

Week 10 Overview

Tuesday, November 08, 2016

2:39 PM

- ☒ Set up team log book
- ☒ Find S4 motor model
 - ☒ Add to git repo
 - ☒ Update with specs from S5 motor
- ☒ Set up GIT repo for project
 - ☒ Add datasheets, charter, S4 motor model, etc

Motor Modelling

Thursday, November 10, 2016 3:48 PM

Semester 4 motor model parameters

From: <http://ctms.engin.umich.edu/CTMS/index.php?example=MotorSpeed§ion=SimulinkModeling>

J = moment of inertia of the rotor = $\text{kg} \cdot \text{m}^2$

b = motor viscous friction constant (damping) = $\text{N} \cdot \text{m} \cdot \text{s}$

K_e = electromotive force constant = $\text{V}/\text{rad}/\text{sec}$

K_t = motor torque constant = $\text{N} \cdot \text{m} / \text{Amp}$

R = motor electrical resistance = ohms

L = motor electrical inductance = henrys

Semester 5 Motor:

ElectroCraft DPP240-29V48

$J = 282.5 \text{ g} \cdot \text{cm}^2 = 2.825 \times 10^{-5}$

b = NOT LISTED = USE 0.1 for now per Bill instructions

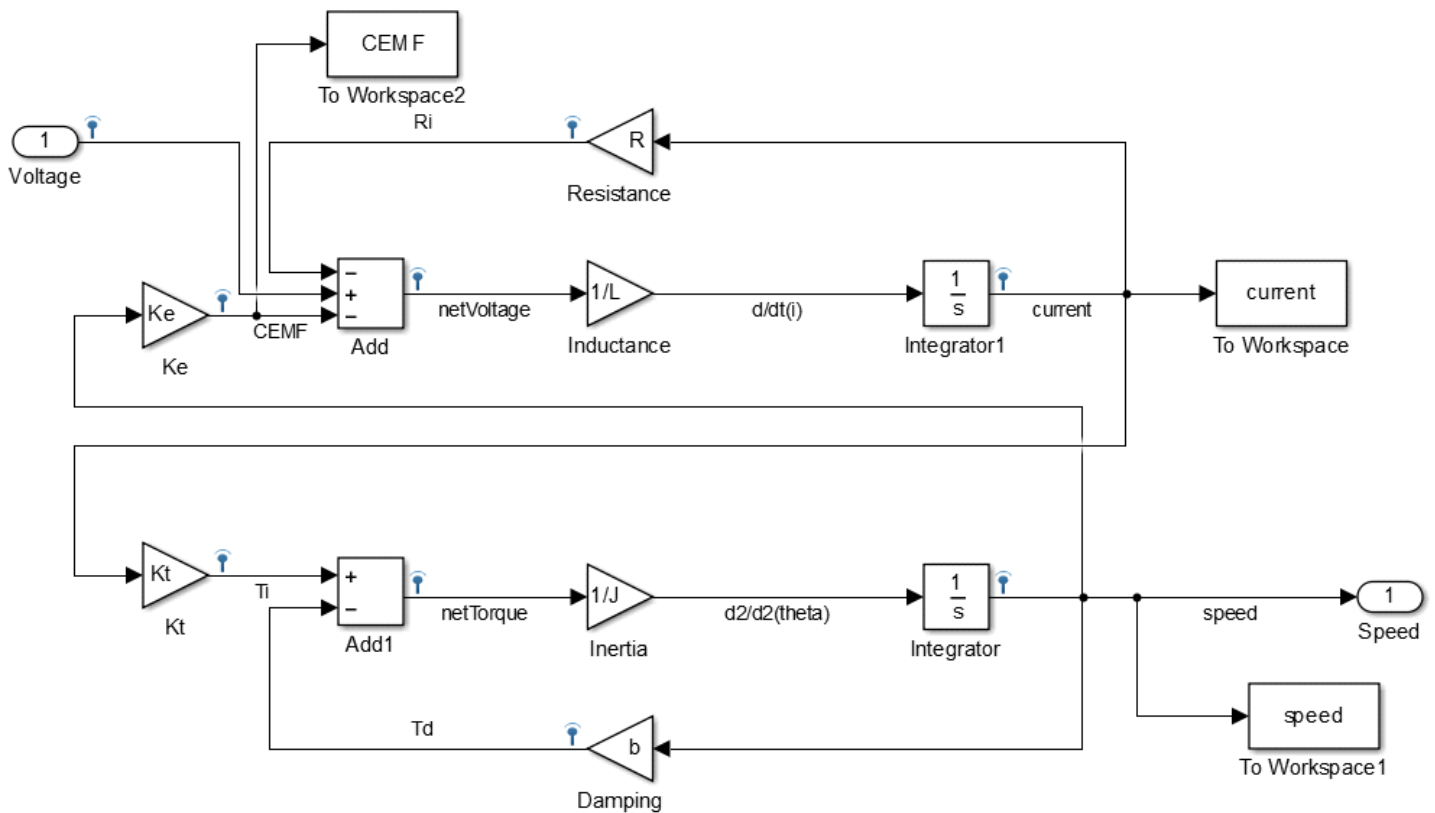
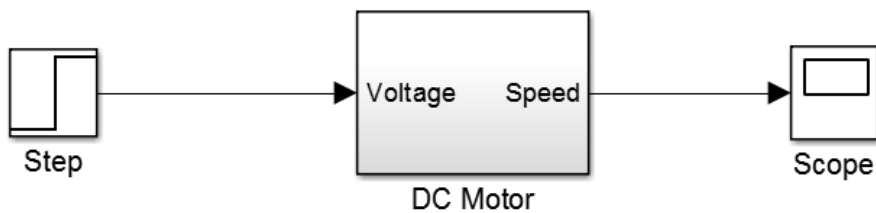
$K_e = 10.9 \text{ V}/\text{krpm} = 0.0109 \text{ V}/\text{rpm} = 0.00141 \text{ V}/\text{rad}/\text{s} = 1.141 \times 10^{-3} \text{ V}/\text{rad}/\text{s}$

$K_t = 14.7 \text{ oz-in} / \text{Amp} = 0.1038 \text{ N} \cdot \text{m} / \text{Amp}$

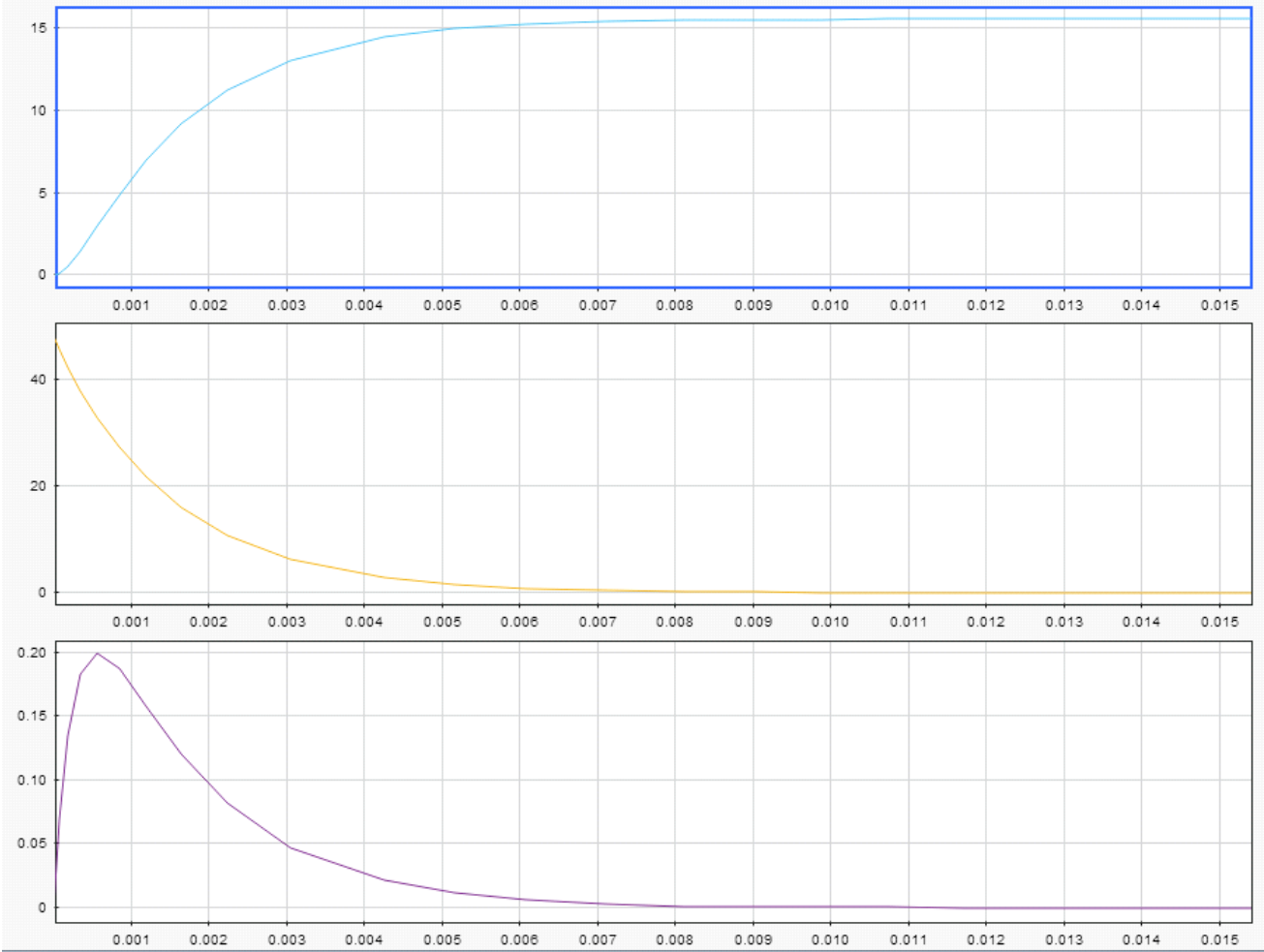
$R = 3.2 \text{ Ohms}$

$L = 4.8 \text{ mH} = 4.8 \times 10^{-3} \text{ H}$

Model In Simulink



Model Output - Driven by 48V Step Input - Speed - Net Voltage - Net Torque



Week 11 Overview

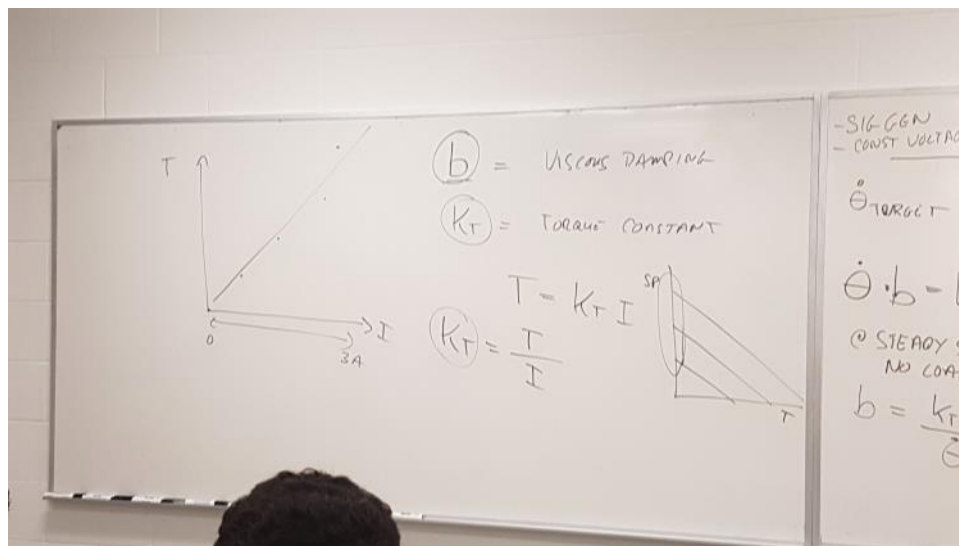
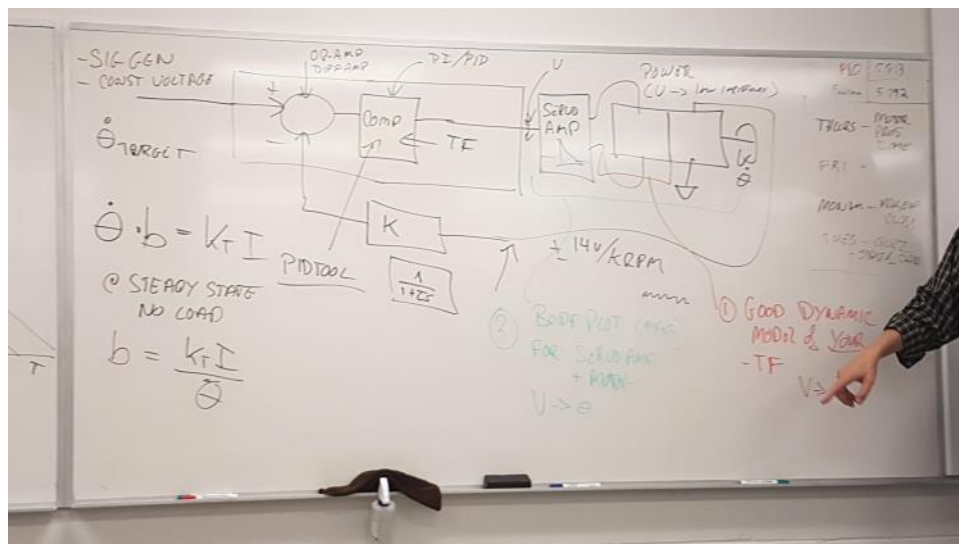
Thursday, November 17, 2016 2:56 PM

Objectives

- ☒ Measure real-world motor characteristics
- ☒ Update motor model with real-world measurements
- ☒ Tune motor model to match real-world step response
 - ☒ Plot of step response from MATLAB model
 - ☒ Scope shot of step response from real motor

Op-amp controlled motor

November 16, 2016 11:49 AM



Motor Measurements

Thursday, November 17, 2016 2:57 PM

Parameter	Units	Datasheet	Measurement	Method of Measurement
Rotor Inertia	Kg*m ²	$2.825 \times 10^{-5} + 9.886 \times 10^{-6} = 3.8136 \times 10^{-5}$	2.5×10^{-5}	MATLAB Tuning
Rotor Damping	N*m*s	(0.1 estimate)	0.000232	Current vs Speed measurements
Voltage Constant	V/rad/s	0.104087	0.100	Torque watch
Torque Constant	N*m /A	0.1038	0.100	Torque watch
R	Ohms	3.2	4	Fluke DMM
Inductance	Henry	4.8×10^{-3}	5.270×10^{-3} Or 7.169×10^{-3}	LCR

Making sure it spins (no load):

PSU Voltage: 10V

Measured Current: 0.23A

Measured Tach Voltage: 12.5V

Motor Speed: $14/12.5 \times 1000 = 1120$ rpm

Torque Constant Estimate

$K_T = 0.100$ N*m/A

Measurements in Excel Spreadsheet.

Damping Estimate

$$b = \frac{K_T I}{W} = \frac{\text{Torque Constant} * \text{Current}}{\text{Angular Speed}} = 0.000232 \text{ N} * \text{m} * \text{s}$$

Measurements in Excel Spreadsheet.

To do list

November 17, 2016

3:47 PM

$K_T = N \cdot m \cdot A$

$K_B = \frac{V \cdot s}{rad}$

① measure R, L

② validate K_T motor

⇒ ② update J ESTIMATE → TACH

③ estimate b (I_n)

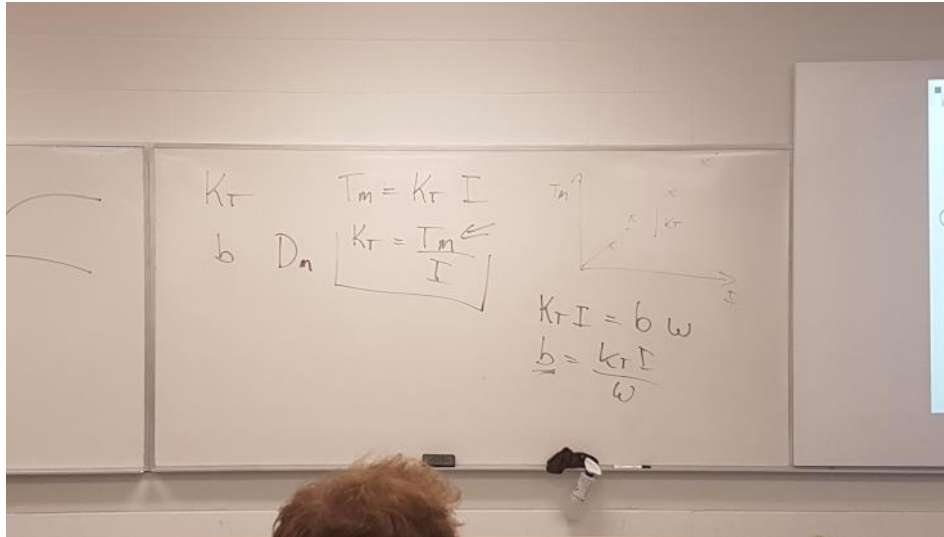
④ update MATLAB model
- SIMULINK
- TF → FLM SCRIPTS

