# TDDD12 - Assignment 3

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# 1 Task 1

Given relation schema R(A,B,C,D,E,F) and functional dependencies:

- FD1: {A} → {B,C}
- FD2: {C} → {A,D}
- FD3: {D,E} → {F}

Derive {C} → {B} and {A,E} → {F}

**{C} → {B}**:
Decomposition of FD2: {C} → {A} (FD4)
Transivity of FD1 and FD3: {C} → {B,C} (FD5)
Decomposition of FD5: {C} → {B} (FD6)
Q.E.D.

**{A,E} → {F}**:
Decomposition of FD1: {A} → {C} (FD7)
Transivity of FD2 and FD7: {A} → {A,D} (FD8)
Decomposition of FD8: {A} → {D} (FD9)
Pseudo-transivity of FD9 and FD3: {A,E} → {F} (FD10)
Q.E.D

# 2 Task 2

$X^+ = X = \{A\}$
FD1: $\{A\} \subseteq X^+$ AND $\{B,C\} \nsubseteq X^+ \rightarrow X^+ = \{A,B,C\}$
FD2: $\{C\} \subseteq X^+$ AND $\{A,D\} \nsubseteq X^+ \rightarrow X^+ = \{A,B,C,D\}$
FD3: $\{D,E\} \nsubseteq X^+$
$\therefore X^+ = \{A,B,C,D\}$ w.r.t. FD1-FD3 of $X = \{A\}$

$X^+ = X = \{C,E\}$
FD2: $\{C\} \subseteq X^+$ AND $\{A,D\} \nsubseteq X^+ \rightarrow X^+ = \{A,C,D,E\}$
FD1: $\{A\} \subseteq X^+$ AND $\{B,C\} \nsubseteq X^+ \rightarrow X^+ = \{A,B,C,D,E\}$
FD3: $\{D,E\} \subseteq X^+$ AND $\{F\} \nsubseteq X^+ \rightarrow X^+ = \{A,B,C,D,E,F\}$
$\therefore X^+ = \{A,B,C,D,E,F\}$ w.r.t. FD1-FD3 of $X = \{C,E\}$

# 3 Task 3

Given relation schema R(A,B,C,D,E,F) and functional dependencies:

- FD1: {A,B} → {C,D,E,F}
- FD2: {E} → {F}
- FD3: {D} → {B}

### 3.1 a)

The candidate keys for R are

- Candidate key 1: $\{A,B\}^+ = \{A,B,C,D,E,F\}$, due to FD1
- Candidate key 2: $\{A,D\}^+ = \{A,B,C,D,E,F\}$, due to FD3 and FD1.

### 3.2 b)

The closure for the FDs are the following

- FD1: $\{A,B\}^+ = \{A,B,C,D,E,F\}$
- FD2: $\{E\}^+ = \{E,F\}$
- FD3: $\{D\}^+ = \{B,D\}$

therefor FD2 and FD3 violate BCNF due to the closure not containing all attributes in R.

### 3.3 c)

Following the algorithm for decomposing R and with knowing that FD1 satisfies BCNF, R is decomposed into

- R1(A,B,C,D,E)
- R2(E,F)

Since the closure of E is known due to non trivial relation $E \rightarrow F$, this gives that E is a key for R2. In R1 there is still the violation of BCNF with $D \rightarrow B$ so it's decomposed again with the final result of

- R1X(D,B)
- R1Y(A,C,D,E)
- R2(E,F).

This decomposition has however lost the original constraint FD1.

## 4 Task 4

Given relation schema R(A,B,C,D,E) and functional dependencies:

- FD1: $\{A,B,C\} \rightarrow \{D,E\}$
- FD2: $\{B,C,D\} \rightarrow \{A,E\}$
- FD3: $\{C\} \rightarrow \{D\}$

### 4.1 a)

The closure for the following FDs are

- FD1: $\{A,B,C\}^+ = \{A,B,C,D,E\}$
- FD2: $\{B,C,D\}^+ = \{A,B,C,D,E\}$
- FD3: $\{C\}^+ = \{C,D\}$

Since FD3 does not contain all attributes BCNF is violated.

### 4.2 b)

The candidate key is $\{B,C\}$ since the closure is $\{B,C\}^+ = \{A,B,C,D,E\}$ due to FD3 and FD1 and it's the only candidate key since there is no way through FDs to obtain B or C. The relation is then decomposed with respect to FD3 since it violates BCNF.

- R1(C,D)
- R2(A,B,C,E)

FD2 is however not fully preserved but the decomposition is now in BCNF.