MAXBAND

A Versatile Program for Setting Signals on
Arteries and Triangular Networks

John D.C. Little*
Mark D. Kelson*
Nathan H. Gartner**

WP# 1185-81   January 1981

*   E53-350, Massachusetts Institute of Technology, Cambridge, MA   02139

**   Department of Civil Engineering, University of Lowell, Lowell, MA   01854

ABSTRACT

MAXBAND is a portable, off-line, FORTRAN IV computer program for setting arterial signals to achieve maximal bandwidth. Special features of the program include (1) automatically choosing cycle time from a given range, (2) permitting the design speed to vary within given tolerances, (3) selecting the best lead or lag pattern for left turn phases from a specified set, (4) allowing a queue clearance time for secondary flow accumulated during red, (5) accepting user-specified weights for the green bands in each direction, and (6) handling a simple network in the form of a three artery triangular loop. Green splits can be provided or, alternatively, flows and capacities given and splits calculated using Webster's theory.

The program produces cycle time, offsets, speeds, and order of left turn phases to maximize the weighted combination of bandwidths. The optimization employs Land and Powell's MPCODE branch and bound algorithm. Up to 12 signals can be handled efficiently. The MAXBAND program is available from the Federal Highway Administration.

Signal setting methods for fixed-time systems separate broadly into two classes. The first, and historically oldest, consists of methods that maximize bandwidth and progression. This group includes, among others, Little and Morgan (1964), Little (1966) and Messer, Whitson, Dudek, and Romano (1974). The second group contains methods that seek to minimize delay, stops or other measures of disutility. Examples are the combination method (Hillier 1966), SIGOP (Traffic Research Corporation 1966), TRANSYT (Robertson 1969), MITROP (Gartner, Little, and Gabbay 1975) and SIGOP II (Lieberman and Woo 1976).

Although disutility-oriented methods have now been available for some time, many traffic engineers continue to prefer maximal bandwidth settings because they have certain inherent advantages. For one thing, bandwidth methods use relatively little input, the basic requirements being street geometry, traffic speeds, and green splits. Secondly, progression systems are operationally robust. Space time diagrams let the traffic engineer visualize easily the quality of the results. Through accumulated experience, engineers with knowledge of the specific streets can spot problems and, if necessary, make modifications to the settings. In addition, various studies (e.g., Wagner, Gerlough and Barnes (1969)) and much practical experience have shown that bandwidth systems give good results in the field. It may even be that drivers expect signal progression and take it as a measure of setting quality. In any case, we take the position here that, if engineers are going to use bandwidth systems, they should have the best.

Morgan and Little (1964) first computerized the setting of arterial signals for maximal bandwidth. The widely distributed program of Little, Martin and Morgan (1966) efficiently finds offsets for maximal bandwidth given cycle time, red times, signal distances and street speed. The total bandwidth attained can be allocated between directions on the basis of flow.

Little (1966) subsequently generalized the computation in several ways: The cycle time could be automatically selected from a given range and so could speed. Networks could be solved. These and several further extensions became possible through a mixed-integer formulation of the problem.

The flexibility thereby introduced has several advantages. For example, maximal bandwidth calculations frequently have a disconcerting feature. On a long street the signals that constrict bandwidth may turn out to be far apart. A small change in design speed or cycle time can

produce quite different signal settings and bandwidth. Yet drivers do not hold their speeds exactly constant and, as shown by Desrosiers and Leighty (1966) tend to adjust their speeds to the signals. Therefore, it makes sense to let design speed between signals be a variable, at least within certain limits. Similarly, it is helpful to be able to consider a range of possible cycle times automatically and determine which one combines best with the street geometry to yield maximal bandwidth. The mixed-integer formulation permits this.

The approach has not become popular, however, for two principal reasons: First, a person must invest substantial effort to learn how to formulate and solve problems in this way. Second, at the time the paper appeared, the solving of mixed-integer problems was inefficient and expensive. Since then, however, methods for solving mixed-integer problems have become better and large scale computations have become cheaper.

Further research described here reveals that the mixed-integer formulation extends to multiphase signals. For example, asymmetric reds occurring when green is delayed for left turns can easily be introduced into the formulation. The decision whether to put a left turn arrow at the beginning or end of green can be assigned to the optimization and resolved in whichever way maximizes total bandwidth.

Messer, Whitson, Dudek, and Romano (1974) have also developed a program, PASSER II, to consider this last issue. An advantage of the present approach is that the multiphase feature is combined with the flexibilities mentioned earlier into a single formulation which can be pursued to a mathematically proven optimum. A further extension is of potential value when secondary flows are significant. Turn traffic entering the artery at a previous intersection may build a queue that interferes with the progression. In this case a time advance can be put into the through-band to permit the queue to clear the intersection before the through platoon arrives.

A portable FORTRAN IV computer program has been developed that maximizes bandwidth according to the methods described in Little (1966)

and extended here. The program, called MAXBAND, is designed to handle arteries and simple 3-artery networks that contain up to 17 signals. The program has been documented in a series of reports to the Federal Highway Administration (Little and Kelson, 1980).

OPTIMIZATION

The optimization formulation draws and generalizes on Little (1966). The basic geometry is shown in Figure 1.

Let

$b(\bar{b})$ = outbound (inbound) bandwidth (cycles).

S_i = ith signal, $i = 1,\ldots,n$.

$r_i(\bar{r}_i)$ = outbound (inbound) red time at S_i (cycles).

$w_i(\bar{w}_i)$ = time from right (left) side of red at S_i to left (right) edge of outbound (inbound) green band (cycles).

$t(h,i)[\bar{t}(h,i)]$ = travel time from S_i to S_h outbound [S_h to S_i inbound] (cycles).

$\phi(h,i)[\bar{\phi}(h,i)]$ = time from center of an outbound [inbound] red at S_h to the center of a particular outbound [inbound] red at S_i. The two reds are chosen so that each is immediately to the left [right] of the same outbound [inbound] green band. $\phi(h,i)[\bar{\phi}(h,i)]$ is positive if S_i's center of red lies to the right [left] of S_h's (cycles).

Δ_i = time from center of $\bar{r}_i$ to nearest center of r_i. Positive if center of r_i is to right of center of $\bar{r}_i$ (cycles).

$\tau_i(\bar{\tau}_i)$ = queue clearance time, an advance of the outbound (inbound) bandwidth upon leaving S_i (cycles).

