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Integrated optimization of transit priority operation at isolated intersections: A person-capacity-based approach

Wanjing Ma [a,*], K. Larry Head [b], Yiheng Feng [b]

[a] Key Laboratory of Road and Traffic Engineering of the Ministry of Education, Tongji University, 4800 Cao'an Road, Shanghai, PR China
[b] Department of Systems and Industrial Engineering, The University of Arizona, 1127 E. James E. Rogers Way, P.O. Box 210020, Tucson, AZ 85721-0012, United States

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ABSTRACT

In this paper, a person-capacity-based optimization method for the integrated design of lane markings, exclusive bus lanes, and passive bus priority signal settings for isolated intersections is developed. Two traffic modes, passenger cars and buses, have been considered in a unified framework. Person capacity maximization has been used as an objective for the integrated optimization method. This problem has been formulated as a Binary Mixed Integer Linear Program (BMILP) that can be solved by a standard branch-and-bound routine. Variables including, allocation of lanes for different passenger car movements (e.g., left turn lanes or right turn lanes), exclusive bus lanes, and passive bus priority signal timings can be optimized simultaneously by the proposed model. A set of constraints have been set up to ensure feasibility and safety of the resulting optimal lane markings and signal settings. Numerical examples and simulation results have been provided to demonstrate the effectiveness of the proposed person-capacity-based optimization method. The results of extensive sensitivity analyses of the bus ratio, bus occupancy, and maximum degree of saturation of exclusive bus lanes have been presented to show the performance and applicable domain of the proposed model under different composition of inputs.

1. Introduction

Traffic congestion has long been one of the pressing issues in many cities. It is highly recommended by many researches and authorities that providing high level of service of public transit system would encourage more travelers to choose transit mode for their travels hence mitigate traffic congestion. Granting signal priority and designing exclusive bus lanes are two typical strategies which can be used to improve speed and reliability of public transit system. Exclusive bus lanes can be implemented with relatively low cost and short implementation time, and they are considered a cost-effective approach for providing a high-quality transit service (Deng and Nelson, 2011); in addition, they can effectively improve the reliability and increase the speed of buses by avoiding the need for them to share road space with congested urban traffic. Since the 1930s, when the idea of exclusive bus lanes was first introduced, several studies have specifically examined bus prioritization measures (Currie, 2006; Eichler and Daganzo, 2006; Fuhs and Obenberger, 2002; Hounsell and McDonald, 1988; Levinson et al., 2003; Mesbah et al., 2008; Song, 2000; Viegas and Lu, 2004). Although exclusive bus lanes effectively improve bus prioritization, a major potential limitation in their implementation is the reduction in road capacity for other types of vehicles, which results in increased levels of congestion at signalized intersections.

The prioritization problem can also be addressed by transit signal priority (TSP) strategies. TSP strategies are of three main types: passive priority strategy, active priority strategy, and real-time priority strategy (Balke et al., 2000). Passive

* Corresponding author. Tel.: +86 21 69584674; fax: +86 21 69589475.
E-mail addresses: mawanjing@tongji.edu.cn (W. Ma), klhead@email.arizona.edu (K.L. Head), yihengfeng@email.arizona.edu (Y. Feng).

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priority strategies operate continuously regardless of whether transit vehicles are present, and they do not require a transit detection/priority request generation system (Skabardonis, 2000). Active priority strategies prioritize a specific transit vehicle following detection or upon receiving a priority request from the vehicle/system (Furth and Muller, 2000; Yagar and Han, 1994). Adaptive/real-time TSP strategies provide priority while simultaneously trying to optimize signal timings under given performance criteria such as person delay, transit delay, vehicle delay, and/or a combination of these criteria (Baker et al., 2002; Chang et al., 1995; Christofa and Skabardonis, 2011; Furth et al., 2010; Ma et al., 2010; Mesbah et al., 2011; Mirchandani et al., 2001; Stevanovic et al., 2008).

On the one hand, in all transit signal priority strategies, it is assumed that the lane function (e.g., straight-ahead or left-turn) of both exclusive bus lanes and passenger car lanes are provided as exogenous inputs. On the other hand, these exclusive lane design methods did not address the interactions between lane assignments and signal timings. Moreover, the design of lane markings for the passenger lanes is also usually considered a prerequisite for calculating signal timings. The conventional approach is to design lane markings on a trial-and-error basis in which an initial set of lane markings is first assumed, and then, the signal settings are determined based on this lane configuration. After assessing the performance of different approaches at the signal-controlled intersection with optimal settings, the lane markings are revised (if necessary) based on the engineer's experience. The procedure is repeated until the performance of the intersection meets the requirement (Wong and Wong, 2003).

However, for complicated intersections, it is very difficult to determine an optimal set of lane markings for transit and traffic movements with corresponding transit priority. For example, consider one approach of a signal-controlled intersection with four traffic lanes, as shown in Fig. 1. If one lane is marked with an exclusive bus lane, the lanes available for passenger cars decrease, as a result of which the saturation flow of passenger car lanes decreases substantially and a longer cycle length is required to discharge the same level of traffic volume, which might induce longer delay of buses. Even if the exclusive bus lane is fixed (e.g., in a Bus Rapid Transit corridor), the lane markings are still very difficult to optimize. Therefore, the conventional isolated signal timing optimization and lane assignment methods may not always produce a truly optimal set of lane markings and signal timings for the intersection, especially if transit priority operation is considered.

Several previous attempts to combine the design of lane markings and the calculation of signal timing (Lam et al., 1997; Wong and Heydecker, 2011; Wong and Wong, 2003) optimized lane markings and signal timings simultaneously. However, they did not simultaneously address transit priority issues. If transit demands and exclusive bus lanes are also considered, the single mode traffic control problem transforms into a multi-mode traffic control problem that requires specific consideration of the features, level of priority, and performance of each mode. In one of our previous attempts to combine the design of lane markings and signal timings for transit priority operations (Ma and Yang, 2007), it was shown that the average delay of transit vehicles can be reduced with limited negative impact on general traffic at an intersection with an exclusive bus lane. However, only one approach was considered in this study. The problem will become more complex if the existence of an exclusive bus lane and all approaches at the entire intersection are taken into account. Moreover, it combines the objectives of transit priority and traffic operations into a single objective function by setting different weights, and the optimal solution obtained depends on the relative values of the weights specified.

