MA 550 – Homework 3

Due Wed, 11:30am, October 14, 2020

Homework Assignment Policy and Guidelines

(a) Homework assignment should be submitted electronically through `Canvas`, and your submission should be combined into ONE PDF file. Credit will not be given for homework turned in late.

(b) Homework assignments should be well organized. It is required that you show your work in order to receive credit.

(c) It's highly recommended to use R Markdown to write up your homework solutions, especially for problems that involve data analysis. But this is not required.

(d) When writing up solutions via R Markdown, please refrain from showing lengthy results/output for data analysis problems. Please display only relevant results.

(e) If you are not to use R Markdown, please keep the R code and/or R output for all the relevant problems in a *well organized appendix*, and please refrain from copying and pasting R code and/or R output directly into your answers.

(f) For problems that involve technical writing, strive to be clear, concise, and cogent. Organize figures and tables in an efficient manner while maintaining clarity.

(g) For problems that involve data analysis, always interpret the results in the context of the study. This may include comments on whether the results are meaningful or not. Think of your answers as a mini technical report of your findings and follow the guidelines on technical writing.

(h) You may discuss most homework problems with others including your peers and instructor, but you must write up your homework solutions by yourself in order to receive credit. Similarly, you must write your own computer code and obtain your computer output independently.

1. Define an invertible MA(q) process $\{Y_t\}$ such that the autocovariance function of Y_t satisfies

$$\gamma(0) = 4.0, \quad \gamma(2) = -1.6, \quad \gamma(k) = 0, \text{ for } k \neq 0, \pm 2.$$

2. For the AR(2) process

$$Y_t = \frac{1}{3}Y_{t-1} + \frac{2}{9}Y_{t-2} + \epsilon_t, \quad \epsilon_t \sim WN(0, 2)$$

(a) Verify that the process is causal and compute the ACF $\rho(k)$ of $\{Y_t\}$ for $k = 1, 2, 3$.

(b) Suppose that ϵ_t follows $N(0, 2)$ for each t. In `R`,

(i) plot the theoretical ACF and PACF up to lag 30;

(ii) simulate three times 100 observations from this model. Each time, plot the time series, and the sample ACF and the sample PACF up to lag 30.

(iii) Comment on the figures in (b-i) and (b-ii).

(c) Express the ACF explicitly in the form

$$\rho(k) = c_1 m_1^k + c_2 m_2^k, \quad k = 0, 1, 2, \ldots,$$

where m_1, m_2 are the roots of $m^2 - \frac{1}{3}m - \frac{2}{9} = 0$.

3. Consider the AR(2) process

$$Y_t = 1.3Y_{t-1} - 0.8Y_{t-2} + 3.5 + \epsilon_t, \quad \epsilon_t \sim WN(0, 3).$$

(a) Verify that the process is causal and compute the mean and variance.

(b) Solve the first two Yule-Walker equations for $\rho(1)$ and $\rho(2)$, and then compute $\rho(k)$ for $k = 3, 4, 5$, using the appropriate recursive relation.

(c) Express the ACF explicitly in the form

$$\rho(k) = R^k \left[c_1 \cos(\lambda_0 k) + c_2 \sin(\lambda_0 k) \right], \quad k = 0, 1, 2, \ldots.$$

Check that the values for $\rho(k), k = 1, \ldots, 5$ obtained by (b) and (c) match.

(d) Verify your results in (e) and (f) using `R` function `ARMAacf`.

(e) Express this AR(2) process in the infinite MA form

$$Y_t = \mu + \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$$

by providing the numerical values for the constant mean μ and for $\psi_1, \psi_2, \ldots, \psi_5$.

4. Assume that the stationary process $\{Y_t\}$ is generated by a low order AR model, and that $\{Y_t\}$ has the following variance and autocorrelations.

$$\gamma(0) = 6,\ \rho(1) = 0.80,\ \rho(2) = 0.46,\ \rho(3) = 0.152,\ \rho(4) = -0.0476.$$

(a) Solve the first sets of Yule-Walker equations for each of the possible AR orders 1, 2, and 3 to find the corresponding values of the ϕ_i.

(b) Determine the value of $\sigma^2 = Var(\epsilon_t)$ in each case.

(c) What is the actual AR order p of this process?

5. Consider Slides #8, Page 39. Verify that $\cos(\lambda) = \frac{\phi_1}{2\sqrt{-\phi_2}} = \frac{m_1+m_2}{2\sqrt{m_1 m_2}}$.