4) $X_n = \frac{n+1}{n^2+1} + \frac{n+2}{n^2+4} + \cdots + \frac{n+n}{n^2+n^2}$

$$X_n = \sum_{K=1}^{n} \frac{n+K}{n^2+K^2} = \frac{1}{n^2}\sum_{K=1}^{n} \frac{n+K}{1+\left(\frac{K}{n}\right)^2} = \frac{1}{n}\sum_{K=1}^{n} \frac{1+\frac{K}{n}}{1+\left(\frac{K}{n}\right)^2}$$

Consideramos $f:[0,1] \longrightarrow \mathbb{R}$, como:

$$f(x) = \frac{1+x}{1+x^2} \quad \forall x \in [0,1]$$

Como f es continua en $[0,1]$, entonces es integrable, y se puede aplicar la regla de Barrow:

$$\int_0^1 f(x)dx = \int_0^1 \frac{1+x}{1+x^2}dx = \int_0^1 \frac{1}{1+x^2}dx + \int_0^1 \frac{x}{1+x^2}dx =$$

$$= \operatorname{arctg}(x) + \frac{1}{2}\operatorname{Ln}|1+x^2| \Big|_0^1 = \frac{\pi}{4} + \frac{1}{2}\operatorname{Ln}(2) - 0 - \frac{1}{2}\operatorname{Ln}(1)$$

$$= \frac{\pi}{4} + \frac{1}{2}\operatorname{Ln}(2)$$

Por último, se tiene:

$$\int_0^1 f(x)dx = \lim_{n\to\infty} \frac{1}{n}\sum_{K=1}^{n} f\left(\frac{K}{n}\right) = \lim_{n\to\infty} \frac{1}{n}\sum_{K=1}^{n} \frac{1+\frac{K}{n}}{1+\left(\frac{K}{n}\right)^2} =$$

$$= \lim_{n\to\infty} X_n = \boxed{\frac{\pi}{4} + \frac{1}{2}\operatorname{Ln}(2)}$$