1. a) $A \cdot A^{-1} = I$

$$\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a+c=1$ $\quad b+d=0$

$-a=0$ $\quad -b=1$

$a=0$
$b=-1$
$c=1$
$d=1$

$$A^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

1. b) $B \cdot B^{-1} = I$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$-2\,(2a+c=1)$ $\quad -2(2b+d=0)$

$+\ a+2c=0$ $\quad +\ b+2d=1$

$a=2/3$ $\quad b=-1/3$

$a=2/3$
$b=-1/3$
$c=-1/3$
$d=2/3$

$$B^{-1} = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

1. c) $C \cdot C^{-1} = I$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$-2a+2c=1$ $\quad -2b+2d=0$

$+\ 2a-2c=0$ $\quad +\ 2b-2d=1$

$0=1$ $\quad 0=1$

C'nin tersi yoktur.

1. d) $D \cdot D^{-1} = I$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$c=1$ $\quad b=1$

$a=0$ $\quad d=0$

$$D^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

1. e) $E \cdot E^{-1} = I$

$$\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$-a+c=1$ $\quad -b+d=0$

$+\ a+c=0$ $\quad +\ b+d=1$

$c=1/2$ $\quad d=1/2$

$a=-1/2$
$b=1/2$
$c=1/2$
$d=1/2$

$$E^{-1} = \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

(2)

$$\left[\begin{array}{ccccc|ccccc} 2 & 1 & -3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 8 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_4 \to R_4 - 4R_5$

$R_2 \to R_2 - R_5$

$$\left[\begin{array}{ccccc|ccccc} 2 & 1 & -3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_4 \to R_4 / 4$

$R_5 \to R_5 / 2$

$$\left[\begin{array}{ccccc|ccccc} 2 & 1 & -3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/4 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/2 \end{array}\right]$$

$R_1 \to R_1 - R_5$

$R_3 \to R_3 - 3R_5$

$$\left[\begin{array}{ccccc|ccccc} 2 & 1 & -3 & 1 & 0 & 1 & 0 & 0 & 0 & -1/2 \\ 0 & -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 & -3/2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/4 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/2 \end{array}\right]$$

$R_3 \to R_3 - 2R_4$

$R_2 \to R_2 - R_4$

$D_1 \to R_1 - R_4$

$$\left[\begin{array}{ccccc|ccccc} 2 & 1 & -3 & 0 & 0 & 1 & 0 & 0 & -1/4 & 1/2 \\ 0 & -1 & 2 & 0 & 0 & 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/4 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/2 \end{array}\right]$$

$R_2 \to R_2 - 2R_3$

$$\left[\begin{array}{ccccc|ccccc} 2 & 1 & -3 & 0 & 0 & 1 & 0 & 0 & -1/4 & 1/2 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & -2 & 3/4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/4 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/2 \end{array}\right]$$

$R_1 \to R_1 + R_2$

$$\left[\begin{array}{ccccc|ccccc} 2 & 0 & -3 & 0 & 0 & 1 & 1 & -2 & 1/2 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & -2 & 3/4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/4 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/2 \end{array}\right]$$

$R_1 \to R_1 + 3R_3$

$$\left[\begin{array}{ccccc|ccccc} 2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & -2 & 3/4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/4 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/2 \end{array}\right]$$

$R_1 \to R_1 / 2$

$R_2 \to R_2 \cdot (-1)$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 2 & -3/4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/4 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1/2 \end{array}\right]$$

Cevap = B

3-

$$\frac{1}{2}\begin{bmatrix} 1 & -2 & 0 \\ 4 & 6 & 1 \\ 6 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 2\cdot\begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 1 \\ 3 & 0 & a-b \\ 2 & a+b & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & -3 & 2 \\ -2 & 1 & -3/2 \\ -3 & 4 & 0 \\ -4 & 2 & 1/2 \end{bmatrix}$$

$$\frac{1}{2}\cdot 0 - 2(a-b) = 0 \longrightarrow -2a+2b = 0$$

$$\frac{1}{2}\cdot 0 - 2(a+b) = 2 \longrightarrow -2a-2b = 2$$

$$-4a = 2$$

$$a = \frac{-1}{2} \qquad b = \frac{-1}{2}$$