⑤ GET STEP RESP FROM MODEL

370

Finding k_t and b

November 17, 2016 3:48 PM



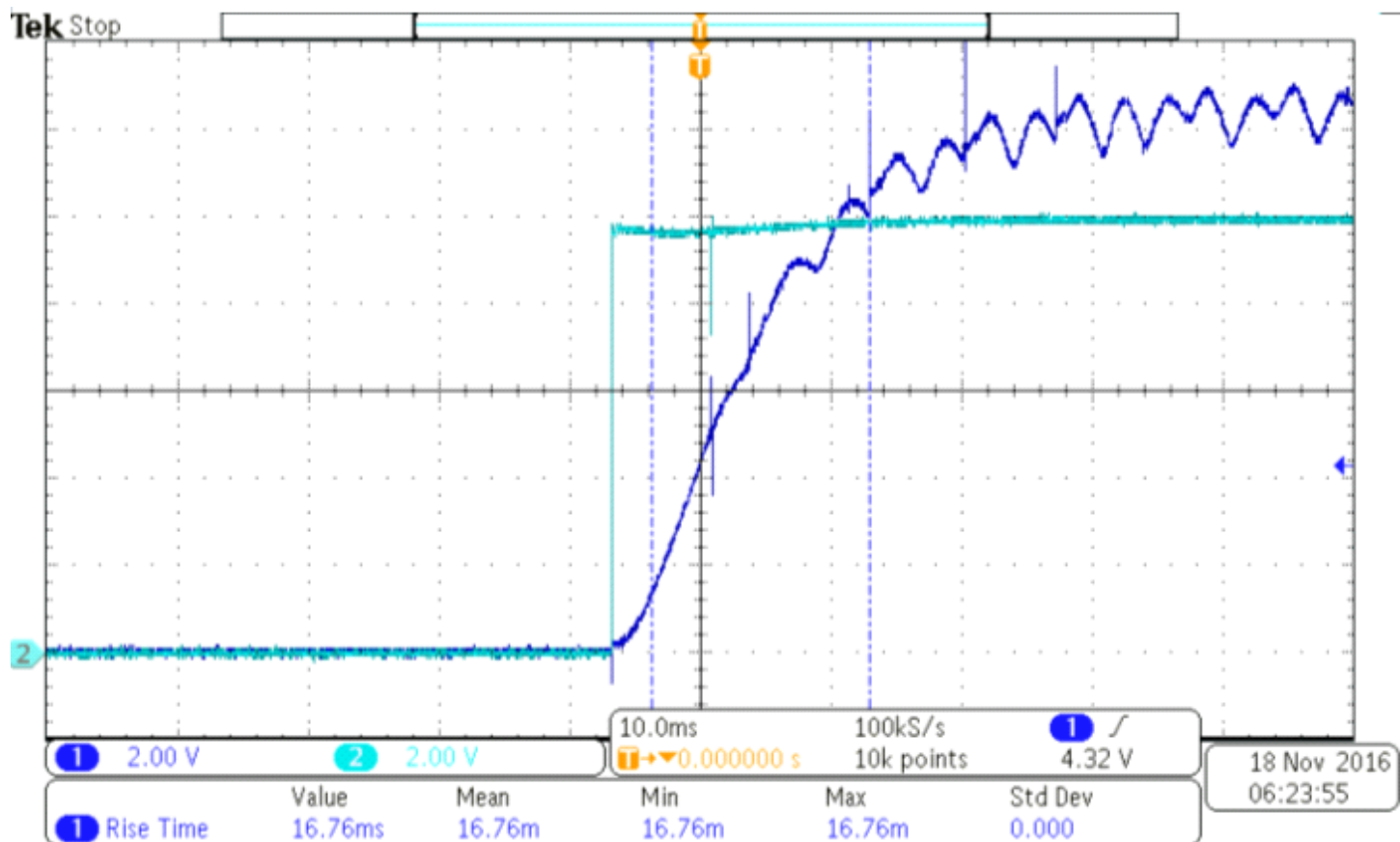
Step-Response Measurements & Tuning

Thursday, November 17, 2016 5:52 PM

Real-World Step Response

Tach Speed = 91.25 rad/s

10-90 Rise Time: 16.76ms

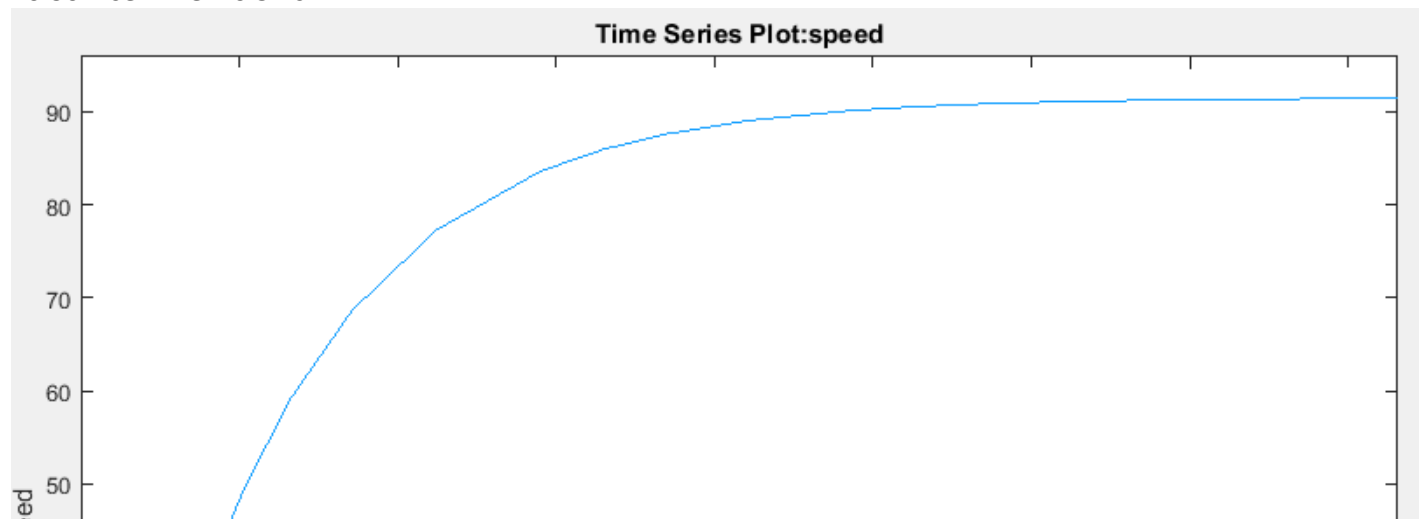


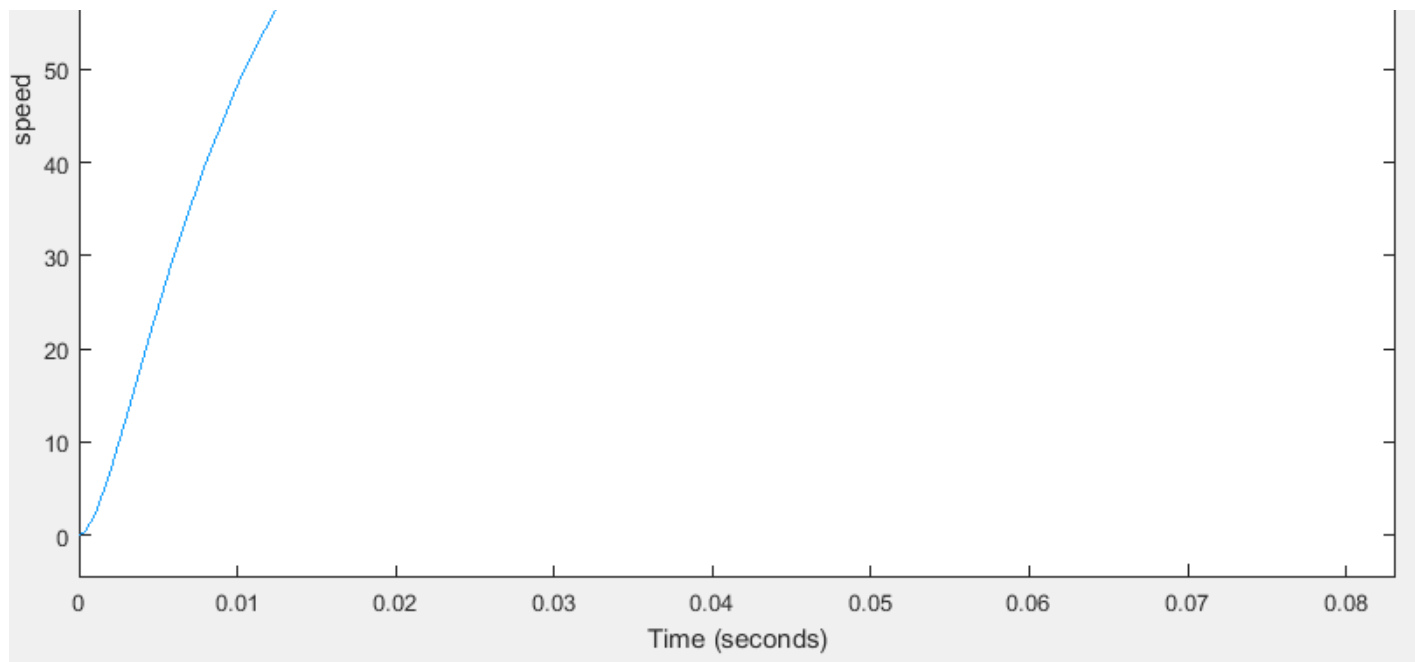
10V Step Response - Dark Blue = Tach Voltage

MATLAB Model Step Response

Steady State Speed = 91.3 rad/s

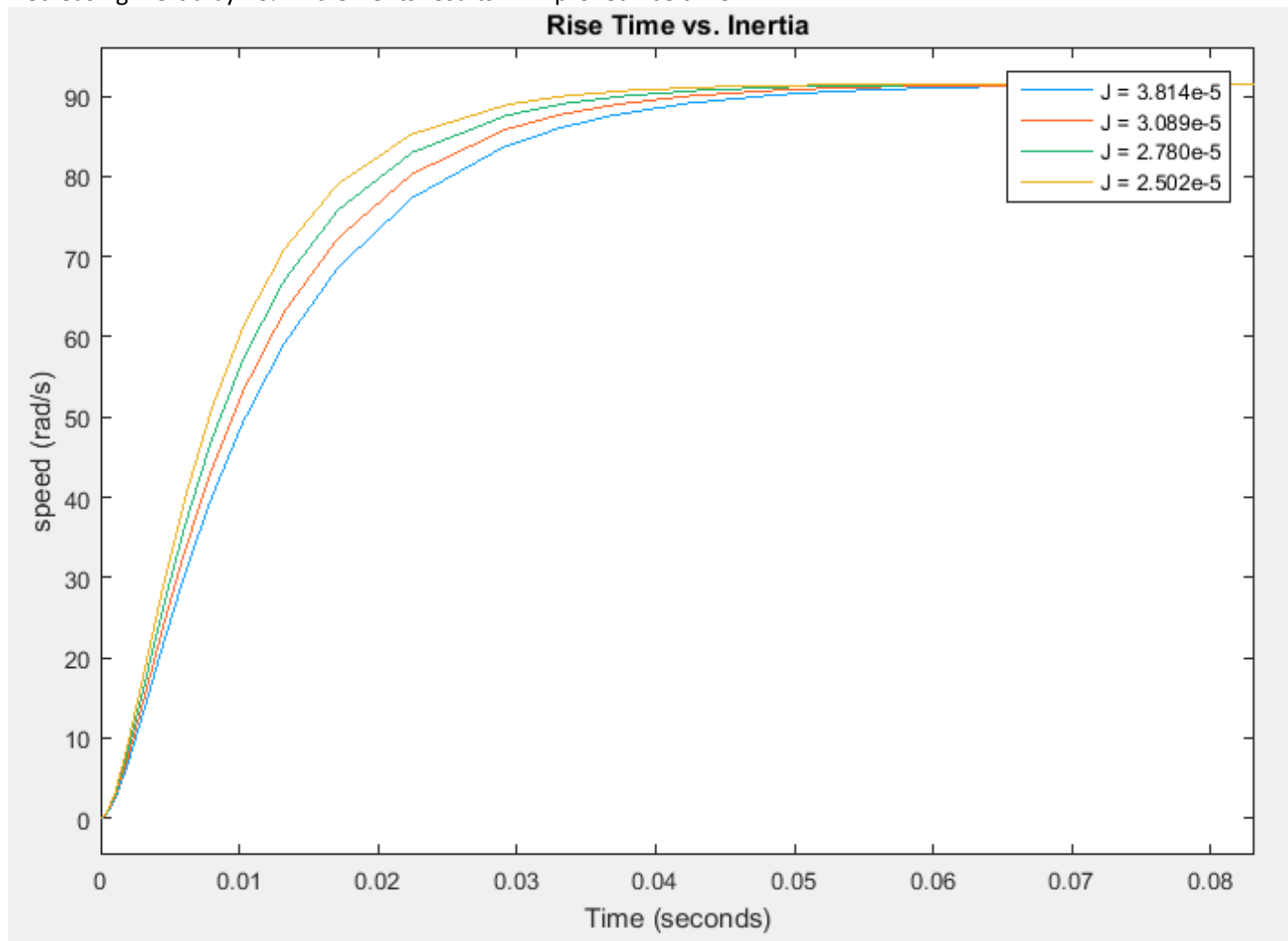
10-90 Rise Time: 26.5ms



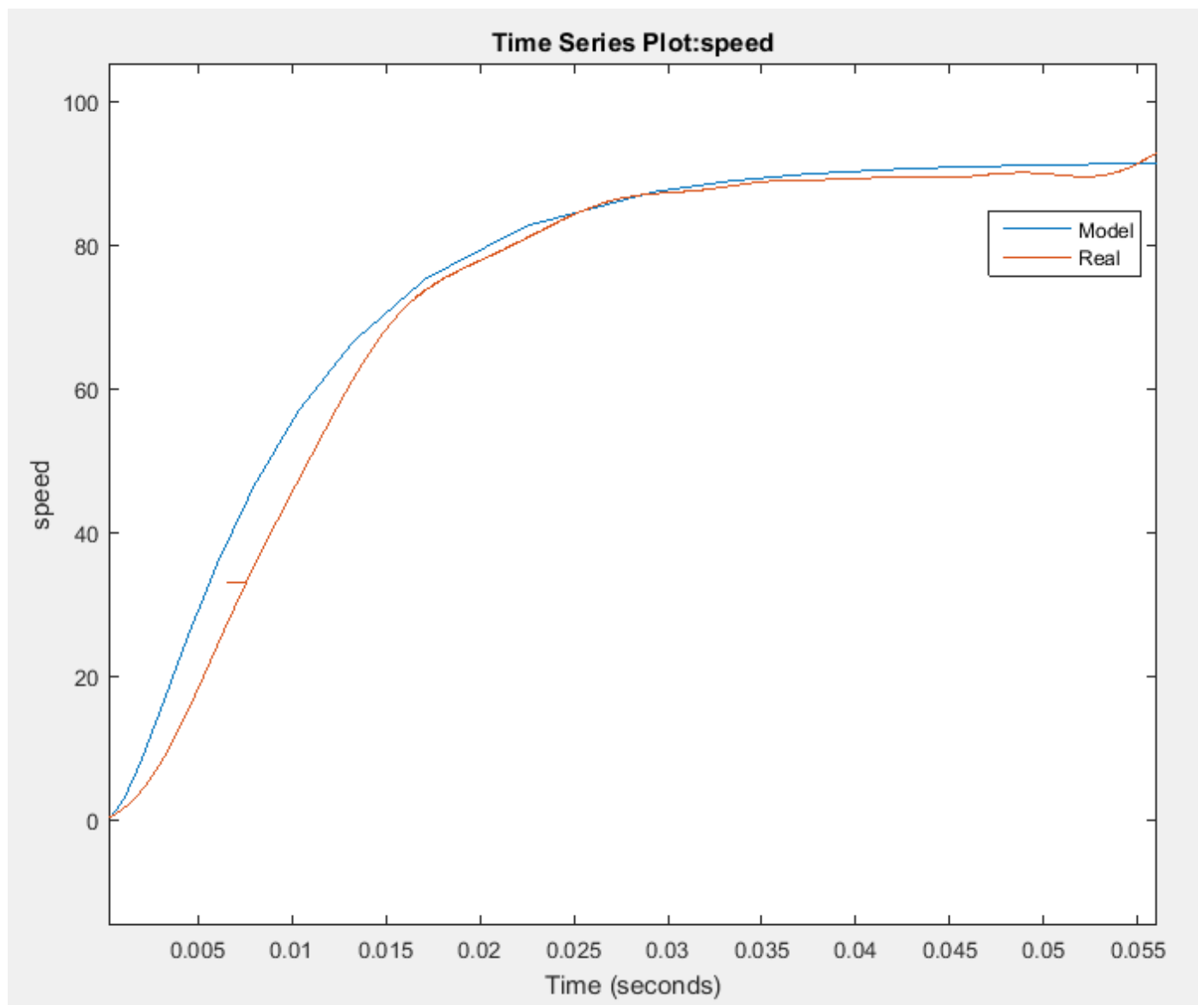


Tuning MATLAB Model (Inertia)

Decreasing Inertia by 10% increments results in improved rise time.



With Inertia set to be just the motor inertia, not adding the tach inertia, it looks good.



11/18/2016 2:10 PM - Screen Clipping

Transfer Function of Motor Model

Friday, November 18, 2016 12:38 PM

Transfer Function from Bill's Slides

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{\frac{K_t}{LJ}}{s^2 + \frac{(RJ + bL)}{LJ}s + \frac{Rb + K_e K_t}{LJ}}$$

Transfer Function as entered into Matlab

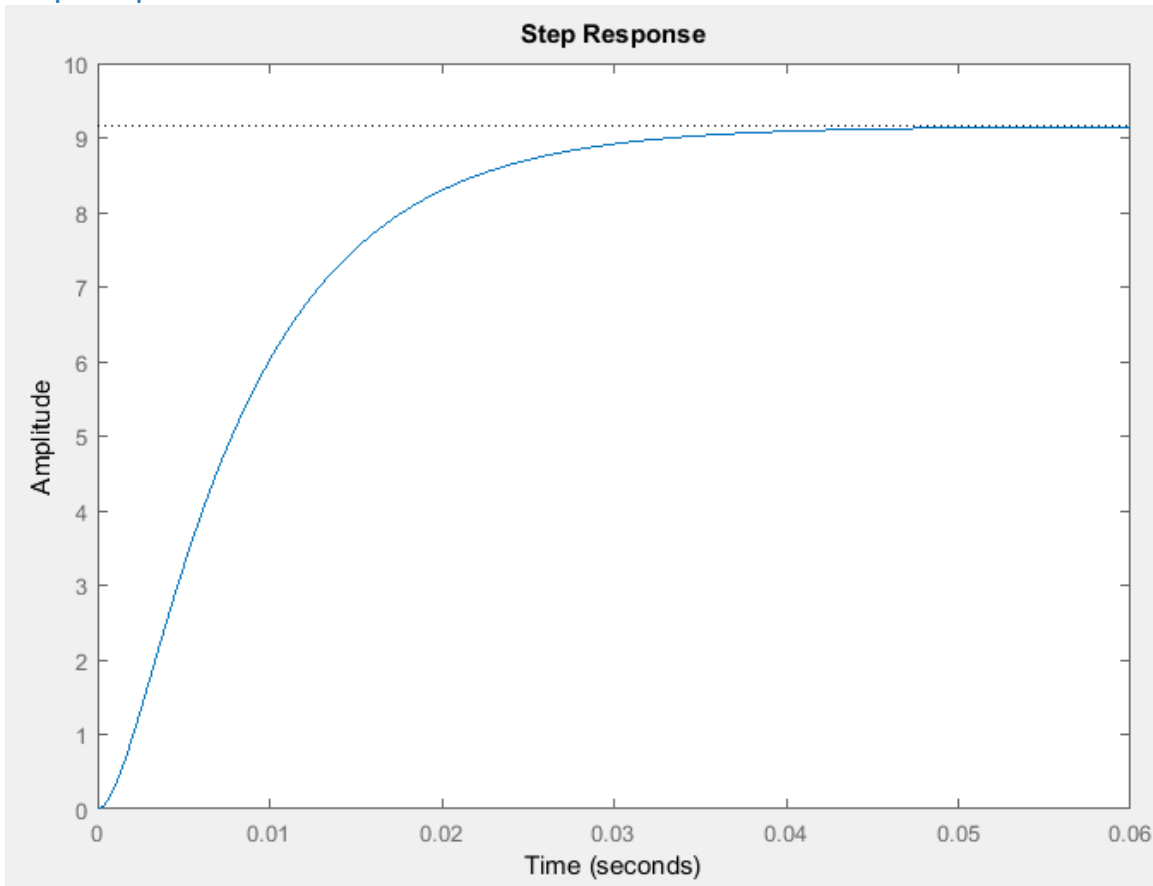
```
>> H = ( (Kt / (L * J)) / ( (s^2) + ( ( (R * J) + (b * L)) / (L * J) ) * s + ( (R * b) + (Ke * Kt)) / (L * J) ) )
```

H =

```
      7.59e05  
-----  
s^2 + 768.3 s + 8.294e04
```

Continuous-time transfer function.

