

### Question 1

A, B, C, D are filled equally in that array.

So P(probability) will be 1/4.

**A)** The expected number (average number) of trials for a success =  $1/P$

So it's  $4/1 = 4$

Reference: Slide number 13.

**B)** The expected number (average number) of trials for k successes =  $k/P$

$k = 10$ ,  $P = 1/4$

So the answer is 40.

Reference: Slide number 15.

**C)** Worst case is scan whole array, best case is all k elements are found at first k scan. Average time complexity will be average number of array locations to inspect before find k D. So it is  $\frac{k}{p} = 4k$

### Question 2

$$1 + (1/2 + 1/2) + (1/4 + 1/4 + 1/4 + 1/4) + \dots = \log(n+1) =$$

$$S = 1 + \left(\frac{1}{2+0} + \frac{1}{2+0}\right) + \left(\frac{1}{4+0} + \frac{1}{4+0} + \frac{1}{4+0} + \frac{1}{4+0}\right)$$

$$S = 1 + (1/2 + 1/3) + (1/4 + 1/5 + 1/6 + 1/7) + \dots + 1/n =$$

$$S = 1 + \left(\frac{1}{2+0} + \frac{1}{2+1}\right) + \left(\frac{1}{4+0} + \frac{1}{4+1} + \frac{1}{4+2} + \frac{1}{4+3}\right)$$

So, it is clear that  $S \leq \log(n+1)$ . This S is  $O(\log(n+1))$

### Question 3

$$S = 1/2 + 2/4 + 3/8 + 4/16 + 5/32 + \dots$$

$$S/2 = 1/4 + 2/8 + 3/16 + 4/32 + \dots$$

$$S - S/2 = 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + \dots$$

$$S/2 = 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + \dots$$

$$S = 1 + 1/2 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + \dots \Rightarrow$$

$$S = a + ar + ar^2 + ar^3 + \dots + ar^{n-1} \text{ where } a=1, r=1/2$$

Let us apply geometric series formula:

$$S = \frac{a(1-r^n)}{1-r} = \frac{1(1-1/2^n)}{1-1/2} = 2(1 - 1/2^n) \quad \text{where } n \text{ is infinite} \Rightarrow S=2(1-0)=2$$