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Transportmetrica A: Transport Science

ISSN: 2324-9935 (Print) 2324-9943 (Online) Journal homepage: http://www.tandfonline.com/loi/ttra21

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Transportmetrica A: Transport Science, 2015
Vol. 11, No. 7, 579–602, http://dx.doi.org/10.1080/23249935.2015.1049675

A bandwidth approach to arterial signal optimisation with bus priority

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(*Received 3 July 2014; accepted 6 May 2015*)

A bandwidth approach to arterial signal optimisation with bus priority is proposed in this paper. This approach can generate green bands for both bus systems and general vehicles with the same timing plan. First, the characteristics of the bus green band and the general vehicle green band and the relationships between the bus green band and the general vehicle green band are analysed to construct the basic linear constraints of the models. Second, the elements of bus systems are adjusted to expand the feasible region of the models. The maximum and minimum values of some decision variables are established to satisfy the demands of bus systems and general vehicles. Finally, a case study is presented to illustrate the application of the proposed approach with VISSIM simulations. The results show that the proposed approach generated significant improvements for bus systems, general vehicles and the entire traffic system.

Keywords: bus green band; general vehicle green band; MAXBAND; bandwidth optimisation

1. Introduction

The bandwidth optimisation approach is an important method for improving traffic progression along a main street. It coordinates the traffic signals of successive intersections along a main street, which generates green bands for both directions to ensure that vehicles can start at one end of the street and travel at a defined speed to the other end of the street without stopping. However, the green bands generated from the majority of bandwidth optimisation approaches are designed for general vehicles and only one green band exists in the entire traffic system. Typical bandwidth optimisation approaches fail to consider the characteristics of bus systems, which significantly influence the operational efficiency of bus systems. In this paper, an approach that can generate green bands for both bus systems and general vehicles with the same timing plan is proposed.

Numerous studies of bandwidth optimisation approaches have been conducted during the past 50 years. Morgan and Little (1964) and Little (1966) developed a half-time synchronisation to maximise total bandwidths on a two-way main street, and Little, Kelson, and Gartner (1981) proposed a mixed-integer linear programming model (MAXBAND) that can achieve a weighted combination of bandwidths. The model employed a branch-and-bound algorithm. Baass (1983) developed two programs to create curves that show the continuous relation between a uniform progression speed and the corresponding maximum bandwidth over an extensive range of speeds and cycles and applied the methods to 18 data sets for a maximum of 24 intersections. Considering the traffic flow patterns, Gartner et al. (1990, 1991) and Stamatiadis and Gartner (1996)

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proposed a MULTIBAND approach that can generate different bandwidths on different segments in each direction along a main street. After a few years, Gartner and Stamatiadis (2002) applied MULTIBAND to grid networks. Another typical bandwidth optimisation approach is the PASSER program. Messer et al. (1973), Chang and Messer (1991), and Chaudhary and Messer (1996) developed the PASSER program to optimise multiphase signals to obtain maximum bandwidths. Because the solution generated by MAXBAND may not exist if the number of intersections along the main street is too large, Tian and Urbanik (2007) introduced a system partition technique to the bandwidth optimisation approaches. This approach partitions the intersections along the main street into a number of subsets and optimises each subsystem to achieve individual maximum bandwidths. Then, it retains the maximum bandwidth in the more important direction based on the optimisations of all subsets. Li (2014) also developed a two-phase approach, in which the first phase solves the MAXBAND models with perturbation controlled by a parameter and generates a number of optimal or suboptimal plans. In the second phase, Monte Carlo analysis is used to simulate the random progression time, evaluate the generated plans and rank them by their reliability.

Another possible method for adjusting bus progression is by real-time bus priority systems. With transit signal priority (TSP), buses can request the green phase of traffic signals and progress unimpeded along a main street. Typical TSP strategies consist of the green extension, red truncation and phase insertion. These TSP strategies are based on background signal timings. To facilitate the bus progression along a main street, background signal timings should be designed and optimised in advance. Two methods can be employed to achieve the green extension, red truncation and phase insertion. The first method is to adjust the cycle length while maintaining a constant split of the crossing street, which may influence the bus progression along the main street. The second method is to adjust the split of the crossing street while maintaining a constant cycle length, which may influence the traffic along the crossing street. Bandwidth optimisation approaches can generate basic background signal timings along a main street and maintain constant cycle lengths and splits with adjustments to the offsets. Therefore, the proposed approach is based on bandwidth optimisation approaches.

Some studies have focused on the characteristics of bus systems during the last two decades. Quiroga and Bullock (1998) described a new methodology for performing travel time studies using global positioning system (GPS) and geographic information system (GIS) technologies. The methodology documented the data collection, data reduction, and data reporting procedures, as well as the analysis of the capabilities of the GPS/GIS methodology. Zhang (2003) constructed velocity–volume models of the relationship between buses and general vehicles on various roads according to traffic surveys and traffic flow models. Cortésa et al. (2011) presented a method based on GPS-generated data to systematically monitor average commercial bus speeds. The framework can be applied to an entire bus route and segments of arbitrary lengths and can be divided into time intervals of arbitrary duration. Cathey and Daily (2003) presented a general prescription for the prediction of transit vehicle arrival/departure. The prescription identified three components – a tracker, a filter, and a predictor – which are necessary to employ automatic vehicle location (AVL) data to position a vehicle in space and time and predict the arrival/departure at a selected location. Rajbhandari, Chien, and Daniel (2003) investigated the effect of boarding and alighting passengers, standees, time of day, and service type on bus dwell time and estimated bus dwell time with data collected from an archived database. Dueker, Kimpel, and Strathman (2004) used archived AVL and automatic passenger counter data that were reported at the level of individual bus stops to analyse bus dwell times. Li et al. (2006) used data collected from Florida's Broward County Transit system to develop a binary door choice model to predict the proportion of alighting passengers who will use the front or rear doors to disembark from the bus to estimate dwell times.

Other than bandwidth maximisation, the algorithms of signal timings of signalised intersections for area traffic control have been studied for decades (Hillier 1965, 1966; Allsop 1968a, 1968b; Robertson 1969). Wong (1996) proposed a group-based optimisation of fixed-time signal timings for area traffic control with a performance index, which is a weighted combination of estimated delay and number of stops. Then, parallel algorithms and a dynamic load balancing scheme were developed for these optimisation heuristics, Wong (1997) used parallel computing to improve the computational efficiency. Wong and Yang (1999) presented an iterative scheme for a combined signal optimisation and assignment problem, and the group-based optimisation was extended to traffic equilibrium networks. Wong et al. (2000, 2002, 2003) developed a time-dependent TRANSYT traffic model for the evaluation of a performance index, in which the whole period of analysis is divided into a number of short time intervals. The group-based specification of signal timings is employed for the traffic model and the set of constraints on these variables is formed to ensure feasibility and safety of settings. The derivatives of the performance index with respect to the group-based variables are derived, based on which optimisation heuristics are developed to solve the time-dependent problem.

Although typical bandwidth optimisation approaches have been proven to facilitate traffic progression along the main street for general vehicles, they fail to consider the characteristics of bus systems. Bus-related studies have demonstrated that the characteristics of bus systems differ from the characteristics of general vehicles. In particular, the elements of bus systems, such as the bus speed and the dwell time, have their own characteristics. In this paper, the fundamental model of the new approach can be generated from typical bandwidth approaches. The characteristics of bus systems and general vehicles need to be separately considered in the same timing plan and relevant adjustments need to be added to the fundamental model. In addition, a trade-off between bus systems and general vehicles is necessary.

The bus systems in this paper have exclusive bus lanes; thus, all buses can travel along the bus lanes with less interference from general vehicles. Generally, the average headway of buses is greater than the average headway of general vehicles due to the bus frequency; thus, all buses can travel with less interference from each other. Bus speed guidance can be conveniently achieved by communication between a bus control centre and bus drivers. Therefore, bus drivers are given more flexibility in their control of buses. Compared with general vehicles, the adjustment for the operational speed of bus systems is more feasible. Considering the characteristics of bus systems, this paper proposes a new bandwidth approach to arterial signal optimisation with bus priority. In addition, the operational efficiency of general vehicles is also considered in this new approach. The approach can generate a bus green band and a general vehicle green band to ensure that both bus systems and general vehicles can share the same timing plan from which the entire traffic system can benefit. In the proposed approach, a new model is developed based on MAXBAND, whereas the objective function of the proposed model differs from MAXBAND considering the adjustment of the speed and dwell time of bus systems. Instead of maximising bandwidths, this new model pursues the minimum total bus travel times for both directions.

The remainder of this paper is organised as follows: Section 2 introduces the constraints, mixed-integer linear programming models, and solution algorithms. In Section 3, a case study is presented to evaluate the performance of the proposed approach. Some important conclusions and suggestions for future studies are summarised in Section 4.

2. Methodology

In this section, an approach is introduced to construct a mixed-integer linear programming model for generating green bands for both bus systems and general vehicles. The proposed approach consists of 6 steps:

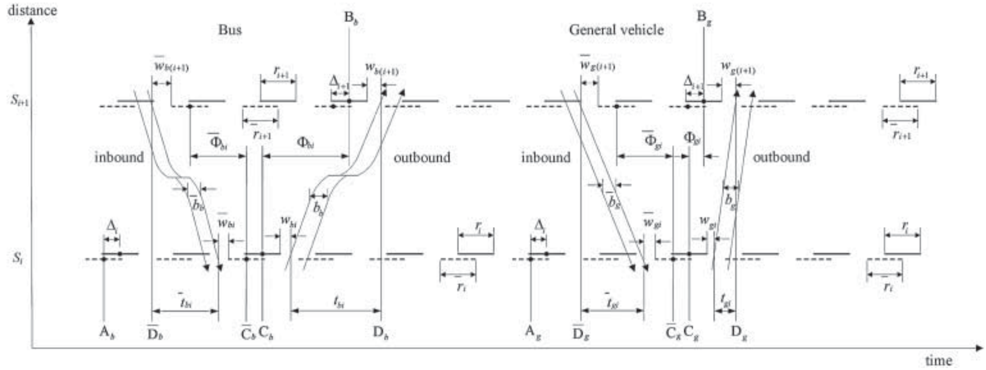

Figure 1. Relationships between adjacent intersections for bus systems and general vehicles.

