MA 202: Numerical Methods
Semester–II, Academic Year 2022-23
Tutorial set -4

Instructions

- **Submission deadline:** April 24, 2023.
- Any request for extension will not be entertained. Late submission (even by 1 second) will not be accepted.
- Some selected problems from question 2 will be solved during the tutorial. The students have to solve the rest of the problems on their own and submit the answers to all problems.
- You have to write computer programs for each problem separately. You may use MATLAB or Python to write the programs.
- Submission will be through google classroom.
- Please put all the programs in a single 'zip' file and upload on google classroom.
- Each program should be named as follows: `Tut<Insert-Number>Prob<Insert-Number>.m/py`.
- The 'zip' file should be named as: `Tut<Insert-Number><Your-Name><Roll-Number>.zip /rar/...`. If additional information is required to run your program, please submit that as a PDF file contained within the same 'zip' file. Handwritten solutions, wherever appropriate must also be included in this same 'zip' file.
- Please make sure that your programs do not contain any error. In case there is a run time error, not credit for that particular problem will be given.
- Please do not submit screenshots of your programs. You must submit executable files.

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(1) The outflow concentration from a reactor is measured at discrete times over a 24-hr period and the values are as follows:

t (hr.)	0	1	5.5	10	12	14	16	18	20	24
c, (mg/l)	1	1.5	2.3	2.1	4	5	5.5	5	3	1.2

The outflow rate in m^3/s may be computed using the relation:

$$Q(t) = 20 + 10 \sin \left\{ \frac{2\pi}{24} (t - 10) \right\}$$

where t is in seconds. Compute the average concentration leaving the reactor over the 24-hr. period. The avg. concentration is defined as:

$$\bar{c} = \frac{\int_0^t dt' Q(t') c(t')}{\int_0^t dt' Q(t')}$$

(2) Perform the following integrals using the Trapezoidal and the Simpson's 1/3 rule, for various values of the arguments (x, y, etc.) of your choice. Ensure that the values that you obtain are accurate upto $\varepsilon_a < 0.001$.

$$\text{(a)} \qquad I(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$$

$$\text{(b)} \qquad Ei(x) = \int_{x}^{\infty} \frac{e^{-t}}{t} dx$$

$$\text{(c)} \qquad Si(z) = \int_{0}^{z} \frac{\sin(t)}{t} dt$$

$$\text{(d)} \qquad I = \int_{0}^{\pi} \sqrt{\sin(x)} dx$$

$$\text{(e)} \qquad I = \int_{1}^{10} \sqrt{\log(x)} dx$$

(3) Develop a computer program(s) to perform the following tasks.

(a) Create $N+1$ uniformly spaced grid points between -1 and 1 (take $N = 16$) and compute the function $f(x) = 1/(1+16x^2)$.

(b) Now generate a Lagrange interpolating polynomial $g(x)$ of degree $N = 16$ using the data points you have created in part (a). Plot $g(x)$ between -1 and 1 along with the discrete data points generated in part (a). Set the y-limits of the graph between -1 and 1.5. Define $E_{t,\max} = |(f(x) - g(x))|$. Compute the value of $E_{t,\max}$. Do you observe the Runge Phenomena?

(c) Now evaluate the same function $f(x) = 1/(1+16x^2)$ at the following set of points: $x_j = \cos(j\pi/N)$, $j = 0, 1, 2, ..., N$.

(d) Repeat part (b), but now with the newly generated data points in part (c). Do you still observe the Runge Phenomena? Also report $E_{t,\max}$ in this case. The set of points chosen in part (c) are called *Chebyshev-Lobatto points* and they form the basis for Spectral Methods.

(4) The following data were collected for a cross-section of a river (y = distance from the bank, H = depth and U = velocity):

y (m)	0	1	3	5	7	8	9	10
H (m)	0	1	1.5	3	3.5	3.2	2	0
U (m/s)	0	0.09	0.12	0.23	0.243	0.27	0.18	0

Use a numerical computation technique of your choice to compute the (a) average depth (defined as $\bar{H} = (b-a)^{-1} \int_a^b H(y)dy$, where $a = \min(y)$ and $b = \max(y)$); (b) the cross-sectional area, defined as: $A_c = \int_a^b H(y)dy$; (c) The flow rate, defined as: $Q = \int_a^b H(y)U(y)dy$.