Step Response of Motor Model Transfer Function



Week 12 Overview

Tuesday, November 22, 2016 10:11 AM

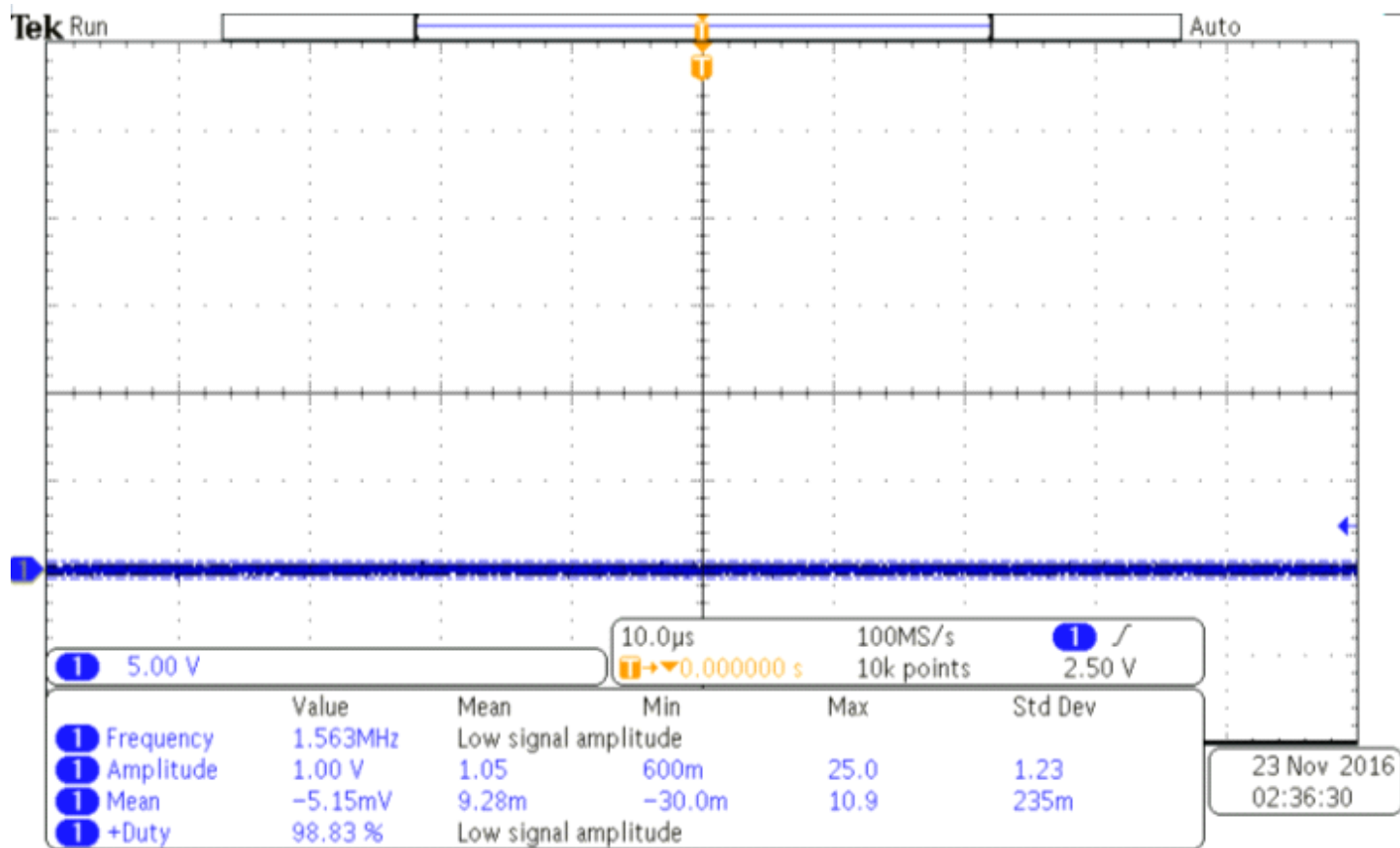
Objectives

- ☒ Design & Build Op-Amp Speed Controller
 - ☒ MATLAB Model of Servo-Amp
 - ☒ MATLAB Model of Tachometer
 - ☒ Updated Transfer Function
 - ☒ PIDTOOL controller development
 - ☒ Op-Amp Controller build/test

Servo-Amp Setup

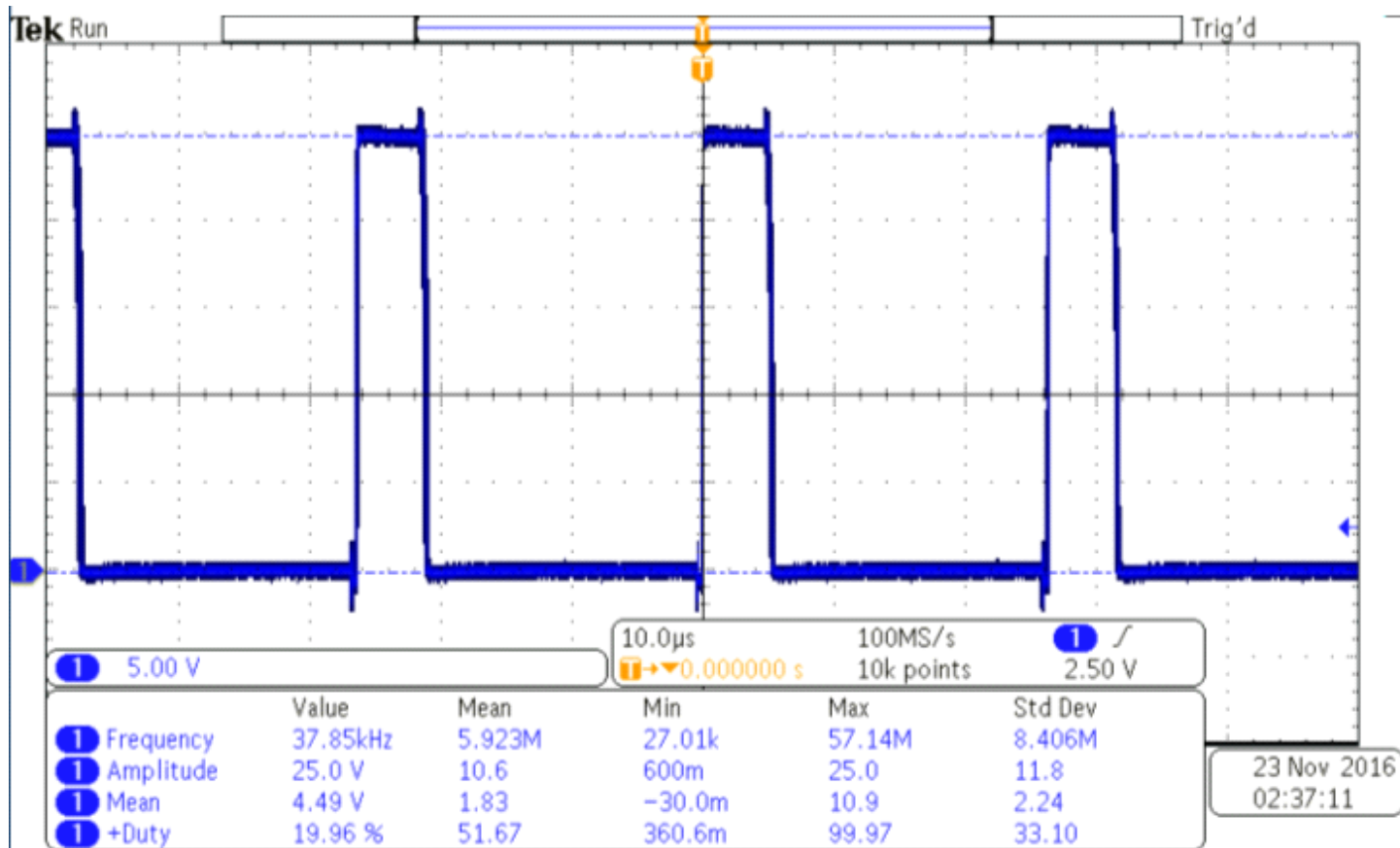
Tuesday, November 22, 2016 11:55 AM

Measuring Input Offset



Basically zero.

Setting Servo-Amp Gain = 1



5V Signal Input = 5V Output (20% * 25V)

PSU Set to 25V

Input Signal	Output Duty Cycle %	Effective Output Voltage
1	5.12	1.28
2	8.35	2.09
3	12.10	3.03
4	16.25	4.06
5	20.00	5.00
6	24.10	6.03
7	28.05	7.01
8	32.00	8.00
9	36.00	9.00
10	40.0	10.00

Conclusion: Good relationship between input signal & output voltage.

Transfer Function

Tuesday, November 22, 2016 2:43 PM

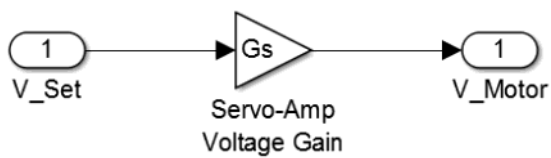
Modelling Tachometer

Tachometer Gain = 14 V / 1000 rpm
rad/s to RPM = 9.54929659643

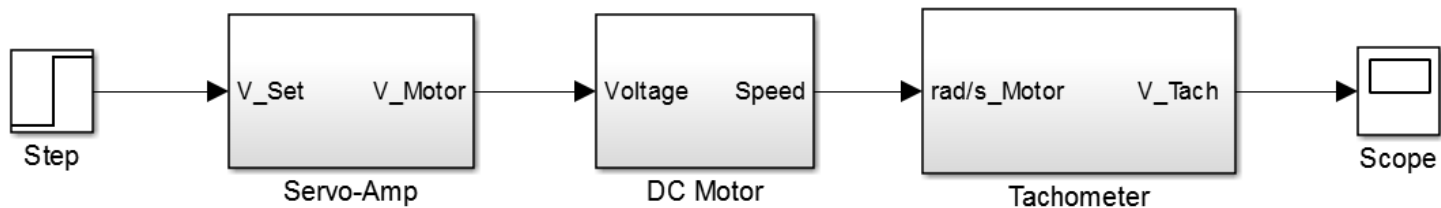


Modelling Servo-Amp

Servo-Amp Gain = 1 V/V



Entire Motor Model



Updating Transfer Function

Just motor (from last week):

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{\frac{K_t}{LJ}}{s^2 + \frac{(RJ + bL)}{LJ}s + \frac{Rb + K_e K_t}{LJ}}$$

Slide 49 - S&CS Systems of DE's Slides

MATLAB Function:

H=((Kt/(L*J))/((s^2)+(((R*J)+(b*L))/(L*J))*s+(((R*b)+(Ke*Kt))/(L*J))))

Adding servo-amp:

$$\text{Motor: } G(s) = \frac{W(s)}{V(s)} = \frac{\frac{K_t}{LJ}}{s^2 + \left(\frac{RJ + bL}{LJ}\right)s + \frac{Rb + K_e K_t}{LJ}} \quad \frac{\text{rad/s}}{V}$$

$$\text{Servo-Amp: } H(s) = \frac{V_{\text{motor}}(s)}{V_{\text{set}}(s)} = 1$$

$$\text{Tachometer: } I(s) = \frac{V_{\text{tach}}(s)}{W(s)} = \left(\frac{14V}{1000 \text{ rpm}}\right) \left(9.5493 \frac{\text{rpm}}{\text{rad/s}}\right) = 0.13369 \frac{V}{\text{rad/s}}$$

$$(\text{Servo-Amp})(\text{Motor})(\text{Tachometer}) = H(s)G(s)I(s)$$

$$\boxed{T.F. = (1)G(s)(0.13369)}$$

MATLAB Function:

$H = G_s * (G_t * G_{rr}) * ((K_t/(L*J))/((s^2 + (((R*J)+(b*L))/(L*J))*s + (((R*b)+(K_e*K_t))/(L*J))))$

H =

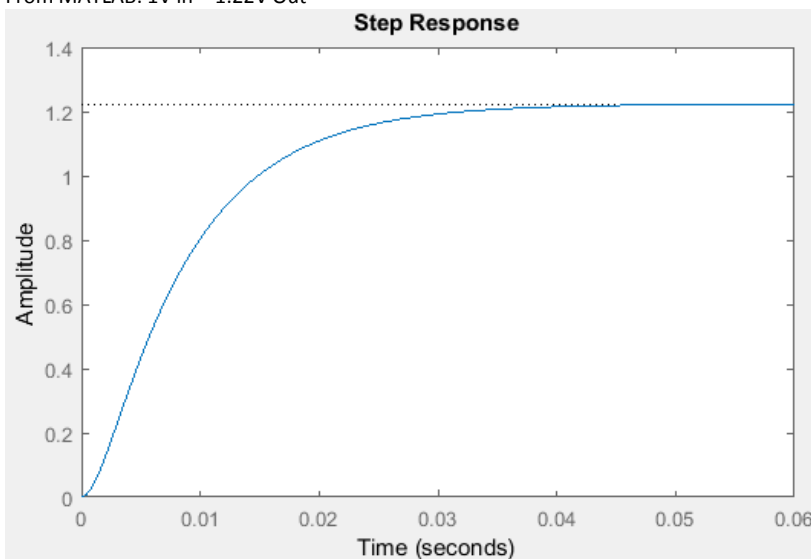
```

1.014e05
-----
s^2 + 768.3 s + 8.288e04

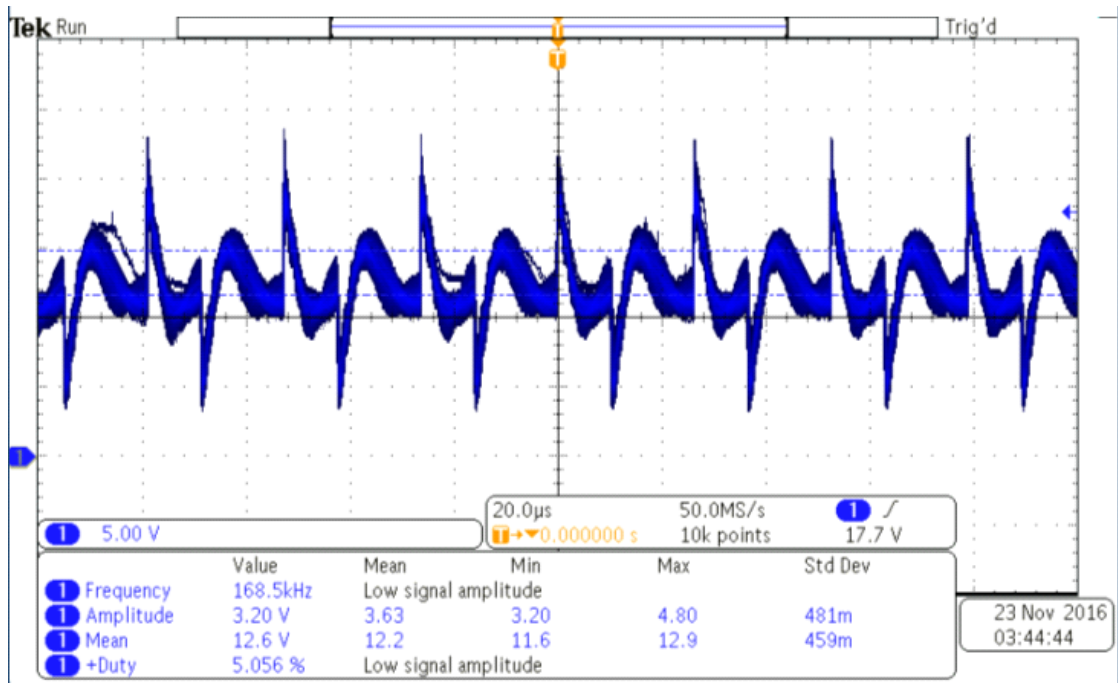
```

Step-Response of H(s):

From MATLAB: 1V in = 1.22V Out



From Real-World: 10V in = 12.V Out



CONCLUSION: Our transfer function is a good estimation of the real world system.

