Ζητούμενα

Μέρος Α: Θεωρητικό (20%)

1. Δίδεται η πιο κάτω εικόνα :

2	1
1	0

Υπολογίσετε με το χέρι το μέτρο και τη φάση του DFT.

$$\tilde{I}_{real}(0,0) = \sum_{i=0}^{N-1}\sum_{j=0}^{m-1} I(i,j) * \cos\left(2\pi * \left(\frac{ui}{N} + \frac{vj}{m}\right)\right)$$

$$= I(0,0) * \cos\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right) \quad 1 \quad 1$$

$$= 2*1 + I(0,1) * \cos\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right)$$

$$= 2+1*1 + I(1,0) * \cos\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right) \quad 1$$

$$= 2+1+1*1 + I(1,1) * \cos\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right) \quad 1$$

$$= 2+1+1+0 = 4$$

$$\tilde{I}_{real}(0,1) = I(0,0) * \cos\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right) \quad 1$$

$$= 2*1 + I(0,1) * \cos\left(2\pi * \left(\frac{0}{2} + \frac{1}{2}\right)\right) \quad -1$$

$$= 2+1*(-1) + I(1,0) * \cos\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right) \quad 1$$

$$= 2-1+1 + I(1,1) * \cos\left(2\pi * \left(\frac{0}{2} + \frac{1}{2}\right)\right) \quad -1$$

$$= 2-1+1+0*(-1)$$

$$= 2-1+1+0 = 2$$

$$\tilde{I}_{real}(1,0) = I(0,0) * \cos\left(2\pi * \left(\frac{0}{2}+\frac{0}{2}\right)\right) \quad 1$$
$$= 2*1 + I(0,1) * \cos\left(2\pi * \left(\frac{0}{2}+\frac{0}{2}\right)\right) \quad 1$$
$$= 2+1*1 + I(1,0) * \cos\left(2\pi * \left(\frac{0}{2}+\frac{0}{2}\right)\right) \quad -1$$
$$= 2+1+1*(-1) + I(1,1) * \cos\left(2\pi * \left(\frac{0}{2}+\frac{1}{2}\right)\right) \quad -1$$
$$= 2+1-1+0*(-1)$$
$$= 2+1-1+0 = 2$$

$$\tilde{I}_{real}(1,1) = I(0,0) * \cos\left(2\pi * \left(\frac{0}{2}+\frac{0}{2}\right)\right) \quad 0$$
$$= 2*1 + I(0,1) * \cos\left(2\pi * \left(\frac{0}{2}+\frac{1}{2}\right)\right) \quad -1$$
$$= 2+1*(-1) + I(1,0) * \cos\left(2\pi + \left(\frac{1}{2}+\frac{0}{2}\right)\right) \quad -1$$
$$= 2-1+1*(-1) + I(1,1) * \cos\left(2\pi + \left(\frac{1}{2}+\frac{1}{2}\right)\right) \quad 1$$
$$= 2-1-1+0*1$$
$$= 2-1-1+0 = 0$$

$$\tilde{I}_{imag}(0,0) = -\sum_{i=0}^{N-1}\sum_{j=0}^{m-1} I(i,j) * \sin\left(2\pi * \left(\frac{ui}{N}+\frac{vj}{m}\right)\right)$$
$$= -I(0,0) * \sin\left(2\pi * \left(\frac{0}{2}+\frac{0}{2}\right)\right) \quad 0$$
$$= -2*0 - I(0,1) * \sin\left(2\pi * \left(\frac{0}{2}+\frac{0}{2}\right)\right) \quad 0$$
$$= 0-1*0 - I(1,0) * \sin\left(2\pi * \left(\frac{0}{2}+\frac{0}{2}\right)\right) \quad 0$$
$$= 0-0-1*0 - I(1,1) * \sin\left(2\pi * \left(\frac{0}{2}+\frac{0}{2}\right)\right) \quad 0$$
$$= 0-0-0-0 =$$
$$= 0$$

$$\tilde{I}_{imag}(0,1) = -I(0,0) * \sin\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right)$$
$$= -2 * 0 - I(0,1) * \sin\left(2\pi * \left(\frac{0}{2} + \frac{1}{2}\right)\right)$$
$$= 0 - 1 * 0 - I(1,0) * \sin\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right)$$
$$= 0 - 0 - 1 * 0 - I(1,1) * \sin\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right)$$
$$= 0 - 0 - 0 - 0$$
$$= 0 \quad //$$

$$\tilde{I}_{imag}(1,0) = -I(0,0) * \sin\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right)$$
$$= -2 * 0 - I(0,1) * \sin\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right)$$
$$= 0 - 1 * 0 - I(1,0) * \sin\left(2\pi * \left(\frac{1}{2} + \frac{0}{2}\right)\right)$$
$$= 0 - 0 - 1 * 0 - I(1,1) * \sin\left(2\pi * \left(\frac{1}{2} + \frac{0}{2}\right)\right)$$
$$= 0 - 0 - 0 - 0$$
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$$\tilde{I}_{imag}(1,1) = -I(0,0) * \sin\left(2\pi * \left(\frac{0}{2} + \frac{0}{2}\right)\right)$$
$$= -2 * 0 - I(0,1) * \sin\left(2\pi * \left(\frac{0}{2} + \frac{1}{2}\right)\right)$$
$$= 0 - 1 * 0 - I(1,0) * \sin\left(2\pi * \left(\frac{1}{2} + \frac{0}{2}\right)\right)$$
$$= 0 - 0 - 1 * 0 - I(1,1) * \sin\left(2\pi * \left(\frac{1}{2} + \frac{1}{2}\right)\right)$$
$$= 0 - 0 - 0 - 0$$
$$= 0 \quad //$$

Μέτρο $\rightarrow |\tilde{I}(u,v)| = \sqrt{I_{real}^2(u,v) + I_{imag}^2(u,v)}$

$$|\tilde{I}(0,0)| = \sqrt{4^2+0} = 4$$

$$|\tilde{I}(0,1)| = \sqrt{2^2+0} = 2$$

$$|\tilde{I}(1,0)| = \sqrt{2^2+0} = 2$$

$$|\tilde{I}(1,1)| = \sqrt{0+0} = 0$$

$$|\tilde{I}| = \begin{array}{|c|c|} \hline 4 & 2 \\ \hline 2 & 0 \\ \hline \end{array}$$

Φάση $\rightarrow \angle \tilde{I}(u,v) = \tan^{-1}\left[\dfrac{I_{imag}(u,v)}{I_{real}(u,v)}\right]$

$$\angle \tilde{I}(0,0) = \tan^{-1}[0] = 0$$

$$\angle \tilde{I}(0,1) = \tan^{-1}[0] = 0$$

$$\angle \tilde{I}(1,0) = \tan^{-1}[0] = 0$$

$$\angle \tilde{I}(1,1) = \tan^{-1}\left[\tfrac{0}{0}\right] \Big\} \text{ Μη Επιτρεπτή Πράξη.}$$