Transpn. Res.-B, Vol. 25B, No. 1, pp. 55-74, 1991
Printed in Great Britain.

0191-2615/91 $3.00+.00

A MULTI-BAND APPROACH TO ARTERIAL TRAFFIC SIGNAL OPTIMIZATION

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(*Received* 31 *August* 1989; *in revised form* 10 *July* 1990)

Abstract — Progression schemes are widely used for traffic signal control in arterial streets. Under such a scheme a continuous green band of uniform width is provided in each direction along the artery at the desired speed of travel. A basic limitation of existing bandwidth-based programs is that they do not consider the actual traffic volumes and flow capacities on each link in their optimization criterion. Consequently they cannot guarantee the most suitable progression scheme for different traffic flow patterns. In this paper we present a new optimization approach for arterial progression that incorporates a systematic traffic-dependent criterion. The method generates a variable bandwidth progression in which each directional road section can obtain an individually weighted bandwidth (hence, the term *multi-band*). Mixed-integer linear programming is used for the optimization. Simulation results indicate that this method can produce considerable gains in performance when compared with traditional progression methods. It also lends itself to a natural extension for the optimization of grid networks.

1. INTRODUCTION

Coordination of traffic lights along arterial streets provides numerous advantages as indicated in the *Transportation and Traffic Engineering Handbook* (Inst. of Transp. Engrs., 1982):

1. A higher level of traffic service is provided in terms of higher overall speed and reduced number of stops.
2. Traffic flows more smoothly, often with an improvement in capacity.
3. Vehicle speeds are more uniform because there is no incentive to travel at excessively high speeds to reach a signalized intersection within a green interval that is not in step. Also, the slow driver is encouraged to speed up in order to avoid having to stop for a red light.
4. There are fewer accidents because the platoons of vehicles arrive at each signal when it is green, thereby reducing the possibility of red signal violations or rear-end collisions.
5. Greater obedience to the signal commands is obtained from both motorists and pedestrians because the motorist tries to keep within the green interval and the pedestrian stays at the curb because the vehicles are more tightly spaced.
6. Through traffic tends to stay on the arterial street instead of diverting onto parallel minor streets.

A widely used method for arterial coordination, both in the U.S. and in other countries, is the *progression method* (FHWA, 1985; FGSV, 1981). Traffic signals tend to group vehicles into a "platoon" with more uniform headways than would otherwise occur. The platooning effect is accentuated on the major streets which have signalized intersections at frequent intervals. It seems desirable, in these circumstances, to encourage the platooning so that continuous movement (or progression) of vehicle platoons through successive traffic lights can be maintained. The signal timings, in this case, are designed to maximize the width of continuous green bands in both directions along the artery at the expected speed of travel. In general, such signal systems operate best when the main-street flow is predominantly through traffic and when the number of vehicles turning onto the main street is small.

With the advent of the computer age the manual design of time-space diagrams was replaced by computerized models. A large variety of such models were developed, including SIGART (Metropolitan Toronto, 1965), SIGPROG (Bleyl, 1967), the IBM models (Brooks, undated; Yardeni, 1964), NO-STOP-1 (Leuthardt, 1975), PASSER-II (Messer *et al.*, 1973), and Bandwidth Maximization (Morgan and Little, 1964; Little, 1966). Advances in optimization techniques and computational capabilities have steadily increased the sophistication of these models. The most advanced and versatile of these models today is MAXBAND, the latest version of which is described by Little, Kelson, and Gartner (1981). The model uses a powerful optimization method (mixed-integer linear programming), and is an extension of the earlier work by Little (1966). It can determine a global optimal solution and calculates cycle time, offsets, progression speeds and order of left-turn phases to maximize the weighted combination of the bandwidths in the two directions along the artery. It was also recently extended for application to multi-arterial closed networks (Chang *et al.*, 1988).

The U.S. Federal Highway Administration (FHWA) has been promoting in recent years the systematic optimization of traffic signal timings in urban areas (Euler, 1983). As part of this effort, TRANSYT-7F, a delay-based signal network optimization method was adopted for use in North America (Wallace *et al.*, 1981). Yet many traffic agencies find it necessary to impose upon the TRANSYT solution some arterial progression scheme for the main streets. The objective is to obtain a smoother flow of traffic on the principal arteries than is allowed by TRANSYT settings alone. This has led to the development of "hybrid" versions in which the TRANSYT program is constrained by a bandwidth solution for the arterial street (Cohen and Liu, 1986; Liu, 1988). Other methods for combining the advantages of bandwidth-based models with delay-minimizing models were proposed by Wallace and Courage (1982) and by Hisai (1987). Tsay and Lin (1988) present an "inverted funnel" progression scheme for the same purpose. Cohen (1983) and Skabardonis and May (1985) have also shown that substantial benefits are to be gained by concurrent use of delay-based methods (such as TRANSYT-7F, SIGOP-III) and bandwidth-based methods (such as MAXBAND, PASSER-II). These benefits accrue primarily due to the phase-sequence decision capabilities of the progression methods. In this paper we describe a different approach, which combines the advantages of the progression methods with traffic flow optimization criteria into a simultaneous global optimization model.

2. PROBLEM DESCRIPTION

A basic limitation of existing bandwidth-based programs is that their progression design criteria do not depend on the actual traffic flows on the arterial links and, therefore, is insensitive to variations in such flows. The total bandwidth that is obtained for the arterial can be allocated in any desired ratio among the two directions of travel. A common practice is to apportion it according to the directional volume ratio, k. Possible choices for this parameter include the ratio between the highest link volumes in each direction, or the ratio between the average (or total) link volumes in each direction. Neither of these choices can guarantee the best (or even a good) progression (in terms of delays and stops) for different traffic flow patterns.

Because of turn-in and turn-out traffic we do not, generally, have constant volumes along each direction of the arterial. Consequently, the idea of a uniform platoon moving through all the signals in one direction, which forms the conceptual basis for the bandwidth approach, does not always hold. Moreover, the ratio of volumes on opposing road sections between each pair of adjacent signals is also varying. It is, therefore, inconceivable that a single parameter for the entire arterial (k) can adequately reflect this diversity. The situation is readily illustrated by an example taken from Hawthorne Boulevard in Los Angeles (FHWA, 1983). The ratio of total traffic volumes in the two directions along the arterial is 2/1. Table 1 presents NETSIM simulation results for different bandwidth ratios. We can see that the bandwidth ratios that produce the lowest delay (6/1, or even 10/1) bear no relationship to the actual traffic volume ratio. A recent study proposed to determine the directional bandwidth ratio using a crude estimate of link delays (Change *et al.*, 1986).

Table 1. Hawthorne Boulevard NETSIM simulation results

E/W Volume Ratio	E/W Band Ratios	Delay (Sec/Veh)	Deviation from Optimal (%)
	1/1	80.73	26.9
2/1	2/1	71.44	12.3
	3/1	68.09	7.0
	4/1	67.07	5.4
	5/1	66.39	4.4
	6/1	63.61 (minimum)	0.0
	7/1	64.11	0.8
	8/1	64.06	0.7
	10/1	63.85	0.4

While some improvements were obtained, the results are not substantially different from the conventional volume ratio or equal weighting methods. It is apparent that existing bandwidth maximization programs are lacking a suitable traffic-dependent optimization criterion and, therefore, cannot guarantee an optimum result for varying traffic flow patterns.

In this paper we present a new optimization approach that is designed to remedy the deficiencies mentioned above. This approach places the arterial bandwidth optimization concept on a more solid foundation by incorporating into the calculation procedure a systematic traffic-dependent criterion. The volume on each link of the artery, together with other traffic parameters (such as capacity, speed, etc.), will have an effect on the optimization outcome through suitably chosen individual weighting factors, as contrasted with a single weight for the entire arterial in existing programs. Thus, we can achieve the objective of providing volume-weighted progression while reducing delay (or travel time) and stops. In the next section we first formulate the arterial bandwidth maximization problem and then describe the *multi-band* approach, which was originally developed for the Massachusetts Department of Public Works (Gartner *et al.*, 1989).

3. ARTERIAL BANDWIDTH MAXIMIZATION

To develop the multi-band approach we first introduce the basic arterial bandwidth maximization problem and then augment it with additional decision variables. Since all models use mixed-integer linear programming (MILP) for the optimization, we present three different MILP programs with progressively increasing complexity. MILP-1 is the basic, symmetric, uniform-width bandwidth maximization problem. MILP-2 extends the basic problem to include asymmetric bandwidths in opposing directions, variable left-turn phase sequences, as well as decisions on cycle time length (in terms of frequency) and link specific progression speeds. MILP-3 presents the new multi-band, multi-weight approach, which also incorporates all previous decision capabilities.

