9)

$$\lim_{x \to 0} \frac{1}{x^4} \left(2x^3 \sqrt{1+x^3} + 2\sqrt{1+x^2} - 2 - 2x - x^2 \right) = \frac{5}{12} ?$$

Para resolverlo, calcularemos el polinomio de Taylor centrado en 0 de grado 4 de $g(x) = 2x^3\sqrt{1+x^3}$ y $h(x) = 2\sqrt{1+x^2}$

- $g(x) = 2x^3\sqrt{1+x^3} \to \boxed{g(0) = 0}$
- $g'(x) = 2(1+x^3)^{\frac{1}{3}} + \frac{2x}{3} \cdot (1+x^3)^{-\frac{2}{3}} \cdot 3x^2 = 2(1+x^3)^{\frac{1}{3}} + 2x^3(1+x^3)^{-\frac{2}{3}} \to \boxed{g'(0) = 2}$
- $g''(x) = \frac{2}{3}(1+x^3)^{-\frac{2}{3}} \cdot 3x^2 + 6x^2(1+x^3)^{-\frac{2}{3}} + 6x^5 \cdot -\frac{2}{3}(1+x^3)^{-\frac{5}{3}}$
$= 8x^2(1+x^3)^{-\frac{2}{3}} - 4x^5(1+x^3)^{-\frac{5}{3}} \to \boxed{g''(0) = 0}$
- $g'''(x) = 16x(1+x^3)^{-\frac{2}{3}} - \frac{2}{3} \cdot 8x^2(1+x^3)^{-\frac{5}{3}} \cdot 3x^2 - 20x^4(1+x^3)^{-\frac{5}{3}} - 4x^5 \cdot -\frac{5}{3}(1+x^3)^{-\frac{8}{3}} \cdot 3x^2 = 16x(1+x^3)^{-\frac{2}{3}} - 36x^4(1+x^3)^{-\frac{5}{3}} + 20x^7(1+x^3)^{-\frac{8}{3}} \to \boxed{g'''(0) = 0}$
- $g''''(x) = 16(1+x^3)^{-\frac{2}{3}} - \frac{32}{2}x(1+x^3)^{-\frac{5}{3}} \cdot 3x^2 - 144x^3(1+x^3)^{-\frac{5}{3}} + 36x^6 \cdot 5(1+x^3)^{-\frac{8}{3}} + 140x^6(1+x^3)^{-\frac{8}{3}} - \frac{160}{3}x^7(1+x^3)^{-\frac{11}{3}} \cdot 3x^2$
$\to \boxed{g''''(0) = 16}$

$$P_4(g,x) = g(0) + \frac{g'(0)}{1!} \cdot x + \frac{g''(0)}{2!}x^2 + \frac{g'''(0)}{3!}x^3 + \frac{g''''(0)}{4!}x^4$$

$$P_4(g,x) = 2x + \frac{2}{3}x^4$$

- $h(x) = 2\sqrt{1+x^2} \rightarrow \boxed{h(0) = 2}$
- $h'(x) = 2x(1+x^2)^{-\frac{1}{2}} \rightarrow \boxed{h'(0) = 0}$
- $h''(x) = 2(1+x^2)^{-\frac{1}{2}} - 2x^2 \cdot (1+x^2)^{-\frac{3}{2}} \rightarrow \boxed{h''(0) = 2}$
- $h'''(x) = 6x^3(1+x^2)^{-\frac{5}{2}} - 6x(1+x^2)^{-\frac{3}{2}} \rightarrow \boxed{h'''(0) = 0}$
- $h''''(x) = -6(1+x^2)^{-\frac{3}{2}} + 36x^2(1+x^2)^{-\frac{5}{2}} - 30x^4(1+x^2)^{-\frac{7}{2}}$

$\rightarrow h''''(x) = -6$

$P_4(h, x) = 2 + x^2 - \frac{1}{4} \cdot x^4$

<u>lim</u>

$$\lim_{x \to 0} \frac{1}{x^4} \cdot \left(\cancel{2x} + \frac{2}{3}x^4 + \cancel{2} + \cancel{x^2} - \frac{x^4}{4} - \cancel{2} - \cancel{2x} - \cancel{x^2} \right) =$$

$$\lim_{x \to 0} \frac{1}{\cancel{x^4}} \left(\frac{5}{12} \cdot \cancel{x^4} \right) = \frac{5}{12} \checkmark$$