Exercise 1.3.17 aka Failed Horse Induction Example

$P(n)$:= "All groups of n horses are the same color" } These are okay

WTS $P(n)$ is true for $n \in \mathbb{N}^+$

BC: $P(1)$ is true because one horse is the same color as itself * $P(1)$ is fine, but what about $P(2)$?!?!

IH: Fix $K \in \mathbb{N}^+$. Assume $P(K)$ i.e. Assume that all groups of K horses are the same color

IS: WTS $P(K+1)$ i.e. WTS that all groups of $K+1$ horse are the same color

Consider a set of $K+1$ horses.
Remove one horse from the set.
By the IH, the remaining K horses are the same color
Remove a different horse from the set.
By the IH, the remaining K horses are the same color.
Thus they must all be the same color. ■

This induction step only works if there are at least 2 horses
After you pick 1 horse, you still need "a different horse" to pick next — you need at least 2

Conclusion: This proof fell into the trap of not having enough base cases! If you could somehow show that $P(2)$ is true, then it would actually work!

But of course, $P(2)$ isn't true, so it fails.

Counterexample