Transportation Research Part C 71 (2016) 215–230

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Signal coordination models for long arterials and grid networks

Lihui Zhang [a], Ziqi Song [b,*], Xiaojun Tang [c], Dianhai Wang [a]

[a] Institute of Transportation Engineering, Zhejiang University, Hangzhou 310058, China
[b] Department of Civil and Environmental Engineering, Utah State University, Logan, UT 84322, United States
[c] School of Transportation and Logistics, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Received 16 April 2016
Received in revised form 28 July 2016
Accepted 30 July 2016

Keywords:
Signal coordination
Long arterial
Grid network
Bandwidth maximization
Mixed-integer linear program

ABSTRACT

This paper proposes two models to tackle traffic signal coordination problems for long arterials and grid networks. Both models, denoted as MaxBandLA and MaxBandGN, are built based on Little's bandwidth maximization model, and the resulting formulations are both small-sized mixed-integer linear programs. Model MaxBandLA can optimize arterial partition plan and signal coordination plans of all the subsystems simultaneously. Model MaxBandGN directly optimizes the offsets for all the signals in a grid network, and as such, no 'cycle constraints' need to be constructed. Numerical tests are presented to show that both models have the potential to produce coordination plans that are comparable to signal plans optimized by Synchro.

1. Introduction

Signal coordination is a traffic control strategy that determines the timings of grouped signals together so as to improve overall traffic flow propagation. It has long been recognized as one of the most efficient and economical methods to avoid or mitigate traffic congestion.

Extensive research can be found in the literature that investigated various signal coordination problems. A recent review of relevant studies can be found in Zhang et al. (2015). Generally, the models proposed are of two types: bandwidth-based models and performance-based models. Little's bandwidth maximization model maximizes two-way green bandwidths of a given arterial so that vehicles may have larger chances to traverse the arterial without any stops (Little, 1966). The model is in the form of a mixed-integer linear program (MILP), and can be solved to global optimum very efficiently with readily available commercial solvers. This seminal model lays the foundation for many studies carried out over several decades. For example, Chang et al. (1988) introduced an extension of the model to consider left-turn phase sequence in general networks, Gartner et al. (1991) proposed another variant that maximized the bandwidth of each individual road section, Zhang and Yin (2008) and Li (2014) considered the uncertainties in arterial signal coordination, and Gomes (2015) introduced vehicle arrival functions to maximize the benefit from signal coordination. Performance-based models try to optimize signal settings to directly improve measures of effectiveness relating to delay, stop or queue, see, e.g., Park et al. (1999), Zhang et al. (2010), Hu et al. (2013), He et al. (2014) and Ye et al. (2015). Yang (2001) applied both bandwidth-based and delay-based models to an arterial with nine signalized intersections in Lawrence, Kansas and found that the bandwidth-based approach generally outperformed the delay-based approach. The models proposed in this paper are built based on Little's MILP formulation, thus the introduction primarily focuses on bandwidth-based models.

* Corresponding author.
E-mail addresses: lihuizhang@zju.edu.cn (L. Zhang), ziqi.song@usu.edu (Z. Song), tangxiaojun@mail.dlut.edu.cn (X. Tang), wangdianhai@zju.edu.cn (D. Wang).

http://dx.doi.org/10.1016/j.trc.2016.07.015
0968-090X/

According to the formulations of bandwidth-based models, it's not hard to observe that when the number of signals in a system increases, the green bandwidths generated will decrease, due to that more constraints need to be imposed on the bandwidths. Under their problem settings, Ma et al. (2011) actually found that two-way green bandwidths were not achievable when the number of signals increased to sixteen, indicating that it may not be a wise strategy to coordinate very long arterials as a whole system. A natural idea that came up was to divide a long arterial into several small subsystems and then perform signal coordination for each individual subsystem. Conventional approaches to determine the partition plan of a long arterial are mainly based on evaluation indexes, such as Coupling Index, Strength of Attraction and Coordinatability Factor, which consider factors like traffic volume, signal distance, travel speed and cycle length (Hook and Albers, 1999). These approaches may still produce long subsystems when the evaluated signal indexes along the arterial are similar. Tian and Urbanik (2007) proposed a heuristic that divided an arterial into several subsystems with three to five intersections based on distance, volume, queue length and saturation degree between adjacent intersections. Zhang and Zhang (2014) applied improved K-means clustering to search for feasible partition plans, and then utilized PASER-II to produce coordination plans for all the subsystems. These above studies provide different methodologies to coordinate signals on long arterials, but none proposes an explicit mathematical formulation for the problem.

While studies on long arterials are rare, much attention has been paid to the signal optimization problem for area-wide networks. Algorithms developed for adaptive control systems optimize network signal timings step-by-step with coordination plans explicitly or inexplicitly embedded. Their control objectives are usually performance-based, see, e.g., Timotheou et al. (2015), Yang and Jayakrishnan (2015), Di Febbraro et al. (2016) and Guilliard et al. (2016). Some other methodologies are more suitable for off-line optimization, e.g., Ozan et al. (2015) combined a reinforcement learning algorithm with TRANSYT-7F to optimize area-wide signal timings to minimize a weighted sum of delay and stop. Cantarella et al. (2015) proposed two strategies that employed meta-heuristics to solve the network signal setting problem, aimed at minimizing total system delay. Recent studies are mostly performance-based, while few look at green bandwidths. Actually, Little (1966) also proposed a methodology to coordinate signals at the network level, with an aim to maximize the weighted sum of the bandwidths of all the arterials in the network. The network program mainly consists of the individual arterial coordination programs and the 'cycle constraints'. When coordinated arterials form a closed loop, the offsets of those signals at the intersections of the arterials will sum up to be integer numbers of the half cycle length, which is called 'cycle constraint'. One 'cycle constraint' needs to be introduced whenever one closed loops is formed by coordinated arterials. If the analyzed network is too large, it can be expected that the procedure to build all possible cycle constraints can be time consuming. Most of the bandwidth-based network coordination studies follow the above methodology, see, e.g., Chaudhary et al. (1991) and Gartner and Stamatiadis (2002, 2004).

This paper attempts to synchronize signalized long arterials and gird networks along the line of bandwidth maximization. The model to be built for long arterials is able to simultaneously generate an optimal network partition plan, as well as the coordination plans for all the subsystems generated. And the model for grid networks takes the offsets of all the signals as decision variables, thus that no efforts are required to build 'cycle constraints'. The both models obtained are explicit mathematical models in the form of MILPs, which are small-sized and can be solved to global optimum in seconds by commercial solvers. The remainder of the paper is organized as follows: Section 2 briefly introduces Little's bandwidth maximization formulation. Section 3 builds the coordination model for long arterials, and three numerical tests are presented. Section 4 gives the coordination model for grid networks with another set of numerical tests. Concluding remarks are provided in the last section.

2. Introduction to Little's MILP formulation

Little's bandwidth maximization model is covered here first since the models proposed later are built upon this model. Given an arterial with fixed number of signals, whose split information are known in advance, the bandwidth maximization model optimizes the common cycle length and offsets for these signals to maximize the sum of inbound and outbound green bandwidths.

