Istanbul Technical University

Faculty of Electrical and Electronics Engineering

KON301E - System Modeling & Simulation

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Term Project Report

Modeling and Simulation of a Plastic Bottle Crusher System

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1 Introduction

This project focuses on the mathematical modeling, simulation, and analysis of a hydraulic plastic bottle crusher system. The system comprises a DC motor, a rack-and-pinion mechanism, a hydraulic servo valve, and a piston. The primary objective is to control the piston position $x(t)$ using a reference voltage input $V_r(t)$[cite: 7].

The study involves deriving the block diagram, implementing a linear model in MATLAB/Simulink, and developing a physical component model in MATLAB/Simscape. Finally, the response of both models is compared to validate the design parameters.

2 Parameter Selection and Rationale

Since the system design was open-ended, the parameters were selected to represent a realistic industrial prototype. The rationale behind the key variable choices is detailed below:

2.1 DC Motor Dynamics and Stability (B_m)

- **Viscous Friction ($B_m = 0.1\, N \cdot m \cdot s/rad$):** Ideally, motors are often modeled with negligible friction. However, in this design, a specific viscous friction coefficient was introduced intentionally. This term provides necessary mechanical damping to the system. Without sufficient B_m, the interaction between the high-pressure hydraulic stage and the motor could lead to oscillatory behavior. The selected value ensures system stability and a smoother steady-state response.

- **Inertia (J_{eq}):** A low equivalent inertia was chosen for the rotor and gear system to ensure a fast mechanical time constant, allowing the system to react quickly to voltage changes.

2.2 Hydraulic Parameters and Linearization

- **Supply Pressure ($P_s = 50\,$Bar):** A standard medium-pressure industrial value was selected to ensure sufficient force generation for crushing plastic bottles.

- **Valve Coefficients (K_x, K_p):** Instead of assigning arbitrary gains, the flow gain (K_x) and pressure-flow coefficient (K_p) were calculated based on the physical *Orifice Equation.*

$$K_x = C_d \cdot w \cdot \sqrt{\frac{P_s}{\rho}} \tag{1}$$

  Using fluid density ($\rho = 850\, kg/m^3$) and valve geometry, these calculated values ensure that the linear Simulink model closely approximates the physical behavior of the valve modeled in Simscape.

3 System Modeling

3.1 Block Diagram

The mathematical model was derived by combining the electrical equations of the motor, the kinematic relationships of the gear system, and the linearized hydraulic equations.

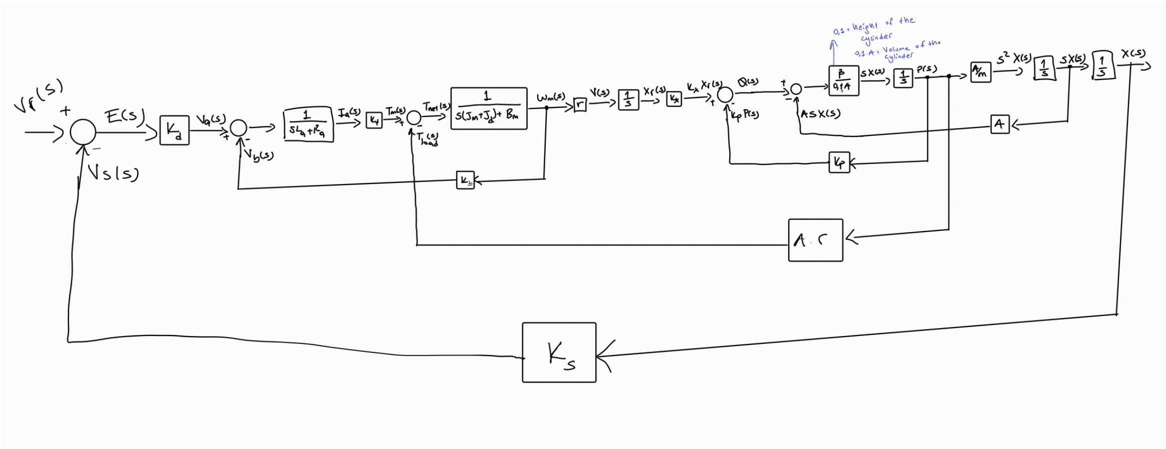

Figure 1: System Block Diagram (Hand-derived)

3.2 Simulink Model (Linear)

Based on the derived transfer functions and the linearized valve equation $Q_L = K_x x_v - K_p P_L$, the system was implemented in Simulink.

