Physical Layer Security in Multi-Antenna Cellular Systems: Joint Optimization of Feedback Rate and Power Allocation

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Abstract

This paper comprehensively studies the physical layer security in frequency division duplex multi-antenna cellular systems, where multi-antenna base stations (BSs), legitimate users (LUs), and eavesdroppers are all randomly located. Each BS employs a secure multi-user multi-antenna transmission based on artificial-noise (AN)-aided linear zero-forcing beamforming, with limited channel state information feedback from multiple distributed LUs. Based on the stochastic geometry theory, the secrecy performance of the proposed scheme is analytically investigated and optimized, which have rarely been studied before. Specifically, we first develop a new analytical expression of a lower bound on the ergodic secrecy rate (ESR) of each LU without assuming asymptotes for any system parameter. Given a fixed power allocation coefficient between message-bearing signals and AN, we develop a tight closed-form approximation on the optimal number of feedback bits to maximize a lower bound on the per-LU net ESR, which captures the overall system secrecy performance by taking into consideration the cost of uplink spectral efficiency for limited feedback. To further optimize the power allocation coefficient, we develop another analytical lower bound on per-LU ESR, which, with the above closed-form approximation on the optimal number of feedback bits being substituted into, can be proved to be a concave function of the power allocation coefficient when the channel coherent time is sufficiently large. Then, the optimal power allocation can be obtained by using a bisection search method based on the obtained analytical lower bound on per-LU ESR. The approximation on the optimal number of feedback bits follows by substituting the obtained optimal coefficient into the above closed-form approximation. It was proved that the optimum number of feedback bits scales linearly with the number of antennas and path-loss exponent, and logarithmically with the channel coherence time. The derived analytical results can also provide system-level insights into the optimum power allocation coefficients and the achievable secrecy rate performance of the multi-antenna random cellular networks with limited feedback. Numerical results verify that secrecy rate performances achieved with the obtained optimum power allocation coefficient and number of feedback bits can be very close to the real optimum values.

Index Terms

Physical layer security, secrecy (sum) rate, artificial noise, limited rate feedback, stochastic geometry

I. Introduction

Deployment of ultra-dense wireless networks along with multiple-antenna technologies is a promising solution to provide seamless coverage and support unprecedented growth of mobile data traffics in 5G mobile communications [1]. This rapid evolution toward the dynamic and large-scale

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networks brings more and more challenges for secure communications, due to the broadcast nature of wireless communications and the complicated secret key generation and management with the traditional cryptography-based protocols in modern cellular systems. As the alternatives to the cryptography techniques, physical layer security (PLS) can guarantee perfect secrecy by exploiting the randomness of wireless channels without using secret keys, and thus have attracted much attention from the research community recently [2–4]. It was shown in the pioneer works [2, 5] that a positive perfect information rate (secrecy rate, SR) can be achieved when the legitimate channel is "more capable" than the eavesdropping channel.

PLS has been extensively studied for various contemporary communication architectures in static single-cell and multi-cell systems considering fixed spatial positions of network nodes, especially in multi-antenna systems [4, 6–11]. In a practical wireless systems, the eavesdroppers (Eves) are usually passive, and thus it is almost impossible to obtain the instantaneous channel sate information (CSI) of Eves at transmitter (CSIT). Moreover, for the widely-deployed frequency division duplex (FDD) systems, CSIT of the legitimate channel is typically obtained at each receiver through pilot training, and conveyed to transmitter through digital feedback [12, 13]. When the CSI of illegitimate channels is not available at transmitter, the artificial-noise (AN)-aided secure transmission is commonly employed to degrade link quality of any illegitimate channel, where a portion of total power is deliberately allocated to AN injected into the null space of the main channel. Various AN-based secure transmissions have been studied in previous literature [6–8, 10, 11]. Specifically, [6] and [7] studied the secrecy performance of AN-aided single-user beamforming (BF) for the systems with perfect CSIT and limited CSI feedback respectively, and obtained the optimal power allocation between message-bearing signals and AN to maximize the ergodic secrecy rate (ESR) for some asymptotic cases. While [8] maximized the secrecy throughput under a connection outage constraint and a secrecy outage constraint. Our previous works in [10, 11] generalized these previous design and performance results of AN-aided transmissions to the systems where multiple distributed users were simultaneously supported through spatial-multiplexing capability.

Recently, the study of PLS has been extended to dynamic large-scale cellular networks by taking the random spatial positions of network nodes into account. Unlike static scenarios, the communication between nodes in dynamic large-scale networks highly depends on the distributions of nodes's locations and the interactions between nodes. By modeling the positions of different types of nodes according to some independent Poisson Point Processes (PPPs), stochastic geometry has been extensively used as an analytical tool to investigate the average performance of the networks with or without secrecy, which can closely approximate realistic cellular systems [14–26]. Specifically, [21] obtained the analytical results of the achievable secrecy rate of cellular networks under different assumptions on the information of Eves' locations. The ESR with regularized channel inversion precoding *without* AN was analyzed from large-antenna perspective in downlink cellular networks in [22]. [23] proposed an access threshold-based secrecy mobile association policy for heterogeneous cellular network employing the AN-aided single-user BF as that in [4, 6], and provided the tractable expressions for the connection and secrecy outage probabilities of a legitimate user (LU). [24] investigated PLS in three-tier wireless sensor networks and derived analytical expressions for the ESR. [25] investigated the secrecy performance for AN-aided single-user BF and multiuser (MU) BF transmission in low and high cell-load scenarios of multi-antenna small-cell networks. The authors developed the analytical expressions of the connection and secrecy outage probabilities and the ESR, and studied the impact of different parameters on the secrecy performance from asymptotic large-antenna perspective. [26] analyzed the average secrecy throughput of AN-aided single-user BF in multi-antenna cellular networks with training-based channel estimation and studied the impact of channel estimation error on the performance thorough some numerical results.

Although AN-aided secure transmissions in dynamic large-scale cellular networks were exploited in some previous works of [23, 25, 26], all assumed perfect CSIT of legitimate channels or considered only estimated CSIT with estimation error. However, as we have illustrated above that it is impossible for a transmitter to obtain neither perfect nor estimated CSI in a practical FDD system but only quantized CSI through limited feedback. As far as we know, the design and system-level secrecy performance of AN-aided secure transmission with limited CSI feedback have never been investigated in large-scale random cellular networks before. It is still unknown how the quantized CSI (or the limited feedback rate) affects the secrecy performance of a random cellular network. In addition, the joint optimization of feedback rate and power allocation for AN-aided transmission scheme to maximize net secrecy rate performance when the cost of uplink CSI feedback of each LU is taken into account has never been studied before in a large-scale random network. Actually, to the best of our knowledge, even assuming perfect CSIT, the problem of the optimal power allocation between the message-bearing signal and AN to maximize the secrecy performance in a large-scale random network has never been *analytically* solved before. Although this issue was mentioned in [23, 25, 26], it was only shown by some numerical results therein how the secrecy performance varied with the power allocation coefficient.

We think the lack of *analytical* solutions to the above problems with multi-antenna cellular networks even with perfect CSIT is mainly due to the difficulties that arise from the more complicated distributions of the useful signals and aggregate interference compared with those in single-antenna systems. These distributions are determined by the channel fading distribution and the concrete adopted AN-aided multi-antenna transmission scheme. Moreover, when it comes to the systems that employ transmissions based on quantized CSIT, the distributions of the useful signals and aggregate interference are also affected by CSI feedback rate (or CSI quantization resolution). Thus, these distributions become extremely complicated, which leads to a variety of highly challenging mathematical analysis problems. The above facts also make it very difficult to provide a system-level study of the effect of ICI on limited feedback design. The corresponding analytical system-level performance results is also very complicated to develop. In this paper we propose an analytical framework to effectively overcome the afore-mentioned challenges and provide some answers for these problems.

In this paper, we aim to investigate the system-level secrecy performance of AN-aided secure multiuser zero-forcing BF (ZFBF) based on limited feedback as well as the corresponding analytical solutions to the optimal feedback rate design and the optimization of power allocation to maximize the per-LU net ESR, which measures the normalized downlink per-LU ESR minus the cost of uplink CSI feedback for each channel coherence time T_c. The specific definition of the per-LU net ESR and a lower bound are respectively provided in (22) and (23). The main contributions of this paper are completely new for large-scale random networks and can be summarized below.

- We develop, for the first time, an analytical framework for the optimum design and performance analysis of physical-layer security in a FDD multi-antenna cellular network with only quantized CSIT through limited CSI feedback, where the BSs, LUs and non-colluding Eves are all randomly distributed on a plane modeled by independent PPPs. We establish a new analytical lower bound on the ergodic rate of a typical LU, and also a new analytical upper bound on the ergodic rate of the typical LU's messages over the most detrimental Eve's channel among all Eves, both of which are in integral form as functions of the related system parameters. Subsequently, an analytical expression of a lower bound on the per-LU ESR follows. Some useful insights for practical system design can be observed from the obtained results that the ergodic rate of a typical LU is not related to the densities of BSs and LUs and the ergodic rate of a typical LU's messages over the most detrimental Eve's channel is only related to the ratio of λ_b/λ_e and is monotonic decreasing with respect to (w.r.t.) λ_b/λ_e, where λ_b and λ_e are the densities of BS and Eves respectively.
- Given a fixed power allocation coefficient ϕ between signals and AN, we obtain respectively a lower bound and an upper bound on the optimal number of feedback bits $B^*_{\text{real}}(\phi)$ that maximize a lower bound on the per-LU net ESR. Our analytical result illustrates that the gap

between the obtained lower and upper bounds said above asymptotically converges to zero as $T_c \to +\infty$. Then, we develop an accurate analytical approximation on $B^*_{\text{real}}(\phi)$ for sufficiently large T_c, which is denoted as $B^*_{ax}(\phi)$. We prove that, if the system parameters other than ϕ and T_c are fixed, $B^*_{ax}(\phi)$ is monotonic decreasing w.r.t. ϕ and is monotonic increasing w.r.t. T_c.

- To optimize the power allocation coefficient, we develop another upper bound on the ergodic rate of the typical LU's messages over the most detrimental Eve's channel among all Eves. Then, another lower bound on the per-LU net ESR follows, which can be proved to be a concave function of ϕ with the above analytical approximation on the optimal number of feedback bits being substituted into. Based on these results, we propose a bisection search method to obtain the optimized power allocation coefficient $\phi^\star$. We prove that, when the system parameters other than B and T_c remain fixed, $\phi^\star$ is monotonic decreasing in T_c in the region where T_c takes the sufficiently large value. Moreover, as $T_c \to +\infty$, $\phi^\star$ converges to a limit value which is determined by the system parameters other than B and T_c.
- We can prove that, when the system parameters other than B and ϕ remain fixed, the approximation on the optimal number of feedback bits $B^*_{ax}(\phi^\star)$ and the corresponding practical integer value $B^\star$ is monotonic increasing in T_c in the region where T_c takes sufficiently large value. Moreover, as $T_c \to \infty$, $B^*_{ax}(\phi^\star)$ and $B^\star$ scale linearly with the term $\frac{\alpha}{2}(M-1)$ and scales logarithmically with T_c.

Notations: $\mathcal{C}$, $\mathcal{R}$, $\mathcal{Z}$ and $\mathcal{N}$ denote the sets of complex numbers, real numbers, integer number and natural number respectively. $\mathbb{E}_X\{\cdot\}$ represents expectation with respect to random variable (RV) X. $\mathcal{L}_X(s)$ denotes Laplace transform of RV X. $[x]^+ = \max\{x, 0\}$.

II. System and Signal Model

We consider secure communications in a downlink cellular network operating in FDD mode, where each base station (BS) wants to deliver confidential messages to its associated LUs in the presents of the randomly distributed eavesdroppers (Eves). The BSs, LUs and Eves are distributed according to three independent homogeneous PPPs in $\mathcal{R}^2$ with the densities λ_b, λ_u and λ_e respectively, which are respectively denoted as the sets Φ_b, Φ_u and Φ_e. Each BS in the network has a coverage region characterized by a Voronoi tessellation [14–16]. For fully-loaded systems (i.e., $\lambda_u \gg \lambda_b$), there are at least K LUs in each Voronoi region with very high probability, and thus each BS can randomly choose K LUs to serve. Note that this is a common assumption in the analysis of random cellular networks [15–21]. Each Eve only passively overhears the confidential messages transmitted by the BSs and they are assumed to be non-colluding, where each Eve individually decodes secret messages. We assume that each BS is equipped with M antennas and the cellular

LUs and the Eves are all equipped with single antenna. In addition, we assume each LU is associated with the nearest BS and universal frequency reuse is adopted.

According to Slivnyak's theorem, we only need to focus on the performance of the LU located at the origin o (i.e., typical LU) [16], which is denoted as LU k $(1 \leq k \leq K)$ throughout this paper. Let $b_0 \in \mathcal{N}$ denote the BS that serves LU k, which is the nearest to origin in Φ_b, and the set of the K LUs (denoted as $\{1, 2, \ldots, K\}$) are all served by BS b_0. The channel vector from BS i to LU $k \in \{1, 2, \ldots, K\}$ is represented as $r_{i,k}^{-\alpha/2}\mathbf{h}_{i,k}$, where $\mathbf{h}_{i,k} \in \mathcal{C}^{1\times M}$ is fast fading, $r_{i,k}$ denotes the distance between them and $\alpha > 2$ is the path-loss exponent. Similarly, the channel vector from BS i to Eve j is represented as $d_{i,j}^{-\alpha/2}\mathbf{g}_{i,j}$, where $\mathbf{g}_{i,j} \in \mathcal{C}^{1\times M}$ is fast fading and $d_{i,j}$ denotes the distance between them. We assume all channels in the cellular system to be block frequency-flat faded. Then, the received signals at LU k associated with BS b_0 and Eve j are respectively given as

$$
\begin{aligned}
y_{u,k} &= r_{b_0,k}^{-\alpha/2}\mathbf{h}_{b_0,k}\mathbf{x}_{b_0} + \sum_{i=1, i\neq b_0}^{\infty} r_{i,k}^{-\alpha/2}\mathbf{h}_{i,k}\mathbf{x}_i + n_{b,k} \quad (1)\\
y_{e,j} &= d_{b_0,j}^{-\alpha/2}\mathbf{g}_{b_0,j}\mathbf{x}_{b_0} + \sum_{i=1, i\neq b_0}^{\infty} d_{i,j}^{-\alpha/2}\mathbf{g}_{i,j}\mathbf{x}_i + n_{e,j}, \quad (2)
\end{aligned}
$$

where $\mathbf{x}_i \in \mathcal{C}^{M\times 1}$ is the transmitted confidential message-bearing signal vector with the total transmit power constraint $\mathbb{E}\left[\|\mathbf{x}_i\|^2\right] = P$, and $n_{b,k} \sim \mathcal{CN}(0, \sigma_{u,k}^2)$ and $n_{e,j} \sim \mathcal{CN}(0, \sigma_{e,j}^2)$ are respectively the additive white Gaussian noises received at LU k and Eve j.

A. *Channel and CSI Feedback Models*

To implement appropriate BF transmission at the transmitter, each BS requires certain form of downlink CSI. For the considered FDD system, to focus on the effect of limited CSI feedback, we follow many previous works (e.g. [7, 8, 12, 13, 19, 20]) to assume that each LU k can estimate the downlink CSI from the associated BS (i.e., $\mathbf{h}_{b_0,k}$) perfectly, and then the quantized version of the channel direction vector (CDV) $\widetilde{\mathbf{h}}_{b_0,k} = \mathbf{h}_{b_0,k}/\|\mathbf{h}_{b_0,k}\|$ is fed back to the BS through an error-free feedback channel with a rate constraint of B bits. LU k quantizes the CDV by using a predefined codebook $\mathcal{C}_k$ which consists of 2^B codewords and is known to both of LU k and the associated BS. Since the optimal vector quantization is generally unknown, we employ the same random vector quantization (RVQ) codebooks [12, 13] in many previous PLS works [7, 8, 10]. The rule for LU k to quantize $\widetilde{\mathbf{h}}_{b_0,k}$ is given by $\hat{\mathbf{h}}_{b_0,k} = \arg\max_{\mathbf{c}\in\mathcal{C}_k} \left|\widetilde{\mathbf{h}}_{b_0,k}\mathbf{c}^H\right|$. Then, $\widetilde{\mathbf{h}}_{b_0,k}$ can be decomposed as [12]

$$
\tilde{\mathbf{h}}_{b_0,k} = \hat{\mathbf{h}}_{b_0,k}\cos\theta_k + \mathbf{e}_{b_0,k}\sin\theta_k, \quad (3)
$$

where $\sin^2\theta_k$ is the quantization error with $\theta_k = \angle(\tilde{\mathbf{h}}_{b_0,k}, \hat{\mathbf{h}}_{b_0,k})$ and $\mathbf{e}_{b_0,k}$ is a unit norm vector that is isotropically distributed in the null-space of $\hat{\mathbf{h}}_{b_0,k}$. $\hat{\mathbf{h}}_{b_0,k}$ and $\mathbf{e}_{b_0,k}$ are independent with $\sin^2\theta_k$

(and $\cos^2\theta_k$). Moreover, the Eves's CSI is unknown at each BS due to passive Eves, but we assume each BS knows the statistical information of Eves' CSI.

B. Artificial-Noise-Aided Multiuser Linear ZFBF

We employ in this paper the widely used MU linear ZFBF with the assistance of AN at each BS based on the quantized CSIT of the K supported LUs for simultaneously broadcasting confidential message-bearing signals [12, 13, 19, 20]. Let $\mathbf{W}_i = [\mathbf{w}_{i,1}, \mathbf{w}_{i,2}, \ldots, \mathbf{w}_{i,K}] \in \mathbb{C}^{M\times K}$ denote the ZFBF matrix for all LUs at BS i with $\mathbf{w}_{i,k}$ being the BF vector of the k-th LU. Let $\hat{\mathbf{H}}_i = [\hat{\mathbf{h}}_{i,1}^H, \hat{\mathbf{h}}_{i,2}^H, \ldots, \hat{\mathbf{h}}_{i,K}^H]^H \in \mathcal{C}^{K\times M}$ denote the composite channel matrix consisting of the quantized CDVs of all LUs. Then, the BF vector $\mathbf{w}_{i,k}$ can be obtained by normalizing the k-th columns of matrix $\hat{\mathbf{H}}_i^H(\hat{\mathbf{H}}_i\hat{\mathbf{H}}_i^H)^{-1}$ [12, 13, 19, 20], which satisfies $\hat{\mathbf{h}}_{i,j}\mathbf{w}_{i,k} = 0\ \forall j \neq k$. $\mathbf{Z}_i \in \mathcal{C}^{M\times(M-K)}$ is the precoding matrix for the AN vector used by BS i whose columns form the orthogonal basis of the null space of $\hat{\mathbf{H}}_i$, i.e., $\hat{\mathbf{H}}_i\mathbf{Z}_i = \mathbf{0}_{K\times(M-K)}$ and $\mathbf{Z}_i^H\mathbf{Z}_i = \mathbf{I}_{(M-K)}$. It is obvious that $M > K$ is necessary for each BS to employ the AN. For simplicity, we follow many previous works [10, 12, 13, 19, 25] to assume that a fraction ϕ of the total transmit power of each BS, i.e., ϕP, is equally allocated among multiple LUs' message-bearing signals, and the remaining power is used for AN vector. Then, the useful signals in conjunction with AN transmitted by BS i can be written as

$$\mathbf{x}_i = \sqrt{\frac{\phi P}{K}}\mathbf{W}_i\mathbf{s}_i + \sqrt{\frac{(1-\phi)P}{M-K}}\mathbf{Z}_i\mathbf{v}_i = \sqrt{\frac{\phi P}{K}}\sum_{k=1}^{K}\mathbf{w}_{i,k}s_{i,k} + \sqrt{\frac{(1-\phi)P}{M-K}}\mathbf{Z}_i\mathbf{v}_i, \tag{4}$$

where $\mathbf{s}_i = [s_{i,1}, s_{i,2}, \ldots, s_{i,K}]^T$ consists of the confidential messages of all LUs at BS i with $E[\mathbf{s}_i\mathbf{s}_i^H] = \mathbf{I}_K$ and $\mathbf{v}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{M-K})$ denotes the Gaussian distributed AN vector.

