homework 2

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Problem 1

Since a strictly stationary depends only on the time lag k, and a Guassian stationary process has the multivariate normal distributions: $f(y) = \frac{1}{2\pi^{n/2}|\Gamma|^{1/2}} e^{-1/2(y-\mu)'\Gamma^{-1}(y-\mu)}$,

it has the same distribution at $ t+k $. So a Guassian stationary is a strictly stationary.

Problem 2

(a) $\mu_t = Y_t - Z_t = \beta_0 + \beta_1 t$

(b) No, since $Y_t = \beta_0 + \beta_1 t + Z_t$, $\beta_1 t$ is a factor of Y_t, which depend on the time t. So it is not stationary.

(c) $W_t = Y_t - Y_{t-1} = [\beta_0 + \beta_1 t + Z_t] - [\beta_0 + \beta_1(t-1) + Z_{t-1}]$
$= \beta_1 + Z_t - Z_{t-1}$,which is not depend on t, so W_t is stationary.

Problem 3

(a) $Cov(Y_{t+k}, Y_t) = E[Y_{t+k}Y_t] - E[Y_t]E[Y_t + k]$
$= E[Y_{t+k}Y_t] - \mu^2$
$= E[(\mu + \sum_{j=-\infty}^{\infty} \psi_j Z_{t-j})(\mu + \sum_{j=-\infty}^{\infty} \psi_{j+k} Z_{t-j-k})] - \mu^2$
$= E[\mu^2 + \mu(\sum_{j=-\infty}^{\infty} \psi_j Z_{t-j} + \sum_{j=-\infty}^{\infty} \psi_{j+k} Z_{t-j-k}) + (\sum_{j=-\infty}^{\infty} \psi_j Z_{t-j})(\sum_{j=-\infty}^{\infty} \psi_{j+k} Z_{t-j-k})] - \mu^2$
$= \mu^2 + \mu E[\sum_{j=-\infty}^{\infty} \psi_j Z_{t-j}] + \mu E[\sum_{j=-\infty}^{\infty} \psi_{j+k} Z_{t-j-k}] + E[(\sum_{j=-\infty}^{\infty} \psi_j Z_{t-j})(\sum_{j=-\infty}^{\infty} \psi_{j+k} Z_{t-j-k})] -$
μ^2
$= E[(\sum_{j=-\infty}^{\infty} \psi_j Z_{t-j})(\sum_{j=-\infty}^{\infty} \psi_{j+k} Z_{t-j-k})]$
since $E[\sum_{j=-\infty}^{\infty} \psi_j Z_{t-j}] = 0$ and $E[\sum_{j=-\infty}^{\infty} \psi_{j+k} Z_{t-j-k}] = 0$.
Since $Z_t \sim WN(0, \sigma^2)$,
$Cov(Y_{t+k}, Y_t) = \sigma^2 \sum_{j=-\infty}^{\infty} \psi_j \psi_{j+k}$

(b) $\gamma(k) = Cov(Y_t, Y_{t+k}) = \sigma^2 \sum_{j=-\infty}^{\infty} \psi_j \psi_{j+k}$ Since $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$ implys $\sigma^2 \sum_{j=-\infty}^{\infty} \psi_j \psi_{j+k} < \infty$ and $\sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \psi_{j+k} < \infty$
So $\sum_{k=-\infty}^{\infty} |\gamma(k)|$
$= \sigma^2 \sum_{j=-\infty}^{\infty} \psi_j \psi_{j+k} * \sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \psi_{j+k} < \infty * \infty < \infty$

(c) Since $\sum_{j=-\infty}^{\infty} \psi_j$has the same variables with $\sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \psi_{j+k}$, so$\sum_{j=-\infty}^{\infty} \psi_j = \sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \psi_{j+k}$,
then$v = \sum_{k=-\infty}^{\infty} \gamma(k) = \sigma^2 \sum_{j=-\infty}^{\infty} \psi_j \sum_{j=-\infty}^{\infty} \psi_j$
$= \sigma^2 (\sum_{j=-\infty}^{\infty} \psi_j)^2$