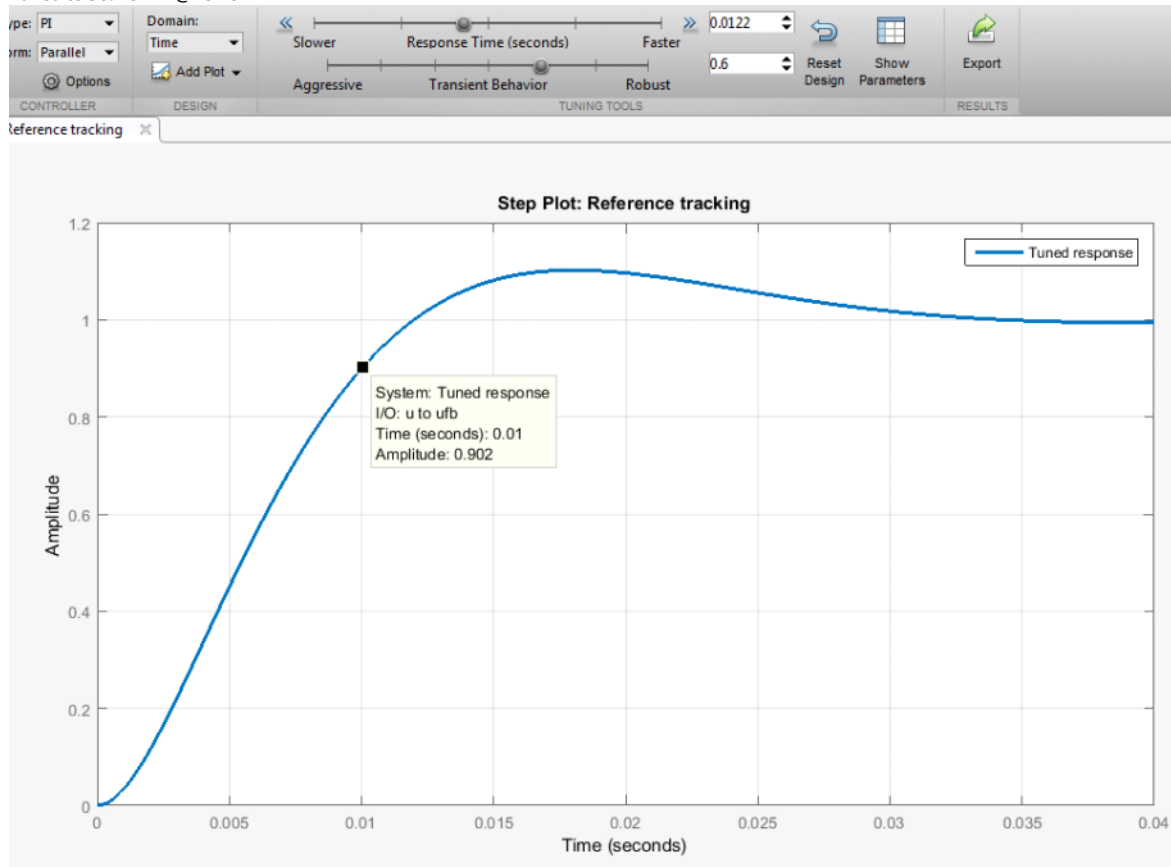
PI Controller Development

Tuesday, November 22, 2016 3:15 PM

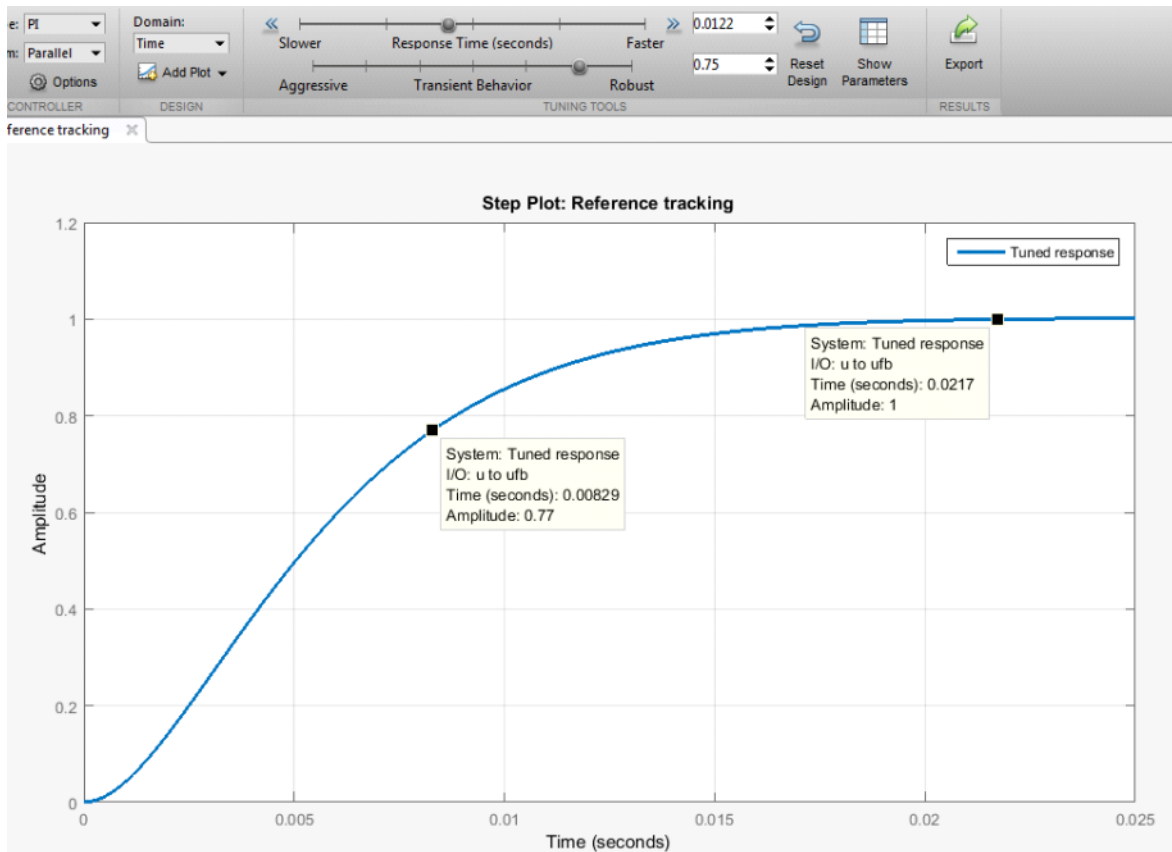
PIDTOOL Controller

Goals: 90% of final value in 10ms

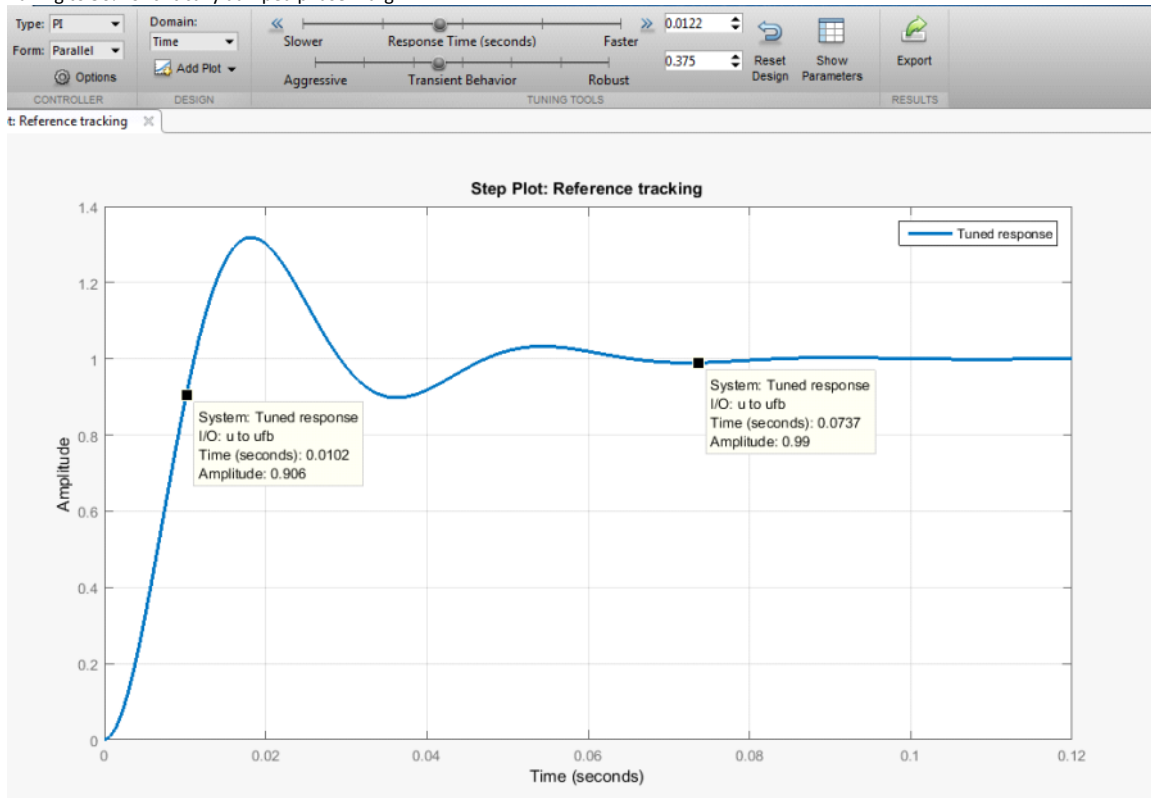
Tuned to 90% of FV @10ms



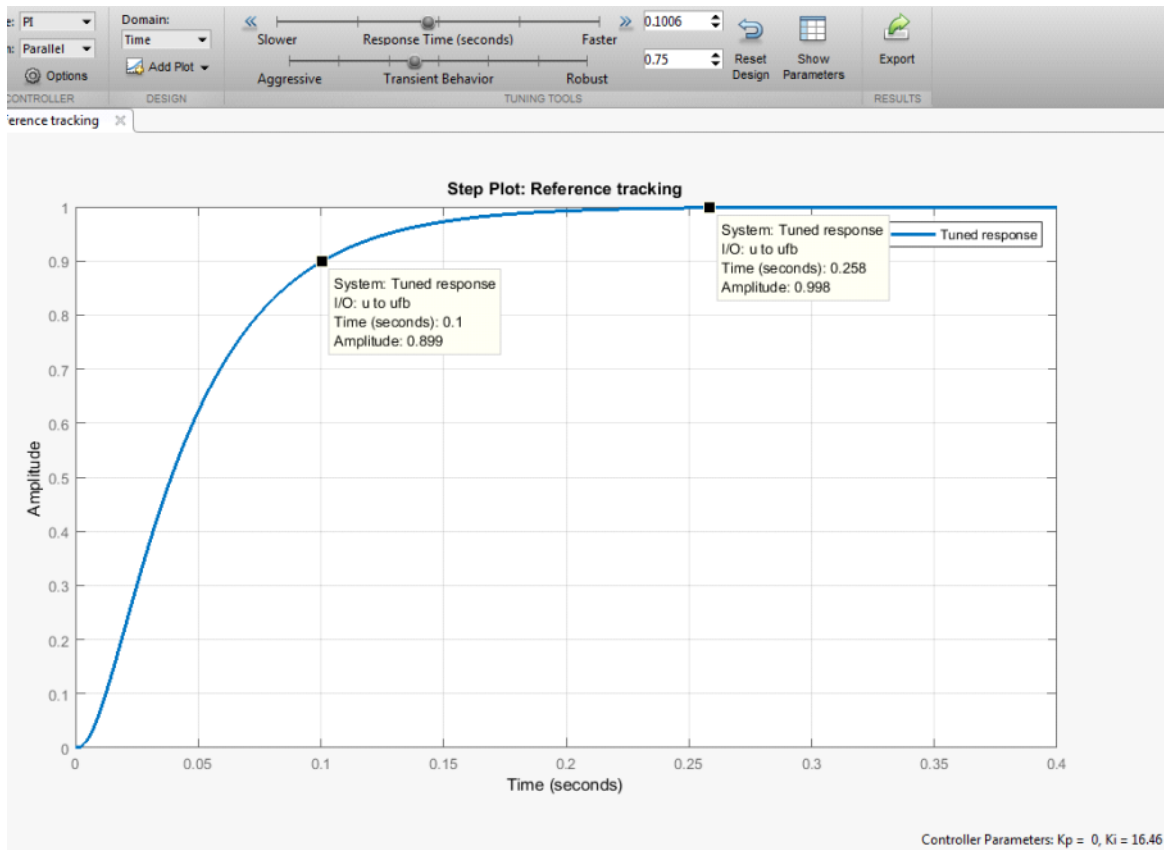
Tuning to be critically damped



Tuning to 50% of critically damped phase margin



Tuning to be close to critically damped with 10ms rise time to 90% F.V.



Adding PIDTOOL Controller to Model

Transfer function of feedback system

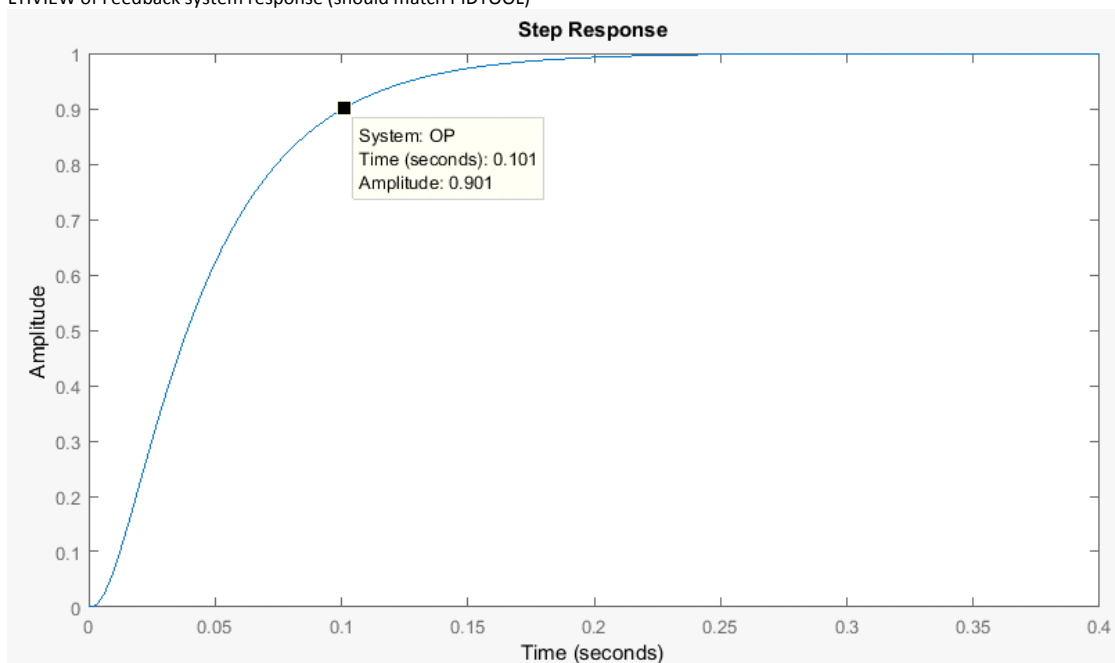
```
>> OP = (C*H) / (1 + C*H)
```

OP =

$$\frac{1.668e06 s^3 + 1.282e09 s^2 + 1.383e11 s}{s^6 + 1537 s^5 + 7.56e05 s^4 + 1.29e08 s^3 + 8.15e09 s^2 + 1.383e11 s}$$

Continuous-time transfer function.

LTIVIEW of Feedback system response (should match PIDTOOL)



OUR PI CONTROLLER IS JUST AN I CONTROLLER ACTUALLY

>> PI = zpk(C)

PI =

16.456

s

Continuous-time zero/pole/gain model.

Op-Amp Controller Design

$$\text{Motor: } G(s) = \frac{W(s)}{V(s)} = \frac{\frac{K_T}{LJ}}{s^2 + \left(\frac{RJ + b}{LJ}\right)s + \frac{Rb + K_T K_T}{LJ}} \frac{\text{rad/s}}{V}$$

$$\text{Servo-Amp: } H(s) = \frac{V_{\text{motor}}(s)}{V_{\text{set}}(s)} = 1$$

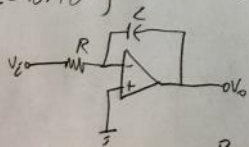
$$\text{Tachometer: } I(s) = \frac{V_{\text{tach}}(s)}{W(s)} = \left(\frac{K_V V_{\text{rpm}}}{1000}\right) \left(\frac{9.5493 \text{ rpm/rad/s}}{1}\right) = 0.13369 \text{ V/rad/s}$$

$$(\text{Servo-Amp})(\text{Motor})(\text{Tachometer}) = H(s)G(s)I(s)$$

$$T.F. = (1)G(s)(0.13369)$$

PI Controller Design:

$$\left. \begin{array}{l} P=0 \\ I=16.456 \end{array} \right\} G(s) = \frac{16.456}{s} = \frac{-z_z}{z_p} = \frac{-1/s_c}{R}$$



$$G(s) = \frac{16.456}{s} = \frac{-1}{sRC}$$

$$\frac{-1}{RC} = 16.456$$

$$R = \frac{-1}{16.456(C)}$$

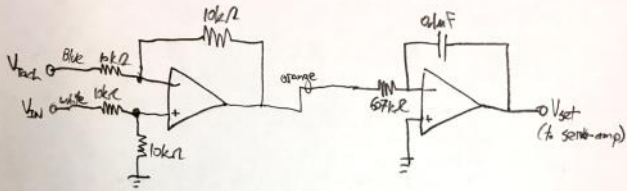
$$\text{Try } C = 0.1 \mu F$$

$$R = 607.681 \text{ k}\Omega$$

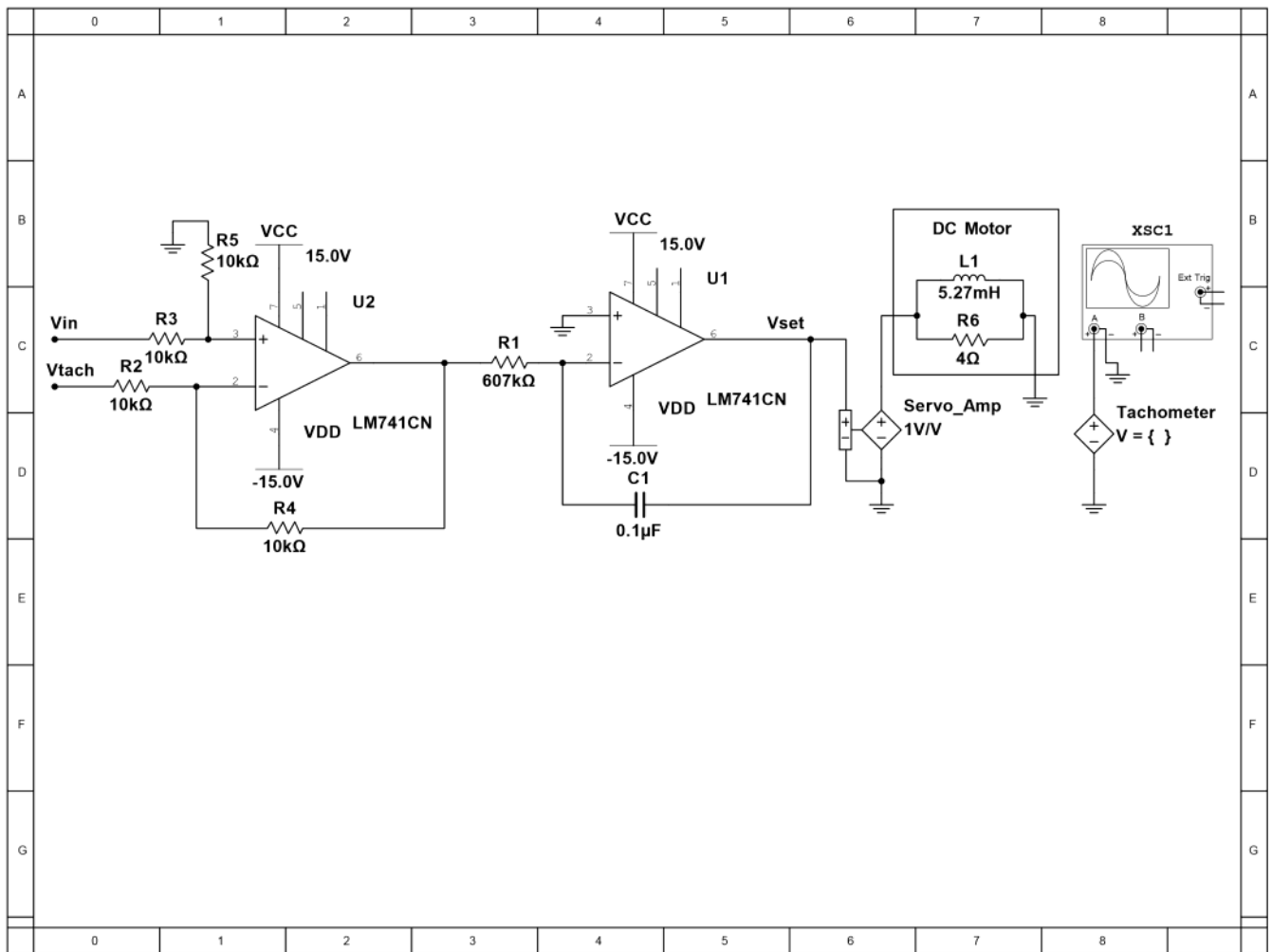
$$370 \text{ k}\Omega + 220 \text{ k}\Omega = 0.606 \text{ M}\Omega$$

