7} \approx 0.29$$