(1) Analyse the basic constraints for the bus green band.
(2) Analyse the basic constraints for the general vehicle green band.
(3) Analyse the supplementary constraints for bus systems and the bus green band.
(4) Analyse the relationships between the bus green band and the general vehicle green band.
(5) Construct the mixed-integer linear programming model.
(6) Present the algorithm of the model.

The relationships between adjacent intersections for bus systems and general vehicles are shown in Figure 1. The relevant variables are listed. In addition, the construction of a mixed-integer linear programming model by expressing some time variables in units of cycle time (the cycle times of all signalised intersections are uniform) is more convenient, according to Little, Kelson, and Gartner (1981).

C = cycle length (s). C_{min} and C_{max} are the lower limit and upper limit, respectively, for the cycle length.

Z = $1/C$ (cycles/s). Reciprocal cycle length is used to ensure that all constraints are linear.

S_i = signalised intersection i, where $i = 1, \ldots, n$.

n = the number of signalised intersections.

L_i = distance between the intersections S_i and S_{i+1} (m).

$N_i(\bar{N}_i)$ = the outbound (inbound) number of bus stops between the intersections S_i and S_{i+1}.

$b_{\mathrm{b}}(\bar{b}_{\mathrm{b}})$ = outbound (inbound) bandwidth for a bus (cycles). $b_{\mathrm{b\,max}}(\bar{b}_{\mathrm{b\,max}})$ and $b_{\mathrm{b\,min}}(\bar{b}_{\mathrm{b\,min}})$ represent the lower limit and upper limit, respectively, for the outbound (inbound) bandwidth for a bus.

$b_{\mathrm{g}}(\bar{b}_{\mathrm{g}})$ = outbound (inbound) bandwidth for a general vehicle (cycles). $b_{\mathrm{g\,max}}(\bar{b}_{\mathrm{g\,max}})$ and $b_{\mathrm{g\,min}}(\bar{b}_{\mathrm{g\,min}})$ are the lower limit and upper limit, respectively, for the outbound (inbound) bandwidth for a general vehicle.

$r_i(\bar{r}_i)$ = outbound (inbound) red time at S_i (cycles).

$l_i(\bar{l}_i)$ = outbound (inbound) green time for the left-turn movement at S_i of the main street (cycles).

$\delta_i(\bar{\delta}_i)$ = 0–1 variables at S_i.

$w_{\mathrm{b}\,i}(\bar{w}_{\mathrm{b}\,i})$ = time from the right (left) side of $r_i(\bar{r}_i)$ at S_i to the left (right) edge of $b_{\mathrm{b}}(\bar{b}_{\mathrm{b}})$ for a bus (cycles).

$w_{\mathrm{g}i}(\bar{w}_{\mathrm{g}i})$ = time from the right (left) side of $r_i(\bar{r}_i)$ at S_i to the left (right) edge of $b_{\mathrm{g}}(\bar{b}_{\mathrm{g}})$ for a general vehicle (cycles).

Δ_i = offset between the outbound red time and the inbound red time at S_i = time from the centre of $\bar{r}_i$ to the nearest centre of r_i. This value is positive if the centre of r_i is located to the right of the centre of $\bar{r}_i$ (cycles). $\Delta_i = ((2\delta_i - 1) * l_i - (2\bar{\delta}_i - 1) * \bar{l}_i)/2$, $\delta_i = 1, 0$ and $\bar{\delta}_i = 1, 0$.

$\Phi_{\mathrm{b}i}(\bar{\Phi}_{\mathrm{b}i})$ = offset between the outbound (inbound) red time at S_i and S_{i+1} for a bus = time from the centre of $r_i(\bar{r}_i)$ at S_i to the centre of a particular $r_{i+1}(\bar{r}_{i+1})$ at S_{i+1}. The two red times are selected such that each red time is located immediately to the left (right) of the same $b_{\mathrm{b}}(\bar{b}_{\mathrm{b}})$. This value is positive if S_{i+1}'s centre of $r_{i+1}(\bar{r}_{i+1})$ is located to the right (left) of S_i's centre of $r_i(\bar{r}_i)$ (cycles).

$\Phi_{\mathrm{g}i}(\bar{\Phi}_{\mathrm{g}i})$ = offset between the outbound (inbound) red time at S_i and S_{i+1} for a general vehicle = time from the centre of $r_i(\bar{r}_i)$ at S_i to the centre of a particular $r_{i+1}(\bar{r}_{i+1})$ at S_{i+1}. The two red times are selected such that each red time is located immediately to the left (right) of the same $b_{\mathrm{g}}(\bar{b}_{\mathrm{g}})$. This value is positive if S_{i+1}'s centre of $r_{i+1}(\bar{r}_{i+1})$ is located to the right (left) of S_i's centre of $r_i(\bar{r}_i)$ (cycles).

$v_{\mathrm{b}i}(\bar{v}_{\mathrm{b}i})$ = average speed of a bus from S_i to S_{i+1} outbound (S_{i+1} to S_i inbound) (m s^{-1}). $v_{\mathrm{b\,min}} \leq v_{\mathrm{b}i}(\bar{v}_{\mathrm{b}i}) \leq v_{\mathrm{b\,max}}$, where $v_{\mathrm{b\,min}}$ and $v_{\mathrm{b\,max}}$ represent the lower limit and upper limit, respectively, for the average speed of a bus.

$a_{\mathrm{b}1}$ = average acceleration of a bus (m s^{-2}).

$a_{\mathrm{b}2}$ = average deceleration of a bus (m s^{-2}).

$t_{\mathrm{b}i}(\bar{t}_{\mathrm{b}i})$ = travel time for a bus from S_i to S_{i+1} outbound (S_{i+1} to S_i inbound).

$t_{\mathrm{p}i}(\bar{t}_{\mathrm{p}i})$ = first part of the travel time for a bus from S_i to S_{i+1} outbound (S_{i+1} to S_i inbound), which consists of the travel time with an average bus speed, acceleration time when a bus leaves a bus stop, the deceleration time when a bus arrives at a bus stop (cycles).

$T_i(\bar{T}_i)$ = first part of the travel time for a bus from S_i to S_{i+1} outbound (S_{i+1} to S_i inbound) (s). $T_{i\,\mathrm{max}}(\bar{T}_{i\,\mathrm{max}})$ and $T_{i\,\mathrm{min}}(\bar{T}_{i\,\mathrm{min}})$ denote the lower limit and upper limit, respectively, for $T_i(\bar{T}_i)$.

$\sum_0^{N_i} t_{\mathrm{d}ij} \left(\sum_0^{\bar{N}_i} \bar{t}_{\mathrm{d}ij}\right)$ = second part of the travel time for a bus from S_i to S_{i+1} outbound (S_{i+1} to S_i inbound) = sum of dwell times at all outbound (inbound) bus stops between S_i and S_{i+1} (cycles), where $t_{\mathrm{d}ij}$ is the dwell time at the outbound bus stop j and $\bar{t}_{\mathrm{d}ij}$ is the dwell time of at the inbound bus stop j; $t_{\mathrm{d}ij\,\mathrm{max}}(\bar{t}_{\mathrm{d}ij\,\mathrm{max}})$ and $t_{\mathrm{d}ij\,\mathrm{min}}(\bar{t}_{\mathrm{d}ij\,\mathrm{min}})$ are the lower limit and upper limit, respectively, for $t_{\mathrm{d}ij}(\bar{t}_{\mathrm{d}ij})$, where $j = 1, \ldots, N_i(\bar{N}_i)$.

$t_{\mathrm{g}i}(\bar{t}_{\mathrm{g}i})$ = travel time for a general vehicle from S_i to S_{i+1} outbound (S_{i+1} to S_i inbound). The travel time can be derived from the distance and the general vehicle speed between S_i and S_{i+1} (cycles).

k_{b} = the ratio of the total inbound bus volumes to the total outbound bus volumes.

k_{g} = the ratio of the total inbound general vehicle volumes to the total outbound general vehicle volumes.

$m_{\mathrm{b}i}$ = integer variable for $b_{\mathrm{b}}(\bar{b}_{\mathrm{b}})$ at S_i (cycles).

$m_{\mathrm{g}i}$ = integer variable for $b_{\mathrm{g}}(\bar{b}_{\mathrm{g}})$ at S_i (cycles).

$m_i(\bar{m}_i)$ = outbound (inbound) integer variable at S_i (cycles).

2.1. *Basic constraints for the bus green band*

The basic constraints for the bus green band can be obtained from the left part of Figure 1.

The time from A_b to B_b can be derived, as shown in Figure 1:

$$\text{Time}(A_b \text{ to } B_b) = \Delta_i + m_{bA} + \Phi_{bi} = \Delta_{i+1} + m_{bB} - \bar{\Phi}_{bi}, \tag{1}$$

where m_{bA}, m_{bB} are two integers; each integer represents integer multiples of the uniform cycle time. Adjust Equation (1) with $m_{bi} = m_{bB} - m_{bA}$ to obtain the relations among Φ_{bi}, $\bar{\Phi}_{bi}$, Δ_i, and $\overline{\Delta}_i$

$$\Phi_{bi} + \bar{\Phi}_{bi} + \Delta_i - \Delta_{i+1} = m_{bi}, \tag{2}$$

where m_{bi} is an integer variable related to S_i and S_{i+1} for a bus.

