ANIMATION AND SIMULATION 09

CS184: COMPUTER GRAPHICS AND IMAGING

March 31, 2020

1 Animation and Physical Simulation

1. What are keyframes and what do we do with them?

   Solution: Keyframes, as their name suggests, are the important moments in some transition or motion, usually the starting and ending points. We usually interpolate between them (though not linearly, because usually too complex) in order to create the frames between the keyframes to form fluid video - the process of creating the intermediate frames is known as tweening.

2. What is the difference between forward and inverse kinematics? What are some problems associated with the latter?

   Solution: Forward - we provide angles (e.g. for joints), computer determines final position (e.g. of end of limbs).

   Inverse - we provide ending position, need to compute the joint angles to reach the position.

   Difficulties with inverse kinematics - sometimes has multiple possible solutions (sometimes connected to each other, sometimes separate), sometimes has no solutions, want to make sure solution found is realistic, etc.

3. Recall the forward, or explicit Euler method, which uses the following update rules:

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^t + \Delta t \dot{\boldsymbol{x}}^t$$
$$\dot{\boldsymbol{x}}^{t+\Delta t} = \dot{\boldsymbol{x}}^t + \Delta t \ddot{\boldsymbol{x}}^t$$

where $\boldsymbol{x}^t, \dot{\boldsymbol{x}}^t, \ddot{\boldsymbol{x}}^t$ respectively denote the position, velocity, and acceleration at time t.

(a) Give some pros and cons of using the explicit Euler method.

(b) Say we have a particle with mass 1 starting at position $\boldsymbol{x}^0 = (0, 1)$ with an initial velocity $\dot{\boldsymbol{x}}^0 = (-1, 0)$ and no initial acceleration. The particle is at one end of a spring, whose other end is the origin $(0, 0)$, and whose spring constant is $k = 1$ and rest length is 1. Calculate particle's position at $t = 3$ using the explicit Euler method with timestep $\Delta t = 1$.

Solution:

(a) Pros - simple and easy to compute, we don't always care about precision in graphics (i.e. close enough is good enough).

Cons - inaccurate and unstable, which frequently leads to divergent results, especially as time progresses and errors accumulate.

(b)

$$\boldsymbol{x}^0 = (0, 1) \qquad \dot{\boldsymbol{x}}^0 = (-1, 0) \qquad \ddot{\boldsymbol{x}} = (0, 0)$$

$$\boldsymbol{x}^1 = (-1, 1) \qquad \dot{\boldsymbol{x}}^1 = (-1, 0)$$

$$\ddot{\boldsymbol{x}}^1 = \frac{F_s}{m} * \frac{(0,0) - (-1,1)}{||(0,0) - (-1,1)||} = \frac{1 * (\sqrt{2} - 1)}{1} * (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}) = (1 - \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - 1)$$

$$\boldsymbol{x}^2 = (-2, 1) \qquad \dot{\boldsymbol{x}}^2 = (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - 1) \qquad \ddot{\boldsymbol{x}}^2 = \text{unneeded}$$

$$\boldsymbol{x}^3 = (-2 - \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) \qquad \dot{\boldsymbol{x}}^3 = \text{unneeded} \qquad \ddot{\boldsymbol{x}}^3 = \text{unneeded}$$