Problem 4

(a) $E(Y_t) = E(e_1 cos(t) + e_2 sin(t)) = E(e_1)cos(t) + E(e_2)sin(t)]$

(b) Since $e1$, $e2$ have mean 0, then $E(Y_t) = 0$
$Cov(Y_{t+k}, Y_t) = E(Y_{t+k}Y_t) - E(Y_{t+k})E(Y_t)$
$= E(Y_{t+k}Y_t)$
$= E[(e_1 cos(t+k) + e_2 sin(t+k))(e_1 cos(t) + e_2 sin(t))]$
$= E[e_1^2 cos(t+k)cos(t) + e_1 e_2(cos(t+k)sin(t) + sin(t+k)cos(t)) + e_2^2 sin(t+k)sin(t)]$
Since e_1,e_2 are independent, $E[e_1 e_2] = E[e_1]E[e_2] = 0$,and $E[e_1^2] = E[e_2^2] = \sigma^2$
then $Cov(Y_{t+k}, Y_t) = \sigma^2 cos(t+k)cos(t) + \sigma^2 sin(t+k)sin(t)$
$= \sigma^2(cos(t+k)cos(t) + sin(t+k)sin(t))$
$= \sigma^2 cos(t+k-t)$
$= \sigma^2 cos(k)$

(c) Since $E[Y_t] = 0$ is not depent on t, and $Cov(Y_{t+k}, Y_t)$ depends only on lag k, So $\{Y_t\}$ is stationary.

Problem 5

(a) $E(Y_t) = \mu = 5.0$
$\gamma(0) = \sigma_{MA}^2 = (1 - 1.4 + 0.7)\sigma^2 = 0.3 * 2.4 = 0.72$
Since MA(2), when k>2, $\gamma(k) = 0$,
$\rho(k) = \gamma(k)/\gamma(0) = 0$

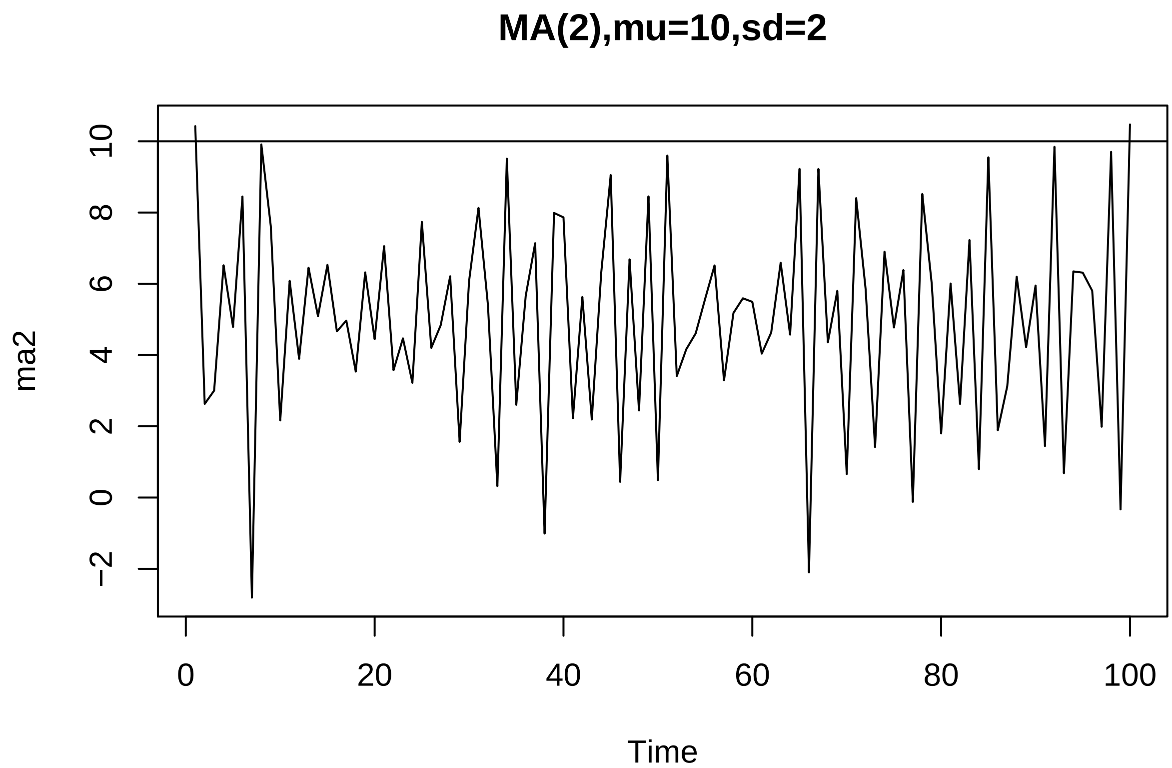
(b)

```
(acf.ma2 = ARMAacf(ma=c(-1.4,0.7), lag.max=7))
```