3.1 The basic bandwidth maximization problem

We refer to the time-space diagram shown in Fig. 1. We have an arterial with n signals. S_i denotes the signal at node (intersection) i, where $i = 1, \ldots, n$. All time variables are in units of the cycle time. We define the following variables:

$b(\bar{b})$ = outbound (inbound) bandwidth (cycles)

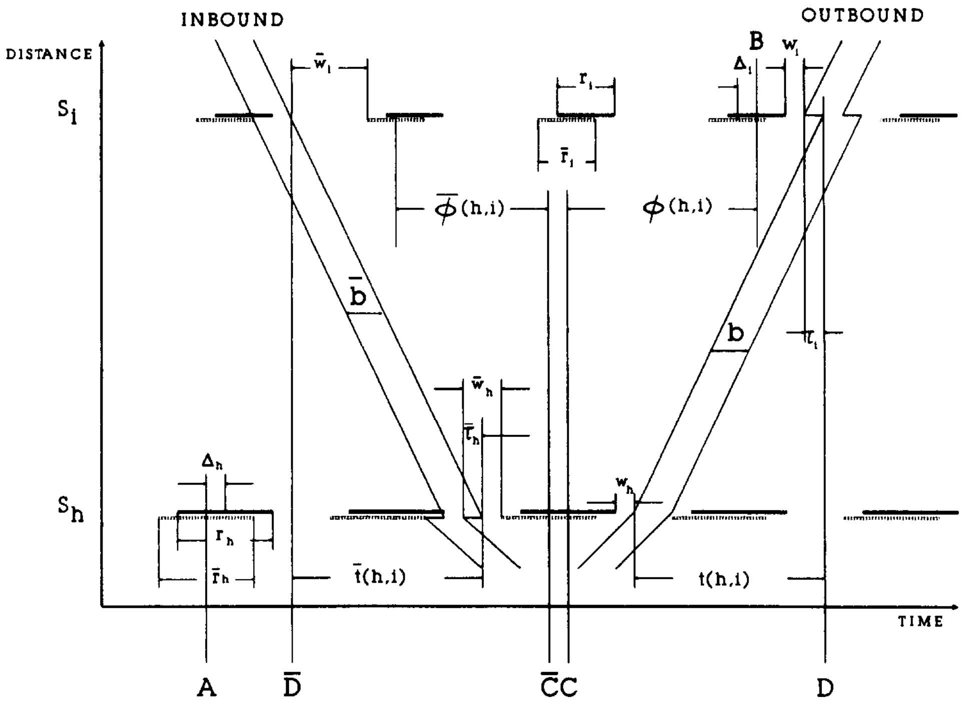

Fig. 1. Time-space diagram for MAXBAND model.

$r_i(\bar{r}_i)$	=	outbound (inbound) red time at S_i (cycles)
$w_i(\bar{w}_i)$	=	interference variables = time from right (left) side of red at S_i to left (right) edge of outbound (inbound) green band (cycles)
$t(h,i)[\bar{t}(h,i)]$	=	travel time from S_i to S_h outbound [S_h to S_i inbound] (cycles)
$\phi(h,i)[\bar{\phi}(h,i)]$	=	internode offsets = time from center of an outbound [inbound] red at S_h to the center of a particular outbound [inbound] red at S_i. The two reds are chosen so that each is immediately to the left [right] of the same outbound [inbound] green band. $\phi(h,i)[\bar{\phi}(h,i)]$ is positive if S_i's center of red lies to the right [left] of S_h's (cycles).
Δ_i	=	intranode offset = time from center of $\bar{r}_i$ to nearest center of r_i. Positive if center of r_i is to right of center of $\bar{r}_i$ (cycles)
$\tau_i(\bar{\tau}_i)$	=	queue clearance time, an advance of the outbound (inbound) bandwidth upon leaving S_i (cycles); time to clear traffic turned-in from side streets.

First we introduce the *directional interference constraints*. These constraints make sure that the progression bands use only the available green time and do not infringe upon any of the red times. From Fig. 1 we see that:

$$w_i + b \leq 1 - r_i \tag{1a}$$

$$\bar{w}_i + b \leq 1 - \bar{r}_i. \tag{1b}$$

Next, we calculate the *loop integer constraint*. This constraint is due to the fact that the signals of the arterial are synchronized (i.e. they operate in a common cycle time). If we start at the center of the outbound red at S_h and proceed along a loop consisting of the following points:

Center of outbound red at S_i
Center of inbound red at S_i
Center of inbound red at S_h

Center of outbound red at S_h,

we must end up at a point that is removed an integral number of cycle times from the point of departure. Summing algebraically the appropriate internode and intranode offsets along the loop, we obtain

$$\phi(h,i) + \bar{\phi}(h,i) + \Delta_h - \Delta_i = m(h,i) \tag{2}$$

where $m(h,i)$ is the corresponding loop integer variable.

Adding variables from (C) to (D) in Fig. 1, we observe the following identity:

$$\phi(h,i) + (1/2)r_i + w_i + \tau_i = (1/2)r_h + w_h + t(h,i). \tag{3a}$$

Similarly, from ($\bar{\mathrm{C}}$) to ($\bar{\mathrm{D}}$) we observe:

$$\bar{\phi}(h,i) + (1/2)\bar{r}_i + \bar{w}_i = (1/2)\bar{r}_h + \bar{w}_h - \bar{\tau}_h + \bar{t}(h,i). \tag{3b}$$

Substituting (3) into (2) to eliminate ϕ and $\bar{\phi}$ gives:

$$\begin{aligned} &t(h,i) + \bar{t}(h,i) + (1/2)(r_h + \bar{r}_h) + (w_h + \bar{w}_h) \\ &\quad - (1/2)(r_i + \bar{r}_i) - (w_i + \bar{w}_i) - (\tau_i + \bar{\tau}_h) + \Delta_h - \Delta_i = m(h,i). \end{aligned} \tag{4}$$

So far we have assumed that S_i follows S_h in the outbound direction, but this restriction is not necessary. Let x represent any of the variables t, $\bar{t}$, m, ϕ, $\bar{\phi}$; then the following relationship is satisfied:

$$x(h,j) = x(h,i) + x(i,j) \tag{5}$$

whence, by setting $h = j$, we obtain $x(h,i) = -x(i,h)$. Thus eqns (2) and (3) hold for arbitrary S_h and S_i.

To simplify notation we number the signals sequentially from 1 to n in the outbound direction, and we define $x_i = x(i,i + 1)$. We can now rewrite (4) as follows:

$$\begin{aligned} &t_i + \bar{t}_i + (w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + \Delta_i - \Delta_{i+1} \\ &\quad = -(1/2)(r_i + \bar{r}_i) + (1/2)(r_{i+1} + \bar{r}_{i+1}) + (\bar{\tau}_i + \tau_{i+1}) + m_i. \end{aligned} \tag{6}$$

To obtain the basic bandwidth maximization problem we also require $b = \bar{b}$ (i.e. a symmetric progression).

MILP-1. Find b, w_i, $\bar{w}_i$, m_i to
Max $b = \bar{b}$, subject to

$$\left.\begin{aligned} w_i + b &\le 1 - r_i \\ \bar{w}_i + \bar{b} &\le 1 - \bar{r}_i \end{aligned}\right\} i = 1, \ldots, n$$

$$\begin{aligned} &(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) + \Delta_i - \Delta_{i+1} = -(1/2)(r_i + \bar{r}_i) \\ &\qquad + (1/2)(r_{i+1} + \bar{r}_{i+1}) + (\bar{\tau}_i + \tau_{i+1}) + m_i, \qquad i = 1, \ldots, n - 1 \\ &\qquad m_i = \text{integer} \\ &\qquad b, \bar{b}, w_i, \bar{w}_i \ge 0, \qquad\qquad i = 1, \ldots, n. \end{aligned}$$

MILP-1 has $3n$ constraints, $(2n + 2)$ continuous variables and $(n - 1)$ unrestricted integer variables. Note that this optimization program only calculates bandwidths and interference variables. Using eqns (3a) and (3b) we can calculate the offsets, provided the red splits are known. The cycle time is also assumed to be given.

