homework 5

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Problem 1

(i) $\frac{\phi_1(\phi_2-1)}{4*\phi_2} = 0.72 < 1$ $f_Y(\lambda) = \frac{\sigma^2}{2\pi}\frac{|\theta(e^{-i\lambda})|^2}{|\phi(e^{-i\lambda})|^2}$
$= \frac{6}{2\pi}\frac{|1+0.7e^{-i\lambda}|^2}{|1-1.2e^{-i\lambda}+0.7e^{-2i\lambda}|^2}$

```
library(TSA)
```

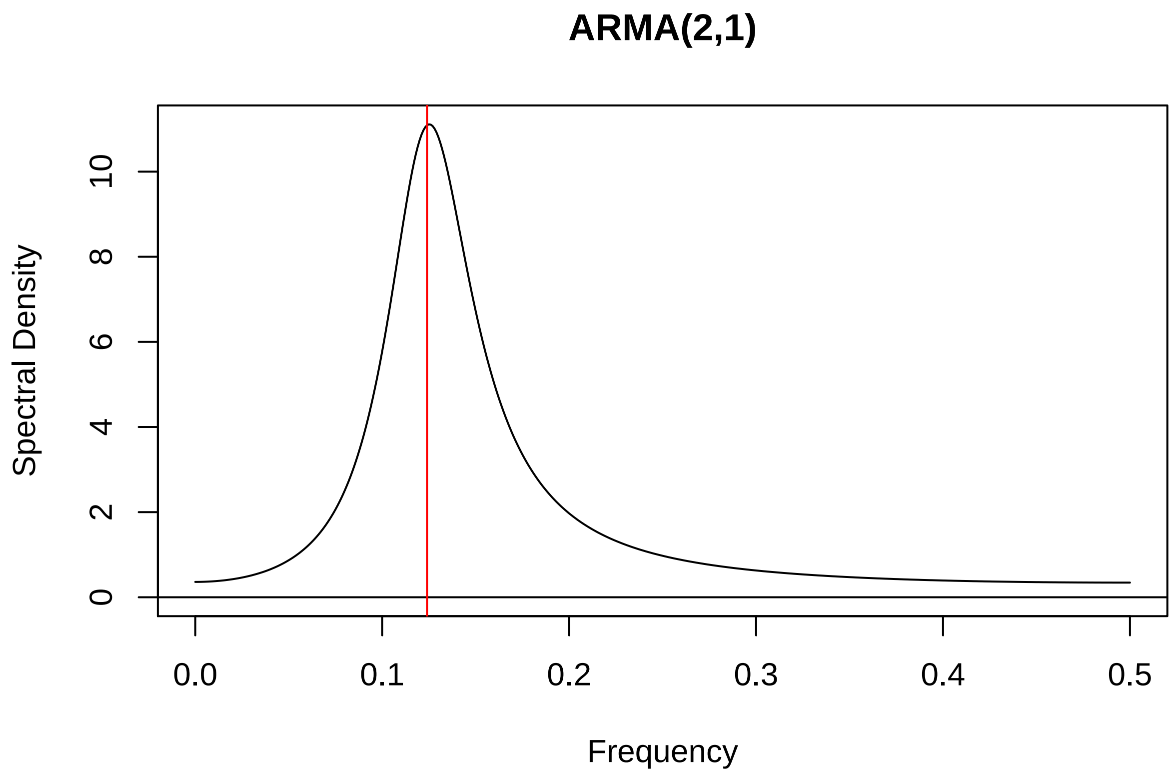
```
##
## Attaching package: 'TSA'

## The following objects are masked from 'package:stats':
##
##     acf, arima

## The following object is masked from 'package:utils':
##
##     tar
```

```
sd= ARMAspec(model=list(ar=c(1.2,-0.7),ma=-0.7), main="ARMA(2,1)")
for(i in 1:500){
  if(sd$spec[i+1,] >sd$spec[i,] ){
    maxfreq <- sd$freq[i]
  }
}

abline(v = maxfreq, col = "red")
```




```
maxfreq
```

```
## [1] 0.124
```

Problem 2

(a) $(1-\phi B)Z_t = a_t$

Then $Y_t = \frac{1}{1-\phi B}a_t + e_t$

Then $(1-\phi B)Y_t = a_t + (1-\phi B)e_t$

Since a_t is White Noise, and $(1-\phi B)e_t$ is MA(1), then $a_t + (1-\phi B)e_t$ is MA(1).

