HW-9

1 -

$$\langle \vec{x}, \vec{y} \rangle = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \\ -6 \end{bmatrix} = 2.1 + (-3).5 + 1.(-6)$$

$$= (-19)$$

2 -

$$\int_0^1 (3x^2 + 2x + 5)(x+1)\,dx$$

$$= \int_0^1 (3x^3 + 3x^2 + 2x^2 + 2x + 5x + 5)\,dx$$

$$= \int_0^1 (3x^3 + 5x^2 + 7x + 5)\,dx$$

$$= \left(3\frac{x^4}{4} + 5\frac{x^3}{3} + \frac{7x^2}{2} + 5x \right) \Big|_0^1$$

$$= \underset{(3)}{\frac{3}{4}} + \underset{(4)}{\frac{5}{3}} + \underset{(6)}{\frac{7}{2}} + \underset{(12)}{5}$$

$$= \frac{9 + 20 + 42 + 60}{12}$$

$$= \frac{131}{12}$$

3- $\langle \vec{A}, \vec{B} \rangle = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix}$

$= 1.2 + 2.0 + 3.1 + 4.(-3)$

$= (-7)$

4- (cosine law)

$\|\vec{A} - \vec{B}\|^2 = \|\vec{A}\|^2 + \|\vec{B}\|^2 - 2\,\|\vec{A}\|.\|\vec{B}\|.\cos\theta$

$A - B = \begin{bmatrix} 2 & 2 \\ -2 & -4 \end{bmatrix}$ $\quad \|A - B\| = \sqrt{2^2 + 2^2 + (-2)^2 + (-4)^2}$

$= \sqrt{28}$

$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ $\quad \|A\| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$

$B = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$ $\quad \|B\| = \sqrt{(-1)^2 + 2^2 + 3^2} = \sqrt{14}$

$(\sqrt{28})^2 = (\sqrt{6})^2 + (\sqrt{14})^2 - 2.\sqrt{6}.\sqrt{14}.\cos\theta$

$28 = 6 + 14 - 2.\sqrt{6}.\sqrt{14}.\cos\theta$

$2.\sqrt{6}.\sqrt{14}\cos\theta = -8$

$\cos\theta = -0{,}436$

$\arccos(-0{,}436) = \theta$

$\theta = 115{,}88^\circ$

5 _ $\vec{x_1} = (1, 0, 3)$

$\vec{x_2} = (0, 2, 0)$

$\vec{x_3} = (-3, 0, 1)$

$$\langle \vec{x_1}, \vec{x_2}, \vec{x_3} \rangle = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$$= 1.0.(-3) + 0.2.0 + 3.0.1$$

$$= 0$$

* $\vec{x_1}$, $\vec{x_2}$ and $\vec{x_3}$ are orthogonal vectors because inner product of $\vec{x_1}$, $\vec{x_2}$ and $\vec{x_3}$ is zero.

$\vec{x_1} \Rightarrow \|\vec{x_1}\| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$$\frac{1}{\sqrt{10}} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{10} \\ 0 \\ 3/\sqrt{10} \end{bmatrix}$$

$\vec{x_2} \Rightarrow \|\vec{x_2}\| = \sqrt{4} = 2$

$$\frac{1}{2} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\vec{x_3} \Rightarrow \|\vec{x_3}\| = \sqrt{(-3)^2 + 1^2} = \sqrt{10}$

$$\frac{1}{\sqrt{10}} \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3/\sqrt{10} \\ 0 \\ 1/\sqrt{10} \end{bmatrix}$$

* orthonormal basis vectors are

$\vec{x_1} \Rightarrow \left(\frac{1}{\sqrt{10}}, 0, \frac{3}{\sqrt{10}} \right)$ $\quad \vec{x_3} \Rightarrow \left(\frac{-3}{\sqrt{10}}, 0, \frac{1}{\sqrt{10}} \right)$

$\vec{x_2} \Rightarrow (0, 1, 0)$

6 – $u_1 = (1,1,1)$ $\quad u_2 = (1,0,2)$ $\quad u_3 = (1,2,3)$

$$v_1 = (1,1,1)$$

$$v_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{\| v_1 \|^2} \cdot v_1$$

$$= (1,0,2) - \frac{\langle (1,0,2)(1,1,1) \rangle}{(\sqrt{3})^2} (1,1,1)$$

$$= (1,0,2) - (1,1,1) = (0,-1,1)$$

$$v_3 = u_3 - \frac{\langle u_3, v_1 \rangle}{\| v_1 \|^2} \cdot v_1 - \frac{\langle u_3, v_2 \rangle}{\| v_2 \|^2} v_2$$

$$= (1,2,3) - \frac{\langle (1,2,3) \cdot (1,1,1) \rangle}{(\sqrt{3})^2} (1,1,1) - \frac{\langle (1,2,3)(0,-1,1) \rangle}{(\sqrt{2})^2} (0,-1,1)$$

$$= (1,2,3) - 2(1,1,1) - \frac{1}{2}(0,-1,1)$$

$$= \left(1-2+0,\ 2-2+\frac{1}{2},\ 3-2-\frac{1}{2}\right)$$

$$= \left(-1, \frac{1}{2}, \frac{1}{2}\right)$$

$\| v_1 \| = \sqrt{3}$ $\qquad \| v_2 \| = \sqrt{2}$ $\qquad \| v_3 \| = \sqrt{\frac{3}{2}}$

$$q_1 = \frac{1}{\sqrt{3}}(1,1,1) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$q_2 = \frac{1}{\sqrt{2}}(0,-1,1) = \left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$q_3 = \frac{1}{\frac{\sqrt{3}}{\sqrt{2}}}\left(-1, \frac{1}{2}, \frac{1}{2}\right) = \left(-\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{2\sqrt{3}}\right)$$