The fundamental equation for formulating the arterial problem arises from a physical constraint. We derive it with the help of Fig. 1 by expressing the difference in time from (A) to (B) in two
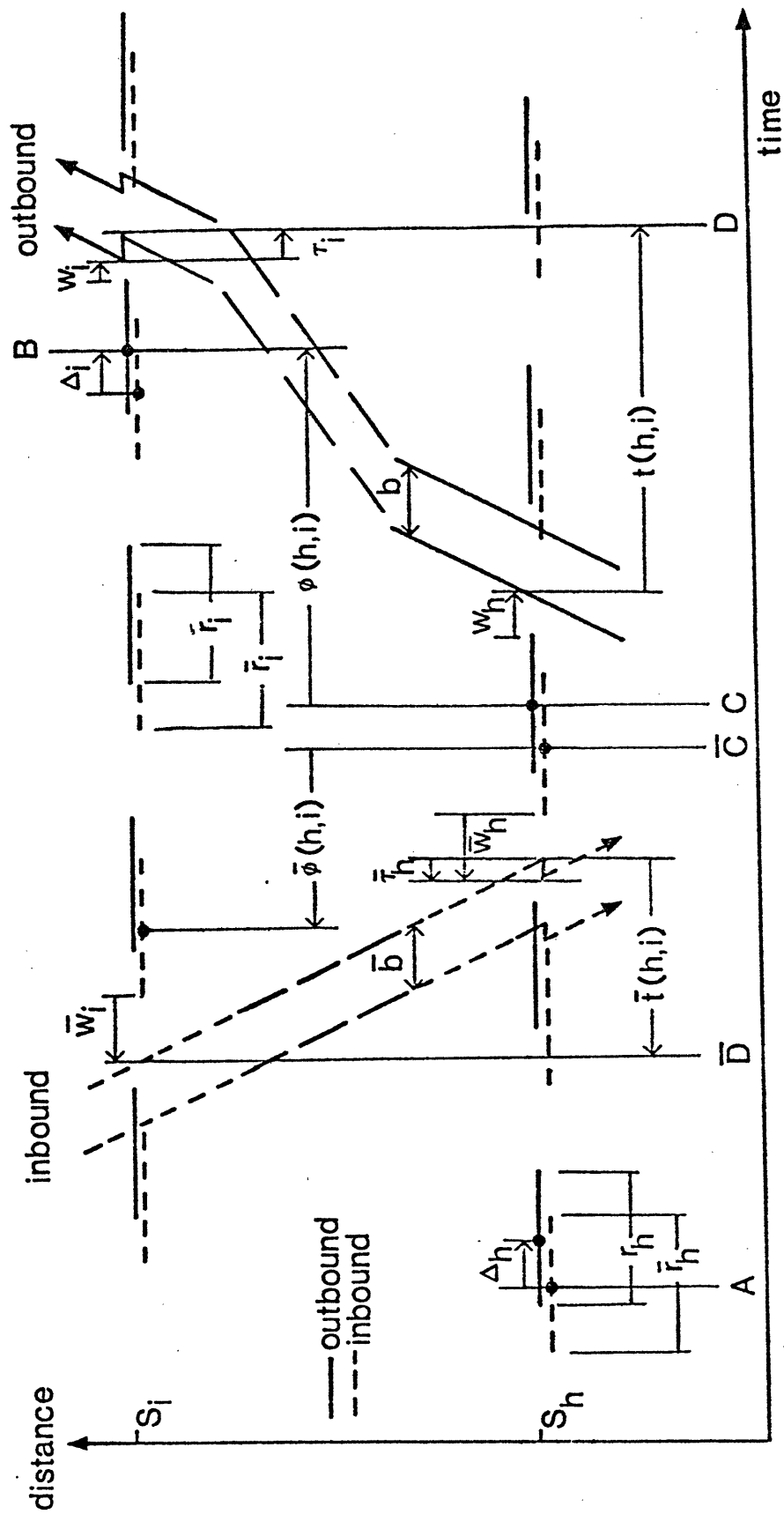
Figure 1.   Space-time diagram showing greenbands.   Inbound and outbound greenbands pass through signals S_h and S_i.   Quantities with bars refer to inbound direction; those without, outbound.   Outbound reds are drawn solid and above inbound reds which are dashed.   In the general case shown inbound and outbound reds need not coincide.

different ways: First, using outbound-defined quantities,

$$\text{time (A) to (B)} = \Delta_h + \text{integer no. of cycles} + \phi(h,i).$$

Then, using inbound-defined quantities,

$$\text{time (A) to (B)} = \text{integer no. of cycles} - \bar{\phi}(h,i)$$

$$+ \text{another integer no. of cycles} + \Delta_i.$$

Setting these times equal, rearranging and coalescing the integers into a single variable, $m(h,i)$:

$$\phi(h,i) + \bar{\phi}(h,i) + \Delta_h - \Delta_i = m(h,i). \qquad (1)$$

We call $m(h,i)$ the <u>loop integer</u> in recognition of the more general case of networks. The terminology applies in the present case because the links S_h to S_i and S_i to S_h form a loop and (1) states that the sum of times around the loop is an integer number of cycles.

From Figure 1, we also read from (C) to (D).

$$\phi(h,i) + (1/2)r_i + w_i + \tau_i = (1/2)r_h + w_h + t(h,i), \qquad (2a)$$

and from $(\bar{C})$ to $(\bar{D})$

$$\bar{\phi}(h,i) + (1/2)\bar{r}_i + \bar{w}_i = (1/2)\bar{r}_h + \bar{w}_h - \bar{\tau}_h + \bar{t}(h,i). \qquad (2b)$$

Substituting (2) into (1) to eliminate ϕ and $\bar{\phi}$ gives:

$$t(h,i) + \bar{t}(h,i) + (1/2)(r_h + \bar{r}_h) + (w_h + \bar{w}_h)$$

$$- (1/2)(r_i + \bar{r}_i) - (w_i + \bar{w}_i) - (\tau_i + \bar{\tau}_h) + \Delta_h - \Delta_i$$

$$= m(h,i). \qquad (3)$$

So far we have required that S_i follow S_h in the outbound direction but this restriction is not necessary. For physical reasons we wish $t(.)$ to satisfy

$$t(h,j) = t(h,i) + t(i,j);$$

whence, setting $h = j$, we shall require

$$t(i,h) = -t(h,i)$$

and by a similar argument $\bar{t}(i,h) = -\bar{t}(h,i)$. With these relations (2) and (3) hold for arbitrary S_h and S_i and

$$\phi(h,j) = \phi(h,i) + \phi(i,j) , \qquad \phi(h,i) = -\phi(i,h)$$

$$m(h,j) = m(h,i) + m(i,j) , \qquad m(h,i) = -m(i,h)$$

along with corresponding expressions for $\bar{\phi}$.

