HW-5

1) a_ $v_1 = (2,2,2)$ $v_2 = (0,0,3)$ $v_3 = (0,1,1)$

$$\begin{vmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{vmatrix} = \begin{array}{c} \begin{vmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{vmatrix} \\ \begin{matrix} 2 & 0 & 0 \\ 2 & 0 & 1 \end{matrix} \end{array} = 0-(6) = -6$$

* linearly independent

b_ $v_1 = (2,-1,3)$ $v_2 = (4,1,2)$ $v_3 = (8,-1,8)$

$$\begin{vmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{vmatrix} = \begin{array}{c} \begin{vmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{vmatrix} \\ \begin{matrix} 2 & 4 & 8 \\ -1 & 1 & -1 \end{matrix} \end{array} = (16-16-12)-(24-4-32)$$

$$= (-12)-(-12) = 0$$

* linearly dependent

2) $k_1 v_1 + k_2 v_2 + k_3 v_3 = w \rightarrow w = (a,b,c,d)$

$$\begin{aligned} 2k_1 + 3k_2 - k_3 &= a \\ k_1 - k_2 \quad &= b \\ 5k_2 + 2k_3 &= c \\ 3k_1 + 2k_2 + k_3 &= d \end{aligned} \qquad \begin{bmatrix} 2 & 3 & -1 & a \\ 1 & -1 & 0 & b \\ 0 & 5 & 2 & c \\ 3 & 2 & 1 & d \end{bmatrix}$$

$$R_2 \leftrightarrow R_1 \quad \begin{bmatrix} 1 & -1 & 0 & b \\ 2 & 3 & -1 & a \\ 0 & 5 & 2 & c \\ 3 & 2 & 1 & d \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_4 \to R_4 - 3R_1$

$$\begin{bmatrix} 1 & -1 & 0 & b \\ 0 & 5 & -1 & a-2b \\ 0 & 5 & 2 & c \\ 0 & 5 & 4 & d-3b \end{bmatrix}$$

$R_3 \to R_2 - R_3$
$R_4 \to R_2 - R_4$

$$\begin{bmatrix} 1 & -1 & 0 & b \\ 0 & 5 & -1 & a-2b \\ 0 & 0 & -3 & a-2b-c \\ 0 & 0 & -5 & a-2b-d+3b \end{bmatrix}$$

$R_4 \to R_4 - \frac{5}{3} R_3$

$$\begin{bmatrix} 1 & -1 & 0 & b \\ 0 & 5 & -1 & a-2b \\ 0 & 0 & -3 & a-2b-c \\ 0 & 0 & 0 & a-2b-d+3b - \frac{5(a-2b-c)}{3} \end{bmatrix}$$

$$a-2b-d+3b - \frac{5a-10b-5c}{3} = \frac{3a-6b-3d+9b-5a+10b+5c}{3}$$

$$\frac{-2a+13b+5c-3d}{3} = 0 \qquad \underline{\underline{2a+3d = 13b+5c}}$$

A- $2.2 + 3.3 = 13.3 + 5(-7) \Rightarrow 13 \neq 4$ X

B- $0.0 + 0.0 = 0.0 + 0.0$ ✓

C- $2.1 + 3.1 = 13.1 + 5.1 \Rightarrow 5 \neq 18$ X

D- $2.(-4) + 3.4 = 13.6 + 5(-13) \Rightarrow 4 \neq 13$ X

v_1, v_2 and v_3 span $\vec{b}\,(0,0,0,0)$

3) $\vec{u} = (0,-2,2)$ $\quad \vec{v} = (1,3,-1)$ $\quad \vec{w} = (a,b,c)$

$k_1 \vec{u} + k_2 \vec{v} = \vec{w}$

$$\begin{aligned} k_2 &= a \\ -2k_1 + 3k_2 &= b \\ 2k_1 - k_2 &= c \end{aligned}$$

$R_2 + R_1$

$\hookrightarrow 2k_2 = b+c$

$k_2 = \frac{b+c}{2}$

$k_2 = a$

$\frac{b+c}{2} = a \quad \boxed{2a = b+c}$

a =) $2.2 = 2+2$ ✓ $\longrightarrow$ linear combination of $\vec{u}$ and $\vec{v}$

b =) $2.0 = 4+5$ ✗ $\longrightarrow$ not linear combination of $\vec{u}$ and $\vec{v}$

c =) $2.0 = 0+0$ ✓ $\longrightarrow$ linear combination of $\vec{u}$ and $\vec{v}$

4) $k_1 \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix} + k_2 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{aligned} 4k_1 + k_2 \quad &= a \\ -k_2 + 2k_3 &= b \\ -2k_1 + 2k_2 + k_3 &= c \\ -2k_1 + 3k_2 + 4k_3 &= d \end{aligned}$$

$$\begin{bmatrix} 4 & 1 & 0 & a \\ 0 & -1 & 2 & b \\ -2 & 2 & 1 & c \\ -2 & 3 & 4 & d \end{bmatrix}$$

$R_3 \rightarrow R_1 + 2R_3$

$R_4 \rightarrow 2R_4 + R_1$

$$\begin{bmatrix} 4 & 1 & 0 & a \\ 0 & -1 & 2 & b \\ 0 & 5 & 2 & 2c+a \\ 0 & 7 & 8 & 2d+a \end{bmatrix}$$

$$R_3 \to R_3 + 5R_2 \qquad R_4 \to R_4 + 7R_2 \qquad \begin{bmatrix} 4 & 1 & 0 & a \\ 0 & -1 & 2 & b \\ 0 & 0 & 12 & 2c+a+5b \\ 0 & 0 & 22 & 2d+a+7b \end{bmatrix}$$

$$R_4 \to R_4 - \frac{22}{12} R_3 \qquad \begin{bmatrix} 4 & 1 & 0 & a \\ 0 & -1 & 2 & b \\ 0 & 0 & 12 & 2c+a+5b \\ 0 & 0 & 0 & 2d+a+7b - \frac{11}{6}(2c+a+5b) \end{bmatrix}$$

$$2d + a + 7b - \frac{22c + 11a + 55b}{6} = \frac{12d + \cancel{6a} + \cancel{42b} - \cancel{22c} - \cancel{11a} - \cancel{55b}}{6} = 0$$

$$-5a - 13b - 22c + 12d = 0$$

$$5a + 13b + 22c - 12d = 0$$

a $\Rightarrow$ $5.6 + 13(-5) + 22(-1) - 12(-8) = 0$ ✓

b. $\Rightarrow$ $5.0 + 13.0 + 22.0 - 12.0 = 0$ ✓

c $\Rightarrow$ $5.(-1) + 13.5 + 22.7 - 12.1 \neq 0$ X

a and b are linear combinations of A, B and C