Since the additive noise variance at any Eve is generally unknown to BS due to passive Eve, we follow many previous works to consider a "worst-case" scenario with $\sigma_{e,j}^2 \to 0\ \ \forall j$ [6, 8]. Moreover, since cellular systems with universal frequency reuse are usually interference-limited [14–20, 25], we can focus on the received SIR instead of the SINR at each LU and Eve. It is noted that there is a leakage of AN at each LU due to the quantized CSIT. According to (3), the output SIR of LU k ($k \in \{1, \ldots, K\}$) and Eve j can be respectively expressed as

$$\begin{aligned}\gamma_{u,k} &= \frac{\frac{\phi}{K}r_{b_0,k}^{-\alpha}|\mathbf{h}_{b_0,k}\mathbf{w}_{b_0,k}|^2}{\sum_{k'=1,k'\neq k}^{K}\frac{\phi}{K}r_{b_0,k}^{-\alpha}|\mathbf{h}_{b_0,k}\mathbf{w}_{b_0,k'}|^2 + \frac{1-\phi}{M-K}r_{b_0,k}^{-\alpha}\|\mathbf{h}_{b_0,k}\mathbf{Z}_{b_0}\|^2 + I_{u,k}} \\ &= \frac{\frac{\phi}{K}\|\mathbf{h}_{b_0,k}\|^2|\tilde{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2}{\|\mathbf{h}_{b_0,k}\|^2\sin^2\theta_{b_0,k}\left(\frac{\phi}{K}\sum_{k'=1,k'\neq k}^{K}|\mathbf{e}_{b_0,k}\mathbf{w}_{b_0,k'}|^2 + \frac{1-\phi}{M-K}\|\mathbf{e}_{b_0,k}\mathbf{Z}_{b_0}\|^2\right) + r_{b_0,k}^{\alpha}I_{u,k}},\end{aligned} \tag{5}$$

$$\gamma_{e,j} = \frac{\frac{\phi}{K}d_{b_0,j}^{-\alpha}\|\mathbf{g}_{b_0,j}\mathbf{W}_{b_0}\|^2}{\frac{1-\phi}{M-K}d_{b_0,j}^{-\alpha}\|\mathbf{g}_{b_0,j}\mathbf{Z}_{b_0}\|^2 + I_{e,j}} = \frac{\frac{\phi}{K}\|\mathbf{g}_{b_0,j}\mathbf{W}_{b_0}\|^2}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j}^{\alpha}I_{e,j}},\ \ j \in \Phi_e \tag{6}$$

where (5) is obtained by using (3) with some simple manipulations, and the ICI terms in (5) and (6) are respectively given by

$$I_{u,k} = \sum_{i=1, i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\phi\left[\frac{1}{K}\sum_{l=1}^{K}|\mathbf{h}_{i,k}\mathbf{w}_{i,l}|^2 + \tau\|\mathbf{h}_{i,k}\mathbf{Z}_i\|^2\right], \tag{7}$$

$$I_{e,j} = \sum_{i=1, i\neq b_0}^{\infty} d_{i,j}^{-\alpha}\phi\left[\frac{1}{K}\sum_{l=1}^{K}|\mathbf{g}_{i,j}\mathbf{w}_{i,l}|^2 + \tau\|\mathbf{g}_{i,j}\mathbf{Z}_i\|^2\right], \tag{8}$$

where $\tau \triangleq \frac{1-\phi}{\phi(M-K)}$. In the following section, we will first analyze the per-LU ESR performance of a typical LU.

III. Per-LU and Net Ergodic Secrecy Rate Analysis

We consider delay-tolerant traffics in this paper, where the wiretap coding block of each LU's confidential messages can be sufficiently long to cover multiple channel coherent time periods such that the ESR can be used as the performance metric. For the ESR analysis of a typical LU, we consider all small-scale fading channels follow the typical spatially uncorrelated Rayleigh fading, i.e., independent and identically distributed (i.i.d.) with zero-mean and unit-variance complex Gaussian elements, which has been widely considered in the literature of PLS [6–8, 22, 23, 25]. The per-LU ESR can be expressed as[1] [3]

$$R_{sec} = [R_u - R_e]^+, \tag{9}$$

where $R_u = \mathbb{E}\{\log_2(1+\gamma_{u,k})\}$ with $\gamma_{u,k}$ given by (5) is the ergodic rate of a typical LU, and R_e is the maximum achievable ergodic rate of a typical LU's (i.e., LU k's) messages over the most detrimental Eve's channel among all Eves by any possible method. We notice that, since the message-bearing symbols of all LUs that communicate with the same BS are i.i.d. with equal power, it easily follows that R_e for all LUs' messages are equal. If we let $R_{e,sum}$ denote the maximum achievable ergodic sum rate of the confidential messages of all LUs $\in \{1, 2, \ldots, K\}$ over the most detrimental Eve's channel by any possible method, R_e can be expressed as

$$R_e = \frac{1}{K}R_{e,sum}. \tag{10}$$

Before deriving the results of R_u and R_e, we first present the following lemma which easily follows from [27, *Lemma* 1] and is useful to express ergodic rates R_u, R_e.

Lemma 1: Let $x_1, \ldots, x_N, y_1, \ldots, y_M$ be arbitrary non-negative RVs. Then

$$\mathbb{E}\left[\ln\left(1 + \frac{\sum_{n=1}^{N} x_n}{\sum_{m=1}^{M} y_m}\right)\right] = \int_0^{\infty} \frac{\mathcal{M}_y(z) - \mathcal{M}_{x,y}(z)}{z}\mathrm{d}z, \tag{11}$$

[1]Note that, when the full CSI of the legitimate channels is known by the transmitter, the transmitter can vary the transmission rate in every channel fading block. Then, there can be an alternative definition of ESR as in [2]. However, this is impossible for the settings considered here, since only quantized CDVs are available at each BS.

where $\mathcal{M}_y(z) = \mathbb{E}\left[e^{-z\sum_{m=1}^{M} y_m}\right]$ and $\mathcal{M}_{x,y}(z) = \mathbb{E}\left[e^{-z\left(\sum_{n=1}^{N} x_n + \sum_{m=1}^{M} y_m\right)}\right]$.

First, we consider the result of R_u. However, being the same as the single-cell case in [10], we can observe from (5) that there exists multiuser interference (MUI) within each cell, and there is the substantial statistical dependence (correlation) among the received message-bearing signal ($|\mathbf{h}_{b_0,k}\mathbf{w}_{b_0,k}|^2$), the MUI signals ($\|\mathbf{h}_{b_0,k}\|^2 \sin^2\theta_{b_0,k}|\mathbf{e}_{b_0,k}\mathbf{w}_{b_0,k'}|^2\ \forall k' \neq k$) and the AN leakage signal ($\|\mathbf{h}_{b_0,k}\|^2 \sin^2\theta_{b_0,k}\|\mathbf{e}_{b_0,k}\mathbf{Z}_{b_0}\|^2$), which makes the exact theoretical analysis intractable. In spite of the afore-mentioned difficulty, we try to develop as accurate a closed-form approximation (lower bound) of R_u as possible, which is given in the following theorem.

Theorem 1: R_u can be lower bounded as $R_u \geq R_u^L$, where

$$R_u^L = \log_2 e \int_0^\infty \frac{(1+A_1\delta z)^{-(M-1)} - (1+A_2 z)^{-1}(1+[A_1\delta + A_2(1-\delta)]z)^{-(M-1)}}{z\big(1+\Xi(z)\big)}\mathrm{d}z \quad (12)$$

with $A_1 = \frac{K-\phi}{K(M-1)}$, $A_2 = \frac{\phi(M-K)}{K(M-1)}$ and $\delta = 2^{-\frac{B}{M-1}}$. The function $\Xi(z)$ is given by

$$\Xi(z) = \begin{cases} \sum_{m=0}^{K-1} C_1(m)\left[\frac{1}{(K+\phi z)^{K-m}} + \frac{(\phi z)^\beta (K-m)}{K^{K-m+\beta}}\mathcal{B}\left(\frac{\phi z}{K+\phi z}; 1-\beta, K-m+\beta\right)\right] & \\ \quad -\sum_{m=0}^{K-1}\sum_{n=0}^{M-K+m-1} C_2(m,n)\left[\frac{1}{(1+\tau\phi z)^{K-m+n}} + (\tau\phi z)^\beta (K-m+n)\right. & \\ \quad \left.\times\mathcal{B}\left(\frac{\tau\phi z}{1+\tau\phi z}; 1-\beta, K-m+n+\beta\right)\right] - 1, & \phi \neq \frac{K}{M} \\ \frac{K^M}{(K+\phi z)^M} + \frac{M(\phi z)^\beta}{K^\beta}\mathcal{B}\left(\frac{\phi z}{K+\phi z}; 1-\beta, M+\beta\right) - 1, & \phi = \frac{K}{M} \end{cases}, \quad (13)$$

where $\beta \triangleq \frac{2}{\alpha}$, and $C_1(m)$ and $C_2(m,n)$ are defined as

$$C_1(m) = \binom{K-1}{m}\frac{(-1)^m \tau^m K^K \Gamma(M-K+m)\Gamma(K-m)}{(1-K\tau)^{M-K+m}\Gamma(K)\Gamma(M-K)}, \quad (14)$$

$$C_2(m,n) = \binom{K-1}{m}\frac{(-1)^m \tau^K K^K \Gamma(M-K+m)\Gamma(K-m+n)}{n!(1-K\tau)^{M-K+m-n}\Gamma(K)\Gamma(M-K)}. \quad (15)$$

$\mathcal{B}(x;p,q) = \int_0^x t^{p-1}(1-t)^{q-1}\mathrm{d}t$ is the incomplete Beta function [28, 8.39].

Proof: See Appendix A. □

It can be easily seen that the lower bound on R_u is independent of BS density λ_b. Actually, due to the fact that a BS is active with probability of 1 in a dense (over-loaded) network, the received signal power, AN power and all interference power all scale as λ_b^α. Thus, SIR and ergodic rate R_u (and also R_u^L) are independent of λ_b. This result also agrees with those in many pervious works, such as [15, 18, 19, 25], which infers that deploying more BSs in a dense network will not improve the ergodic rate performance of legitimate channel.

Remark 1: Although the RVs $|\mathbf{h}_{i,k}\mathbf{w}_{i,l}|^2$ ($l = 1, 2, \ldots, K$; $i \in \Phi_b\backslash\{b_0\}$) and $\|\mathbf{h}_{i,k}\mathbf{Z}_i\|^2$ associated with the RV X_i defined in (57) are generally not independent with each other, for analytical tractability, all the theoretical results in this paper are obtained by assuming that these RVs are all independent as it was done in [19, 20, 25] with (as in [20]) or without explicit illustrations (as in [19, 25]). The reasonability of this assumption was validated by the numerical results in [19, 20].

In the following, we consider deriving the result of R_e. We first need to determine the most detrimental Eve to overhear BS b_0, which can be determined by using (6) as

$$j^\star = \max_{j\in\Phi_e} \mathbb{E}_{\Phi_b\setminus\{b_0\}} \left\{ \mathbb{E}\left[\log_2\left(1+\frac{\frac{\phi}{K}\|\mathbf{g}_{b_0,j}\mathbf{W}_{b_0}\|^2}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j}^{\alpha} I_{e,j}}\right) \middle| \Phi_b, j\in\Phi_e \right] \middle| b_0 \right\}, \quad (16)$$

where the inner expectation is on the fast fading of all links from BSs to Eve j. It is easy to observe that each random term in (16) that is related to the fast fading for each $j \in \Phi_e$ follows the same distribution respectively. In addition, according to Slivnyak's theorem, the random term $I_{e,j}$ $\forall j \in \Phi_e$ has the same distribution and is independent with $d_{b_0,j}$ [16]. Thus, it follows that

$$j^\star = \min_{j\in\Phi_e} d_{b_0,j}, \quad (17)$$

or equivalently, the Eve which is the nearest to a BS is the most detrimental to this BS for the system scenario considered in this paper. We note that Eve $j^\star$ is the nearest Eve to BS b_0, however, BS b_0 is *not necessarily* the nearest BS to Eve $j^\star$ in Φ_b. Then, $R_{e,sum}$ can be written as

$$R_{e,sum} = \mathbb{E}_{d_{b_0,j^\star}} \left\{ \mathbb{E}_{\Phi_b\setminus\{b_0\}} \left\{ \mathbb{E}\left[\log_2\left(1+\frac{\frac{\phi}{K}\|\mathbf{g}_{b_0,j^\star}\mathbf{W}_{b_0}\|^2}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j^\star}^{\alpha} I_{e,j^\star}}\right) \middle| \Phi_b \right] \middle| b_0 \right\}\right\}. \quad (18)$$

We note that the result of $R_{e,sum}$ in (18) is different from those in some previous papers [22, 23, 25] which can be explained as follows. As we have noted above, since all Eves are non-colluding and the coding block of each LU's messages are sufficiently long, an Eve has to overhear the whole coding block which may last for multiple coherent time periods. $R_{e,sum}$ expressed in (18) can be obtained in the follow theorem.

Theorem 2: $R_{e,sum}$ can be obtained as

$$R_{e,sum} = \log_2 e \int_0^\infty \frac{1-\left(1+\frac{\phi}{K}z\right)^{-K}}{z}\left(1+\frac{1-\phi}{M-K}z\right)^{-(M-K)} \frac{1}{1+\frac{\lambda_b}{\lambda_e}A_3(\phi z)^{\frac{2}{\alpha}}}\mathrm{d}z, \quad (19)$$

where A_3 is given by

$$A_3 = \Gamma\left(1-\frac{2}{\alpha}\right)\times\begin{cases} \sum_{m=0}^{K-1} C_0(m)\Big\{\frac{\Gamma\left(K-m+\frac{2}{\alpha}\right)}{K^{K-m+\frac{2}{\alpha}}} - \sum_{n=0}^{M-K+m-1}\frac{(1-K\tau)^n\tau^{K-m+\frac{2}{\alpha}}}{n!} \\ \quad\times\Gamma\left(K-m+n+\frac{2}{\alpha}\right)\Big\}, & \phi\in(0,1], \phi\neq\frac{K}{M} \\ \frac{\Gamma\left(M+\frac{2}{\alpha}\right)}{K^{\frac{2}{\alpha}}\Gamma(M)}, & \phi = \frac{K}{M}\end{cases}, \quad (20)$$

with $C_0(m) = \binom{K-1}{m}\frac{(-1)^m\tau^m K^K\Gamma(M-K+m)}{\Gamma(K)\Gamma(M-K)(1-K\tau)^{M-K+m}}$.

Proof: See Appendix B. □

Then, the expression of R_e follows by substituting (19) into (10). We note that, although the results of R_u^L in (12) and $R_{e,sum}$ in (19) seem to be rather unwieldy due to its integral form, they are actually very easy to calculate and can avoid the time-consuming computer simulations in evaluating system performance. With the analytical results of a lower bound on R_u given by (12) and R_e given by (10) and (19), a lower bound on the per-LU ESR can be obtained as

$$R_{sec} \geq R_{sec}^L \triangleq \left[R_u^L - R_e\right]^+. \tag{21}$$

The following results can be easily observed from (12) and (19).

Corollary 1: R_e is a monotonic decreasing function of the ratio of the BS–Eve density, $\frac{\lambda_b}{\lambda_e}$, and naturally is also a monotonic increasing function of λ_e and a monotonic decreasing function of λ_b.

Corollary 2: R_{sec} (and also R_{sec}^L) is a monotonic increasing function of $\frac{\lambda_b}{\lambda_e}$, and naturally is also a monotonic decreasing function of λ_e and a monotonic increasing function of λ_b.

Proof: The results can be easily proved by using *Corollary* 1 and noticing the fact that R_u (and also R_u^L) does not depend on λ_b and λ_e. □

We note that, although the properties shown in *Corollary* 1 and *Corollary* 2 comply with the corresponding results obtained in [25], it was assumed in [25] that the different most detrimental Eves that overheard different parts of one coding block of the typical LU's messages can collude[2], which is different from ours in this work.

Moreover, the per-LU net ESR is defined in this paper to measure the overall system secrecy rate performance, as the downlink ESR improvement comes at the cost of the uplink spectral efficiency used for CSI feedback. Specifically, the per-LU net ESR is defined as

$$R_{\text{Net}} = R_{sec} - B/T_c = [R_u - R_e]^+ - B/T_c, \tag{22}$$

where T_c is specifically defined as the number of each LU's downlink symbols that experience the same channel fading. Similar to R_{sec}, it is mathematically intractable to obtain the exact result of R_{Net}. Instead, we can obtain a lower bound on R_{Net} as

$$R_{\text{Net}} \geq R_{\text{Net}}^L \triangleq \left[R_u^L - R_e\right]^+ - B/T_c. \tag{23}$$

In the following section, we will consider the optimization of the number of feedback bits of each LU for one coherent time period to maximize the per-LU net ESR.

[2]Recall that we consider delay-tolerant traffics and thus the coding block can be sufficiently long, and the most detrimental Eve in [25] was determined by taking consideration of the instantaneous fast fading.

IV. The Optimization of the Number of Feedback Bits Given the Power Allocation Coefficient

In this section, we consider maximizing the per-LU net ESR given a fixed power allocation coefficient by optimizing feedback bits allocation of each LU. Since the exact analytical result of the per-LU net ESR R_{Net} is unknown, we take the obtained lower bound R_{Net}^L in (23) as the objective. In addition, it is very inconvenient to directly consider R_{Net}^L as the objective function. Instead, we define $\tilde{R}_{\text{Net}}^L \triangleq R_u^L - R_e - B/T_c$. We first note that $\tilde{R}_{\text{Net}}^L$ is a continuous function of B without the non-negative integer constraint on B. Moreover, the optimal B depends on the concrete power allocation coefficient ϕ. Thus, before obtaining the optimal solution of ϕ in the next section and without causing any confusion, we will relax the integer B to be a real non-negative number throughout in this and next sections. The practical solution of optimized B can be determined after the optimization of ϕ. Then, the original feedback optimization problem can be transformed as

$$\max_{B \in \mathcal{R}^+ \cup \{0\}} \quad \tilde{R}_{\text{Net}}^L. \tag{24}$$

Before solving this problem, we firstly need to investigate the convexity/concavity property of $\tilde{R}_{\text{Net}}^L$ as a function of B. Since R_e is not related to B, we can obtain

$$\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B} = \frac{\partial R_u^L}{\partial B} - \frac{1}{T_c}, \ \frac{\partial^2 \tilde{R}_{\text{Net}}^L}{\partial B^2} = \frac{\partial^2 R_u^L}{\partial B^2}. \tag{25}$$

The properties of the first-order and the second-order derivatives of R_u^L w.r.t. B are given in the following lemma.

Lemma 2: The first-order derivative of R_u^L is given by

$$\frac{\partial R_u^L}{\partial B} = \frac{\delta}{M-1} \mathbb{E}\left\{ \frac{A_1 Y}{\delta A_1 Y + r_{b_0,k}^{\alpha} I_{u,k}} - \frac{(A_1 - A_2) Y}{\delta (A_1 - A_2) Y + A_2 (X+Y) + r_{b_0,k}^{\alpha} I_{u,k}} \right\} \tag{26}$$

$$= \int_0^1 (1-x)^{M-2} \left[1 + \Xi\left(\frac{x}{\delta A_1 (1-x)} \right) \right]^{-1} \mathrm{d}x - \frac{\delta (A_1 - A_2)}{\delta (A_1 - A_2) + A_2} \times \int_0^1 \frac{(1-x)^{M-2}}{1 + \frac{A_2}{\delta(A_1 - A_2) + A_2} \frac{x}{1-x}} \left[1 + \Xi\left(\frac{x}{\left[\delta(A_1 - A_2) + A_2 \right] (1-x)} \right) \right]^{-1} \mathrm{d}x, \tag{27}$$

and the second-order derivative of R_u^L is given by $\frac{\partial^2 R_u^L}{\partial B^2} = \mathbb{E}\left\{ g_1(B) g_2(B) \right\}$ with

$$g_1(B) = -\frac{\ln 2}{(M-1)^2} \left[\frac{A_1 Y}{A_1 Y + 2^{\frac{B}{M-1}} r_{b_0,k}^{\alpha} I_{u,k}} - \frac{(A_1 - A_2) Y}{(A_1 - A_2) Y + 2^{\frac{B}{M-1}} \left[A_2 (X+Y) + r_{b_0,k}^{\alpha} I_{u,k} \right]} \right] \tag{28}$$

$$g_2(B) = 1 - \left[\frac{A_1 Y}{A_1 Y + 2^{\frac{B}{M-1}} r_{b_0,k}^{\alpha} I_{u,k}} + \frac{(A_1 - A_2) Y}{(A_1 - A_2) Y + 2^{\frac{B}{M-1}} \left[A_2 (X+Y) + r_{b_0,k}^{\alpha} I_{u,k} \right]} \right], \tag{29}$$

where the coefficients A_1, A_2 are as defined in *Theorem* 1, $I_{u,k}$ is given by (7), and the function

$\Xi(z)$ is given by (13). X and Y are two independent Gamma-distributed RVs as defined in (47).

Moreover, we have the following results: (a) There exits at most one finite positive root of the equation $\frac{\partial^2 R_u^L}{\partial B^2} = 0$ (denoted as B_0), such that

$$\frac{\partial^2 R_u^L}{\partial B^2} \geq 0 \quad \text{for} \quad 0 \leq B \leq B_0 \quad \text{and} \quad \frac{\partial^2 R_u^L}{\partial B^2} < 0 \quad \text{for} \quad B > B_0. \tag{30}$$

(b) If the equation $\frac{\partial^2 R_u^L}{\partial B^2} = 0$ does not have root in $B \in (0, \infty)$,

$$\frac{\partial^2 R_u^L}{\partial B^2} \leq 0 \quad \text{for} \quad B \geq 0. \tag{31}$$

Proof: See Appendix C. □

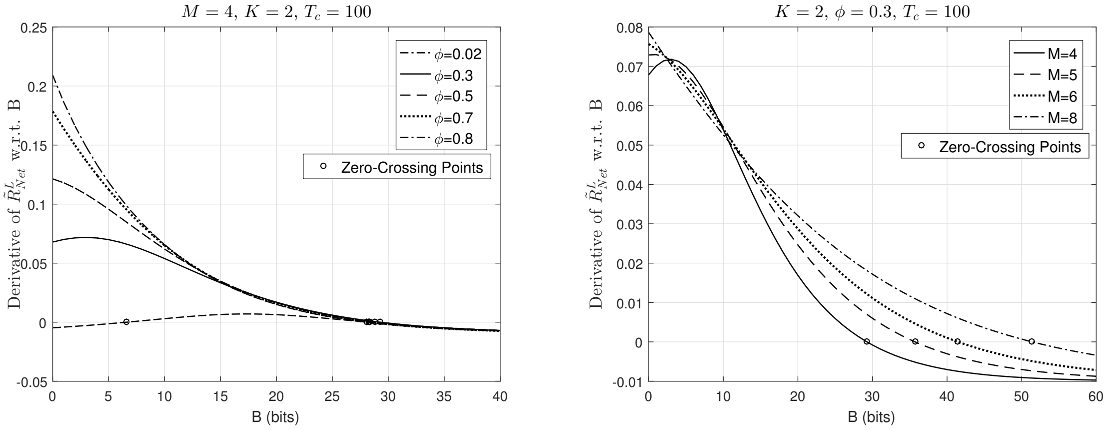
As we will see from the numerical results below that which situation ((a) or (b)) it is depends on the concrete system parameter setting. It can be easily observed from (26) that $\frac{\partial R_u^L}{\partial B} > 0$ for $B \in [0, \infty)$ and $\lim_{B\to\infty} \frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B} = -\frac{1}{T_c}$. Then, the properties of $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B}$ given in the following corollary follow easily from *Lemma* 2 by noticing the relation that $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B} = \frac{\partial R_u^L}{\partial B} - \frac{1}{T_c}$.