Notation becomes simplier if the signals are numbered sequentially from 1 to n in the outbound direction. Then define

$$x_i = x(i,i+1) \quad \text{for} \quad x = t,\bar{t},m,\phi,\bar{\phi}.$$

Now (3) gives

$$t_i + \bar{t}_i + (w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + \Delta_i - \Delta_{i+1}$$
$$= -(1/2)(r_i + \bar{r}_i) + (1/2)(r_{i+1} + \bar{r}_{i+1}) + (\bar{\tau}_i + \tau_{i+1}) + m_i . \tag{4}$$

From Fig. 1 we also see that

$$w_i + b \leq 1 - r_i \tag{5a}$$
$$\bar{w}_i + \bar{b} \leq 1 - \bar{r}_i . \tag{5b}$$

If for the moment we also require $b = \bar{b}$, we can collect (4) on (5) into a basic mixed integer linear program for setting arterial signals.

<u>LP1</u>: Find $b, \bar{b}, w_i, \bar{w}_i, m_i$ to

$$\max b$$

Subject to $\quad \bar{b} = b$

$$\left.\begin{array}{l} w_i + b \leq 1 - r_i \\ \bar{w}_i + \bar{b} \leq 1 - \bar{r}_i \end{array}\right\} \qquad i = 1,\ldots,n$$

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) + \Delta_i - \Delta_{i+1}$$

$$= -(1/2)(r_i + \bar{r}_i) + (1/2)(r_{i+1} + \bar{r}_{i+1})$$

$$+ (\bar{\tau}_i + \tau_{i+1}) + m_i , \qquad\qquad i = 1,\ldots,n-1$$

$$m_i = \text{integer}$$

$$b, \bar{b}, w_i, \bar{w}_i \geq 0 , \qquad\qquad\qquad i = 1,\ldots,n$$

LP1 has 3n constraints, $2n + 2$ continuous variables and $n - 1$ unrestricted integer variables.

In the formulation of LP1 the green band is defined upon departure from the signal. Therefore, when queue clearance times are introduced, the jog put into the band will, under some circumstances, cause the tail of the arriving band to hit red. If desired, the green band can be defined so as to require room for both arriving greenband and the queue clearance jog. This can be done by adding t_i and $\bar{t}_i$ respectively to the left hand sides of (5a) and (5b) (or (11a) and (11b) below). This change may reduce the bandwidth somewhat.

We next introduce a generalization that permits the optimization program to pick when the left turn phase (if one is present) will occur with respect to the through green at any signal. The left turn green can be picked to lead or lag, whichever gives the most total bandwidth. At the same time, however, the traffic engineer must be able to specify which of the possible combination of leads or lags will be permitted in a given instance.
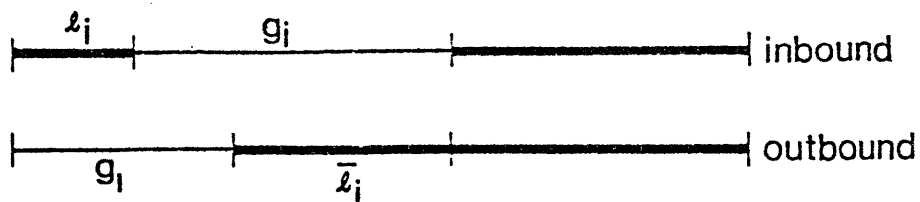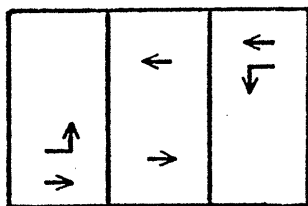
Figure 2 shows the four possible patterns of left turn green phases. Let

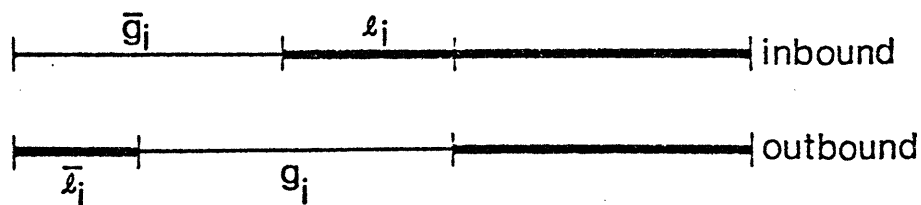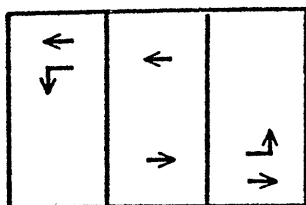$g_i(\bar{g}_i)$ = outbound (inbound) green time for through traffic at S_i (cycles).

$\ell_i(\bar{\ell}_i)$ = time allocated for outbound (inbound) left turn green at S_i (cycles).

R = common red time in both directions to provide for cross street movements (cycles).
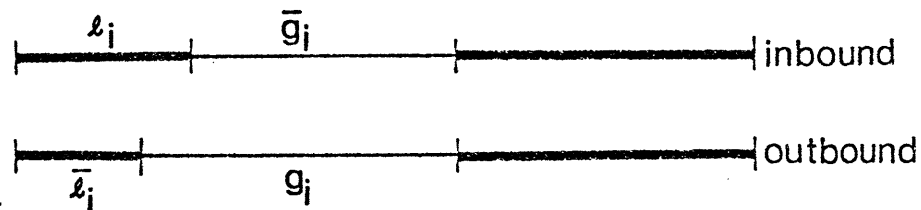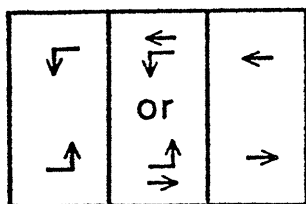
1. Outbound left leads; inbound lags

