Instructions

- The judge we have selected to check the submissions is very impatient. Most of the time he flips table when a program takes time to execute. Pro Tip: Avoid for loops where possible. Use vectorized calculations.
- **You must fill up the file *Team_Info.txt* before you submit.** This file is basically a comma separated file for obvious reasons. Delete the Xs and fill up the file with appropriate info. Email the zipped folder/all the contents to eeedaymatlabcontest@gmail.com before the contest ends.
- Complete a problem by editing the given template.
- If you think there is any error in any of the problems please report in the event page.
- In case of multiple submissions from a single team, only the final submission will be accepted.

EEE DAY MATLAB CONTEST 2016

EEE DAY REDEFINED
8th-10th December, 2016

Bangladesh University of Engineering and Technology

1. Beautiful Triangle

Fitz has invented a triangle — a beautiful triangle. He calls a *beautiful* if it meets the following conditions:

1. The triangle is right angled.
2. The legs of the triangle are integer.
3. The hypotenuse of the triangle is non integer.

Consider all values of the legs (x, y) which satisfy the inequalities:

$$0 < x \leq n$$
$$0 < y \leq m$$
$$x + y > 0$$

Find a set of all possible legs of beautiful triangle. You have to write a function which takes two integers n and m ($1 \leq n, m \leq 100$) as inputs and gives the size k of the found set as output. It also gives 2d matrix s as output which contains the possible sets. Note that: (1, 2) and (2, 1) are same here. You may return any of them.

Edit the template in *beautifind.m*.

Example:

Input:
n=2
m=2

Output:
k=1
s=[1 2]

2. Degenerate Matrix

A matrix is called *degenerate* if its determinant is equal to zero. The *norm* A of a matrix A is defined as a maximum of absolute values of its elements. You are given a matrix 2×2 matrix, A. Consider any degenerate matrix B such that *norm* $A - B$ is minimum possible. Determine *norm* $A - B$.

Your function should take the 2×2 matrix, A as input and give norm (A-B) as output.

Edit the template in *normfind.m*.

Example:

Input:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Output:

n=0.2

3. Flanders' Pool

Ned Flanders wants to build a swimming pool in the shape of a rectangle in his backyard. He has set up coordinate axes, and he wants the sides of the rectangle to be parallel to them. Of course, the area of the rectangle must be positive. Ned had all four vertices of the planned pool written on a paper, until Bart Simpsons came along and erased some of the vertices.

Now Ned is wondering, if the remaining n vertices of the initial rectangle give enough information to restore the area of the planned swimming pool.

Your function should take vertices that were **not** erased by Bart as the input. The input should be a 2d matrix with $\boldsymbol{n}$ $(1 \leq n \leq 4)$ rows containing two integers $\boldsymbol{x_i}$ and $\boldsymbol{y_i}$ $(-1000 \leq x_i, y_i \leq 1000)$ —the coordinates of the $\boldsymbol{i}$-th vertex that remains. Vertices are given in an arbitrary order. It should return the area of the initial rectangle if it could be uniquely determined by the points remaining. Otherwise, return - 1.

It's guaranteed that these points are distinct vertices of some rectangle, that has positive area and which sides are parallel to the coordinate axes.

Edit the template in *areafind.m*.

Example:

Input:

$$V = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Output:

Area = 1

4. Matrix Revolution

Mr. Anderson has a row matrix $a_1, a_2, ..., a_n$ initially consisting of n zeros. At one step, he can choose any index i and either add 1 to all elements $a_i, a_{i+1}, ..., a_n$ or subtract 1 from all elements $a_i, a_{i+1}, ..., a_n$. His goal is to end up with the row matrix $b_1, b_2, ..., b_n$.

He wants to achieve this goal in the minimum number of steps and asks you to compute this value. Your function should take a row matrix containing n integers $b_1, b_2, ..., b_n$ $(-10^9 \leq b_i \leq 10^9\ ,\ \ 1 \leq n \leq 200\,000)$ and return the minimum number of steps that he needs to make in order to achieve $a_i = b_i$ for all i. Note that: initially $a_i = 0$ for all i, so it is not given as input.

