homework 3

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Problem 1

Since $\gamma(k) = 0$ for $k > 2$, it can apply to MA(2).

$Y_t = \mu + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2}$

Since $\gamma(1) = 0$, $\sigma^2(-\theta_1 + \theta_1\theta_2) = 0$, implys $-\theta_1 + \theta_1\theta_2 = 0$ then $\theta_2 = 1$

$\sigma^2(-\theta_2) = -1.6$ implys $\sigma^2 = 1.6$

$\sigma^2(1 + \theta_1^2 + \theta_2^2) = 4$

$1.6(1 + \theta_1^2 + 1) = 4$

$\theta_1 = \frac{\sqrt{2}}{2}$

let $\mu = 0$, $Y_t = \epsilon_t - \frac{\sqrt{2}}{2}\epsilon_{t-1} - \epsilon_{t-2}$, $\{\epsilon_t\} \sim WN(0, 1.6)$

Problem 2

(a) $Y_t = \frac{1}{3}Y_{t-1} + \frac{2}{9}Y_{t-2} + \epsilon_t$, then $\phi_1 = \frac{1}{3}$ and $\phi_2 = \frac{2}{9}$

the root of $1 - 1/3z - 2/9z^2 = 0$are $z_1 = -3$ and $z_2 = 3/2$, both$|z_1|, |z_2| > 1$, then $\{Y_t\}$ is causal.

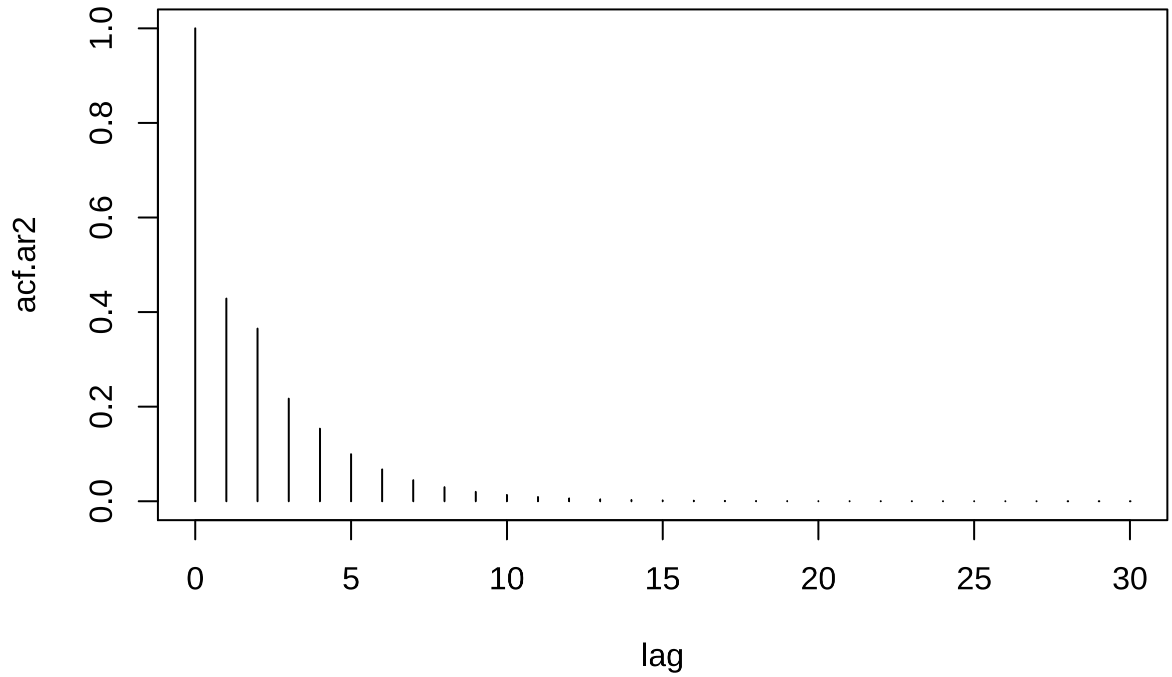
$\rho(1) = \frac{\phi_1}{1-\phi_2} = \frac{3}{7} = 0.429$

$\rho(2) = \phi_1\rho(1) + \phi_2\rho(0) = \frac{1}{3} \times 0.429 + \frac{2}{9} \times 1 = 0.365$