The time from C_b to D_b can also be derived from Figure 1:

$$\text{Time}(C_b \text{ to } D_b) = \Phi_{bi} + \frac{r_{i+1}}{2} + w_{b(i+1)} = \frac{r_i}{2} + w_{bi} + t_{bi}, \tag{3}$$

$$\Phi_{bi} = \frac{r_i}{2} - \frac{r_{i+1}}{2} + w_{bi} - w_{b(i+1)} + t_{bi}. \tag{4}$$

The time from $\bar{C}_b$ to $\bar{D}_b$ is expressed as follows:

$$\text{Time}(\bar{C}_b \text{ to } \bar{D}_b) = \bar{\Phi}_{bi} + \frac{\bar{r}_{i+1}}{2} + \bar{w}_{b(i+1)} = \frac{\bar{r}_i}{2} + \bar{w}_{bi} + \bar{t}_{bi}, \tag{5}$$

$$\bar{\Phi}_{bi} = \frac{\bar{r}_i}{2} - \frac{\bar{r}_{i+1}}{2} + \bar{w}_{bi} - \bar{w}_{b(i+1)} + \bar{t}_{bi}. \tag{6}$$

Substituting Equations (4) and (6) into Equation (2), the integer variable constraint equation is derived:

$$\frac{r_i + \bar{r}_i}{2} - \frac{r_{i+1} + \bar{r}_{i+1}}{2} + (w_{bi} + \bar{w}_{bi}) - (w_{b(i+1)} + \bar{w}_{b(i+1)}) + (t_{bi} + \bar{t}_{bi}) + (\Delta_i - \Delta_{i+1}) = m_{bi}. \tag{7}$$

To ensure that the bandwidth does not exceed the green time at any signalised intersection, continuous variable constraint inequalities can be deduced from Figure 1:

$$w_{bi} + b_b \leq 1 - r_i, \tag{8}$$

$$\bar{w}_{bi} + \bar{b}_b \leq 1 - \bar{r}_i. \tag{9}$$

To ensure that the ratio of the inbound bandwidth to the outbound bandwidth reflects the ratio of the total inbound volumes to the total outbound volumes, continuous variable constraint inequalities can be derived:

$$(1 - k_b)k_b b_b \leq (1 - k_b)\bar{b}_b. \tag{10}$$

2.2. *Basic constraints for the general vehicle green band*

The basic constraints for the general vehicle green band can be obtained from the right part of Figure 1. Similar to the constraints for a bus, the constraint equations and inequalities for a general vehicle can be derived as follows:

$$\frac{r_i + \bar{r}_i}{2} - \frac{r_{i+1} + \bar{r}_{i+1}}{2} + (w_{gi} + \bar{w}_{gi}) - (w_{g(i+1)} + \bar{w}_{g(i+1)}) + (t_{gi} + \bar{t}_{gi}) + (\Delta_i - \Delta_{i+1}) = m_{gi}, \tag{11}$$

$$w_{gi} + b_g \leq 1 - r_i, \tag{12}$$

$$\bar{w}_{gi} + \bar{b}_g \leq 1 - \bar{r}_i, \tag{13}$$

$$(1 - k_g)k_g b_g \leq (1 - k_g)\bar{b}_g. \tag{14}$$

2.3. *Additional constraints for the bus green band*

The travel time for a bus between adjacent signalised intersections consists of four components: the travel time with an average bus speed, the acceleration time when a bus leaves a bus stop, the deceleration time when a bus arrives at a bus stop and the dwell time at a bus stop. In the proposed approach, bus systems have exclusive bus lanes.

To ensure that the constraints of the bus travel time between adjacent signalised intersections are linear in the models, the total travel time is formulated as the sum of two parts: the travel time without dwell time and the travel time with dwell time.

$$t_{bi} = \begin{pmatrix} t_{pi} + \sum_{1}^{N_i} t_{dij}, \ N_i > 0 \\ t_{pi}, \ N_i = 0 \end{pmatrix}, \tag{15}$$

$$\bar{t}_{bi} = \begin{pmatrix} \bar{t}_{pi} + \sum_{1}^{\bar{N}_i} \bar{t}_{dij}, \ \bar{N}_i > 0 \\ \bar{t}_{pi}, \ \bar{N}_i = 0 \end{pmatrix}, \tag{16}$$

$$t_{pi} = \begin{pmatrix} \left\{ \frac{[L_i - N_i(v_i^2/2a_{b1} + v_i^2/2a_{b2})]}{v_i} + \frac{N_i v_i}{a_{b1}} + \frac{N_i v_i}{a_{b2}} \right\} Z, N_i > 0 \\ \left(\frac{L_i}{v_i} \right) Z, N_i = 0 \end{pmatrix} = T_i Z, \tag{17}$$

$$\bar{t}_{pi} = \begin{pmatrix} \left\{ \frac{[L_i - \bar{N}_i(\bar{v}_i^2/2a_{b1} + \bar{v}_i^2/2a_{b2})]}{\bar{v}_i} + \frac{\bar{N}_i \bar{v}_i}{a_{b1}} + \frac{\bar{N}_i \bar{v}_i}{a_{b2}} \right\} Z, \bar{N}_i > 0 \\ \left(\frac{L_i}{\bar{v}_i} \right) Z, \bar{N}_i = 0 \end{pmatrix} = \bar{T}_i Z. \tag{18}$$

For bus systems, $v_{b\,\min}$ and $v_{b\,\max}$ can be obtained according to the bus type, the traffic conditions, and the characteristics of the bus lane, whereas $T_{i\,\max}(\bar{T}_{i\,\max})$ and $T_{i\,\min}(\bar{T}_{i\,\min})$ can be derived by considering the range of $v_{bi}(\bar{v}_{bi})$.

Because the dwell time can be adjusted in the proposed models, the maximum and minimum values of the outbound and inbound dwell times need to be established in advance.

If $N_i > 0$, set $t_{dij\,\min}$ to the minimum outbound dwell time. Because the largest dwell times can be obtained from the field survey, the lower limits of the dwell times can be set as the largest dwell times during peak hours to satisfy the demands of boarding and alighting passengers. Generally, the actual dwell times may not exceed the dwell times obtained from the model; thus, the bus drivers can hold the buses at the bus stops. For off-peak hours, the lower limits of the dwell times can be set to other values with less conservation, such as the mean of the investigation data. In the proposed approach, $t_{dij\,\max}$ can be established as follows:

$$t_{dij\,\max} = t_{dij\,\min} + \frac{r_{i+1}}{N_i}. \tag{19}$$

If $\bar{N}_i > 0$, set $\bar{t}_{dij\,\min}$ to the minimum inbound dwell time at the inbound bus stop j; similarly,

$$\bar{t}_{dij\,\max} = \bar{t}_{dij\,\min} + \frac{\bar{r}_i}{\bar{N}_i}, \tag{20}$$

where r_{i+1} is the outbound red time at intersection $i+1$ and $\bar{r}_i$ is the inbound red time at intersection i. Although the subscripts are similar, t_{dij} and $\bar{t}_{dij}$ differ. t_{dij} is the dwell time at the jth outbound bus stop between intersection i and intersection $i+1$, whereas $\bar{t}_{dij}$ is the dwell time at the jth inbound bus stop between intersection i and intersection $i+1$.

To ensure that the ratio of the inbound total travel time to the outbound total travel time reflects the ratio of the total inbound volumes to the total outbound volumes, continuous variable constraint inequalities for total travel times can be derived. Compared with (10), b_b and $\bar{b}_b$ are replaced by $\sum_{i=1}^{n-1} t_{bi}$ and $\sum_{i=1}^{n-1} \bar{t}_{bi}$.

$$(1-k_b)k_b \sum_{i=1}^{n-1} t_{bi} \le (1-k_b) \sum_{i=1}^{n-1} \bar{t}_{bi}. \tag{21}$$

2.4. *The relationships between the bus green band and the general vehicle green band*

Only one green band can exist in traditional bandwidth optimisation approaches. However, in this paper, both the bus green band and the general vehicle green band exist in the same timing plan, which is a core design in the proposed approach. Therefore, the relationships between these two green bands should be discussed.

In Figure 2, the solid lines represent the bus green band, whereas the dashed lines represent the general vehicle green band. To ensure that the bus green band and the general vehicle green band share the same timing plan, an integer variable constraint equation can be derived

$$\Phi_{bi} - \Phi_{gi} = m_i, \tag{22}$$

$$w_{bi} - w_{b(i+1)} + t_{bi} - (w_{gi} - w_{g(i+1)} + t_{gi}) = m_i, \tag{23}$$

$$\bar{\Phi}_{bi} - \bar{\Phi}_{gi} = \bar{m}_i, \tag{24}$$

$$\bar{w}_{bi} - \bar{w}_{b(i+1)} + \bar{t}_{bi} - (\bar{w}_{gi} - \bar{w}_{g(i+1)} + \bar{t}_{gi}) = \bar{m}_i, \tag{25}$$

where $m_i(\bar{m}_i)$ represents integer multiples of the uniform cycle time.

2.5. *Development of the mixed-integer linear programming model*

In the following sections, Models X and Y are constructed to obtain the maximum bandwidths of bus systems and general vehicles. These two models are MAXBAND models: Model X is for bus systems, and Model Y is for general vehicles. The minimum bandwidths of bus systems and general vehicles are also discussed. Then, Model V, which is a mixed-integer linear programming model for both the bus green band and the general vehicle green band, can be constructed. All previously described constraints are considered in Models X, Y, and V, and $C_{\min}$ and $C_{\max}$ are the lower limit and upper limit, respectively, for the uniform cycle length.

(1) Obtain $b_{b\,\max}$, $\bar{b}_{b\,\max}$, $b_{b\,\min}$, and $\bar{b}_{b\,\min}$.

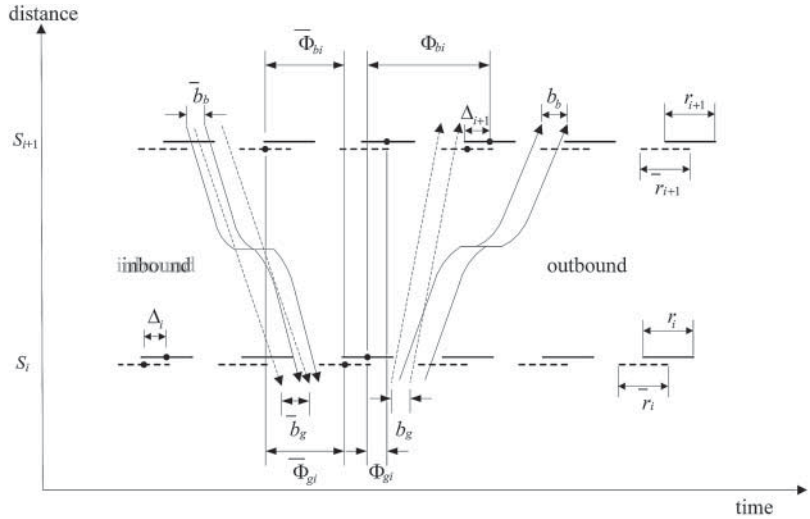

Figure 2. Relationships between the bus green band and the general vehicle green band.

Here, $b_{\mathrm{b\,max}}$ and $\bar{b}_{\mathrm{b\,max}}$ can be derived by solving Model X, whereas $b_{\mathrm{b\,min}}$ and $\bar{b}_{\mathrm{b\,min}}$ represent the minimum bandwidths that satisfy the outbound bus demands and inbound bus demands, respectively, within a cycle.