3.2 The extended bandwidth maximization model

The basic model described above is extended by adding a number of important decision capabilities for improved traffic control. The first extension concerns the directional weighting of the two bands. In many cases the traffic engineer may wish to favor one direction of traffic over the other (e.g. the "inbound" direction during the morning peak period and the "outbound" direction during the afternoon peak period). A "balanced" progression may be desirable during off-peak periods. Let k = target ratio of inbound to outbound bandwidth (taken as the ratio of total inbound to total outbound volumes along the arterial). We can set up the objective function and the *ratio constraint* as:

$$\begin{aligned}
&\mathbf{max}(b + k\bar{b}), \textbf{ subject to} \\
&\bar{b} \geq kb \quad \text{if } k < 1 \text{ (outbound favored)} \\
&\bar{b} \leq kb \quad \text{if } k > 1 \text{ (inbound favored)} \\
&\bar{b} = b \quad \text{if } k = 1 \text{ (balanced progression).}
\end{aligned}$$

The first two inequalities can be replaced by a single inequality as follows:

$$(1 - k)\bar{b} \geq (1 - k)kb. \tag{7}$$

We use an inequality rather than a strict equality since it is not necessary to restrict the larger band to a specific ratio once the smaller band has reached its maximum potential.

Another important extension is to let both the common signal cycle time C (seconds) and the link specific progression speed $v_i(\bar{v}_i)$ (feet/second) be optimizable variables. This introduces considerable flexibility in the calculation of the best arterial progression. Each of these variables is constrained by upper and lower limits. In addition, changes in speed from one link to the next can also be limited. Let the limits be as follows:

C_1, C_2 = lower and upper limits on cycle length
$e_i, f_i(\bar{e}_i, \bar{f}_i)$ = lower and upper limits on outbound (inbound) speed (feet/second)
$g_i, h_i(\bar{g}_i, \bar{h}_i)$ = lower and upper limits on change in outbound (inbound) speed (feet/second).

To obtain constraints that are linear in the decision variables we define the inverse of the cycle time (i.e. the signal frequency $z = 1/C$ (cycles/second), such that:

$$1/C_2 \leq z \leq 1/C_1. \tag{8}$$

The progression speeds and the changes in speeds are also expressed in terms of their reciprocals. For the outbound direction we obtain:

$$1/f_i \leq 1/v_i \leq 1/e_i$$
$$1/h_i \leq (1/v_{i+1}) - (1/v_i) \leq 1/g_i.$$

Let $d(h,i)[\bar{d}(h,i)]$ = distance between S_h and S_i outbound [inbound] (feet). Using the relationship $d_i = d(i,i + 1)$ and $\bar{d}_i = \bar{d}(i,i + 1)$, we obtain $t_i = (d_i/v_i)z$ and $\bar{t}_i = (\bar{d}_i/\bar{v}_i)z$. Substituting these expressions above and manipulating the variables, we obtain

$$(d_i/f_i)z \leq t_i \leq (d_i/e_i)z \tag{9}$$

$$(d_i/h_i)z \leq (d_i/d_{i+1})t_{i+1} - t_i \leq (d_i/g_i)z. \tag{10}$$

In the extended formulation, t_i, $\bar{t}_i$ and z are decision variables which, once known, determine progression speeds. Another important decision capability that is offered by the mixed-integer linear programming technique is to determine the sequence of the left turn phase (if one is present) with respect to the through green at any signal. The left turn green can be chosen to lead or lag, whichever gives the most total bandwidth. At the same time, however, it is necessary to give the traffic engineer the ability to specify which combination of leads or lags in each direction will be permitted. Figure 2 shows the four possible patterns of left turn green phases. Let

$G_i(\bar{G}_i)$ = outbound (inbound) green time for through traffic at S_i (cycles).
$L_i(\bar{L}_i)$ = time allocated for outbound (inbound) left turn green at S_i (cycles)
R_i = common red time in both directions to provide for cross street movement at S_i (cycles).

Since time allocated to outbound left turn green is inbound red time we have (see Fig. 2):

$$r_i = R_i + \bar{L}_i \qquad r_i + G_i = 1$$
$$\text{and}$$
$$\bar{r}_i = R_i + L_i \qquad \bar{r}_i + \bar{G}_i = 1.$$

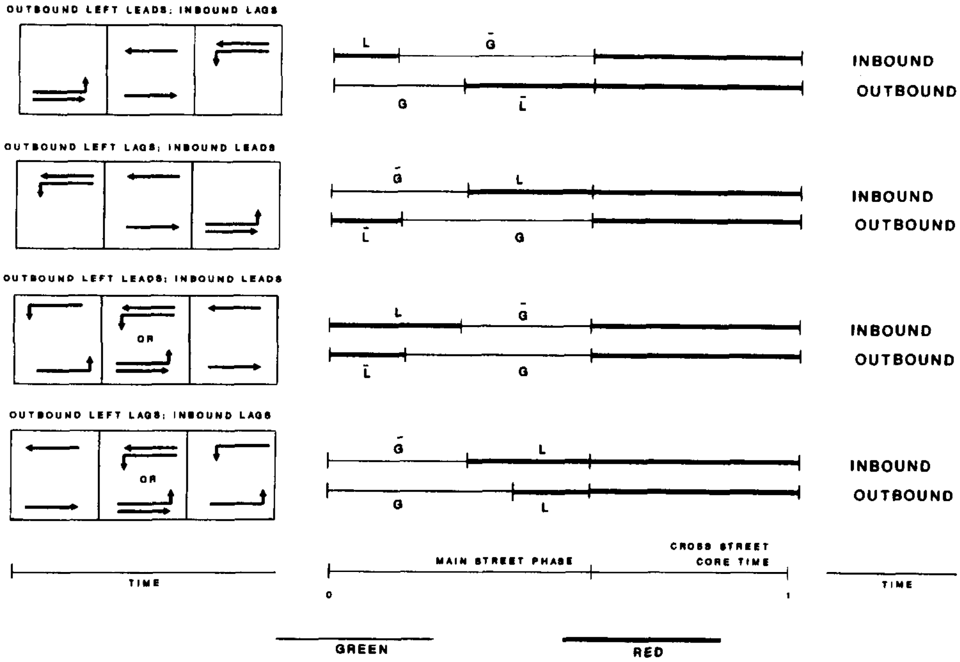

Fig. 2. The four possible patterns of left-turn phases.

Moreover, we can express Δ_i, the time from the center of $\bar{r}_i$ to the next center of r_i, in terms of L_i and $\bar{L}_i$ for each case as shown in the Table 2.

The different cases can be combined in the following formula:

$$\Delta_i = (1/2)[(2\delta_i - 1)L_i - (2\bar{\delta}_i - 1)\bar{L}_i] \tag{11}$$

where δ_i and $\bar{\delta}_i$ are 0–1 variables as shown. If only certain patterns are to be permitted, restrictions can be placed on the δ_i and $\bar{\delta}_i$ to enforce these requirements. For example, if only patterns 1 and 2 are permitted the constraint $\delta_i + \bar{\delta}_i = 1$ is added.

Incorporating all these extensions into MILP-1 yields a more versatile mixed-integer linear program:

MILP-2. Find $b, \bar{b}, z, w_i, \bar{w}_i, t_i, \bar{t}_i, \delta_i, \bar{\delta}_i, m_i$ **to**

Max $(b + k\bar{b})$, **subject to**

$$(1 - k)\bar{b} \geq (1 - k)kb$$

$$1/C_2 \leq z \leq 1/C_1$$

$$\left.\begin{aligned} w_i + b &\leq 1 - r_i \\ \bar{w}_i + \bar{b} &\leq 1 - \bar{r}_i \end{aligned}\right\} i = 1, \ldots, n$$

$$\begin{aligned} (w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) + \delta_i L_i - \bar{\delta}_i \bar{L}_i - \delta_{i+1} L_{i+1} + \bar{\delta}_{i+1} \bar{L}_{i+1} - m_i \\ = (r_{i+1} - r_i) + (\tau_i + \bar{\tau}_i), \qquad i = 1, \ldots, n - 1 \end{aligned}$$

Table 2. Left turn patterns

Pattern	Δ_i	δ_i	$\bar{\delta}_i$
1	$-(1/2)(L_i + \bar{L}_i)$	0	1
2	$(1/2)(L_i + \bar{L}_i)$	1	0
3	$-(1/2)(L_i - \bar{L}_i)$	0	0
4	$(1/2)(L_i - \bar{L}_i)$	1	1

$$\left.\begin{array}{l}(d_i/f_i)z \le t_i \le (d_i/e_i)z \\ (\bar{d}_i/\bar{f}_i)z \le \bar{t}_i \le (\bar{d}_i/\bar{e}_i)z\end{array}\right\} \quad i = 1, \ldots, n - 1$$

$$\left.\begin{array}{l}(d_i/h_i)z \le (d_i/d_{i+1})t_{i+1} - t_i \le (d_i/g_i)z \\ (\bar{d}_i/\bar{h}_i)z \le (\bar{d}_i/\bar{d}_{i+1})\bar{t}_{i+1} - \bar{t}_i \le (\bar{d}_i/\bar{g}_i)z\end{array}\right\} \quad i = 1, \ldots, n - 2$$

$b, \bar{b}, z, w_i, \bar{w}_i, t_i, \bar{t}_i \ge 0$

m_i integer

$\delta_i, \bar{\delta}_i$ zero/one variables.