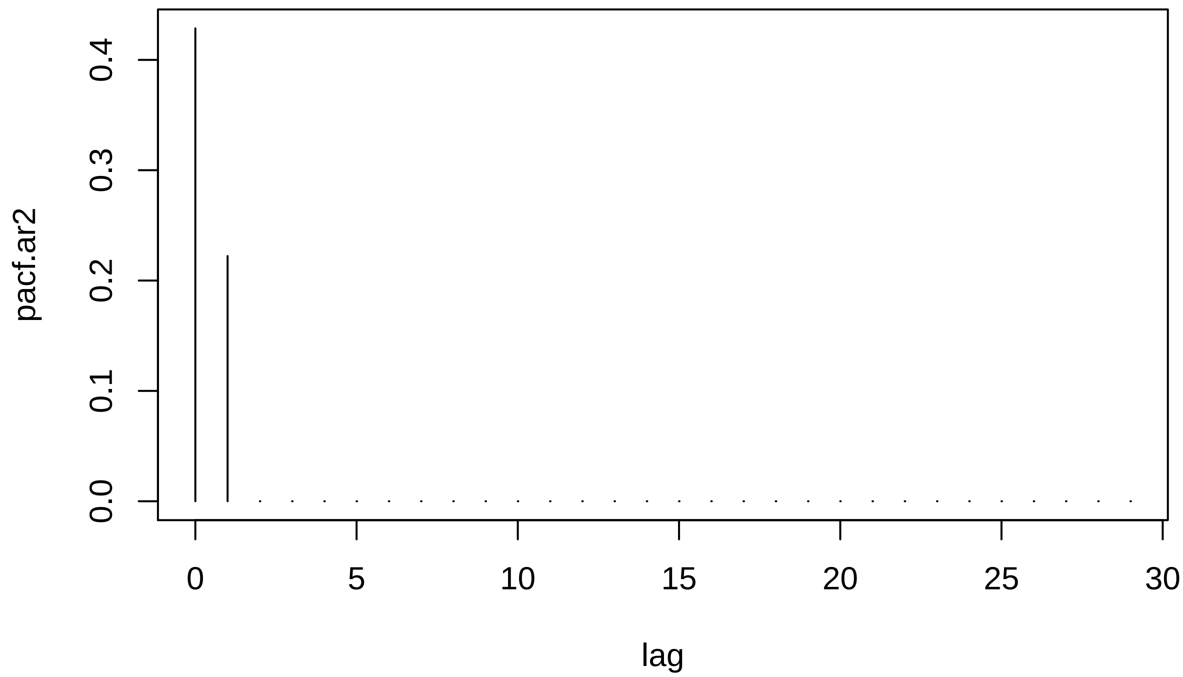
$\rho(3) = \phi_1\rho(2) + \phi_2\rho(1) = \frac{1}{3} \times 0.365 + \frac{2}{9} \times 0.429 = 0.217$

(b)

