MA 202: Numerical Methods
Semester–II, Academic Year 2022-23
Tutorial set -2

Instructions

- **Submission deadline:** April 4, 2023.
- Any request for extension will not be entertained. Late submission (even by 1 second) will not be accepted.
- Problem 1 will be solved during the tutorial. The students have to solve the rest of the problems on their own and submit the answers to all problems.
- You have to write computer programs for each problem separately. You may use MATLAB or Python to write the programs.
- Submission will be through google classroom.
- Please put all the programs in a single 'zip' file and upload on google classroom.
- Each program should be named as follows: `Tut<Insert-Number>Prob<Insert-Number>.m/py`.
- The 'zip' file should be named as: `Tut<Insert-Number><Your-Name><Roll-Number>.zip /rar/...`. If additional information is required to run your program, please submit that as a PDF file contained within the same 'zip' file. Handwritten solutions, wherever appropriate must also be included in this same 'zip' file.
- Please make sure that your programs do not contain any error. In case there is a run time error, not credit for that particular problem will be given.
- Please do not submit screenshots of your programs. You must submit executable files.

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(1) Determine the roots of the simultaneous non-linear equations:

$$\begin{aligned}(x-4)^2+(y-4)^2 &= 5\\ x^2+y^2 &= 16\end{aligned}$$

using the Newton's method. Can you take an initial guess such that $x_0 = y_0$? Explain.

(2) Let $f(x) = e^x - x - 1$. Show that f has a zero of multiplicity 2 at $x = 0$ and that the Newton's method with $x_0 = 1$, converges to this zero but not quadratically. Moreover, show that the modified newton's method improves the rate of convergence.

(3) Solve the following equations using the Newton's method.

$$\begin{aligned}x_1^2 - x_2 + x_3^3 &= 0\\ x_1x_2 - x_3^2 - 1 &= 0\\ x_1^2x_3^2 - x_2 + 1 &= 0\end{aligned}$$

For now, you may use an in built MATLAB/Python function only to invert a matrix. Does the Newton's method converge for all choices of initial guess? Give an example of an initial guess that causes the iterations to diverge. How many sets of roots (i.e., values of $\mathbf{x} = [x_1, x_2, x_3]$) are you able to find using Newton's method? - Prepare a list of roots you have been able to find and also mention the initial guesses. Are there any other roots that this method is unable to find?

(4) It may be shown that inside a thermoflask, the temperature (T's, measured in degree Celsius) of various layers and the heat fluxes (q's, in mW/m^2) through those layers are related approximately by the following equations:

$$q_1 = \epsilon\left[(T_0 + 273)^4 - (T_1 + 273)^4\right]$$
$$q_2 = A(T_1 - T_2)$$
$$q_3 = B(T_2 - T_3)^{4/3}$$

where $T_0 = 500^0$C, $T_3 = 25^0$C, $\epsilon = 10^{-9}$, $A = 4$ and $B = 1.3$. At steady state, $q_1 = q_2 = q_3$. Find T_2, T_3 and the heat flux in the steady state.

(5) Try finding complex roots of the cubic polynomial $f(x) = x^3 - 1$ using the Newton's method. Take complex numbers as your initial guess and check whether you the method converges to the correct roots. If so, try this for other polynomials also.