2. Outbound left lags; inbound leads

3. Outbound left leads; inbound leads
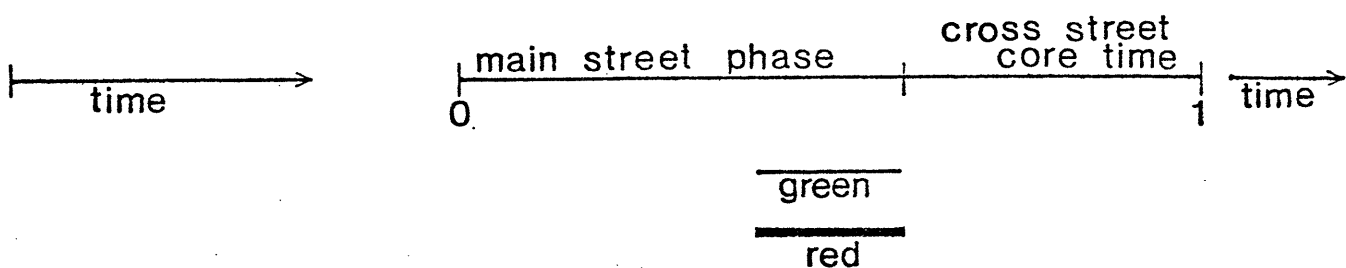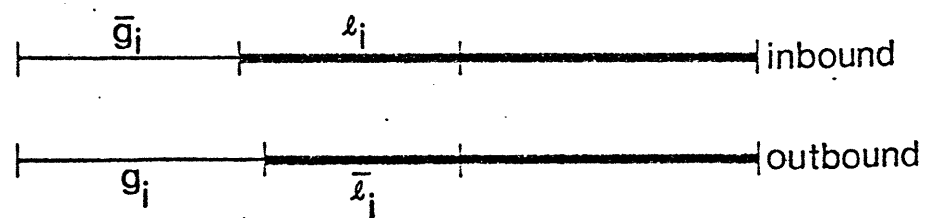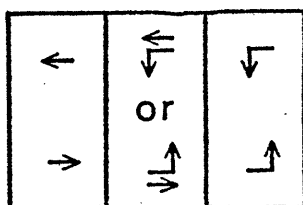

4. Outbound left lags; inbound lags

Figure 2. The 4 possible patterns of left turn phases. ℓ_i is the length of the outbound left turn phase, $\bar{\ell}_i$ of inbound.

Since time allocated to outbound left turn green is inbound red time
we have (see Figure 2):

$$r_i = R + \bar{\ell}_i \qquad \qquad r_i + g_i = 1$$

$$\text{and}$$

$$\bar{r}_i = R + \ell_i \qquad \qquad \bar{r}_i + \bar{g}_i = 1$$

Moreover we can express Δ_i, the time from the center of $\bar{r}_i$ to the next
center of r_i, in terms of ℓ_i and $\bar{\ell}_i$ for each case as follows:

Pattern	Δ_i
1	$-(1/2)(\ell_i + \bar{\ell}_i)$
2	$(1/2)(\ell_i + \bar{\ell}_i)$
3	$-(1/2)(\ell_i - \bar{\ell}_i)$
4	$(1/2)(\ell_i - \bar{\ell}_i)$

All of these can be expressed in the form:

$$\Delta_i = (1/2)[(2\delta_i - 1)\ell_i - (2\bar{\delta}_i - 1)\bar{\ell}_i] \qquad \qquad (6)$$

where δ_i and $\bar{\delta}_i$ are 0-1 variables and the previous cases are now
picked out by

Pattern	δ_i	$\bar{\delta}_i$
1	0	1
2	1	0
3	0	0
4	1	1

Therefore we can use the mixed integer program to select the pattern
that will maximize bandwidth. If only certain patterns are to be
permitted, restrictions can be placed on the δ_i and $\bar{\delta}_i$ to enforce the
requirements. For example, if only patterns 1 and 2 are permitted
the constraint $\delta_i + \bar{\delta}_i = 1$ is added.

We often wish to let the user favor one direction, say, by
manipulating the ratio of inbound to outbound bandwidth. For example,

this ratio might be set to that of the two flows. Such a requirement is easily built into the LP as a constraint. However, in making one green band larger than the other, we can never make it larger than the smallest green in that direction. Once this is achieved, it is foolish to restrict the opposite direction further just to satisfy the ratio. Therefore we speak of a _target_ ratio. Let

k = target ratio of inbound to outbound bandwidth .

For the case of $k < 1$ (outbound favored) we can set up the objective function and ratio constraint as

$$\max\ b + k\bar{b}$$

subject to

$$\bar{b} \geq kb \ .$$

The $k > 1$ case is also accommodated if we change the formulation to

$$\max\ b + k\bar{b}$$

$$(1 - k)\bar{b} \geq (1 - k)kb \ . \tag{7}$$

For $k = 1$, the inequalities (7) must be replaced by $b = \bar{b}$.

A further set of generalizations is possible. One of the most important is to let signal period (cycle length) and speed be variables. Each will be constrained by upper and lower limits. In addition, changes in speed from one street segment to the next can be limited.

Let

T = cycle length (signal period) (seconds). ·

$\bar{z}$ = $1/T$ = signal frequency (cycles/second).

T_1, T_2 = lower and upper limits on cycle length, i.e.
$T_1 \leq T \leq T_2$ (seconds).

$d(h,i)\ [\bar{d}(h,i)]$= distance between S_h and S_i outbound (inbound) (meters).

$$d_i = d(i,i+1), \quad \bar{d}_i = \bar{d}(i, i+1)$$

$$e_i, f_i (\bar{e}_i, \bar{f}_i) = \text{lower and upper limits on outbound (inbound)}$$
$$\text{speed (meters/second).}$$

$$1/h_i, 1/g_i (1/\bar{h}_i, 1/\bar{g}_i) = \text{lower and upper limits on change in outbound}$$
$$\text{(inbound) reciprocal speed, i.e.,}$$

$$1/h_i \leq (1/v_{i+1}) - (1/v_i) \leq 1/g_i \text{ (meters/sec)}^{-1}.$$

We are constraining change in speed by putting upper and lower limits on change in reciprocal speed. Although the two are not quite the same, either serves to prevent large abrupt speed changes. Reciprocal speed is used because it enters linearly in the constraints and can be transformed into t_i. Thus

$$t_i = (d_i/v_i)z \quad \text{and} \quad \bar{t}_i = (\bar{d}_i/\bar{v}_i)z . \qquad (8)$$

In the expanded formulation, t_i, $\bar{t}_i$, and z are decision variables which, once known, determine progression speeds.

We add to LP1 all these generalizations to yield a more versatile mixed-integer linear program:

$\underline{\text{LP2}}$. Find $b, \bar{b}, z, \bar{w}_i, t_i, \bar{t}_i, \delta_i, \bar{\delta}_i, m_i$

subject to $\qquad$ to max $b + k\bar{b}$

$$(1 - k)\bar{b} \geq (1 - k)kb \qquad\qquad (9)$$

$$1/T_2 \leq z \leq 1/T_1 \qquad\qquad (10)$$

$$\left.\begin{array}{l} w_i + b \leq 1 - r_i \\[2mm] \bar{w}_i + \bar{b} \leq 1 - \bar{r}_i \end{array}\right\} \qquad i = 1,\ldots,n \qquad \begin{array}{l} (11a) \\[2mm] (11b) \end{array}$$

