# HW-8

1 - $T(x,y,z) = (x+2y+3z, -y+2z, 2z)$

$= x(1,0,0) + y(2,-1,0) + z(3,2,2)$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$|A - \lambda| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & -1-\lambda & 2 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (1-\lambda).(-1-\lambda).(2-\lambda) = 0$$

$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = -1$

$\lambda_1 = 1 \Rightarrow$

$$\begin{pmatrix} 0 & 2 & 3 \\ 0 & -2 & 2 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$2x_2 + 3x_3 = 0$

$-2x_2 + 2x_3 = 0$

$x_3 = 0$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix} = k \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\lambda_2 = 2 \quad \Rightarrow$

$$\begin{pmatrix} -1 & 2 & 3 \\ 0 & -3 & 2 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} -x_1 + 2x_2 + 3x_3 &= 0 \\ -3x_2 + 2x_3 &= 0 \end{aligned} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 13k \\ 2k \\ 3k \end{pmatrix} = k \begin{pmatrix} 13 \\ 2 \\ 3 \end{pmatrix}$$

$\lambda_3 = -1 \quad \Rightarrow$

$$\begin{pmatrix} 2 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} 2x_1 + 2x_2 + 3x_3 &= 0 \\ 2x_3 &= 0 \\ 3x_3 &= 0 \end{aligned} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} k \\ -k \\ 0 \end{pmatrix} = k \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$P = \left( \begin{bmatrix} \text{1.} \\ \text{eigen} \\ \text{vector} \end{bmatrix} \begin{bmatrix} \text{2. eigen} \\ \text{vector} \end{bmatrix} \begin{bmatrix} \text{3.} \\ \text{eigen} \\ \text{vector} \end{bmatrix} \right)$$

$$= \begin{pmatrix} 1 & 13 & 1 \\ 0 & 2 & -1 \\ 0 & 3 & 0 \end{pmatrix}$$

$$B = P^{-1} \cdot A \cdot P$$

for $P^{-1}$

$$\begin{bmatrix} (-1)^2 \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} & (-1)^3 \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix} & (-1)^4 \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} \\ (-1)^3 \begin{vmatrix} 13 & 1 \\ 3 & 0 \end{vmatrix} & (-1)^4 \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} & (-1)^5 \begin{vmatrix} 1 & 13 \\ 0 & 3 \end{vmatrix} \\ (-1)^4 \begin{vmatrix} 13 & 1 \\ 2 & -1 \end{vmatrix} & (-1)^5 \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} & (-1)^6 \begin{vmatrix} 1 & 13 \\ 0 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 3 & 0 & -3 \\ -15 & 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & 3 & -15 \\ 0 & 0 & 1 \\ 0 & -3 & 2 \end{bmatrix}$$

$$\det(P) = 3$$

$$P^{-1} = \frac{1}{3} \begin{bmatrix} 3 & 3 & -15 \\ 0 & 0 & 1 \\ 0 & -3 & 2 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 1 & -5 \\ 0 & 0 & 1/3 \\ 0 & -1 & 2/3 \end{bmatrix}$$

$$B = P^{-1} . A . P$$

$$= \begin{pmatrix} 1 & 1 & -15 \\ 0 & 0 & 1/3 \\ 0 & -1 & 2/3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 13 & 1 \\ 0 & 2 & -1 \\ 0 & 3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & -5 \\ 0 & 0 & 2/3 \\ 0 & 1 & -2/3 \end{pmatrix} \begin{pmatrix} 1 & 13 & 1 \\ 0 & 2 & -1 \\ 0 & 3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

* Eigenvalues of given linear transformation are

$$\lambda_1 = 1 \ , \ \lambda_2 = 2 \ , \ \lambda_3 = -1$$

* Eigenvectors of given linear transformation are

1. eigenvector $(1, 0, 0)$
2. eigenvector $(13, 2, 3)$
3. eigenvector $(1, -1, 0)$

2- $T(x,y) = (x, -2x+y)$

$$= x(1,-2) + y(0,1)$$

$$A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$|A - \lambda| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 \\ -2 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 = 0 \qquad \lambda = 1$$

$\lambda = 1 \Rightarrow$

$$\begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-2x_1 = 0 \qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ k \end{pmatrix} = k \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_2 = k$$

$$P = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \qquad \det(P) = 0$$

* Not exist inverse matrix of $P$ because $\det(P) = 0$.

* Not exist diagonalization matrix of given linear transformation because not exist inverse matrix of $P$.

3 _ $A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 6 & 0 \\ 3 & 0 & 3 \end{bmatrix}$

$|A - \lambda| = 0$

$$\begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 6-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = (6-\lambda) \cdot (-1)^4 \begin{vmatrix} 3-\lambda & 3 \\ 3 & 3-\lambda \end{vmatrix}$$

$$= (6-\lambda) \left( (3-\lambda)^2 - 9 \right)$$

$$= (6-\lambda) \left( \cancel{9} - 6\lambda + \lambda^2 - \cancel{9} \right)$$

$$= (6-\lambda) \cdot \lambda \cdot (-6+\lambda) = 0$$

$$\lambda_1 = 0 \quad \lambda_2 = 6 \quad \lambda_3 = 6$$

$\lambda_1 = 0 \Rightarrow$

$$\begin{pmatrix} 3 & 0 & 3 \\ 0 & 6 & 0 \\ 3 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$3x_1 + 3x_3 = 0$

$6x_2 = 0$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} k \\ 0 \\ -k \end{pmatrix} = k \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$\lambda_{2,3} = 6 \Rightarrow$

$$\begin{pmatrix} -3 & 0 & 3 \\ 0 & 0 & 0 \\ 3 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-3x_1 + 3x_3 = 0$

$x_2 = R$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} k \\ R \\ k \end{pmatrix} = k \begin{pmatrix} 1 \\ R \\ 1 \end{pmatrix}$$

$x_1 = 1 \quad x_2 = 0 \longrightarrow$ 2. eigen vector $(1,0,1)$

$x_1 = 3 \quad x_2 = 1 \longrightarrow$ 3. eigen vector $(3,1,3)$

$$P = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$P^{-1} \Rightarrow$

$$\begin{bmatrix} (-1)^2 \begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} & (-1)^3 \begin{vmatrix} 0 & 1 \\ -1 & 3 \end{vmatrix} & (-1)^4 \begin{vmatrix} 0 & 0 \\ -1 & 1 \end{vmatrix} \\ (-1)^3 \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} & (-1)^4 \begin{vmatrix} 1 & 3 \\ -1 & 3 \end{vmatrix} & (-1)^5 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \\ (-1)^4 \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} & (-1)^5 \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} & (-1)^6 \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -1 & 0 \\ 0 & 6 & -2 \\ 1 & -1 & 0 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 6 & -1 \\ 0 & -2 & 0 \end{bmatrix}$$

$\det(P) = -2$

$$P^{-1} = \frac{1}{-2} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 6 & -1 \\ 0 & -2 & 0 \end{bmatrix}$$

$B = P^{-1} . A . P$

$$= \frac{1}{-2} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 6 & -1 \\ 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 3 \\ 0 & 6 & 0 \\ 3 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= \frac{-1}{2} \begin{bmatrix} 0 & 0 & 0 \\ -6 & 36 & -6 \\ 0 & -12 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= \frac{-1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \Rightarrow$$ Diagonalization matrix of given transformation matrix.