Consider the words A_n and B_n defined as follows: $A_0 = 0$, $B_0 = 1$, $A_n = A_{n-1}B_{n-1}$ and $B_n = A_{n-1}A_{n-1}$, for $n \geq 1$.

Lemma 1. *Let*

$$N_n = \{01\} \cup \bigcup_{k=0}^{n-2} \{B_k A_k, A_k A_k\}.$$

For $n \geq 2$ and any factor w of A_n, there exists $s \in N_n$ such that some occurrence of w in s contains the centre of s.

Proof. As in Lemma 1 of Kutsukake et al., we define a binary tree with root labeled by A_n, where any node labeled A_k has left child labeled A_{k-1} and right child labeled B_{k-1}, and any node labeled B_k has left and right children labeled A_{k-1}. We abuse notation and refer to nodes by their labels.

Consider an occurrence $w = A_n[i..j]$ and let A be the least common ancestor of $A_n[i]$ and $A_n[j]$.

Case 1: $A = A_k$. If $k \geq 2$, then $A_k = A_{k-2}B_{k-2}A_{k-2}A_{k-2}$. Now the occurrence of w contains the centre of A_k and hence contains the centre of $B_{k-2}A_{k-2} \in N_n$. If $k = 1$, then $A_k = 01 \in N_n$.

Case 2: $A = B_k$. Then $k < n$ and $B_k = A_{k-1}A_{k-1}$. Thus, the occurrence of w contains the centre of $A_{k-1}A_{k-1} \in N_n$. □

Theorem 2. *For $n \geq 3$ the set*

$$K_n = \{1, 2^{n-1}, 3 \cdot 2^{n-2}\}$$

is a string attractor of A_n.

Proof. By Lemma 1 it suffices to show that every element $s \in N_n$ has an occurrence in A_n whose centre is an element of K_n. If $s = 01$, then its centre 1 is in K_n. Next, notice that $A_n = A_{n-2}B_{n-2}A_{n-2}A_{n-2}$, so the centre 2^{n-1} of $B_{n-2}A_{n-2}$ is in K_n and the centre $3 \cdot 2^{n-2}$ of $A_{n-2}A_{n-2}$ is in K_n. Now we observe that for $1 \leq k \leq n-2$ we have $B_kA_k = A_{k-1}A_{k-1}A_{k-1}B_{k-1}$ and $A_kA_k = A_{k-1}B_{k-1}A_{k-1}B_{k-1}$. Thus there is an occurrence of $A_{k-1}A_{k-1}$ with the same centre as B_kA_k and there is an occurrence of $B_{k-1}A_{k-1}$ with the same centre as A_kA_k. It follows by induction that every element of N_n has an occurrence whose centre is an element of K_n. □