MA 202: Numerical Methods
Semester–II, Academic Year 2022-23
Tutorial set -1

Instructions

- **Submission deadline:** March 31, 2023.
- Any request for extension will not be entertained. Late submission (even by 1 second) will not be accepted.
- Problems 1, 4 and 5 will be solved during the tutorial. The students have to solve the rest of the problems on their own and submit the answers to all problems.
- You have to write computer programs for each problem (except question 3) separately. You may use MATLAB or Python to write the programs.
- Submission will be through google classroom.
- Please put all the programs in a single 'zip' file and upload on google classroom.
- Each program should be named as follows: `Tut<Insert-Number>Prob<Insert-Number>.m/py`.
- The 'zip' file should be named as: `Tut<Insert-Number><Your-Name><Roll-Number>.zip/rar/....` If additional information is required to run your program, please submit that as a PDF file contained within the same 'zip' file. Handwritten solutions, wherever appropriate must also be included in this same 'zip' file.
- Please make sure that your programs do not contain any error. In case there is a run time error, not credit for that particular problem will be given.
- Please do not submit screenshots of your programs. You must submit executable files.

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(1) The Redlich-Kwong equation of state is given by

$$p = \frac{RT}{v-b} - \frac{a}{v(v+b)\sqrt{T}}$$

where R = the gas constant [= 0.518 kJ/(kg K)], T = absolute temperature (K), p = absolute pressure (kPa), and v = the specific volume of 1 kg of gas (m^3/kg). The parameters a and b are calculated by

$$a = \frac{0.427 R^2 T_c^{2.5}}{p_c}, \qquad b = 0.0866 R \frac{T_c}{p_c}$$

where p_c = critical pressure (kPa) and T_c = critical temperature (K). It is known that for Methane p_c = 4580 kPa and T_c = 191 K. Use the bisection method to calculate the specific volume v of Methane when $T = -50$°C and p = 65000 kPa. Use $v_l = 1$ m^3/kg and $v_u = 5$ m^3/kg as your initial interval boundaries.

(2) Develop a fixed point iteration scheme to find the root of any given number A. Can you comment on the choices for $g(x)$? Also comment on when the iterations converge and when they diverge.

(3) Prove that for Newton's method, the true errors in the successive iterations ($\epsilon_{t,i}$ and $\epsilon_{t,i+1}$, say) are related as:

$$\epsilon_{t,i+1} = \mathcal{B}\epsilon_{t,i}^2$$

Can you find an expression for the constant $\mathcal{B}$? Assume that the equation to be solved is of the form: $f(x) = 0$.

(4) The Manning equation may be written for a rectangular open channel as:

$$Q = \frac{\sqrt{S}(BH)^{5/3}}{n(B+2H)^{2/3}}$$

where Q is the flow rate (m^3/s), S is the slope (m/m), H is the depth (m) and n is the Mannign roughness coefficient. Solve the above equation for H, when $Q = 5$, $S = 0.0002$, $B = 20$ and $n = 0.03$, using fixed point iteration.

(5) Locate the first positive root of:

$$f(x) = \sin(x) + \cos(1 + x^2) - 1$$

using Newton's method. Observe what you get when you start with the following initial guesses: (a) $x_0 = 0.1$, (b) $x_0 = 2$, (c) $x_0 = 0.2$ and (d) $x_0 = 1$.

(6) Use the Newton's method to find the root of:

$$f(x) = e^{-0.5x}(4 - x) - 2$$

Employ initial guesses, (a) 2, (b) 6 and (c) 8 and explain your results.

(7) Find the lower positive root of

$$f(x) = 7\sin(x)e^{-x} - 1$$

using the Secant method. Try the initial guesses $x_{-1} = 0.4$ and $x_0 = 0.5$.

(8) Solve for the root of:

$$f(x) = x^5 + 16.05x^4 + 88.75x^3 - 132.0375x^2 + 116.35x + 31.6875$$

using the Modified Secant method with $x_0 = 0.5825$ and $\delta x = 0.05$. Can you find other roots of the above equation starting with different initial guesses?