Thus, Y_t is a ARMA1,1 model.

(b) let $\tilde{Z}_t = a_t$ and $\tilde{W}_t = (1-\phi B)e_t$

then $f_Y(\lambda) = f_{\tilde{Z}_t}(\lambda) + f_{\tilde{W}_t}(\lambda)$

then $\frac{\sigma^2}{2\pi}\frac{1-2\theta cos(\lambda)+\theta^2}{1-2\phi cos(\lambda)+\phi^2} = \frac{\sigma_a^2}{2\pi} + \frac{\sigma_e^2}{2\pi}(1-2\phi cos(\lambda)+\phi^2)$

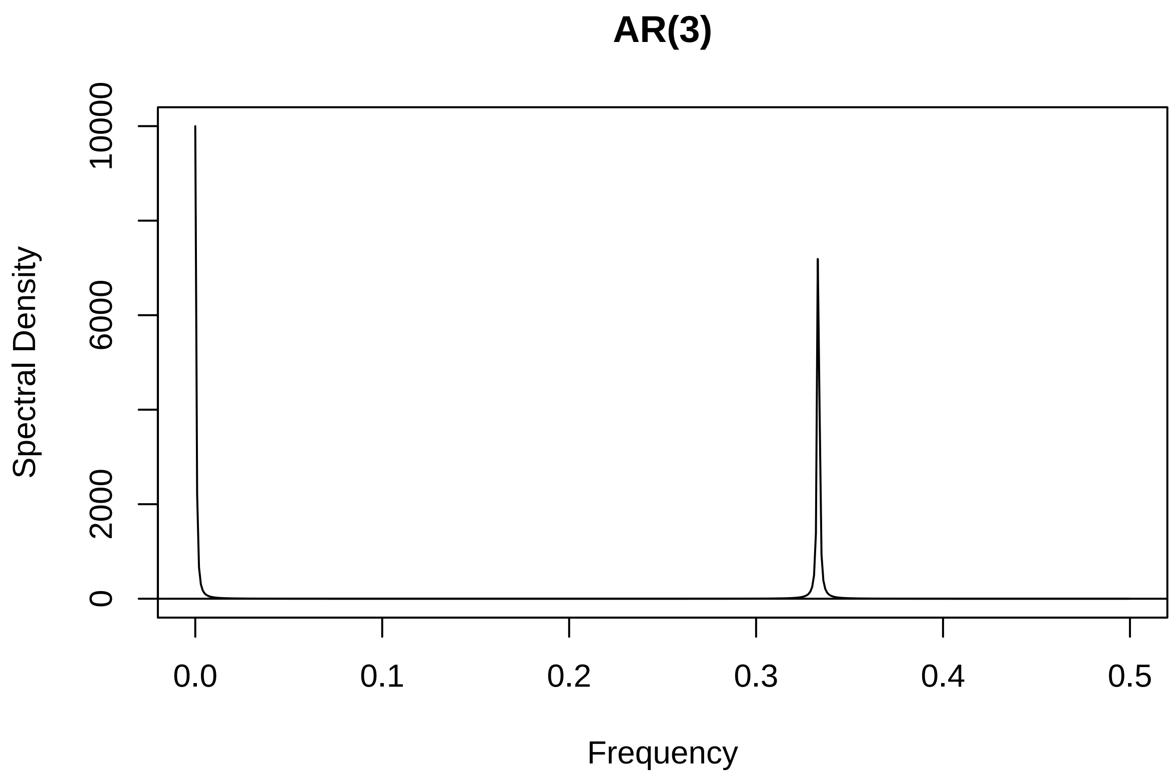
$\sigma^2(1-2\theta cos(\lambda)+\theta^2) = \sigma_a^2(1-2\phi cos(\lambda)+\phi^2) + \sigma_e^2(1-2\phi cos(\lambda)+\phi^2)^2$

(c) $f_Y(\lambda) = \frac{\sigma_a^2}{2\pi} + \frac{\sigma_e^2}{2\pi}(1-2\phi cos(\lambda)+\phi^2)$

Problem 3

$f_X(\lambda) = \frac{1}{2\pi}|1-0.99e^{-3i\lambda}|^2$

```r
ARMAspec(model=list(ar=c(0,0,0.99)), main="AR(3)")
```



The sample paths of X_t will exhibit oscillatory behavior, since there's crest at frequency 0, and about 0.33.