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) + \delta_i \ell_i - \bar{\delta}_i \bar{\ell}_i$$
$$- \delta_{i+1}\ell_{i+1} + \bar{\delta}_{i+1}\bar{\ell}_{i+1} - m_i$$
$$= (r_{i+1} - r_i) + (\bar{r}_i + r_{i+1}) , \qquad i = 1,\ldots,n-1 \qquad (12)$$

$$\left.\begin{array}{l} (d_i/f_i)z \leq t_i \leq (d_i/e_i)z \\[2mm] (\bar{d}_i/\bar{f}_i)z \leq \bar{t}_i \leq (\bar{d}_i/\bar{e}_i)z \end{array}\right\} \qquad i = 1,\ldots,n-1 \qquad \begin{array}{l} (13a) \\[2mm] (13b) \end{array}$$

$$(d_i/h_i)z \le (d_i/d_{i+1})t_{i+1} - t_i \le (d_i/g_i)z$$ (14a)

$$(\bar{d}_i/\bar{h}_i)z \le (\bar{d}_i/\bar{d}_{i+1})\bar{t}_{i+1} - \bar{t}_i \le (\bar{d}_i/\bar{g}_i)z$$ (14b)

$$i = 1,\ldots,n-2$$

$$b, \; \bar{b}, \; z, \; w_i, \; \bar{w}_i, \; t_i, \; \bar{t}_i \ge 0$$

$$m_i \text{ integer}$$

$$\delta_i, \bar{\delta}_i \; 0,1$$

where if $k = 1$ (9) is replaced by $\bar{b} = b$.

LP2 involves $(11n-10)$ constraints and $(4n + 1)$ continuous variables, up to $2n$ 0-1 variables and $n-1$ unrestricted integer variables, not counting slack variables. In addition, if the user decides to require or prohibit certain left turn patterns, constraints on δ_i and $\bar{\delta}_i$ are added up to a maximum of $2n$.

LP2 describes how the arterial case is solved. The triangular loop consists of three arteries. Its mathematical program consists of (1) an objective function which is a weighted combination of the objective functions of the individual LP2's (the weights being set by the user to express the relative importance of bandwidth on the three arteries), (2) all the constraints of the individual LP2's and (3) the loop constraint. The loop constraint is

$$\sum_{(i,j)\epsilon L} \psi_{ij} = n_L$$

where ψ_{ij} is the offset (in cycles) for link (i,j) in loop L and n_L is the loop integer.

DETERMINING GREEN SPLITS

One option in MAXBAND is for the user to supply the green splits.

Alternatively, the user can provide traffic volume and capacity information and the program will calculate the splits. This is done essentially by using the theory of Webster (1958). Webster has shown that under certain circumstances, total delay at an intersection is minimized by dividing the available cycle time among competing streams of traffic proportional to their volumes divided by their capacities.

In MAXBAND, the user who wishes to use this option provides volume and capacity information for the traffic, classified into 4 through movements and 4 left turn movements of each intersection. Let

$TRAT(i)$ = through-traffic ratio of volume to capacity in direction i.

$LRAT(i)$ = left turn traffic ratio of volume to capacity in direction i.

i = OUT, IN, OUTC, INC = outbound main, inbound main, outbound cross street, inbound cross street.

The procedure calculates

$MAIN$ = $\max\{TRAT(OUT) + LRAT(IN), TRAT(IN) + LRAT(OUT)\}$ = the larger of through volume/capacity plus opposite left turn volume/capacity for the two directions on the main street.

$CROSS$ = $\max\{TRAT(OUTC) + LRAT(OUTC), TRAT(INC) + LRAT(OUTC)\}$ = the same quantity for the cross street.

The basic split between main street and cross street is

$$MM = \frac{MAIN}{MAIN + CROSS} = \text{total time allocated to main street (fraction of cycle)}$$

$$CC = \frac{CROSS}{MAIN + CROSS} = \text{total time allocated to cross street (fraction of cycle)}$$

-14-

Let

L(OUT) [L(IN)] = outbound [inbound] left split.

G(OUT) [G(IN)] = outbound [inbound] through split.

Then

$$L(OUT) = \frac{LRAT(OUT)}{TRAT(OUT) + LRAT(IN)} \times MM$$

$$L(IN) = \frac{LRAT(IN)}{TRAT(OUT) + LRAT(IN)} \times MM$$

$$G(OUT) = MM - L(IN)$$

$$G(IN) = MM - L(OUT)$$

If necessary, these splits are then modified slightly to meet minimum green requirements.

COMPUTER PROGRAM

The computer program consists of (1) a control section, (2) an input section that accepts the problem data from the user, (3) a matrix generator that transforms the input into a form usable by the mixed integer linear program, (4) a mathematical programming package and, (5) an output routine that interprets the mathematical programming results and prints them out in a form usable by a traffic engineer. The program contains approximately 11,000 lines of FORTRAN IV code, broken out as follows: control program, 100; input program, 3,200; matrix generator, 3,200; MPCODE, 2,500; and output program, 2,000.

CONTROL PROGRAM

The control program manages the overall computation, calling each of the other programs as needed.

INPUT

Input to the program is on IBM cards or a corresponding card image file on another medium such as magnetic tape or disk. The basic inputs are:

1. **Overall problem information.** This includes a name for the run, an indicator for whether it is a loop problem or an artery problem, an indicator for whether metric or English units are used, the acceptable range of cycle lengths, and the target ratio for the bandwidths on each artery and their weights, unless these are to be computed from volume information. Usually default values are used in the mathematical programming package for certain parameters, such as the maximum number of iterations and reinversions, but, if the user wishes, these can be supplied as part of the overall problem information.

2. <u>Network geometry</u>. The order of signals on each artery is given, along with the distances between them (which may be different in each direction), and the names of their intersections. In the case of a 'loop the intersection numbers at artery meetings are specified.

3. <u>Green splits or traffic flows and capacities</u>. The user may specify the green splits at each signal as a fraction of the overall cycle time. Alternatively, traffic flows and capacities can be given or each link, including cross streets and turning movements, and the program will calculate green splits using Webster's formula.

4. <u>Left turning patterns</u>. Left turn phases can occur at the beginning or end of green in either direction, creating four possible patterns for the through direction at each intersection. The user can specify which of the patterns are acceptable and the program will choose among them to maximize bandwidth.

5. <u>Queue clearance time</u>. Queues may build up during red time as a result of turning movements onto the artery at previous inter-sections. Such queues may impede the flow of vehicles in the through band. The user may, therefore, specify at any intersection in either direction a queue clearance time as a fraction of the cycle length. The program will adjust the through band to arrive at the intersection after the queue has cleared and leave the intersection with the queue included as part of the band. In effect, this puts a jog into the through band, advancing it upon leaving the intersection by an amount equal to the queue clearance time.