Thus, this paper aims to formulate the intersection design problem with the consideration of transit priority operation into a mathematical program and includes the intersection's geometric layout, individual lane usages, exclusive bus lanes, and signal timings as design variables that can be optimized simultaneously to achieve higher intersection reserve person capacity. The reserve capacity maximization idea is a well-known concept. Based on the assumption that the traffic flows for the traffic movements in the intersection will increase in proportion to the demand matrix, the maximum reserve vehicle capacity is obtained by determining the largest common multiplier (Allsop, 1972; Gallivan and Heydecker, 1988; Wong and Heydecker, 2011; Wong and Wong, 2003; Wong and Yang, 1997). However, in previous studies, only one mode, namely, passenger car, was considered, and neither the design of exclusive bus lanes nor bus signal priority was addressed. The proposed person capacity maximization method considers all of these factors. Because the final objective for the design of intersections is to accommodate more passengers rather than vehicles, person capacity maximization is employed as the objective of this integrated optimization model. The person capacity is defined as the total number of people that can be accommodated by the intersection. It can be calculated based on the capacity of passenger cars and buses multiplied by their

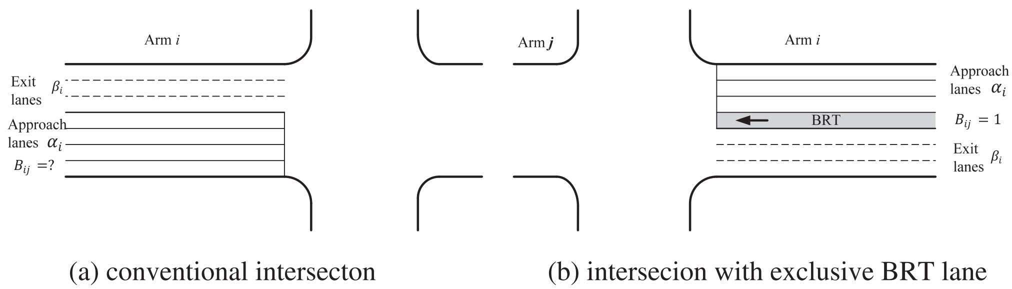


Fig. 1. Numbering convention for destination arms in an example junction.

occupancy. This problem is formulated as a Binary Mixed Integer Linear Program (BMILP) that can be solved by any standard branch-and-bound routine.

The remainder of this paper is organized as follows. Section 2 summarizes the notations and parameters used in the optimization framework. Section 3 presents the necessary governing constraints confining the feasible solution region for all of the control variables. Section 4 describes the optimization problem for maximizing the intersection person capacity. Section 5 demonstrates two numerical examples and corresponding sensitivity analysis. Section 6 presents the conclusions and discusses future works.

2. General notation and terminology

Consider the signalized intersection shown in Fig. 1 to facilitate the model presentation; Table 1 lists the parameters used hereafter.

3. Optimization model

3.1. Objective function

Because the ultimate objective of the design of intersections is to accommodate more people rather than cars, person capacity maximization is employed as the objective of the integrated optimization model. We employed the concept of reserve capacity to formulate a linear model. Conventionally, the concept of reserve capacity has been applied to individual signal-controlled intersections, and is measured by the greatest common multiplier of existing flows that can be accommodated subject to approach capacity constraints, cycle time and minimum green constraints and others. Based on the common used assumption that the traffic flows for the turning movements in the intersection would increase in proportion to the demand matrix (Wong and Heydecker, 2011), the problem becomes one of determining the largest common multiplier, μ and μ^b, without violating any of the constraints specified in the previous section. μ_{ij}^b is introduced because the factors that impact the demand and degree of saturation for exclusive bus lanes and passenger lanes are different. Note that, besides μ^b, the difference in bus volume and traffic volume can also be regulated by giving different maximum degree of saturation of bus lanes and passenger car lanes as illustrated in constraints part, respectively. Then, the intersection capacity maximization problem transforms into one of maximizing the person capacity, and it can be effectively formulated as a BMILP as shown below:

$$\max \mu \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} Q_{ij}''^{v} + \mu^b \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} Q_{ij}''^{b} \tag{1}$$

3.2. Decision variables

As summarized in Sections 2 and 3, the set of control variables can now be specified as follows:

Geometric layout

Δ_{ijk}	permission of movement j on lane k in arm i
b_{ijk}	permission of bus movement j on exclusive lane k in arm i
B_{ij}	permission of exclusive lanes for bus movement j in arm i

Intersection signal timing

q_{ijk}	assigned passenger car flow
q_{ijk}^b	assigned bus flow
ξ	reciprocal of cycle length
μ	common multiplier
θ_{ij}	start of green signal for movement j in arm i
ϕ_{ij}	green signal time ratio for movement j in arm i
Θ_{ik}	start of green signal on lane k in arm i
Φ_{ik}	green signal time ratio on lane k in arm i
$\Omega_{ij,lm}$	order of signal phase for a pair of mutually incompatible traffic movements (i, j) and (l, m)

3.3. Constraints

3.3.1. Lane assignment constraints

(1) Minimum number of permitted movements on traffic lanes: each traffic lane should permit at least one turning or through movement of passenger cars or buses, which can be specified as

Table 1
Symbols and parameters.

b_{ijk}	b_{ijk} = 1 means that bus movement is permitted from arm i to arm j on exclusive bus lane k in arm i, b_{ijk} = 0 means that bus movement is not permitted from arm i to arm j on exclusive bus lane k in arm i
B_{ij}	B_{ij} = 1 if an exclusive bus lane exists for buses from arm i to arm j, B_{ij} = 0 if an exclusive bus lane does not exist for buses for arm i to arm j
C	cycle length (s)
$C_{\max}$	maximum cycle length (s)
$C_{\min}$	minimum cycle length (s)
e	extra effective green signal time derived from the difference between actual and effective green signal times (s)
g_{ij}	minimum duration of green signal for movement from arm i to arm j (s)
H_i	set of traffic movements that should use the exit lanes of arm i
i	numbering for arms
j	numbering for arms defined locally with respect to arm i along clockwise direction
k	numbering for lanes; the numbering convention is lane k is located to the left of lane k + 1, as shown in Fig. 1
M	arbitrary large positive constant
N_T	number of arms
O^v	occupancy of car
O^b	occupancy of bus
q_{ijk}	assigned passenger car flow of movement from arm i to arm j on lane k in arm i (veh/h)
q^b_{ijk}	assigned bus flow of movement from arm i to arm j on lane k in arm i (veh/h)
Q'^v_{ij}, Q''^v_{ij}	total traffic volume and person volume of movement from arm i to arm j excluding bus flow on exclusive bus lane (pcu/h, person/h)
Q'^b_{ij}, Q''^b_{ij}	total bus volume and person volume of movement from arm i to arm j on exclusive bus lane (pcu/h, person/h)
Q^b_{ij}	bus demand flow from arm i to arm j (veh/h)
Q^v_{ij}	passenger car demand flow from arm i to arm j (pcu/h)
s_{ik}	saturation flow rate of lane k in arm i (veh/h)
$x_{\max}$	upper boundary of the saturation degree of passenger lanes
$x^b_{\max}$	upper boundary of the saturation degree of exclusive bus lanes
y_{ik}	flow ratio of lane k in arm i
y^b_{ik}	flow ratio of exclusive bus lane k in arm i
ξ	reciprocal of cycle length (1/s)
β_{ij}	number of exit lanes of the corresponding exit arm of movement from arm i to arm j
α_i	number of approach lanes in arm i
β_i	number of exit lanes in arm i
$\varDelta_{ijk}$	$\varDelta_{ijk}$ = 1 means that movement is permitted from arm i to arm j on lane k in arm i, $\varDelta_{ijk}$ = 0 means that movement is not permitted from arm i to arm j on lane k in arm i
θ_{ij}	start of green signal for movement from arm i to arm j (fraction of cycle length)
ϕ_{ij}	green signal time ratio for movement from arm i to arm j
$\varTheta_{ik}$	start of green signal on lane k in arm i (fraction of cycle length)
$\varPhi_{ik}$	green signal time ratio on lane k in arm i
$\varOmega_{ij,lm}$	order of signal phase for a pair of mutually incompatible traffic movements (i, j) and (l, m), $\varOmega_{ij,lm}$ = 0 if the start of green signal phase (l, m) follows that of signal phase (i, j), and $\varOmega_{ij,lm}$ = 1 vice versa
$\omega_{ij,lm}$	minimum clearance time for a pair of mutually incompatible traffic movements (s)
ε	conversion factors between pcu and bus
μ	common passenger car flow multiplier
μ^b	multiplier of bus flow for each bus movement from arm i to arm j operating on exclusive bus lane
$\varPsi_s$	set of mutually incompatible traffic movements pair