2.5.1. *Model X for bus green band*

Obtain $b_\mathrm{b}, \bar{b}_\mathrm{b}$, to

$$\max = b_\mathrm{b} + k_\mathrm{b}\bar{b}_\mathrm{b}$$

Subject to

$$(1 - k_\mathrm{b})k_\mathrm{b}b_\mathrm{b} \le (1 - k_\mathrm{b})\bar{b}_\mathrm{b},$$

$$\begin{pmatrix} w_{\mathrm{b}i} + b_\mathrm{b} \le 1 - r_i \\ \bar{w}_{\mathrm{b}i} + \bar{b}_\mathrm{b} \le 1 - \bar{r}_i \end{pmatrix}, \quad i = 1, \dots, n,$$

$$(w_{\mathrm{b}i} + \bar{w}_{\mathrm{b}i}) - (w_{\mathrm{b}(i+1)} + \bar{w}_{\mathrm{b}(i+1)}) + (t_{\mathrm{b}i} + \bar{t}_{\mathrm{b}i}) + (\delta_i l_i - \bar{\delta}_i \bar{l}_i) - (\delta_{i+1} l_{i+1} - \bar{\delta}_{i+1}\bar{l}_{i+1}) + (r_i - r_{i+1}) = m_{\mathrm{b}i}, \quad i = 1, \dots, n-1,$$

$$t_{\mathrm{b}i} = \begin{pmatrix} t_{\mathrm{p}i} + \sum_{1}^{N_i} t_{\mathrm{d}ij}, N_i > 0 \\ t_{\mathrm{p}i}, N_i = 0 \end{pmatrix}, \quad i = 1, \dots, n-1;\ j = 1, \dots N_i,$$

$$\bar{t}_{\mathrm{b}i} = \begin{pmatrix} \bar{t}_{\mathrm{p}i} + \sum_{1}^{\bar{N}_i} \bar{t}_{\mathrm{d}ij}, \bar{N}_i > 0 \\ \bar{t}_{\mathrm{p}i}, \bar{N}_i = 0 \end{pmatrix}, \quad i = 1, \dots, n-1;\ j = 0, \dots \bar{N}_i,$$

$$b_\mathrm{b}, \bar{b}_\mathrm{b} \ge 0,$$

$$\frac{1}{C_{\max}} \le Z \le \frac{1}{C_{\min}},$$

$$T_{i\,\min} Z + t_{\mathrm{d}ij\,\min} \le t_{\mathrm{b}i} \le T_{i\,\max} Z + t_{\mathrm{d}ij\,\max}, \quad i = 1, \dots, n-1;\ j = 1, \dots N_i,$$

$$\bar{T}_{i\,\min}Z + \bar{t}_{\mathrm{d}ij\,\min} \le \bar{t}_{\mathrm{b}i} \le \bar{T}_{i\,\max}Z + \bar{t}_{\mathrm{d}ij\,\max}, \quad i = 1, \ldots, n-1;\ j = 1, \ldots \bar{N}_i,$$
$$w_{\mathrm{b}i}, \bar{w}_{\mathrm{b}i} \ge 0, \quad i = 1, \ldots, n,$$
$$m_{\mathrm{b}i} \quad \text{integer}, \quad i = 1, \ldots, n-1,$$
$$\delta_i = 1, 0, \quad i = 1, \ldots, n,$$
$$\bar{\delta}_i = 1, 0, \quad i = 1, \ldots, n,$$

where if $k_{\mathrm{b}} = 1$, $(1 - k_{\mathrm{b}})k_{\mathrm{b}}b_{\mathrm{b}} \le (1 - k_{\mathrm{b}})\bar{b}_{\mathrm{b}}$ is transferred to $b_{\mathrm{b}} = \bar{b}_{\mathrm{b}}$.

$t_{\mathrm{p}i}(\bar{t}_{\mathrm{p}i})$ and $t_{\mathrm{d}ij}(\bar{t}_{\mathrm{d}ij})$ are decision variables in Model X; details of these variables are described in ‘Analyse the supplementary constraints for bus systems and bus green band’.

If the solution of Model X exists, b_{b} in this solution is $b_{\mathrm{b}\,\max}$, whereas $\bar{b}_{\mathrm{b}}$ is $\bar{b}_{\mathrm{b}\,\max}$, $b_{\mathrm{b}\,\min}$ and $\bar{b}_{\mathrm{b}\,\min}$ can be obtained by the schedule and frequency of the bus systems. We may define $N_{\mathrm{bus}}(\bar{N}_{\mathrm{bus}})$ as the outbound (inbound) number of buses that arrive during one cycle length, $H_{\min}(\bar{H}_{\min})$ as the minimum outbound (inbound) average headway, and $\varepsilon(\bar{\varepsilon})$ as the outbound (inbound) adjusted variable. We can set $b_{\mathrm{b}\,\min}(\bar{b}_{\mathrm{b}\,\min})$ to $(N_{\mathrm{bus}} - 1)H_{\min} + \varepsilon((\bar{N}_{\mathrm{bus}} - 1)\bar{H}_{\min} + \bar{\varepsilon})$, where $H_{\min}(\bar{H}_{\min})$ can be achieved by the adjustment of the schedule and frequency of the bus systems and $\varepsilon(\bar{\varepsilon})$ can be adjusted to modify the bandwidth.

(2) Obtain $b_{\mathrm{g}\,\max}$, $\bar{b}_{\mathrm{g}\,\max}$, $b_{\mathrm{g}\,\min}$, and $\bar{b}_{\mathrm{g}\,\min}$.

$b_{\mathrm{g}\,\max}$ and $\bar{b}_{\mathrm{g}\,\max}$ can be derived by solving Model Y for the general vehicle green band.

2.5.2. *Model Y for general vehicle green band*

Obtain $b_{\mathrm{g}}, \bar{b}_{\mathrm{g}}$, to

$$\max = b_{\mathrm{g}} + k_{\mathrm{g}}\bar{b}_{\mathrm{g}}$$

Subject to

$$(1 - k_{\mathrm{g}})k_{\mathrm{g}}b_{\mathrm{g}} \le (1 - k_{\mathrm{g}})\bar{b}_{\mathrm{g}},$$
$$\begin{pmatrix} w_{\mathrm{g}i} + b_{\mathrm{g}} \le 1 - r_i \\ \bar{w}_{\mathrm{g}i} + \bar{b}_{\mathrm{g}} \le 1 - \bar{r}_i \end{pmatrix}, \quad i = 1, \ldots, n,$$
$$(w_{\mathrm{g}i} + \bar{w}_{\mathrm{g}i}) - (w_{\mathrm{g}(i+1)} + \bar{w}_{\mathrm{g}(i+1)}) + (t_{\mathrm{g}i} + \bar{t}_{\mathrm{g}i}) + (\delta_i l_i - \bar{\delta}_i \bar{l}_i) - (\delta_{i+1} l_{i+1} - \bar{\delta}_{i+1}\bar{l}_{i+1})$$
$$+ (r_i - r_{i+1}) = m_{\mathrm{g}i}, \quad i = 1, \ldots, n-1,$$
$$b_{\mathrm{g}}, \bar{b}_{\mathrm{g}} \ge 0,$$
$$\frac{1}{C_{\max}} \le Z \le \frac{1}{C_{\min}},$$
$$w_{\mathrm{g}i}, \bar{w}_{\mathrm{g}i} \ge 0, \quad i = 1, \ldots, n,$$
$$m_{\mathrm{g}i} \quad \text{integer}, \quad i = 1, \ldots, n-1,$$
$$\delta_i = 1, 0, \quad i = 1, \ldots, n,$$
$$\bar{\delta}_i = 1, 0, \quad i = 1, \ldots, n,$$

where if k_{g}=1, $(1 - k_{\mathrm{g}})k_{\mathrm{g}}b_{\mathrm{g}} \le (1 - k_{\mathrm{g}})\bar{b}_{\mathrm{g}}$ is transferred to $b_{\mathrm{g}} = \bar{b}_{\mathrm{g}}$.

$t_{\mathrm{g}i}$ and $\bar{t}_{\mathrm{g}i}$ are parameters, and the values have been set in advance.

Generally, the volumes of bus systems are lower than the volumes of general vehicles. Therefore, we define $b_{\mathrm{g}\,\min} = b_{\mathrm{b}}$ and $\bar{b}_{\mathrm{g}\,\min} = \bar{b}_{\mathrm{b}}$ to ensure that the minimum green band for general vehicles is wider than the minimum green band for bus systems. By doing so, we can enlarge the general vehicle green band when the bus demands are satisfied.

(3) Construct a mixed-integer linear programming model 'Model V' to generate an optimal timing plan for the bus green band and the general vehicle green band. Model V not only searches for bandwidths of buses and general vehicles, but also coordinates operations of bus systems with the signal timing plan.

Objective functions of traditional bandwidth optimisation approaches, such as MAXBAND and MULTIBAND, primarily obtain the maximum weighted bandwidths for both directions. In the proposed approach, two different green bands need to share the same timing plan. Because a global optimal solution cannot be achieved by pursuing the maximum weighted bandwidths for an individual system, a trade-off between the bus green band and the general vehicle green band is necessary. Eight variables – $b_{\mathrm{b\,max}}$, $\bar{b}_{\mathrm{b\,max}}$, $b_{\mathrm{b\,min}}$, $\bar{b}_{\mathrm{b\,min}}$, $b_{\mathrm{g\,max}}$, $\bar{b}_{\mathrm{g\,max}}$, $b_{\mathrm{g\,min}}$, and $\bar{b}_{\mathrm{g\,min}}$ – which have been solved in Model X and Model Y, can be established as the lower and upper bounds for the bus green band and general vehicle band in Model V. Therefore, the constraints$b_{\mathrm{b\,min}} \le b_{\mathrm{b}} \le b_{\mathrm{b\,max}}$ and $\bar{b}_{\mathrm{b\,min}} \le \bar{b}_{\mathrm{b}} \le \bar{b}_{\mathrm{b\,max}}$ are utilised to ensure that the outbound and inbound bus green bands can satisfy the bus demands in both directions. In addition, the constraints $b_{\mathrm{b}} \le b_{\mathrm{g}} \le b_{\mathrm{g\,max}}$ and $\bar{b}_{\mathrm{b}} \le \bar{b}_{\mathrm{g}} \le \bar{b}_{\mathrm{g\,max}}$ are utilised to enlarge the outbound and inbound general vehicle green bands to serve additional general vehicles in both directions.