6. <u>Range of speed</u>. For each link or, if preferred, for the artery as a whole, the user specifies a design speed and a speed tolerance. The program then chooses speeds for each link from this range so as to maximize bandwidth. In addition, the user can constrain the change in speed from one link to the next. If the user does not set limits on speeds and speed changes, default values of 10% are used.

Further, more detailed specification of inputs is found in the USERS MANUAL.

MATRIX GENERATOR

The mathematical program LP2 is a special case of the general linear mixed integer problem:

$$\begin{aligned}
\max \quad & cx \\
\text{subject to} \quad & Ax \lessgtr b \\
& x \geq 0 \\
& x_j \text{ integer } j \in J
\end{aligned}$$

where x is an n-vector of variables whose values are sought, c is an n-vector of objective functions coefficients, A is an m x n matrix of coefficients, b is an m-vector of right hand side constants and J is a set of subscripts identifying the variables required to be integer.

The traffic problem described earlier as LP2 requires specific values for A, b, c, and J. These will change as the traffic problem changes. It is necessary to have a generalized program, called the matrix generator, which will take the traffic input data and convert it into the appropriate vectors and matrices for input to the mathematical programming package.

MATHEMATICAL PROGRAMMING PACKAGE

A key part of the computer program is the routine for solving the mixed integer linear program. After examing several linear programming codes and making comparative runs on two mixed integer packages, MPCODE of Land and Powell (1973) was selected. MPCODE is available in FORTRAN IV source code and is superbly documented in a hard cover book.

OUTPUT

The output is divided into three parts:

1. Input cards.
2. Data summary.
3. Solution report.

The <u>input card</u> section is a simple printing of the input cards.

The <u>data summary</u> report contains the following information:

(a) MPCODE values used by the Land and Powell system.

(b) For an artery:

(1) General information such as the name of the artery, the number of signals, limits on cycle length, units, target bandwidth ratio, and bandwidth weights.

(2) Artery-wide values such as design speed, tolerances, and limits on changes between links.

(3) Intersection values including splits with an indication of their origin, queue clearance times, minimum greens, and the permitted patterns for left turns.

(4) Link values as actually used including length, design speed and speed tolerances.

(5) Volumes and capacities on all approaches, when provided.

(c) For a loop:

(1) General loop information including upper and lower limits on cycle time and where the arteries meet.

(2) The same artery information described above but for each of the three arteries.

The <u>solution report</u> presents the following data:

(a) An indicator whether the problem has been solved successfully.

(b) MPCODE statistics describing the number of iterations, etc., used by the Land and Powell algorithm to solve the problem. "Number of solutions" is the number of integer solutions (including the optimal integer solution) found for the problem.

(c) For an artery:

(1) General information including name of artery, number of signals, and type of units.

(2) Cycle time and bandwidths.

(3) Left patterns selected as optimal.

(4) Duration and offsets of splits in both fractions of a cycle and seconds.

(5) Traversal times and speeds on links.

(d) For a loop:

(1) Loop information including chosen cycle time, bandwidths, and objective function.

(2) Same information as for a single artery for each of the three arteries.

(3) Repeat of duration and offsets of splits for signals at artery meetings.

Examples of outputs for several test problems are found in Little and Kelson (1980).

TESTING

The testing of the program has included runs on a wide variety of problems and operation on several computers.

Table 1 displays run statistics for 10 arterial problems and 3 loop problems. The number of variables, constraints, integer variables (total and free) relate to the mixed integer program and represent measures of program difficulty. The number of branch and bound iterations is another measure of how much computation the program required. The number of solutions is the number of feasible integer solutions discovered up to and including the optimal one. Input data for several of the problems can be found in Little and Kelson (1980).

The CPU time is the number of seconds required on the MIT IBM 370-168 and is the primary performance measure. As can be seen, most problems have been solved in a few seconds, although one problem, the 15 signal FHWA test network loop problem, took 10.5 minutes. The cost shown is the cost charged by the MIT Information Processing Service.

The settings developed by MAXBAND have been put into a traffic simulation program (NETSIM) for several of the test networks with good results. In addition, the MAXBAND output can be used to construct a space-time plot. Figure 3 depicts Main Street, Waltham, Massachusetts. Figure 4 shows space-time diagrams for its MAXBAND generated settings.

The MAXBAND program has been transmitted to FHWA on magnetic tape and has been operated on computers to which FHWA has access.

Table 1. MAXBAND performance statistics.

Problem	Problem Characteristics					Solution Characteristics			
	# Signals	# Vars.	# Cons.	# Int.Vars Total	Free	# Iter.	# Sols.	CPU time (sec)	Cost ($)
ARTERIES									
1. Broadway, Cambridge	5	25	45	4	4	36	1	2.17	0.73
2. Voorhees Scenario 1	6	30	56	5	5	668	2	5.81	1.07
3. Short version of Waltham artery	6	33	56	8	8	197	3	3.23	0.76
4. Voorhees Scenario 2	6	36	56	11	11	957	6	7.01	1.23
5. Voorhees Scenario 2 - computed splits	6	36	56	11	11	1457	5	9.74	1.52
6. Modified Voorhees Scenario 2 - Computed splits	6	42	56	17	17	607	7	6.26	1.35
7. Voorhees Scenario 1 - Computed splits	6	42	56	17	17	2089	6	14.08	2.03
8. Modified Waltham artery	11	60	116	15	10	1296	4	17.16	2.44
9. Waltham artery	11	60	111	15	15	3781	7	44.79	5.69
10. Wisconsin Ave., Wash. DC	17	88	177	19	19	8700	12	~210	~25.00
LOOPS									
11. Modified Attleboro loop	4	36	44	5	5	286	3	3.69	0.84
12. Attleboro loop	4	37	44	5	5	432	4	4.44	0.92
13. FHWA Test Network	15	93	168	19	16	32885	15	628.81	74.87