$$\sum_{j=1}^{N_T-1}(\varDelta_{ijk}+b_{ijk}) \geqslant 1, \quad \forall\, i=1,\ldots,N_T; \quad k=1,\ldots,\alpha_i \tag{2}$$

(2) Maximum permitted movements at exit: due to safety and operational considerations, for each turning movement from arm i, the number of exit lanes in the corresponding exit arm should always be at least as many as the total number of lanes assigned to permit such a movement:

$$\beta_{ij} \geqslant \sum_{k=1}^{\alpha_i}(\varDelta_{ijk}+b_{ijk}), \quad \forall\, i=1,\ldots,N_T; \quad j=1,\ldots,N_T-1 \tag{3}$$

(3) Prohibited movement: if a lane is the exclusive bus lane, passenger car movement should be prohibited in this lane, which can be expressed as:

$$M(1-b_{ijk}) \geqslant q_{ijk} \geqslant 0 \quad \forall\, i=1,\ldots,N_T; \quad j=1,\ldots,N_T-1; \quad k=1,\ldots,\alpha_i \tag{4}$$

Note that, the exclusive bus lanes cannot be used for turning vehicles. In practice of many cities, especially for the BRT system, turning vehicles usually are forbidden from the exclusive bus lane. Typically there are two ways to manage turning vehicles. One is using special traffic signal (protected phase for left turns if bus lane located in the center of the road or special signal for right turns if bus lane located in the most right lane of the road). The other one is designing weaving sections at upstream of intersection in which right turning vehicles can change into most right lane and the second right lane turning to be exclusive bus lane. In this paper, if bus lane can be used by turning vehicles, it will be considered as a general purpose lane instead of exclusive bus lane.

(4) Right-of-way constraint: if $\Delta_{ijk} = 0$, the movement j on lane k in arm i does not have the right-of-way so the assigned lane flow q_{ijk} will be 0. This condition can be realized by the following inequality:

$$M\Delta_{ijk} \geqslant q_{ijk} \geqslant 0, \quad \forall\ i = 1, \ldots, N_T; \quad j = 1, \ldots, N_T - 1; \quad k = 1, \ldots, \alpha_i \tag{5}$$

(5) Conflict avoidance within an arm: for any two adjacent traffic lanes, k (left-hand) and $k + 1$ (right-hand) lanes from arm i, if the traffic movement of turning j is permitted on lane $k + 1$, then traffic movements of all other turns, $j + 1, \ldots, N_T - 1$, should be prohibited on lane k to forbid potential internal-cross conflicts within an arm. This can be specified by Eqs. (6)–(9).

$$1 - \Delta_{ij(k+1)} \geqslant \Delta_{imk}, \forall\ i = 1, \ldots, N_T; \quad j = 1, \ldots, N_T - 2; \quad m = j + 1, \ldots, N_T - 1; \quad k = 1, \ldots, \alpha_i - 1 \tag{6}$$

$$1 - b_{ij(k+1)} \geqslant b_{imk}, \forall\ i = 1, \ldots, N_T; \quad j = 1, \ldots, N_T - 2; \quad m = j + 1, \ldots, N_T - 1; \quad k = 1, \ldots, \alpha_i - 1 \tag{7}$$

$$1 - b_{ij(k+1)} \geqslant \Delta_{imk}, \forall\ i = 1, \ldots, N_T; \quad j = 1, \ldots, N_T - 2; \quad m = j + 1, \ldots, N_T - 1; \quad k = 1, \ldots, \alpha_i - 1 \tag{8}$$

$$1 - \Delta_{ij(k+1)} \geqslant b_{imk}, \forall\ i = 1, \ldots, N_T; \quad j = 1, \ldots, N_T - 2; \quad m = j + 1, \ldots, N_T - 1; \quad k = 1, \ldots, \alpha_i - 1 \tag{9}$$

3.3.2. Assigned flow constraints

The total traffic flow Q_{ij} and assigned flows on different lanes q_{ijk} should obey the following set of constraints.

(1) Flow conservation for two modes: the sum of flows from origin i to destination j should be equal to the total demand for that movement, as given by the following equations:

$$Q'^{v}_{ij} = Q^{v}_{ij} + (1 - B_{ij})\varepsilon Q^{b}_{ij} \tag{10}$$

$$Q'^{b}_{ij} = B_{ij} Q^{b}_{ij} \tag{11}$$

$$Q''^{v}_{ij} = Q^{v}_{ij} O^{v} + (1 - B_{ij}) Q^{b}_{ij} O^{b} \tag{12}$$

$$Q''^{b}_{ij} = B_{ij} Q^{b}_{ij} O^{b} \tag{13}$$

If an exclusive bus lane exists for bus movement from i to j, then $B_{ij} = 1$. All buses from approach i to approach j will operate on the exclusive bus lane, and therefore, the value of the second term on the right-hand side of Eqs. (10) and (11) becomes zero.

(2) Flow conservation for each movement: the sum of the assigned flows on different lanes should be equal to the increased demand for that movement, as given by the following equations:

$$\mu Q'^{v}_{ij} = \sum_{k=1}^{\alpha_i} q_{ijk}, \quad \forall\ i = 1, \ldots, N_T; \quad j = 1, \ldots, N_T - 1 \tag{14}$$

$$\mu^{b} Q'^{b}_{ij} = \sum_{k=1}^{\alpha_i} q^{b}_{ijk} \quad \forall\ i = 1, \ldots, N_T; \quad j = 1, \ldots, N_T - 1 \tag{15}$$

In Eq. (14), if no exclusive bus lane exists for movement from arm i to arm j, the bus flow will be converted to PCU by multiplying a conversion factor ε and sharing a common multiplier μ. In Eq. (15), if an exclusive bus lane exists for movement from arm i to arm j, all exclusive bus lanes use a common multiplier μ^{b} because the factors that impact traffic demands of passenger car lanes and exclusive bus lanes may be different.