The objective functions of traditional bandwidth optimisation approaches have been transferred into constraints in the proposed approach, and a new objective function is adopted. The outbound and inbound travel times for general vehicles are constant variables, whereas the outbound and inbound travel times for buses can be adjusted. Therefore, different combinations of outbound and inbound travel times for buses may generate the same timing plan and bandwidths, some combinations may not be reasonable and travel times are too long to be accepted. To confine the outbound and inbound travel times for buses within a reasonable region and improve the performance of bus systems, the minimum total weighted travel times for buses in both directions is established as the new objective function.

2.5.3. *Model V for both bus green band and general vehicle green band*

Obtain $Z, w_{\mathrm{b}i}, \bar{w}_{\mathrm{b}i}, w_{\mathrm{g}i}, \bar{w}_{\mathrm{g}i}, \delta_i, \bar{\delta}_i, b_{\mathrm{g}}, \bar{b}_{\mathrm{g}}, b_{\mathrm{b}}, \bar{b}_{\mathrm{b}}, t_{\mathrm{b}i}, \bar{t}_{\mathrm{b}i}$ to

$$\min = \sum_{i=1}^{n-1} t_{\mathrm{b}i} + k_{\mathrm{b}} \sum_{i=1}^{n-1} \bar{t}_{\mathrm{b}i}$$

Subject to

$$(1-k_{\mathrm{b}})k_{\mathrm{b}} \sum_{i=1}^{n-1} t_{\mathrm{b}i} \le (1-k_{\mathrm{b}}) \sum_{i=1}^{n-1} \bar{t}_{\mathrm{b}i},$$

$$(1-k_{\mathrm{b}})k_{\mathrm{b}} b_{\mathrm{b}} \le (1-k_{\mathrm{b}})\bar{b}_{\mathrm{b}},$$

$$(1-k_{\mathrm{g}})k_{\mathrm{g}} b_{\mathrm{g}} \le (1-k_{\mathrm{g}})\bar{b}_{\mathrm{g}},$$

$$\begin{pmatrix} w_{\mathrm{b}i} + b_{\mathrm{b}} \le 1 - r_i \\ \bar{w}_{\mathrm{b}i} + \bar{b}_{\mathrm{b}} \le 1 - \bar{r}_i \end{pmatrix}, \quad i = 1, \ldots, n,$$

$$(w_{\mathrm{b}i} + \bar{w}_{\mathrm{b}i}) - (w_{\mathrm{b}(i+1)} + \bar{w}_{\mathrm{b}(i+1)}) + (t_{\mathrm{b}i} + \bar{t}_{\mathrm{b}i}) + (\delta_i l_i - \bar{\delta}_i \bar{l}_i) - (\delta_{i+1} l_{i+1} - \bar{\delta}_{i+1} \bar{l}_{i+1}) + (r_i - r_{i+1}) = m_{\mathrm{b}i}, \quad i = 1, \ldots, n-1,$$

$$\begin{pmatrix} w_{gi} + b_g \leq 1 - r_i \\ \bar{w}_{gi} + \bar{b}_g \leq 1 - \bar{r}_i \end{pmatrix}, \quad i = 1, \ldots, n,$$

$$(w_{gi} + \bar{w}_{gi}) - (w_{g(i+1)} + \bar{w}_{g(i+1)}) + (t_{gi} + \bar{t}_{gi}) + (\delta_i l_i - \bar{\delta}_i \bar{l}_i) - (\delta_{i+1} l_{i+1} - \bar{\delta}_{i+1} \bar{l}_{i+1}) + (r_i - r_{i+1}) = m_{gi}, \quad i = 1, \ldots, n-1,$$

$$\begin{pmatrix} w_{bi} - w_{b(i+1)} + t_{bi} - (w_{gi} - w_{g(i+1)} + t_{gi}) = m_i \\ \bar{w}_{bi} - \bar{w}_{b(i+1)} + \bar{t}_{bi} - (\bar{w}_{gi} - \bar{w}_{g(i+1)} + \bar{t}_{gi}) = \bar{m}_i \end{pmatrix}, \quad i = 1, \ldots, n-1,$$

$$t_{bi} = \begin{pmatrix} t_{pi} + \sum_{1}^{N_i} t_{dij}, N_i > 0 \\ t_{pi}, N_i = 0 \end{pmatrix}, \quad i = 1, \ldots, n-1; j = 1, \ldots N_i,$$

$$\bar{t}_{bi} = \begin{pmatrix} \bar{t}_{pi} + \sum_{1}^{\bar{N}_i} \bar{t}_{dij}, \bar{N}_i > 0 \\ \bar{t}_{pi}, \bar{N}_i = 0 \end{pmatrix}, \quad i = 1, \ldots, n-1; j = 0, \ldots \bar{N}_i,$$

$$b_{b\min} \leq b_b \leq b_{b\max},$$

$$\bar{b}_{b\min} \leq \bar{b}_b \leq \bar{b}_{b\max},$$

$$b_b \leq b_g \leq b_{g\max},$$

$$\bar{b}_b \leq \bar{b}_g \leq \bar{b}_{g\max},$$

$$\frac{1}{C_{\max}} \leq Z \leq \frac{1}{C_{\min}},$$

$$T_{i\min} Z + t_{dij\min} \leq t_{bi} \leq T_{i\max} Z + t_{dij\max}, \quad i = 1, \ldots, n-1; j = 1, \ldots N_i,$$

$$\bar{T}_{i\min} Z + \bar{t}_{dij\min} \leq \bar{t}_{bi} \leq \bar{T}_{i\max} Z + \bar{t}_{dij\max}, \quad i = 1, \ldots, n-1; j = 1, \ldots \bar{N}_i,$$

$$w_{bi}, \bar{w}_{bi}, w_{gi}, \bar{w}_{gi} \geq 0, \quad i = 1, \ldots, n,$$

$$m_{bi}, m_{gi}, m_i, \bar{m}_i \quad \text{integer}, \quad i = 1, \ldots, n-1,$$

$$\delta_i = 1, 0, \quad i = 1, \ldots, n,$$

$$\bar{\delta}_i = 1, 0 \quad i = 1, \ldots, n$$

where if $k_b = 1$, $(1 - k_b)k_b b_b \leq (1 - k_b)\bar{b}_b$ is transferred to $b_b = \bar{b}_b$ and $(1 - k_b)k_b \sum_{i=1}^{n-1} t_{bi} \leq (1 - k_b) \sum_{i=1}^{n-1} \bar{t}_{bi}$ is transferred to $\sum_{i=1}^{n-1} t_{bi} = \sum_{i=1}^{n-1} \bar{t}_{bi}$. If $k_g = 1$, $(1 - k_g)k_g b_g \leq (1 - k_g)\bar{b}_g$ is transferred to $b_g = \bar{b}_g$.

If the solution of Model V exists, $Z, w_{bi}, \bar{w}_{bi}, w_{gi}, \bar{w}_{gi}, \delta_i, \bar{\delta}_i, b_g, \bar{b}_g, b_b, \bar{b}_b, t_{bi}, \bar{t}_{bi}$ can be obtained. According to the discussions in the section ‘Analyse the supplementary constraints for bus systems and bus green band’, $t_{bi}(\bar{t}_{bi})$ consists of $t_{pi}(\bar{t}_{pi})$ and $t_{dij}(\bar{t}_{dij})$ and $v_{bi}(\bar{v}_{bi})$ can be derived from $t_{pi}(\bar{t}_{pi})$. Therefore, one $t_{bi}(\bar{t}_{bi})$ can generate different combinations of $v_{bi}(\bar{v}_{bi})$ and $t_{dij}(\bar{t}_{dij})$ without changing the value of $t_{bi}(\bar{t}_{bi})$. Within the value ranges of $v_{bi}(\bar{v}_{bi})$ and $t_{dij}(\bar{t}_{dij})$, we can select different combinations of bus speeds and dwell times to satisfy operational demands of the entire traffic system.

Although we can change the bus speeds and dwell times and elect different combinations of bus speeds and dwell times, the travel times, the timing plan, and the global optimal solution remain constant. In the proposed approach, we can select different combinations of bus speeds and dwell times with the same timing plan to achieve multiple objectives.

Figure 3. Map of the case study.

In addition, when $t_{pi}(\bar{t}_{pi})$ and $t_{dij}(\bar{t}_{dij})$ simultaneously achieve the maximum or minimum values, only one combination of bus speeds and dwell times may exist with the same timing plan.

2.6. *Solving the mixed-integer linear programming model*

All constraints in Models X, Y, and V are linearly expressed. These mixed-integer linear programming models are generally solved by adopting the branch-and-bound technique. With the advance of optimisation packages and computing power, the problem of a practical size can be readily solved using commercial or free MIP solvers within an acceptable amount of time.

The linear interactive and general optimiser (LINGO) is a simple software package that is developed and maintained by LINDO SYSTEMS INC. for utilising the power of linear and nonlinear optimisation to concisely formulate large problems, solve them, and analyse the solutions. LINGO 11 is selected to obtain the global optimal solution of these models.

3. A case study

The case study in this section consists of two components: applying the proposed approach to a main street in Foshan, China, and utilising VISSIM simulation to examine the improvement in traffic delays and stoppages after the application of the proposed approach.

3.1. *Application of the proposed approach*

The proposed approach is tested on a main street with five fixed-time signalised intersections. The main street encompasses a part of Fenjiang Street in Foshan, China. The map of the study area, in which the streets and intersections are labelled, is shown in Figure 3. We define the southbound direction of the street as the outbound direction and the northbound direction as the inbound direction.