Edit the template in *matrixfind.m.*

Example:

Input:

B= [1, 2, 3, 4, 5]

Output:

steps = 5

5. Find the Shortcut

Baaljit has built a strange calculator hoping that he would get an A++ in digital logic design course. In this calculator, there are only two buttons-red and blue and a display. Initially, the display shows number *n* which is a positive integer. After clicking the red button, device multiplies the displayed number by two. After clicking the blue button, device subtracts one from the number on the display. If at some point the number stop being positive, Math ERROR is shown. The display can show arbitrarily large numbers.

He wants to get number ***m*** on the display. Write a MATLAB function which calculates the minimum number of clicks he has to make in order to achieve this result.

The function should take two distinct integers ***n*** **and** ***m*** ($1 \leq n, m \leq 100$), as inputs.

The function should give a single number — the minimum number of times he needs to push the button required to get the number *m* out of number *n*.

Edit the template in *stepsfind.m*.

Example:

Input:

n=4
m=6

Output:

steps = 2

6. Count the Stars

Silent Sardarji wants to count all the stars in the universe. For that he has to give each star a tag number i (i=1, 2,). But which order should he number them? He wants to know all the possible ways to number all the stars. Write a function which takes the number of stars, n ($1<n<10^{300}$) and gives output the number of all possible way of numbering them. The output should contain two variables: one should be mantissa and other should be exponent.

Edit the template in *starfind.m*.

Example:

Input:

n = 10^{10}

Output:

m= 4.5730
e= 9.5657e+10

7. Game of Thrones

Behold! Our beloved Khaleesi has decided at last to attack Westeros, as she has to break the wheels of tyranny by the four families (Lannister, Baratheon, Stark and Tyrell) and set foot on her homeland again.

All of Westeros' might vs Khaleesi's army. To have a fair(!) fight both army have assembled exactly n troops each (we don't care whether its dragon, others, undead, unsullied, warg etc) and they will engage in *1 vs 1* fight. Each troop has exactly two attributes **attack(A) & defense(D).** Troop i (with attack Ai and defense Di) can defeat troop j(Aj,Dj) 'iff' Ai-Dj > Aj-Di. Tyrion has managed to get the stats for all the troops of Westeros (of course he knows all about Khaleesi's army). Before the battle starts he will place the troops 'optimally' so that maximum number of Khaleesi's troops will win their battle. Your (because Tyrion has gone to you know 'where') job is to find out the optimal strategy and calculate how many of Khaleesi's troop can win their battle.

So, given A1, D1 arrays representing attack and defense of Khaleesi's Army and A2 and D2 are the corresponding arrays of Westeros' Army stats. You have to calculate maximum number of khaleesi's troops that can win the battle if you 'place' the troops optimally.

Edit the file *game_of_thrones.m*.

Example:

Input:

A1=[6,1,2,4,8];
D1=[5,7,1,2,4];
A2=[5,1,2,8,4];
D2=[2,5,4,1,5];

Output:

c=3

Explanation:
Let's number the troops from 1 to 5. One possible arrangement is (K-khaleesi,W-Westeros):
K2 - W1 , K wins (1-5>1-7)
K5 - W5 , K wins
K1 - W4 , K wins
K3 - W2 , W wins
K4 - W3 , W wins
(note: if Ai-Dj = Aj-Di it is 'not' counted as win).

8. Uzumaki

The image below shows a 3x3 spiral matrix.

1	2	3
8	9	4
7	6	5

Let us clarify a bit further about an NxM spiral matrix. It's like we start with 1,2,3,4,… row wise and when we reach the end of the row we go downwards. Then when we hit the bottom we go back left along the row. Then after reaching another corner we go up. We repeat the process and fill up the table in this way.

Now, you will be given the values N and M (N, M≤10,000). And, we are interested in the value at the location (P, Q) [1 ≤ (P, Q) ≤ (N, M)]. That is the P^{th} Row and Q^{th} Column.

Edit the template in *uzumaki.m*.