In addition to bandwidths and interference variables, this program also calculates optimal cycle length (1/z), progression speeds, and phase sequences. To calculate the offsets, green splits need to be given or, alternatively, the user can provide traffic volume and capacity information for each intersection and the program will calculate the splits. MILP-2 is essentially the current version of the MAXBAND program (Little *et al.*, 1981). It involves $(11n - 10)$ constraints and $(4n + 1)$ continuous variables, up to $2n$ zero/one variables and $n - 1$ unrestricted integer variables, not counting slack variables. In addition, if the user decides to require or prohibit certain left turn patterns, constraints on δ_i and $\bar{\delta}_i$ are added up to a maximum of $2n$.

3.3 *The multi-band/multi-weight approach*

In this section we describe the new multi-band/multi-weight approach (the computer program is named MULTIBAND). We now define a different bandwidth for each directional road section of the arterial which is individually weighted with respect to its contribution to the overall objective function. (Note: The band is continuous; only the width can vary.) Thus we obtained a method that is sensitive to varying traffic conditions and can tailor the progression scheme to the different possible traffic flow patterns. The user can still choose uniform bandwidth progression if he so desires, but this is now only one of the many user options.

Referring to the geometry in Fig. 3, we define the following variables:

$b_i(\bar{b}_i)$ = Outbound (inbound) bandwidth between signals S_i and S_{i+1}; there is now *a specific band* for each directional road section or link.

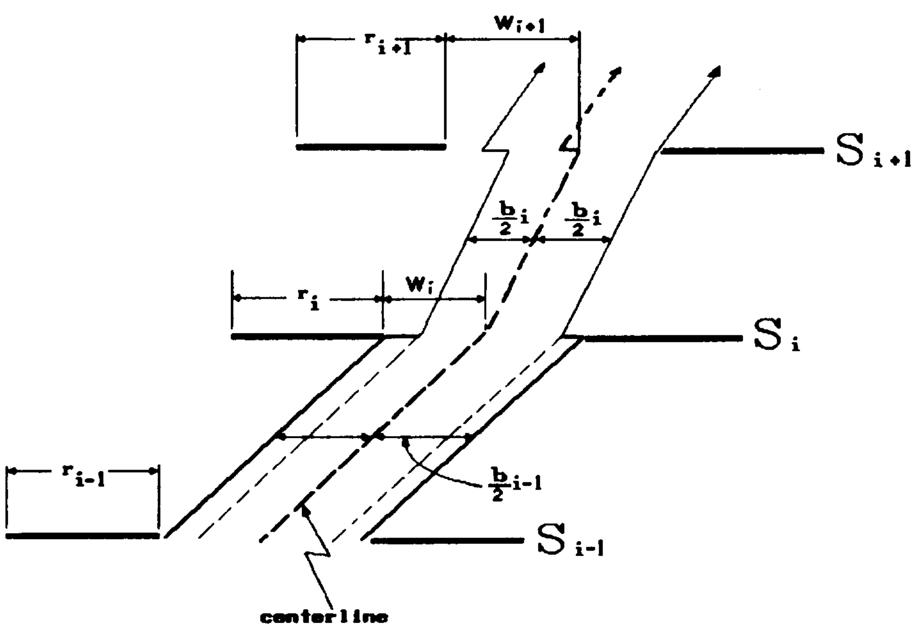

Fig. 3. Geometric relations for MULTIBAND formulation.

$w_i(\bar{w}_i)$ = Time from right (left) side of red at S_i to centerline of outbound (inbound) green band; the time reference point at each signal is moved from the edges to the *centerline* of the band.

The following constraints apply in the outbound directions at signal S_i:

$$w_i + (1/2)b_i \leq 1 - r_i \tag{12a}$$

$$w_i - (1/2)b_i \geq 0. \tag{12b}$$

The pair of constraints can be combined as follows:

$$(1/2)b_i \leq w_i \leq (1 - r_i) - (1/2)b_i. \tag{13a}$$

A similar relation must be observed at S_{i+1}, since band b_i must be constrained at both ends:

$$(1/2)b_i \leq w_{i+1} \leq (1 - r_{i+1}) - (1/2)b_i. \tag{13b}$$

Corresponding relationships exist in the inbound direction (marked by a bar on all variables):

$$(1/2)\bar{b}_i \leq \bar{w}_i \leq (1 - \bar{r}_i) - (1/2)\bar{b}_i \tag{14a}$$

$$(1/2)\bar{b}_i \leq \bar{w}_{i+1} \leq (1 - \bar{r}_{i+1}) - (1/2)\bar{b}_i. \tag{14b}$$

We call these constraints the *directional interference constraints*. Noting that we redefined the time reference points to the *centerline* of the bands (or, the *progression* line), rather than the edges, the "loop integer constraint" give by eqn (6) remains unchanged and so do the travel time and speed-change constraints.

The *ratio constraint* (7) is now also changed to reflect the multi-band situation:

$$(1 - k_i)\bar{b}_i \geq (1 - k_i)k_i b_i \tag{15}$$

where, k_i = target ratio of inbound to outbound bandwidth on section i (taken as the ratio of the corresponding volumes in each direction).

The most important change occurs in the objective function. Since the bands are link-specific, they can be weighted disaggregately to reflect desirable traffic objectives for each link. The new objective function has the following form:

$$\text{MAX B} = \frac{1}{n-1} \sum_{i=1}^{n-1} (a_i b_i + \bar{a}_i \bar{b}_i) \tag{16}$$

where $a_i(\bar{a}_i)$ are the link-specific weights in the two directions. There are a multitude of options available for determining the weighting coefficients. We chose to investigate the following weighting options:

$$a_i = \left(\frac{\mathcal{V}_i}{\mathcal{S}_i}\right)^p \qquad \bar{a}_i = \left(\frac{\bar{\mathcal{V}}_i}{\bar{\mathcal{S}}_i}\right)^p$$

where, $\mathcal{V}_i(\bar{\mathcal{V}}_i)$ = directional volume on section i, outbound (inbound); either total volume or through (platoon) volume can be used.

$\mathcal{S}_i(\bar{\mathcal{S}}_i)$ = saturation flow on section i, outbound (inbound).

p = exponential power; the values p = 0,1,2,4 were used.

To obtain an objective function value that is consistent with those used previously, we normalized the weighting coefficients to obtain

$$\sum_{i=1}^{n-1} a_i = n - 1; \qquad \sum_{i=1}^{n-1} \bar{a}_i = n - 1$$

The multi-band/multi-weight optimization program is summarized in MILP-3.

MILP-3. Find $b_i, \bar{b}_i, z, w_i, \bar{w}_i, t_i, \bar{t}_i, \delta_i, \bar{\delta}_i, m_i$ to

$$\text{MAX B} = \frac{1}{n-1} \sum_{i=1}^{n-1} (a_i b_i + \bar{a}_i \bar{b}_i)$$

Subject to

$$(1 - k_i)\bar{b}_i \geq (1 - k_i)k_i b_i \qquad i = 1, \ldots, n-1$$

$$1/C_2 \leq z \leq 1/C_1$$

$$\left.\begin{aligned}(1/2)b_i &\leq w_i \leq (1 - r_i) - (1/2)b_i \\ (1/2)b_i &\leq w_{i+1} \leq (1 - r_{i+1}) - (1/2)b_i \\ (1/2)\bar{b}_i &\leq \bar{w}_i \leq (1 - \bar{r}_i) - (1/2)\bar{b}_i \\ (1/2)\bar{b}_i &\leq \bar{w}_{i+1} \leq (1 - \bar{r}_{i+1}) - (1/2)\bar{b}_i\end{aligned}\right\} \qquad i = 1, \ldots, n-1$$

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) + \delta_i l_i - \bar{\delta}_i \bar{l}_i - \delta_{i+1} l_{i+1} + \bar{\delta}_{i+1} \bar{l}_{i+1} - m_i$$
$$= (r_{i+1} - r_i) + (\bar{\tau}_i + \tau_i), \qquad i = 1, \ldots, n-1$$

$$\left.\begin{aligned}(d_i/f_i)z &\leq t_i \leq (d_i/e_i)z \\ (\bar{d}_i/\bar{f}_i)z &\leq \bar{t}_i \leq (\bar{d}_i/\bar{e}_i)z\end{aligned}\right\} \qquad i = 1, \ldots, n-1$$

$$\left.\begin{aligned}(d_i/h_i)z &\leq (d_i/d_{i+1})t_{i+1} - t_i \leq (d_i/g_i)z \\ (\bar{d}_i/\bar{h}_i)z &\leq (\bar{d}_i/\bar{d}_{i+1})\bar{t}_{i+1} - \bar{t}_i \leq (\bar{d}_i/\bar{g}_i)z\end{aligned}\right\} \qquad i = 1, \ldots, n-2$$

$b_i, \bar{b}_i, z, w_i, \bar{w}_i, t_i, \bar{t}_i \geq 0$

m_i integer

$\delta_i, \bar{\delta}_i$ zero/one variables

MILP-3 involves $(18n - 20)$ constraints and $(6n - 3)$ continuous variables, up to $2n$ zero/one variables and $n - 1$ unrestricted integer variables, not counting slack variables. In addition, left turn pattern constraints on δ_i and $\bar{\delta}_i$ may add up to a total of $2n$. Green or red splits are determined the same as in MILP-2. Table 3 gives a comparison of the number of constraints and variables for MAXBAND and MULTIBAND. The increased decision capabilities of MULTIBAND require a corresponding increase in the size of the mathematical program.