Corollary 3: When the equation $\frac{\partial^2 R_u^L}{\partial B^2} = 0$ does have a root $B_0 > 0$, $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B}$ is a convex increasing function of B for $0 \leq B \leq B_0$ and is a concave decreasing function of B for $B > B_0$. When the equation $\frac{\partial^2 R_u^L}{\partial B^2} = 0$ does not have any root in $(0, +\infty)$, $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B}$ is a concave decreasing function of B for $B \in [0, \infty)$.

According to *Lemma* 2 and *Corollary* 3, when $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B}\Big|_{B=0} = \frac{\partial R_u^L}{\partial B}\Big|_{B=0} - \frac{1}{T_c} > 0$, the equation $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B} = 0$ has an unique root in $(0, \infty)$ which is the global optimal solution to (24). Moreover, for a given fixed ϕ, it is possible that $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B}\Big|_{B=0} < 0$ holds and at the same time the equation $\frac{\partial^2 \tilde{R}_{\text{Net}}^L}{\partial B^2} = 0$ has an unique root $B_0 \in (0, \infty)$. In this case, there are two local maxima of $\tilde{R}_{\text{Net}}^L$ in $[0, \infty)$ at $B = 0$ and $B = b^* > 0$ which is given by the larger root of the equation $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B} = 0$ of the two roots in $(0, \infty)$. Then, the global optimal B (denoted as B_{real}^*) can be determined by comparing the values of $\tilde{R}_{\text{Net}}^L$ at $B = 0$ and $B = b^*$. For both scenarios, we denote *the largest root* of the equation $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B} = 0$ as b^*. Moreover, we can obtain $\frac{\partial R_u^L}{\partial B}\Big|_{B=0}$ from (26) as

$$\begin{aligned}
\frac{\partial R_u^L}{\partial B}\Big|_{B=0} &= \mathbb{E}\left[\frac{\frac{1}{M-1}A_1 Y}{A_1 Y + r_{b_0,k}^\alpha I_{u,k}} - \frac{\frac{1}{M-1}(A_1 - A_2)Y}{A_1 Y + A_2 X + r_{b_0,k}^\alpha I_{u,k}}\right] \\
= & \int_0^1 (1-x)^{M-2} \mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{I_{u,k}}\left(\frac{r_{b_0,k}^\alpha x}{A_1(1-x)}\right)\right\} \mathrm{d}x - \frac{A_1 - A_2}{A_1} \\
& \times \int_0^1 \frac{(1-x)^{M-2}}{1 + \frac{A_2}{A_1}\frac{x}{1-x}} \mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{I_{u,k}}\left(\frac{r_{b_0,k}^\alpha x}{A_1(1-x)}\right)\right\} \mathrm{d}x, \qquad (32) \\
= & \int_0^1 \frac{(1-x)^{M-2}}{1 + \Xi\left(\frac{x}{A_1(1-x)}\right)} \mathrm{d}x - \frac{A_1 - A_2}{A_1} \int_0^1 \frac{(1-x)^{M-2}}{1 + \frac{A_2}{A_1}\frac{x}{1-x}} \frac{1}{1 + \Xi\left(\frac{x}{A_1(1-x)}\right)} \mathrm{d}x \triangleq \epsilon(\phi), \qquad (33)
\end{aligned}$$

where (32) is obtained by using [20, *Lemma* 2], and (33) is obtained by using the result in (76). With (33), the values of $\epsilon(\phi)$ and $1/\epsilon(\phi)$ for different sets of system parameters can be efficiently



(a) $M = 4$, $K = 2$ and $T_c = 100$ with different ϕs

(b) $K = 2$, $\phi = 0.3$ and $T_c = 100$ with different Ms

Fig. 1. The first-order derivative of $\tilde{R}^L_{\text{Net}}$ w.r.t. B according to (27).

calculated and given in Table I and II. We can see from these numerical results that, the condition that $\frac{\partial \tilde{R}^L_{\text{Net}}}{\partial B}\big|_{B=0} > 0$ can be generally satisfied with $T_c \geq 100$ for some typical system settings. Then, there is a global optimal solution to the problem in (24). Therefore, in the following we will only focus on the scenario where the condition $T_c > \frac{1}{\epsilon(\phi)}$ is satisfied, and thus $B^*_{\text{real}} = b^*$ is the global optimal solution.

TABLE I. $M = 4$, $\alpha = 4$.

$\epsilon(\phi)$ $\left[\frac{1}{\epsilon(\phi)}\right]$	$K = 2$	$K = 3$
$\phi = 0.3$	0.0779 [12.84]	0.0271 [36.92]
$\phi = 0.5$	0.1313 [7.61]	0.0459 [21.78]
$\phi = 0.8$	0.2190 [4.57]	0.0764 [13.09]

TABLE II. $M = 6$, $\alpha = 4$.

$\epsilon(\phi)$ $\left[\frac{1}{\epsilon(\phi)}\right]$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
$\phi = 0.3$	0.0856[11.68]	0.0434[23.02]	0.0220[45.49]	0.0089[112.12]
$\phi = 0.5$	0.1450[6.90]	0.0734[13.62]	0.0372[26.87]	0.0151[66.25]
$\phi = 0.8$	0.2422[4.13]	0.1212[8.25]	0.0614[16.30]	0.0249[40.18]

Fig. 1(a), Fig. 1(b) and Fig. 2 show the first-order derivative of $\tilde{R}^L_{\text{Net}}$ w.r.t. B, $\frac{\partial \tilde{R}^L_{\text{Net}}}{\partial B}$, for various system setups. Specifically, Fig. 1(a) shows the numerical results for the systems with $M = 4$, $K = 2$, $T_c = 100$ and the different ϕs. Fig. 1(b) and Fig. 2 show the numerical results for the systems with $\phi = 0.3$, $T_c = 100$ and different combinations of M and K. It can be seen from the figures that the analytical properties of $\frac{\partial \tilde{R}^L_{\text{Net}}}{\partial B}$ obtained in *Lemma* 2 and *Corollary* 3 are all reflected in the figures.

As we have mentioned above that $\frac{\partial \tilde{R}^L_{\text{Net}}}{\partial B}\big|_{B=0} = \frac{\partial R^L_u}{\partial B}\big|_{B=0} - \frac{1}{T_c} > 0$, or equivalently $T_c > \frac{1}{\epsilon(\phi)}$, is a sufficient condition for $B^*_{\text{real}} = b^*$ to hold, where $\epsilon(\phi)$ in (33) is given by the analytical single-

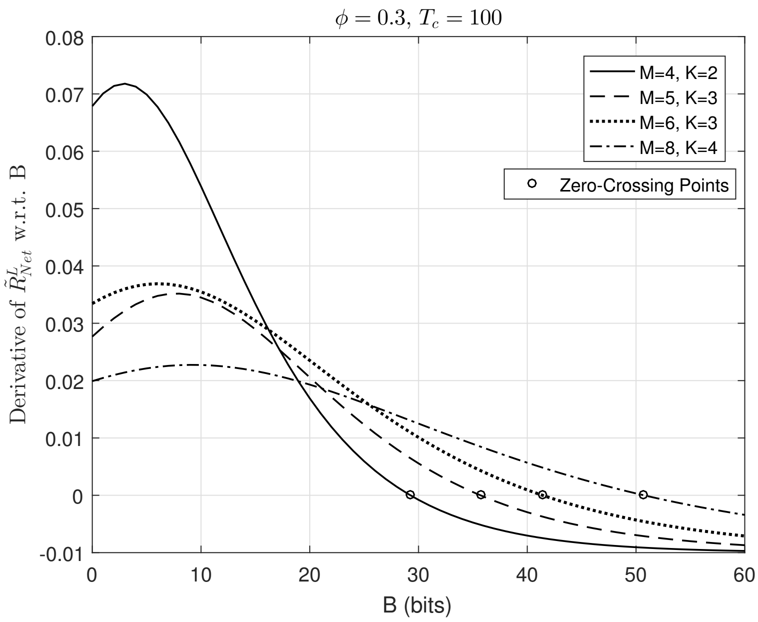


Fig. 2. The first-order derivative of $\tilde{R}_{\text{Net}}^{L}$ w.r.t. B according to (27) for different pairs of (M, K) when $\phi = 0.3$ and $T_c = 100$.

variable integral form. Unfortunately, an explicit sufficient and necessary condition is rather difficult to obtain. Moreover, in order to further optimize the power allocation coefficient in an analytical way, an explicit closed-form expression of b^* (as a function of ϕ) is required for all scenarios discussed above. Although the exact result of b^* can be obtained *numerically* from the equation $\frac{\partial R_u^L}{\partial B} - \frac{1}{T_c} = 0$ with the exact result of $\frac{\partial R_u^L}{\partial B}$ given by (27), it appears intractable to obtain an exact closed-form expression of b^*. Thus, we would like to develop some analytical approximation on b^*. We first develop some upper and lower bounds on $\frac{\partial R_u^L}{\partial B}$ and $\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial B}$.

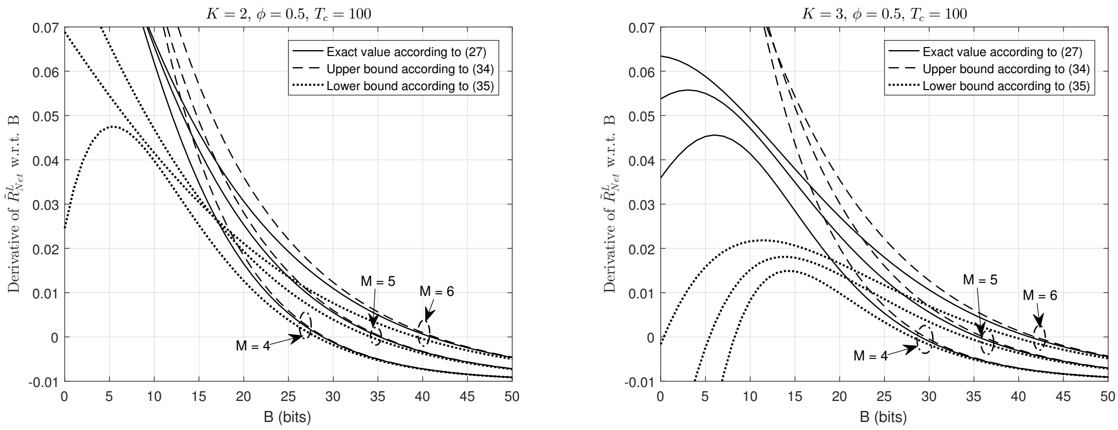
Theorem 3: Define the following two functions of variable B as

$$D_{er}^{up}(B) \triangleq \begin{cases} \left(\frac{\delta A_1}{\phi}\right)^{\frac{2}{\alpha}} A_3^{-1} \beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right), & 0<\phi<\frac{K}{M-K+1} \\ \left(\frac{\delta A_1}{\phi}\right)^{\frac{2}{\alpha}} A_3^{-1} \beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right) \\ \quad -\frac{\delta(A_1-A_2)}{[\delta(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}}} \frac{\beta\left(1-\frac{2}{\alpha}, M+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}} A_3}, & \frac{K}{M-K+1} \leq \phi \leq 1 \end{cases}, \tag{34}$$

and

$$D_{er}^{low}(B) \triangleq \begin{cases} \int_0^1 \frac{(\delta A_1)^{\frac{2}{\alpha}}(1-x)^{M-2}}{(\delta A_1)^{\frac{2}{\alpha}}+\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}} A_3} \mathrm{d}x - \frac{\delta(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)}{[\delta(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}} \phi^{\frac{2}{\alpha}} A_3}, & 0<\phi<\frac{K}{M-K+1} \\ \int_0^1 \frac{(\delta A_1)^{\frac{2}{\alpha}}(1-x)^{M-2}}{(\delta A_1)^{\frac{2}{\alpha}}+\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}} A_3} \mathrm{d}x, & \frac{K}{M-K+1} \leq \phi \leq 1 \end{cases}, \tag{35}$$

where $\delta = 2^{-\frac{B}{M-1}}$, the coefficients A_1, A_2 are as given in *Theorem* 1 and A_3 is as given in *Theorem* 2. $\beta(a, b)$ is a beta function with parameters (a, b). When the system parameters other than B



(a) $K = 2$, $\phi = 0.5$ (b) $K = 3$, $\phi = 0.8$

Fig. 3. The first-order derivative of $\tilde{R}^L_{\text{Net}}$ w.r.t. B according to (27) with $\phi = 0.5$ and $T_c = 100$ for different combinations of M and K, as well as the corresponding upper bound in (34) and lower bound in (35).

are fixed, $D^{up}_{er}(B)$ and $D^{low}_{er}(B)$ are respectively an upper bound and a lower bound on $\frac{\partial R^L_u}{\partial B}$, i.e., $D^{low}_{er}(B) \leq \frac{\partial R^L_u}{\partial B} \leq D^{up}_{er}(B)$ and $D^{low}_{er}(B) - \frac{1}{T_c} \leq \frac{\partial \tilde{R}^L_{\text{Net}}}{\partial B} \leq D^{up}_{er}(B) - \frac{1}{T_c}$.

Moreover, $D^{up}_{er}(B)$ is a monotonic decreasing function of B in $[0, +\infty)$. $D^{low}_{er}(B)$ is a monotonic decreasing function of B in $[0, +\infty)$ when $\frac{K}{M-K+1} \leq \phi \leq 1$. However, when $0 < \phi < \frac{K}{M-K+1}$, $D^{low}_{er}(B)$ is not necessarily to be a monotonic function of B in $[0, +\infty)$.

Proof: See Appendix D. □

Then, an upper bound and a lower bound on b^* can be obtained in the following lemma by using *Theorem* 3.

Lemma 3: When both of the equations $D^{up}_{er}(B) - \frac{1}{T_c} = 0$ and $D^{low}_{er}(B) - \frac{1}{T_c} = 0$ of variable B have root in $(0, \infty)$ for a given set of system parameters other than B, an upper bound (denoted as b^*_u) and a lower bound (denoted as b^*_l) on b^* can be obtained respectively from the unique root of the equation $D^{up}_{er}(B) - \frac{1}{T_c} = 0$ and the largest root of the equation $D^{low}_{er}(B) - \frac{1}{T_c} = 0$, i.e., $b^*_l < b^* < b^*_u$. Moreover, we have $\lim\limits_{T_c \to +\infty} b^*_u = \lim\limits_{T_c \to +\infty} b^*_l = b^* \to \infty$.

Proof: See Appendix E. □

Remark 2: Based on *Lemma* 3, when the system parameters other than B are given, both b^*_u and b^*_l can be respectively found analytically or numerically by using the expressions of $D^{up}_{er}(B)$ in (34) and $D^{low}_{er}(B)$ in (35). Moreover, the result with *Lemma* 3 illustrates that the gap between the obtained upper bound and lower bound asymptotically converges to zero as $T_c \to +\infty$.

Fig. 3(a) and Fig. 3(b) show the first-order derivative of $\tilde{R}^L_{\text{Net}}$ w.r.t. B according to (27) for various system setups, as well as the corresponding upper bound in (34) and lower bound in (35). The numerical results show that the bounds are tight around the zero-crossing points, which corresponding to the upper and the lower bound on the optimal feedback bits B^*_{real} given fixed ϕ

and T_c. All numerical results are consistent with the analytical results in *Theorem* 3.

We note that, according to the expressions of $D_{er}^{up}(B)$ in (34) and $D_{er}^{low}(B)$ in (35) given in *Theorem* 3 and the results in *Lemma* 3, it is still intractable to obtain an explicit closed-form expression of neither of the bounds b_u^* and b_l^* for a general ϕ, except for b_u^* when $0 < \phi < \frac{K}{M-K+1}$. Moreover, we notice that for $0 < \phi < \frac{K}{M-K+1}$,

$$\frac{\partial R_u^L}{\partial B} \leq D_{er}^{up}(B) < c_1 \delta^{\frac{2}{\alpha}} + c_2 \delta \triangleq f(B), \tag{36}$$

where the coefficients of δ are given by $c_1 = \phi^{-\frac{2}{\alpha}} (A_1)^{\frac{2}{\alpha}} A_3^{-1} \beta\left(1 - \frac{2}{\alpha}, M - 1 + \frac{2}{\alpha}\right)$ and $c_2 = \phi^{-\frac{2}{\alpha}} (A_2 - A_1) A_1^{\frac{2}{\alpha}-1} A_3^{-1} \beta\left(1 - \frac{2}{\alpha}, M + \frac{2}{\alpha}\right)$. Moreover, according to *Lemma* 3 we have $\lim\limits_{T_c \to +\infty} b_u^* = b^* \to \infty$ and $\lim\limits_{T_c \to +\infty} \delta = 0$. Thus, when $T_c \to +\infty$, the term $f(B)$ can be arbitrarily close to $\tilde{f}(B) \triangleq c_1 \delta^{\frac{2}{\alpha}}$ in a neighborhood of B_{real}^*. Denote the root of the equation $\tilde{f}(B) - \frac{1}{T_c} = 0$ as $\tilde{b}_u^*$. Then, for any given $\varepsilon > 0$, there always exists a finite $T_0(\varepsilon)$, such that $b_u^* - B_{\text{real}}^* < \tilde{b}_u^* - B_{\text{real}}^* < \varepsilon$ when $T_c > T_0(\varepsilon)$. We note that, when $0 < \phi < \frac{K}{M-K+1}$, $\tilde{f}(B) = D_{er}^{up}(B)$ exactly. Therefore, we can obtain in the following theorem an analytical asymptotic approximation on B_{real}^* based on the expression of $D_{er}^{up}(B)$ when $0 < \phi < \frac{K}{M-K+1}$ as $T_c \to +\infty$.

Theorem 4: When T_c is sufficiently large to satisfy the condition that $T_c > \frac{1}{\epsilon(\phi)}$, B_{real}^* can be approximated as $B_{\text{real}}^* \approx B_{ax}^*(\phi)$ given a fixed ϕ, where $\epsilon(\phi)$ is given by (33) and

$$B_{ax}^*(\phi) \triangleq \frac{\alpha}{2}(M-1) \log_2 \left\{ \phi^{-\frac{2}{\alpha}} A_1^{\frac{2}{\alpha}} A_3^{-1} \beta\left(1 - \frac{2}{\alpha}, M - 1 + \frac{2}{\alpha}\right) T_c \right\}. \tag{37}$$

A_1 and A_3 are respectively defined as in *Theorem* 1 and *Theorem* 2.

The numerical results in Fig. 4, Fig. 6, Fig. 7 and Fig. 8 in Section VI will illustrate that this approximation on B_{real}^* is very accurate. One important observation is that, as R_u^L and R_u, neither B_{real}^* nor B_{ax}^* is relevant to λ_b. Moreover, some properties of B_{ax}^* as the function of some main system parameters can be developed in the following corollary.

Corollary 4: When the other system parameters remain fixed, $B_{ax}^*(\phi)$ given by *Theorem* 4 is monotonic decreasing w.r.t. ϕ and is monotonic increasing w.r.t. T_c.

Proof: See Appendix F. □

Remark 3: At this stage, we note that it appears mathematically intractable to strictly prove the monotonic properties of B_{ax}^* w.r.t. some other system parameters, such as α and M. To show this, we can observe from some numerical results in Section VI that B_{ax}^* monotonically increases w.r.t. $\alpha > 2$ and M. In contrast, we notice it is easy to prove that the term $\log_2(\beta\left(1 - \frac{2}{\alpha}, M - 1 + \frac{2}{\alpha}\right))$ within the expression of $B_{ax}^*(\phi)$ monotonically decreases w.r.t. both α and M.

V. The Optimization of Power Allocation Coefficient

In this section, we consider the optimization of the power allocation coefficient between MU message-bearing signals and AN to maximize the per-LU net ESR R_{Net}. Unfortunately, as we have illustrated in Section III that it is mathematically intractable to obtain an exact closed-form result of R_{Net}. Moreover, by using the obtained analytical lower bound R_{Net}^L of R_{Net}, we can definitely obtain sub-optimal solution(s) of ϕ by using some numerical method. However, due to the complicated form of R_{Net}^L given by *Theorem* 1 and 2, we find it still mathematically intractable to guarantee that the global optimal solution can be obtained based on R_{Net}^L. Therefore, we will first develop another analytical lower bound on R_{Net} as the objective function in the following theorem, which can be proved to be concave in ϕ.