Let S_h and S_i be two successive intersections in the outbound direction. Fig. 1 shows the green bandwidths between the two signals. Some notations used in the formulation are given first as follows:

$b/\bar{b}$	outbound /inbound bandwidth (cycle)
r_i	red time of the synchronized phase of signal i (cycle)
$t(h,i)/\bar{t}(i,h)$	travel time from signal $h(i)$ to signal $i(h)$; $t_i = t(i,i+1)$
$w_i/\bar{w}_i$	time from the right/left side of $S_i's$ red to the green band (cycle)
$\phi(h,i)/\bar{\phi}(i,h)$	time from the center of red at S_h to the center of a particular red at S_i. The two reds are chosen so that each is immediately to the left /right of the same outbound /inbound green band (cycle)
$\tau(h,i)$	$= \phi(h,i) + \bar{\phi}(i,h)$; $\tau_i = \tau(i,i+1)$
T_1, T_2	lower and upper bounds on cycle length (s)
z	signal frequency, or the inverse of cycle length
$d(h,i)$	distance from S_h to S_i (s); $d_i = d(i,i+1)$
$e_i, f_i/\bar{e}_i, \bar{f}_i$	lower and upper bounds on outbound /inbound speed (m/s)

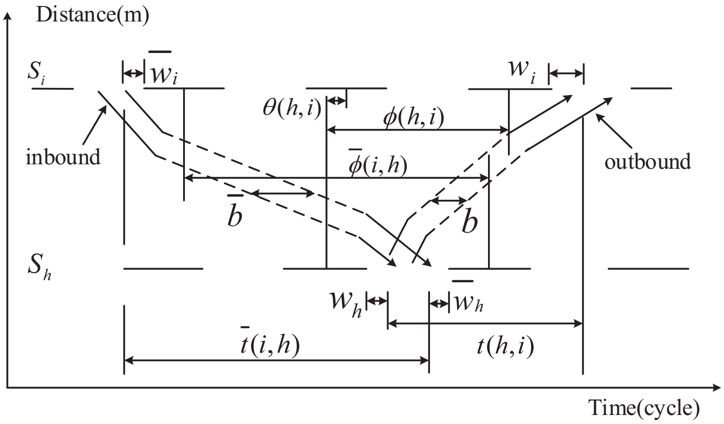

Fig. 1. Illustration of green bands.

In Fig. 1, the horizontal lines mean the red durations of the phases to be synchronized, and the zigzag lines are the trajectories of vehicles passing the arterial without meeting any red signals. All such continuous trajectories form a green band for each travel direction, whose horizontal width (b or $\bar{b}$) is called green bandwidth, measured in time or cycle. The green bands repeat once per cycle in parallel bands along the time horizon.

According to the geometry information shown in Fig. 1, the bandwidth maximization problem can be explicitly written as follows:

$$\max_{(b,\bar{b},w,\bar{w},t,\bar{t},\tau)} \quad B = b + \bar{b} \tag{1}$$

$$s.t. \quad 1/T_2 \leqslant z \leqslant 1/T_1 \tag{2}$$

$$w_i + b \leqslant 1 - r_i \quad \forall i = 1, \ldots, I \tag{3}$$

$$\bar{w}_i + \bar{b} \leqslant 1 - r_i \quad \forall i = 1, \ldots, I \tag{4}$$

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) = \tau_i - (r_i - r_{i+1}) \quad \forall i = 1, \ldots, I-1 \tag{5}$$

$$\tau_i = integer \quad \forall i \in [1, I-1] \tag{6}$$

$$(d_i/f_i)z \leqslant t_i \leqslant (d_i/e_i)z \quad \forall i = 1, \ldots, I-1 \tag{7}$$

$$(d_i/\bar{f}_i)z \leqslant \bar{t}_i \leqslant (d_i/\bar{e}_i)z \quad \forall i = 1, \ldots, I-1 \tag{8}$$

$$b, \bar{b}, w, \bar{w}, \geqslant 0 \tag{9}$$

For presentation simplicity, the above system does not contain constraints on the speed changes between adjacent road segments. The objective function is to maximize the sum of the two-way green bandwidths, where $b, \bar{b}, w, \bar{w}, t, \bar{t}$ and τ represent vectors whose elements are the scalar decision variables $b_i, \bar{b}_i, w_i, \bar{w}_i, t_i, \bar{t}_i$ and τ_i. Eq. (2) limits the range of feasible cycle lengths. Eqs. (3) and (4) are constraints on green bandwidths. Eqs. (5) and (6) are integer constraints due to the fact that $\phi(h,i) + \bar{\phi}(i,h)$ must be integer. Eqs. (7) and (8) are constraints on two-way travel speeds, and the last constraint (9) is the non-negativity constraint. With all the constraints being linear, the above formulation is a mixed-integer linear program, which can be solved to the global optimum with algorithms like the branch-and-bound algorithm. After solving the program, offsets of the signals can be obtained as:

$$\theta_{i+1} = [w_i - w_{i+1} + t_i + 1/2(r_i - r_{i+1})]_+ \quad \forall i = 1, \ldots, n-1 \tag{10}$$

where

θ_{i+1}	the offset of signal $i+1$ with respect to signal i
$[x]_+$	$= x - \text{int}(x)$; int(x) is the largest integer not greater than x

To be noted, Eq. (10) is not linear in the present form.

3. Signal coordination of long arterials

3.1. Model formulation

According to the system (1)–(9), when more signals are coordinated, smaller two-way bandwidths will be generated. This section expands the above formulation to deal with long arterials. To be noted, such arterials do not contain intersections with the surrounding traffic flow patterns obviously different from other intersections, or are far distanced from other intersections along the arterials. These intersections will be the natural break points of the long arterials, making the long arterials into shorter ones.

For arterials without such intersections, we propose an explicit mathematical model below to optimize coordination plans. Subscript m is used to denote variables and parameters associated with subsystem m. For an arterial with a total number of I signals, there must be $I-1$ road segments in total. To represent the arterial partition plan, we introduce two sets of binary variables β_m^i and α_m^i, defined as follows:

$$\beta_m^i = \begin{cases} 1 & \text{signal } i \text{ belongs to subsystem } m \\ 0 & \text{signal } i \text{ doesn't belong to subsystem } m \end{cases} \quad \forall\ i\ \in [1, I], m \in [1, M]$$

$$\alpha_m^i = \begin{cases} 1 & \text{link } i \text{ belongs to subsystem } m \\ 0 & \text{link } i \text{ doesn't belong to subsystem } m \end{cases} \quad \forall i \in [1, I-1], m \in [1, M]$$

It is apparent that one signal can only belong to one subsystem, thus we have:

$$\sum_m \beta_m^i = 1 \quad \forall i \in [1, I] \tag{11}$$

In order to obtain ideal two-way green bandwidths, we limit the number of signals in each subsystem. Tian and Urbanik (2007) divided an arterial into subsystems each with three to five signals. We here restrict the number of signals in each subsystem to be in an interval [3,6], which can be easily adjusted to consider other intervals. Thus, constraint (12) is introduced into the formulation:

$$3 \leqslant \sum_i \beta_m^i \leqslant 6 \quad \forall m \in [1, M] \tag{12}$$

Each coordinated subsystem has a background cycle length, which may vary from one subsystem to another. Constraint (13) sets the upper and lower bounds on these cycle lengths:

$$1/T_2 \leqslant z_m \leqslant 1/T_1 \quad \forall m \in [1, M] \tag{13}$$

For each subsystem, the number of signals equals the number of links plus one, thus constraint (14) holds:

$$\sum_i \alpha_m^i = \sum_i \beta_m^i - 1 \quad \forall m \in [1, M] \tag{14}$$

Constraint (14) can also prevent the coordination of two separated components.