4. IMPLEMENTATION AND COMPUTATIONAL RESULTS

The MULTIBAND computer program was created from the existing MAXBAND program (Little *et al.*, 1981). The core of MAXBAND is MPCODE, a mathematical programming package which uses a branch-and-bound technique to solve the mixed-integer linear programs (Land and Powell, 1973). The Matrix Generator module in MAXBAND prepares the input file for MPCODE from the traffic data. It was revised to generate the new multi-band/multi-weight objective function as well as the new link-

Table 3. Constraints and variables for MAXBAND compared to MULTIBAND

Type		MAXBAND		MULTIBAND	
Constraint	Variable	No. of Constraints	No. of Variables	No. of Constraints	No. of Variables
Bands	$b_i, \bar{b}_i$	—	2	—	2(n − 1)
Bandwidths	k_i	1	—	n − 1	—
Cyclic rate	z	2	1	2	1
Interference	$w_i, \bar{w}_i$	2n	2n	8(n − 1)	2n
Loop integer	m_i	n − 1	n − 1	n − 1	n − 1
Travel times (speed)	$t_i, \bar{t}_i$	4(n − 1)	2(n − 1)	4(n − 1)	2(n − 1)
Speed changes	—	4(n − 2)	—	4(n − 2)	—
Totals		$\boxed{11n - 10}$	5n	$\boxed{18n - 20}$	7n − 4
Example: n = 5		45	25	70 (+55%)	31 (+24%)
n = 10		100	50	160 (+60%)	66 (+32%)

specific bandwidth variables, interference variables, bandwidth ratio constraints and interference constraints. MPCODE was altered to handle the larger number of expected variables and constraints necessary to solve the multi-band model as shown in Table 3. Tolerances for round-off errors had to be carefully readjusted. These are critical in comparisons to zero in the branch-and-bound procedure and in obtaining the integer variable values. The output module in MAXBAND, which also calculates the cycle time, offsets and phase sequences from the MPCODE results, was modified to provide time-space plots with variable bandwidths, as shown in the figures below.

To assess the performance of the multi-band approach we compared it with the performance of MAXBAND on a data set representing Canal Street in New Orleans. A diagram of the Canal Street network as prepared for NETSIM simulation appears in Fig. 4. Canal Street itself is represented by the vertical arrows. The side streets are represented by horizontal arrows. On each section of street we have given a list of three numbers, such as (0, 432, 120). This indicates that in each hour approximately 552 cars travel on that section of street and that when they get to the intersection, 0% turn left, 78% go straight, and 22% turn right. There is a wide variation in sectional volume ratio values, k_i. They vary from 1 : 3.1 in the top section to 1.8 : 1 in the bottom; however, the ratio of the total volumes in the two directions is 1 : 1.2. Conventional progression schemes, such as MAXBAND or PASSER-II, cannot take account of this variability. In this particular data set,

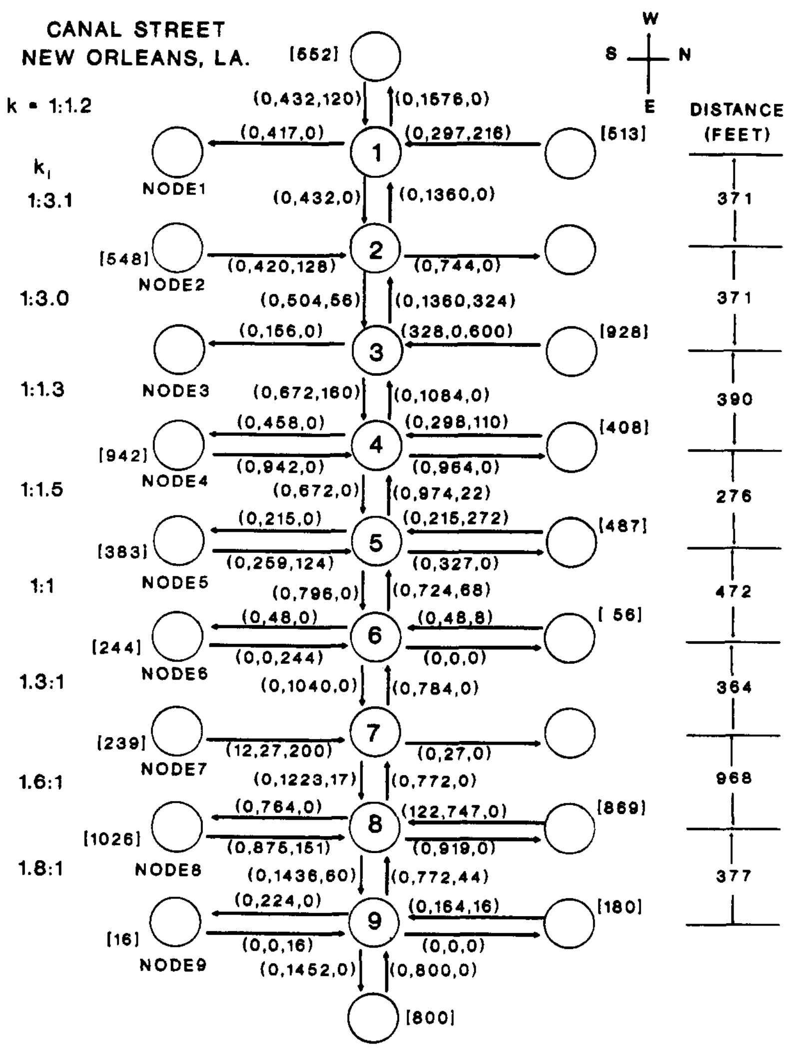

Fig. 4. Data for Canal Street, New Orleans, LA.

Table 4. Simulation results for Canal Street (without side streets)*

Method	Weighting Coefficient	Average Delay	Average# of Stops	Average Speed	Average M.P.G.	Delay + 20 (Stops)
MAXBAND	1	29.69	1.35	15.98	10.70	56.65
MAXBAND	TVR	28.84	1.35	16.11	10.78	55.84
MULTIBAND	1	25.62	1.13	16.82	11.20	48.30
MULTIBAND	TVC	25.20	1.07	16.96	11.28	46.68
MULTIBAND	PVC	25.08	1.02	16.99	11.30	45.40
MULTIBAND	$(TVC)^2$	25.35	1.20	16.80	11.15	49.31
MULTIBAND	$(TVC)^4$	24.11	1.14	17.08	11.30	46.87
MULTIBAND	$(PVC)^4$	25.25	1.01	16.95	11.29	45.53

*Each entry is the average from five simulation runs of the same traffic light settings with different seed numbers. Settings for $(PVC)^2$ were the same as those for $(PVC)^4$.

there are no left turns allowed from Canal Street, although cars from some of the side streets do turn left onto Canal Street.

We ran MULTIBAND on the Canal Street data set using seven different weighting schemes as detailed in Tables 4 and 5. The following notation is used for the weighting coefficients (for each directional road section): TVC = total volume/total capacity, PVC = platoon volume/platoon capacity ("platoon" refers to through-going traffic only). We also ran MAXBAND using its two possible weighting schemes: $k = 1$ and $k = \text{TVR}$ (total volume ratio). Both programs incorporate a centering routine which centers the progression bands within the available green space if there is leeway to do so. Our own simulations and those of others show that centering usually improves network performance. Time-space plots for each of the different signal settings are shown in Figs. 5 through 12. The MULTIBAND weighting schemes $(PVC)^2$ and $(PVC)^4$ resulted in the same settings. It is evident that different multi-band options generate different progression configurations, which in turn are quite different from those generated by MAXBAND.