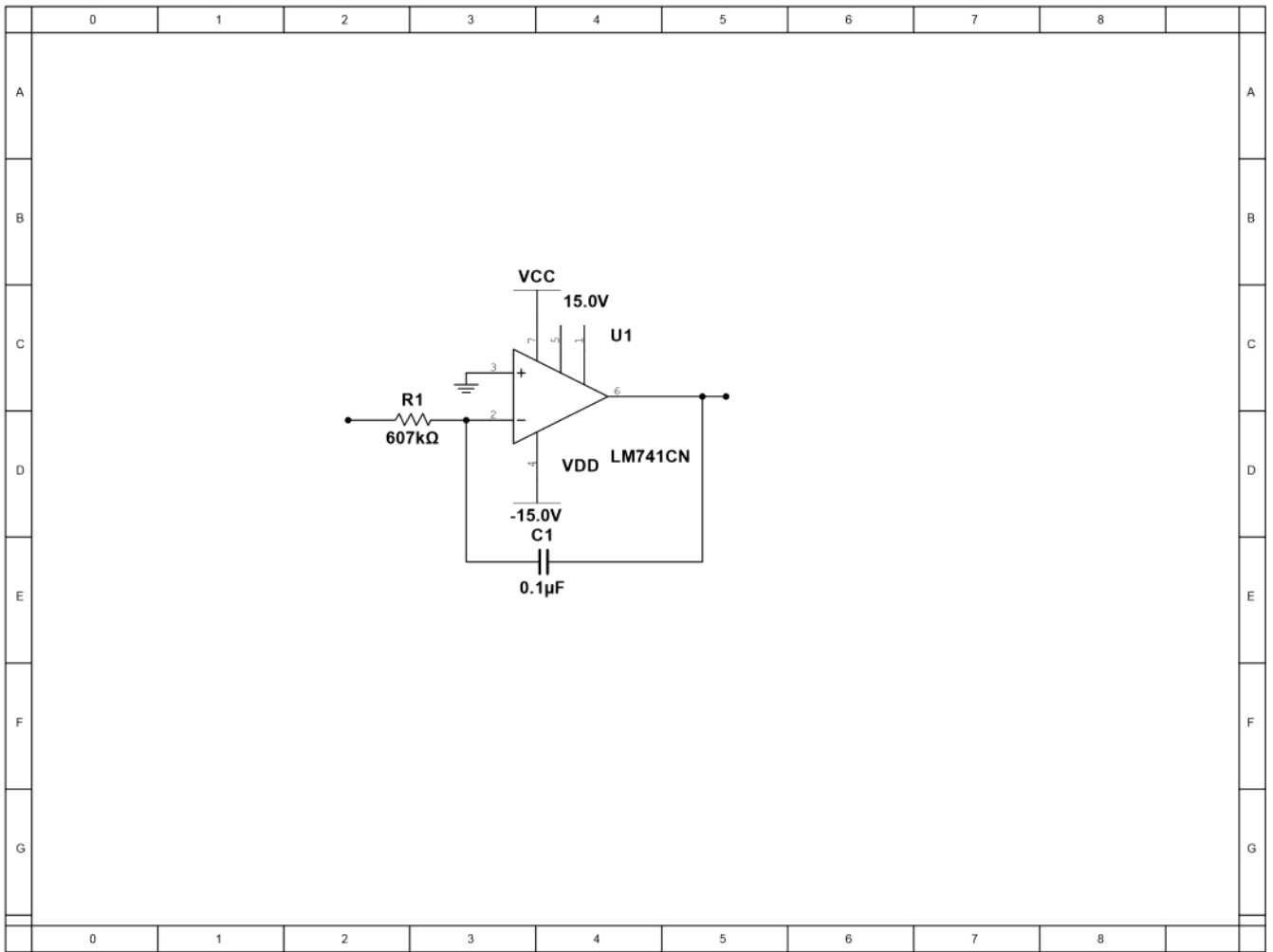
$$562 \text{ k}\Omega + 47 \text{ k}\Omega = 0.610 \text{ M}\Omega$$

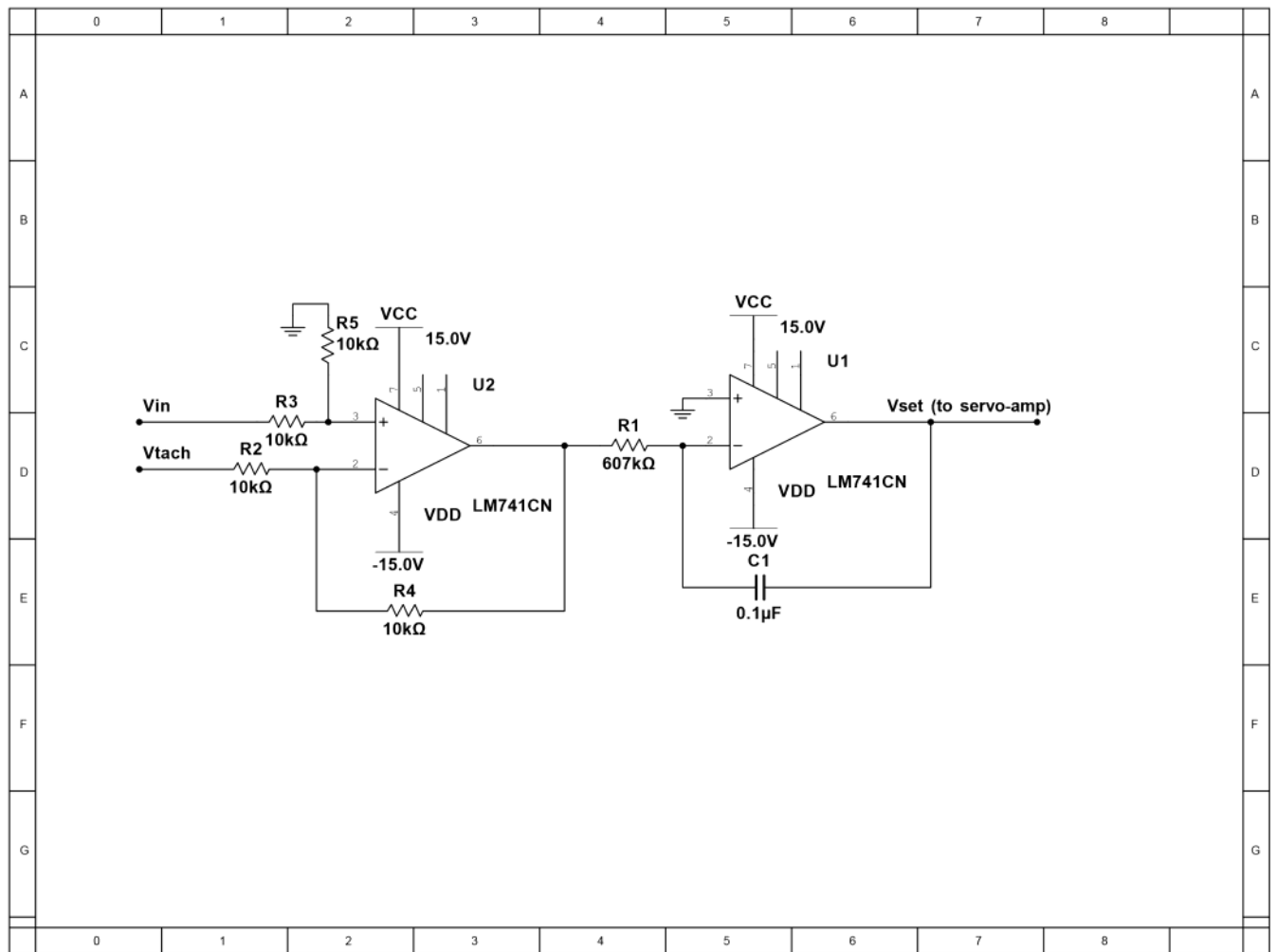
Controller Design:-



$$G(s) = \frac{16.456}{s} = -\frac{Z_f}{Z_i} = -\frac{1}{sC/R}$$







Controller Performance Measurements

Tuesday, November 22, 2016 4:57 PM

Measurement Setup

Channel 1 - Yellow - Setpoint

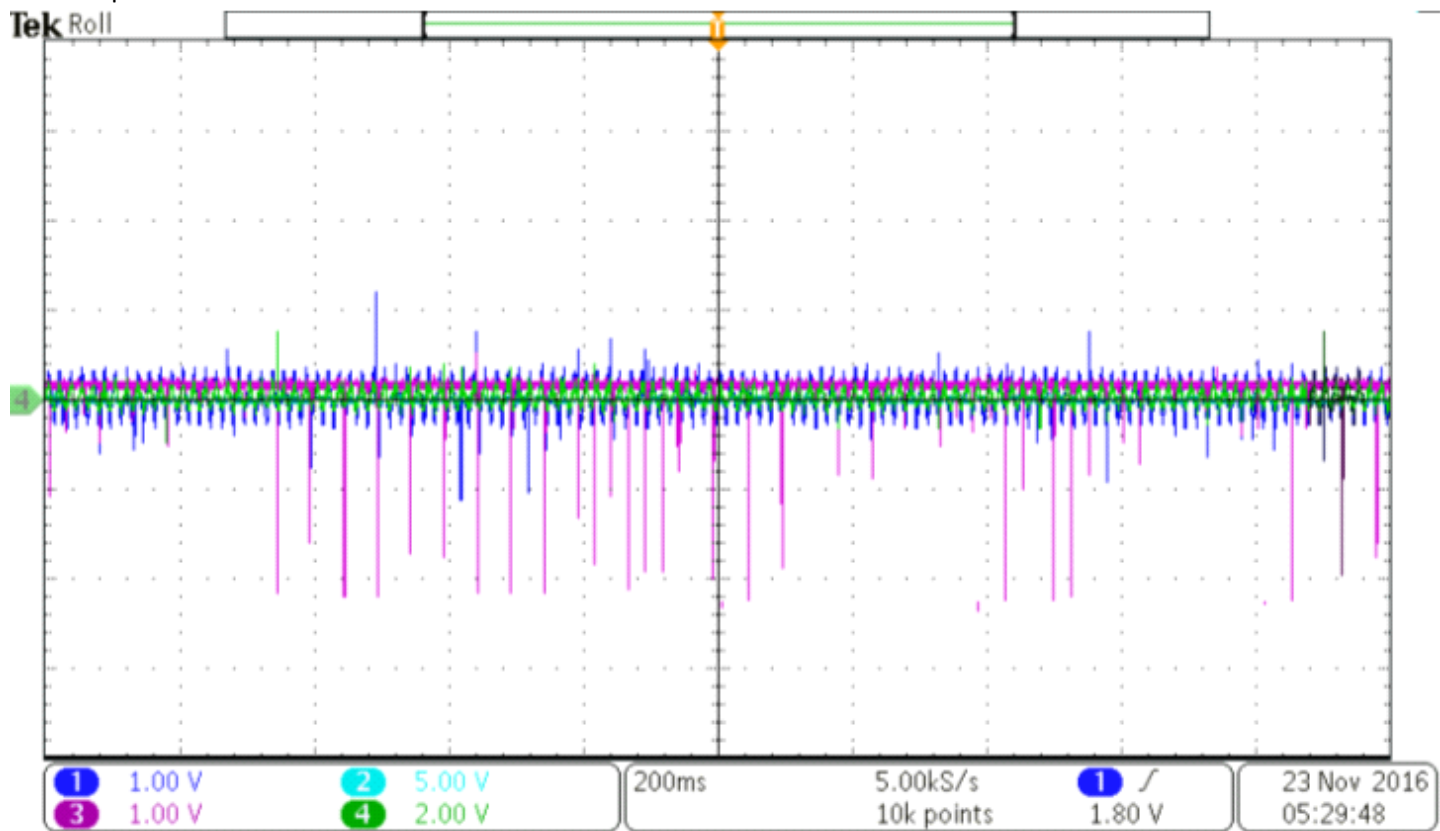
Channel 2 - Blue - Error

Channel 3 - Pink - Controller Output

Channel 4 - Green - Tachometer Output

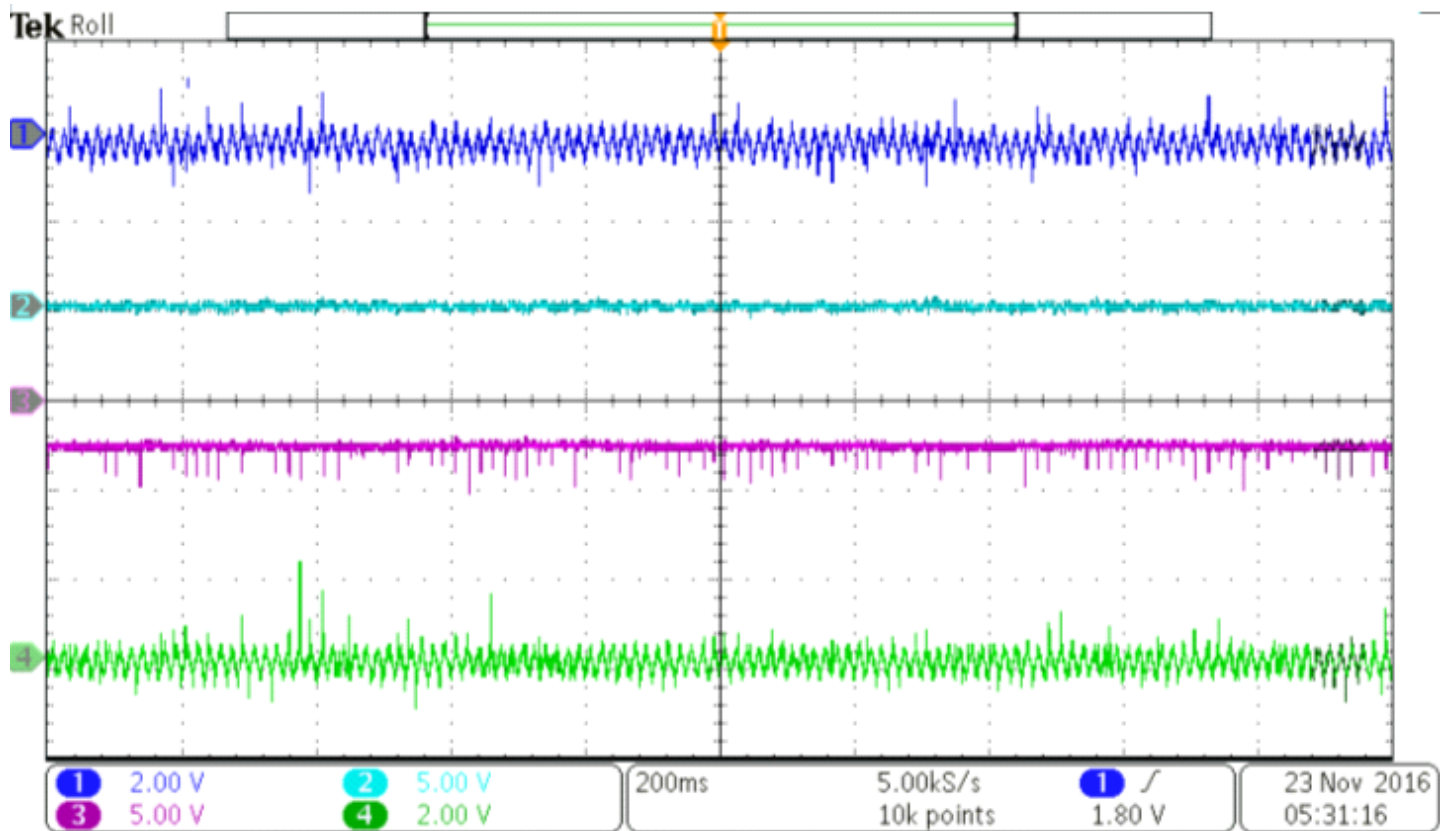
Zero Input

Zero output! Zero error! Zero Tach!