It is also observed that if one link belongs to a subsystem, then both ends of this link must belong to the same subsystem; on the other hand, if one link doesn't belong to a subsystem, there is at least one end of this link that doesn't belong to the same subsystem. Such logic can be captured by the two sentences below:

$$\begin{aligned} &\text{if} \quad \alpha_m^i = 1, \quad \text{then } \beta_m^i + \beta_m^{i+1} = 2 \\ &\text{elseif} \quad \alpha_m^i = 0, \quad \text{then } \beta_m^i + \beta_m^{i+1} \leqslant 1 \end{aligned}$$

The if-then relationship can then be transformed into the following linear equations:

$$2 \cdot \alpha_m^i \leqslant (\beta_m^i + \beta_m^{i+1}) \leqslant 1 + \alpha_m^i \quad \forall i \in [1, I-1], m \in [1, M] \tag{15}$$

When $\alpha_m^i = 1$, constraint (15) leads to $2 \leqslant \beta_m^i + \beta_m^{i+1} \leqslant 2$, or equivalently $\beta_m^i + \beta_m^{i+1} = 2$; when $\alpha_m^i = 0$, (15) leads to $\beta_m^i + \beta_m^{i+1} \leqslant 1$.

If signal i belongs to subsystem m or $\beta_m^i = 1$, then the green bandwidth of subsystem m will be constrained by the green time of signal i, which can be described by the following linear equations:

$$(w_i + b_m) - (1 - r_i) \leqslant U(1 - \beta_m^i) \quad \forall i \in [1, I], m \in [1, M] \tag{16}$$

$$(\bar{w}_i + \bar{b}_m) - (1 - r_i) \leqslant U(1 - \beta_m^i) \quad \forall i \in [1, I], m \in [1, M] \tag{17}$$

where U is a sufficiently large number. When $\beta_m^i = 1$, $w_i + b_m \leqslant 1 - r_i$ and $\bar{w}_i + \bar{b}_m \leqslant 1 - r_i$; otherwise, when $\beta_m^i = 0$, $(w_i + b_m) - (1 - r_i) \leqslant U$ and $(\bar{w}_i + \bar{b}_m) - (1 - r_i) \leqslant U$, which always hold.

Integer constraints (5) and (6) ought to hold for each subsystem m. They are slightly modified as follows:

$$U\cdot(\alpha_m^i-1)\leqslant(w_i+\bar{w}_i)-(w_{i+1}+\bar{w}_{i+1})+(t_i+\bar{t}_i)-[\tau_i-(r_i-r_{i+1})]\leqslant U\cdot(1-\alpha_m^i)\quad\forall i\in[1,I-1];m\in[1,M]\tag{18}$$

$$\tau(i)=integer\qquad\forall i\in[1,I-1]\tag{19}$$

If $\alpha_m^i=1$ for certain m, then signal i and $i+1$ are coordinated in the same subsystem. In this case, constraint (18) reduces to

$$(w_i+\bar{w}_i)-(w_{i+1}+\bar{w}_{i+1})+(t_i+\bar{t}_i)=\tau_i-(r_i-r_{i+1}),$$

which is in the same form of constraint (5). If $\alpha_m^i=0$ for all m, then signal i and $i+1$ are not coordinated. In this case, no integer constraint will be imposed on this pair of signals. To be noted, the dimension of integer variable τ is not increased in constraint (18) compared to constraint (5), as one signal can only belong to one subsystem.

Similar to constraint (18), the following two constraints are further added to limit the speeds on different road segments.

$$U\cdot(\alpha_m^i-1)+(d_i/f_i)z_m\leqslant t(i)\leqslant(d_i/e_i)z_m+U\cdot(1-\alpha_m^i)\quad\forall i\in[1,I-1],m\in[1,M]\tag{20}$$

$$U\cdot(\alpha_m^i-1)+(d_i/\bar{f}_i)z_m\leqslant\bar{t}(i)\leqslant(d_i/\bar{e}_i)z_m+U\cdot(1-\alpha_m^i)\quad\forall i\in[1,I-1],m\in[1,M]\tag{21}$$

To this end, the bandwidth-based coordination model for long arterials can be derived as follows:

$$\textbf{MaxBandLA}:\qquad\max_{(b,\bar{b},z,w,\bar{w},t,\bar{t},\tau)}\sum_m(b_m+\bar{b}_m)/M$$

$$s.t.\quad\sum_m\beta_m^i=1\quad\forall i\in[1,I]\tag{11}$$

$$3\leqslant\sum_i\beta_m^i\leqslant 6\quad\forall m\in[1,M]\tag{12}$$

$$1/T_2\leqslant z_m\leqslant 1/T_1\quad\forall m\in[1,M]\tag{13}$$

$$\sum_i\alpha_m^i=\sum_i\beta_m^i-1\quad\forall m\in[1,M]\tag{14}$$

$$2\cdot\alpha_m^i\leqslant(\beta_m^i+\beta_m^{i+1})\leqslant 1+\alpha_m^i\quad\forall i\in[1,I-1],m\in[1,M]\tag{15}$$

$$(w_i+b_m)-(1-r_i)\leqslant U(1-\beta_m^i)\quad\forall i\in[1,I],m\in[1,M]\tag{16}$$

$$(\bar{w}_i+\bar{b}_m)-(1-r_i)\leqslant U(1-\beta_m^i)\quad\forall i\in[1,I],m\in[1,M]\tag{17}$$

$$U\cdot(\alpha_m^i-1)\leqslant(w_i+\bar{w}_i)-(w_{i+1}+\bar{w}_{i+1})+(t_i+\bar{t}_i)-[\tau_i-(r_i-r_{i+1})]\leqslant U\cdot(1-\alpha_m^i)\quad\forall i\in[1,I-1],m\in[1,M]\tag{18}$$

$$\tau_i=integer\quad\forall i\in[1,I-1]\tag{19}$$

$$U\cdot(\alpha_m^i-1)+(d_i/f_i)z_m\leqslant t(i)\leqslant(d_i/e_i)z_m+U\cdot(1-\alpha_m^i)\quad\forall i\in[1,I-1],m\in[1,M]\tag{20}$$

$$U\cdot(\alpha_m^i-1)+(d_i/\bar{f}_i)z_m\leqslant\bar{t}(i)\leqslant(d_i/\bar{e}_i)z_m+U\cdot(1-\alpha_m^i)\quad\forall i\in[1,I-1],m\in[1,M]\tag{21}$$

$$U:\ \text{a sufficient large number}$$

$$b_m,\bar{b}_m,w_i,\bar{w}_i\geqslant 0$$

The objective function of the MaxBandLA model is to maximize the mean two-way bandwidth of all the subsystems generated. The program is able to simultaneously produce an optimal network partition plan and the optimal signal coordination plans for all the subsystems. The constraints and the objective function are all linear, thus MaxBandLA is also a mixed-integer linear program. The size of the program is roughly M times of that of the system (1)–(9), thus it is of no difficulty to solve it to global optimum.

3.2. Numerical tests

Three numerical tests were carried out on a virtual arterial with 20 signalized intersections. Test 1 used the geometric information and the demand scenario given in Table 1. All these data were randomly generated. The free-flow speed of the whole arterial was set to be 50 km/h.

Table 1
Data input for long arterial in test 1.