Theorem 5: R_{sec} can also be lower-bounded as

$$R_{sec} \geq \hat{R}_{sec}^L \triangleq \left[\hat{R}_u^L - R_e^U\right]^+, \tag{38}$$

where $\hat{R}_u^L$ is a lower bound on R_u^L (and also R_u) which is given by

$$\hat{R}_u^L = \log_2 e \int_0^\infty \frac{(1+A_1\delta z)^{-(M-1)} - (1+A_2 z)^{-1}\big(1+[A_1\delta + A_2(1-\delta)]z\big)^{-(M-1)}}{z\left(1+\tilde{\Xi}(z)\right)} \mathrm{d}z \tag{39}$$

with $\tilde{\Xi}(z) \triangleq -1 + e^{-z} + z^{\frac{2}{\alpha}}\gamma\left(1-\frac{2}{\alpha}, z\right)$, and A_1, A_2 are defined in *Theorem* 1. R_e^U is an upper bound on R_e which is given by

$$\begin{aligned} R_e^U = \frac{\log_2 e}{K}\Bigg\{ &\int_0^\infty \frac{1-\left[1+\frac{\lambda_b}{\lambda_e}\Gamma\left(1-\frac{2}{\alpha}\right)z^{\frac{2}{\alpha}}\right]^{-1}}{z}\left(1+\frac{z}{M}\right)^{-M}\mathrm{d}z - \ln M - \ln\left(\frac{1-\phi}{M-K}\right) \\ &- \int_0^\infty \frac{1-\left[1+\frac{\lambda_b}{\lambda_e}A_3(\phi z)^{\frac{2}{\alpha}}\right]^{-1}}{z}\left(1+\frac{1-\phi}{M-K}z\right)^{-(M-K)}\mathrm{d}z + \psi(M) - \psi(M-K)\Bigg\}, \end{aligned} \tag{40}$$

where A_3 is defined in *Theorem* 2. $\psi(\cdot)$ is the Euler's digamma function given by $\psi(m) = \psi(1) + \sum_{l=1}^{m-1}\frac{1}{l}$ for a positive integer m with $-\psi(1) = 0.57721566490\ldots$ being the Euler-Mascheroni constant [28, 8.367.1]. Then, another lower bound on the per-LU net ESR is given by

$$R_{\text{Net}} \geq \hat{R}_{\text{Net}}^L \triangleq \left[\hat{R}_u^L - R_e^U\right]^+ - B/T_c. \tag{41}$$

Proof: See Appendix G. □

Similar as the problem of feedback optimization, we define $\check{R}_{\text{Net}}^L \triangleq \hat{R}_u^L - R_e^U - B/T_c$ as the objective function. Then, the power allocation optimization problem can be formulated as

$$\max_{0<\phi\leq 1} \quad \check{R}_{\text{Net}}^L\big|_{B=B_{ax}^*(\phi)}, \tag{42}$$

where we have substituted the analytical approximation on the optimized number of feedback bits $B_{ax}^{*}(\phi)$ into the expression of $\check{R}_{\text{Net}}^{L}$. Before solving this problem, we first investigate the convexity/concavity property of $\check{R}_{\text{Net}}^{L}$ w.r.t. ϕ and the result is given in the following lemma.

Lemma 4: When T_c is sufficiently large and satisfies the condition that $T_c > \frac{1}{\epsilon(\phi)}$, where $\epsilon(\phi)$ is given by (33), $\check{R}_{\text{Net}}^{L}\big|_{B=B_{ax}^{*}(\phi)}$ is a concave function of ϕ for $0 \leq \phi \leq 1$.

Proof: See Appendix H. □

It follows from *Lemma* 4 that the global optimal solution to (42) is given by the unique root of the equation $\frac{\mathrm{d}\check{R}_{\text{Net}}^{L}[\delta^*,\phi]}{\mathrm{d}\phi} = 0$, which is given by (131) in Appendix H. Unfortunately, it seems mathematically intractable to obtain the closed-form expressions of the terms $\frac{\partial \check{R}_{\text{Net}}^{L}[\delta^*,\phi]}{\partial \phi}$ and $\frac{\partial \check{R}_{\text{Net}}^{L}[\delta^*,\phi]}{\partial \delta^*}$ in (131). In contrast, we propose to obtain the global optimal $\phi^{\star}$ with arbitrary accuracy by using a bisection search method. Moreover, we can obtain the property of $\phi^{\star}$ as a function of T_c in the following corollary.

Corollary 5: When the system parameters other than ϕ, B and T_c remain fixed, $\phi^{\star}$ is monotonically decreasing in T_c in the region where T_c takes the sufficiently large value, and $\lim_{T_c \to \infty} \phi^{\star} = \phi_{\infty}^{\star}$ which is a constant value related to other system parameters.

Proof: See Appendix I. □

With the optimized power allocation coefficient $\phi^{\star}$, the exact optimal feedback bits $B_{\text{real}}^{\star}$ without integer constraint certainly can be obtained by numerically solving the equation $\frac{\partial R_u^L}{\partial B} = 0$ given by (27). Also, applying (37) with $\phi^{\star}$, the optimal feedback bits can be approximated as $B_{\text{real}}^{\star} \approx B_{ax}^{*}(\phi^{\star})$. The results of *Lemma* 3 support that, when T_c is sufficiently large, the exact value of $B_{\text{real}}^{\star}$ can be arbitrarily close to the approximation $B_{ax}^{*}(\phi^{\star})$. The practical optimal feedback bits $B^{\star}$ can be obtained by comparing the values of R_{Net}^{L} given by (23) with $B^{\star} \in \{\lfloor B_{\text{real}}^{\star} \rfloor, \lceil B_{\text{real}}^{\star} \rceil, 0\}$.

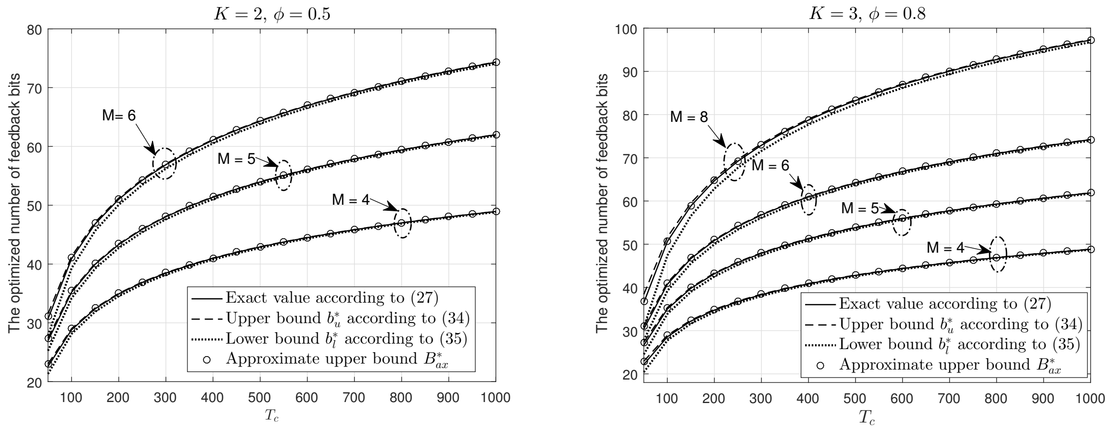
Corollary 6: When the system parameters other than B and ϕ remain fixed, $B_{\text{real}}^{\star} \approx B_{ax}^{*}(\phi^{\star})$ and $B^{\star}$ are monotonic increasing in T_c in the region where T_c takes sufficiently large value. Moreover, as $T_c \to \infty$, $B_{\text{real}}^{\star}$ and $B^{\star}$ go to infinity without bound.

Proof: The proof is easily completed by combing the results of *Theorem* 4, *Corollary* 5 and *Corollary* 4. □

According to *Corollary* 5, when $T_c \gg 1$ and the other parameters are given and fixed, we have the following approximation that

$$\begin{aligned} B_{\text{real}}^{\star} \approx B_{ax}^{*}(\phi^{\star}) &\approx \frac{\alpha}{2}(M-1)\log_2(T_c) + (M-1)\log_2\left[A_1 \beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)(\phi_{\infty}^{\star})^{-1} A_3^{-\frac{\alpha}{2}}\right] \\ &\approx \frac{\alpha}{2}(M-1)\log_2(T_c), \end{aligned} \tag{43}$$

where (43) follows from $\frac{\alpha}{2}\log_2(T_c) \gg \log_2\left[A_1\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)(\phi_{\infty}^{\star})^{-1}A_3^{-\frac{\alpha}{2}}\right]$. We observe that the optimal number of feedback bits scales linearly with the term $\frac{\alpha}{2}(M-1)$ and scales



(a) $K = 2$, $\phi = 0.5$ (b) $K = 3$, $\phi = 0.8$

Fig. 4. The exact values of the optimized number of feedback bits as a function of T_c obtained according to (27), and also the corresponding upper bound and the lower bound obtained according to (34) and (35) and the approximated upper bound obtained from (37) for different combinations of M, K and ϕ.

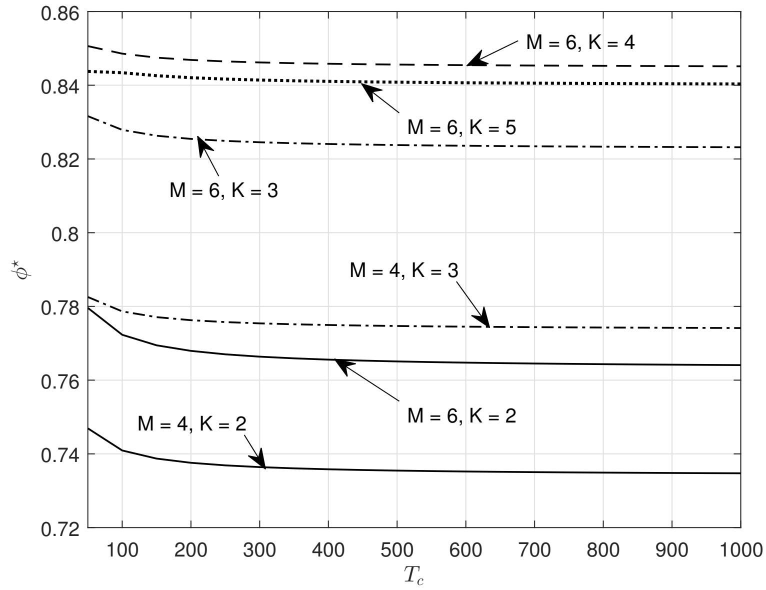


Fig. 5. The optimal $\phi^\star$ as a function of T_c for various combinations of M and K.

logarithmically with T_c. In the next section, we will verify our theoretical results by numerical results

VI. Numerical Results

In this section, numerical results are provided for various system setups to verify our main theoretical results obtained in the previous sections. If not specified, we assume the densities of BSs and Eves are $\lambda_b = 10^{-4}$ and $\lambda_e = 2 \times 10^{-4}$ respectively throughout this section. The path-loss exponent is set to be 4. We follow [19] and [20] to set the coherence time $T_c \geq 50$ to guarantee that the sufficient condition $T_c > \frac{1}{\epsilon(\phi)}$ in *Theorem* 4 can be generally satisfied.

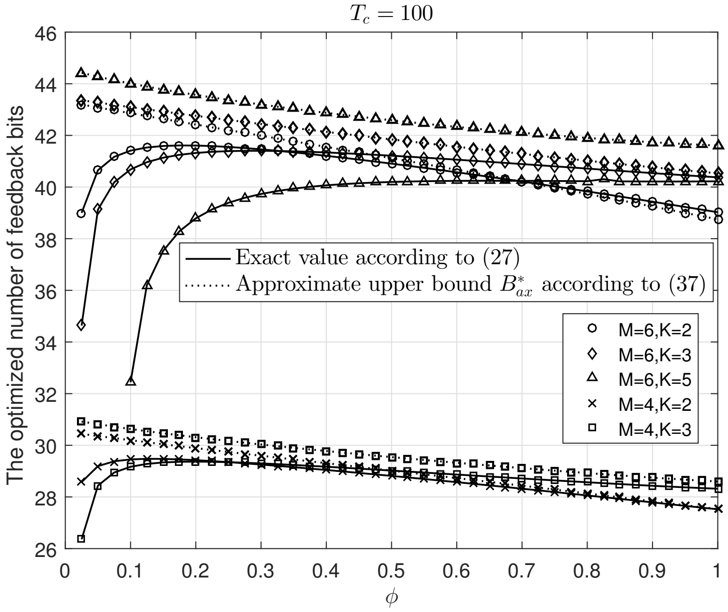


Fig. 6. The optimal number of feedback bits and the approximation given by (37) w.r.t. ϕ for different combinations of M and K with $T_c = 100$.

The exact values of the optimized number of feedback bits with respect to coherence time T_c that are obtained according to (27) are illustrated in Fig. 4 for different combinations of M, K and ϕ, and also the corresponding upper bound and the lower bound obtained according to (34) and (35) and the approximated upper bound obtained from (37). Specifically, Fig. 4(a) shows the case of $K = 2$ and Fig. 4(b) shows the case of $K = 3$. As we have explained in Section IV that, without optimization of power allocation coefficient ϕ, $B^*_{\text{real}} \in \mathcal{R}^+$ and $B^*_{ax} \in \mathcal{R}^+$ are considered in the numerical results rather than finding without the integer values. According to *Lemma* 3, we know that the gap between the upper and lower bounds converges to zero as T_c increases sufficiently large. This result is reflected in all curves in Fig. 4. Moreover, we can see that B^*_{ax} increases as M increases for a fixed T_c and increases logarithmically with T_c for all system setups considered, which is consistent with our analysis in Section IV. But very interestingly K affects B^*_{ax} very little.

Fig. 5 shows the optimized power allocation coefficient $\phi^\star$ w.r.t. T_c for various combinations of M and K obtained using a bisection search method due to *Lemma* 4. We can observe from the figure that the obtained $\phi^\star$ for each system setup decreases as T_c increases large and converges to a finite value, which verifies the results of *Corollary* 5. In addition, we find that $\phi^\star$ increases as the K increases when the system parameters other than K are fixed.

Fig. 6 shows the exact optimized number of feedback bits B^*_{real} and the approximation B^*_{ax} given by (37) w.r.t. ϕ with $T_c = 100$ for different combinations of M and K. We can observe from the figure that the approximation B^*_{ax} for each system setup decreases as ϕ increases large, which

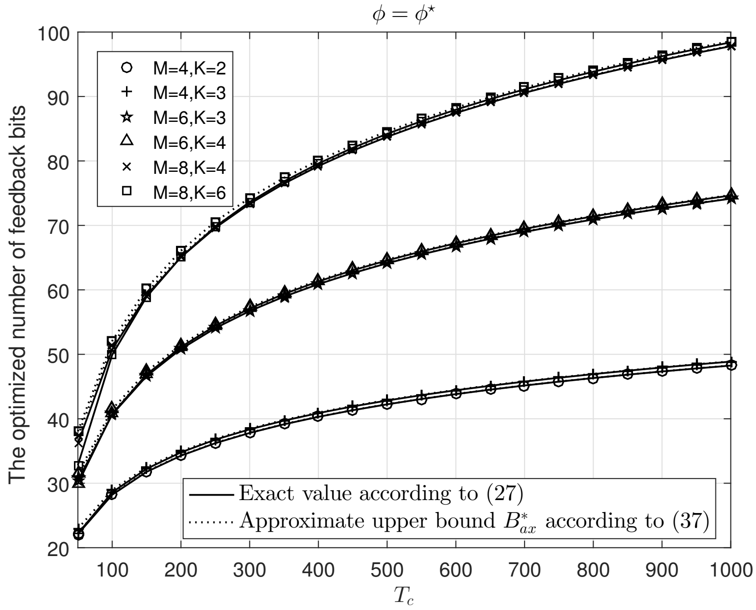


Fig. 7. $B_{\text{real}}^\star \approx B_{ax}(\phi^\star)$ as a function of T_c with $\phi = \phi^\star$ for different combinations of M and K.

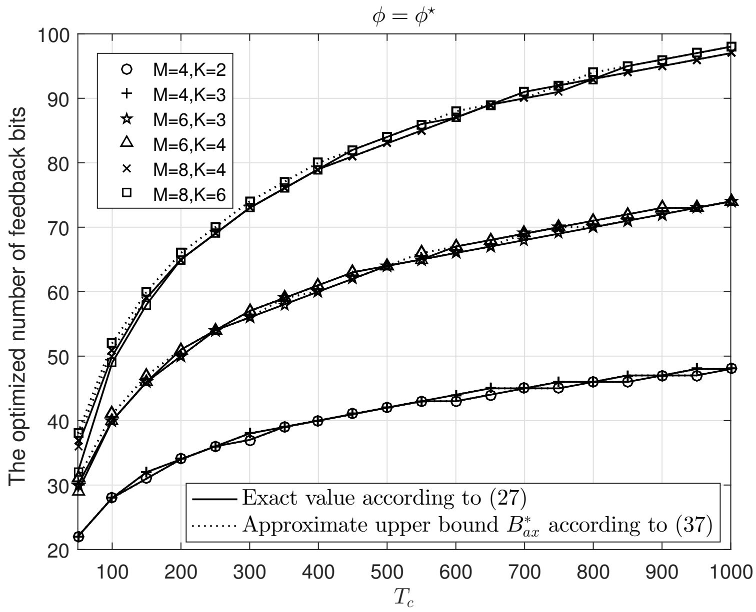
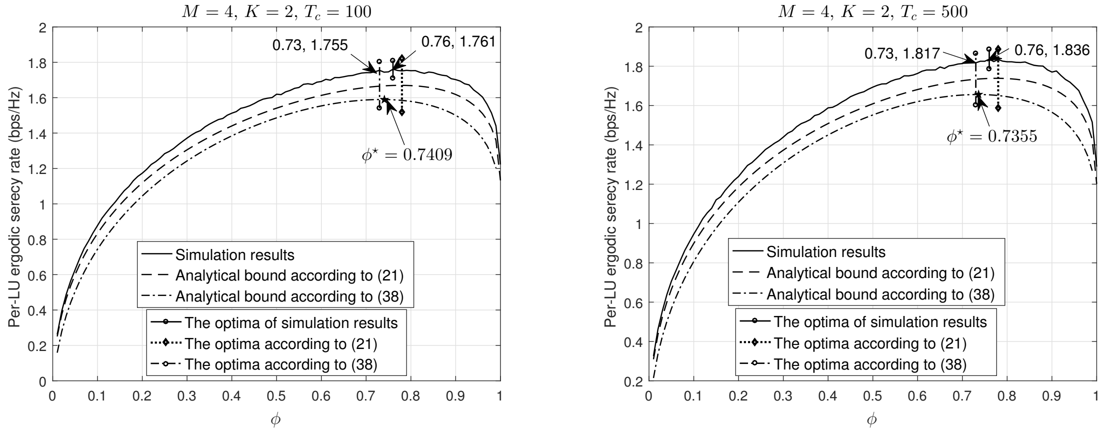


Fig. 8. The approximated $B^\star$ as a function of T_c with $\phi = \phi^\star$ for different combinations of M and K.

verifies our results in *Corollary* 4. Moreover, we can see that the gap between the exact result and the approximation for each system setup generally decreases as ϕ increases and becomes very small when ϕ is not very small. We know from the results in Fig. 5 that $\phi^\star > 0.7$ in general. Thus, the approximation B_{ax}^* can be very accurate for general system setups. This will be further verified by the following numerical results.

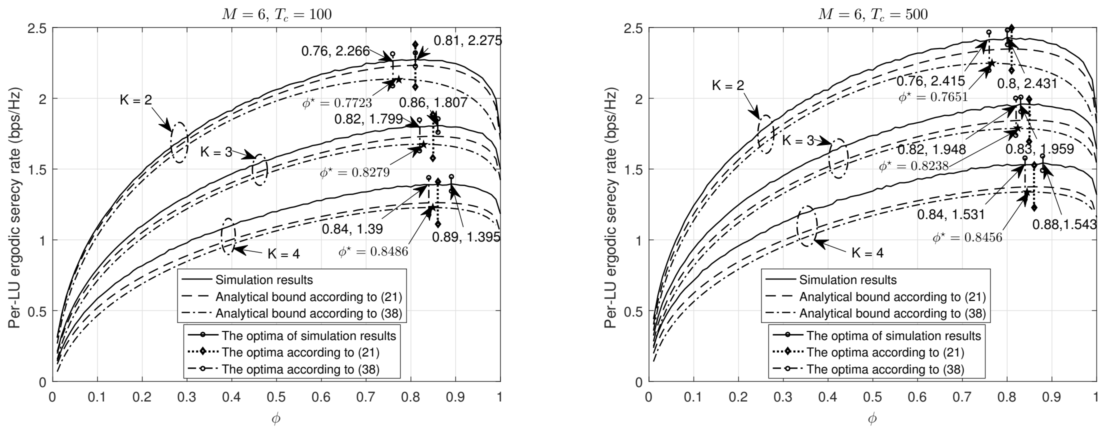
Fig. 7 shows respectively the exact optimized feedback bits $B_{\text{real}}^\star$ without integer constraint and the approximation $B_{\text{real}}^\star \approx B_{ax}^*$ as functions of T_c for different combinations of M and K, where



(a) $M = 4$, $K = 2$, $T_c = 100$

(b) $M = 4$, $K = 2$, $T_c = 500$

Fig. 9. Per-LU ESR as a function of ϕ with the optimized integer feedback bits obtained from B^*_{real} and $M = 4$, $K = 2$, as well as two analytical lower bounds in (21) and (38).



(a) $M = 6$, $T_c = 100$

(b) $M = 6$, $T_c = 500$

Fig. 10. Per-LU ESR as a function of ϕ with the optimized integer feedback bits obtained from B^*_{real} for $M = 6$ and different Ks, as well as two analytical lower bounds in (21) and (38).

the power allocation coefficient ϕ has been also optimized (i.e., $\phi = \phi^\star$). Fig. 8 shows curves of the corresponding practical integer values. We can see from the figures that $B^\star_{\text{real}}$ is very close to the approximation B^*_{ax} for all system setups, and the practical integer solutions of $B^\star$ obtained from $B^\star_{\text{real}}$ and B^*_{ax} are also very close to each other. All curves increase without bound as T_c increases large, which verify our results in *Corollary* 6. Moreover, similar to Fig. 4(a), we can see that $B^\star_{\text{real}}$ and B^*_{ax} increases as M increases for a fixed T_c and increases logarithmically with T_c.

