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$$(a) (\forall x \exists y p(x, y) \wedge (\exists y q(y) \rightarrow q(a))) \vee \forall y (\exists y \forall x p(x, y) \vee \exists z p(y, a))$$

$$(\forall x \exists y p(x, y) \wedge (\neg \exists y q(y) \vee q(a))) \vee \forall y (\exists y \forall x p(x, y) \vee \exists z p(y, a))$$

$$(\forall x \exists y p(x, y) \wedge (\forall y \neg q(y) \vee q(a))) \vee \forall y (\exists y \forall x p(x, y) \vee \exists z p(y, a))$$

$$(\forall x \exists y p(x, y) \wedge (\forall w \neg q(w) \vee q(a))) \vee \forall y (\exists y \forall x p(x, y) \vee p(y, a))$$

$$\forall w (\forall x \exists y p(x, y) \wedge (\neg q(w) \vee q(a))) \vee \forall y \exists y \forall x (p(x, y) \vee p(y, a))$$

$$\forall w \forall x \exists y (p(x, y) \wedge (\neg q(w) \vee q(a))) \vee \forall y_2 \exists y \forall x (p(x, y) \vee p(y_2, a))$$

$$\forall w \forall x \exists y \forall y_2 \exists y \forall x \left(\left(p(x, y) \wedge (\neg q(w) \vee q(a)) \right) \vee \left(p(x, y) \vee p(y_2, a) \right) \right)$$

$$\forall w \forall x \exists y \forall y_2 \exists y \forall x \left((p(x, y) \vee p(x, y_2) \vee p(y_2, a)) \wedge (\neg q(w) \vee q(a) \vee p(x, y) \vee p(y_2, a)) \right)$$

$$c) \forall x p(x, y) \rightarrow (\forall y p(y, x) \rightarrow \forall x (q(x) \wedge \exists y \forall z r(a, y, z)))$$

$$\neg \forall x p(x, y) \vee (\neg \forall y p(y, x) \vee \forall x (q(x) \wedge \exists y \forall z r(a, y, z)))$$

$$\exists x \neg p(x, y) \vee (\exists y \neg p(y, x) \vee \forall x (q(x) \wedge \exists y \forall z r(a, y, z)))$$

$$\exists x \neg p(x, y) \vee (\exists y \neg p(y, x) \vee \forall x \exists y \forall z (q(x) \wedge r(a, y, z)))$$

$$\exists x \neg p(x, y) \vee \exists y (\neg p(y, x) \vee \forall x \forall z (q(x) \wedge r(a, y, z)))$$

$$\exists x \neg p(x, y) \vee \exists y \forall z (\neg p(y, x) \vee \forall w (q(w) \wedge r(a, y, z)))$$

$$\exists x \neg p(x, y) \vee \exists y \forall z \forall w (\neg p(y, x) \vee (q(w) \wedge r(a, y, z)))$$

$$\exists x, \neg p(x, y) \vee \exists y, \forall z \forall w (\neg p(y, x) \vee (q(w) \wedge r(a, y, z)))$$

$$\exists x, \exists y, \forall z \forall w (\neg p(x, y) \vee (\neg p(y, x) \vee (q(w) \wedge r(a, y, z))))$$

$$\exists x, \exists y, \forall z \forall w (\neg p(x, y) \vee ((\neg p(y, x) \vee q(w)) \wedge (\neg p(y, x) \vee r(a, y, z))))$$

$$\exists x, \exists y, \forall z \forall w ((\neg p(x, y) \vee \neg p(y, x) \vee q(w)) \wedge (\neg p(x, y) \vee \neg p(y, x) \vee r(a, y, z)))$$

$$\begin{aligned}
 & \textcircled{f} \left(\forall w \left(\forall x (r(x, w) \rightarrow (\forall x p(x) \rightarrow \exists x (q(x) \vee p(w))) \right) \right) \vee \forall z (p(z) \vee \exists z q(z)) \\
 & \left(\forall w \left(\neg \forall x (r(x, w) \vee (\neg \forall x p(x) \vee \exists x (q(x) \vee p(w))) \right) \right) \vee \forall z (p(z) \vee \exists z q(z)) \\
 & \left(\forall w \left(\exists x (r(x, w) \vee (\exists x \neg p(x) \vee \exists x (q(x) \vee p(w))) \right) \right) \vee \forall z (p(z) \vee \exists z q(z)) \\
 & \left(\forall w \exists x (\neg r(x, w) \vee \neg p(x) \vee q(x) \vee p(w)) \right) \vee \forall z (p(z) \vee \exists z q(z)) \\
 & \forall w \exists x (\neg r(x, w) \vee \neg p(x) \vee q(x) \vee p(w)) \vee \forall z \exists z_1 (p(z) \vee q(z_1)) \\
 & \forall w \exists x \forall z \forall z_1 \left(\neg r(x, w) \vee \neg p(x) \vee q(x) \vee p(w) \right) \vee (p(z) \vee q(z_1))
 \end{aligned}$$