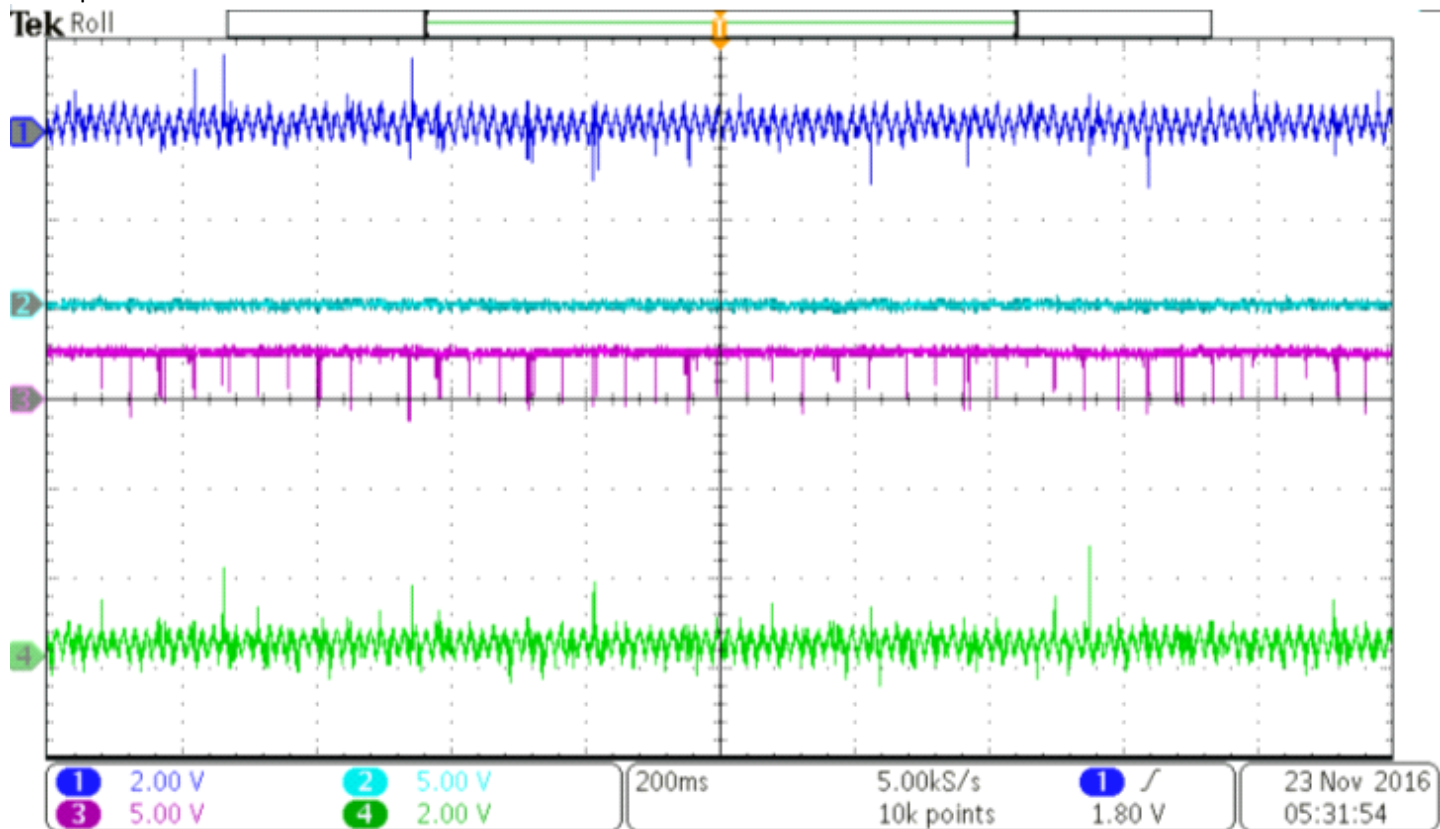


+/- 1V Input

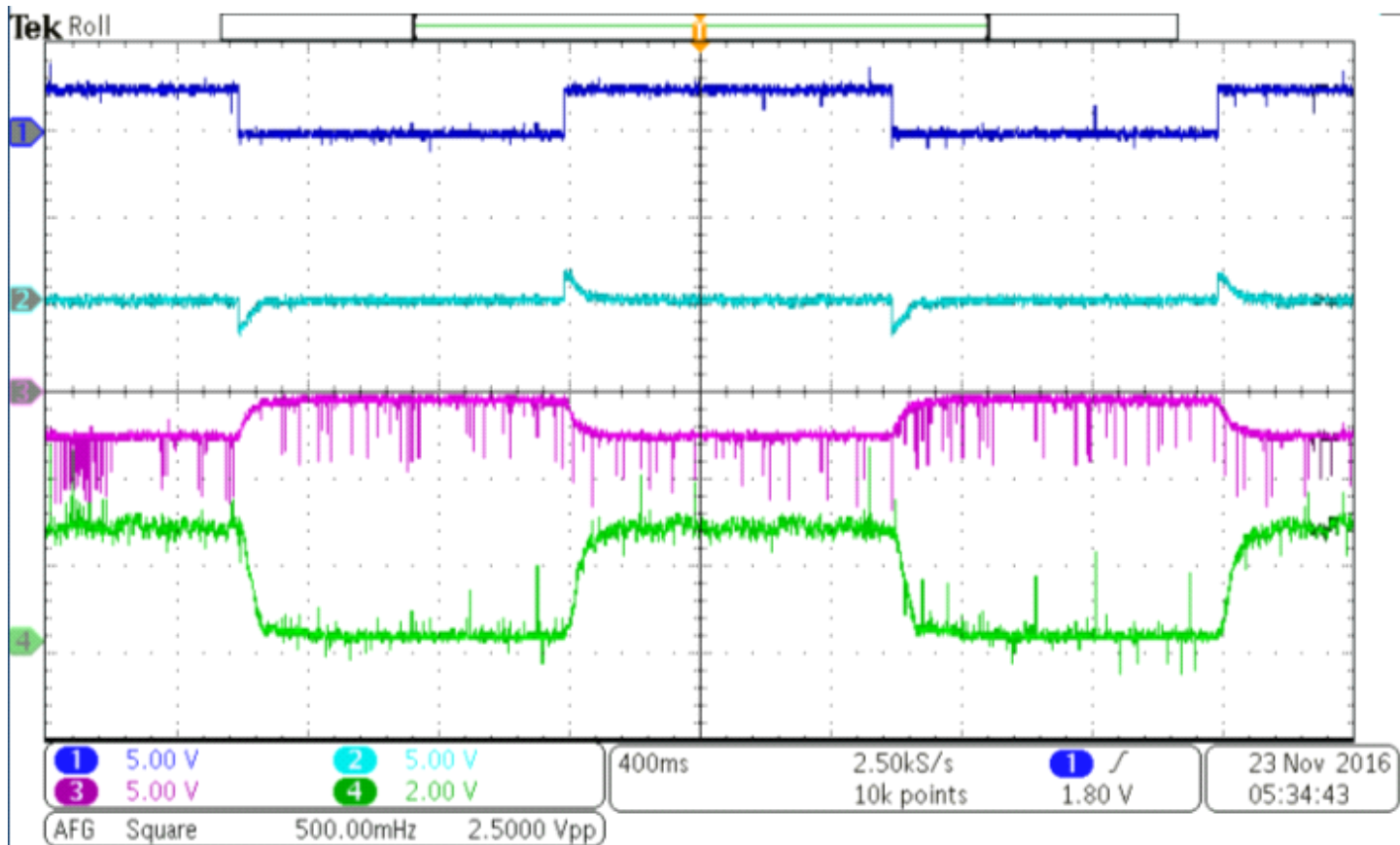
+1V Input - Motor Spins at Constant Speed



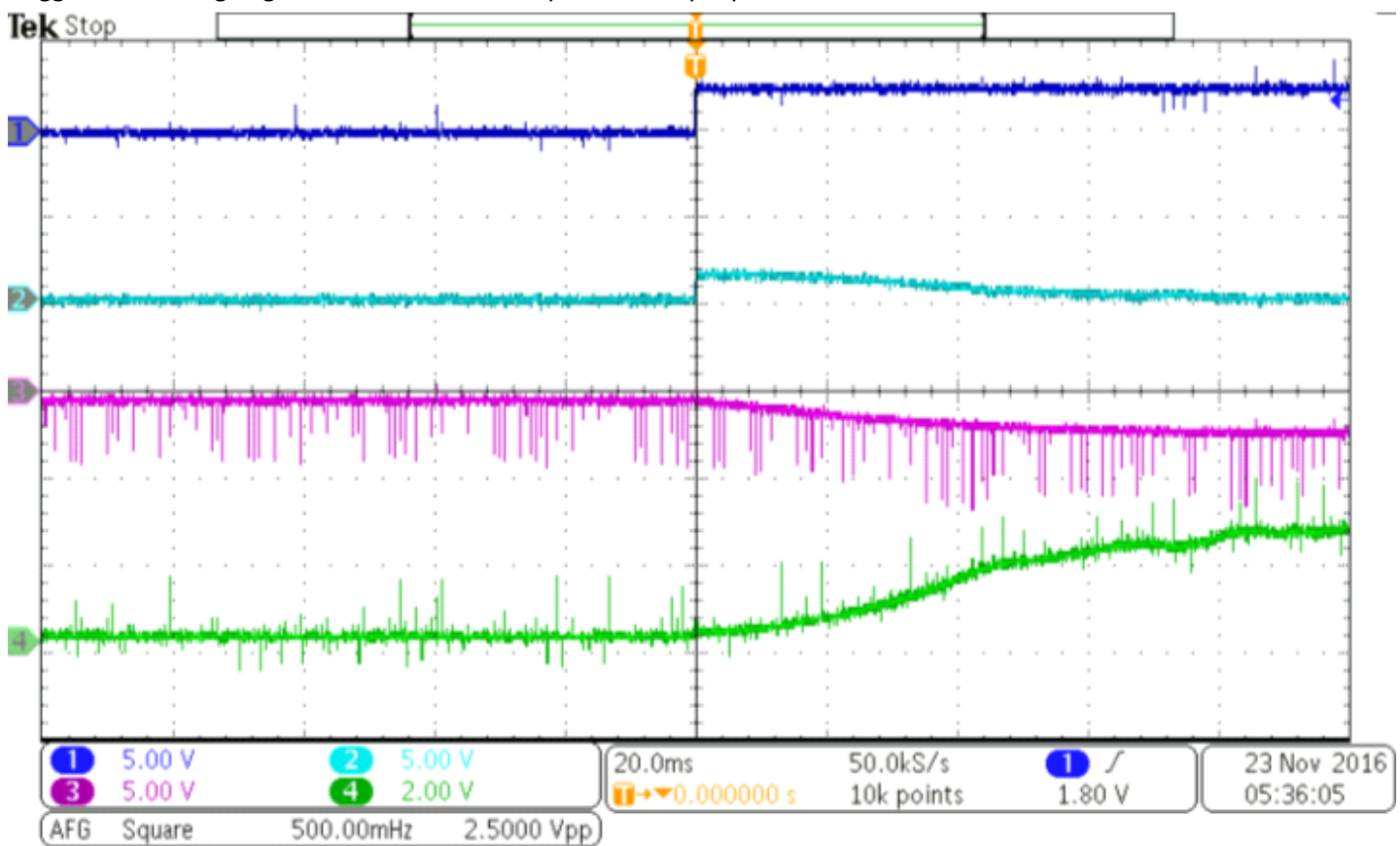
-1V Input



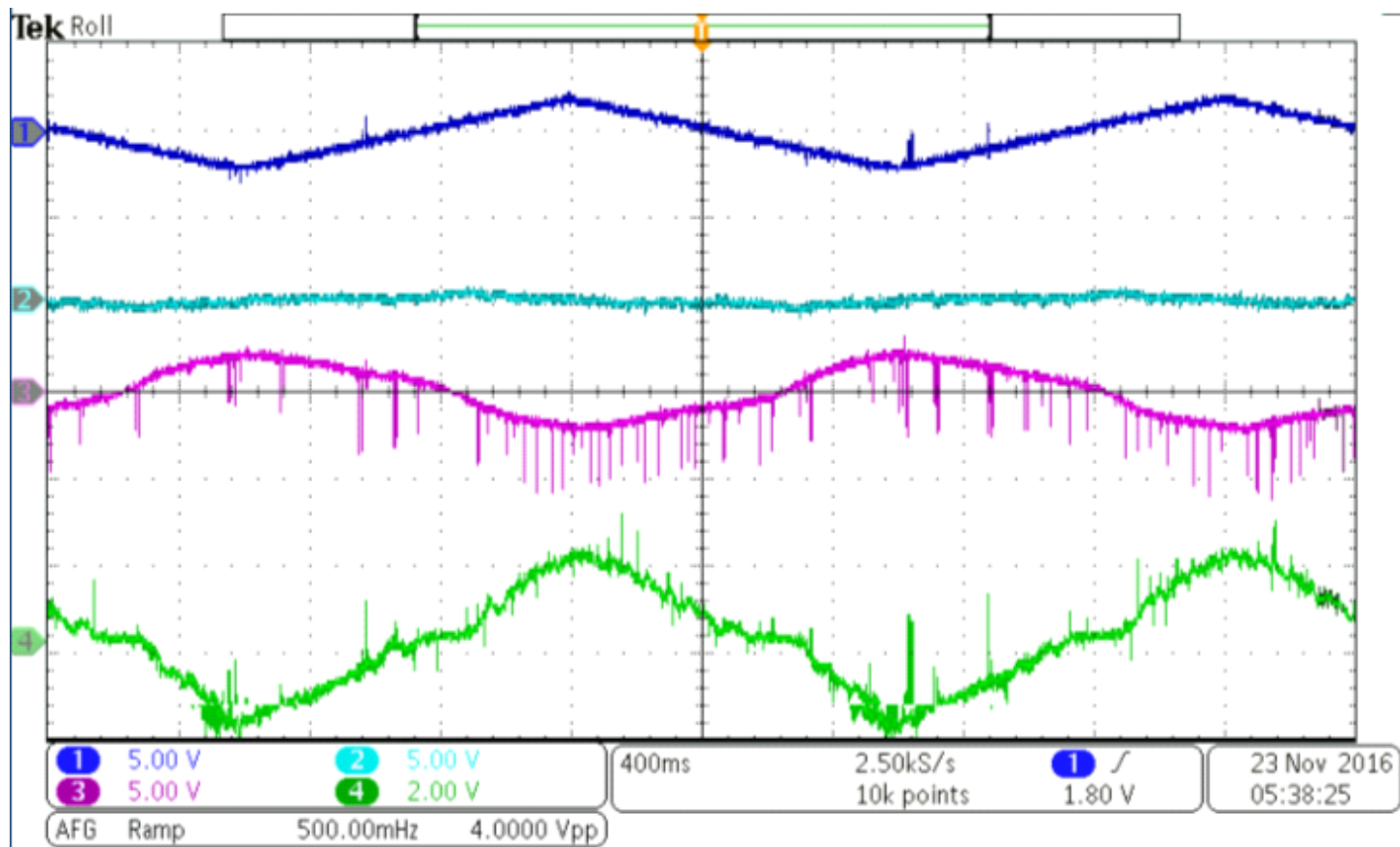
2.5V Square Wave - 0.5Hz



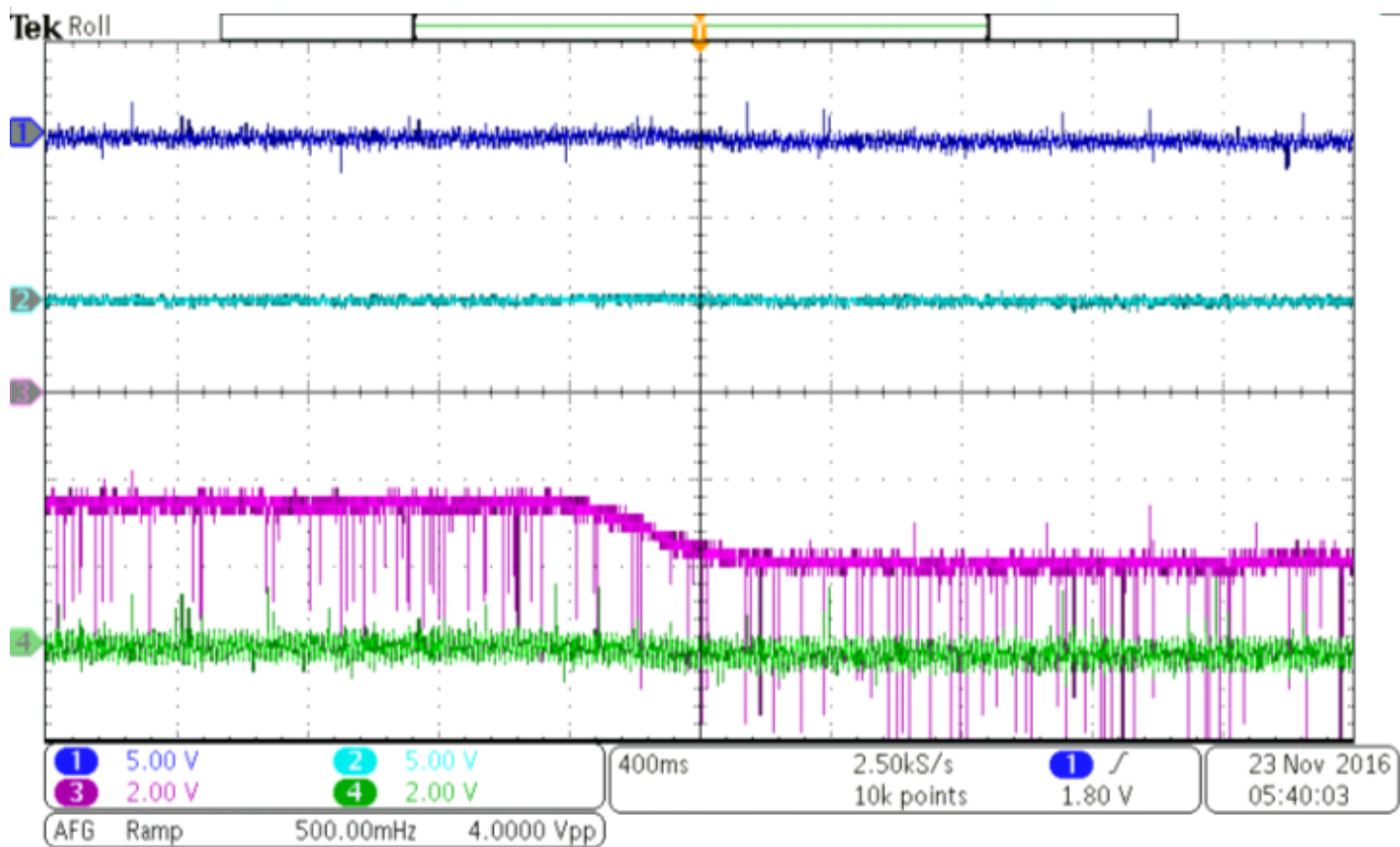
Triggered on Rising Edge to show controller response to step input



+/- 2V Triangle Wave - 0.5Hz



1V Input - Loaded by hand



Week 13 Overview

Tuesday, November 29, 2016 12:11 PM

We won't be doing testing with the elevators this week.

