10.6

1) $x_n = \frac{n}{n^2+1} + \frac{n}{n^2+4} + \dots + \frac{n}{n^2+n^2}$

$$x_n = \sum_{K=1}^{n} \frac{n}{n^2+K^2} = \frac{1}{n}\sum_{K=1}^{n} \frac{n^2}{n^2+K} = \frac{1}{n}\sum_{K=1}^{n} \frac{1}{1+\left(\frac{K}{n}\right)^2}$$

Consideramos $f: [0,1] \to \mathbb{R}$ como

$$f(x) = \frac{1}{1+x^2} \quad \forall x \in [0,1]$$

Como f es continua, entonces es integrable y se puede aplicar la regla de Barrow:

$$\int_0^1 f(x)dx = \int_0^1 \frac{1}{1+x^2}dx = \operatorname{arctg}(x)\Big|_0^1 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Por último, se tiene:

$$\int_0^1 f(x)dx = \lim_{n\to+\infty} \frac{1}{n}\sum_{K=1}^{n} f\left(\frac{K}{n}\right) = \lim_{n\to\infty} \frac{1}{n}\sum_{K=1}^{n} \frac{1}{1+\left(\frac{K}{n}\right)^2} =$$

$$= \lim_{n\to\infty} \sum_{K=1}^{n} \frac{n}{n^2+K^2} = \lim_{n\to\infty} x_n = \boxed{\frac{\pi}{4}}$$