Signal number	Distance (m)	East bound (pcu/h)			West bound (pcu/h)			South bound (pcu/h)			North bound (pcu/h)		
		LT	TH	RT	LT	TH	RT	LT	TH	RT	LT	TH	RT
1	–	62	1409	82	56	1361	138	40	200	52	38	219	109
2	341	85	1254	42	131	1291	114	24	179	42	54	150	51
3	695	50	1084	85	98	908	68	15	150	38	31	168	75
4	594	50	1328	68	90	1426	128	50	500	48	40	480	50
5	533	133	1245	69	90	1342	48	50	180	50	50	180	50
6	296	32	1053	100	85	1246	120	50	800	120	42	650	62
7	497	59	1095	156	100	1200	96	75	350	123	52	520	48
8	366	49	1208	81	86	1164	250	–	440	72	–	520	156
9	774	120	1325	150	68	1209	100	–	850	150	–	1000	150
10	475	64	1672	64	60	1528	100	56	240	38	32	134	51
11	359	36	1483	250	45	1250	360	45	700	67	40	680	78
12	643	102	1345	157	100	1230	141	60	450	59	86	349	87
13	403	72	1600	152	105	1644	134	51	321	37	50	260	50
14	357	64	1500	321	53	1589	78	45	289	60	68	450	78
15	524	45	1526	167	98	1614	58	–	750	75	–	700	140
16	518	100	1456	120	104	1389	100	–	700	169	–	700	152
17	344	68	1500	52	100	1324	156	42	500	35	36	324	72
18	518	103	1675	110	120	1582	150	–	289	58	–	442	64
19	398	58	1329	213	64	1439	156	34	304	112	25	256	144
20	780	64	1350	64	76	1292	120	28	504	76	48	544	128

First, Synchro Studio 7 (Husch and Albeck, 2006) was applied to generate all the individual signal timing plans including cycle lengths and splits. The results are shown in Table 2. The cycle lengths vary from 40 s to 60 s. The synchronized phases for the main street always share the majority parts of the cycle.

Synchro also provides procedures for network partition, whose methodology is based on the coordinatability factors of adjacent intersections. Four strategies are available in Synchro: 'one system', 'divide rarely', 'divide sometimes', and 'divide often'. For the given network settings, both options of 'one system' and 'divide rarely' only generated one coordination system, which means no network partitions were performed. For the 'divide often' option, a total of 11 subsystems were generated, many containing only one or two signals. For illustration purpose, the detailed partition plan generated by the 'divide sometimes' option is presented in Table 3. The arterial is divided into four subsystems, with a varying number of signals. The green bandwidths of the subsystems both in second and in cycle can be collected from the time–space diagram provided by Synchro. Subsystem 2 contains six signals, and Synchro assigns it a one-way bandwidth. Subsystem 3 is even longer with 11 signals, and the two-way bandwidths formed are relatively small compared to subsystems 1 and 4. The mean of all the two-way bandwidths is 0.84 cycles.

The MaxBandLA model was solved for the problem in the General Algebraic Modeling System (GAMS) (Brooke et al., 1992) using the CPLEX solver (CPLEX, 2004), on a Dell computer with 3.6 GHz Intel i7 CPU and 12 GB of RAM. The split infor-

Table 2
Individual signal plans on long arterial.

Signal number	Cycle (s)	Split of synchronized phase	Split of non-synchronized phase
1	45	0.756	0.244
2	45	0.800	0.200
3	40	0.725	0.275
4	40	0.700	0.300
5	40	0.775	0.225
6	55	0.600	0.400
7	40	0.675	0.325
8	40	0.675	0.325
9	60	0.650	0.350
10	40	0.775	0.225
11	55	0.655	0.345
12	40	0.700	0.300
13	55	0.800	0.200
14	60	0.733	0.267
15	55	0.727	0.273
16	55	0.709	0.291
17	45	0.733	0.267
18	50	0.820	0.180
19	50	0.740	0.260
20	45	0.667	0.333

Table 3
Coordination plan generated by synchro in test 1.

Subsystem number	Cycle length (s)	Signal number	Offset (s)	Two-way bandwidth	
				(s)	(cycle)
1	45	1	20	32 + 33	1.44
		2	44		
2	46	3	37	0 + 14	0.30
		4	34		
		5	17		
		6	36		
		7	20		
		8	0		
3	55	9	39	18 + 3	0.38
		10	18		
		11	43		
		12	36		
		13	8		
		14	36		
		15	54		
		16	18		
		17	45		
		18	27		
		19	0		
4	50	20	0	31 + 31	1.24

mation from Synchro were the same as in Table 2. It only took 2.7 s to derive an optimal coordination plan, as shown in Table 4. The plan divides the 20 signals into five groups each with three to six signals, as required by constraint (12). The mean of all the two-way bandwidths is 1.29 in cycle, much larger than that produced by Synchro. The reason may lie in two aspects: (1) some of the subsystems generated by Synchro are still quite large thus that only limited two-way green bandwidths can be formed and (2) Synchro does not always apply the coordination plan with the largest bandwidth (Husch and Albeck, 2006).

Another two tests were performed by modifying the data input in test 1. Test 2 randomly changed the geometry input, or the distances between signals, while kept the demand same as in Table 1. Test 3 modified the demand input in Table 1, while kept the distances unchanged. The major difference was that the demands for the side streets were roughly reduced by half. Table 5 presents the new geometry information and new demand scenario.

Tables 6–9 report the optimal coordination plans generated by Synchro and MaxBandLA in tests 2 and 3. For test 2, the phase splits kept the same as in test 1, because the vehicle arrivals at the intersections were not changed. For test 3, the phase splits generated by Synchro changed due to modified demand input, which are not presented here for simplicity.

Table 4
Coordination plan generated by MaxBandLA in test 1.

Section number	Cycle length (s)	Signal number	Offset (s)	Two-way bandwidth	
				(s)	(cycle)
1	47	1	0	33 + 31	1.35
		2	22		
		3	25		
		4	24		
2	40	5	0	23 + 24	1.19
		6	21		
		7	21		
3	60	8	0	39 + 39	1.30
		9	0		
		10	30		
		11	0		
4	57	12	0	40 + 40	1.40
		13	26		
		14	0		
5	43	15	0	28 + 24	1.22
		16	42		
		17	22		
		18	21		
		19	2		
		20	21		

Table 5
Modified data input for long arterial.

Signal number	Distance (m)	East bound (pcu/h)			West bound (pcu/h)			South bound (pcu/h)			North bound (pcu/h)		
		LT	TH	RT	LT	TH	RT	LT	TH	RT	LT	TH	RT
1	–	62	1441	83	40	1527	91	15	68	38	11	68	31
2	472	108	876	28	167	963	168	15	102	23	29	74	34
3	529	66	826	48	129	1259	89	6	107	28	17	72	27
4	665	34	1942	84	104	777	175	26	234	24	15	223	23
5	577	131	1480	36	103	1884	38	26	61	24	23	116	35
6	428	18	1351	104	94	1652	98	35	353	45	23	467	46
7	255	43	679	177	127	1085	110	26	260	83	35	295	24
8	454	40	1450	42	88	1743	247	–	148	32	–	271	50
9	938	122	1710	197	67	1657	83	–	570	86	–	530	73
10	507	77	1853	86	42	1311	146	15	99	28	14	86	15
11	192	29	1706	193	36	1007	199	15	472	19	26	235	45
12	784	106	1402	139	60	767	113	21	276	42	45	206	43
13	313	93	2060	91	68	1354	143	36	188	24	29	144	36
14	250	76	2147	243	41	2215	88	26	99	41	35	167	25
15	627	37	972	170	74	2041	33	–	406	47	–	222	43
16	630	144	992	132	90	1174	116	–	241	73	–	385	84
17	247	85	1203	29	115	1670	200	16	262	14	27	89	35
18	538	101	841	131	112	1889	189	–	73	23	–	315	21
19	508	80	819	175	86	1801	178	9	138	63	15	72	38
20	854	47	1407	80	91	1069	126	16	357	33	35	223	92

Table 6
Coordination plan generated by synchro in test 2.

