HW-6

1) .

a- $2-x+4x^2,\ 3+6x+2x^2,\ 2+10x-4x^2$

$$k_1v_1 + k_2v_2 + k_3v_3 = 0$$

$$2k_1 + 3k_2 + 2k_3 = 0$$

$$-k_1 + 6k_2 + 10k_3 = 0$$

$$4k_1 + 2k_2 - 4k_3 = 0$$

$$\left.\begin{array}{l} R_1 + 2R_2 \rightarrow 15k_2 + 22k_3 = 0 \\ R_3 + 4R_2 \rightarrow 26k_2 - 36k_3 = 0 \end{array}\right\} R_2 + \frac{36}{22} R_1$$

$$\left(26 + 15 \cdot \frac{36}{22}\right) k_2 + \underset{\circ}{0}\,\cancel{k_3} = 0 \qquad k_2 = 0,\ k_3 = 0,\ k_1 = 0$$

* linearly independent

b- $k_1(1+3x+x^2) + k_2(x+4x^2) + k_3(5+6x+3x^2) + k_4(7+2x-x^2) = 0$

$$k_1 + \qquad + 5k_3 + 7k_4 = 0$$

$$3k_1 + k_2 + 6k_3 + 2k_4 = 0$$

$$k_1 + 4k_2 + 3k_2 - k_4 = 0$$

unknowns $>$ equation $\longrightarrow$ linearly dependent

2)

a- $v_1=(-1,2,3)$ $v_2=(2,-4,-6)$ $v_3=(-3,6,0)$

$$k_1v_1+k_2v_2+k_3v_3=0$$
$$-k_1+2k_2-3k_3=0$$
$$2k_1-4k_2+6k_3=0$$
$$3k_1-6k_2\qquad=0$$

R_2+2R_1 $\to 0=0$

$3k_1=6k_2$

$\boxed{k_1=2k_2}$

* v_1 and v_2 lie on the same line but v_3 doesn't lie

b- $v_1=(2,-1,4)$ $v_2=(4,2,3)$ $v_3=(2,7,-6)$

$$\begin{vmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ -1 & 2 & 7 \\ 4 & 3 & -6 \end{vmatrix} \begin{matrix} 2 & 4 & 2 \\ -1 & 2 & 7 \end{matrix} = (-24-6+112) - (16-42+24)$$
$$=82-(-2)$$
$$=84$$

* v_1, v_2 and v_3 doesn't lie on the same line

c- $v_1=(4,6,8)$ $v_2(2,3,4)$ $v_3(-2,-3,-4)$

$$\begin{vmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{vmatrix} = \begin{vmatrix} 4 & 2 & -2 \\ 6 & 3 & -3 \\ 8 & 4 & -4 \end{vmatrix} \begin{matrix} 4 & 2 & -2 \\ 6 & 3 & -3 \end{matrix} = (-48-48-48) - (-48-48-48)$$
$$=0$$

* v_1, v_2 and v_3 lie on the same line.

3) $v_1 = (\lambda, \frac{-1}{2}, \frac{-1}{2})$

$v_2 = (\frac{-1}{2}, \lambda, \frac{-1}{2})$

$v_3 = (\frac{-1}{2}, \frac{-1}{2}, \lambda)$

If $k_1 v_1 + k_2 v_2 + k_3 v_3 = 0$ is linearly dependent, v_1, v_2 and v_3 lie on the same line or $v_1 = v_2 = v_3$

$$\lambda = \frac{-1}{2}$$

4) $k_1 \begin{bmatrix} 1 & 0 \\ 1 & k \end{bmatrix} + k_2 \begin{bmatrix} -1 & 0 \\ k & 1 \end{bmatrix} + k_3 \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} = 0$

$$k_1 - k_2 + 2k_3 = 0$$
$$k_1 + k.k_2 + k_3 = 0$$
$$k.k_1 + k_2 + 3k_3 = 0$$

* If it is linearly independent, k_1, k_2 and k_3 are zero

$$0.k = 0$$

$\rightarrow$ * $k \in R$