

$$b) \quad x_n = \frac{1}{\sqrt{n(n+1)}} + \frac{1}{\sqrt{n(n+2)}} + \dots + \frac{1}{\sqrt{n(n+k)}}$$

$$\begin{aligned} x_n &= \sum_{k=1}^n \frac{1}{\sqrt{n(n+k)}} = \frac{1}{n} \sum_{k=1}^n \frac{\sqrt{n}}{\sqrt{n+k}} = \frac{1}{n} \sum_{k=1}^n \sqrt{\frac{n}{n+k}} = \\ &= \frac{1}{n} \sum_{k=1}^n \sqrt{\frac{1}{1+\frac{k}{n}}} \end{aligned}$$

Consideremos  $f: [0,1] \rightarrow \mathbb{R}$

$$f(x) = \frac{1}{\sqrt{1+x}} \quad \forall x \in [0,1]$$

Por la regla de Barrow, la función es continua para todo el dominio, y por tanto integrable.

$$\int_0^1 f(x) dx = \int_0^1 \frac{dx}{\sqrt{1+x}} = 2(1+x)^{1/2} \Big|_0^1 = 2\sqrt{2} - 2$$

Por último, se tiene:

$$\begin{aligned} \int_0^1 f(x) dx &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \sqrt{\frac{1}{1+\frac{k}{n}}} = \\ &= \lim_{n \rightarrow \infty} x_n = \boxed{2\sqrt{2} - 2} \end{aligned}$$