The approximate period of the oscillation is 0.33.

Since $(1-0.99B^3)X_t = e_t$, $X_t = 1/(1-0.99B^3)e_t$

$Y_t = 1/3(1+B+1/B)X_t = 1/3\frac{1+B+1/B}{1-0.99B^3}e_t$

then $(1-0.99B^3)Y_t = 1/3(1+B+1/B)e_t$

$f_Y(\lambda) = \frac{1}{2\pi}\frac{|1/3+e^{-i\lambda}/3+e^{i\lambda}/3|^2}{|1-0.99e^{-3i\lambda}|^2}$

$f_X(2\pi/3) = \frac{1}{2\pi}|1-0.99e^{-2\pi i}|^2 = 1.59*10^{-5}$

$f_Y(2\pi/3) = \frac{1}{2\pi}\frac{|1/3+e^{-2\pi i/3}/3+e^{2\pi i/3}/3|^2}{|1-0.99e^{-2\pi i}|^2} = 1591.54$

The oscillations of $\{X_t\}$ gets lower.

Problem 4

(a)

$$\begin{cases} \gamma(0) = \phi_1\gamma(1) + \phi_2\gamma(2) + \sigma^2(1-\theta\psi_1) \\ \gamma(1) = \phi_1\gamma(0) + \phi_2\gamma(1) - \sigma^2\theta \\ \rho(2) = \phi_1\rho(1) + \phi_2\rho(0) \\ \rho(3) = \phi_1\rho(2) + \phi_2\rho(1) \end{cases}$$

then

$$\begin{cases} -0.0106 = \phi_1 * 0.4894 + \phi_2 \\ -0.2341 = \phi_1 * (-0.0106) + \phi_2 * 0.4894 \end{cases}$$

$\phi_1 = 0.915$, and $\phi_2 = 0.458$.

Since $\tilde{\gamma}(1) = \tilde{\gamma}(0)\tilde{\rho}(1) = 2.74$

$\tilde{\gamma}(2) = \tilde{\gamma}(0)\tilde{\rho}(2) = -0.059$

$\psi_1 = \phi_1 - \theta$,

$$\begin{cases} 5.6 = 0.915 * 2.74 + 0.458 * (-0.059) + \sigma^2(1 - \theta(0.915 - \theta)) \\ 2.74 = 0.915 * 5.6 + 0.458 * 2.74 - \sigma^2\theta \end{cases}$$

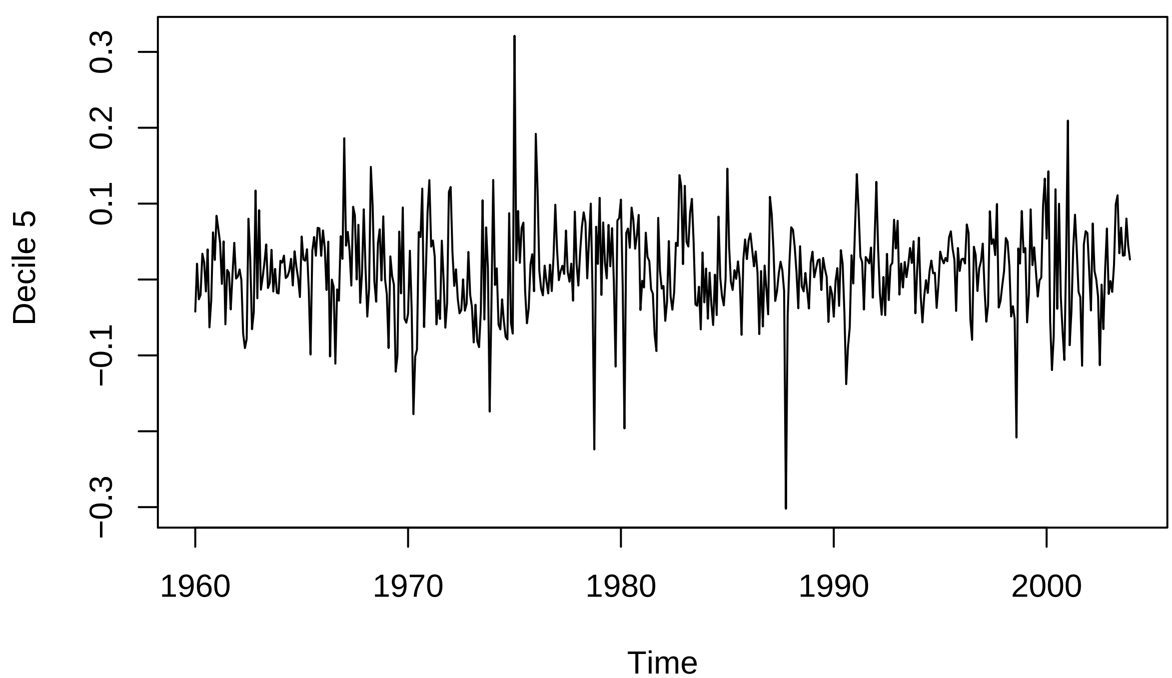
$\theta = 0.27 \pm 0.96i$ and $\sigma^2 = 2.44 \pm 0.66i$