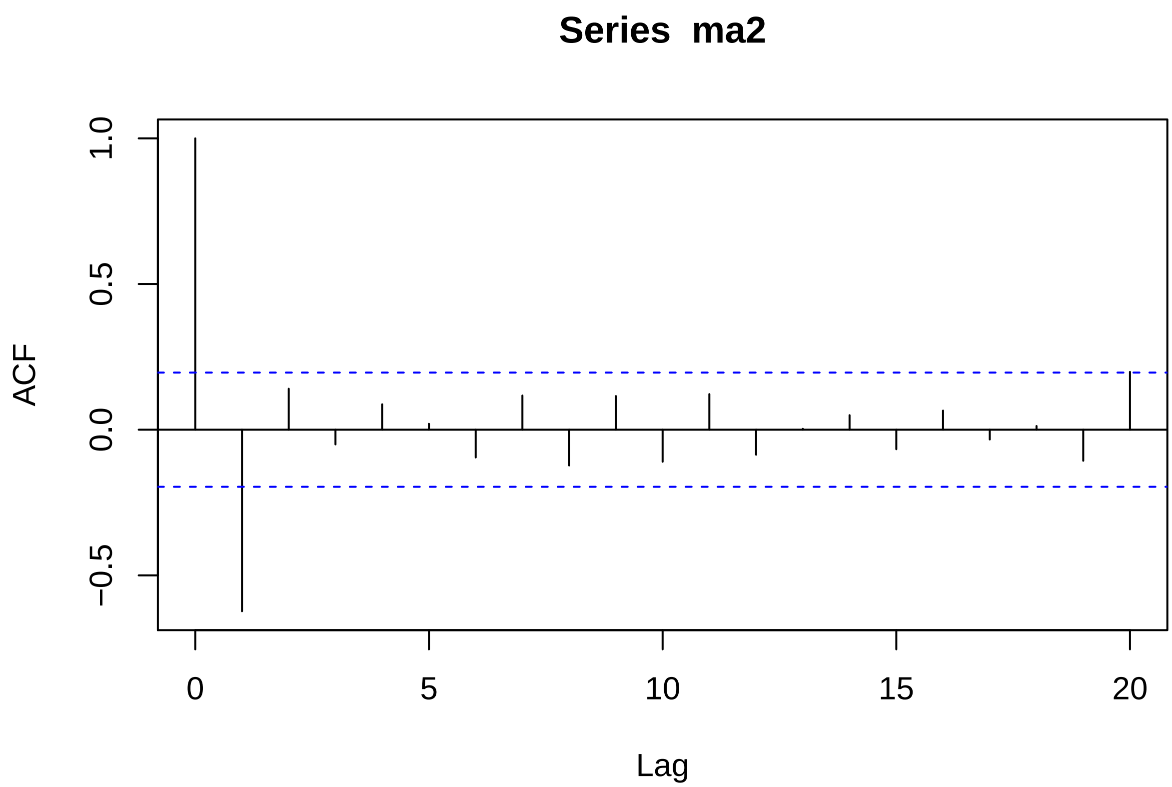
```
##         0          1          2          3          4          5          6
##  1.0000000 -0.6898551  0.2028986  0.0000000  0.0000000  0.0000000  0.0000000
##         7
##  0.0000000
```

(c)