3.3.3. Flow ratio constraints

The flow ratio of a lane is the ratio of the flow rate to the saturation flow rate, as given by the following equation:

$$y_{ik} = \frac{\sum_{j=1}^{N_T - 1} q_{ijk}}{s_{ik}}, \quad \forall\ i = 1, \ldots, N_T; \quad k = 1, \ldots, \alpha_i \tag{16}$$

The flow ratio of an exclusive bus lane can be calculated be the following equation:

$$y^{b}_{ik} = \frac{\varepsilon \sum_{j=1}^{N_T - 1} q^{b}_{ijk}}{s_{ik}}, \quad \forall\ i = 1, \ldots, N_T; \quad k = 1, \ldots, \alpha_i \tag{17}$$

For all approach lanes, including both passenger car lanes and exclusive bus lanes, it is required that the flow ratios must be identical for a pair of adjacent lanes that have a common lane marking. This constraint can be specified by the following equations:

$$M(2-\Delta_{ijk}-\Delta_{ij(k+1)}) \geqslant y_{i(k+1)}-y_{ik} \geqslant -M(2-\Delta_{ijk}-\Delta_{ij(k+1)}),\quad \forall\, i=1,\ldots,N_T;\ \ j=1,\ldots,N_T-1;\ \ k=1,\ldots,\alpha_i \tag{18}$$

$$M(2-b_{ijk}-b_{ij(k+1)}) \geqslant y^b_{i(k+1)}-y^b_{ik} \geqslant -M(2-b_{ijk}-b_{ij(k+1)}),\quad \forall\, i=1,\ldots,N_T;\ \ j=1,\ldots,N_T-1;\ \ k=1,\ldots,\alpha_i-1 \tag{19}$$

In this model framework, we allow more than one adjacent exclusive lanes to include the extreme cases in which bus volume is very high. The exact number of exclusive bus lanes is decided by the model itself.

3.3.4. Degree of saturation constraints

(1) Degree of saturation of passenger lane: In order to ensure the performance of the intersection, the degree of saturation of each traffic lane should be no greater than the maximum acceptable upper boundary.

$$x_{max}(\Phi_{ik}+e\xi) \geqslant y_{ik},\quad \forall\ i=1,\ldots,N_T;\quad k=1,\ldots,\alpha_i \tag{20}$$

(2) Degree of saturation of exclusive bus lane: In order to ensure that buses have higher priority than passenger cars, the degree of saturation of every exclusive bus lane should be no greater than that of passenger lanes with the same origin and destination.

$$x^b_{max}(\Phi_{ik}+e\xi) \geqslant y^b_{ik},\quad \forall\ i=1,\ldots,N_T;\quad k=1,\ldots,\alpha_i \tag{21}$$

3.3.5. Signal timing constraints

(1) Cycle length, start of green signal, order of signal displays, duration of green signal time, and clearance time constraints: these constraints have been discussed in previous study (Wong and Wong, 2003) as shown below:

$$\frac{1}{C_{\min}} \geqslant \xi \geqslant \frac{1}{C_{\max}} \tag{22}$$

$$1 \geqslant \theta_{ij} \geqslant 0,\quad \forall\ i=1,\ldots,N_T;\quad j=1,\ldots,N_T-1 \tag{23}$$

$$1 \geqslant \phi_{ij} \geqslant \xi g_{ij},\quad \forall\ i=1,\ldots,N_T;\quad j=1,\ldots,N_T-1 \tag{24}$$

$$\Omega_{ij,lm}+\Omega_{lm,ij}=1,\quad \forall\ (i,j),(l,m)\in\Psi_s \tag{25}$$

$$\theta_{lm}+\Omega_{ij,lm}+M(2-\Delta_{ijk}-\Delta_{lmn}) \geqslant \theta_{ij}+\phi_{ij}+\xi\omega_{ij,lm},\quad \forall\ (i,j),(l,m)\in\Psi_s \tag{26}$$

(2) Lane signal timing: if a lane is shared by more than one movement, these movements must receive identical signal indications to avoid ambiguity. Considering lane k from arm i, if a movement j is permitted in this lane, then the following two constraint sets can be established to fulfill the above condition.

$$M(1-\Delta_{ijk}) \geqslant \Theta_{ik}-\theta_{ij} \geqslant -M(1-\Delta_{ijk}),\forall\ i=1,\ldots,N_T;\quad j=1,\ldots,N_T-1;\quad k=1,\ldots,\alpha_i \tag{27}$$

$$M(1-\Delta_{ijk}) \geqslant \Phi_{ik}-\phi_{ij} \geqslant -M(1-\Delta_{ijk}),\forall\ i=1,\ldots,N_T;\quad j=1,\ldots,N_T-1;\quad k=1,\ldots,\alpha_i \tag{28}$$

$$M(1-b_{ijk}) \geqslant \Theta_{ik}-\theta_{ij} \geqslant -M(1-b_{ijk}),\forall\ i=1,\ldots,N_T;\quad j=1,\ldots,N_T-1;\quad k=1,\ldots,\alpha_i \tag{29}$$

$$M(1-b_{ijk}) \geqslant \Phi_{ik}-\phi_{ij} \geqslant -M(1-b_{ijk}),\forall\ i=1,\ldots,N_T;\quad j=1,\ldots,N_T-1;\quad k=1,\ldots,\alpha_i \tag{30}$$

4. Numerical examples and sensitivity analysis

4.1. Numerical examples

To illustrate the applicability of the proposed model, the Wuyingshan intersection of Beiyuan Road, Jinan city, is considered as an example through two numerical tests. The basic layout of the intersection is shown in Fig. 2. Two cases are studied for this intersection:

- *Case 1:* An intersection without a constraint on exclusive bus lanes. In other words, all arms can be designed with an exclusive bus lane, and all lane markings and signal timings need to be determined.
- *Case 2:* A intersection with two exclusive bus lanes (lane function is straight-ahead) on two of the four arms. As in most intersections with BRT systems, none of the rest arms needs exclusive bus lanes, and only the rest lane markings and signal timings need to be determined.