(b) $\rho(4) = \phi_1\rho(3) + \phi_2\rho(2) = 0.915 * -0.2341 + 0.458 * -0.0106 = -0.219$ $\rho(5) = \phi_1\rho(4) + \phi_2\rho(3) = 0.915 * -0.219 + 0.458 * -0.2341 = -0.3076$ $\rho(6) = \phi_1\rho(5) + \phi_2\rho(4) = 0.915 * -0.3076 + 0.458 * -0.219 = -0.381$

Problem 5

(a)

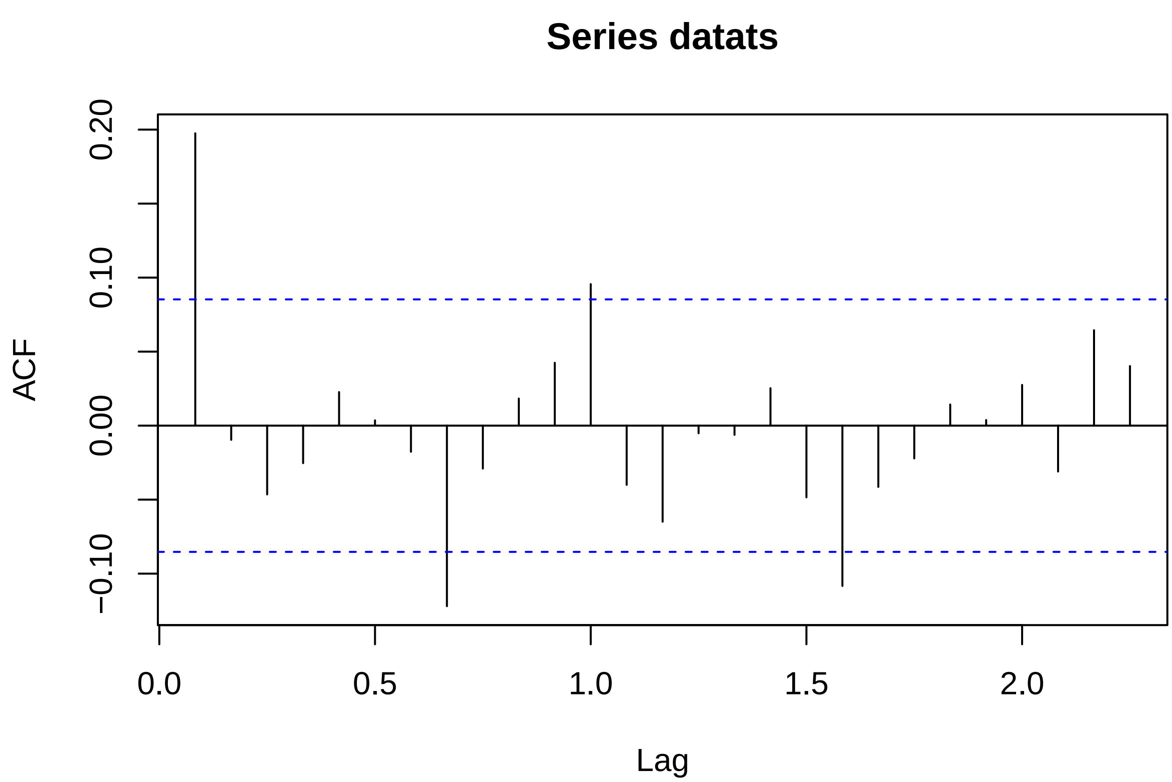
```
dat<-read.csv("m-decile1510.txt",sep = "",skip = 2)
datats <- ts(dat$D5, start=c(1960,1), freq = 12)
plot(datats, ylab=' Decile 5') #ts plot
```



