HW-7

1 _ $T(x) = (x, 3x) \quad \mathbb{R} \longrightarrow \mathbb{R}^2$

$T(x) = x(1, 3)$

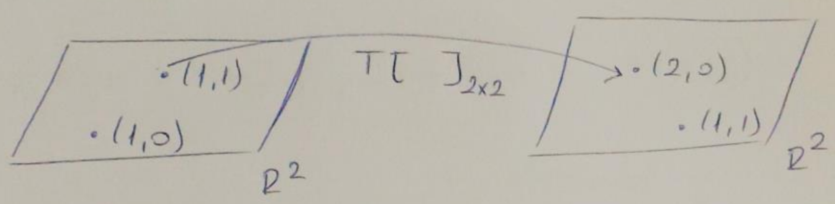
Matrix of linear transformation is $[1 \ 3]_{1\times 2}$

2 _ I. method

$T(x, y) = (x+y, x-y) \quad \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$T(1,1) = (2, 0)$

$T(1,0) = (1, 1)$



$$[1 \ 1] \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2} = [2 \ 0]$$

$a + c = 2 \qquad b + d = 0$

$$[1 \ 0] \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2} = [1 \ 1]$$

$a = 1 \qquad b = 1$

$c = 1 \qquad d = -1$

Matrix of linear transformation is $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}_{2\times 2}$

2 _ II. method

$$T(x,y) = (x+y\ ,\ x-y)$$

$$T(x,y) = x(1,1) + y(1,-1)$$

Matrix of linear transformation is $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}_{2\times 2}$

3 _

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ -1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+2y \\ 2x+3y+z \\ -x+y \end{pmatrix}$$

$$T(x,y,z) = (x+2y\ ,\ 2x+3y+z\ ,\ -x+y)$$

Answer = 'C'

4 _ $[E' \mid E]$

$R_3 \to R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & 1 \end{array}\right]$$

$R_2 \to R_3 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & 1 \end{array}\right]$$

The base transformation matrix from E to E' is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

5 - $T(x,y) = (x+y, x)$

$S(x,y) = (y, x+y)$

$(S \circ T)(x,y) =$

$S(T(x,y)) = S(x+y, x)$

$S(x+y, x) = (x, 2x+y)$