The purpose of studying these two cases is to evaluate the applicability performance of the proposed Person Capacity (PC)-based model for normal intersections and intersections in a BRT system in which the existence of an exclusive bus lane is previously determined before the optimization of lane markings and signal timings. The optimized results obtained from the proposed model will be compared with those of the Vehicle Capacity Maximization (VC) model by changing the objective of the model to be vehicle reserve capacity as done by (Wong and Wong, 2003). To be specific, in the VC model, $O^b = \varepsilon O^v$ and $\mu^b = \mu$. Therefore the buses are treated as same (multiply a PCU conversion factor) as cars and the objective function, Eq. (1) can be represented as:

$$\max\ RC = \mu \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} Q_{ij}^{\prime\prime v} + \mu^b \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} Q_{ij}^{\prime\prime b} = \mu \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} (Q_{ij}^{\prime\prime v} + Q_{ij}^{\prime\prime b}) \tag{31}$$

where RC is used to represent the objective function defined in Eq. (1). Putting Eqs. (12) and (13) into Eq. (31), one can get:

$$\begin{aligned} RC &= \mu \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} (Q_{ij}^{\prime\prime v} + Q_{ij}^{\prime\prime b}) = \mu \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} [Q_{ij}^v O^v + (1 - B_{ij}) Q_{ij}^b O^b + B_{ij} Q_{ij}^b O^b] = \mu \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} [Q_{ij}^v O^v + Q_{ij}^b O^b] \\ &= \mu \sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} [Q_{ij}^v + \varepsilon Q_{ij}^b] O^v \end{aligned} \tag{32}$$

In Eq. (32), $Q_{ij}^v + \varepsilon Q_{ij}^b$ is total traffic demand in PCU. Moreover, the term $\sum_{i=1}^{N_T} \sum_{j=1}^{N_T-1} [Q_{ij}^v + \varepsilon Q_{ij}^b] O^v$ is a constant once traffic demand is given. Therefore, the objective function (Eq. (32)) can be simplified as:

$$\max\ RC = \mu \tag{33}$$

The Eq. (33) is the standard objective function of maximum vehicle reserve capacity problem. All the constraints in VC model are same with those in PC model.

The same inputs are used for the two models to obtain a fair comparison results.

For each case, the number of exit lanes on each arm is 4, and it is equal to the number of approaching lanes on that arm, except for Arms 2 and 4 in Case 2 where one through lane is dedicated to buses (BRT lane). The minimum and maximum cycle length are set to be 60 and 120 s, respectively, minimum green signal time is set to be 5 s, effective green signal time is always equal to the actual green signal time in the calculation, clearance time is set to be 4 s for a pair of mutually incompatible traffic movements, and saturation flow for each lane is set to be 1800 pcu/h. The upper boundaries of the degree of saturation for vehicle lanes and exclusive bus lanes are 0.9. The occupancy of cars and buses is 3 and 40, respectively. The traffic demands are listed in Table 2. The program was coded in C++ and tested on an AMD Turion 2.2 GHz processor and 2.0G RAM, running under Windows. The computing times for all two cases take only less than 10 s (9.2 s for Case 1 and 4.3 s for Case 2).

4.1.1. Model results

Table 3 summarizes the optimization results of the two case studies of the PC model and the comparison with VC model. The parameters include the optimized reciprocal of cycle lengths, maximized common flow multipliers, and corresponding optimized reserve vehicle capacities and person capacities. It can be observed that the person capacity of the intersection can be improved by more than 44% under the proposed PC model for Case 1 and 22% for Case 2 compared with VC model. This

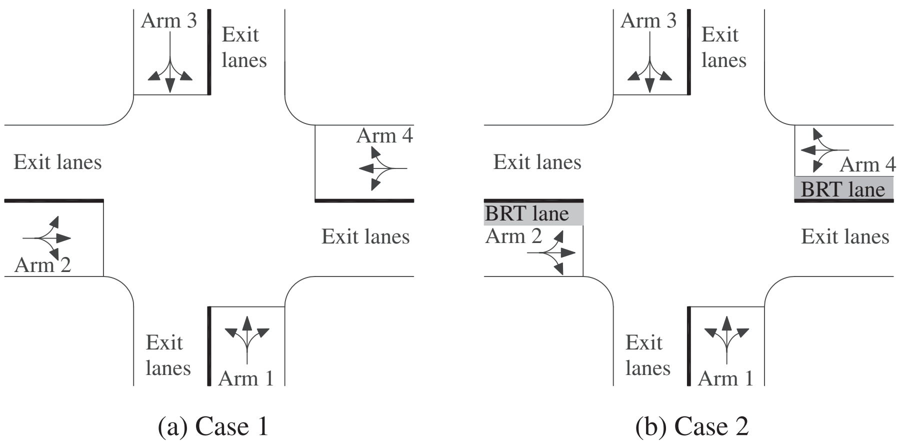


Fig. 2. Layout of a four-arm intersection for the examples.

means that more persons can be served by the intersection while restricting the degrees of saturation of all approaching traffic lanes to less than 90% under the PC model in Cases 1 and 2. In contrast, in the PC model, the vehicle capacity is decreased by 8.7% for Case 1 and 10.8% for Case 2 compared with VC model. Note that in Case 2, with the pre-determined exclusive lanes, the PC model can also improve the person capacity significantly. This confirms that the PC model increases the capacity of buses not only by optimizing exclusive bus lane existence variables (B_{ij}) but also by integrated optimizing lane markings and signal timings. The results of lane markings (Fig. 3) and signal timings (Fig. 4) further reveal how the person capacity increases.

Fig. 3 shows the results of the optimal lane configurations for the two cases under the PC and VC models, respectively. The lane markings are optimized efficiently by the proposed model for both cases. Compared with Case 1 in the VC model (Fig. 3a), the PC model generates one exclusive bus lane at each of the four arms (Fig. 3b). In contrast, in Case 2 (Fig. 3d), the PC model does not add new exclusive bus lanes, compared with the results of the VC model (Fig. 3c) because they are predetermined. In Case 1, the number of lanes for left turns increases from one to two, whereas the number of lanes for straight-ahead vehicles decreases from three to one at all arms as the control model changes from VC (Fig. 3a) to PC (Fig. 3b). Similarly, in Case 2 under PC the model, one additional lane is allocated to left turns at arms with exclusive lanes whereas the optimal lane markings for arms without exclusive bus lanes is kept the same compared with the results of the VC model. This can be explained by the fact that the green signal split for straight-ahead movements as well as that for exclusive bus lanes in the same phases must be extended with the reduction in the number of lanes allocated. Therefore, the person capacity of the intersection is increased accordingly.

Fig. 4 shows the optimal signal timings for the two case studies under the PC and the VC model. The bold lines represent green time and the thin lines represent inter-green time and red time. It can be seen that a complicated cycle structure is formed automatically during optimization. Optimized signal stages and groups are determined based on the input data. For all the scenarios shown in Fig. 3a–d, the optimal cycle lengths are at most 120 s, but the splits for each movement are different. Compared with the VC model, the PC model allocates more green signal time to signal groups that serve higher bus demand in both cases. This means that the proposed model can either improve the reserve person capacity or decrease the delay of buses with the given traffic demand.

From the calculations, the optimized lane configuration and signal timings can be automatically deduced from the optimization results and the existence of exclusive bus lanes can be determined based on the performance of the entire intersection. This is particularly useful for a large intersection with many lanes, where the design of lane markings, exclusive bus lanes, and transit priority signal timings for optimal operation of traffic is not trivial and it is complicated.