The distances between adjacent intersections and the volumes of general vehicles are shown in Figure 4. The vertical arrows represent the outbound and inbound directions of the main street, whereas the horizontal arrows represent the two directions of the cross streets. The number in [ ] represent the volumes that originate from the corresponding node, whereas the numbers in ( ) represent the volumes of the corresponding approaches at the downstream intersection. The three numbers in ( ) correspond to the left-turn traffic volume, the centre through traffic volume and the outer right-turn traffic volume, respectively. The volumes for all intersections were investigated

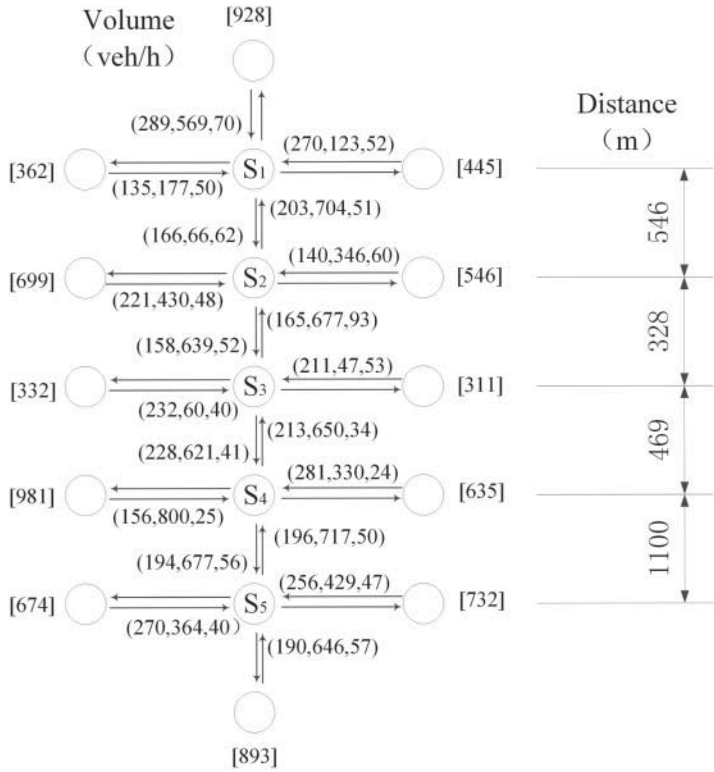

Figure 4. Distances and general vehicle volumes of the case study.

Table 1. Travel times for general vehicles in the case study.

i	t_{gi} (cycles)	$\bar{t}_{gi}$ (cycles)
1	32.8*Z	32.8*Z
2	19.7*Z	19.7*Z
3	28.1*Z	28.1*Z
4	66*Z	66*Z

during the same peak hour. The posted speed limit of the main street is $60\,\mathrm{km\,h^{-1}}$, which is employed as the average speed of the general vehicle along the main street in this study. Due to the symmetry of flows, k_g is set to 1. The travel times for general vehicles between adjacent intersections are shown in Table 1.

The bus system along the main street is shown in Figure 5. The main street in each direction contains an exclusive bus lane (i.e. rightmost lane) with five bus stops along the bus. The bus lanes and bus stops serve three bus routes in each direction along the main street. According to the characteristics of the bus system and the field data along the main street, the values of the variables relevant to the bus green band in the proposed approach can be initialised. The volume of the southbound bus flow is equivalent to the volume of the northbound bus flow, which is 72 buses $\mathrm{h^{-1}}$. k_b is set to 1. The minimum bus speed $v_{b\,max}$ is $40\,\mathrm{km\,h^{-1}}$, and the $v_{b\,min}$ $30\,\mathrm{km\,h^{-1}}$.

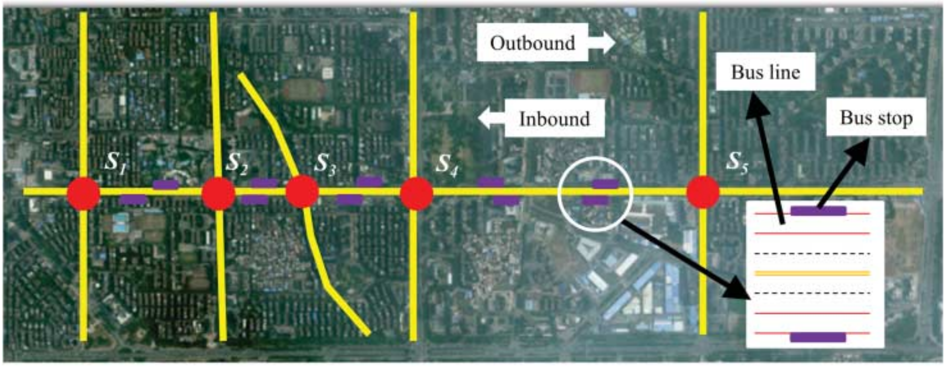

Figure 5. Bus systems in the case study.

Table 2. Travel times for buses in the case study.

	t_{bi} (cycles)				$\bar{t}_{bi}$ (cycles)			
	t_{pi} (cycles)		t_{dij} (cycles)		$\bar{t}_{pi}$ (cycles)		$\bar{t}_{dij}$ (cycles)	
i	Max	Min	Max	Min	Max	Min	Max	Min
1	72.3*Z	58.2*Z	r_2 + 16*Z	16*Z	72.3*Z	58.2*Z	$\bar{r}_1 + 16 * Z$	16*Z
2	46.1*Z	38.5*Z	r_3 + 15*Z	15*Z	46.1*Z	38.5*Z	$\bar{r}_2 + 15 * Z$	15*Z
3	63.1*Z	51.2*Z	r_4 + 20*Z	20*Z	63.1*Z	51.2*Z	$\bar{r}_3 + 20 * Z$	20*Z
4	145.5*Z	117.1*Z	r_5/2 + 23*Z	23*Z	145.5*Z	117.1*Z	$\bar{r}_4/2 + 23 * Z$	23*Z
			r_5/2 + 24*Z	24*Z			$\bar{r}_4/2 + 24 * Z$	24*Z

Table 3. Relevant variables for all signals in the case study.

i	l_i (cycles)	$\bar{l}_i$ (cycles)	r_i (cycles)	$\bar{r}_i$ (cycles)	$C_{\max}$ (s)	$C_{\min}$ (s)
1	0.227	0.227	0.533	0.533	150	60
2	0.140	0.140	0.540	0.540		
3	0.213	0.213	0.453	0.453		
4	0.133	0.133	0.667	0.667		
5	0.147	0.147	0.593	0.593		

The minimum bandwidth is 30 s. The travel times for buses between adjacent intersections are shown in Table 2.

The red splits for each signal in the current timing plan were determined according to the flow ratio; therefore, we only need to adjust the cycle length. The variables r_i, $\bar{r}_i$, l_i, and $\bar{l}_i$ are expressed as the ratio of the cycle times, which can be obtained from a field survey. $C_{\max}$ is set to 150 s, and $C_{\min}$ is set to 60 s. The relevant variables for all signals are shown in Table 3.

The maximum bandwidths of the buses and general vehicles in Models X and Y were solved by programming in LINGO, from which we obtained $b_{\mathrm{b\,max}} = \bar{b}_{\mathrm{b\,max}} = 50$ s and $b_{\mathrm{g\,max}} = \bar{b}_{\mathrm{g\,max}} = 43$ s. The lower limits of the dwell times were set to the largest dwell times among all bus routes at the bus stops.

Using these variables, Model V was solved by programming in LINGO. The cycle lengths, bandwidths, average speed of buses, and dwell times at the bus stops were also derived, as shown in Table 4. According to the results, the time–space diagrams for the bus green band and the general vehicle green band were created, as presented in Figures 6 and 7.

Table 4. Results of the proposed approach of the case study.

i	v_{bi} (km h^{-1})	$\bar{v}_{bi}$ (km h^{-1})	t_{dij} (s)	$\bar{t}_{dij}$ (s)	b_b (s)	$\bar{b}_b$ (s)	b_g (s)	$\bar{b}_g$ (s)	C (s)
1	40	40	16	16	30	30	30	30	150
2	40	40	15	15					
3	40	40	55	59					
4	40	40	27	23					
			68	68					

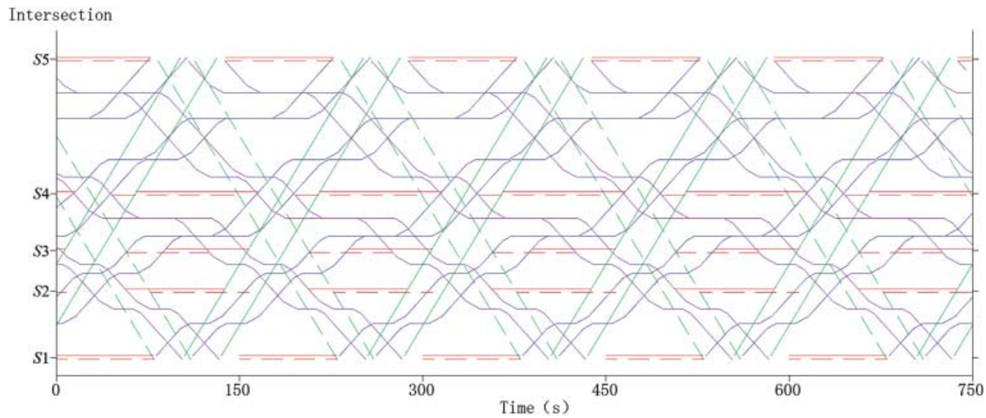

Figure 6. Original time–space diagram for bus green and general vehicle bands of Model V.

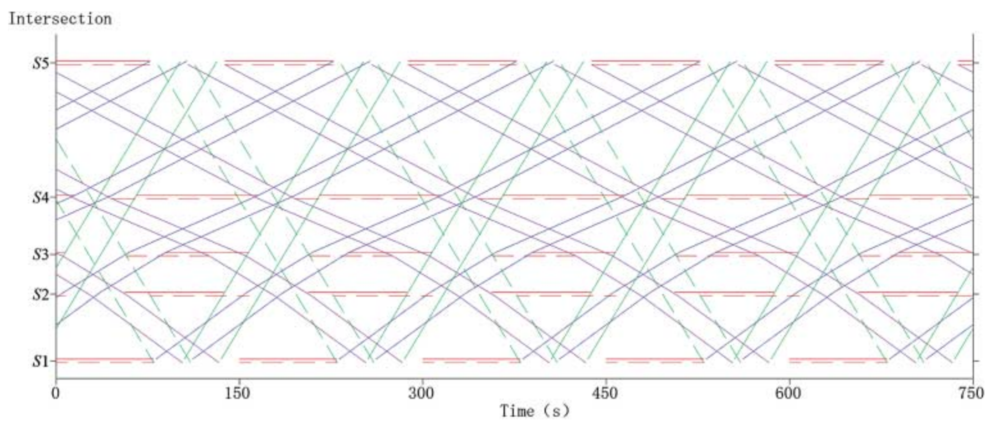

Figure 7. Smooth time–space diagram for bus green and general vehicle bands of Model V.