Fig. 9 and Fig. 10 show the simulation results of the per-LU ESR as a function of ϕ with the optimized integer feedback bits obtained from B^*_{real} in Section IV given each ϕ for the different combinations of system parameters. For comparisons and verification of our analytical results, we also plot the corresponding two analytical lower bounds respectively given by (21) and (38) by using the results of *Theorem* 1, 2 and 5. The concrete parameters are shown in each figure and caption. We have also marked down the maximum of the simulation result and that corresponding

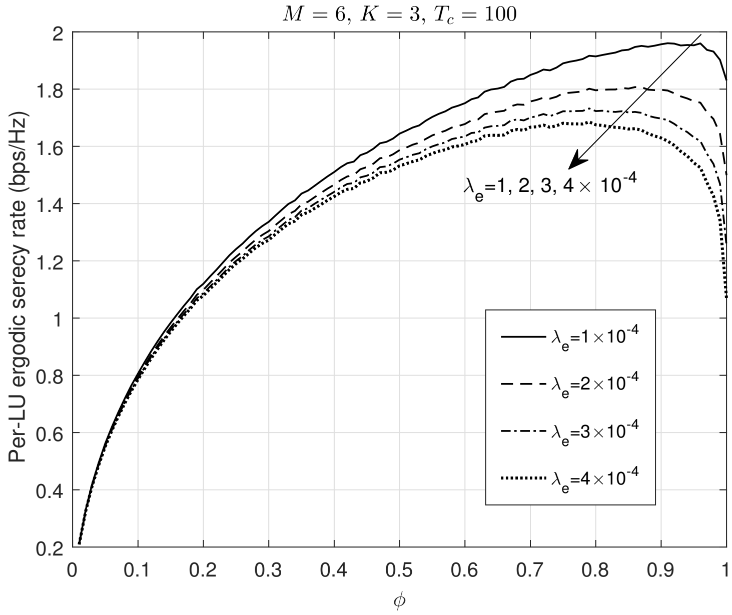


Fig. 11. The simulated per-LU ESR as a function of ϕ for different λ_es with the optimized integer feedback bits obtained from B^*_{real} and $M = 6$, $K = 3$ and $T_c = 100$.

to the optimized power allocation $\phi^\star$ obtained the proposed bisection search method in Section V on each curve of simulation. We can see from the figures that the analytical lower bound given by (21) can approximate the actual values well for all the system setups considered. In addition, the per-LU ESR achieved by the optimized $\phi^\star$ can be so close to the actual optimal that the gap between the two can be negligible (within 1 percent of the real optimal value) for all system parameters considered. Moreover, we can observe from all curves that, when the systems work without AN, the per-LU ESR performance is much worse than the optimal for all the system setups considered.

Fig. 11 compares the simulated per-LU ESR as a function of ϕ for the systems with different λ_es, where the optimized integer number of feedback bits B^*_{real} is obtained from $B^*_{ax}(\phi)$ given by *Theorem* 4 and $M = 6, K = 3, T_c = 100$. We can see that each per-LU ESR performance degrades as λ_e increases, which is consistent with the conclusion given by *Corollary* 2 about our analytical results on per-LU ESR.

Fig. 12 shows the exact optimized number of feedback bits without integer constraint $B^\star_{\text{real}}$ as a function of T_c which is obtained according to (27) with $\phi = \phi^\star$. For verification, we also plot the curves according to the upper bound in (34), the lower bound in (35) and the approximated upper bound in (37). Fig. 13 shows the optimized practical integer feedback bits corresponding to each curve in Fig. 12. We can see from the figures that both the optimized feedback bits without integer constraint and the corresponding practical integer solutions monotonically increases as α increase. Moreover, the lower bound, the upper bound and the approximation on the optimal feedback bits

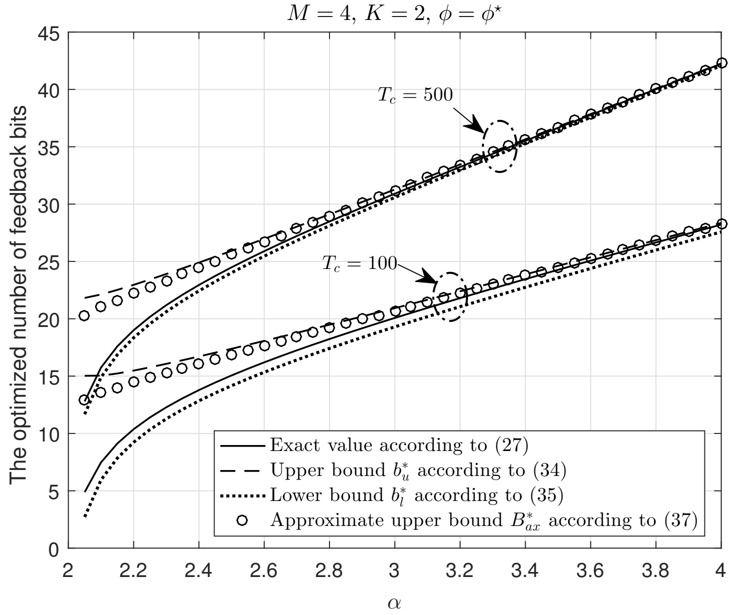


Fig. 12. The optimized number of feedback bits without integer constraint as a function of T_c with $\phi = \phi^\star$ which is obtained according to (27), and also those according to the upper bound and the lower bound in (34) and (35), and the approximated upper bound in (37).

become closer to the exact values as α becomes large.

VII. Conclusion

In this paper, we have proposed an analytical framework for the optimum design and secrecy performance of AN-aided multiuser secure transmission in practical FDD downlink multi-antenna random cellular systems considering limited CSI feedback and delay-tolerant traffics. Semi-closed-form analytical expressions of two lower bounds on the net ESR of each typical LU have been derived without assuming asymptotes for any system parameter for the first time. Given a fixed power allocation coefficient ϕ, a tight closed-form approximation on the optimal number of feedback bits has been developed that maximizes the first per-LU net ESR lower bound. Moreover, the convexity property of the second per-LU net ESR lower bound as a function of ϕ has been theoretically proved when the above said approximation on the optimal number of feedback bits was substituted into. Then, we have proposed a bisection search method to obtain the power allocation solution. We have also theoretically studied the monotonic properties of the optimized ϕ and the approximated optimal number of feedback bits as functions of some other main system parameters. From the results, it has been proved that, when the length of the coherence block T_c goes large, the optimal number of feedback bits scales linearly with the number of transmit antennas and path-loss exponent, and logarithmically with T_c. Numerical results presented have verified our main

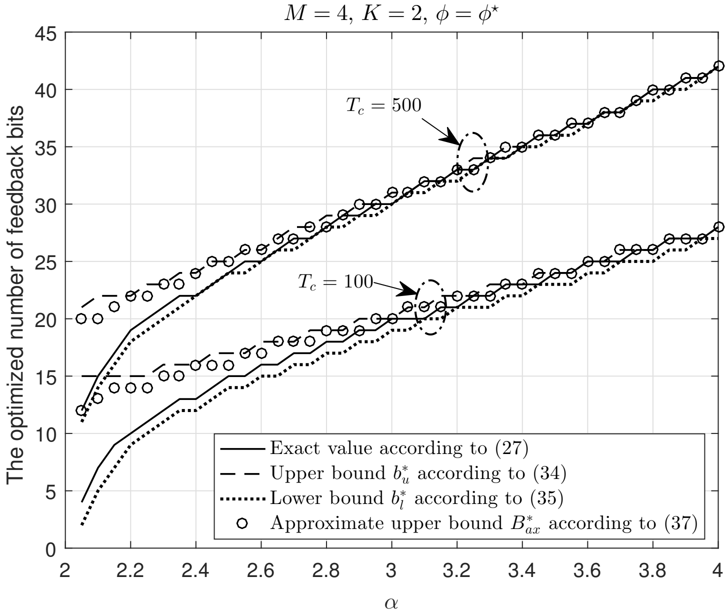


Fig. 13. The practical solution of the optimized number of feedback bits as a function of T_c with $\phi = \phi^\star$ which is obtained according to (27), and also those according to the upper bound and the lower bound in (34) and (35), and the approximated upper bound in (37).

theoretical results obtained.

APPENDIX

A. *Proof of Theorem 1*

First, using the result from [29, Appendix B], the numerator term $|\tilde{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2$ in (5) can be lower bounded as $|\tilde{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2 \geq \cos^2\theta_{b_0,k}|\hat{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2$, which can become very accurate as B increases large. Then, the ergodic rate of the typical LU can be lower bounded by substituting this result into (5) as

$$R_u \geq \mathbb{E}\Bigg\{\log_2\Bigg[1+\left(\frac{\phi}{K}\|\mathbf{h}_{b_0,k}\|^2\cos^2\theta_{b_0,k}|\hat{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2\right)\Bigg/$$
$$\left(\|\mathbf{h}_{b_0,k}\|^2\sin^2\theta_{b_0,k}\left[\frac{\phi}{K}\sum_{k'=1,k'\neq k}^{K}|\mathbf{e}_{b_0,k}\mathbf{w}_{b_0,k'}|^2+\frac{1-\phi}{M-K}\|\mathbf{e}_{b_0,k}\mathbf{Z}_{b_0}\|^2\right]+r_{b_0,k}^{\alpha}I_{u,k}\right)\Bigg]\Bigg\}$$
$$\geq \mathbb{E}\Bigg\{\log_2\Bigg[1+\left(\frac{\phi}{K}\|\mathbf{h}_{b_0,k}\|^2\cos^2\theta_{b_0,k}|\hat{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2\right)\Bigg/$$
$$\left(\|\mathbf{h}_{b_0,k}\|^2\sin^2\theta_{b_0,k}\mathbb{E}\left[\frac{\phi}{K}\sum_{k'=1,k'\neq k}^{K}|\mathbf{e}_{b_0,k}\mathbf{w}_{b_0,k'}|^2+\frac{1-\phi}{M-K}\|\mathbf{e}_{b_0,k}\mathbf{Z}_{b_0}\|^2\right]+r_{b_0,k}^{\alpha}I_{u,k}\right)\Bigg]\Bigg\}, \quad (44)$$

where (44) is obtained by using Jensen's inequality. According to [30, Lemma 1], we have $\|\mathbf{e}_{b_0,k}\mathbf{Z}_{b_0}\|^2 \sim \text{Beta}(1, M-2)$ and $|\hat{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2 \sim \text{Beta}(M-K+1, K-1)$, when $1 < K < M$. In addition,

$|\mathbf{e}_{b_0,k}\mathbf{w}_{b_0,k'}|^2 \sim \mathrm{Beta}(1, M-2)$ [12]. Then, it is easy to obtain

$$\mathbb{E}\left\{\frac{\phi}{K}\sum_{k'=1,k'\neq k}^{K}|\mathbf{e}_{b_0,k}\mathbf{w}_{b_0,k'}|^2+\frac{1-\phi}{M-K}\|\mathbf{e}_{b_0,k}\mathbf{Z}_{b_0}\|^2\right\}=\frac{\phi}{K}\frac{K-1}{M-1}+\frac{1-\phi}{M-K}\frac{M-K}{M-1}$$
$$=\frac{K-\phi}{K(M-1)}\triangleq A_1, \tag{45}$$
$$\mathbb{E}\left\{\frac{1}{|\hat{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2}\right\}=\int_0^1\frac{1}{x}\frac{x^{M-K}(1-x)^{K-2}}{B(M-K+1,K-1)}\mathrm{d}x=\frac{M-1}{M-K}. \tag{46}$$

Moreover, according to [13, *Lemma* 1], we can have

$$\|\mathbf{h}_{b_0,k}\|^2\cos^2\theta_{b_0,k}\overset{d.}{=}X+(1-\delta)Y;\ \|\mathbf{h}_{b_0,k}\|^2\sin^2\theta_{b_0,k}\overset{d.}{=}\delta Y, \tag{47}$$

where X and Y are two independent Gamma-distributed RVs, i.e., $X\sim\mathrm{Gamma}(1,1)$ and $Y\sim\mathrm{Gamma}(M-1,1)$. It follows that

$$R_u \geq \mathbb{E}\left\{\log_2\left(1+\frac{\frac{\phi}{K}\|\mathbf{h}_{b_0,k}\|^2\cos^2\theta_{b_0,k}}{\mathbb{E}\left\{\frac{1}{|\hat{\mathbf{h}}_{b_0,k}\mathbf{w}_{b_0,k}|^2}\right\}\left(A_1\|\mathbf{h}_{b_0,k}\|^2\sin^2\theta_{b_0,k}+r_{b_0,k}^{\alpha}I_{u,k}\right)}\right)\right\}, \tag{48}$$
$$= \mathbb{E}\left\{\log_2\left(1+\frac{A_2[X+(1-\delta)Y]}{A_1\delta Y+r_{b_0,k}^{\alpha}I_{u,k}}\right)\right\}\triangleq R_u^L. \tag{49}$$

where (48) follows by applying the inequality $\mathbb{E}\left\{\log_2\left(1+ax\right)\right\}\geq\log_2\left[1+\frac{a}{\mathbb{E}\left(\frac{1}{x}\right)}\right]$ to (44) with (45), and (49) follows by substituting (46) and (47) into (48). A_2 is as defined in the theorem.

Since the random terms X, Y, $r_{b_0,k}$ and $I_{u,k}$ are independent with each other, R_u^L can be obtained by using *Lemma* 1 as

$$R_u^L=\log_2 e\int_0^\infty\frac{\mathcal{M}_1(z)-\mathcal{M}_2(z)}{z}\mathcal{M}_3(z)\mathrm{d}z, \tag{50}$$

where

$$\mathcal{M}_1(z) = \mathbb{E}_Y\left\{e^{-A_1\delta zY}\right\}=\mathcal{L}_Y(A_1\delta z), \tag{51}$$
$$\mathcal{M}_2(z) = \mathbb{E}_{X,Y}\left\{e^{-z[A_1\delta Y+A_2(X+(1-\delta)Y)]}\right\}=\mathbb{E}_X\left\{e^{-A_2zX}\right\}\mathbb{E}_Y\left\{e^{-[A_1\delta+A_2(1-\delta)]zY}\right\}$$
$$= \mathcal{L}_X(A_2z)\mathcal{L}_Y\left\{[A_1\delta+A_2(1-\delta)]z\right\}, \tag{52}$$
$$\mathcal{M}_3(z) = \mathbb{E}_{r_{b_0,k},I_{u,k}}\left\{e^{-zr_{b_0,k}^{\alpha}I_{u,k}}\right\}=\mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{I_{u,k}}\left(r_{b_0,k}^{\alpha}z\right)\right\}. \tag{53}$$

Firstly, the Laplace transforms of X and Y can be respectively obtained as [28, 3.351.3]

$$\mathcal{L}_X(s)=\int_0^\infty e^{-sx}e^{-x}\mathrm{d}x=\frac{1}{1+s},\qquad \mathcal{L}_Y(s)=\int_0^\infty e^{-sy}\frac{y^{M-2}e^{-y}}{\Gamma(M-1)}\mathrm{d}y=\frac{1}{(1+s)^{M-1}}. \tag{54}$$

It follows that

$$\mathcal{L}_X(A_2 z) = \frac{1}{1+A_2 z}, \qquad \mathcal{L}_Y(A_1\delta z) = \frac{1}{(1+A_1\delta z)^{M-1}}, \tag{55}$$

$$\mathcal{L}_Y\left\{[A_1\delta + A_2(1-\delta)]z\right\} = \frac{1}{\big(1+[A_1\delta + A_2(1-\delta)]z\big)^{M-1}}. \tag{56}$$

We then derive the Laplace transform of $I_{u,k}$. We let

$$X_i \triangleq \frac{1}{K}\sum_{l=1}^{K} |\mathbf{h}_{i,k}\mathbf{w}_{i,l}|^2 + \tau\|\mathbf{h}_{i,k}\mathbf{Z}_i\|^2,\ i \neq b_0. \tag{57}$$

For analytical tractability, we follow the previous papers [19, 20, 25] to assume that the RVs $|\mathbf{h}_{i,k}\mathbf{w}_{i,l}|^2$ ($l = 1, 2, \ldots, K$) and $\|\mathbf{h}_{i,k}\mathbf{Z}_i\|^2$ associated with X_i are independent with each other. Then, it follows that $X_i \overset{d.}{=} \text{Gamma}\left(K, \frac{1}{K}\right) + \text{Gamma}\left(M-K, \tau\right)$, whose PDF can be obtained in the following two different cases.

When $\phi \neq \frac{K}{M}$, i.e. $\tau \neq \frac{1}{K}$, we have

$$\begin{aligned}
f_{X_i}(x) &= \int_0^x \frac{(x-y)^{K-1}e^{-K(x-y)}K^K}{\Gamma(K)}\frac{y^{M-K-1}e^{-\frac{y}{\tau}}}{\Gamma(M-K)\tau^{M-K}}\mathrm{d}y \\
&= \int_0^x \frac{\sum_{m=0}^{K-1}\binom{K-1}{m}(-1)^m y^m x^{K-1-m}e^{-K(x-y)}K^K}{\Gamma(K)}\frac{y^{M-K-1}e^{-\frac{y}{\tau}}}{\Gamma(M-K)\tau^{M-K}}\mathrm{d}y \\
&= \sum_{m=0}^{K-1}\binom{K-1}{m}\frac{(-1)^m x^{K-1-m}e^{-Kx}K^K}{\Gamma(K)\Gamma(M-K)\tau^{M-K}}\int_0^x y^{M-K-1+m}e^{-\frac{(1-K\tau)y}{\tau}}\mathrm{d}y \\
&= \sum_{m=0}^{K-1} C_0\frac{x^{K-m-1}e^{-Kx}}{\Gamma(M-K+m)}\gamma\left(M-K+m, \frac{(1-K\tau)x}{\tau}\right),
\end{aligned} \tag{58}$$

where C_0 is as defined in the *Theorem* 2 and (58) follows from [28, 3.381.1]. When $\phi = \frac{K}{M}$, i.e., $\tau = \frac{1}{K}$. In this case, $X_i \overset{d.}{=} \text{Gamma}\left(M, \frac{1}{K}\right)$. Then, we have $f_{X_i}(x) = \frac{K^M x^{M-1}e^{-Kx}}{\Gamma(M)}$. Thus, the PDF of X_i can be written as

$$f_{X_i}(x) = \begin{cases} \sum_{m=0}^{K-1} C_0\frac{x^{K-m-1}e^{-Kx}}{\Gamma(M-K+m)}\gamma\left(M-K+m, \frac{(1-K\tau)x}{\tau}\right), & \phi \neq \frac{K}{M} \\ \frac{K^M x^{M-1}e^{-Kx}}{\Gamma(M)}, & \phi = \frac{K}{M} \end{cases}. \tag{59}$$

We denote $\mathfrak{B}(x, r)$ as the ball of radius r centered at point x. The Laplace transform of $I_{u,k}$ can be obtained as

$$\mathcal{L}_{I_{u,k}}(s) = \mathbb{E}_{I_{u,k}}\left[\exp\left(-s\sum_{i=1, i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\phi X_i\right)\right] = \mathbb{E}_{\Phi_b}\left\{\prod_{i\in\Phi_b\setminus\{b_0\}}\mathbb{E}_{X_i}\left[\exp\left(-\phi s r_{i,k}^{-\alpha}X_i\right)\right]\right\}$$

$$= \exp\left(-\lambda_b\int_{\mathcal{R}^2\setminus\mathfrak{B}(o, r_{b_0,k})}\left[1-\chi\big(\phi s r^{-\alpha}\big)\right]\Lambda(\mathrm{d}r)\right) \tag{60}$$

$$= \exp\left(-2\pi\lambda_b\int_{r_{b_0,k}}^{\infty}\left[1-\chi\big(\phi s r^{-\alpha}\big)\right]r\mathrm{d}r\right), \tag{61}$$

where (60) follows from the probability generating functional (PGFL) over PPP Φ_b [14, 16] with $\chi(\omega) \triangleq \int_0^\infty e^{-\omega x} f_{X_i}(x)\mathrm{d}x$, and (61) is obtained by changing cartesian coordinates to polar coordinates. Thus, we need to first obtain the result of $\chi(\omega)$.