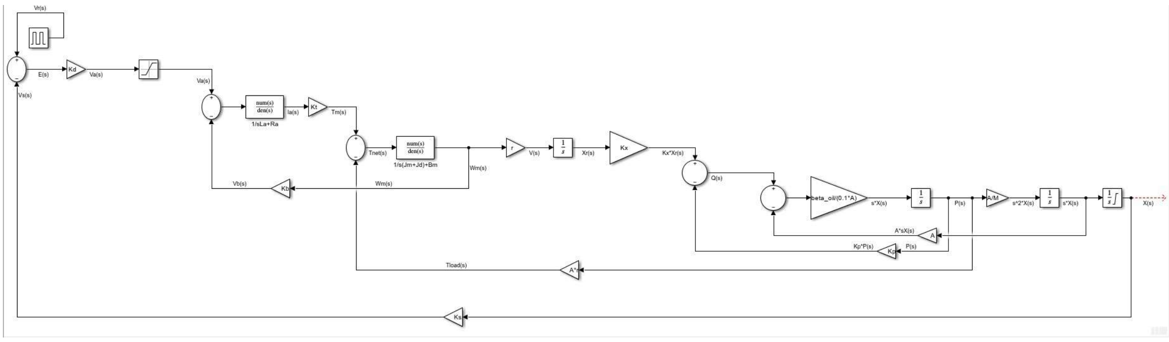

Figure 2: MATLAB/Simulink Model Implementation

3.3 Simscape Model (Physical)

An equivalent physical model was created using Simscape Fundamental Libraries. This model utilizes physical connections (hydraulic fluid, mechanical rotation) rather than signal flows, providing a higher fidelity representation of non-linearities.

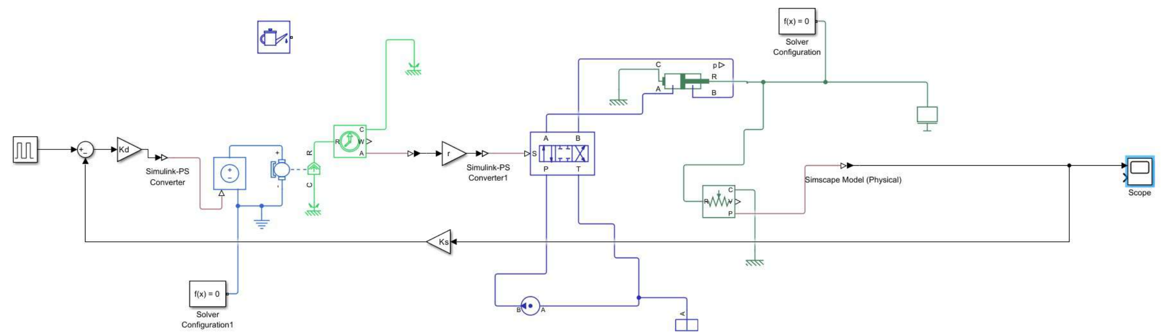

Figure 3: MATLAB/Simscape Physical Model

4 Simulation Results and Comparison

Both models were simulated for 10 seconds with identical parameters. The position output $x(t)$ for both the Simulink (Blue) and Simscape (Yellow/Orange) models is presented below.