- ☒ Updated Motor Model
 - ☒ Add inertia of elevator cars
 - ☒ Output = linear position (instead of rotational speed)
- ☒ New PI Position Controller
 - ☒ Continuous transfer function
 - ☒ Discrete transfer function for compensator

Bill briefing

Wednesday, November 30, 2016 1:34 PM

Key Points

- Mass of elevator cars: 400g each
- Drive pulley = A 6P 9-0802408
- Controller effort limited to $\pm 2.5V$

$M_{CAR} = 400g$
 $= 0.4 kg$

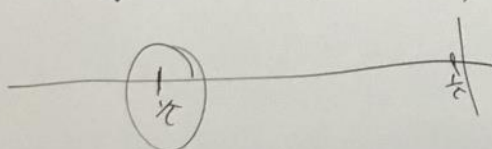
STOCK DRIVE PULLEY
A 6P 9-0802408

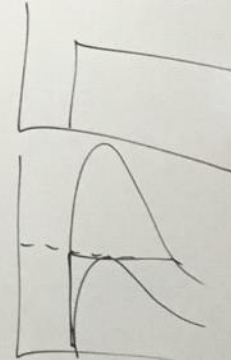
PROJECT

$J_{eq} = J_m + \frac{1}{4} R_p^2 M_{CAR} + J_p$

$J_m = \text{MOTOR/TACH INERTIA}$ $kg \cdot m^2$
 $R_p = \text{DRIVE PULLEY RADIUS}$ m
 $M_{CAR} = \text{MASS OF EITHER CAR}$ kg
 $J_p = \text{INERTIA OF DRIVE PULLEY}$

$\pm 2.5V$





① MODIFY MODEL TO INCLUDE ELEVATOR CAR
 ② MODIFY MODEL TO $V_{in}/\text{Position Out}$
 - ANGULAR POSITION IS INTEGRAL OF ANGULAR VELOCITY
 - LINEAR POSITION IS INTEGRAL OF LINEAR VELOCITY
 - CONVERT ANGULAR VELOCITY TO LINEAR VELOCITY USING PULLEY RADIUS + SINGLE WRAP CABLE
 ③ DESIGN CONTROLLER FOR CRIT DAMPED RESPONSE
 CONTROLLER EFFORT $< \pm 2.5V$
 ④ CONVERT TO DISCRETE (50Hz SAMPLING)

Updated Motor Model

Wednesday, November 30, 2016 1:40 PM

Effective Inertia with Elevator Cars

Inertia of Motor + Tachometer: (from Weeks 10-12)

$$J_m = 2.5021 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

Effective Inertia of Elevator Cars:

$$J_c = \frac{1}{4} R_p^2 M_c$$

R_p = Pitch Radius of Drive Pulley (m) = 0.0145542m

Drive Pulley = A 6P-0802408

Pitch Diameter = 1.146" = 29.1084mm = 2.91084×10^{-2} m

M_c = Mass of each elevator car (kg) = 0.4kg

Mass = 400g each = 0.4kg

$$J_c = \frac{1}{4} (0.0145542 \text{ m}^2) (0.4 \text{ kg}) = 1.45542 \times 10^{-3} \text{ kgm}^2$$

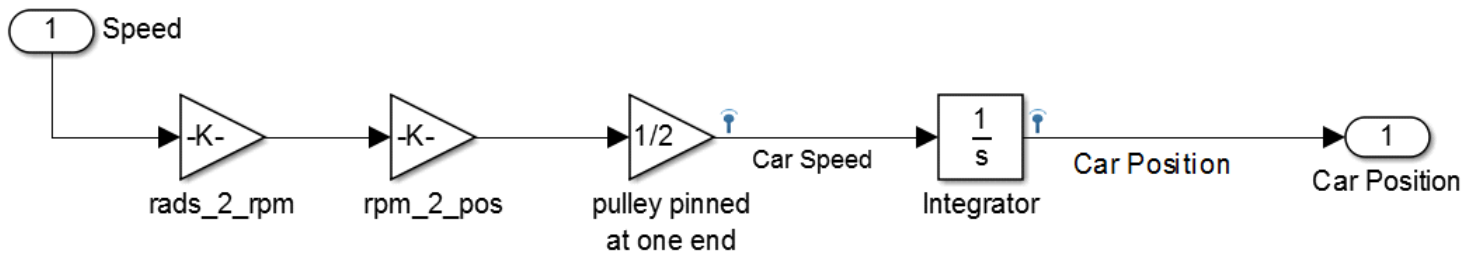
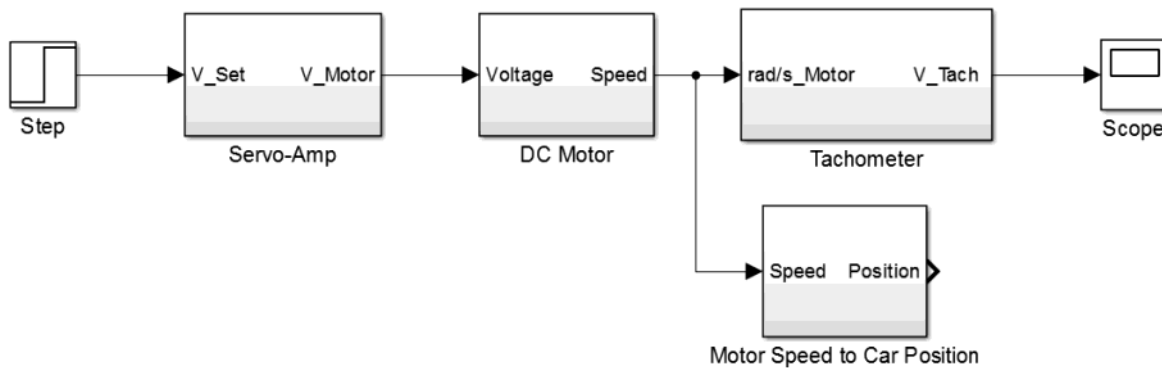
Effective Inertia of the System:

$$J_{eq} = J_m + J_c = 1.480441 \times 10^{-3} \text{ kgm}^2$$

Output Position Calculated as a function of Rotational Speed

Elevator Position = Rotational Speed of Motor * (Factor)

Factor = (1 Rad/s) * (1/2pi rev/rad) * (2piR m/rev) * 1/2 (factor for pinned end of elevator pulley cables)



```
rads_2_rpm = 1/(2*pi); % Radians/second to RPM
rpm_2_pos = 2*pi*Rp; % Angular speed in RPM to position in Metres
Rp = 29.1084/1000; % Pitch radius of drive pulley
pos_car = speed * rads_2_rpm * rpm_2_pos * 1/2; % Car position calculation, 1/2 for pinned end.
```

Updated Transfer Function

Thursday, December 01, 2016 2:00 PM

Just motor (from week 11):

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{\frac{K_t}{LJ}}{s^2 + \frac{(RJ + bL)}{LJ}s + \frac{Rb + K_e K_t}{LJ}}$$

Slide 49 - S&CS Systems of DE's Slides

MATLAB Function of just motor + tach:

$$H = (G_s * (K_t / (L * J))) / ((s^2 + (((R * J) + (b * L)) / (L * J)) * s + (((R * b) + (K_e * K_t)) / (L * J))))$$

Adding car position:

Motor: $G(s) = \frac{\omega(s)}{V(s)} = \frac{\text{Angular Velocity}}{\text{Voltage Input}}$

Elevator: $H(s) = \frac{d(s)}{V(s)} = \frac{\text{Distance of elevator car}}{\text{Voltage Input}}$

Speed to distance: $\frac{d(s)}{\omega(s)} = \left(\frac{1 \text{ rpm}}{2\pi \text{ rad/s}} \right) \left(\frac{2\pi R_p \text{ rpm}}{m/s} \right) \left(\frac{1}{2} \right) \left(\frac{1}{s} \right) = \left(\frac{R_p}{2s} \right) \frac{m}{\text{rad/s}}$, $R_p = \text{drive pulley pitch radius}$
 speed to distance.

$H(s) = G(s) \left(\frac{d(s)}{\omega(s)} \right) = \left(\frac{\omega(s)}{V(s)} \right) \left(\frac{d(s)}{\omega(s)} \right) = \frac{d(s)}{V(s)}$

$H(s) = \frac{d(s)}{V(s)} = \left[\frac{\frac{R_t}{LJ}}{s^2 + \frac{RJ + bL}{LJ}s + \frac{Rb + K_e K_t}{LJ}} \right] \left(\frac{R_p}{2s} \right)$

MATLAB Transfer Function:

$$H = (G_s * ((K_t / (L * J))) / ((s^2 + (((R * J) + (b * L)) / (L * J)) * s + (((R * b) + (K_e * K_t)) / (L * J)))) * (R_p / (2 * s)))$$

H =

$$\frac{373.1}{2 s^3 + 1518 s^2 + 2801 s}$$

Continuous-time transfer function.

Factored transfer function:

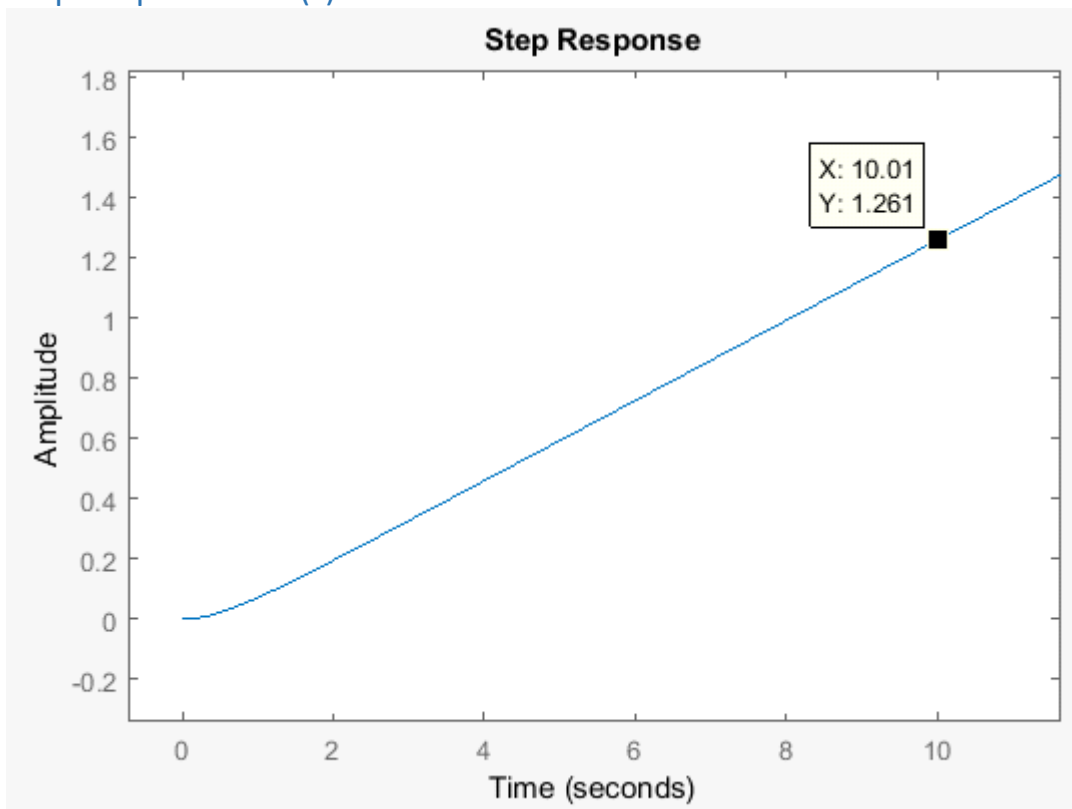
```
>> TF = zpk(H)
```

TF =

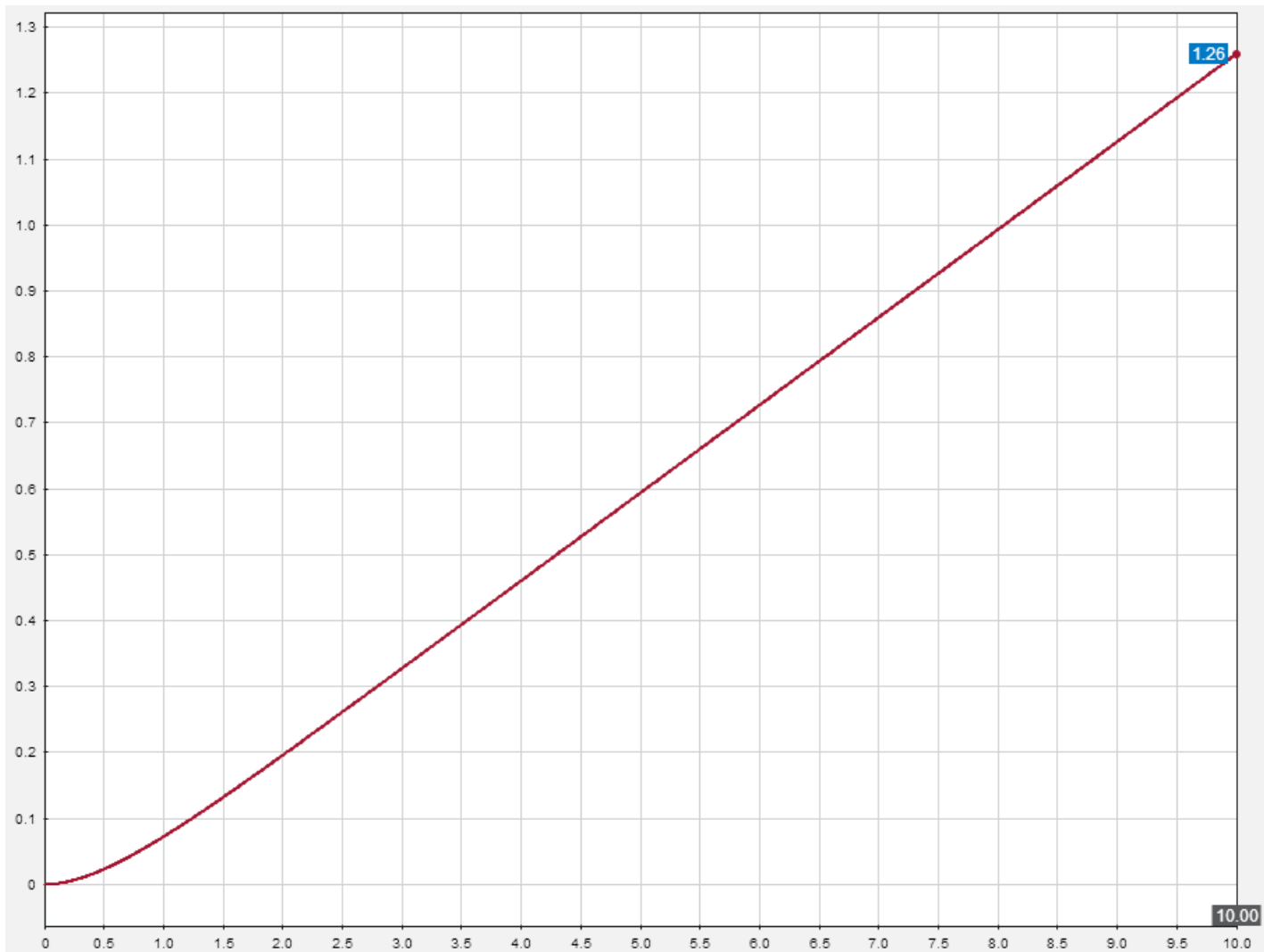
$$\frac{186.55}{s (s+757.3) (s+1.85)}$$

Continuous-time zero/pole/gain model.

Step Response of H(s) Transfer Function



Step Response of Simulink Model for System



They match. This is good.

PI Controller Development

Thursday, December 01, 2016 2:51 PM

PIDTOOL Controller

Goals:

- Critically damped response.
- Settling time ~5 seconds --> 20cm/s elevator speed. Seems reasonable.