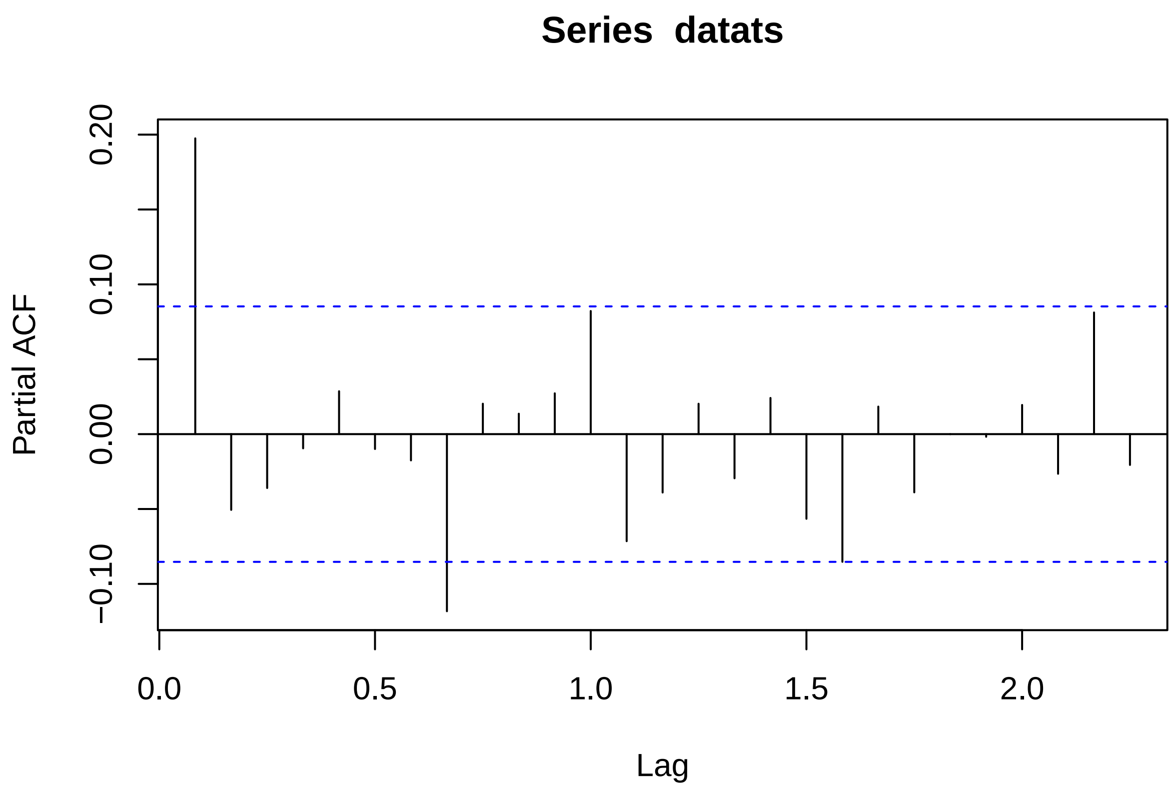
```r
summary(datats) #summary
```

```
##      Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -0.30210 -0.02182  0.01293  0.01137  0.04435  0.32119
```

```r
acf(datats) # acf plot
```




```r
pacf(datats)# pacf plot
```




```
eacf(datats, ar.max = 10, ma.max = 10) #eacf ARMA(2,1)
```

```
## AR/MA
##    0 1 2 3 4 5 6 7 8 9 10
## 0  x o o o o o o x o o o
## 1  x o o o o o o x o o o
## 2  x o o o o o o x o o o
## 3  x x x o o o o x o o o
## 4  x x o o o o o x o o o
## 5  x x o x o o o o o o o
## 6  x o x x x o o o o o o
## 7  x o x x x x x o o o o
## 8  x x x x x x o o o o o
## 9  x o o x x x x o o o o
## 10 x o o x o o o o x o o
```