Subsystem number	Cycle length (s)	Signal number	Offset (s)	Two-way bandwidth	
				(s)	(cycle)
1	49	1	29	16 + 10	0.53
		2	15		
		3	35		
		4	34		
		5	35		
		6	5		
		7	35		
		8	13		
2	61	9	24	28 + 19	0.77
		10	0		
		11	46		
3	55	12	38	0 + 0	0
		13	6		
		14	46		
		15	1		
		16	40		
		17	36		
		18	20		
		19	42		
4	50	20	0	30 + 30	1.20

In test 2, the Synchro plan again divides the network into four shorter sections, and the mean two-way bandwidth is 0.63 in cycle, and the MaxBandLA plan contains five sections, with a mean bandwidth of 1.29 in cycle. The third section with eight signals generated by Synchro is assigned no two-way bandwidths. In test 3, Synchro divides the arterial into three sections, and the mean two-way bandwidth is 1.04 in cycle. MaxBandLA again divides the arterial into five sections, whose mean two-way bandwidth is 1.48 in cycle. It can be observed that, in all the three tests, MaxBandLA generated coordination plans with relatively larger mean two-way bandwidths.

To further examine the coordination plans generated by MaxBandLA model, they were fed into the SimTraffic simulation module in Synchro, and compared with the Synchro plans. The MaxBandLA plans used the same yellow and all red intervals as Synchro plans. The aggregated microscopic simulation results for the entire network, the main street and the side streets, under both sets of coordination plans, are presented in Table 10. Three performance measures were chosen: delay per vehicle, stop per vehicle and average travel speed.

Table 7
Coordination plan generated by MaxBandLA in test 2.

Section number	Cycle length (s)	Signal number	Offset (s)	Two-way bandwidth	
				(s)	(cycle)
1	78	1	0	47 + 47	1.20
		2	42		
		3	4		
		4	43		
		5	4		
		6	42		
2	68	7	0	45 + 45	1.30
		8	35		
		9	37		
3	120	10	0	66 + 79	1.20
		11	114		
		12	60		
4	44	13	0	30 + 32	1.41
		14	23		
		15	22		
5	40	16	0	27 + 27	1.33
		17	21		
		18	19		
		19	21		
		20	2		

Table 8
Coordination plan generated by Synchro in test 3.

Subsystem number	Cycle length (s)	Signal number	Offset (s)	Two-way bandwidth	
				(s)	(cycle)
1	50	1	43	16 + 16	0.64
		2	20		
		3	19		
		4	37		
		5	24		
		6	2		
		7	30		
		8	4		
2	68	9	43	25 + 42	
		10	11		
		11	37		
		12	15		
		13	44		
		14	1		0.99
		15	34		
		16	3		
		17	40		
		18	3		
		19	42		
3	42	20	0	31 + 31	1.48

It can be observed that the MaxbandLA plans generally reduce the average vehicle delay for the main street, but increase that for the side streets. When examined at the network level, in tests 1 and 2 (higher demands on the side streets), the delays produced by the MaxBandLA plans are larger, and in test 3 (lower demands on the side streets), the MaxBandLA plan generates smaller delay than the Synchro plan. This indicates that the possible loss of the side-street traffic needs to be carefully considered when performing the bandwidth-based signal coordination.

In the three tests, the MaxBandLA plans generally result in smaller average numbers of stops than Synchro plans at the network level, which, however, does not lead to the increase of the network-level average travel speeds in all the three cases. Similar to the average delay, the average speeds on the main arterial are improved under MaxBandLA plans in all the cases.

We checked these performance measures at the approach level, at the signal level and at the network level, to find that improvement in one measure may not necessarily leads to improvement in another.

Table 9
Coordination plan generated by MaxBandLA in test 3.

Section number	Cycle length (s)	Signal number	Offset (s)	Two-way bandwidth (s)	Two-way bandwidth (cycle)
1	51	1	0	40 + 40	1.60
		2	26		
		3	27		
2	40	4	0	27 + 29	1.40
		5	39		
		6	19		
		7	21		
3	62	8	0	48 + 48	1.55
		9	62		
		10	31		
4	51	11	0	37 + 37	1.47
		12	0		
		13	25		
		14	1		
5	43	15	0	31 + 28	1.37
		16	41		
		17	21		
		18	20		
		19	0		
		20	20		

Table 10
Comparison of MaxBandLA plans and Synchro plans with simtraffic.

Test 1		Delay/vehicle (s)	Stop/vehicle	Average speed (km/h)
Synchro plan	Entire network	253.4	3.42	17
	Main street	388.0	8.79	23.6
	Side streets	197.3	1.64	7.5
MaxBandLA plan	Entire network	266.9	3.30	16
	Main street	372.6	8.85	24.4
	Side streets	220.9	1.42	7.2
Test 2				
Synchro plan	Entire network	277.4	3.75	16
	Main street	450.1	10.15	22.3
	Side streets	209.0	1.67	6.9
MaxBandLA plan	Entire network	294.2	3.09	16
	Main street	370.9	8.18	24.0
	Side streets	258.1	1.31	7.1
Test 3				
Synchro plan	Entire network	240.6	2.88	20
	Main street	330.3	4.41	27.7
	Side streets	127.5	1.06	10.3
MaxBandLA plan	Entire network	228.2	2.81	19
	Main street	225.5	4.11	28.3
	Side streets	276.0	1.07	9.8

4. Signal coordination of grid networks

4.1. Model formulation

Consider a grid network as in Fig. 2, each signal is denoted either as a single number or a pair (i,j), standing for the intersection of ith street (horizontal) and jth road (vertical), $i \in \{1,2,\ldots,I\}$, $j \in \{1,2,\ldots,J\}$. Same as in Section 3, a simple case is considered where each signal only comprises of two phases. The left turn traffic and right turn traffic share the same phase with the through traffic. East–west bound traffic propagates along the horizontal streets, and north–south bound traffic propagates along the vertical roads.

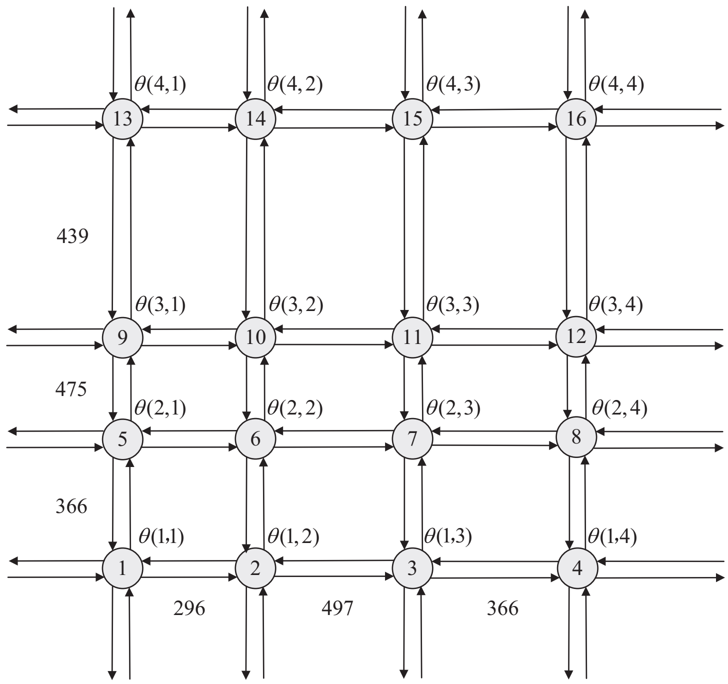

Fig. 2. Grid network.

