Ray Tracing and Radiometry 5

CS184: Computer Graphics and Imaging

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1 Ray-Triangle Intersection

Given a mesh representation of an object, we would like to render it onto a display. To do so, we need to know which parts of the object are visible, where to put shadows, how to apply the scene's lighting, and more. The simplest idea to handle these problems is to take a ray and intersect it with each triangle in the mesh.

Recall that a ray is defined by its origin $\mathbf{O}$ and a direction vector $\mathbf{D}$ and varies with "time" t for $0 \leq t < \infty$.

$$\mathbf{r}(t) = \mathbf{O} + t\mathbf{D}. \tag{1}$$

A point within a triangle $\mathbf{P_0P_1P_2}$ can be represented as

$$\mathbf{P} = \alpha\mathbf{P}_0 + \beta\mathbf{P}_1 + \gamma\mathbf{P}_2, \tag{2}$$

where $\alpha + \beta + \gamma = 1$. Since α, β and γ are related, we can also write P as

$$\mathbf{P} = (1 - b_1 - b_2)\mathbf{P}_0 + b_1\mathbf{P}_1 + b_2\mathbf{P}_2. \tag{3}$$

1. Let's solve for the intersection of a ray and a triangle. Specifically, if we arrange the unknowns t, b_1 and b_2 into a column vector $\mathbf{x} = [t, b_1, b_2]^T$, can you get a matrix $\mathbf{M}$ and a column vector $\mathbf{b}$ so that $\mathbf{Mx} = \mathbf{b}$?

2. Now let's derive the **Möller-Trumbore algorithm**!

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\mathbf{S_1} \cdot \mathbf{E_1}} \begin{bmatrix} \mathbf{S_2} \cdot \mathbf{E_2} \\ \mathbf{S_1} \cdot \mathbf{S} \\ \mathbf{S_2} \cdot \mathbf{D} \end{bmatrix} \tag{4}$$

where $\mathbf{E_1} = \mathbf{P_1} - \mathbf{P_0}$, $\mathbf{E_2} = \mathbf{P_2} - \mathbf{P_0}$, $\mathbf{S} = \mathbf{O} - \mathbf{P_0}$, $\mathbf{S_1} = \mathbf{D} \times \mathbf{E_2}$, $\mathbf{S_2} = \mathbf{S} \times \mathbf{E_1}$.

Hint 1: (Cramer's rule) Linear equations $\mathbf{Mx} = \mathbf{b}$ can be simply solved using determinants of matrices as:

$$\mathbf{x} = \frac{1}{|\mathbf{M}|} \begin{bmatrix} |\mathbf{M_1}| \\ |\mathbf{M_2}| \\ |\mathbf{M_3}| \end{bmatrix}, \tag{5}$$

where $\mathbf{M_i}$ is the matrix $\mathbf{M}$ with its i-th column replaced by $\mathbf{b}$.

Hint 2: Suppose $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are column vectors, the determinant of the 3×3 matrix $[\mathbf{A}, \mathbf{B}, \mathbf{C}]$ satisfy:

$$|\mathbf{A}, \mathbf{B}, \mathbf{C}| = -(\mathbf{A} \times \mathbf{C}) \cdot \mathbf{B} = -(\mathbf{C} \times \mathbf{B}) \cdot \mathbf{A} = -(\mathbf{B} \times \mathbf{A}) \cdot \mathbf{C}. \tag{6}$$

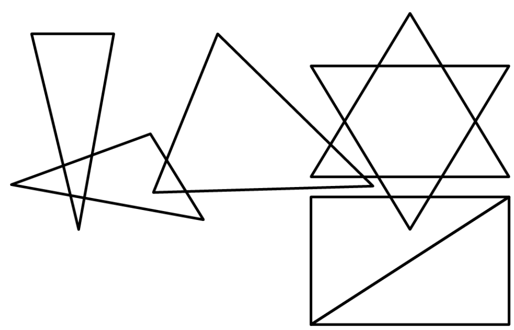
3. Once you've solved for t, b_1 and b_2, what conditions must be satisfied so that you have a valid ray-triangle intersection?

2 Bounding Volume Hierarchy

In ray tracing, bounding volumes are used to accelerate ray-triangle intersection tests. If the ray does not intersect a bounding volume, it cannot intersect the triangles contained within, allowing us to perform a batch rejection.

A bounding volume hierarchy (BVH) is simply a tree of bounding volumes. The bounding volume at a given node encloses the bounding volumes of its children. The ray tracing algorithm traverses this hierarchy to determine if the ray intersects an object.

1. Given a set of planar triangles, build a BVH following these rules:
    - Always pick the longest axis to divide.
    - Use barycenters of triangles to decide their relative positions.
    - Keep the BVH as balanced as possible, i.e. try to ensure the same number of triangles for children nodes.



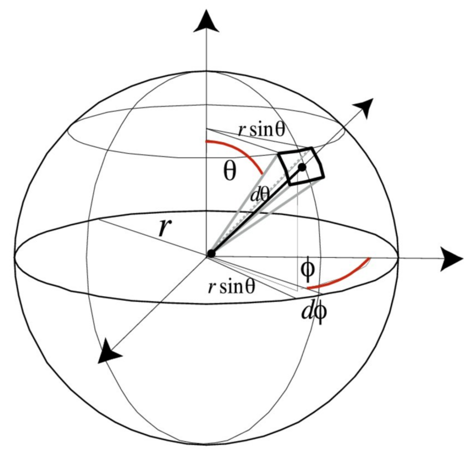
2. Given a box with corners $(-2, -2, -2)$ and $(2, 2, 2)$. Compute the entry and exit point of this box for a ray that has origin $(-3, 4, 5)$ and direction $(1, -1, -2)$.

3 Radiometry & Photometry

In computer graphics, we study radiometry and photometry to accurately simulate how much light is emitted and received, so that we can generate photo-realistic images.

1. What's the difference between **radiant flux / power** (Φ), **radiant intensity** (I), **irradiance** (E) and **radiance** (L)? How does increasing the distance from the light source affect these values?

2. Supppose we use (θ, ϕ)-parameterization of directions. Recall that the solid angle represents the ratio of the subtended area on a sphere to the radius squared, $\Omega = \frac{A}{r^2}$. Estimate the solid angle subtended by a patch that covers $\theta \in [\pi/6 - \pi/12, \pi/6 + \pi/12]$ and $\phi \in [\pi/5 - \pi/24, \pi/5 + \pi/24]$? (Hint: you may assume that the patch is small enough. Recall or derive the differential solid angle $d\omega$, then use the values given.)



3. Calculate the irradiance at point p from a disk area light overhead with uniform radiance L. (Hint: irradiance is an integral of incoming radiance over the hemisphere: $E(p) = \int_{H^2} L_i(p, \omega) \cos\theta \, \mathrm{d}\omega$.)