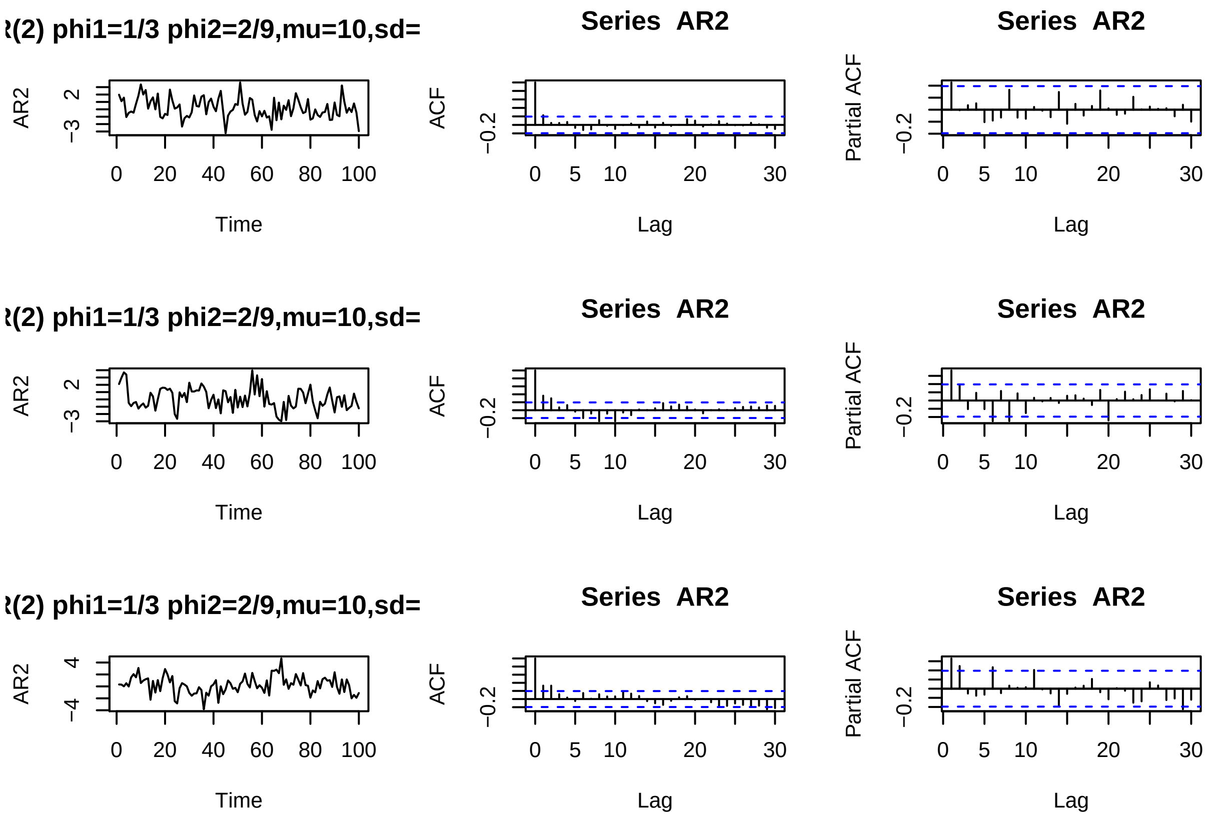
```
# i
nlag =30
acf.ar2 = ARMAacf(ar=c(1/3,2/9), lag.max=nlag)
plot(0:nlag, acf.ar2, type="h",xlab="lag");
```




```r
pacf.ar2 = ARMAacf(ar=c(1/3,2/9), lag.max=nlag,pacf = TRUE)
plot(0:29, pacf.ar2, type="h",xlab="lag");
```




```r
#ii
par(mfrow=c(3,3))
for(i in 1:3){
  AR2 = arima.sim(list(order=c(2,0,0), ar=c(1/3,2/9)),n =100,sd=sqrt(2))
  ts.plot(AR2, main="AR(2) phi1=1/3 phi2=2/9,mu=10,sd=sqrt(2)")
  acf(AR2, lag.max=nlag)
  pacf(AR2, lag.max=nlag)
  abline(h=0)
}
```




```
# iii: these figures plot the therretical ACF and PACF to lag 30. There is a acf until lag 10, and pacf
# For three times simulate of 100 observations with ts plot, acf and pacf,  most of the acf and pacf ar
```

(c) By solveing the equation, $m_1 = \frac{2}{3}$ and $m_2 = -\frac{1}{3}$ Since both $m_1, m_2 < 1$

$\rho(k) = c_1 * (\frac{2}{3})^k + c_2 * (-\frac{1}{3})^k$

apply to $\rho(0), \rho(1)$,$c_1 + c_2 = 1$,$2/3c_1 - 1/3c_2 = 0.429$ $c_1 = 0.762, c_2 = 0.237$

$\rho(k) = 0.762 * (\frac{2}{3})^k + 0.237 * (-\frac{1}{3})^k$, $k = 1, 2....$

Problem 3

(a) Since $1 - 1.3z + 0.8z^2 = 0$ have the root $z = 13/16 \pm i\sqrt{151}/16 \approx 0.8 + 0.7i$,$|z_1| = |z_2| = 1.11 > 1$ so the process is causal.

$\mu = E(Y_t) = 3.5/(1 - 1.3 + 0.8) = 7$

$\gamma(0) = Var(Y_t) = \frac{1+0.8}{1-0.8}\frac{3^2}{(1+0.8)^2-1.3^2} = 52.2$

(b) $\rho(1) = \phi_1/(1 - \phi_2) = 1.3/(1 + 0.8) = 0.722$

$\rho(2) = \phi_1\rho(1) + \phi_2\rho(0) = 1.3 * 0.722 + (-0.8) * 1 = 0.139$

$\rho(3) = 1.3 * 0.139 + (-0.8) * 0.722 = -0.397$

$\rho(4) = 1.3 * (-0.397) - 0.8 * 0.139 = -0.6275$

$\rho(5) = 1.3 * (-0.6275) - 0.8 * (-0.397) = -0.497$

(c) $m^2 - 1.3m + 0.8 = 0$

$m = 0.65 \pm 0.614i$, $R = |m_1| = |m_2| = 0.894$

$cos(\lambda) = 0.65/0.894 = 0.727$, $\lambda = 0.757$

$\rho(k) = 0.894^k[c_1 cos(0.757k) + c_2 sin(0.757k)]$

apply to ρ_0, ρ_1, $c_1 = 1$, $c_2 = 0.117$ $\rho(k) = 0.894^k[cos(0.757k) + 0.117sin(0.757k)]$,for $k = 0, 1, 2...$