(b)

```
Box.test(dat$D1, lag = 12, type = "Ljung-Box", fitdf=1)
```

```
##
##  Box-Ljung test
##
## data:  dat$D1
## X-squared = 78.781, df = 11, p-value = 2.538e-12
```

```
Box.test(dat$D5, lag = 12, type = "Ljung-Box", fitdf=1)
```

```
##
##  Box-Ljung test
##
## data:  dat$D5
## X-squared = 37.289, df = 11, p-value = 0.0001031
```

```r
Box.test(dat$D10, lag = 12, type = "Ljung-Box", fitdf=1)
```

```
##
##  Box-Ljung test
##
## data:  dat$D10
## X-squared = 7.0184, df = 11, p-value = 0.7976
```

(c)

```r
est1<-arima(dat$D5, order=c(0,0,1), include.mean = T)
est1
```

```
##
## Call:
## arima(x = dat$D5, order = c(0, 0, 1), include.mean = T)
##
## Coefficients:
##          ma1  intercept
##       0.2050     0.0114
## s.e.  0.0421     0.0031
##
## sigma^2 estimated as 0.003505:  log likelihood = 743.29,  aic = -1482.58
```

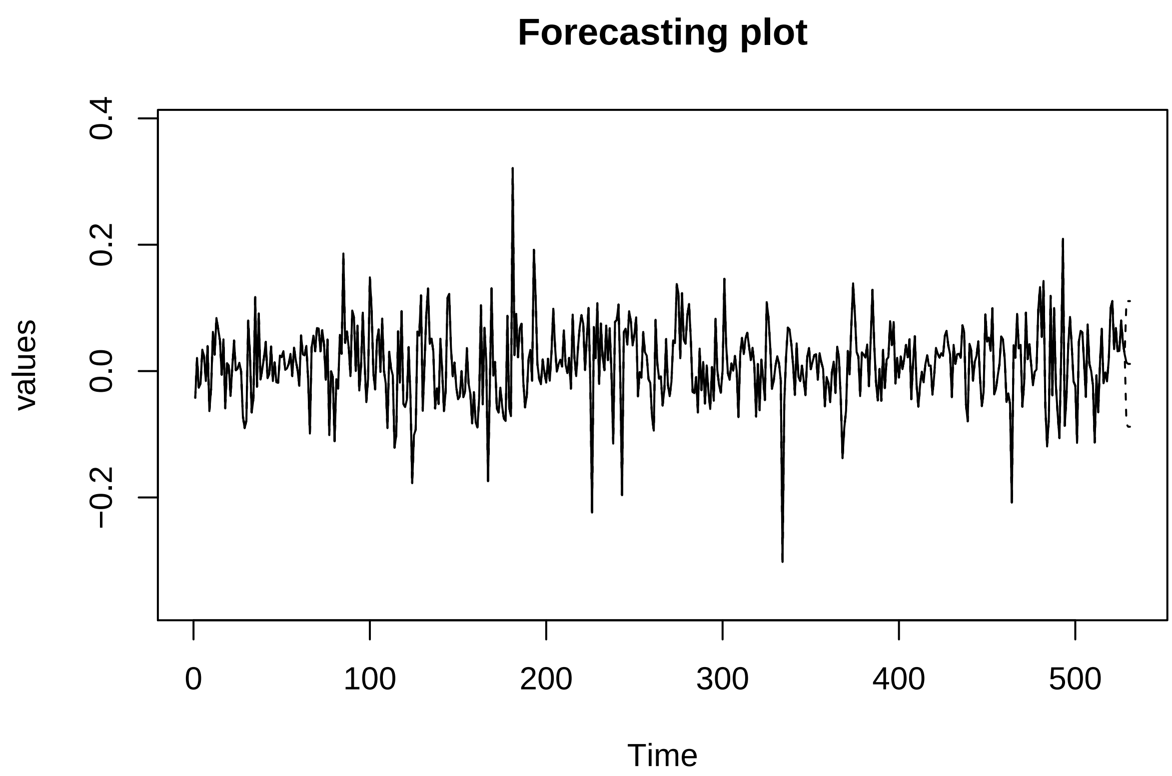
```r
source("r-backtest.txt")
backtest(est1,datats,100,3)
```

```
## [1] "RMSE of out-of-sample forecasts"
## [1] 0.06143779 0.06273377 0.06280184
```

```r
pred.dat <- predict(est1, n.ahead=3); pred.dat
```

```
## $pred
## Time Series:
## Start = 529
## End = 531
## Frequency = 1
## [1] 0.01359002 0.01135644 0.01135644
##
## $se
## Time Series:
## Start = 529
## End = 531
## Frequency = 1
## [1] 0.05920642 0.06043824 0.06043824
```

```r
source("foreplot.R")
foreplot(pred.dat, datats, orig=528,start=1,p=0.95)
```