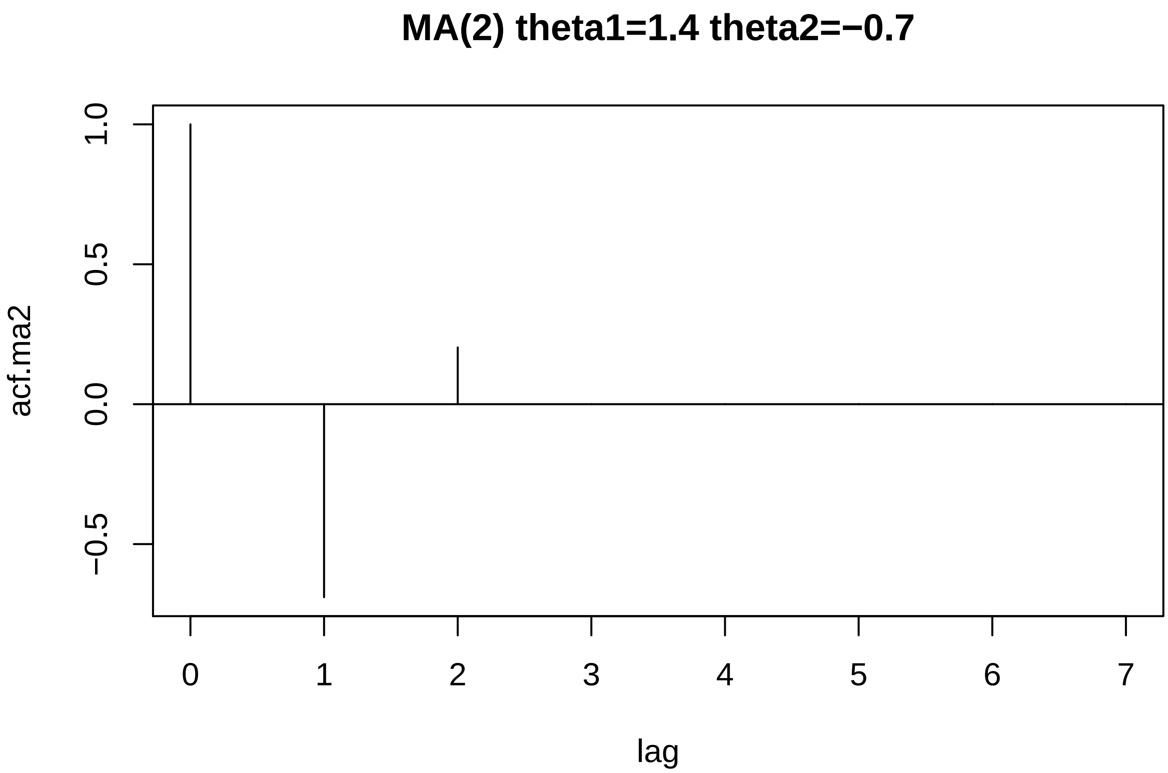
```
ma2 = 5.0+arima.sim(list(order=c(0,0,2), ma=c(-1.4,0.7)), n=100, sd=sqrt(2.4))
ts.plot(ma2, main="MA(2),mu=10,sd=2"); abline(h=10)
```




```r
acf(ma2)
```




```r
plot(0:7, acf.ma2, type="h",xlab="lag",main="MA(2) theta1=1.4 theta2=-0.7")
abline(h=0)
```



Problem 6

(a) Since $lim_{k->\infty}\rho(k) = 0$,

then exist a $k = n$, where $\gamma(n) = 0$,

then from $t = n$ to $T = \infty$, $\bar{Y_n} = (1/(T - n)) \sum_{t=n}^{T} Y_t = \mu$ then plug $t = 1$ to $t = n - 1$ into $\bar{Y_n}$ with $T - n$ times,

we can get $\bar{Y} = (1/T) \sum_{t=1}^{T} Y_t \approx \mu$

(b) Since $\bar{Y} = (1/T) \sum_{t=1}^{T} Y_t$

then $Var(\bar{Y}) = (1/T)^2 \sum_{s=1}^{T} \sum_{t=1}^{T} Y_{t-s}$, where s is add as row sums.

Then $Var(\bar{Y}) = (1/T) \sum_{k=-(T-1)}^{T-1} (1 - \frac{|k|}{T})\gamma(k)$, where $k = t - s$.

Then $TVar(\bar{Y}) = \sum_{k=-(T-1)}^{T-1} (1 - \frac{|k|}{T})\gamma(k)$,

when T->∞, $\frac{|k|}{T}$->0, and then $(1 - \frac{|k|}{T})$->1

then $TVar(\bar{Y})$->$\sum_{k=-\infty}^{\infty} \gamma(k)$