4.1.2. Simulation results

To further clarify the performance of the proposed model in terms of traffic delay, in this section, the optimized signal plans and lane markings obtained from the proposed PC model are compared with those obtained from the VC model using VISSIM as an unbiased evaluator. To overcome the stochastic nature of a microscopic simulation system, an average of 10 simulation runs was used. From the results listed in Table 4, the following conclusions can be drawn:

- According to pared *T*-test results, the performance of PC model and VC model are significantly different in terms of average car delay (*P*-value = 0.00), average bus delay (*P*-value = 0.00), average person delay (*P*-value = 0.00), and maximum throughput (*P*-value = 0.00).
- Compared with the VC model, the PC model can significantly decrease the average bus delay in both cases. The average bus delay was decreased by 33.1% (12.96 s/bus) and 21.3% (7.78 s/bus) in Cases 1 and 2, respectively. However, the VC model decreases the average car delay. Compared with the PC model, the average car delay was decreased by 17.7% (9.89 s/car) and 13.1% (7.07 s/car) in Cases 1 and 2, respectively.
- The PC model outperforms the VC model in terms of the average person delay. The average person delay was decreased by 4.5% (1.82 s/person) and 6.3% (2.57 s/person) in Cases 1 and 2, respectively.
- The maximum throughputs of the intersection under different cases are given in the last column of Table 4. The results are very similar with the results of Table 3. The VC model outperform PC model in terms of vehicle capacity while PC model can improve the person capacity significantly.

Table 2
Traffic demand for example intersection.

Car demand in pcu/h (bus demand in veh/h)	To arm			
From arm	1	2	3	4
1	–	172	550(50)	52
2	170	–	183	675(100)
3	656(40)	171	–	152
4	168	635(105)	105	–

Table 3
Summary of optimization results.

Case	Model	Degree of saturation		μ	μ^b	ξ (1/s)	Person capacity (per/h)	Vehicle capacity (pcu/h)
		Car	Bus					
Case1	VC	0.9	0.9	1.417	1.417	1/120	36,589	6064
	PC	**0.9**	**0.9**	**0.762**	**3.001**	**1/120**	**52,697**	**4582**
Case2	VC	0.9	0.9	1.319	2.481	1/120	40,730	6118
	PC	**0.9**	**0.9**	**1.001**	**3.939**	**1/120**	**51,985**	**5485**

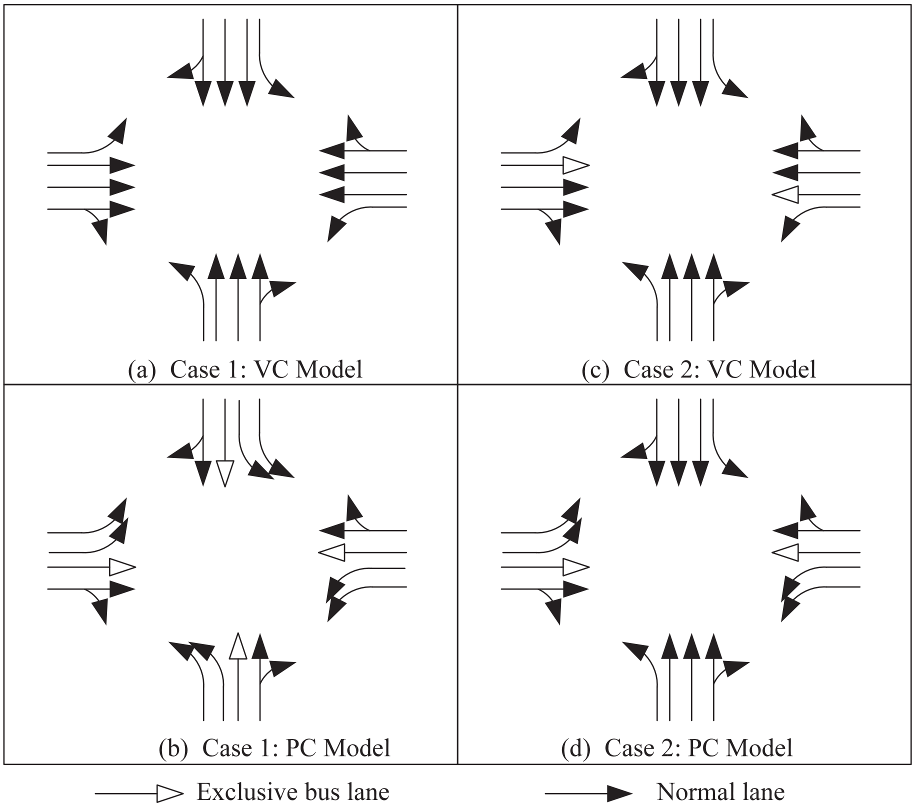


Fig. 3. Optimal lane markings for the two cases.

4.2. Sensitivity analysis

As stated previously, several critical factors, including the bus ratio (i.e., ratio of bus flow in total traffic demand), occupancy of buses and passenger cars, as well as maximum degree of saturation of passenger lanes and exclusive bus lanes will affect the optimization results of the proposed person-capacity-based model. As a result, sensitivity analysis on these parameters are conducted to provide operational guidelines for decision makers in better designing lane markings and signal timings for transit priority operation at signalized intersections. We consider the four arms intersection shown in Fig. 2a with all parameters used in the previous section except that the total traffic demand for each arm is assumed to be 1000 pcu/h and the fraction of left and right turns is set to be 0.3 and 0.1, respectively. In order to review the performance of the proposed model under different situation, following parameters are defined:

$$Person\ Capacity_{ratio} = \frac{Person\ Capacity_{PC} - Person\ Capacity_{VC}}{Person\ Capacity_{VC}}$$

$$Vehicle\ Capacity_{ratio} = \frac{Vehicle\ Capacity_{PC} - Vehicle\ Capacity_{VC}}{Vehicle\ Capacity_{VC}}$$

