homework 4

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Problem 1

(a) when $1-1.2z+0.7z^2=0$, $|z|=3.16$

Since $\phi(z)=1-1.2z+0.7z^2\neq 0$ and $\theta(z)=1+0.7z\neq 0$, for $|z|\leq 1$ it is causal and invertible, and this is a ARMA(2,1)model.

(b) $\psi_1=\phi_1-\theta_1=1.2+0.7=1.9$

$\psi_2=\phi_1\psi_1+\phi_2\psi_0=1.2*1.9-0.7=1.58$

$\psi_3=\phi_1\psi_2+\phi_2\psi_1=1.2*1.58-0.7*1.9=0.566$

$\psi_4=\phi_1\psi_3+\phi_2\psi_2+\phi_3\psi_1-\theta_4=1.2*0.566-0.7*1.58=-0.4268$

(c) $\pi_1=\theta_1\pi_0+\phi_1=-0.7*1+1.2=0.5$

$\pi_2=\theta_1\pi_1+\phi_2=-0.7*0.5-0.7=-1.05$

$\pi_3=\theta_1\pi_2=-0.7*-1.05=0.735$

$\pi_4=\theta_1\pi_3=-0.7*0.735=0.5145$

(d) $\rho(k)=\phi_1\rho(k-1)+\phi_2\rho(k-2)$ for $k>1$

(e)

$$\begin{cases}\gamma(0)=\phi_1\gamma(1)+\phi_2\gamma(2)+\sigma^2(1-\theta_1\psi_1)\\ \gamma(1)=\phi_1\gamma(0)+\phi_2\gamma(1)-\sigma^2(\theta_1\psi_0)\\ \gamma(2)=\phi_1\gamma(1)+\phi_2\gamma(0)\end{cases}$$

then

$$\begin{cases}\gamma(0)=1.2\gamma(1)-0.7\gamma(2)+6*(1+0.7*1.9)\\ \gamma(1)=1.2\gamma(0)-0.7\gamma(1)-6*(-0.7)\\ \gamma(2)=1.2\gamma(1)-0.7\gamma(0)\end{cases}$$

$$\begin{cases}\gamma(0)=58.11\\ \gamma(1)=43.63\\ \gamma(2)=11.67\end{cases}$$

$\rho(1)=\frac{\gamma(1)}{\gamma(0)}=0.495$

$\rho(2)=\frac{\gamma(2)}{\gamma(0)}=-0.104$

$\rho(3)=\phi_1\rho(2)+\phi_2\rho(1)=-0.473$

(f) $\phi_{11} = \rho(1) = 0.49$

$$\begin{bmatrix} 1 & \rho(1) \\ \rho(1) & 1 \end{bmatrix} \begin{bmatrix} \phi_{12} \\ \phi_{22} \end{bmatrix} = \begin{bmatrix} \rho(1) \\ \rho(2) \end{bmatrix}$$

then $\phi_{22} = -0.46$

$$\begin{bmatrix} 1 & \rho(1) & \rho(2) \\ \rho(1) & 1 & \rho(1) \\ \rho(2) & \rho(1) & 1 \end{bmatrix} \begin{bmatrix} \phi_{13} \\ \phi_{23} \\ \phi_{33} \end{bmatrix} = \begin{bmatrix} \rho(1) \\ \rho(2) \\ \rho(3) \end{bmatrix}$$

then$\phi_{33} = -0.28$

(g)

```
ARMAacf(ar=c(1.2,-0.7),ma=-0.7, lag.max=5)
```

```
##          0          1          2          3          4          5
##  1.0000000  0.4958968 -0.1049238 -0.4730363 -0.4941970 -0.2619109
```

```
ARMAacf(ar=c(1.2,-0.7),ma=-0.7, lag.max=5,pacf = TRUE)
```

```
## [1]  0.4958968 -0.4652484 -0.2810857 -0.1843900 -0.1252156
```

(h) $m^2 + 1.2m - 0.7 = 0$ then $m1 = 0.42$, $m2 = -1.62$

$\hat{Y}_t(l) = c_1^{(t)} 0.42^l - 1.62^l c_2^{(t)}$

$\hat{Y}_t(1) = -1.2Y_t + 0.7Yt - 1 - 0.7\epsilon_t = 0.42c_1 - 1.62c_2$

$\hat{Y}_t(2) = 1.44Y_t - 0.84Yt - 1 + 0.84\epsilon_t + 0.7Y_t = 0.42c_1 - 1.62c_2$