We assume there is a global clock. The global offset of signal (i,j) to the global clock is denoted as $\theta(i,j)$. The reference points are selected as the centers of the red signals serving east–west bound traffic, or equivalently the centers of the green signals serving north–south bound traffic. Being synchronized together, all the signals share a common cycle length. The relative offset between two successive signals can be calculated as the difference of their global offsets. For example, the relative offset between signal $(1,2)$ and signal $(1,1)$, located along a horizontal street, is $[\theta(1,2) - \theta(1,1)]\mathrm{mod}(cycle)$; the relative offset between signal $(2,1)$ and signal $(1,1)$, located along a vertical road, is

$$\{[\theta(2,1) + 1/2] - [\theta(1,1) + 1/2]\}\mathrm{mod}(cycle) = [\theta(2,1) - \theta(1,1)]\mathrm{mod}(cycle).$$

Based on the above settings, the model to coordinate the signals in grid networks is proposed as follows:

$$\textbf{MaxBandGN}: \quad \max_{(b,\bar{b},z,w,\overline{w},t,\bar{t},\tau,\mu)} \left[\sum_i (b_i + \bar{b}_i) + \sum_j (b_j + \bar{b}_j)\right] \Big/ (I+J)$$

$$s.t. \quad 1/T_2 \leqslant z \leqslant 1/T_1 \tag{22}$$

$$w_j^i + b^i \leqslant 1 - r_j^i \quad \forall j = 1,\ldots,J,\ \forall i \tag{23}$$

$$\overline{w}_j^i + \bar{b}^i \leqslant 1 - r_j^i \quad \forall j = 1,\ldots,J,\ \forall i \tag{24}$$

$$(w_j^i + \overline{w}_j^i) - (w_{j+1}^i + \overline{w}_{j+1}^i) + (t_j^i + \bar{t}_j^i) = \tau_j^i - (r_j^i - r_{j+1}^i) \quad \forall j = 1,\ldots,J-1,\ \forall i \tag{25}$$

$$\tau_j^i = integer \quad \forall j = 1,\ldots,J-1,\ \forall i \tag{26}$$

$$\theta_{j+1}^i - \theta_j^i = w_j^i - w_{j+1}^i + t_j^i + 1/2(r_j^i - r_{j+1}^i) - \eta_j^i \quad \forall j = 1,\ldots,J-1,\ \forall i \tag{27}$$

$$\eta_j^i = integer \quad \forall j = 1,\ldots,J-1,\ \forall i \tag{28}$$

$$(d_j^i/f_j^i)z \leqslant t_j^i \leqslant (d_j^i/e_j^i)z \quad \forall j = 1,\ldots,J-1,\ \forall i \tag{29}$$

$$(d_j^i/f_j^i)z \leqslant \bar{t}_j^i \leqslant (d_j^i/e_j^i)z \quad \forall j = 1,\ldots,J-1,\ \forall i \tag{30}$$

$$b^i, \bar{b}^i, w^i_j, \overline{w}^i_j \geqslant 0 \quad \forall j = 1, \ldots, J, \ \forall i \tag{31}$$

$$w^j_i + b^j \leqslant 1 - r^j_i \quad \forall i = 1, \ldots, I, \ \forall j \tag{32}$$

$$\overline{w}^j_i + \bar{b}^j \leqslant 1 - r^j_i \quad \forall i = 1, \ldots, I, \ \forall j \tag{33}$$

$$(w^j_i + \overline{w}^j_i) - (w^j_{i+1} + \overline{w}^j_{i+1}) + (t^j_i + \bar{t}^j_i) = \tau^j_i - (r^j_i - r^j_{i+1}) \quad \forall i = 1, \ldots, I-1, \ \forall j \tag{34}$$

$$\tau^j_i = integer \quad \forall i = 1, \ldots, I-1, \ \forall j \tag{35}$$

$$\theta^j_{i+1} - \theta^j_i = w^j_i - w^j_{i+1} + t^j_i + 1/2(r^j_i - r^j_{i+1}) - \eta^j_i \quad \forall i = 1, \ldots, I-1, \ \forall j \tag{36}$$

$$\eta^j_i = integer \quad \forall i = 1, \ldots, I-1, \ \forall j \tag{37}$$

$$(d^j_i / f^j_i) z \leqslant t^j_i \leqslant (d^j_i / e^j_i) z \quad \forall i = 1, \ldots, I-1, \ \forall j \tag{38}$$

$$(d^j_i / \bar{f}^j_i) z \leqslant \bar{t}^j_i \leqslant (d^j_i / \bar{e}^j_i) z \quad \forall i = 1, \ldots, I-1, \ \forall j \tag{39}$$

$$b^j, \bar{b}^j, w^j_i, \overline{w}^j_i \geqslant 0 \quad \forall i = 1, \ldots, I, \ \forall j \tag{40}$$

where $\theta^i_j = \theta^j_i = \theta(i,j)$. Same as the MaxBandLA model, the objective function of the above MaxBandGN model is to maximize the mean of the two-way bandwidths of all the horizontal and vertical arterials. Since all the arterials share the same cycle length, constraint (22) can be directly borrowed from Section 2 to limit the length of the common cycle length. The main input data to the model is the red intervals for all the synchronized phases. Under the simple two-phase setting, for a same signal, the red duration for the vertical road is the green duration for the horizontal street. Thus the red of the jth intersection along the ith street r^i_j and the red of ith intersection along the jth road r^j_i compose a whole cycle, or $r^i_j + r^j_i = 1$. Take signal 7 for example, it is the third signal on street 2, as well as the second signal on road 3. The red time duration of its coordinated phase along the street 2 is r^2_3, and the red time duration of its coordinated phase along the road 3 is r^3_2. The signal only possesses two phases, so $r^2_3 + r^3_2 = 1$ holds.

Eqs. (23)–(31) are constraints on the green bandwidths of the horizontal streets. It can be observed that constraints (23)–(31) are similar to constraints (3)–(9), except that another two constrains (27) and (28) are introduced. According to Eq. (10), $\theta^i_{j+1} - \theta^i_j = [w^i_j - w^i_{j+1} + t^i_j + 1/2(r^i_j - r^i_{j+1})]_+$ holds. To discard the operator $[x]_+$, an integer variable η^i_j is introduced, and $\theta^i_{j+1} - \theta^i_j = [w^i_j - w^i_{j+1} + t^i_j + 1/2(r^i_j - r^i_{j+1})]_+$ is transformed into $\theta^i_{j+1} - \theta^i_j = w^i_j - w^i_{j+1} + t^i_j + 1/2(r^i_j - r^i_{j+1}) - \eta^i_j$, as constraint (27). This is the same technique applied to derive constraints (5) and (6) in Little's formulation. Eqs. (32)–(40) are constraints on the green bandwidths for the vertical roads, which are also similar to constraints (3)–(9), except (36) and (37).

Different from Little's model, we directly incorporate the offset variable θ into the formulation. θ becomes the main decision variable in the model. The idea is straightforward: once the offsets are given, the green bands are formed and bandwidths can be derived. With given split information of all the signals, the two sets of constraints, (23)–(31) and (32)–(40) are essentially connected via the decision variable θ.

The benefit of directly optimizing the offsets is that the 'cycle constraints' as introduced in Little (1966) can be eliminated. For large grid networks, the procedure to compose all possible cycle constraints can be time consuming and will probably lead to inexplicit model formulation. Since each cycle constraint will bring in one integer variable, the number of integer variables via Little's approach will vary when different numbers of cycles are considered. However, the size of η in MaxBandGN is determined once I and J are given.

All the constraints and the objective function in the formulation are linear, thus MaxBandGN is also a mixed-integer linear program. The program contains $4 \cdot I \cdot J$ integer variables.

4.2. Numerical tests

Three numerical tests (tests 4–6) were performed on the gird network shown in Fig. 2, with 16 signals, four horizontal streets and four vertical roads. The signal distances along the streets are 296 m, 497 m and 366 m, while the distances along the roads are 366 m, 475 m and 439 m. The demands for the first 16 signals in Table 1 are used again in test 4.

Due to the changes in network topology, the optimal signal timing plan for each signal changes. The new cycle lengths and splits generated by Synchro are given in Table 11.

