**Question 1**

**A)** Bubble sort

|  |  |  |
| --- | --- | --- |
| **Iteration** | **inversions** | **# inversions** |
| 0 | (34 8), (34 32), (34 21), (64 51), (64 32), (64 21), (51 32), (51 21), (32 21) | 9 |
| 1 | (34 32), (34 21), (51 32), (51 21), (32 21) | 5 |
| 2 | (34 32), (34 21), (32 21) | 3 |
| 3 | (32 21) | 1 |

Total number of inversions for Bubble Sort = 18

**B)** Selection sort

|  |  |  |
| --- | --- | --- |
| **Iteration** | **inversions** | **# inversions** |
| 0 | (34 8), (34 32), (34 21), (64 51), (64 32), (64 21), (51 32), (51 21), (32 21) | 9 |
| 1 | (34 32), (34 21), (64 51), (64 32), (64 21), (51 32), (51 21), (32 21) | 8 |
| 2 | (64 51), (64 32), (64 34), (51 32), (51 34) | 5 |
| 3 | (51 34), (64 34) | 2 |
| 4 | (64 51) | 1 |

Total number of inversions for Selection Sort = 25

**C)** Insertionsort

|  |  |  |
| --- | --- | --- |
| **Iteration** | **inversions** | **# inversions** |
| 0 | (34 8), (34 32), (34 21), (64 51), (64 32), (64 21), (51 32), (51 21), (32 21) | 9 |
| 1 | (34 32), (34 21), (64 51), (64 32), (64 21), (51 32), (51 21), (32 21) | 8 |
| 2 | (34 32), (34 21), (64 51), (64 32), (64 21), (51 32), (51 21), (32 21) | 8 |
| 3 | (34 32), (34 21), (51 32), (51 21), (64 32), (64 21), (32 21) | 7 |
| 4 | (32 21), (34 21), (51 21), (64 21) | 4 |

Total number of inversions for Insertion Sort = 36

**Question 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Item # | Operation | Cost for us | Customer paid | Profit | Balance |
| 1 | Add | We assume we start with 1 slot. We add 1 item at the cost of 1 | 7 | 6 | 6 |
| 2 | Add | 3 to resize (to 2 slots) 1 to add | 7 | 6 | 3 9 |
| 3 | Add | 6 to resize (to 4 slots) 1 to add | 7 | 6 | 3  9 |
| 4 | Add | 1 to add | 7 | 6 | 15 |
| 5 | Add | 12 to resize (to 8 slots) 1 to add | 7 | 6 | 3  9 |
| 6 | Add | 1 to add | 7 | 6 | 15 |
| 7 | Add | 1 to add | 7 | 6 | 21 |
| 8 | Add | 1 to add | 7 | 6 | 27 |
| 9 | Add | 24 to resize (to 16 slots) and 1 to add | 7 | 6 | 3  9 |
| 10 | Add | 1 to add | 7 | 6 | 15 |
| 11 | Add | 1 to add | 7 | 6 | 21 |
| 12 | Add | 1 to add | 7 | 6 | 27 |
| 13 | Add | 1 to add | 7 | 6 | 33 |
| 14 | Add | 1 to add | 7 | 6 | 39 |
| 15 | Add | 1 to add | 7 | 6 | 45 |
| 16 | Add | 1 to add | 7 | 6 | 51 |
| 17 | Add | 48 to resize (to 32 slots) and 1 to add | 7 | 6 | 3  9 |
| 18 | Add | 1 to add | 7 | 6 | 15 |
|  |  |  |  |  |  |

1. Total cost? 1+3+1+6+1+12+1+1+1+1+24+1+1+1+1+1+1+1+1+48+1+1=110
2. Average actual cost: 110/23 = 4.8~
3. Amortized total cost: 7 \* 18 =126
4. Average amortized cost: 126/18 = 7