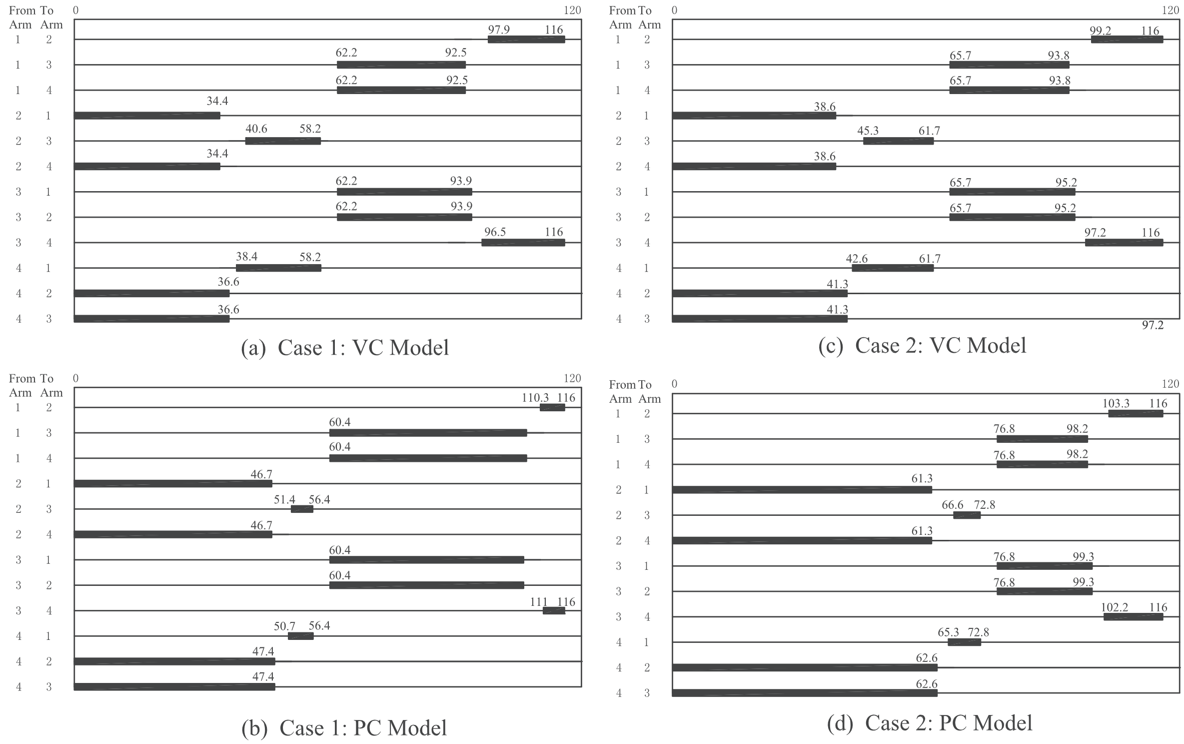


Fig. 4. Optimal signal settings for the two cases.

Table 4
Delay evaluation based on simulation.

	Movements	Average car delay (s/veh)	Average bus delay (s/veh)	Average person delay (s/person)	Maximum throughput (veh/h/per/h)
Case1 PC	WB	55.54	27.34	36.97	4589/52,616
	EB	54.52	26.45	37.16	
	NB	56.11	25.18	40.07	
	SB	57.09	24.09	43.72	
	Average	55.78	26.23	38.90	
Case1 VC	WB	43.13	39.32	40.62	6073/36,533
	EB	41.05	37.83	39.05	
	NB	44.34	40.03	42.11	
	SB	42.97	41.24	42.27	
	Average	45.89	39.19	40.72	
Case2 PC	WB	45.34	18.25	27.51	5493/51,905
	EB	45.79	16.48	27.66	
	NB	55.14	51.74	53.38	
	SB	60.27	57.70	59.23	
	Average	53.78	28.68	38.45	
Case2 VC	WB	42.52	34.78	39.21	6127/41,667
	EB	40.62	32.35	38.27	
	NB	47.08	42.22	44.55	
	SB	45.70	43.92	44.98	
	Average	46.71	36.46	41.02	

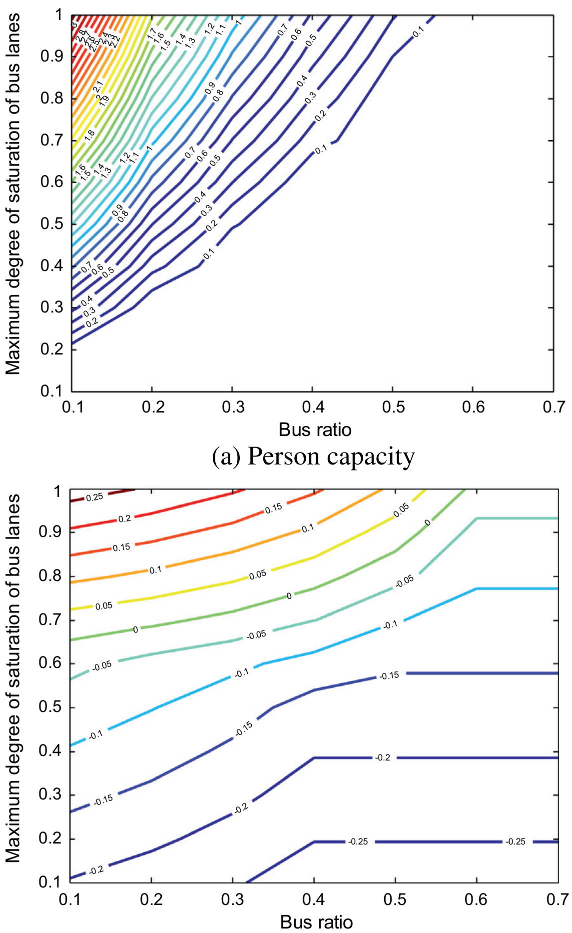
where *Person Capacity*$_{PC}$ and *Person Capacity*$_{VC}$ is the person capacity under PC control and VC control, respectively. *Vehicle Capacity*$_{PC}$ and *Vehicle Capacity*$_{VC}$ is the vehicle capacity under PC control and VC control, respectively. *Person Capacity*$_{ratio}$ and

Vehicle Capacity$_{ratio}$ are the indexes indicating the person capacity difference and vehicle capacity difference of PC model and VC model, respectively.

Fig. 5 shows the impacts of the maximum degree of saturation of bus lanes and bus ratios. Contour lines show the *Person Capacity*$_{ratio}$ and *Vehicle Capacity*$_{ratio}$. The person capacity under the PC model is higher than that under the VC model because all ratios are larger than 0. The highest ratio of person capacity appears at lower bus ratios and in the area with higher maximum degree of saturation of bus lanes (top-left corner in Fig. 5a). This is because the lower the bus ratio, the higher is the incremental person capacity that can be obtained by allocating exclusive bus lanes with higher maximum degree of saturation. It should be noted that the bus volume is not actually very low because the demand is 50 veh/h given a bus ratio of 0.1, and most real-world intersections belong to this category. In contrast, the PC model shows nearly no benefit at the bottom-right corner (Fig. 5a) compared with the VC model. In this area, the permitted maximum degree of saturation is lower and the bus ratio is higher than those in the top-left corner. The lower maximum degree of saturation of bus lanes limits the benefit of the PC model, and all lanes should be permitted the usage of buses under a higher bus ratio situation, which leads to the PC and VC models giving similar results.

Fig. 5b shows the impact of the PC model on the vehicle capacity. Compared with the bus ratio, the impact of the PC model on the vehicle capacity is more sensitive to the maximum degree of saturation of bus lanes because higher maximum degree of saturation of bus lanes could lead to higher capacity of bus lanes and ultimately, an increase in vehicle capacity. It should be noted that vehicle capacity under the PC model could be larger than that under the VC model owing to capacity increases derived from exclusive bus lanes.