For comparison purpose, Synchro was first applied to generate the coordination plan for the grid network. Three steps were performed in Synchro:

Table 11
Individual signal plans in grid network.

Signal number	Cycle (s)	Split of synchronized phase	Split of non-synchronized phase
1	45	0.756	0.244
2	45	0.800	0.200
3	40	0.725	0.275
4	50	0.700	0.300
5	40	0.775	0.225
6	60	0.600	0.400
7	40	0.675	0.325
8	40	0.700	0.300
9	55	0.655	0.345
10	45	0.778	0.222
11	65	0.662	0.338
12	40	0.700	0.300
13	50	0.800	0.200
14	50	0.740	0.260
15	55	0.727	0.273
16	55	0.709	0.291

(1) Optimize cycle length and splits for each individual signal.
(2) Optimize network cycle length.
(3) Optimize network offsets.

The Synchro plan and the associated two-way bandwidths are given in Table 12. The means of the two-way bandwidths for all the horizontal streets, all the vertical roads and all the arterials are 1.06, 0.19 and 0.63 cycles, respectively.

The MaxBandGN model was also solved in GAMS (Brooke et al., 1992) using the CPLEX solver (CPLEX, 2004). It only took 0.4 s to solve the mixed-integer linear formulation. The optimal coordination plan is reported in Table 13. The resulting two-way bandwidths are larger than those produced by Synchro. The means of the two-way bandwidths for all the horizontal streets, all the vertical roads and all the arterials are 1.20, 0.44 and 0.82 cycles, respectively.

Two more tests were performed by modifying the data used in Test 4. Test 5 randomly changed the signal distances to 472 m, 529 m and 665 m along the streets and 577 m, 428 m and 255 m along the roads, while kept the demand input unchanged. Test 6 used the same geometry information, but applied the first 16 demands in Table 5. The optimal coordination plans generated by both Synchro and MaxBandGN are presented in Tables 14–17. It can be observed that the MaxBandGN plans generally result in larger bandwidths, especially for the vertical roads.

SimTraffic microscopic simulation was again employed to further examine the coordination plans generated by Synchro and MaxBandGN. Again, the MaxBandGN plans used the same yellow and all red intervals as Synchro plans. The network-level simulation results for both sets of plans are presented in Table 18. The same performance measures were chosen: delay per vehicle, stop per vehicle and average speed.

In all the three tests, MaxBandGN plans and Synchro plans result in approximately equal average travel speeds. In test 4, MaxBandGN plan results in fewer vehicle stops, but larger vehicle delay, while in test 6, the delay under MaxBandGN plan is smaller, but the number of stops is larger. In test 5, the both performance measures are better under the MaxBandGN plan. It can be observed that the differences in the performance measures are sometimes very small. No conclusion can be drawn that one model is better than the other in terms of the performance measures chosen.

Table 12
Coordination plan for grid network by Synchro in test 4.

Cycle length (s)		Road 1	Road 2	Road 3	Road 4	Street bandwidth	
60						(s)	(cycle)
Street 4		13	14	15	16	35 + 36	1.18
Offset (s)		59	29	59	28		
Street 3		9	10	11	12	28 + 34	1.03
Offset (s)		21	1	28	0		
Street 2		5	6	7	8	28 + 30	0.97
Offset (s)		58	27	1	31		
Street 1		1	2	3	4	27 + 37	1.07
Offset (s)		28	11	38	5		
Road bandwidth	(s)	5 + 7	1 + 3	4 + 9	7 + 10		
	(cycle)	0.20	0.07	0.22	0.28		

Table 13
Coordination plan for grid network by MaxBandGN in test 4.

Cycle length (s)		Road 1	Road 2	Road 3	Road 4	Street bandwidth	
63						(s)	(cycle)
Street 4		13	14	15	16	41 + 38	1.27
Offset (s)		31	61	32	60		
Street 3		9	10	11	12	41 + 32	1.16
Offset (s)		4	31	3	30		
Street 2		5	6	7	8	37 + 32	1.10
Offset (s)		29	0	30	58		
Street 1		1	2	3	4	42 + 39	1.28
Offset (s)		0	30	0	29		
Road bandwidth	(s)	13 + 8	13 + 13	17 + 14	18 + 13		
	(cycle)	0.34	0.40	0.50	0.50		

Table 14
Coordination plan for grid network by Synchro in test 5.

Cycle length (s)		Road 1	Road 2	Road 3	Road 4	Street bandwidth	
60						(s)	(cycle)
Street 4		13	14	15	16	22 + 11	0.55
Offset (s)		58	24	2	30		
Street 3		9	10	11	12	31 + 14	0.75
Offset (s)		20	1	24	12		
Street 2		5	6	7	8	30 + 13	0.72
Offset (s)		58	21	55	41		
Street 1		1	2	3	4	20 + 37	0.95
Offset (s)		54	26	47	0		
Road bandwidth	(s)	0 + 0	0 + 0	0 + 1	16 + 0		
	(cycle)	0.00	0.00	0.02	0.27		

Table 15
Coordination plan for grid network by MaxBandGN in test 5.

Cycle length (s)		Road 1	Road 2	Road 3	Road 4	Street bandwidth	
48						(s)	(cycle)
Street 4		13	14	15	16	22 + 34	1.15
Offset (s)		46	28	22	21		
Street 3		9	10	11	12	19 + 32	1.04
Offset (s)		24	7	1	1		
Street 2		5	6	7	8	17 + 29	0.96
Offset (s)		46	28	22	21		
Street 1		1	2	3	4	25 + 31	1.16
Offset (s)		0	27	25	27		
Road bandwidth	(s)	10 + 5	10 + 4	12 + 7	13 + 4		
	(cycle)	0.30	0.29	0.40	0.34		

Table 16
Coordination plan for grid network by Synchro in test 6.

Cycle length (s)		Road 1	Road 2	Road 3	Road 4	Street bandwidth	
64						(s)	(cycle)
Street 4		13	14	15	16	36 + 39	1.17
Offset (s)		44	32	0	32		
Street 3		9	10	11	12	34 + 32	1.03
Offset (s)		8	1	31	0		
Street 2		5	6	7	8	37 + 38	1.17
Offset (s)		46	26	63	35		
Street 1		1	2	3	4	41 + 39	1.25
Offset (s)		5	62	28	60		
Road bandwidth	(s)	0 + 5	3 + 5	1 + 9	9 + 0		
	(cycle)	0.08	0.13	0.16	0.14		

Table 17
Coordination plan for grid network by MaxBandGN in test 6.

Cycle length (s)		Road 1	Road 2	Road 3	Road 4	Street bandwidth	
62						(s)	(cycle)
Street 4		13	14	15	16	41 + 38	1.26
Offset (s)		62	30	0	31		
Street 3		9	10	11	12	38 + 37	1.18
Offset (s)		32	0	33	62		
Street 2		5	6	7	8	34 + 38	1.14
Offset (s)		62	33	62	28		
Street 1		1	2	3	4	42 + 38	1.27
Offset (s)		32	62	32	61		
Road bandwidth	(s)	10 + 10	10 + 10	17 + 17	16 + 13		
	(cycle)	0.31	0.31	0.54	0.46		

Table 18
Comparison of MaxBandGN plans and synchro plans with SimTraffic.