Example:

Input:

N=3
M=3
P=2
Q=1

Output:

val = 8

9. Derivatives

In mathematical optimizations we often deal with Matrix Multiplications followed by a loss function. However, here we are going a present a mini and truncated version of such class of problems.

We have to optimize using the following function:

$$f(\boldsymbol{X}) = sum(\max(0, \boldsymbol{PXQ}))$$

Here $\boldsymbol{P}, \boldsymbol{X}, \boldsymbol{Q}$ are MxM matrices (1≤M≤2000).

The sum, max functions are elementwise functions. That is,

$$sum\left(max\left(0, \begin{bmatrix} -2 & 4 \\ 9 & -10 \end{bmatrix}\right)\right) = sum\left(\begin{bmatrix} 0 & 4 \\ 9 & 0 \end{bmatrix}\right) = 13$$

You have to find the derivative of the function with respect to $\boldsymbol{X}$.

That is, calculate,

$$\frac{df}{d\boldsymbol{X}}$$

Edit the template in *derivatives.m*.

Example:

Input:

P=Q=$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

X = $\begin{bmatrix} -2 & 4 \\ 9 & -10 \end{bmatrix}$

Output:

df = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

10. Residuum

A series is defined in the following way.

$$u_k = \text{number of palindromic integers having } (k+1) \text{ digits}$$

Speaking of palindromic numbers, we can simply say that, if you need and integer left to right (as you usually do) and then right to left (as in Arabic), surprisingly you find the same number.

No we define,

$$S = \sum_{k=1}^{M} u_k$$

We are interested to find out the remainder **res** of $\frac{S}{M}$. Where M≤2500.

Edit the template *residuum.m*.

Example:

Input:

M = 2

Output:

res = 1

11. Palindrome Again

A palindromic sequence is a sequence of numbers that is read same forward and backward. Example: {23,12,7,12,23},{12,34,34,12}. You are given an array of **n** integers (in the form of a 1D vector). Your task is to transform this sequence into a palindromic sequence. You can change an integer in the array to any other integer (cannot change them into floating point numbers). *The cost for changing an integer x into y is* $(y-x)^2$. Calculate and return the minimum cost to transform the sequence into a palindromic one.

Edit the template in *min_cost_palindrome.m*.

Example:

Input:

A=[11,12,13,11,12]

Output:

c=2

Explanation: One possible palindromic sequence [11,11,13,11,11].

12. Saruman's Uncertain Electron

You have been turned into an electron (yes an electron) by the black wizard Saruman to help him build a new weapon of mass destruction. You may be thinking, how can an electron possibly help? Well Saruman wants to make sure his circuitry is working properly before facing Gandalf and Co. So he wants you to visit **N** nodes in the circuit in a given order. The nodes are labeled using integer in the range **[1,10^5].** To get from node labeled **x to y**, you spend **|x-y|** units of energy where **|a|** denotes absolute value of **a**. Now of course, you have no choice but to follow his order. But being an electron, you can also take advantage of the 'Uncertainty Principle'. You have decided that you won't visit one (and exactly one) particular node at all (Saruman will not be able to detect it because that much uncertainty is inherent in sub atomic world). Now you want to choose a node that minimizes your total energy spent.

Edit the template in *min_energy.m*.

Example:

Input:

A=[2,2,2,5,1,1,3,9,3,6,7,7,7,5,5,9]

Output:

m=15

Explanation:

If you don't visit node 2 the sequence becomes [5,1,1,3,9,3,6,7,7,7,5,5,9] and m=4+0+2+6+6+3+1+0+0+2+0+4=28.