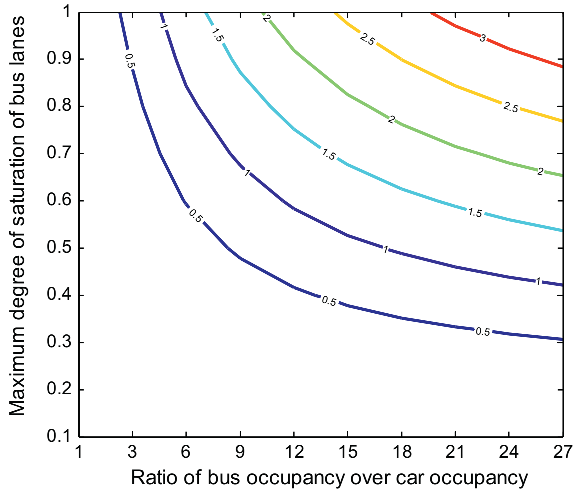
Figs. 6 and 7 shows the impacts of bus occupancy under different maximum degrees of saturation of bus lanes and bus ratios. The contour lines in Fig. 6 show *Person Capacity*$_{ratio}$ and *Vehicle Capacity*$_{ratio}$. Bus occupancy significantly affects the person capacity. With an increase in bus occupancy, the person capacity benefits obtained by the PC model increase significantly by more than 3 times compared to that under the VC model, especially under low bus ratio. It should be noted that



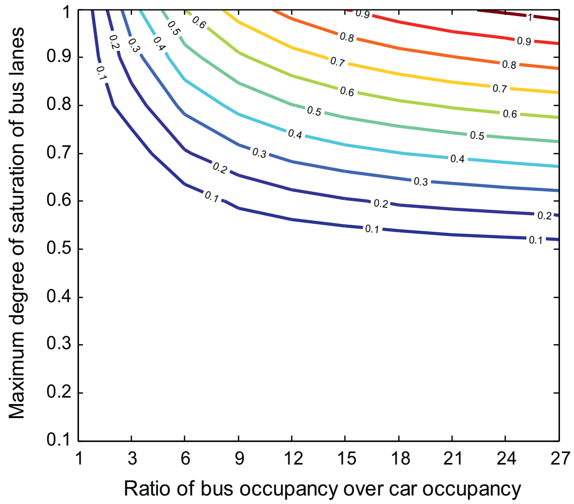
(a) Person capacity

(b) Vehicle capacity

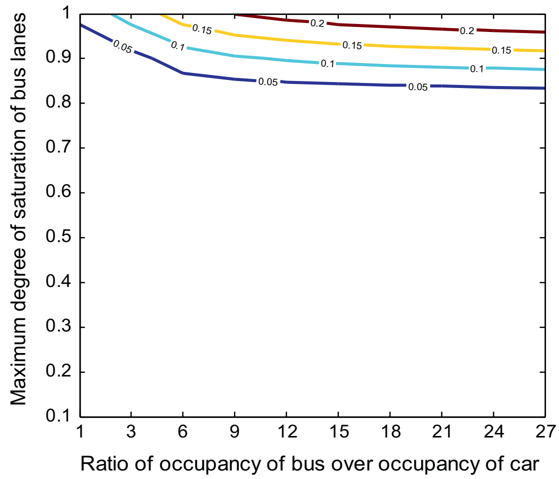
Fig. 5. Impacts of maximum degree of saturation of bus lanes and bus ratio.



(a) Person capacity (bus ratio = 0.1)



(b) Person capacity (bus ratio = 0.3)



(c) Person capacity (bus ratio = 0.5)

Fig. 6. Impacts of maximum degree of saturation of bus lanes and bus occupancy on person capacity.

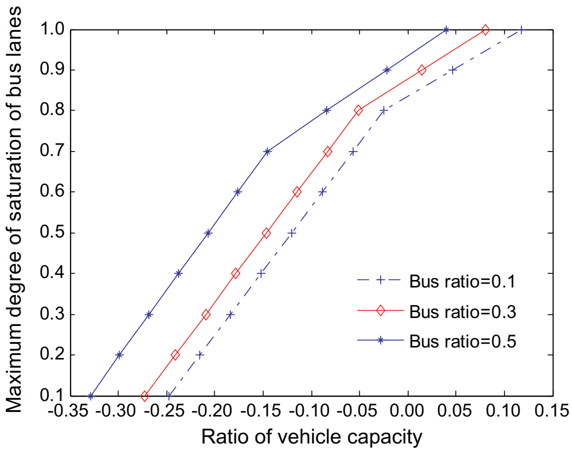


Fig. 7. Impacts of maximum degree of saturation of bus lanes and bus occupancy on vehicle capacity.

the bus volume is not very low because it is 50 veh/h at a bus ratio of 0.1. Hence, Fig. 6a and b represent the most general situations. In contrast, the bus occupancy has no impact on the vehicle capacity. Fig. 7 shows the impact of the maximum degree of saturation on the vehicle capacity. The results further validate that the PC model can also increase the vehicle capacity by increasing the capacity of exclusive bus lanes.

Figs. 5a, b, and 6a–c can also be used to determine whether exclusive bus lanes and passive transit signal timings optimization is required for a given pair of bus ratios and maximum degree of saturation based on person capacity. To obtain more feasible solutions, the maximum degree of saturation of exclusive bus lanes and passenger car lanes can be adjusted according to the real bus demand.

5. Conclusion

In this study, a person-capacity-based optimization method for the integrated design of lane markings, exclusive bus lanes, and passive bus priority signal settings for isolated intersections is developed. Two traffic modes, passenger cars and buses, have been considered in a unified framework. Person capacity maximization has been adopted as a criterion for the integrated optimization model (PC model). The numerical example and sensitivity analysis show:

(1) Lane markings, exclusive bus lanes, and transit signal priority signal timings can be optimized efficiently by the proposed model for intersection with and without predetermined exclusive bus lanes. Moreover, the model tends to allocate less number of lanes and higher green split for the passenger movements with exclusive bus lanes in the same direction to provide priority for buses and increase person capacity.
(2) Compared with vehicle capacity maximization model (VC model), the person capacity of the intersection can be improved significantly by either integrated optimization of lane markings, exclusive bus lanes and signal timings or optimization of lane markings and signal timings for passenger movements only if exclusive bus lanes are predetermined.
(3) The highest benefit (person reserve capacity) of PC model compared with VC mode achieved at highest maximum degree of saturation of bus lanes and bus occupancy situation.

This paper presents preliminary evaluation results of the proposed model. More extensive numerical experiments and/or field tests will be conducted to assess the effectiveness of the proposed model under various traffic demand patterns and intersection geometry configurations. Another possible extension of this study is to employ person delay as an objective and perform a comparative analysis of person delay and person capacity models.

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