When $\phi \neq \frac{K}{M}$, i.e., $\tau \neq \frac{1}{K}$, we can obtain with (59) that

$$\chi(\omega) = \int_0^\infty e^{-\omega x} \sum_{m=0}^{K-1} C_0 x^{K-m-1} e^{-Kx} \left[1 - e^{-\frac{(1-K\tau)x}{\tau}} \sum_{n=0}^{M-K+m-1} \frac{(1-K\tau)^n x^n}{\tau^n n!}\right] \mathrm{d}x \quad (62)$$

$$= \sum_{m=0}^{K-1} C_0 \left\{ \int_0^\infty x^{K-m-1} e^{-(\omega+K)x} \mathrm{d}x - \sum_{n=0}^{M-K+m-1} \frac{(1-K\tau)^n}{\tau^n n!} \int_0^\infty x^{K-m-1+n} e^{-\frac{\tau\omega+1}{\tau}x} \mathrm{d}x \right\}$$

$$= \sum_{m=0}^{K-1} C_0 \left\{ \frac{\Gamma(K-m)}{(K+\omega)^{K-m}} - \sum_{n=0}^{M-K+m-1} \frac{(1-K\tau)^n \tau^{K-m} \Gamma(K-m+n)}{n!(1+\tau\omega)^{K-m+n}} \right\}. \quad (63)$$

where (62) follows by applying [28, 8.352.1] to $\gamma\left(M-K+m, \frac{(1-K\tau)x}{\tau}\right)$, and (63) is obtained by using [28, 3.351.3]. Similarly, when $\phi = \frac{K}{M}$, i.e., $\tau = \frac{1}{K}$, we can obtain

$$\chi(\omega) = \int_0^\infty e^{-\omega x} \frac{x^{M-1} e^{-Kx} K^M}{\Gamma(M)} \mathrm{d}x = \frac{K^M}{\Gamma(M)} \int_0^\infty x^{M-1} e^{-(\omega+K)x} \mathrm{d}x = \frac{K^M}{(K+\omega)^M}. \quad (64)$$

Then, the final result of $\chi(\omega)$ is given by

$$\chi(\omega) = \begin{cases} \sum_{m=0}^{K-1} C_0 \left[\frac{\Gamma(K-m)}{(K+\omega)^{K-m}} - \sum_{n=0}^{M-K+m-1} \frac{(1-K\tau)^n \tau^{K-m} \Gamma(K-m+n)}{n!(1+\tau\omega)^{K-m+n}} \right], & \phi \neq \frac{K}{M} \\ \frac{K^M}{(K+\omega)^M}. & \phi = \frac{K}{M} \end{cases}. \quad (65)$$

The expression of $\mathcal{L}_{I_{u,k}}(s)$ follows by substituting (65) into (61), and the result of $\mathcal{L}_{I_{u,k}}\left(r_{b_0,k}^\alpha z\right)$ can be obtained as

$$\mathcal{L}_{I_{u,k}}\left(r_{b_0,k}^\alpha z\right) = \exp\left(-2\pi\lambda_b \int_{r_{b_0,k}}^\infty \left[1 - \chi(\phi z r_{b_0,k}^\alpha r^{-\alpha})\right] r \mathrm{d}r\right)$$

$$= \exp\left(-2\pi\lambda_b \times \frac{r_{b_0,k}^2 (\phi z)^{\frac{2}{\alpha}}}{2} \int_{(\phi z)^{-\frac{2}{\alpha}}}^\infty \left(1 - \chi(u^{-\frac{\alpha}{2}})\right) \mathrm{d}u\right) \quad (66)$$

$$= \exp\left(-\pi\lambda_b r_{b_0,k}^2 \Xi(z)\right), \quad (67)$$

where (66) follows from the change of variables $u = (\phi z)^{-\frac{2}{\alpha}} r_{b_0,k}^{-2} r^2$ and $\Xi(z) \triangleq (\phi z)^\beta \times \int_{(\phi z)^{-\beta}}^\infty \left(1 - \chi(u^{-\frac{\alpha}{2}})\right) \mathrm{d}u$. Moreover, $\Xi(z)$ can be expressed as

$$\Xi(z) = (\phi z)^\beta \int_{(\phi z)^{-\beta}}^\infty \left(1 - \chi(u^{-\frac{\alpha}{2}})\right) \mathrm{d}u = -(\phi z)^\beta \int_0^{\phi z} (1 - \chi(x)) \mathrm{d}x^{-\frac{2}{\alpha}} \quad (68)$$

$$= -(\phi z)^\beta \left\{ \left[1 - \chi(x)\right] x^{-\frac{2}{\alpha}} \Big|_0^{\phi z} + \int_0^{\phi z} x^{-\frac{2}{\alpha}} \mathrm{d}\chi(x) \right\} \quad (69)$$

$$= (\phi z)^\beta \left\{ -\left[1 - \chi(\phi z)\right] (\phi z)^{-\frac{2}{\alpha}} - \int_0^{\phi z} x^{-\frac{2}{\alpha}} \mathrm{d}\chi(x) \right\} \quad (70)$$

$$= (\phi z)^\beta \left\{ \int_0^{\phi z} \chi(x) \mathrm{d}x^{-\frac{2}{\alpha}} + \lim_{x \to 0^+} \chi(x) x^{-\frac{2}{\alpha}} \right\} - 1, \quad (71)$$

where (68) is obtained by the change of variables, (69) is obtained by integration by part, and (70) follows from $\lim_{x\to 0^+}[1-\chi(x)]\,x^{-\frac{2}{\alpha}}=0$, which is obtained by the Lopida's Law with the definition of $\chi(\omega)$.

According to (65), $\Xi(z)$ can be obtained for the following two cases. When $\phi\neq\frac{K}{M}$, i.e., $\tau\neq\frac{1}{K}$, we have

$$\Xi(z)=(\phi z)^{\beta}\left\{\sum_{m=0}^{K-1}C_0\left\{\Gamma(K-m)\left[\int_0^{\phi z}\frac{1}{(K+x)^{K-m}}\mathrm{d}x^{-\beta}+\lim_{x\to 0}\frac{x^{-\beta}}{(K+x)^{K-m}}\right]\right.\right.$$
$$-\sum_{n=0}^{M-K+m-1}\frac{\Gamma(K-m+n)}{n!}(1-K\tau)^{n}\tau^{K-m}$$
$$\left.\left.\times\left[\int_0^{\phi z}\frac{\mathrm{d}x^{-\beta}}{(1+\tau x)^{K-m+n}}+\lim_{x\to 0}\frac{x^{-\beta}}{(1+\tau x)^{K-m+n}}\right]\right\}\right\}-1. \tag{72}$$

The two terms in (72) can be respectively obtained as

$$\begin{aligned}\int_0^{\phi z}\frac{1}{(K+x)^{K-m}}\mathrm{d}x^{-\beta}+\lim_{x\to 0}\frac{x^{-\beta}}{(K+x)^{K-m}}&=\left.\frac{x^{-\beta}}{(K+x)^{K-m}}\right|_{x=\phi z}+\int_0^{\phi z}\frac{(K-m)x^{-\beta}}{(K+x)^{K-m+1}}\mathrm{d}x\\ =\;&\frac{(\phi z)^{-\beta}}{(K+\phi z)^{K-m}}+\frac{K-m}{K^{K-m+\beta}}\int_0^{\frac{\phi z}{K+\phi z}}t^{-\beta}(1-t)^{K-m+\beta-1}\mathrm{d}t\\ =\;&\frac{(\phi z)^{-\beta}}{(K+\phi z)^{K-m}}+\frac{K-m}{K^{K-m+\beta}}\mathcal{B}\left(\frac{\phi z}{K+\phi z};1-\beta,K-m+\beta\right),\end{aligned} \tag{73}$$

$$\begin{aligned}&\int_0^{\phi z}\frac{1}{(1+\tau x)^{K-m+n}}\mathrm{d}x^{-\beta}+\lim_{x\to 0}\frac{x^{-\beta}}{(1+\tau x)^{K-m+n}}\\ =\;&\left.\frac{x^{-\beta}}{(1+\tau x)^{K-m+n}}\right|_{x=\phi z}+\int_0^{\phi z}\frac{(K-m+n)\tau x^{-\beta}}{(1+\tau x)^{K-m+n+1}}\mathrm{d}x\\ =\;&\frac{(\phi z)^{-\beta}}{(1+\tau\phi z)^{K-m+n}}+(K-m+n)\tau^{\beta}\int_0^{\frac{\tau\phi z}{1+\tau\phi z}}t^{-\beta}(1-t)^{K-m+n+\beta-1}\mathrm{d}t\\ =\;&\frac{(\phi z)^{-\beta}}{(1+\tau\phi z)^{K-m+n}}+(K-m+n)\tau^{\beta}\mathcal{B}\left(\frac{\tau\phi z}{1+\tau\phi z};1-\beta,K-m+n+\beta\right),\end{aligned} \tag{74}$$

where (73) and (74) are obtained by using the change of variables $t=\frac{x}{K+x}$ and $t=\frac{\tau x}{1+\tau x}$ respectively. Then, the results of $\Xi(z)$ follows by substituting (73) and (74) into (72). Similarly, when $\phi=\frac{K}{M}$, i.e., $\tau=\frac{1}{K}$, we have

$$\begin{aligned}\Xi(z)&=(\phi z)^{\beta}\left[-\left.x^{-\beta}\right|_0^{\phi z}+\int_0^{\phi z}\frac{K^M}{(K+x)^M}\mathrm{d}x^{-\beta}+\lim_{x\to 0}\frac{K^M x^{-\beta}}{(K+x)^M}\right]\\ =\;&(\phi z)^{\beta}\left[-(\phi z)^{-\beta}+\left.\frac{K^M x^{-\beta}}{(K+x)^M}\right|_{x=\phi z}+\int_0^{\phi z}\frac{MK^M x^{-\beta}}{(K+x)^{M+1}}\mathrm{d}x\right]\\ =\;&-1+\frac{K^M}{(K+\phi z)^M}+(\phi z)^{\beta}\int_0^{\frac{\phi z}{K+\phi z}}\frac{M(1-t)^{M+\beta-1}}{K^{\beta}t^{\beta}}\mathrm{d}t\\ =\;&\frac{K^M}{(K+\phi z)^M}+\frac{M(\phi z)^{\beta}}{K^{\beta}}\mathcal{B}\left(\frac{\phi z}{K+\phi z};1-\beta,M+\beta\right)-1,\end{aligned} \tag{75}$$

which again is obtained by using the changes of variables $x = u^{-\frac{\alpha}{2}}$ and $t = \frac{x}{K+x}$. Then, $\mathcal{M}_3(z)$ in (53) can be obtained as

$$\begin{aligned}\mathcal{M}_3(z) &= \mathbb{E}_{r_{b_0,k}}\big\{\exp\big(-\pi\lambda_b r_{b_0,k}^2 \Xi(z)\big)\big\} \\ &= \int_0^\infty \exp\big(-\pi\lambda_b r^2 \Xi(z)\big) 2\pi\lambda_b r \exp(-\pi\lambda_b r^2) \mathrm{d}r = \int_0^1 t^{\Xi(z)} \mathrm{d}t = \frac{1}{1+\Xi(z)}, \end{aligned} \tag{76}$$

where (76) is obtained from the change of variables $t = \exp(-\pi\lambda_b r^2)$ with the PDF of $r_{b_0,k}$ $f_{r_{b_0,k}}(r) = 2\pi\lambda_b r \exp(-\pi\lambda_b r^2)$ [16]. Then, the proof is completed by substituting the obtained the results of $\mathcal{M}_1(z)$, $\mathcal{M}_2(z)$ and $\mathcal{M}_3(z)$ given by (51), (52) and (76) into (50).

B. Proof of Theorem 2

Similar to the method to derive R_u^L, we let Eve $j^\star$ be located at origin o, and BS b_0 is the nearest BS in Φ_b. Then, according to (17), $R_{e,sum}$ can be re-expressed as

$$R_{e,sum} = \mathbb{E}\left[\log_2\left(1 + \frac{\frac{\phi}{K}\|\mathbf{g}_{b_0,j^\star}\mathbf{W}_{b_0}\|^2}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j^\star}^\alpha I_{e,j^\star}}\right)\right]. \tag{77}$$

For analytical tractability, again we follow [19, 20, 25] to assume that the RVs $\|\mathbf{g}_{i,j^\star}\mathbf{w}_{i,l}\|^2$ for $l \in \{1, 2\cdots, K\}$ and $\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2$ $\forall i \in \Phi_b$ are independent with each other. It follows that $\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2 \sim \text{Gamma}(K,1)$ and $\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2 \sim \text{Gamma}(M-K,1)$. Thus, $\forall i \in \Phi_b$, $\Big[\frac{1}{K}\sum_{l=1}^K |\mathbf{g}_{i,j^\star}\mathbf{w}_{i,l}|^2 + \tau\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\Big] \overset{d.}{=} X_i$ which is defined as (57). Applying *Lemma* 1 again, $R_{e,sum}$ can be obtained as

$$\begin{aligned} R_{e,sum} &= \log_2 e \int_0^\infty \frac{1-\mathbb{E}\left\{e^{-z\frac{\phi}{K}\|\mathbf{g}_{b_0,j^\star}\mathbf{W}_{b_0}\|^2}\right\}}{z} \mathbb{E}\left\{e^{-z\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2}\right\} \mathbb{E}\left\{e^{-z d_{b_0,j^\star}^\alpha I_{e,j^\star}}\right\} \mathrm{d}z \\ &= \log_2 e \int_0^\infty \frac{1-\mathcal{L}_{\|\mathbf{g}_{b_0,j^\star}\mathbf{W}_{b_0}\|^2}\left(\frac{\phi}{K}z\right)}{z} \mathcal{L}_{\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2}\left(\frac{1-\phi}{M-K}z\right) \mathbb{E}_{d_{b_0,j^\star}}\left\{\mathcal{L}_{I_{e,j^\star}}(d_{b_0,j^\star}^\alpha z)\right\} \mathrm{d}z, \end{aligned} \tag{78}$$

where

$$\mathcal{L}_{\|\mathbf{g}_{b_0,j^\star}\mathbf{W}_{b_0}\|^2}(s) = \int_0^\infty e^{-sy} \frac{y^{K-1}e^{-y}}{\Gamma(K)} \mathrm{d}y = \frac{1}{(1+s)^K}, \tag{79}$$

$$\mathcal{L}_{\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2}(s) = \int_0^\infty e^{-sy} \frac{y^{M-K-1}e^{-y}}{\Gamma(M-K)} \mathrm{d}y = \frac{1}{(1+s)^{M-K}}, \tag{80}$$

The Laplace transform of $I_{e,j^\star}$ can be obtained as

$$\mathcal{L}_{I_{e,j^\star}}(s) = \mathbb{E}_{I_{e,j^\star}}\left[\exp\left(-s\sum_{i=1}^{\infty} d_{i,j^\star}^{-\alpha}\phi X_i\right)\right] = \mathbb{E}_{\Phi_b}\left\{\prod_{i\in\Phi_b}\mathbb{E}_{X_i}\left[\exp\left(-\phi s d_{i,j^\star}^{-\alpha}X_i\right)\right]\right\} \quad (81)$$

$$= \exp\left(-\lambda_b\int_{\mathcal{R}^2}\left\{1-\mathbb{E}_{X_i}\left[\exp\left(-\phi s r^{-\alpha}X_i\right)\right]\right\}\Lambda(\mathrm{d}r)\right)$$

$$= \exp\left(-2\pi\lambda_b\int_0^{\infty}\left\{1-\mathbb{E}_{X_i}\left[\exp\left(-\phi s r^{-\alpha}X_i\right)\right]\right\}r\mathrm{d}r\right) \quad (82)$$

$$= \exp\left(-\pi\lambda_b A_3(\phi s)^{\frac{2}{\alpha}}\right), \quad (83)$$

where $A_3 \triangleq \Gamma\left(1-\frac{2}{\alpha}\right)\mathbb{E}_{X_i}\left[X_i^{\frac{2}{\alpha}}\right]$, (81) follows from Slivnyak's theorem, and (83) follows from [14, (8)] [16, 5.1.7]. The integration in (82) converges to (83) when $\alpha > 2$ [16, *Theorem* 5.3]. With $f_{X_i}(x)$ given by (59), $\mathbb{E}_{X_i}\left[X_i^{\frac{2}{\alpha}}\right]$ can be obtained as follows.

When $\phi \neq \frac{K}{M}$, we have

$$\mathbb{E}_{X_i}\left[X_i^{\frac{2}{\alpha}}\right] = \int_0^{\infty} x^{\frac{2}{\alpha}} f_{X_i}(x)\mathrm{d}x$$

$$= \int_0^{\infty} x^{\frac{2}{\alpha}}\sum_{m=0}^{K-1} C_0\frac{x^{K-m-1}e^{-Kx}}{\Gamma(M-K+m)}\gamma\left(M-K+m,\frac{(1-K\tau)x}{\tau}\right)\mathrm{d}x$$

$$= \sum_{m=0}^{K-1} C_0\int_0^{\infty} x^{K-m-1+\frac{2}{\alpha}}e^{-Kx}\left[1-e^{-\frac{(1-K\tau)x}{\tau}}\sum_{n=0}^{M-K+m-1}\frac{(1-K\tau)^n v^n}{\tau^n n!}\right]\mathrm{d}x$$

$$= \sum_{m=0}^{K-1} C_0\left\{\int_0^{\infty} x^{K-m-1+\frac{2}{\alpha}}e^{-Kx}\mathrm{d}x - \sum_{n=0}^{M-K+m-1}\frac{(1-K\tau)^n}{\tau^n n!}\int_0^{\infty} x^{K-m-1+n+\frac{2}{\alpha}}e^{-\frac{x}{\tau}}\mathrm{d}x\right\}$$

$$= \sum_{m=0}^{K-1} C_0\left\{\frac{\Gamma\left(K-m+\frac{2}{\alpha}\right)}{K^{K-m+\frac{2}{\alpha}}} - \sum_{n=0}^{M-K+m-1}\frac{(1-K\tau)^n\tau^{K-m+\frac{2}{\alpha}}\Gamma\left(K-m+n+\frac{2}{\alpha}\right)}{n!}\right\}. \quad (84)$$

And when $\phi = \frac{K}{M}$, we have

$$\mathbb{E}_{X_i}\left[X_i^{\frac{2}{\alpha}}\right] = \int_0^{\infty} x^{\frac{2}{\alpha}} f_{X_i}(x)\mathrm{d}x = \int_0^{\infty}\frac{K^M x^{M-1+\frac{2}{\alpha}}e^{-Kx}}{\Gamma(M)}\mathrm{d}x = \frac{\Gamma\left(M+\frac{2}{\alpha}\right)}{K^{\frac{2}{\alpha}}\Gamma(M)}. \quad (85)$$

Then, the result of A_3 in (20) follows by combing (84) and (85).

By using (83), we can obtain that

$$\mathbb{E}_{d_{b_0,j^\star}}\left\{\mathcal{L}_{I_{e,j^\star}}\left(d_{b_0,j^\star}^{\alpha}z\right)\right\} = \mathbb{E}_{d_{b_0,j^\star}}\left\{\exp\left(-\pi\lambda_b A_3\left(\phi d_{b_0,j^\star}^{\alpha}z\right)^{\frac{2}{\alpha}}\right)\right\}$$

$$= \int_0^{\infty}\exp\left(-\pi\lambda_b A_3(\phi z)^{\frac{2}{\alpha}}r^2\right)2\pi\lambda_e r\exp(-\pi\lambda_e r^2)\mathrm{d}r$$

$$= \int_0^{1} t^{\frac{\lambda_b}{\lambda_e}A_3(\phi z)^{\frac{2}{\alpha}}}\mathrm{d}t = \frac{1}{1+\frac{\lambda_b}{\lambda_e}A_3(\phi z)^{\frac{2}{\alpha}}}, \quad (86)$$

where (86) is obtained by using change of variables $t = \exp(-\pi\lambda_e r^2)$ with the PDF of $d_{b_0,j^\star}$ $f_{d_{b_0,j^\star}}(r) = 2\pi\lambda_e r\exp(-\pi\lambda_e r^2)$ [16]. The proof is completed by substituting (79), (80) and (86) into (78).