(d)

```r
est2<- arima(dat$D5, order=c(2,0,0), include.mean = T)
est2
```

```
##
## Call:
## arima(x = dat$D5, order = c(2, 0, 0), include.mean = T)
##
## Coefficients:
##          ar1      ar2  intercept
##       0.2075  -0.0506     0.0113
## s.e.  0.0435   0.0434     0.0031
##
## sigma^2 estimated as 0.003503:  log likelihood = 743.48,  aic = -1480.96
```

```r
source("r-backtest.txt")
backtest(est2,datats,100,3)
```

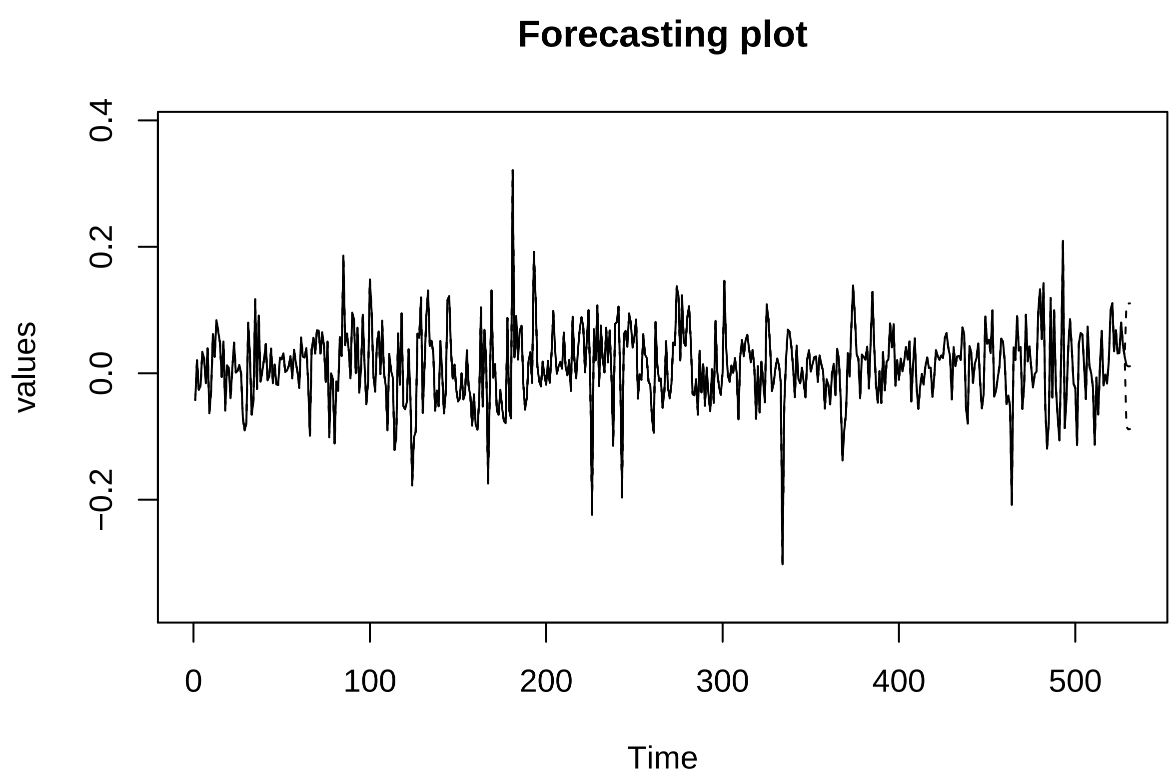
```
## [1] "RMSE of out-of-sample forecasts"
## [1] 0.06162103 0.06290935 0.06280699
```

```r
pred.dat <- predict(est2, n.ahead=3); pred.dat
```

```
## $pred
## Time Series:
## Start = 529
## End = 531
## Frequency = 1
```

```
## [1] 0.01277428 0.01088644 0.01118101
##
## $se
## Time Series:
## Start = 529
## End = 531
## Frequency = 1
## [1] 0.05918516 0.06044564 0.06044729
```

```
source("foreplot.R")
foreplot(pred.dat, datats, orig=528,start=1,p=0.95)
```



(e) MA model, for MA model have same value on the 2 and 3 step.