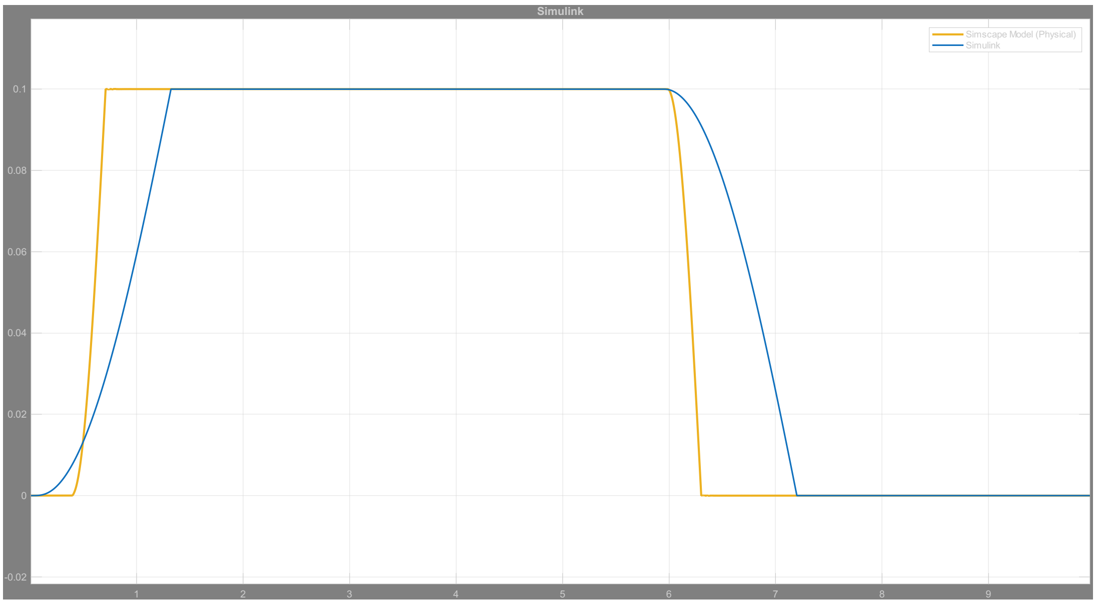

Figure 4: Comparison of Simulation Results: Simulink (Blue) vs. Simscape (Yellow)

4.1 Analysis of the Discrepancy

The comparison plot reveals distinct characteristics between the linear and physical models:

1. **Steady-State Convergence:** Both models converge to the exact same final position. This validates the mathematical derivation of the DC gains; the calculated K_x and K_p values in Simulink correctly correspond to the physical valve properties in Simscape.

2. **Transient Response Differences:**

   - **Simscape (Yellow):** The response is extremely fast, resembling a square wave. This indicates that the physical model, powered by 50 Bar pressure, acts almost like an ideal actuator. The valve opens, and the fluid incompressibility (high Bulk Modulus) drives the piston to saturation almost instantly.
   - **Simulink (Blue):** The response follows a classic second-order curve with a visible rise time. This is due to the *linearization* process. The Simulink model assumes a linear relationship between flow and pressure (K_p) at an operating point. It does not fully capture the "hard" switching nature of the physical valve opening or the extremely high stiffness of the hydraulic fluid as effectively as the Simscape solver.

5 Conclusion

The plastic bottle crusher system was successfully modeled using both analytical (Simulink) and physical (Simscape) approaches. The parameter selection, particularly the inclusion of motor friction (B_m) and calculated valve coefficients, resulted in a stable and realistic system design. The comparison highlights the limitations of linear approximations: while Simulink is excellent for control design and frequency analysis, Simscape provides a more accurate representation of hard non-linearities and hydraulic stiffness.

A MATLAB Parameter Initialization Code

The following script was used to initialize the parameters for both simulations.

```matlab
% Project: Plastic Bottle Crusher System
% Parameter Initialization Script
clear; clc;

%% 1. DC Motor Parameters
Ra = 2.0;        % Armature Resistance [Ohm]
La = 0.02;       % Armature Inductance [H]
Kb = 0.1;        % Back-EMF Constant [V/(rad/s)]
Kt = 0.1;        % Torque Constant [N.m/A]
Jm = 0.002;      % Motor Rotor Inertia [kg.m^2]
Bm = 0.1;        % Motor Viscous Friction [N.m.s/rad] (Added for stability)

%% 2. Driver and Sensor Gains
Kd = 5;          % Motor Driver Gain
Ks = 1;          % Optical Sensor Gain [V/m]

%% 3. Mechanical Transmission
r  = 0.015;      % Pinion Radius [m]
Jd = 0.001;      % Gear Inertia [kg.m^2]

%% 4. Hydraulic System Parameters
Ps = 50e5;       % Supply Pressure [Pa] (50 Bar)
rho = 850;       % Hydraulic Fluid Density [kg/m^3]
beta_oil = 1e7; % Fluid Bulk Modulus [Pa]

%% 5. Actuator
A  = 0.0012;     % Piston Surface Area [m^2]
M  = 10;         % Moving Mass [kg]

% Valve Geometry
valve_max_opening = 0.005; % 5 mm
valve_max_area    = 5e-5;  % Max Area

%% 7. Calculated Coefficients for Linear Model
Cd = 0.7;        % Discharge Coefficient
w_gradient = valve_max_area / valve_max_opening;

% Flow Gain (Kx)
Kx = Cd * w_gradient * sqrt(Ps/rho);

% Pressure-Flow Coefficient (Kp)
Kp = 1e-7;       % Linearized Flow-Pressure Coefficient

disp('Parameters Loaded.');
```