As previously discussed, more than one combination of bus speeds and dwell times may exist in the global optimal solution. Speed is preferred in this case. Within the value ranges, we attempt to obtain the maximum value of the bus speeds between adjacent intersections in the global optimal solution; thus, the dwell time at some bus stops may be longer to serve additional bus demand. As shown in Table 4, the bus speeds in all segments attain the optimal solutions on the bounds. The optimal bus speeds in all segments achieve optimal values on the upper bound (i.e. 40 km h^{-1}). The dwell times for some stops (e.g. the stops between intersections 1 and 2) achieve

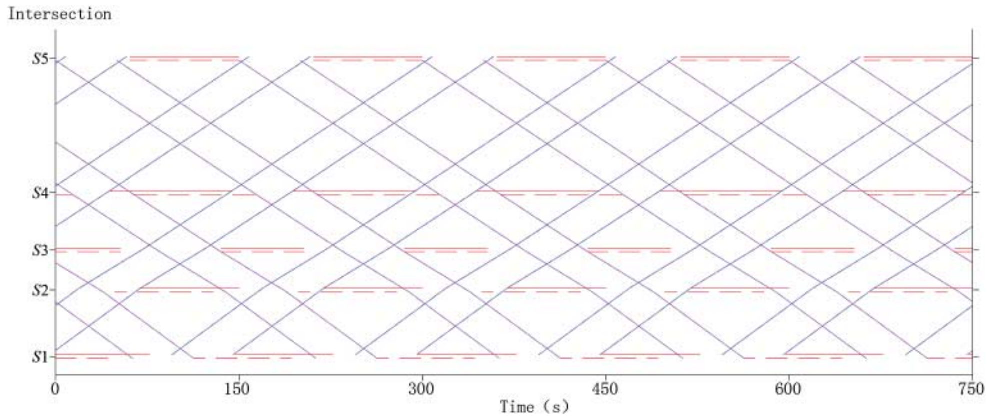

Figure 8. Smooth time–space diagram for bus green band of Model X.

their optimal values on the lower bounds of feasible regions, which comprise the shortest dwell times to satisfy the bus demand. The optimal dwell times for bus stops between intersections 3, 4, and 5 are longer than the lower bounds. However, the travel times and the timing plan remain the same.

In these two figures, the colour red represents the red time for each signalised intersection and the colour green represents the general vehicle green band. For the red time and the general vehicle green band, the solid lines represent the outbound elements and the dashed lines represent the inbound elements. The blue lines represent the outbound bus green band, whereas the pink lines represent the inbound bus green band. Figure 6 details the bus green band, which includes the acceleration, deceleration, and dwell times of the buses. In Figure 7, the average speed of a bus is established as the ratio of the distance to the total travel time between adjacent intersections; therefore, the bus green bands are smooth and brief.

For the comparison, a smooth time–space diagram for the bus green band generated by Model X is shown in Figure 8. As we solve Model X, the bandwidths of bus systems and a timing plan can be generated. Then, we pursue the general vehicle green band in the timing plan generated by Model X, for which no feasible solution exists. The time–space diagram for the general vehicle green band generated by Model Y is also shown in Figure 9. Similar to Model X, we pursue the bus green band in the timing plan generated by Model Y, for which no feasible solution exists. Therefore, only one band exists in either Model X or Model Y.

Some adjustments to Model V and additional experiments have been performed to reveal the relationship between two types of bands. The original objective function of Model V becomes the constraint $M_1 \leq \sum_{i=1}^{n-1} t_{bi} + k_g \sum_{i=1}^{n-1} \bar{t}_{bi} \leq M_2$ and the total bus travel times become a decision variable. M_1 is set to five cycles, which is the possible shortest total bus travel time in this case study. And M_2 should not exceed 10 cycles, which is the possible longest total bus travel time in this case study. M_2 is set to 5.5, 6, 7, and 7.5 cycles. Different relationships between two types of bands can be obtained for different total bus travel times (M_2). Because k_g is set to 1, the new objective function is max $= b_g$ and the bandwidth constraints for general vehicles are removed. Because k_b is set to 1, the bandwidth constraints for bus systems become $b_b = \bar{b}_b = EZ$, and E is set to 1,2,3, ... , 50 s. The bus bandwidth is enumerated with a step length of 1 s to accurately reveal the relationships. The results are shown in Figure 10.

In Figure 10, four kinds of spots represent different general vehicle bands with different total bus travel times (M_2). Generally, general vehicle bandwidths decrease when bus bandwidths increase. As other parameters are determined, the conflicts between two bands are primarily

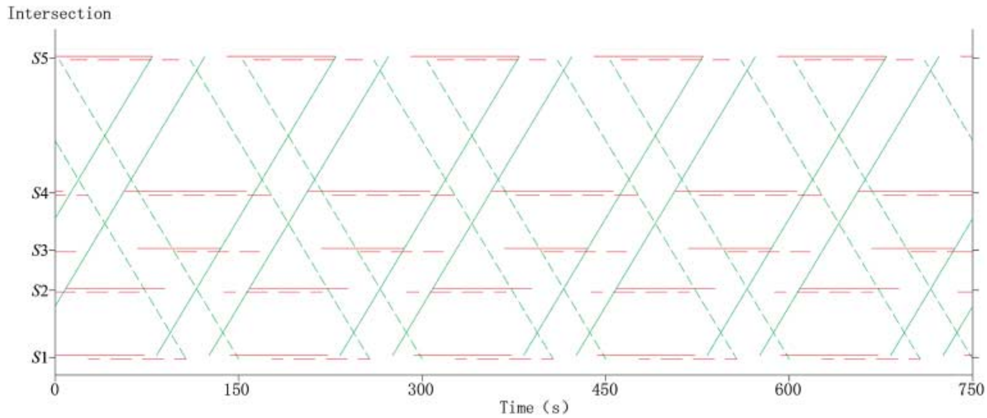

Figure 9. Time–space diagram for general vehicle green band of Model Y.

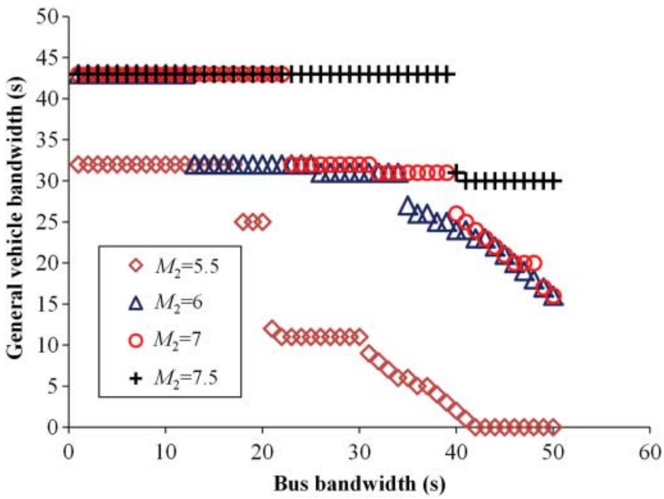

Figure 10. Relationship between two types of bands.

influenced by the total bus travel times. For diamond spots, M_2 is similar to M_1, few adjustments of the total bus travel times can be performed and the general vehicle bandwidth has significantly decreased. For triangle spots and cycle spots, adjustments of the total bus travel times are feasible and the conflict between two bands is mitigated. For cross spots, numerous adjustments can be performed, and minimal interference is observed between two bands. In addition, to mitigate the conflict between two bands requires that the total bus travel times are sacrificed, which may affect the operational efficiency of bus systems.

3.2. *Simulation evaluation*

To evaluate the proposed approach, several schemes such as the current plan, the plan generated by Model X, the plan generated by Model Y, and the plan generated by the proposed approach were coded in VISSIM. The current plan, which was developed by the local traffic management department, focused on isolated signalised intersections without any coordination between intersections. The plan generated by Model X entails the solution of the bus green band

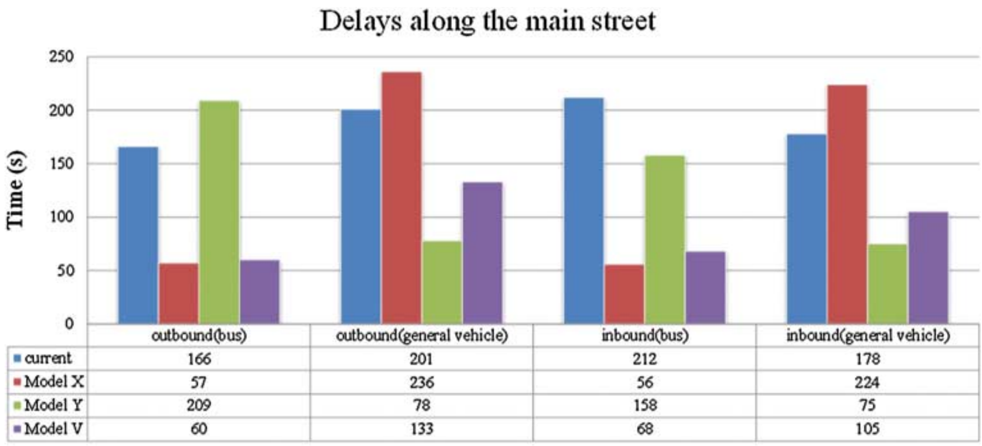

Figure 11. Delays along the main street.

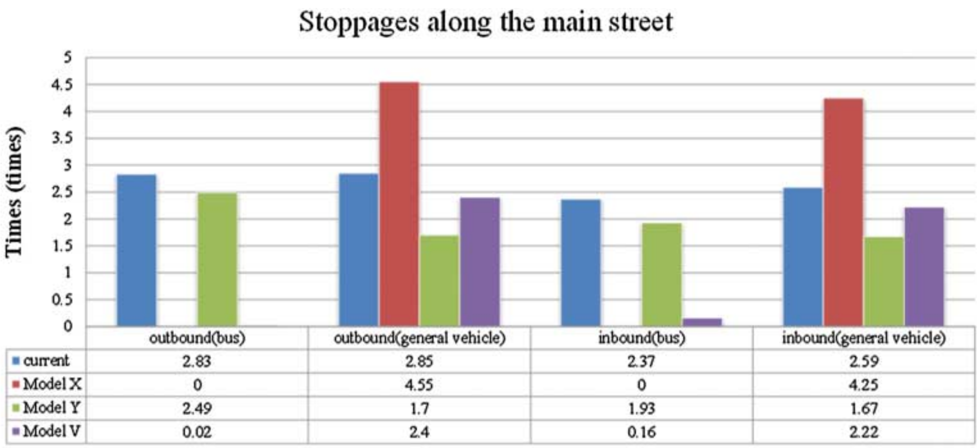

Figure 12. Stoppages along the main street.

of MAXBAND, whereas the plan generated by Model Y entails the solution of the general vehicle green band of MAXBAND. We ran the VISSIM simulations to evaluate the delays and the stoppages of both buses and the general vehicles along the main street. The results are shown in Figures 11 and 12.