Each signal setting scheme was simulated using NETSIM, a program which performs a microscopic statistical simulation of traffic flow in a signalized network and produces figures on such performance characteristics as average delay per vehicle, average number of stops, average speed, and average fuel consumption (FHWA, 1980). These averages can include just the cars on the arterial road or also the cars on the side streets. We did both kinds of simulations. We also used the results to calculate a weighted combination of delay and stops, which is a performance characteristic used by the program TRANSYT.

The simulation results are summarized in Tables 4 and 5. They show a clear advantage for MULTIBAND over MAXBAND in all cases. Additional evaluations were conducted

Table 5. Simulation results for Canal Street (with side streets)*

Method	Weighting Coefficient	Average Delay	Average# of Stops	Average Speed	Average M.P.G.	Delay + 20 (Stops)
MAXBAND	1	26.74	1.15	15.60	9.89	49.74
MAXBAND	TVR	25.78	1.14	15.79	10.01	48.58
MULTIBAND	1	23.09	1.02	16.43	10.35	43.49
MULTIBAND	TVC	22.85	1.01	16.47	10.41	43.05
MULTIBAND	PVC	22.86	0.98	16.41	10.42	42.46
MULTIBAND	$(TVC)^2$	25.23	1.09	15.89	10.12	47.03
MULTIBAND	$(TVC)^4$	23.54	1.04	16.33	10.31	44.34
MULTIBAND	$(PVC)^4$	23.08	0.97	16.40	10.41	42.48

*Each entry is from a single simulation run.

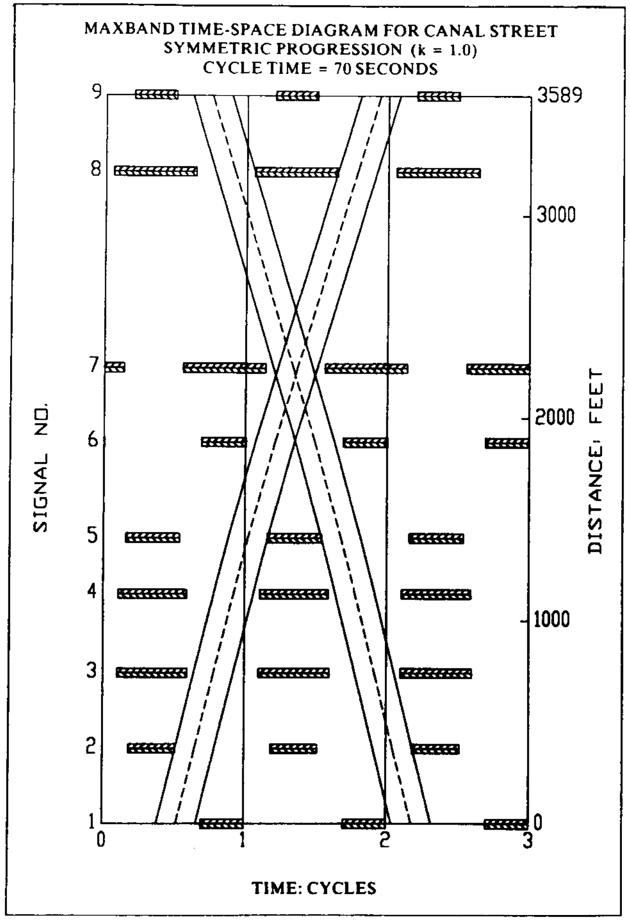

Fig. 5. Maxband time-space diagram for Canal Street; symmetric progression (k = 1.0).

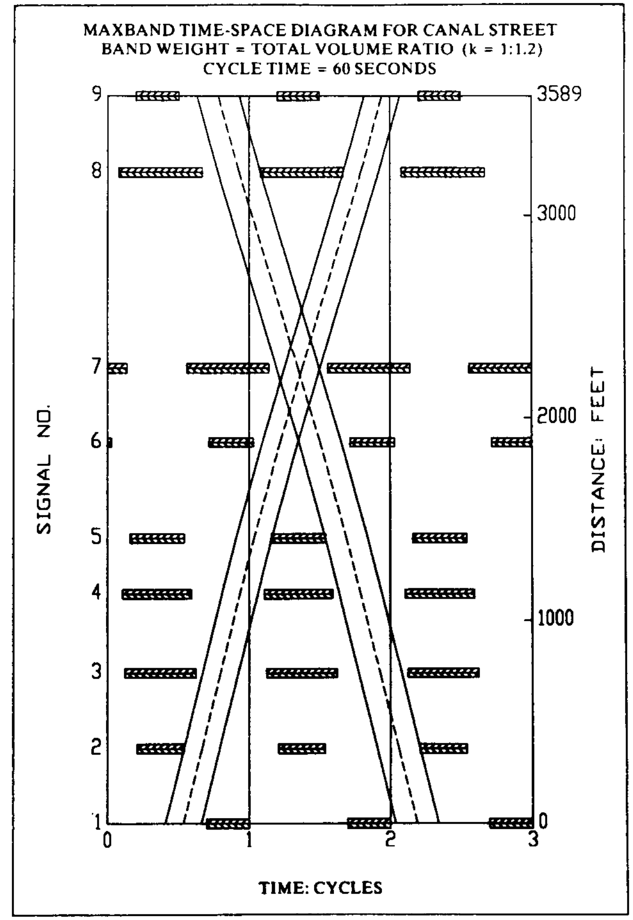

Fig. 6. Maxband time-space diagram for Canal Street; band weight = Total Volume Ratio (k = 1 : 1.2).

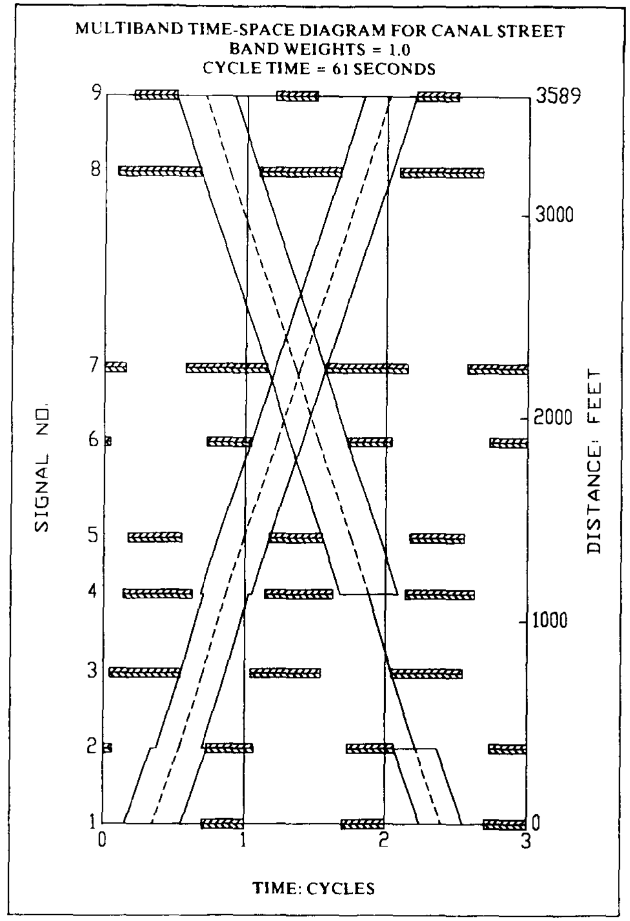

Fig. 7. Multiband time-space diagram for Canal Street; band weights = 1.0.

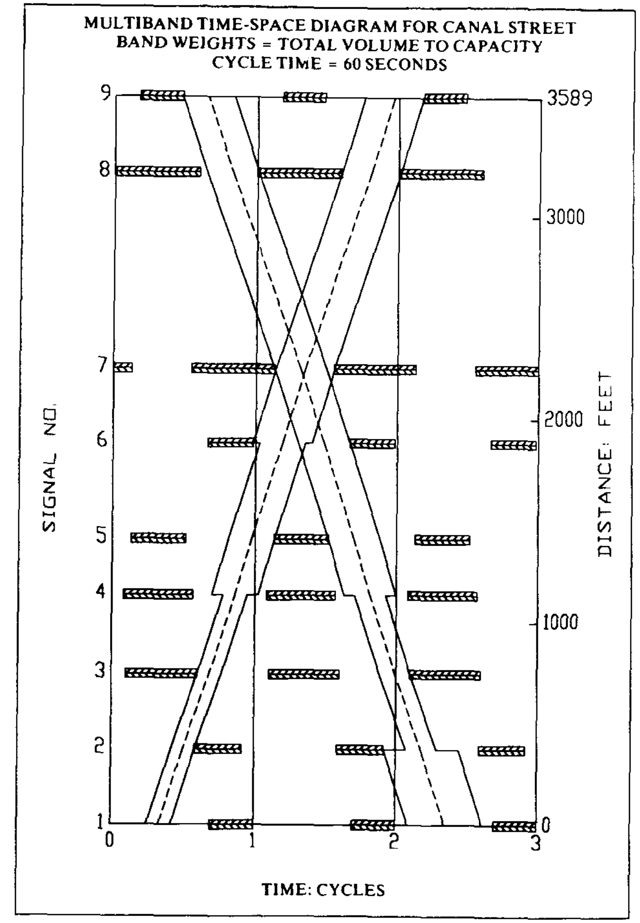

Fig. 8. Multiband time-space diagram for Canal Street; band weights = total volume to capacity.