$\rho(1) = 0.894[cos(0.757) + 0.117sin(0.757)] = 0.722$

$\rho(2) = 0.894^2[cos(0.757 * 2) + 0.117sin(0.757 * 2)] = 0.138$

$\rho(3) = 0.894^3[cos(0.757 * 3) + 0.117sin(0.757 * 3)] = -0.397$

$\rho(4) = 0.894^4[cos(0.757 * 4) + 0.117sin(0.757 * 4)] = -0.627$

$\rho(5) = 0.894^5[cos(0.757 * 5) + 0.117sin(0.757 * 5)] = -0.497$

(d)

```
ARMAacf(c(1.3,-0.8),lag.max = 5)
```

```
##         0          1          2          3          4          5
##  1.0000000  0.7222222  0.1388889 -0.3972222 -0.6275000 -0.4979722
```

(e) $\psi_0 = 1$

$\psi_1 = \phi_1\psi_0 = 1.3$

$\psi_2 = \phi_1\psi_1 + \phi_2\psi_0 = 0.89$ $\psi_3 = \phi_1\psi_2 + \phi_2\psi_1 = 0.917$ $\psi_4 = \phi_1\psi_3 + \phi_2\psi_2 = 0.4801$ $\psi_5 = \phi_1\psi_4 + \phi_2\psi_3 = 0.10947$ $Y_t = 7 + \epsilon_t + 1.3\epsilon_{t-1} + 0.89\epsilon_{t-2} + 0.917\epsilon_{t-3} + 0.4801\epsilon_{t-4} + 0.10947\epsilon_{t-5}$

Problem 4

(a) For AR(1) model $\gamma(1) = \phi\gamma(0) = \rho(1)\gamma(0)$

$\phi = 0.8$

For AR(2) model

$\gamma(1) = \phi_1\gamma(0) + \phi_2\gamma(1) = \rho(1)\gamma(0)$

$\gamma(2) = \phi_1\gamma(1) + \phi_2\gamma(0) = \rho(2)\gamma(0)$

$\phi_1 = 1.2, \phi_2 = -0.5$

For AR(3) model $\gamma(1) = \phi_1\gamma(0) + \phi_2\gamma(1) + \phi_3\gamma(2) = \rho(1)\gamma(0)$

$\gamma(2) = \phi_1\gamma(1) + \phi_2\gamma(0) + \phi_3\gamma(1) = \rho(2)\gamma(0)$

$\gamma(3) = \phi_1\gamma(2) + \phi_2\gamma(1) + \phi_3\gamma(0) = \rho(3)\gamma(0)$

$\phi_1 = 1.2, \phi_2 = -0.5, \phi_3 = 0$

(b) For AR(1) model $\sigma^2 = \gamma(0)[1 - \phi\rho(1)] = 2.16$

For AR(2) model $\sigma^2 = \gamma(0)[1 - \phi_1\rho(1) - \phi_2\rho(2)] = 1.62$

For AR(3) model $\sigma^2 = \gamma(0)[1 - \phi_1\rho(1) - \phi_2\rho(2) - \phi_3\rho(3)] = 1.62$

(c) Since $\phi_3 = 0$ This is a AR(2) process

Problem 5

Since m_1, m_2 are root of $m^2 - \phi_1 m - \phi_2 = 0$

$m_1, m_2 = \frac{\phi_1 \pm \sqrt{\phi_1 + 4\phi_2}}{2}$

Then $m_1 + m_2 = \phi_1$, and $m_1 m_2 = -\phi_2$

Since $m_1 = x + iy$ and $m_2 = x - iy$, $x = \frac{m_1 + m_2}{2}$ and $m_1 m_2 = (x + iy)(x - iy) = x^2 + y^2$

$R = \sqrt{x^2 + y^2} = \sqrt{m_1 m_2}$

By the way $cos(\lambda) = \frac{x}{R} = (\frac{m_1 + m_2}{2})/\sqrt{m_1 m_2} = \frac{m_1 + m_2}{2\sqrt{m_1 m_2}} = \frac{\phi_1}{2\sqrt{-\phi_2}}$