Delays in Figure 11 represent the delays for the intersections and segments between adjacent intersections along the main street for buses and general vehicles. Figure 12 presents the number of stoppages at the intersections along the main street for buses and general vehicles.

The results, which are shown in Figures 11 and 12, show the progression with the proposed approach. Compared with the current plan, the delays and stoppages for either the bus system or general vehicles significantly decrease. In the proposed approach, we have considered the schedule and frequency of bus systems to obtain the minimum bus bandwidth and have adjusted the dwell times and segment speeds of bus systems to share the same timing plan with general vehicles. Therefore, all buses travelling along the main street can be served within the bus bandwidth. Significant improvement in bus operational performance is achieved with a substantial reduction of delays and stops in both directions. The delays in both directions have decreased by more than 60% and the numbers of stoppages in both directions have decreased to nearly 0. Compared with the bus bandwidth, the general vehicle bandwidth cannot serve all general vehicles along the main street due to the high demand of the general vehicle flow. As previously mentioned, the proposed approach is designed to generate green bands for both buses and general vehicles and to benefit the entire traffic system. Similar to buses, a significant improvement is achieved with more than 35% reduction in outbound and inbound delays for general vehicles. In addition, the numbers of stoppages in both directions for general vehicles also decreases by more than 14%.

The plan generated from Model X is to obtain the maximum green bandwidth for buses, in which the outbound and inbound bandwidths are 50 s. Although the bus bandwidths in Model X are longer than the bus bandwidths in the proposed approach, the differences in the delays and stoppages between Model X and the proposed approach are not distinct. The operational performance of the general vehicles in Model X is significantly worse than the operational performance of the general vehicles in the proposed approach.

The plan generated from Model Y is to obtain the maximum bandwidth for general vehicles, in which the outbound and inbound bandwidths are 43 s. Because the plan generated from Model Y is not designed for a bus system, the delays and stoppages from Model Y are not comparable to the proposed approach. However, the operational performance of the general vehicles in the plan generated from Model Y is better than the operational performance of the general vehicles in the plan generated from Model V because when two different green bands (i.e. the bus system and the general vehicles) need to share the same timing plan, one band may be influenced by the other band, which causes the bandwidths of both bands to decrease. For the bus green band, we have defined the minimum bus green bandwidths in the proposed approach to ensure that nearly all buses can be served and the redundant bandwidths in Model X are inefficient. For the general vehicle green band, all general vehicles cannot be served and they will be influenced after the bandwidth decreases. However, the plans generated from Model X and Y are designed for particular vehicles and different vehicles cannot simultaneously share and benefit from the same timing plan. A trade-off is necessary to generate two different green bands in the same timing plan, which enables both bus system and general vehicles to benefit from the proposed approach.

Because we have adjusted the dwell times and speeds of the bus systems in the proposed approach, the travel times for buses along the main street may be influenced. Compared with the current plan, the outbound and inbound travel times for buses have decreased and passengers on buses may experience not only fewer stops at intersections, but also less travel time along the main street, even though the dwell times at some bus stops have increased and the speeds on some segments have decreased.

Additional simulations have been performed to evaluate the influence of the dwell time variance. The new dwell times were generated from the normal distributions and the uniform distributions. The mean of each normal and uniform distribution was established as the corresponding dwell time obtained in the case study. Because the lower limits of the dwell times in this case study were established as the largest dwell times among all bus routes at the bus stops, the standard deviations of the normal distributions and the differences between the upper and lower bounds of the uniform distributions may not be substantial. The upper and lower bounds of the uniform distributions were mean $\pm$ 5 s and mean $\pm$ 10 s; 95% confidence intervals of the normal distributions were determined according to the upper and lower bounds of the uniform distributions, and the standard deviations of the normal distributions were set to the differences between the upper and lower bounds of the uniform distributions by 4, which were 2.5 and 5 s; thus, the possible minimums and maximums of the normal distributions and uniform distributions were similar. General vehicle green bands will not be influenced by the adjustments of the dwell times. The results are shown in Table 5.

Table 5. Results of the dwell time variance.

	Model V						
		Dwell times under the normal distribution		Dwell times under the uniform distribution			
	Original dwell times	Standard deviation 2.5 s	Standard deviation 5 s	Upper and lower bounds mean ± 5 s	Upper and lower bounds mean ± 10 s	Current	Model Y
Outbound delays (s)	60	77	113	79	119	166	209
Inbound delays (s)	68	74	98	83	110	212	158
Outbound stoppages (times)	0.02	0.28	0.87	0.32	0.96	2.83	2.49
Inbound stoppages (times)	0.16	0.22	0.57	0.39	0.76	2.37	1.93

Because the dwell times are no longer constant, the delays and stoppages have increased with an increase in the dwell time variances. However, compared with current plan and the plan generated from Model Y, the bus systems can continue to benefit from the proposed approach. Model V is more sensitive to the uniform distributions of the dwell times.

4. Conclusions

Considering the requirements of the bus priority strategies for urban arterials, an approach is proposed in this paper to generate green bands for bus systems and general vehicles. Based on MAXBAND, this approach considers their own characteristics of bus systems and general vehicles, analyses the relationships between bus systems and general vehicles for the same timing plan and adjusts the elements of bus systems, such as dwell time and speed, to construct a mixed-integer linear programming model. Coding in LINGO, a global optimal solution of this model can be obtained if it exists. The proposed approach was evaluated using a case study with VISSIM simulations.

The proposed approach can generate green bands for bus systems and general vehicles. Two different green bands – the bus green band and the general vehicle band – can share and exist in the same timing plan. The maximum bandwidths of bus systems and general vehicles are primarily influenced by the number and splits of signalised intersections along the main street, whereas the minimum bandwidths can be adjusted to satisfy the demands of bus systems and general vehicles.

Compared with general vehicles, the operations of bus systems can be more feasibly managed. Therefore, the characteristics of bus systems, such as dwell time and speed, can be adjusted to expand the feasible region of the mixed-integer linear programming model. In addition, the maximum and minimum bandwidths of bus systems and general vehicles can be determined. Therefore, the objective function of the model in the proposed approach is to obtain the minimum weighted outbound and inbound travel times for bus systems to guarantee the performance of bus systems, which differs from the objective functions of other bandwidth optimisation approaches. The proposed approach can generate more than one combination of bus speeds and dwell times for the same timing plan and global optimal solution. The proposed approach is significantly more flexible; we can select different combinations of bus speeds and dwell times to achieve multiple objectives.

The entire traffic system, which consists of bus systems and general vehicles, can benefit from the proposed approach. The results of the simulations show that the delays, the number of stoppages and the total travel times along the main street for an entire traffic system have significantly decreased, especially for bus systems. Although dwell times at some bus stops have increased, a significant improvement in the total travel times for bus systems is achieved according to the results of the simulations.

The proposed approach employs a mixed-integer linear programming model that can be solved by coding in LINGO. In practice, once the input parameters, such as the number of signalised intersections, splits, lower and upper limits of the cycle length and bandwidths, have been set in advance, the model can be solved by algorithms embedded in LINGO. LINGO generates two possible results. The first result is that the global solution exists and all decision variables, such as offsets, combinations of bus speeds and dwell times, and exact bandwidths, can be obtained. The second solution is that the global solution does not exist and all decision variables are not obtained. If the global solution does not exist, the relevant input parameters need to be adjusted. Generally, the number of signalised intersections is constant and cannot be adjusted. Then, the splits and lower and upper limits of the cycle length are determined according to the actual traffic flow ratios and the travel times for general vehicles are determined according to the traffic conditions and mechanisms; thus, adjustments of these input parameters are not recommended. The adjustment of bus speeds and dwell times with exclusive bus lines are feasible and bandwidths constraints are set to satisfy peak traffic demands; thus, the lower and upper limits of these input parameters can be adjusted to generate new values of these input parameters. The model with the updated input parameters can be solved by LINGO. If the adjustment of the bus speeds, dwell times and bandwidths constraints have no effect, all input parameters that can be adjusted should be adjusted. The global solution may be solved in this general situation. However, all input parameters have their own valid ranges. For example, an excessively large upper limit for the dwell time will increase the total bus travel time, whereas an excessively small lower limit for the dwell time cannot satisfy the demand of boarding and alighting passengers, and an excessively small lower limit for the bandwidth will have minimal influence on the bus progression. In some cases, no global solution exists for the valid ranges of all input parameters and the proposed approach may not be available.

In this paper, the bus travel time is a decision variable. Because the average value of the acceleration/deceleration of a bus is utilised, the bus travel time is composed of combinations of the average bus speeds and dwell times. The lower and upper limits of the average bus speed and the dwell time are proposed according the rules presented in this paper; the values of these variables are not constants but fall within reasonable regions. The feasible region of the model can be extended. Once the model is solved, the total bus travel time for a particular road segment can be obtained and different combinations of the average speeds and the dwell times can be selected to satisfy different demands. In practice, the model can be solved and a selected combination of the average speed and the dwell time at each road segment can be assigned to the bus drive in advance. A bus can be equipped with an on-board notification device that can display the required average speed and dwell time for each road segment, and the bus driver can follow the corresponding instructions. As previously mentioned, the average speed and dwell time can be controlled with the use of exclusive bus lanes. In the VISSIM simulations, the relevant settings, such as the desired speed decisions, the bus speed and the dwell time distributions embedded in VISSIM can be adjusted to assign different combinations of the average speeds and the dwell times to each road segment; relevant plans have been implemented in the simulations in this paper.

In this paper, we employ a simple strategy to construct constraints for the dwell time at a bus stop. Using a field survey, we set the maximum dwell time of all bus routes at a bus stop as the lower limit. In a future study, we will develop a strategy that can accurately represent the variation in dwell time. In addition, the proposed model should be promoted to handle bus systems without exclusive bus lanes and further efforts to analyse the relationships and interactions among the input parameters are recommended.

Acknowledgements

The authors appreciate the comprehensive data supplied by colleagues in the School of Transportation. The authors are also grateful to the anonymous referees for suggestions to improve the paper.

Funding

This research is supported by the National High Technology Research and Development Program of China [2014AA110303] and Scientific Research Foundation of Graduate School of Southeast University [3221004942].

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