C. Proof of Lemma 2

(26) can be easily obtained by using (49) with $\delta = 2^{-\frac{B}{M-1}}$. Then, it follows that

$$\frac{\partial R_u^L}{\partial B} = \frac{1}{M-1}\mathbb{E}\left\{\frac{Y}{Y+\frac{1}{\delta A_1}r_{b_0,k}^{\alpha}I_{u,k}}\right\} - \frac{\delta(A_1-A_2)}{\left[\delta(A_1-A_2)+A_2\right](M-1)}$$
$$\times\mathbb{E}\left\{\frac{Y}{Y+\frac{A_2}{\delta(A_1-A_2)+A_2}X+\frac{1}{\delta(A_1-A_2)+A_2}r_{b_0,k}^{\alpha}I_{u,k}}\right\}$$
$$= \underbrace{\int_0^1 (1-x)^{M-2}\mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{I_{u,k}}\left(\frac{r_{b_0,k}^{\alpha}x}{\delta A_1(1-x)}\right)\right\}\mathrm{d}x}_{\Psi_1(\delta)} - \frac{\delta(A_1-A_2)}{\delta(A_1-A_2)+A_2} \tag{87}$$
$$\times\underbrace{\int_0^1 \frac{(1-x)^{M-2}}{1+\frac{A_2}{\delta(A_1-A_2)+A_2}\frac{x}{1-x}}\mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{I_{u,k}}\left(\frac{r_{b_0,k}^{\alpha}}{\left[\delta(A_1-A_2)+A_2\right]}\frac{x}{1-x}\right)\right\}\mathrm{d}x}_{\Psi_2(\delta)}, \tag{88}$$

which can be obtained as (27), where (87) and (88) are obtained by using [20, *Lemma* 2], and (27) is obtained using $\mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{I_{u,k}}\left(r_{b_0,k}^{\alpha}z\right)\right\} = \frac{1}{1+\Xi(z)}$ given by (76). Further, we have

$$\frac{\partial}{\partial\delta}\left(\frac{\partial R_u^L}{\partial B}\right) = \mathbb{E}\left\{\frac{\frac{A_1 Y}{M-1}(\delta A_1 Y + r_{b_0,k}^{\alpha}I_{u,k}) - \frac{\delta}{M-1}A_1^2Y^2}{(\delta A_1 Y + r_{b_0,k}^{\alpha}I_{u,k})^2}\right\}$$
$$-\mathbb{E}\left\{\frac{\frac{(A_1-A_2)Y}{M-1}[\delta(A_1-A_2)Y + A_2(X+Y) + r_{b_0,k}^{\alpha}I_{u,k}] - \frac{\delta}{M-1}(A_1-A_2)^2Y^2}{[\delta(A_1-A_2)Y + A_2(X+Y) + r_{b_0,k}^{\alpha}I_{u,k}]^2}\right\}$$
$$= \mathbb{E}\left\{\frac{\frac{1}{M-1}A_1 Y}{\delta A_1 Y + r_{b_0,k}^{\alpha}I_{u,k}} - \frac{\frac{1}{M-1}(A_1-A_2)Y}{\delta(A_1-A_2)Y + A_2(X+Y) + r_{b_0,k}^{\alpha}I_{u,k}}\right.$$
$$\left. - \delta\left[\frac{\frac{1}{M-1}A_1^2Y^2}{(\delta A_1 Y + r_{b_0,k}^{\alpha}I_{u,k})^2} - \frac{\frac{1}{M-1}(A_1-A_2)^2Y^2}{[\delta(A_1-A_2)Y + A_2(X+Y) + r_{b_0,k}^{\alpha}I_{u,k}]^2}\right]\right\}, \tag{89}$$

and hence

$$\frac{\partial^2 R_u^L}{\partial B^2} = \frac{\partial}{\partial\delta}\left(\frac{\partial R_u^L}{\partial B}\right)\times\frac{\partial\delta}{\partial B} = \frac{\partial}{\partial\delta}\left(\frac{\partial R_u^L}{\partial B}\right)\left(-\frac{\delta}{M-1}\right)\ln 2$$
$$= \frac{\ln 2}{(M-1)^2}\mathbb{E}\left\{-\left(\frac{A_1 Y}{A_1 Y + 2^{\frac{B}{M-1}}r_{b_0,k}^{\alpha}I_{u,k}} - \frac{(A_1-A_2)Y}{(A_1-A_2)Y + 2^{\frac{B}{M-1}}\left[A_2(X+Y)+r_{b_0,k}^{\alpha}I_{u,k}\right]}\right)\right.$$
$$\left.+\frac{A_1^2Y^2}{\left(A_1 Y + 2^{\frac{B}{M-1}}r_{b_0,k}^{\alpha}I_{u,k}\right)^2} - \frac{(A_1-A_2)^2Y^2}{\left\{(A_1-A_2)Y + 2^{\frac{B}{M-1}}\left[A_2(X+Y)+r_{b_0,k}^{\alpha}I_{u,k}\right]\right\}^2}\right\}$$
$$\triangleq \mathbb{E}\left\{g_1(B)g_2(B)\right\}, \tag{90}$$

where $g_1(B)$ and $g_2(B)$ are as defined in the lemma. It is easy to see that $g_1(B) < 0$ always holds for any $B \geq 0$. Moreover, it is easy to observe that $g_2(B)$ is a monotonic increasing function of B and $\lim\limits_{B\to+\infty} g_2(B) = 1$. Thus, the equation $g_2(B) = 0$ has most one root in $[0,\infty)$. The properties with $\frac{\partial^2 R_u^L}{\partial B^2}$ described in the lemma easily follows.

D. *Proof of Theorem 3*

Before proceeding, we first obtain both an upper bound and a lower bound on the Laplace transform of $I_{u,k}$ as follows. Based on the definition of $\chi(\omega)$ after (61) and the definition of $\Xi(z)$ after (67), we can lower-bound $1+\Xi(z)$ as

$$\begin{aligned}
&1+\Xi(z) = (\phi z)^{\frac{2}{\alpha}} \int_0^{(\phi z)^{-\frac{2}{\alpha}}} \mathrm{d}u + (\phi z)^{\frac{2}{\alpha}} \int_{(\phi z)^{-\frac{2}{\alpha}}}^{\infty} \left(1-\chi(u^{-\frac{\alpha}{2}})\right) \mathrm{d}u \\
&\geq (\phi z)^{\frac{2}{\alpha}} \int_0^{\infty} \left[1-\chi(u^{-\frac{\alpha}{2}})\right] \mathrm{d}u = \frac{2}{\alpha}(\phi z)^{\frac{2}{\alpha}} \int_0^{\infty} \left[1-\chi(y)\right] \left(\frac{1}{y}\right)^{\frac{2}{\alpha}+1} \mathrm{d}y \\
&= \frac{2}{\alpha}(\phi z)^{\frac{2}{\alpha}} \int_0^{\infty} \left[1-\int_0^{\infty} e^{-yv} f_{X_i}(v)\mathrm{d}v\right] y^{-\frac{2}{\alpha}-1}\mathrm{d}y \\
&= \frac{2}{\alpha}(\phi z)^{\frac{2}{\alpha}} \int_0^{\infty} f_{X_i}(v) \int_0^{\infty} \left(1-e^{-yv}\right) y^{-\frac{2}{\alpha}-1}\mathrm{d}y\mathrm{d}v \\
&= \frac{2}{\alpha}(\phi z)^{\frac{2}{\alpha}} \int_0^{\infty} v^{\frac{2}{\alpha}} f_{X_i}(v)\mathrm{d}v \int_0^{\infty} \frac{(1-e^{-t})}{t^{\frac{2}{\alpha}+1}}\mathrm{d}t \\
&= (\phi z)^{\frac{2}{\alpha}}\Gamma\left(1-\frac{2}{\alpha}\right) \int_0^{\infty} v^{\frac{2}{\alpha}} f_{X_i}(v)\mathrm{d}v = (\phi z)^{\frac{2}{\alpha}} A_3,
\end{aligned} \tag{91}$$

where (91) is obtained from the result that [28, 8.310.1] $\int_0^{\infty} \frac{1-e^{-t}}{t^{\frac{2}{\alpha}+1}}\mathrm{d}t = -\frac{\alpha}{2}\left[(1-e^{-t})t^{-\frac{2}{\alpha}}|_0^{\infty} - \int_0^{\infty} t^{-\frac{2}{\alpha}} e^{-t}\mathrm{d}t\right] = \frac{\alpha}{2}\int_0^{\infty} t^{-\frac{2}{\alpha}} e^{-t}\mathrm{d}t = \frac{\alpha}{2}\Gamma\left(1-\frac{2}{\alpha}\right)$, and the definition of A_3 after (83). With (91) and (76), we have

$$\mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{I_{u,k}}\left(r_{b_0,k}^{\alpha} z\right)\right\} \leq (\phi z)^{-\frac{2}{\alpha}} A_3^{-1}. \tag{92}$$

Moreover, we can upper-bound $\Xi(z)$ as

$$\Xi(z) = (\phi z)^{\frac{2}{\alpha}} \int_{(\phi z)^{-\frac{2}{\alpha}}}^{\infty} \left(1-\chi(u^{-\frac{\alpha}{2}})\right) \mathrm{d}u \leq (\phi z)^{\frac{2}{\alpha}} \int_0^{\infty} \left(1-\chi(u^{-\frac{2}{\alpha}})\right) \mathrm{d}u = (\phi z)^{\frac{2}{\alpha}} A_3. \tag{93}$$

Then, with (93) and (76), we have

$$\mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{I_{u,k}}\left(r_{b_0,k}^{\alpha} z\right)\right\} \geq \frac{1}{1+(\phi z)^{\frac{2}{\alpha}} A_3}. \tag{94}$$

(1) An upper bound on $\frac{\partial R_u^L}{\partial B}$:

Case 1: When $0<\phi<\frac{K}{M-K+1}$, i.e., $A_1-A_2>0$.

Then, an upper bound of $\frac{\partial R_u^L}{\partial B}$ can be obtained by using (87) and (92) as [28, 3.191.1]

$$\begin{aligned}
\frac{\partial R_u^L}{\partial B} &< \Psi_1(\delta) \leq \int_0^1 (1-x)^{M-2} \left(\frac{\phi x}{\delta A_1(1-x)}\right)^{-\frac{2}{\alpha}} A_3^{-1}\mathrm{d}x \\
&= \left(\frac{\delta A_1}{\phi}\right)^{\frac{2}{\alpha}} A_3^{-1} \int_0^1 (1-x)^{M-2+\frac{2}{\alpha}} x^{-\frac{2}{\alpha}}\mathrm{d}x \\
&= \left(\frac{\delta A_1}{\phi}\right)^{\frac{2}{\alpha}} A_3^{-1} \beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right),
\end{aligned} \tag{95}$$

which is $D_{er}^{up}(B)$ in (34) for $0<\phi<\frac{K}{M-K+1}$.

Case 2: When $\frac{K}{M-K+1}\leq\phi<1$, i.e., $A_1-A_2\leq 0$.

According to (87), an upper bound on $\frac{\partial R_u^L}{\partial B}$ can be obtained by developing an upper bound on both $\Psi_1(\delta)$ and $\Psi_2(\delta)$ respectively. Firstly, by using (92) again, an upper bound on $\Psi_2(\delta)$ can be obtained as

$$\begin{aligned}
&\Psi_2(\delta)\leq\int_0^1\frac{(1-x)^{M-2}}{1+\frac{A_2}{\delta(A_1-A_2)+A_2}\frac{x}{1-x}}\left(\frac{\phi x}{\left[\delta(A_1-A_2)+A_2\right](1-x)}\right)^{-\frac{2}{\alpha}}A_3^{-1}\mathrm{d}x\\
=&\left[\frac{\delta(A_1-A_2)+A_2}{\phi}\right]^{\frac{2}{\alpha}}A_3^{-1}\int_0^1\frac{(1-x)^{M-1+\frac{2}{\alpha}}x^{-\frac{2}{\alpha}}}{1-\frac{\delta(A_1-A_2)}{\delta(A_1-A_2)+A_2}x}\mathrm{d}x\\
\leq&\left[\frac{\delta(A_1-A_2)+A_2}{\phi}\right]^{\frac{2}{\alpha}}A_3^{-1}\int_0^1(1-x)^{M-1+\frac{2}{\alpha}}x^{-\frac{2}{\alpha}}\mathrm{d}x\\
=&\left[\frac{\delta(A_1-A_2)+A_2}{\phi}\right]^{\frac{2}{\alpha}}A_3^{-1}\beta\left(1-\frac{2}{\alpha},M+\frac{2}{\alpha}\right)\triangleq\Psi_{2,up_2}(\delta).
\end{aligned}\tag{96}$$

Then, the expression of $D_{er}^{up}(B)$ in (34) for $\frac{K}{M-K+1}\leq\phi<1$ follows by substituting (95) and (96) into (87).

(2) A lower bound on $\frac{\partial R_u^L}{\partial B}$:

Case 1: When $0<\phi<\frac{K}{M-K+1}$, i.e., $A_1-A_2>0$. According to (87), a lower bound on $\frac{\partial R_u^L}{\partial B}$ can be obtained by developing a lower bound on $\Psi_1(\delta)$ and an upper bound on $\Psi_2(\delta)$. Firstly, a lower bound on $\Psi_1(\delta)$ can be obtained as

$$\Psi_1(\delta)\ \geq\ \int_0^1\frac{(1-x)^{M-2}}{1+\left(\frac{\phi x}{\delta A_1(1-x)}\right)^{\frac{2}{\alpha}}A_3}\mathrm{d}x\triangleq\Psi_{1,low}(\delta).\tag{97}$$

In addition, an upper bound on $\Psi_2(\delta)$ can be obtained by using (94) as

$$\begin{aligned}
&\Psi_2(\delta)\leq\int_0^1\frac{(1-x)^{M-2}}{1+\frac{A_2}{\delta(A_1-A_2)+A_2}\frac{x}{1-x}}\left(\frac{\phi x}{\left[\delta(A_1-A_2)+A_2\right](1-x)}\right)^{-\frac{2}{\alpha}}A_3^{-1}\mathrm{d}x\\
=&\left[\frac{\delta(A_1-A_2)+A_2}{\phi}\right]^{\frac{2}{\alpha}}A_3^{-1}\int_0^1\frac{(1-x)^{M-1+\frac{2}{\alpha}}x^{-\frac{2}{\alpha}}}{1-\frac{\delta(A_1-A_2)}{\delta(A_1-A_2)+A_2}x}\mathrm{d}x\\
\leq&\left[\frac{\delta(A_1-A_2)+A_2}{\phi}\right]^{\frac{2}{\alpha}}A_3^{-1}\int_0^1(1-x)^{M-2+\frac{2}{\alpha}}x^{-\frac{2}{\alpha}}\mathrm{d}x\\
=&\left[\frac{\delta(A_1-A_2)+A_2}{\phi}\right]^{\frac{2}{\alpha}}A_3^{-1}\beta\left(1-\frac{2}{\alpha},M-1+\frac{2}{\alpha}\right)\triangleq\Psi_{2,up_1}(\delta).
\end{aligned}\tag{98}$$

Then, the expression of $D_{er}^{low}(B)$ in (35) for $0<\phi<\frac{K}{M-K+1}$ follows by substituting (97) and (98) into (87).

Case 2: When $\frac{K}{M-K+1}\leq\phi<1$, i.e., $A_1-A_2\leq 0$. Since $A_1-A_2\leq 0$, $\frac{\delta(A_1-A_2)}{\delta(A_1-A_2)+A_2}\leq 0$. Then, the expression of $D_{er}^{low}(B)$ for $\frac{K}{M-K+1}\leq\phi<1$ in (35) follows by substituting (97) into

(87) and omitting the second term about $\Psi_2(\delta)$.

In addition, the monotonic decreasing property of $D_{er}^{up}(B)$ w.r.t. $B \in [0,\infty)$ for $\phi \in (0,1]$ and that of $D_{er}^{low}(B)$ w.r.t. $B \in [0,\infty)$ for $\phi \in [\frac{K}{M-K+1}, 1]$ can be easily observed. Then, the proof is completed.

E. *Proof of Lemma 3*

Recall that b^* is the largest root of the equation $\frac{\partial \tilde{R}^L_{\text{Net}}}{\partial B} = 0$ for all scenarios. Then, $b^* < b_u^*$ follows from the fact that the upper bound $D_{er}^{up}(B)$ is monotonic decreasing of B. $b_l^* < b^*$ follows similarly by noticing that the lower bound $D_{er}^{low}(B)$ for $0 < \phi < \frac{K}{M-K+1}$ in (35) may not be a monotonic function of B. First, we let $\delta_l = 2^{-\frac{b_l^*}{M-1}}$ and $\delta_u = 2^{-\frac{b_u^*}{M-1}}$. Obviously, $\delta_u \le \delta_l$. We will prove the result of $\lim\limits_{T_c \to +\infty} b_u^* - b_l^* = 0$ for different regions of ϕ as follows.

When $0 < \phi < \frac{K}{M-K+1}$, according to (34) and (35) we have

$$\int_0^1 \frac{(\delta_u A_1)^{\frac{2}{\alpha}}(1-x)^{M-2}}{A_3\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x - \frac{1}{T_c} = 0, \tag{99}$$

$$\int_0^1 \frac{(\delta_l A_1)^{\frac{2}{\alpha}}(1-x)^{M-2}}{(\delta_l A_1)^{\frac{2}{\alpha}} + A_3\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x - \frac{\delta_l(A_1-A_2)}{[\delta_l(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}}}\frac{\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3} - \frac{1}{T_c} = 0 \tag{100}$$

Then, (99) minus (100) gives

$$\begin{aligned}
&\int_0^1 \frac{(1-x)^{M-2}\left[1+\frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}} - \frac{A_3}{(\delta_u A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\right]}{\frac{A_3}{(\delta_u A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\left[1+\frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\right]}\mathrm{d}x. \\
&+\frac{\delta_l(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3[\delta_l(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}}} = 0 \\
\Rightarrow\ &\left[1-\left(\frac{\delta_u}{\delta_l}\right)^{\frac{2}{\alpha}}\right]\int_0^1 \frac{(1-x)^{M-2}}{1+\frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x - \frac{\delta_l(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3[\delta_l(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}}} \\
&= \left(\frac{\delta_u}{\delta_l}\right)^{\frac{2}{\alpha}}\int_0^1 \frac{(1-x)^{M-2}}{\left[1+\frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\right]\frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x \\
\Rightarrow\ &\left[1-\left(\frac{\delta_u}{\delta_l}\right)^{\frac{2}{\alpha}}\right]\int_0^1 \frac{(1-x)^{M-2}}{\delta_l^{\frac{2}{\alpha}} + A_3 A_1^{-\frac{2}{\alpha}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x - \frac{\delta_l^{1-\frac{2}{\alpha}}(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3[\delta_l(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}}} \\
&= \left(\frac{\delta_u}{\delta_l}\right)^{\frac{2}{\alpha}}\int_0^1 \frac{(1-x)^{M-2}}{\left[1+\delta_l^{-\frac{2}{\alpha}}A_3A_1^{-\frac{2}{\alpha}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\right]A_3A_1^{-\frac{2}{\alpha}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x.
\end{aligned} \tag{101}$$

Let $\psi_1(\delta) \triangleq D_{er}^{low}$ for $0 < \phi < \frac{K}{M-K+1}$, where $\delta \in [0,1]$. Then, it can be easily obtained that

$$\psi_1(\delta) > \int_0^1 \frac{(\delta A_1)^{\frac{2}{\alpha}}(1-x)^{M-2}}{A_1^{\frac{2}{\alpha}} + A_3\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x - \frac{\delta(A_1-A_2)}{A_2^{1-\frac{2}{\alpha}}}\frac{\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3} \triangleq \psi_2(\delta). \quad (102)$$

$\psi_1(\delta)$ and $\psi_2(\delta)$ are both continuous functions of δ. In addition, we can obtain

$$\frac{\mathrm{d}\psi_2(\delta)}{\mathrm{d}\delta} = \frac{2}{\alpha}\delta^{\frac{2}{\alpha}-1}\int_0^1 \frac{A_1^{\frac{2}{\alpha}}(1-x)^{M-2}}{A_1^{\frac{2}{\alpha}} + A_3\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x - \frac{A_1-A_2}{A_2^{1-\frac{2}{\alpha}}}\frac{\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3}. \quad (103)$$

It can be easily observed from (103) that both of the terms $\int_0^1 \frac{A_1^{\frac{2}{\alpha}}(1-x)^{M-2}}{A_1^{\frac{2}{\alpha}}+A_3\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x$ and $\frac{A_1-A_2}{A_2^{1-\frac{2}{\alpha}}}\frac{\beta\left(1-\frac{2}{\alpha},M-1+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3}$ are positive for $0 < \phi < \frac{K}{M-K+1}$. It follows that $\psi_2(\delta)$ is a strictly monotonic increasing function of δ in the region $[0, \min\{\epsilon, 1\}]$, where

$$\epsilon = \left(\frac{2}{\alpha}\frac{A_2^{1-\frac{2}{\alpha}}\phi^{\frac{2}{\alpha}}A_3\int_0^1 \frac{A_1^{\frac{2}{\alpha}}(1-x)^{M-2}}{A_1^{\frac{2}{\alpha}}+A_3\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x}{(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M-1+\frac{2}{\alpha}\right)}\right)^{\frac{\alpha}{\alpha-2}}. \quad (104)$$

In addition, we have $\psi_2(0) = 0$. Thus, the inverse function of $\psi_2(\delta)$, i.e., $\psi_2^{-1}(x)$, is also a strictly monotonic increasing function of x in the region $[0, \psi_2(\min\{\epsilon, 1\})]$. Moreover, since $\psi_1(\delta_l) - \frac{1}{T_c} = 0$, $\psi_2(\delta_l) < \psi_1(\delta_l) = \frac{1}{T_c}$. It follows that $0 < \delta_l < \psi_2^{-1}\left(\frac{1}{T_c}\right)$. Then, according to Sandwich theorem, we have $\lim\limits_{T_c\to+\infty} \delta_l = 0^+$. Moreover, it easily follows from (99) that $\lim\limits_{T_c\to+\infty} \delta_u = 0^+$. By combing the results of $\lim\limits_{T_c\to+\infty} \delta_u = 0^+$ and $\lim\limits_{T_c\to+\infty} \delta_l = 0^+$ with (101), we can obtain

$$\lim_{T_c\to+\infty} \frac{\delta_u}{\delta_l} = 1^-, \quad (105)$$

which is equivalent to $\lim\limits_{T_c\to+\infty} b_u^* - b_l^* \to 0$.

Similarly, when $\frac{K}{M-K+1} \leq \phi \leq 1$, according to (34) and (35) we have

$$\int_0^1 \frac{(\delta_u A_1)^{\frac{2}{\alpha}}(1-x)^{M-2}}{A_3\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x - \frac{\delta_u(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3\left[\delta_u(A_1-A_2)+A_2\right]^{1-\frac{2}{\alpha}}} - \frac{1}{T_c} = 0, \quad (106)$$

$$\int_0^1 \frac{(\delta_l A_1)^{\frac{2}{\alpha}}(1-x)^{M-2}}{(\delta_l A_1)^{\frac{2}{\alpha}} + A_3\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x - \frac{1}{T_c} = 0. \quad (107)$$

Then, (106) minus (107) gives

$$\int_0^1 \frac{(1-x)^{M-2}\left[1 + \frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}} - \frac{A_3}{(\delta_u A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\right]}{\frac{A_3}{(\delta_u A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\left[1 + \frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\right]}\mathrm{d}x$$
$$-\frac{\delta_u(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3[\delta_u(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}}} = 0.$$

$$\Rightarrow \quad \left[1-\left(\frac{\delta_u}{\delta_l}\right)^{\frac{2}{\alpha}}\right]\int_0^1 \frac{(1-x)^{M-2}}{1+\frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x + \frac{\delta_u(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3[\delta_u(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}}}$$

$$= \left(\frac{\delta_u}{\delta_l}\right)^{\frac{2}{\alpha}}\int_0^1 \frac{(1-x)^{M-2}}{\left[1+\frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\right]\frac{A_3}{(\delta_l A_1)^{\frac{2}{\alpha}}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x$$

$$\Rightarrow \quad \left[1-\left(\frac{\delta_u}{\delta_l}\right)^{\frac{2}{\alpha}}\right]\int_0^1 \frac{(1-x)^{M-2}}{\delta_l^{\frac{2}{\alpha}}+A_3A_1^{-\frac{2}{\alpha}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x$$

$$= \left(\frac{\delta_u}{\delta_l}\right)^{\frac{2}{\alpha}}\left\{\int_0^1 \frac{(1-x)^{M-2}}{\left[1+\delta_l^{-\frac{2}{\alpha}}A_3A_1^{-\frac{2}{\alpha}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}\right]A_3A_1^{-\frac{2}{\alpha}}\left(\frac{\phi x}{1-x}\right)^{\frac{2}{\alpha}}}\mathrm{d}x \right.$$
$$\left. -\frac{\delta_u^{1-\frac{2}{\alpha}}(A_1-A_2)\beta\left(1-\frac{2}{\alpha}, M+\frac{2}{\alpha}\right)}{\phi^{\frac{2}{\alpha}}A_3[\delta_u(A_1-A_2)+A_2]^{1-\frac{2}{\alpha}}}\right\}. \tag{108}$$

Moreover, according to (107) it is easy to prove that $\lim\limits_{T_c\to+\infty}\delta_l = 0^+$. It follows from Sandwich theorem that $\lim\limits_{T_c\to+\infty}\delta_u = 0^+$. It follows from (108) that $\lim\limits_{T_c\to+\infty} b_u^* - b_l^* \to 0$ for the same reason as that in the case with $0<\phi<\frac{K}{M-K+1}$.