Test 4	Delay/vehicle (s)	Stop/vehicle	Average speed (km/h)
Synchro plan	419.3	5.19	13
MaxBandGN Plan	420.6	5.13	13
Difference (%)	+0.31	−1.16	0
Test 5			
Synchro plan	395.1	5.42	14
MaxBandGN Plan	362.5	5.36	14
Difference (%)	−8.25	−1.11	0
Test 6			
Synchro plan	315.2	2.25	20
MaxBandGN Plan	313.0	2.32	20
Difference (%)	−0.70	+3.11	0

5. Concluding remarks

This paper delivers two explicit mathematical programs for traffic signal coordination, which are both built upon the classical bandwidth maximization model proposed by Little (1966). The first model called MaxBandLA is for the coordination of signals on very long arterials, like those containing more than 20 signals. The model can simultaneously generate an optimal network partition plan, as well as the individual coordination plans for all the subsystems formed. The second model called MaxBandGN is for the coordination of signals in grid networks. MaxBandGN directly optimizes the offsets of all the signals in a network, and as such, no 'cycle constraints' are required even when coordinated arterials form closed loops. MaxBandLA and MaxBandGN are both models in the form of mixed-integer linear programs. Therefore, the coordination plans produced are all globally optimal solutions.

Numerical tests were carried out for both models, and comparisons were performed against Synchro. The results showed that both models generally produced larger two-way green bandwidths than Synchro. And they also showed potential to produce coordination plans that were comparable to Synchro plans in terms of delay, stop or travel speed. For the arterial coordination problem, the numerical results also revealed that the improvement in the propagation of the main-street traffic will sometimes deteriorate the performance of the side-street traffic. The bandwidth-based signal coordination mainly focuses on the propagation of the main-street traffic, rather than the overall network-level traffic condition. The possible loss of the side-street traffic needs to be carefully considered, especially when the side-street demands are very high. Notably, the MaxBandLA and MaxBandGN models can provide multiple optimal solutions, which leaves the chances to search for coordination plans that exhibit good performance in terms of both green bandwidths and other measures of effectiveness.

The two models are simple in structure, and can be extended or modified without difficulty to consider other traffic situations. For example, if there is a segment with much higher demand than the rest of the long arterial, then signals in this segment may have priority to be synchronized together. Adding constraints to MaxBandLA that ensure the value of β_m^i for all these signals are equal can achieve this control strategy. Similarly, if a path in a grid network exhibits much heavier traffic than the rest of the network, signals on this particular path may be synchronized together. Then, adding constraints to MaxBandGN that fix the relative offsets among all these signals can ensure that traffic along this path will propagate smoothly.

Acknowledgements

This research was supported in part by the National Natural Science Foundation Council of China under projects 71401025, the Key Science and Technology Innovation Team of Zhejiang Province under project 2013TD09 and the Fundamental Research Funds for the Central Universities.

References

Brooke, A., Kendirck, D., Meeraus, A., 1992. GAMS: A User's Guide. The Scientific Press, South San Francisco, California.
Cantarella, G.E., de Luca, S., Di Pace, R., Memoli, S., 2015. Network signal setting design: meta-heuristic optimization methods. Transp. Res. Part C: Emer. 55, 24–45.
Chang, E.C.P., Cohen, S.L., Liu, C., Chaudhary, N.A., Messer, C., 1988. MAXBAND-86: program for optimizing left-turn phase sequence in multi-arterial closed networks. Transp. Res. Rec. 1181, 61–67.
Chaudhary, N.A., Pinnoi, A., Messer, C., 1991. Proposed enhancements to MAXBAND-86 Program. Transp. Res. Rec. 1324, 98–104.
CPLEX, 2004. CPLEX, Version 9.0. CPLEX Optimization Inc., Nevada.
Di Febbraro, A., Giglio, D., Sacco, N., 2016. A deterministic and stochastic petri net model for traffic-responsive signaling control in urban areas. IEEE Trans. Intell. Transp. 17, 510–524.
Gartner, N.H., Assmann, S.F., Lasaga, F.L., Hou, D.L., 1991. A multi-band approach to arterial traffic signal optimization. Transp. Res. Part B: Meth. 25, 55–74.
Gartner, N., Stamatiadis, C., 2002. Arterial-based control of traffic flow in urban grid networks. Math. Comput. Model. 35, 657–671.
Gartner, N., Stamatiadis, C., 2004. Progression optimization featuring arterial- and route-based priority signal networks. J. Intell. Transport. S. 8, 77–86.
Gomes, G., 2015. Bandwidth maximization using vehicle arrival functions. IEEE Trans. Intell. Transp. 16, 1977–1988.
Guilliard, I., Sanner, S., Trevizan, F.W., Williams, B.C., 2016. A non-homogeneous time mixed integer LP formulation for traffic signal control. In: Presented at the 95th Annual Meeting of the Transportation Research Board, Washington, DC.
He, Q., Head, K.L., Ding, J., 2014. Multi-modal traffic signal control with priority, signal actuation and coordination. Transp. Res. Part C: Emer. 46, 65–82.
Hook, D., Albers, A., 1999. Comparison of alternative methodologies to determine breakpoints in signal progression. In: Presented at the 69th Annual Meeting of the Institute of Transportation Engineers, Las Vegas, Nevada.
Hu, H., Wu, X., Liu, H.X., 2013. Managing oversaturated signalized arterials: a maximum flow based approach. Transp. Res. Part C: Emer. 36, 196–211.
Husch, D., Albeck, J., 2006. Synchro Studio 7 User Guide. Trafficware Ltd, Texas.
Li, J.Q., 2014. Bandwidth synchronization under progression time uncertainty. IEEE Trans. Intell. Transp. 15, 749–759.
Little, J.D., 1966. The synchronization of traffic signals by mixed-integer linear programming. Oper. Res. 14, 568–594.
Ma, N., Shao, C.F., Zhao, Y., 2011. Influence factors of coordination control system in signalized intersections. J. Harbin Inst. Technol. 43, 112–117.
Ozan, C., Baskan, O., Haldenbilen, S., Ceylan, H., 2015. A modified reinforcement learning algorithm for solving coordinated signalized networks. Transp. Res. Part C: Emer. 54, 40–55.
Park, B., Messer, C.J., Urbanik, T., 1999. Traffic signal optimization program for oversaturated conditions: genetic algorithm approach. Transp. Res. Rec. 1683, 133–142.
Tian, Z., Urbanik, T., 2007. System partition technique to improve signal coordination and traffic progression. J. Transp. Eng. – ASCE 133, 119–128.
Timotheou, S., Panayiotou, C.G., Polycarpou, M.M., 2015. Distributed traffic signal control using the cell transmission model via the alternating direction method of multipliers. IEEE Trans. Intell. Transp. 16, 919–933.
Yang, X.K., 2001. Comparison among computer packages in providing timing plans for Iowa Arterial in Lawrence, Kansas. J. Transp. Eng. – ASCE 127, 314–318.
Yang, I., Jayakrishnan, R., 2015. Real-time network-wide traffic signal optimization considering long-term green ratios based on expected route flows. Transp. Res. Part C: Emer. 60, 241–257.
Ye, B., Wu, W., Mao, W., 2015. A two-way arterial signal coordination method with queueing process considered. IEEE Trans. Intell. Transp. 16, 3440–3452.
Zhang, C., Xie, Y., Gartner, N.H., Stamatiadis, C., Arsava, T., 2015. AM-Band: an asymmetrical multi-band model for arterial traffic signal coordination. Transp. Res. Part C: Emer. 58, 515–531.
Zhang, L., Yin, Y., 2008. Robust synchronization of actuated signals on arterials. Transp. Res. Rec. 2080, 111–119.
Zhang, L., Yin, Y., Lou, Y., 2010. Robust signal timing for arterials under day-to-day demand variations. Transp. Res. Rec. 2192, 156–166.
Zhang, T., Zhang, Y., 2014. System partition method to improve arterial signal coordination. In: Presented at the 93rd Annual Meeting of the Transportation Research Board, Washington, DC.