Notes: Vars. = Variables
        Cons. = Constraints
        Int. Vars. = Integer Variables
        Iter. = Total number of simplex iterations used
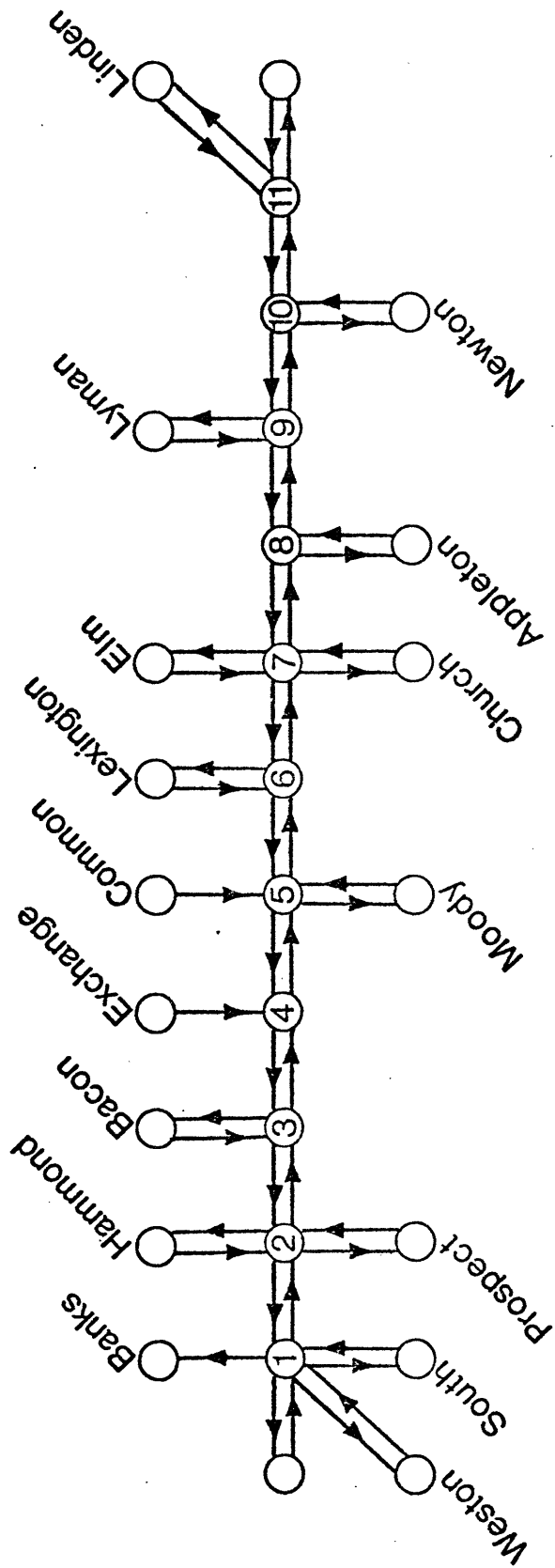   Sols. = Total number of feasible integer solutions found

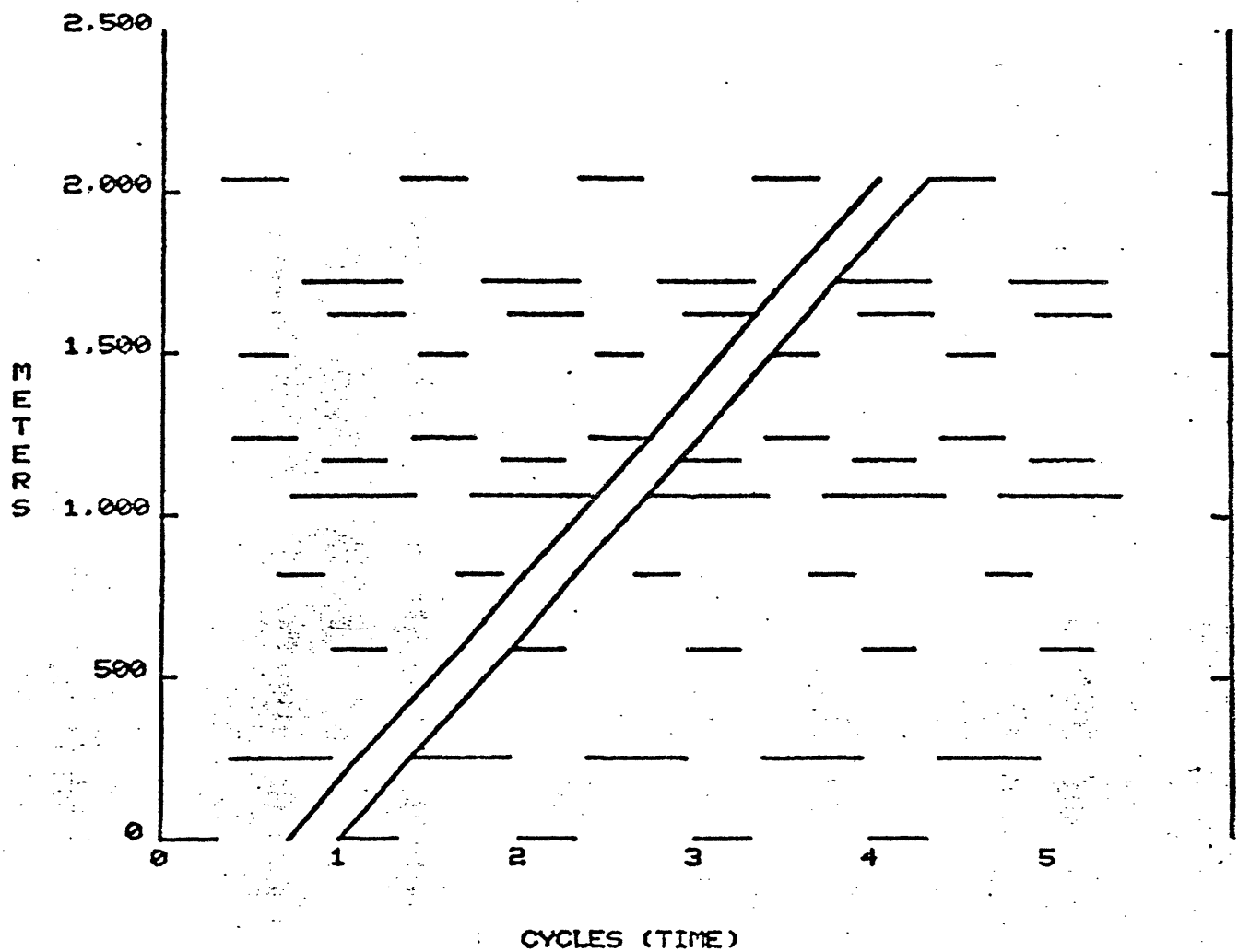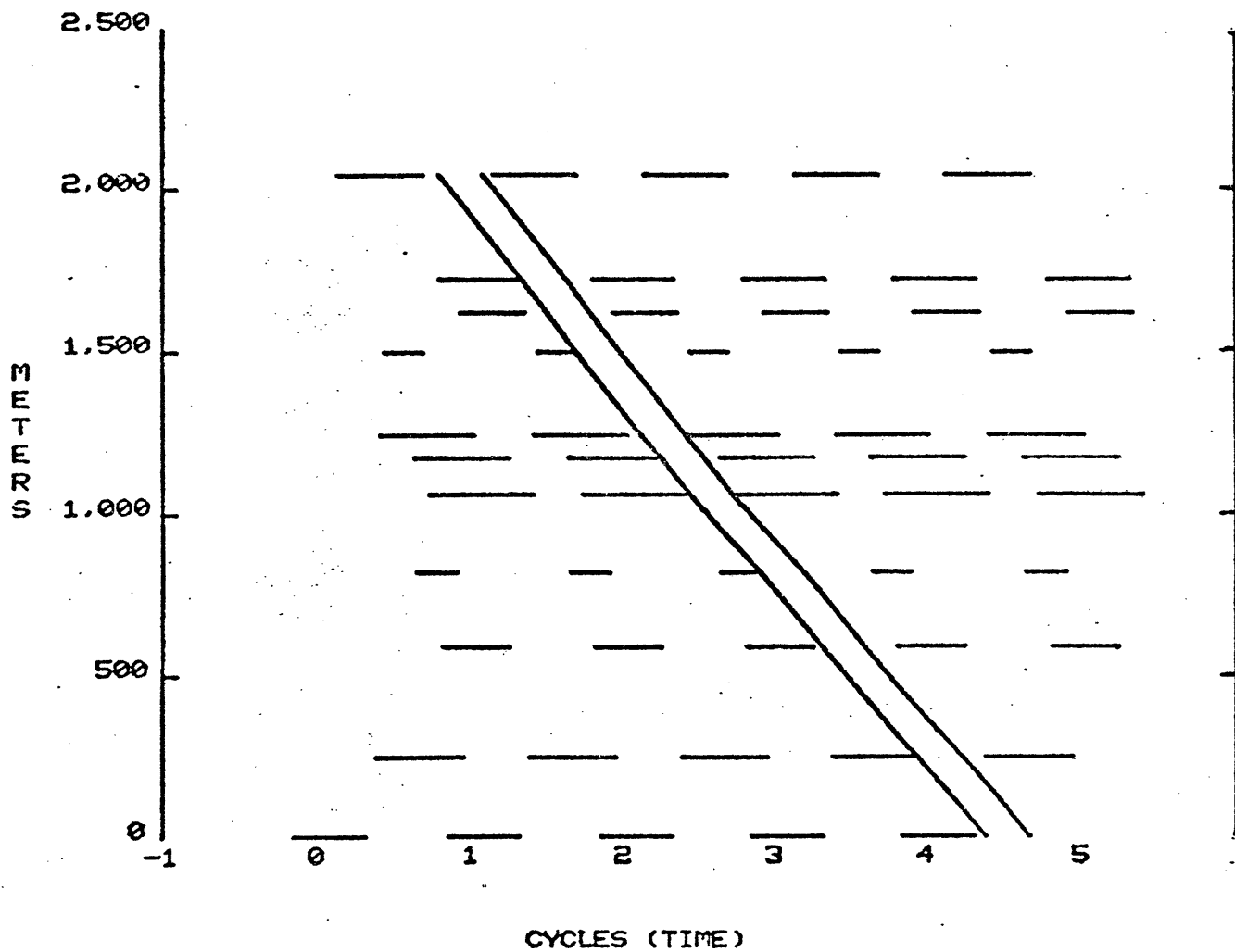Figure 3. Main Street, Waltham, MA. 11-signal test problem.