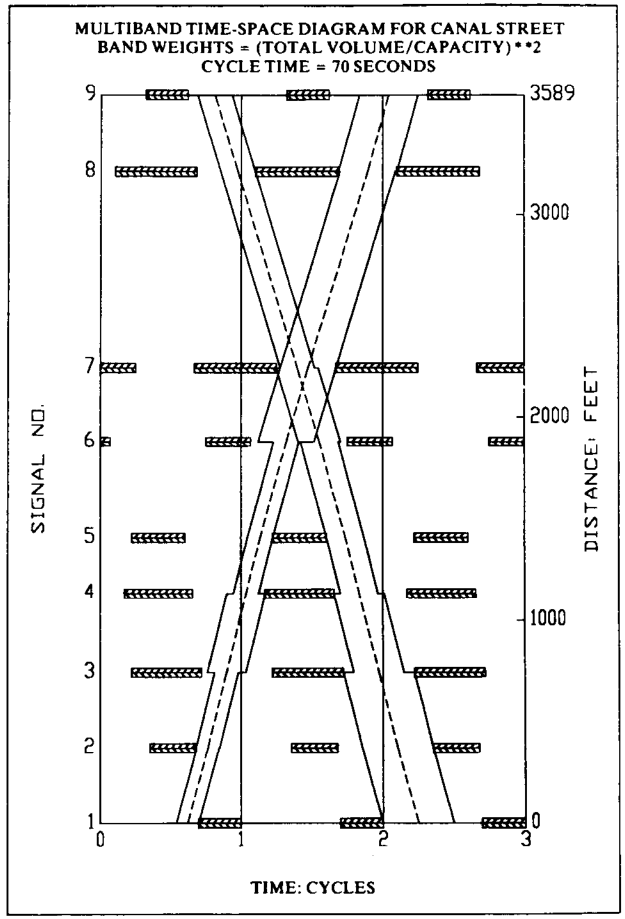

Fig. 9. Multiband time-space diagram for Canal Street; band weights = (total volume/capacity)**2.

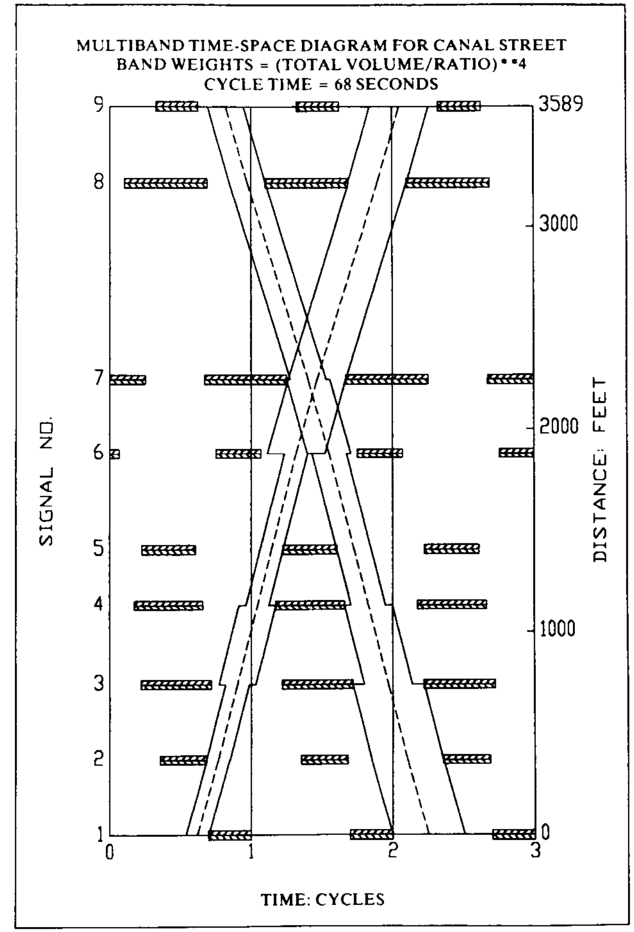

Fig. 10. Multiband time-space diagram for Canal Street; band weights = (total volume/ratio)**4.

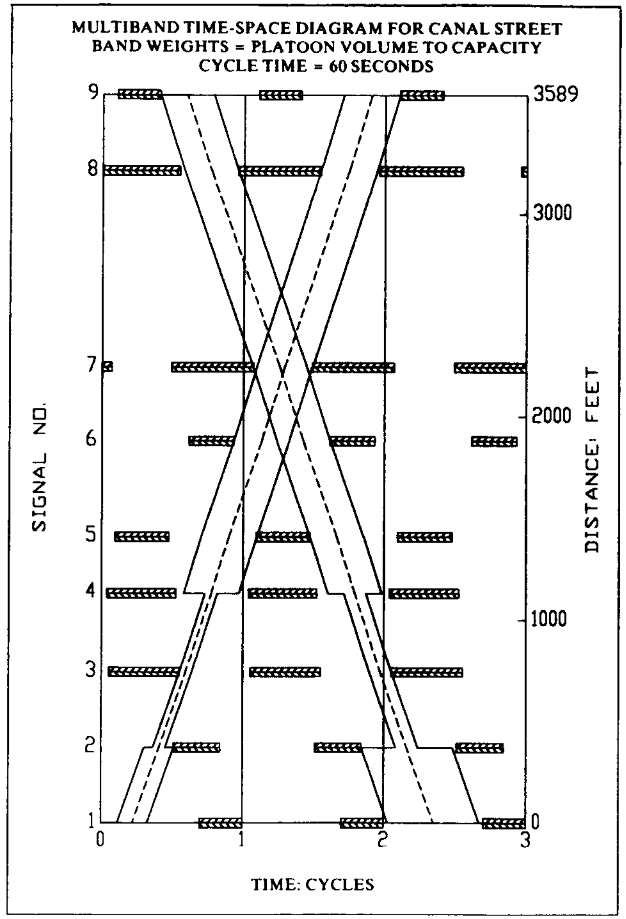

Fig. 11. Multiband time-space diagram for Canal Street; band weights = platoon volume to capacity.

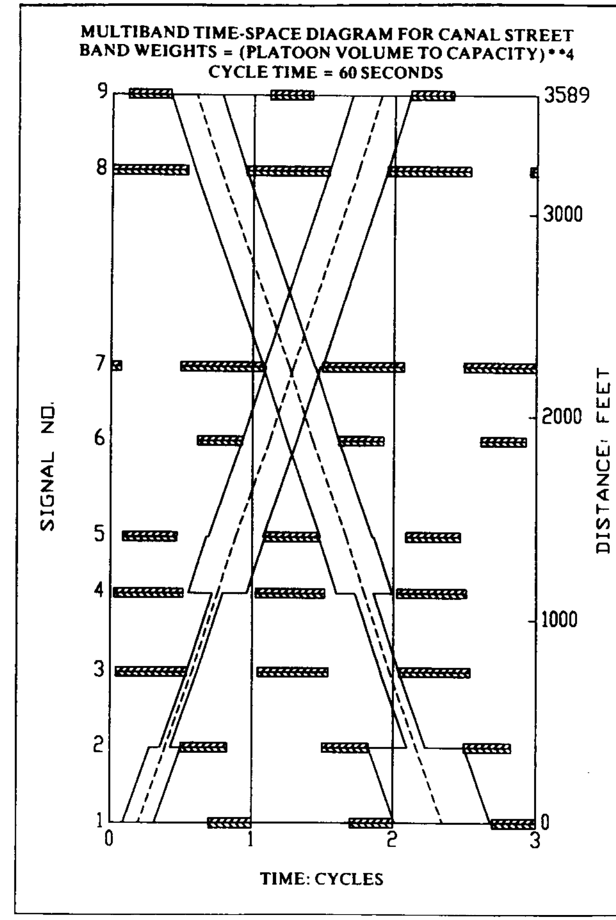

Fig. 12. Multiband time-space diagram for Canal Street; band weights = (platoon volume to capacity)**4.

for other arterial data sets, which also included multi-phase sequences, with comparable results. (They are omitted due to space limitations.)