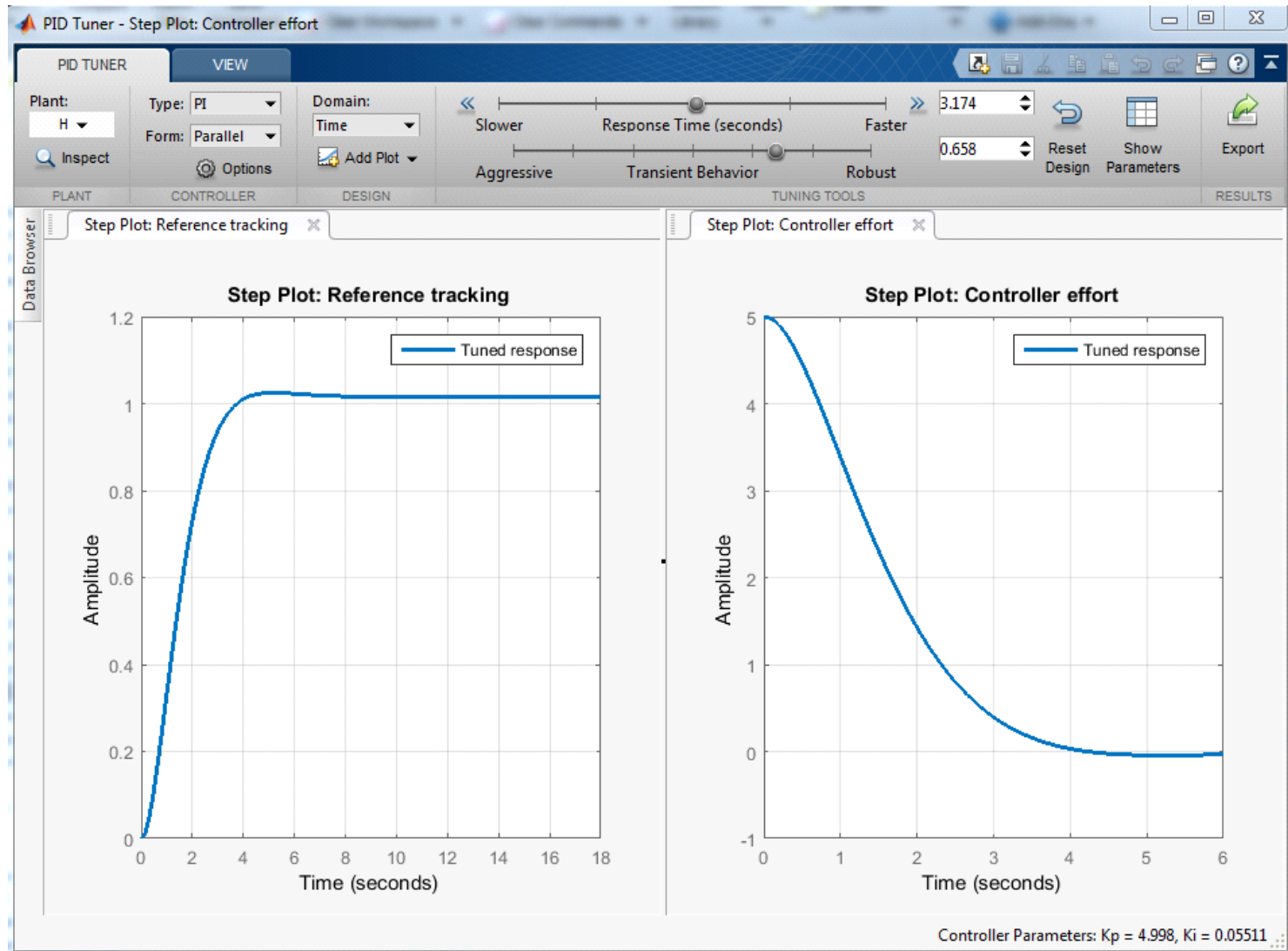
Limits: Controller effort limited to +/- 2.5V

Attempt 2: Servo-Amp Gain of 2

Settling time = 6.7 seconds --> 14.925 cm/s

Can't reach critical damping with PI controller: Overshoot = 2.51%

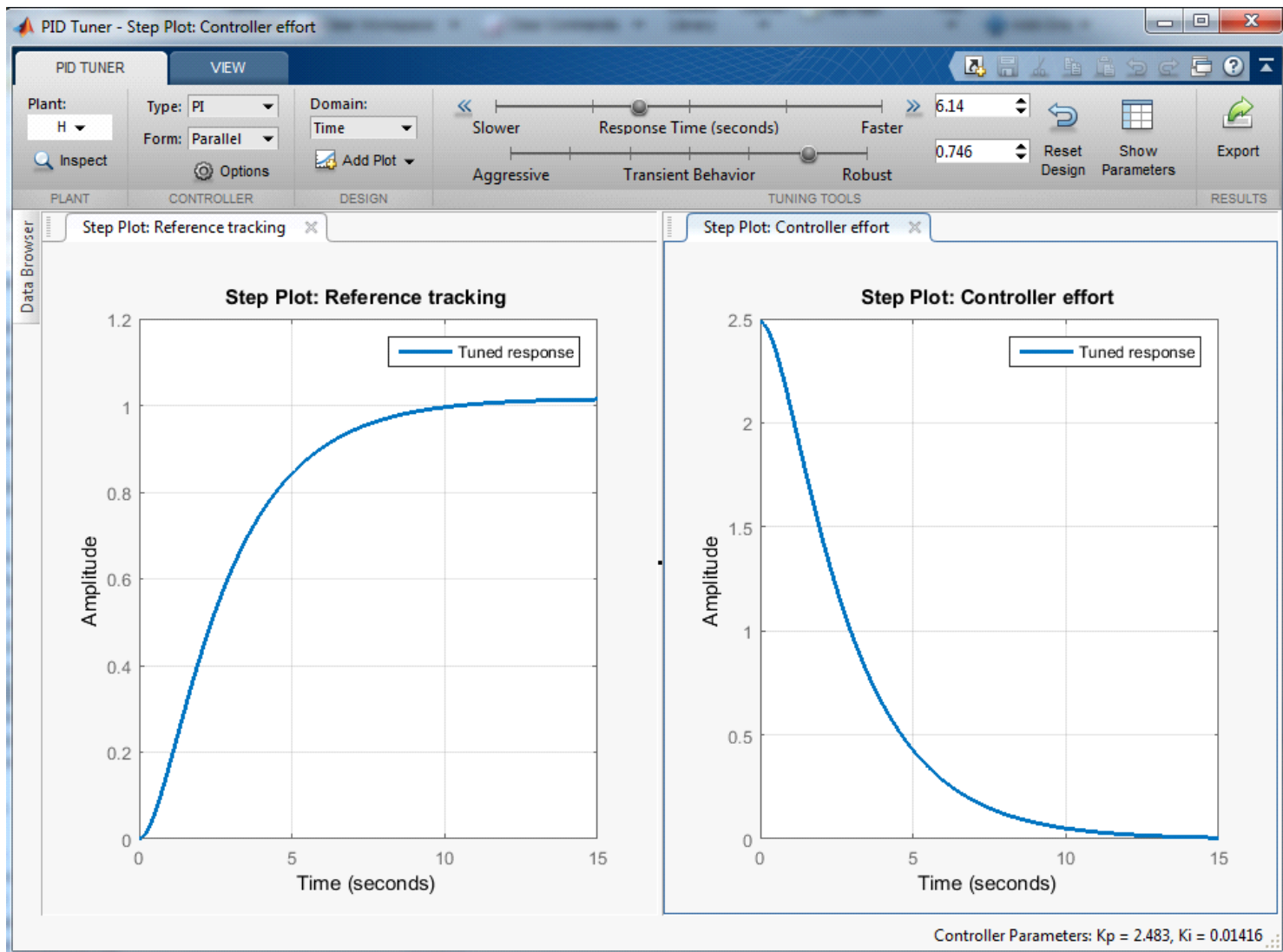
Controller effort required = +5V max



Attempt 3: Servo-Amp Gain of 2 - Tune for controller effort, not settling time

Fastest response with +/-2.5V controller effort limit: **Settling time = 8.65 seconds --> 11.56cm/s**

Can't reach critical damping with PI controller: Overshoot = 1.58%



Adding PIDTOOL Controller to Model

Transfer function of feedback system

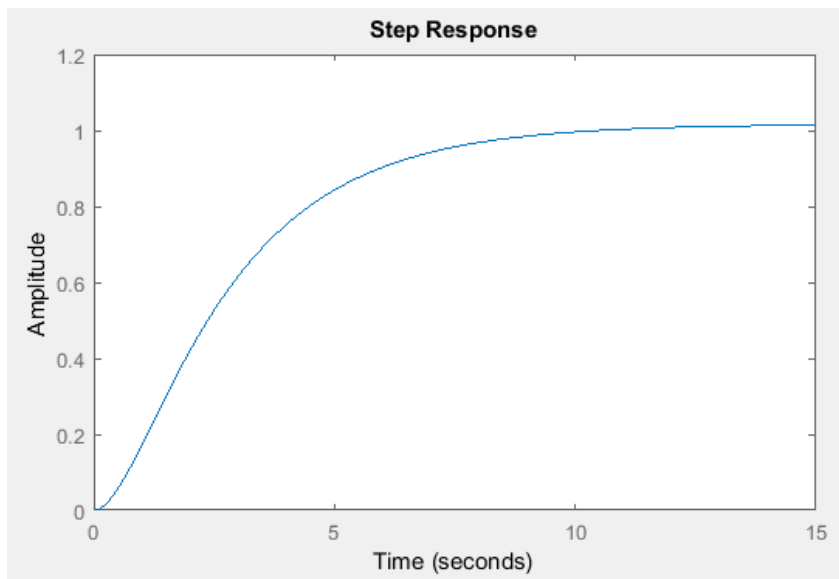
```
>> OP = (C*H) / (1+ C*H)
```

OP =

$$\frac{1853 s^5 + 1.407e06 s^4 + 2.603e06 s^3 + 1.48e04 s^2}{4 s^8 + 6073 s^7 + 2.317e06 s^6 + 8.509e06 s^5 + 9.254e06 s^4 + 2.603e06 s^3 + 1.48e04 s^2}$$

Continuous-time transfer function.

LTIVIEW of Feedback system response (matches PIDTOOL)



Discrete Controller Development

Thursday, December 01, 2016 3:35 PM

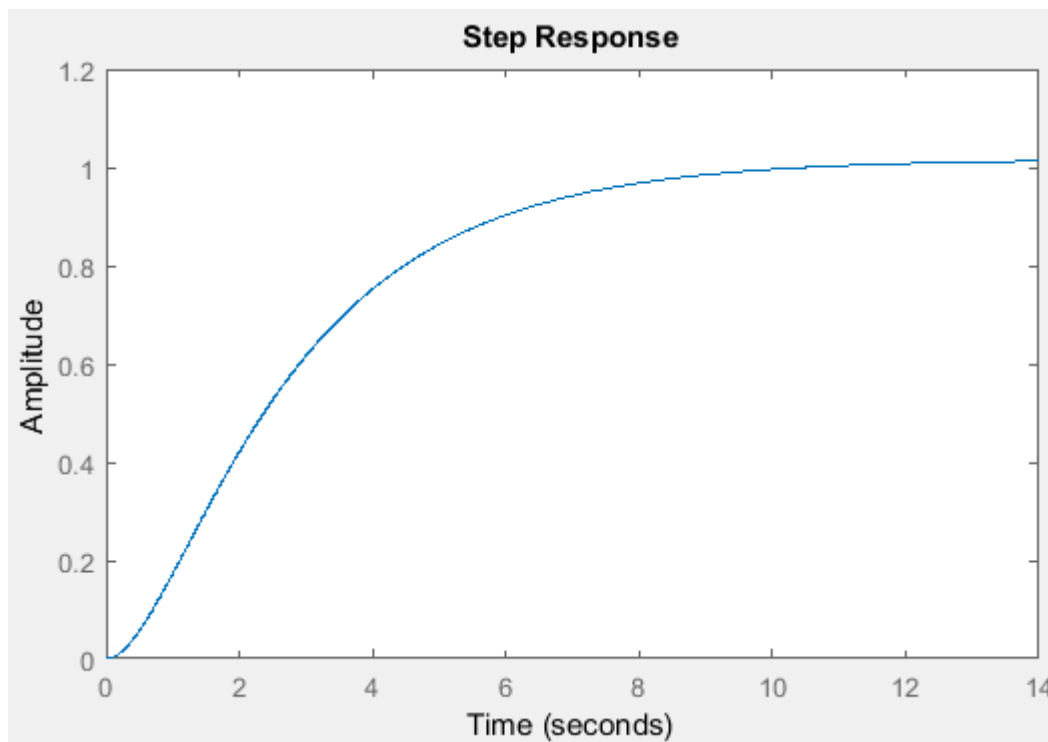
MATLAB Function: c2d does magic things to make a discrete TF from a continuous one.

C = controller model from PIDTOOL (P only controller)

Ts = 1/50 = 0.02 = Sampling time for 50Hz as Bill said to use that in class

```
C_D = c2d(C, Ts, 'tustin'); % Discrete TF for controller from pidtool
H_D = c2d(H, Ts, 'tustin'); % Discrete TF for elevator system model
OP_D = (C_D * H_D) / (1 + C_D*H_D); % Discrete TF for closed loop control system
```

Step-Response of Discrete Transfer Function OP_D



Matches continuous.

Discrete Transfer Function for PI Controller

C_D =

$$K_p + K_i * \frac{T_s * (z+1)}{2 * (z-1)}$$

with $K_p = 2.48$, $K_i = 0.0142$, $T_s = 0.02$

Sample time: 0.02 seconds

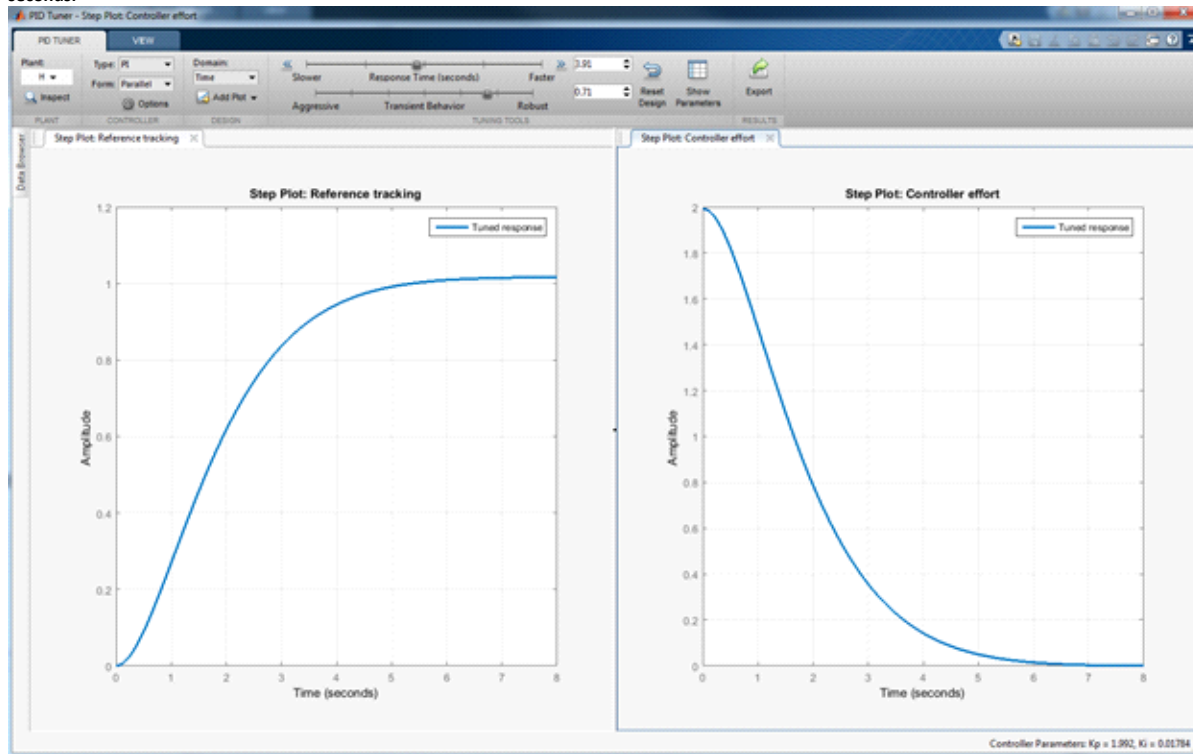
Discrete-time PI controller in parallel form.

Week 14 Overview

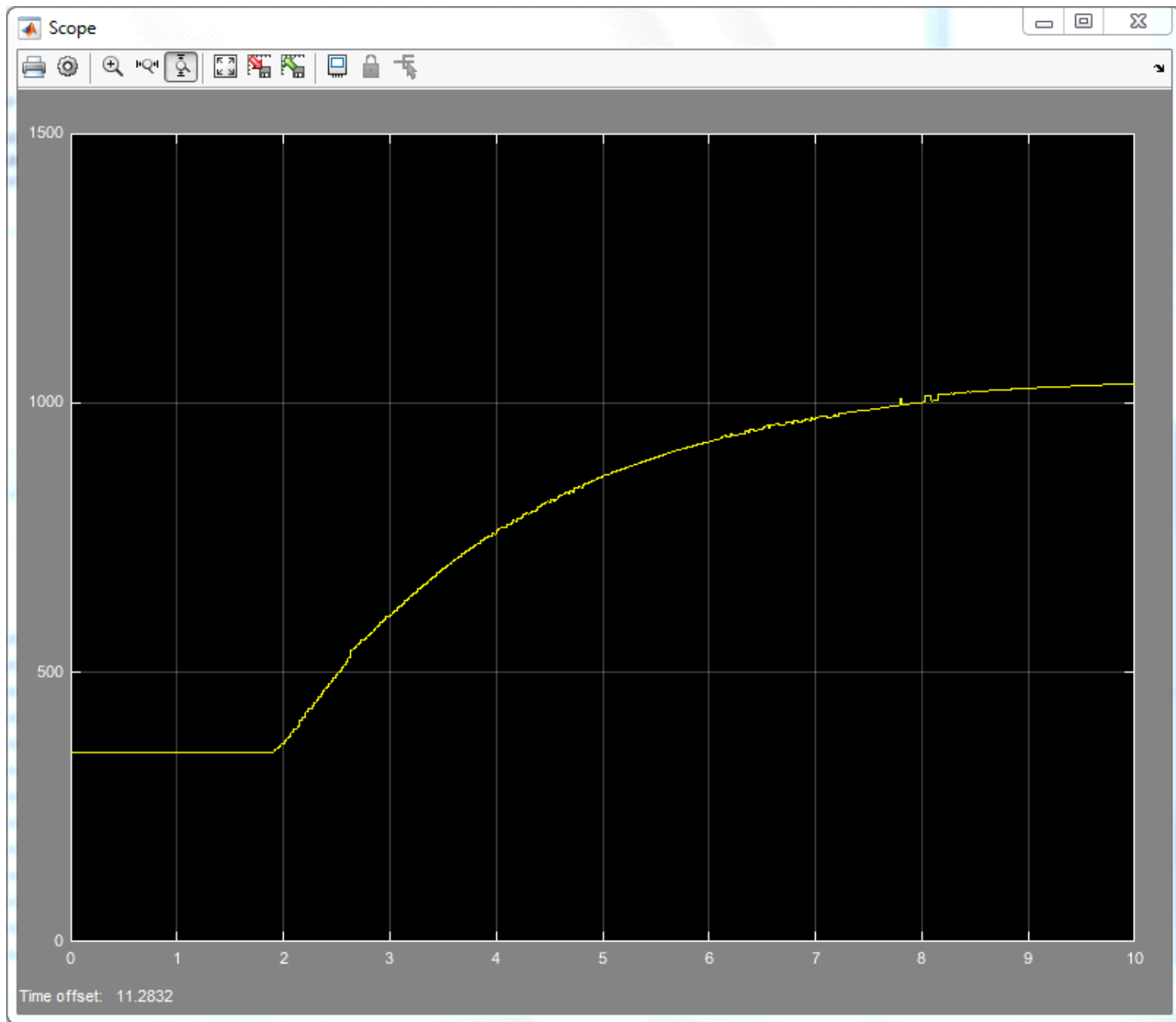
Wednesday, December 07, 2016 1:27 PM

- ☒ Test controller on elevator
- ☒ Project report

This is the controller we ran on the elevator: Rise Time = 2.96 seconds, Settling Time = 4.66 seconds.



This is the step response observed on the actual elevator:



Response time is slower & steady state error is larger due to friction which is not modelled.