Figure 4. Main Street, Waltham. (a) Outbound greenband. Note slight changes in slope resulting from 10% permitted variation in design speed.

(b) Inbound greenband. Red times differ from outbound because of left
turn phases.

DOCUMENTATION

Little and Kelson (1980) provide documentation for MAXBAND in three volumes: Volume 1, SUMMARY REPORT provides an overview of MAXBAND, including complete input and run data on three test networks. Volume 2, the USER'S MANUAL describes the MAXBAND system and how to use it in detail, including worked out examples of a basic symmetric artery, a basic asymmetric artery, a general artery, and a loop. Volume 3, PROGRAMMER'S MANUAL gives computer program documentation. First an overall description of the program is provided, organized by subroutine. Then, a listing is given for each subroutine along with a description of what it does. Additionally, major variables used in the subroutine are listed. No attempt is made to document MPCODE, since excellent documentation already exists in Land and Powell (1973).

ACKNOWLEDGEMENTS

REFERENCES

R. D. Desrosiers and C. H. Leighty, "Traffic Flow Responses to Unannounced Increases in Progression Speeds of Signal Systems," *Public Roads*, 34, Number 1, 1 - 15 (April 1966).

N.H. Gartner, J.D.C. Little and H. Gabbay, "Optimization of Traffic Signal Settings by Mixed-Integer Linear Programming, Parts I and II," *Transportation Science 9*, 321-363 (November 1975).

J.A. Hillier, "Appendix to Glasgow's Experiment in Area Traffic Control," *Traffic Engineering and Control 7*, 569-571 (1966).

A. Land and S. Powell, *FORTRAN Codes for Mathematical Programming*, John Wiley & Sons, Ltd., London, 1973.

E.B. Lieberman and J.L. Woo, "SIGOP II: A New Computer Program to Calculate Optimal Signal Timing Patterns," KLD Associates Inc., January 1976.

J.D.C. Little, "The Synchronization of Traffic Signals by Mixed-Integer Linear Programming," *Operations Research 14*, 568-594 (1966).

J.D.C. Little and M.D. Kelson, "Optimal Signal Timing for Arterial Signal Systems: Volume 1. Summary Report, Volume 2. User's Manual, Volume 3. Programmer's Manual," Report to FHWA on Contract DOT-FH-11-9562, April 1980.

J.D.C. Little, B.V. Martin, and J.T. Morgan, "Synchronizing Traffic Signals for Maximum Bandwidth," *Highway Research Record 118*, 21-47 (1966).

C.J. Messer, R.H. Whitson, C.L. Dudek, and E.J. Romano, "A Variable-Sequence Multiphase Progression Optimization Program," *Highway Research Record 445*, 24-33 (1974).

J.T. Morgan and J.D.C. Little, "Synchronizing Traffic Signals for Maximal Bandwidth," *Operations Research 12*, 896-912 (1964).

D.I. Robertson, "TRANSYT: A Traffic Network Study Tool," Road Research Laboratory Report LR 253, Crowthorne, Berkshire (1969).

Traffic Research Corporation, "SIGOP: Traffic Signal Optimization Program--A Computer Program to Calculate Optimum Coordination in a Grid Network of Synchronized Traffic Signals," New York, September 1966, Clearinghouse PB 173 738.

F.A. Wagner, D.L. Gerlough, and F.C. Barnes, "Improved Criteria for Traffic Signal Systems on Urban Arterials," *National Cooperative Highway Research Program Report 73*, Highway Research Board (1969).

F.V. Webster, "Traffic Signal Settings," Road Research Technical Paper Number 39, Her Majesty's Stationery Office, London (1958).