MULTIBAND produced improvements in all performance characteristics (delay, stops, speed, miles per gallon, and a weighted combination of delay and stops) compared to MAXBAND, no matter which weighting option was chosen, and whether or not data on side street cars were included when calculating the performance characteristics. Depending on which characteristic is considered as being of primary importance, different weighting options produced the best results. However, the difference between various MULTIBAND options or between different MAXBAND options are in general much less than the difference between a MULTIBAND option and a MAXBAND option.

Without side street cars being taken into consideration, the MULTIBAND option with the lowest average delay (24.11 sec) is $(TVC)^4$. The MAXBAND option with the lowest average delay (28.84 sec) is TVR. Here, MULTIBAND produces an improvement of approximately 16% in average delay. If we consider instead the average number of stops, the best MULTIBAND option is $(PVC)^4$ with 1.01 stops per vehicle, whereas the best MAXBAND option has a weighting factor of 1 and produces 1.35 stops per vehicle. Here, MULTIBAND obtains an improvement of approximately 25%.

With side streets included, the MULTIBAND option with the lowest average delay (22.85 sec) is TVC. The MAXBAND option with lowest delay (25.78 sec) is TVR. Here, MULTIBAND produces an improvement of about 11%. If we consider stops instead, the best MULTIBAND option is $(PVC)^4$ with .97 stops, and the best MAXBAND is again TVR, with 1.14 stops. The improvement with MULTIBAND is approximately 15%.

Figures 13 through 16 show graphically the average delay, average speed, average number of stops, and TRANSYT performance index for all the runs with side streets not included, and also show the 95% confidence intervals. In other words, if we ran the NETSIM simulation many times using different random seeds, 95% of the time the results would fall in the areas marked by the horizontal bars. These confidence intervals were calculated using the method of Gafarian and Halati (1986) for the statistical analysis of output ratios in traffic situation. They clearly show the superior performance of MULTIBAND, compared to MAXBAND.

5. CONCLUSIONS

This paper describes a new approach to arterial traffic signal optimization which is based on a multi-band/multi-weight concept. It is shown that this approach can offer numerous advantages and design flexibilities over existing arterial progression methods. It provides a capability to adapt the progression scheme to the specific traffic flow pattern that exists on the links of the arterial. Such capability is not presently available in any other progression scheme. Thus we can determine a global optimal solution that calculates

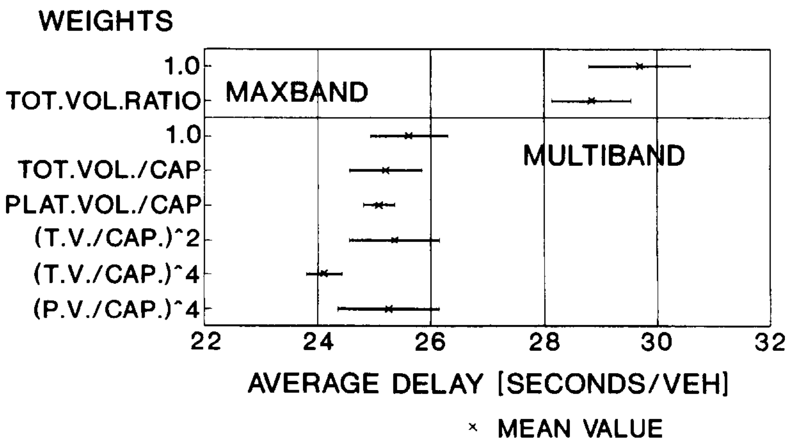

Fig. 13. Canal Street average delay comparison.

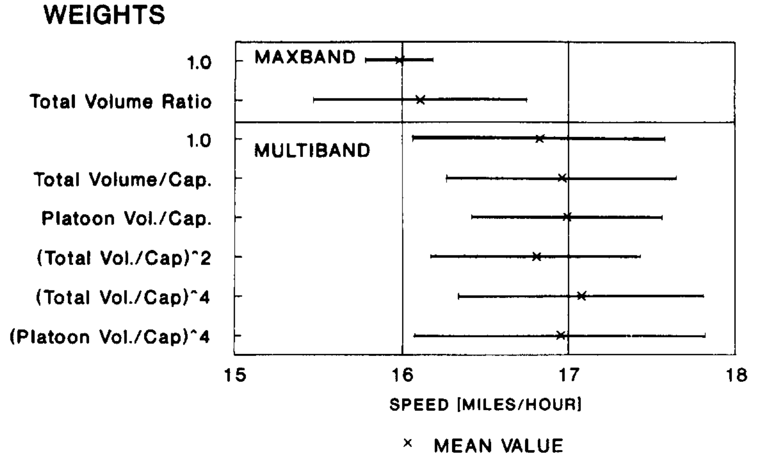

Fig. 14. Canal Street average speed comparison.

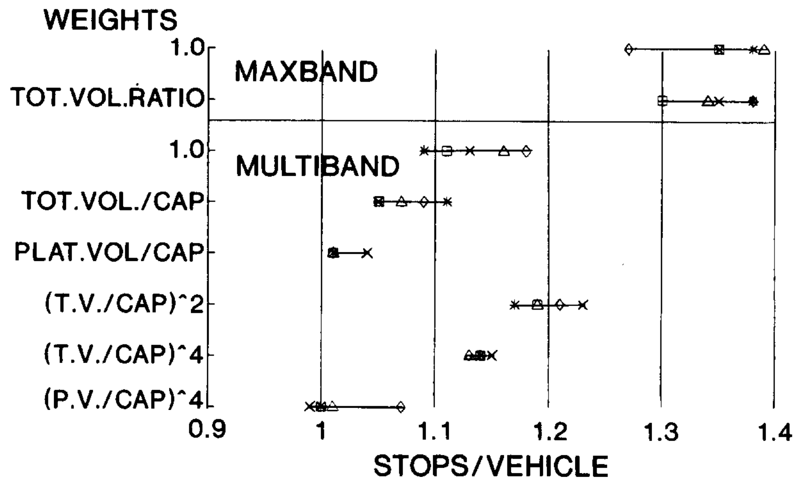

Fig. 15. Canal Street number of stops comparison.

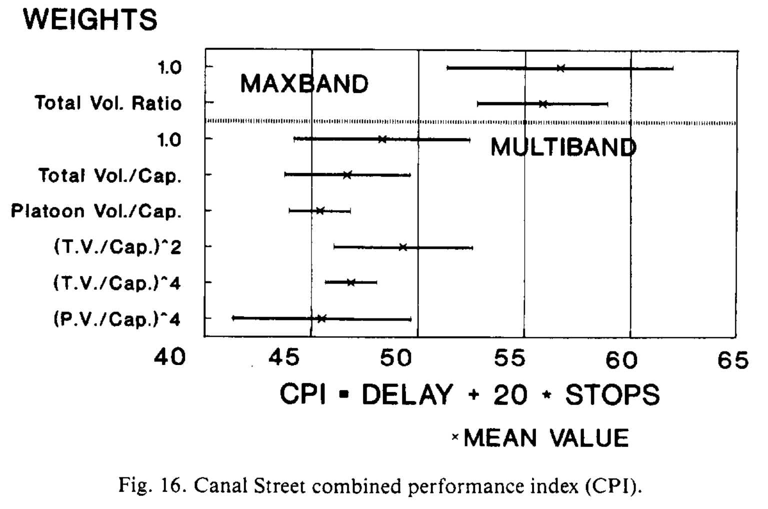

Fig. 16. Canal Street combined performance index (CPI).

cycle time, offsets, progression speeds and phase sequences to maximize a combination of the individually weighted bandwidths in each directional section of the artery. Through progressions with variable bandwidths are maintained in both directions. Evaluation results, using NETSIM simulation, have shown that significant improvements in delays, stops, travel speeds and fuel consumption are possible with this scheme.

The new approach lends itself to extension in a number of directions. Included among these are: progression bands that are asymmetric with respect to the centerline, tailoring the progressions to prevailing origin-destination patterns in an urban area (Keller and Ploss, 1987), and network optimization of the type used in MAXBAND-86 (Chang *et al.*, 1988). Further research in these areas is underway.

Acknowledgement – The work reported in this paper was performed (in part) under Contract No. 87060 between the Massachusetts Department of Public Works and the University of Lowell Research Foundation. Their sponsorship of this work is gratefully acknowledged.

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