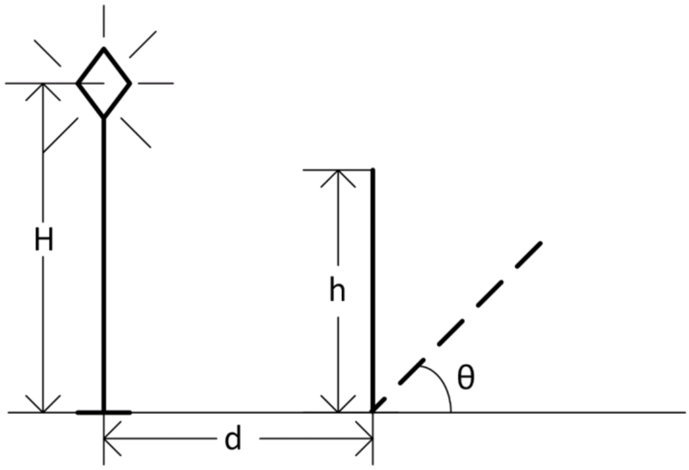
Similarly if you don't visit node 5 A=[2,2,2,1,1,3,9,3,6,7,7,7,9]. m=0+0+1+0+2+6+6+3+1+0+0+2=21.

It turns out if you don't visit node 9 A=[2,2,2,5,1,1,3,3,6,7,7,7,5,5] and m=15 which is minimum possible here.

PS: The author doesn't claim any scientific credibility for the statement ;)

13. Batman's Shadow

Wayne Corp. was given a task to build some walls all around Gotham city. However, Bruce Wayne has a plan. He wants to build the walls so that he can hide well in the shadows when he is doing his job as the masked vigilante Batman. First, let's look at the construction plans.



A wall is **d** length away from a lamp post. The height and/or length of the lamp post and the walls are **H** and **h** respectively. The initial design was that the walls would be perpendicular to the ground. Now, in the name of architectural beauty, Bruce Wayne proposed that the wall should be inclined keeping the length of the wall same as before and that he would tell the site engineers, the inclination of each wall.

As Lucius Fox your job is to find the inclination ($\boldsymbol{\theta}$) of the walls such that the shadow cast by the wall is longest.

Edit the template in *inclination.m*.

Example:

Input:

H=2
h=d=1

Output:

theta=53.13°
punchline = 'I am Batman.'

Note: The output theta is a float. Difference from our solution should be lower than 10^{-6} for all test cases.

14. How much water?

In his birthday, Dipper has decided to invite his friends to a tea party. For that occasion, he has a large teapot with the capacity of w milliliters and $2n$ tea cups; each cup is for one of his friends. The i-th cup can hold at most a_i milliliters of water.

It turned out that among his friends there are exactly n boys and exactly n girls and all of them are going to come to the tea party. To please everyone, Dipper decided to pour the water for the tea as follows:

- Dipper can boil the teapot exactly once by pouring there at most w milliliters of water;
- He pours the same amount of water to each girl;
- He pours the same amount of water to each boy;
- If each boy gets x milliliters of water, then each girl gets $2x$ milliliters of water.

In the other words, each girl should get two times more water than each boy does.

Dipper is very kind and polite, so he wants to maximize the total amount of the water that he pours to his friends. Your task is to write a MATLAB function to help him determine the optimum distribution of cups between his friends.

Your function should take capacity of Dipper's teapot (in milliliters), w ($1 \leq w \leq 10^9$) and the capacities of Dipper's tea cups (in milliliters), a_i ($1 \leq a_i \leq 10^9$, $1 \leq i \leq 2n$) as inputs and give the maximum total amount of water in milliliters that Diper can pour to his friends without violating the given conditions. Your answer will be considered correct if its absolute or relative error doesn't exceed 10^{-6}.

Edit the Matlab code in *sizefind.m*.

Example:

Input:

w =4
a = [1 1 1 1]

Output:

water = 3

15. Penetrate HYDRA

Coulson has decided that it is enough, H.Y.D.R.A. should end now. He will sent his six best agents- May, Daisy, Bobby, Hunter, Daisy, Mack- to infiltrate H.Y.D.R.A., which is a $m \times n$ matrix with all elements being one. The six S.H.I.E.L.D. agents should be denoted by zero.

Place the six agents inside the $m \times n$ matrix such that:

1. Distance between them is equal
2. Distance between them is maximum
3. None of them is outside the matrix.

Take two integers m and n as input, give a six by two matrix agentposition, with each row giving us the coordinate of the agent.

Edit the Matlab code in *penetratehydra.m*.

No example is given for this one. Have fun. ☺