Then $c_1 = 22.9Y_t + 0.343Yt - 1 - 0.343\epsilon_t$, $c_2 = 0.66Y_t + 0.3416Yt - 1 - 0.3416\epsilon_t$

Then $\hat{Y}_t(l) = (22.9Y_t + 0.343Yt - 1 - 0.343\epsilon_t) * 0.42^l + (0.66Y_t + 0.3416Yt - 1 - 0.3416\epsilon_t) - 1.62^l$

Problem 2

let $\psi(z) = \sum_{i=-\infty}^{\infty} \psi_i z^i$, and $\psi(-z) = \sum_{j=-\infty}^{\infty} \psi_j z^{-j}$

let i = j+k

then$\psi(z)\psi(-z) = \sum_{j+k=-\infty}^{\infty} \psi_{j+k} z^{j+k} \sum_{j=-\infty}^{\infty} \psi_j z^{-j}$
$= \sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \psi_k \psi_j z^k$

Problem 3

$\epsilon_t^* = \frac{1-0.5B}{1-0.8B} Y_t$

$\widetilde{\sigma}^2 = G(z) = \sigma^2 \frac{\theta(z)\theta(z^{-1})}{\phi(z)\phi(z^{-1})}$
$= \sigma^2 \frac{(1-1.25z)(1-1.25z^{-1})(1-0.5z)(1-0.5z^{-1})}{(1-2z)(1-2z^{-1})(1-0.8z)(1-0.8z^{-1})}$
$= 0.39\sigma^2$ is a constant,

Then the equation is causal and invertible.

Problem 4

Since $< \hat{Y}_{1-h} - Y_{1-h}, Yk >= E[(\hat{Y}_{1-h} - Y_{1-h})Y_k] = 0$

Then $E(\hat{Y}_{1-h}Y_k) = E(Y_{1-h}Y_k)$ for k = 1,2,3...

Which means $\hat{Y}_{1-h} = \beta_{t,1}Y_1 + ... + \beta_{t,t}Y_t$ for$\beta_t = (\beta_{t,1}, ..., \beta_{t,t})'$ satisfies $\Gamma_t \beta_t = \gamma_t^{(h)}$

Problem 5

$Y_t = e_1 cos(t) + e_2 sin(t)$, e_1,$e_2$are independent

(a) $Y_3 = e_1 cos(3) + e_2 sin(3)$

$Y_2 = e_1 cos(2) + e_2 sin(2)$

$Y_1 = e_1 cos(1) + e_2 sin(1)$

then $cos(3) = \beta_1 cos(2) + \beta_2 cos(1)$

$cos(3) = \beta_1 cos(2) + \beta_2 cos(1)$

then $\beta_1 = 1.08, \beta_2 = -1$

(b) $E(Y_3 - \hat{Y}_2(1))^2 = E(Y_3 - 1.08Y_2 + Y_1)^2$
$= (0.299e_1 + 0.00054e_2)^2$
$= 0.089e_1^2 + 3.02 * 10^{-7} e_2$

Problem 6

(a) $\phi_1 = 1.2$, $\phi_2 = -0.7$

$\hat{Y}_T(1) = \phi_1 Y_T + \phi_2 Y_{T-1} + \mu = 1.2 * 14.4 + 13.9 * -0.7 + 5 = 12.55$

$\hat{Y}_T(2) = \phi_1 \hat{Y}_T(1) + \phi_2 Y_T + \mu = 1.2 * 12.55 + 14.4 * -0.7 + 5 = 9.98$

$\hat{Y}_T(3) = \phi_1 \hat{Y}_T(2) + \phi_1 \hat{Y}_T(1) + \mu = 1.2 * 9.98 - 0.7 * 12.55 + 5 = 8.191$

$\hat{Y}_T(4) = \phi_1 \hat{Y}_T(3) + \phi_1 \hat{Y}_T(2) + \mu = 1.2 * 8.191 - 0.7 * 9.98 + 5 = 9.0432$

(a) $m^2 + 1.2m - 0.7 = 0$ then $m1 = 0.42$, $m2 = -1.62$

$\hat{Y}_t(l) = c_1^{(t)} 0.42^l - 1.62^l c_2^{(t)}$

$0.42c_1^{(t)} - 1.62c_2^{(t)} = 12.55$, and $0.1764c_1^{(t)} + 2.62c_2^{(t)} = 9.98$

$c_1^{(t)} = 35.38, c_2^{(t)} = 1.42$

$\hat{Y}_t(l) = 35.38 * 0.42^l - 1.62^l * 1.42$