F. Proof of Corollary 4

The property of B_{ax}^* as a function of T_c can be easily observed. We then prove the property of B_{ax}^* as a function of ϕ. Only the term $A_1^{\frac{2}{\alpha}}\phi^{-\frac{2}{\alpha}}A_3^{-1}$ inside the logarithm is related to ϕ, where $A_1 = \frac{K-\phi}{K(M-1)}$ as defined in *Theorem* 1 and $A_3 \triangleq \Gamma\left(1-\frac{2}{\alpha}\right)\mathbb{E}_{X_i}\left[X_i^{\frac{2}{\alpha}}\right]$ with X_i given by (57). $X_i \overset{d.}{=} \frac{1}{K}g_{K,1} + \frac{1-\phi}{\phi(M-K)}g_{M-K,1}$, where $g_{K,1}\sim \text{Gamma}(K,1)$ and $g_{M-K,1}\sim\text{Gamma}(K,1)$ are two independent RVs. We define $\vartheta(\phi) \triangleq \left(\frac{\phi}{K-\phi}\right)^{\frac{2}{\alpha}}\mathbb{E}_{X_i}\left\{X_i^{\frac{2}{\alpha}}\right\} = \mathbb{E}\left\{\left(\frac{\frac{\phi}{K}g_{K,1}+\frac{1-\phi}{M-K}g_{M-K,1}}{K-\phi}\right)^{\frac{2}{\alpha}}\right\}$, which is a continuous function of ϕ. Then, $A_1^{\frac{2}{\alpha}}\phi^{-\frac{2}{\alpha}}A_3^{-1} = [K(M-1)]^{-\frac{2}{\alpha}}\left[\Gamma\left(1-\frac{2}{\alpha}\right)\right]^{-1}[\vartheta(\phi)]^{-1}$. It follows that $\frac{\mathrm{d}B_{ax}^*(\phi)}{\mathrm{d}\phi} = -(M-1)\alpha\frac{\vartheta(\phi)}{2}\frac{\mathrm{d}\vartheta(\phi)}{\mathrm{d}\phi}$.

Moreover, we have

$$\frac{\mathrm{d}\vartheta(\phi)}{\mathrm{d}\phi} = \frac{2}{\alpha(K-\phi)^2}\mathbb{E}\left\{\left(\frac{\frac{\phi}{K}g_{K,1}+\frac{1-\phi}{M-K}g_{M-K,1}}{K-\phi}\right)^{\frac{2}{\alpha}-1}\left(g_{K,1}-\frac{K-1}{M-K}g_{M-K,1}\right)\right\}, \tag{109}$$

and when $\alpha>2$, it is easy to observe that

$$\frac{\mathrm{d}}{\mathrm{d}\phi}\mathbb{E}\left\{\left(\frac{\frac{\phi}{K}g_{K,1}+\frac{1-\phi}{M-K}g_{M-K,1}}{K-\phi}\right)^{\frac{2}{\alpha}-1}\left(g_{K,1}-\frac{K-1}{M-K}g_{M-K,1}\right)\right\}$$
$$= \frac{\frac{2}{\alpha}-1}{(K-\phi)^2}\mathbb{E}\left\{\left(\frac{\frac{\phi}{K}g_{K,1}+\frac{1-\phi}{M-K}g_{M-K,1}}{K-\phi}\right)^{\frac{2}{\alpha}-2}\left(g_{K,1}-\frac{K-1}{M-K}g_{M-K,1}\right)^2\right\} < 0. \tag{110}$$

Thus, $\mathbb{E}\left\{\left(\frac{\frac{\phi}{K}g_{K,1}+\frac{1-\phi}{M-K}g_{M-K,1}}{K-\phi}\right)^{\frac{2}{\alpha}-1}\left(g_{K,1}-\frac{K-1}{(M-K)}g_{M-K,1}\right)\right\}$ is a monotonic decreasing function of ϕ, and the minimum of it is achieved when $\phi \to 1^-$, which is given by

$$\begin{aligned}
&\lim_{\phi\to 1^-} \mathbb{E}\left\{\left(\frac{\frac{\phi}{K}g_{K,1}+\frac{1-\phi}{M-K}g_{M-K,1}}{K-\phi}\right)^{\frac{2}{\alpha}-1}\left(g_{K,1}-\frac{K-1}{M-K}g_{M-K,1}\right)\right\}\\
=\;& \left[K(K-1)\right]^{1-\frac{2}{\alpha}}\mathbb{E}\left\{(g_{K,1})^{\frac{2}{\alpha}-1}\left(g_{K,1}-\frac{K-1}{M-K}g_{M-K,1}\right)\right\}\\
=\;& \left[K(K-1)\right]^{1-\frac{2}{\alpha}}\left\{\mathbb{E}\left[(g_{K,1})^{\frac{2}{\alpha}}\right]-(K-1)\mathbb{E}\left[(g_{K,1})^{\frac{2}{\alpha}-1}\right]\right\}.
\end{aligned}$$

Moreover,

$$\mathbb{E}\left[(g_{K,1})^{\frac{2}{\alpha}}\right]-(K-1)\mathbb{E}\left[(g_{K,1})^{\frac{2}{\alpha}-1}\right]=\int_0^\infty x^{\frac{2}{\alpha}}\frac{x^{K-1}e^{-x}}{\Gamma(K)}\mathrm{d}x-(K-1)\int_0^\infty x^{\frac{2}{\alpha}-1}\frac{x^{K-1}e^{-x}}{\Gamma(K)}\mathrm{d}x$$

$$= \frac{1}{\Gamma(K)}\left\{\Gamma\left(K+\frac{2}{\alpha}\right)-(K-1)\Gamma\left(K+\frac{2}{\alpha}-1\right)\right\} \tag{111}$$

$$= \frac{2}{\alpha}\frac{\Gamma\left(K+\frac{2}{\alpha}-1\right)}{\Gamma(K)}>0, \tag{112}$$

where (111) follows from [28, 3.351.3] and (112) follows from [28, 8.331.1]. It follows from (109) that $\frac{\mathrm{d}\vartheta(\phi)}{\mathrm{d}\phi}>0$ for any ϕ, and thus $\vartheta(\phi)$ is a monotonic increasing function of ϕ. Then, the proof is completed by noticing that $A_1^{\frac{2}{\alpha}}\phi^{-\frac{2}{\alpha}}A_3^{-1}$ is a monotonic decreasing function of ϕ.

G. *Proof of Theorem 5*

According to (48) and (49), we can lower-bound R_u^L as

$$R_u^L \geq \mathbb{E}\left\{\log_2\left(1+\frac{A_2[X+(1-\delta)Y]}{A_1\delta Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\phi\mathbb{E}\left[\frac{1}{K}\|\mathbf{h}_{i,k}\mathbf{W}_i\|^2+\frac{1-\phi}{\phi(M-K)}\|\mathbf{h}_{i,k}\mathbf{Z}_i\|^2\right]}\right)\right\} \tag{113}$$

$$= \mathbb{E}\left\{\log_2\left(1+\frac{A_2[X+(1-\delta)Y]}{A_1\delta Y+r_{b_0,k}^{\alpha}\tilde{I}_{u,k}}\right)\right\}\triangleq \tilde{R}_u^L, \tag{114}$$

where we let $\tilde{I}_{u,k}=\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}$, (113) follows from Jensen's inequality and the distributions $\|\mathbf{h}_{i,k}\mathbf{W}_i\|^2\sim$ Gamma$(K,1)$ and $\|\mathbf{h}_{i,k}\mathbf{Z}_i\|^2\sim$ Gamma$(M-K,1)$. X and Y are the RVs defined in (47). Then, by using the independence among the RVs $r_{i,k}$ ($i\in\Phi_b$), X and Y, $\tilde{R}_u^L$ can be obtained with *Lemma* 1 as

$$\tilde{R}_u^L=\log_2 e\int_0^\infty\frac{\mathcal{M}_1(z)-\mathcal{M}_2(z)}{z}\tilde{\mathcal{M}}_3(z)\mathrm{d}z, \tag{115}$$

where $\mathcal{M}_1(z)$ and $\mathcal{M}_2(z)$ are given respectively by (51) and (52), and

$$\tilde{\mathcal{M}}_3(z) \;=\; \mathbb{E}\left\{e^{-zr_{b_0,k}^{\alpha}\tilde{I}_{u,k}}\right\}=\mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{\tilde{I}_{u,k}}\left(r_{b_0,k}^{\alpha}z\right)\right\}. \tag{116}$$

Moreover, we have

$$\mathcal{L}_{\tilde{I}_{u,k}}(s)=\mathbb{E}_{\tilde{I}_{u,k}}\left[\exp\left(-s\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\right)\right]=\mathbb{E}_{\Phi_b}\left\{\prod_{i\in\Phi_b\setminus\{b_0\}}\exp\left(-sr_{i,k}^{-\alpha}\right)\right\}$$

$$=\exp\left(-\lambda_b\int_{\mathcal{R}^2\setminus\mathfrak{B}(o,r_{b_0,k})}\left(1-e^{-sr^{-\alpha}}\right)\Lambda(\mathrm{d}r)\right)=\exp\left(-2\pi\lambda_b\int_{r_{b_0,k}}^{\infty}\left(1-e^{-sr^{-\alpha}}\right)r\mathrm{d}r\right) \quad (117)$$

It follows that

$$\mathcal{L}_{\tilde{I}_{u,k}}\left(r_{b_0,k}^{\alpha}z\right) \;=\; \exp\left(-2\pi\lambda_b\int_{r_{b_0,k}}^{\infty}(1-e^{-zr_{b_0,k}^{\alpha}r^{-\alpha}})r\mathrm{d}r\right)=\exp\left(-\pi\lambda_b r_{b_0,k}^2\tilde{\Xi}(z)\right), \quad (118)$$

where we let $\tilde{\Xi}(z)\triangleq z^{\frac{2}{\alpha}}\int_{z^{-\frac{2}{\alpha}}}^{\infty}\left(1-e^{-u^{-\frac{\alpha}{2}}}\right)\mathrm{d}u$, and (118) is obtained by the change of variables $u=z^{-\frac{2}{\alpha}}r_{b_0,k}^{-2}r^2$. $\tilde{\Xi}(z)$ can be obtained as

$$\begin{aligned}\tilde{\Xi}(z)&=z^{\frac{2}{\alpha}}\int_{z^{-\frac{2}{\alpha}}}^{\infty}\left(1-e^{-u^{-\frac{\alpha}{2}}}\right)\mathrm{d}u=-z^{-\frac{2}{\alpha}}\int_0^z\left(1-e^{-x}\right)\mathrm{d}x^{\frac{2}{\alpha}}\\&=z^{\frac{2}{\alpha}}\left(-x^{-\frac{2}{\alpha}}\Big|_0^z+\int_0^z e^{-x}\mathrm{d}x^{-\frac{2}{\alpha}}\right)=z^{\frac{2}{\alpha}}\left(-x^{-\frac{2}{\alpha}}\Big|_0^z+x^{-\frac{2}{\alpha}}e^{-x}\Big|_0^z+\int_0^z x^{-\frac{2}{\alpha}}e^{-x}\mathrm{d}x\right)\\&=-1+e^{-z}+z^{\frac{2}{\alpha}}\gamma\left(1-\frac{2}{\alpha},z\right),\end{aligned} \quad (119)$$

where the last equality is obtained by using [28, 3.381.1]. Following the similar method to obtain $\mathcal{M}_3(z)$ in (76), $\tilde{\mathcal{M}}_3(z)$ can be obtained as

$$\begin{aligned}\tilde{\mathcal{M}}_3(z)&=\mathbb{E}_{r_{b_0,k}}\left\{\mathcal{L}_{\tilde{I}_{u,k}}\left(r_{b_0,k}^{\alpha}z\right)\right\}=\mathbb{E}_{r_{b_0,k}}\left\{\exp\left(-\pi\lambda_b r_{b_0,k}^2\tilde{\Xi}(z)\right)\right\}\\&=\int_0^{\infty}\exp\left(-\pi\lambda_b r^2\tilde{\Xi}(z)\right)2\pi\lambda_b r\exp(-\pi\lambda_b r^2)\mathrm{d}r=\int_0^1 t^{\tilde{\Xi}(z)}\mathrm{d}t=\frac{1}{1+\tilde{\Xi}(z)}.\end{aligned} \quad (120)$$

Then, $\tilde{R}_u^L$ in (39) is obtained by substituting (51), (52) and (120) into (115).

Moreover, by using (77), an upper bound on $R_{e,sum}$ can be obtained as

$$\begin{aligned}R_{e,sum}=&\,\mathbb{E}\Bigg\{\log_2\Bigg(\frac{\phi}{K}\mathbf{g}_{b_0,j^\star}\mathbf{W}_{b_0}\mathbf{W}_{b_0}^H\mathbf{g}_{b_0,j^\star}^H+\frac{1-\phi}{M-K}\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\mathbf{Z}_{b_0}^H\mathbf{g}_{b_0,j^\star}^H\\&+d_{b_0,j^\star}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}d_{i,j^\star}^{-\alpha}\left[\frac{\phi}{K}\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2+\frac{1-\phi}{M-K}\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\right]\Bigg)\Bigg\}\\&-\mathbb{E}\left\{\log_2\left(\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2+d_{b_0,j^\star}^{\alpha}I_{e,j^\star}\right)\right\}\\\geq&\,\mathbb{E}\Bigg\{\log_2\Bigg(\frac{\phi}{K}\mathbf{g}_{b_0,j^\star}\mathbb{E}\left(\mathbf{W}_{b_0}\mathbf{W}_{b_0}^H\right)\mathbf{g}_{b_0,j^\star}^H+\frac{1-\phi}{M-K}\mathbf{g}_{b_0,j^\star}\mathbb{E}\left(\mathbf{Z}_{b_0}\mathbf{Z}_{b_0}^H\right)\mathbf{g}_{b_0,j^\star}^H\\&+d_{b_0,j^\star}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}d_{i,j^\star}^{-\alpha}\mathbb{E}\left[\frac{\phi}{K}\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2+\frac{1-\phi}{M-K}\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\right]\Bigg)\Bigg\}\\&-\mathbb{E}\left\{\log_2\left(\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2+d_{b_0,j^\star}^{\alpha}I_{e,j^\star}\right)\right\} \quad (121)\\=&\,\mathbb{E}\left\{\log_2\left(\frac{1}{M}\|\mathbf{g}_{b_0,j^\star}\|^2+d_{b_0,j^\star}^{\alpha}\tilde{I}_{e,j^\star}\right)\right\}\\&-\mathbb{E}\left\{\log_2\left(\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2+d_{b_0,j^\star}^{\alpha}I_{e,j^\star}\right)\right\}\triangleq R_{e,sum}^U \quad (122)\end{aligned}$$

where (121) is obtained by using Jensen's inequality, and we let $\tilde{I}_{e,j^\star} \triangleq \sum_{i=1,i\neq b_0}^{\infty} d_{i,j^\star}^{-\alpha}$ in (122). (122) follows by using the results that $\mathbb{E}\left(\mathbf{W}_{b_0}\mathbf{W}_{b_0}^H\right) = \frac{K}{M}\mathbf{I}_M$ and $\mathbb{E}\left(\mathbf{Z}_{b_0}\mathbf{Z}_{b_0}^H\right) = \frac{M-K}{M}\mathbf{I}_M$ which was obtained in [10, Appendix C].

The first term in (122) can be written as

$$\begin{aligned}
&\mathbb{E}\left\{\log_2\left(\frac{1}{M}\|\mathbf{g}_{b_0,j^\star}\|^2 + d_{b_0,j^\star}^{\alpha}\tilde{I}_{e,j^\star}\right)\right\} \\
&= \mathbb{E}\left\{\log_2\left(\|\mathbf{g}_{b_0,j^\star}\|^2\right) + \log_2\left(1+\frac{d_{b_0,j^\star}^{\alpha}\tilde{I}_{e,j^\star}}{\frac{1}{M}\|\mathbf{g}_{b_0,j^\star}\|^2}\right)\right\} - \log_2 M, \qquad (123)
\end{aligned}$$

where

$$\mathbb{E}\left\{\log_2\left(\|\mathbf{g}_{b_0,j^\star}\|^2\right)\right\} = \int_0^\infty \log_2 x \frac{e^{-x}x^{M-1}}{\Gamma(M)}\mathrm{d}x = \frac{\psi(M)}{\ln 2}, \qquad (124)$$

which is obtained by using [28, 4.352.1], and

$$\mathbb{E}\left\{\log_2\left(1+\frac{d_{b_0,j^\star}^{\alpha}\tilde{I}_{e,j^\star}}{\frac{1}{M}\|\mathbf{g}_{b_0,j^\star}\|^2}\right)\right\} = \frac{1}{\ln 2}\int_0^\infty \frac{1-\mathbb{E}\left\{e^{-z d_{b_0,j^\star}^{\alpha}\tilde{I}_{e,j^\star}}\right\}}{z}\mathbb{E}\left\{e^{-z\frac{1}{M}\|\mathbf{g}_{b_0,j^\star}\|^2}\right\}\mathrm{d}z, \qquad (125)$$

which again is obtained using *Lemma* 1. It can be obtained that $\mathbb{E}\left\{e^{-z\frac{1}{M}\|\mathbf{g}_{b_0,j^\star}\|^2}\right\} = \left(1+\frac{1}{M}z\right)^{-M}$. Similarly to Laplace transform of $I_{e,j^\star}$ obtained in (83), $\mathcal{L}_{\tilde{I}_{e,j^\star}}(s)$ can be obtained as

$$\begin{aligned}
\mathcal{L}_{\tilde{I}_{e,j^\star}}(s) &= \mathbb{E}_{\tilde{I}_{e,j^\star}}\left[\exp\left(-s\sum_{i=1}^{\infty} d_{i,j^\star}^{-\alpha}\right)\right] = \mathbb{E}_{\Phi_b}\left\{\prod_{i\in\Phi_b}\exp\left(-s d_{i,j^\star}^{-\alpha}\right)\right\} \qquad (126)\\
&= \exp\left(-\lambda_b\int_{\mathcal{R}^2}\left\{1-\exp\left(-sr^{-\alpha}\right)\right\}\Lambda(\mathrm{d}r)\right) \qquad (127)\\
&= \exp\left(-2\pi\lambda_b\int_0^\infty\left\{1-\exp\left(-sr^{-\alpha}\right)\right\}r\mathrm{d}r\right)\\
&= \exp\left(-\pi\lambda_b\Gamma\left(1-\frac{2}{\alpha}\right)s^{\frac{2}{\alpha}}\right). \qquad (128)
\end{aligned}$$

It follows that

$$\begin{aligned}
&\mathbb{E}_{d_{b_0,j^\star}}\left\{\mathcal{L}_{\tilde{I}_{e,j^\star}}(d_{b_0,j^\star}^{\alpha}z)\right\} = \mathbb{E}_{d_{b_0,j^\star}}\left\{\exp\left(-\pi\lambda_b\Gamma\left(1-\frac{2}{\alpha}\right)d_{b_0,j^\star}^{2}z^{\frac{2}{\alpha}}\right)\right\}\\
=\ &\int_0^\infty \exp\left(-\pi\lambda_b\Gamma\left(1-\frac{2}{\alpha}\right)z^{\frac{2}{\alpha}}r^2\right)2\pi\lambda_e r\exp\left(-\pi\lambda_e r^2\right)\mathrm{d}r\\
=\ &\int_0^1 t^{\frac{\lambda_b}{\lambda_e}\Gamma\left(1-\frac{2}{\alpha}\right)z^{\frac{2}{\alpha}}}\mathrm{d}t = \frac{1}{1+\frac{\lambda_b}{\lambda_e}\Gamma\left(1-\frac{2}{\alpha}\right)z^{\frac{2}{\alpha}}}. \qquad (129)
\end{aligned}$$

Similarly, the second term in (122) can be obtained as

$$\begin{aligned}
&\mathbb{E}\left\{\log_2\left(\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j^\star}^{\alpha}I_{e,j^\star}\right)\right\}\\
&= \mathbb{E}\left\{\log_2\left(\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2\right) + \log_2\left(1+\frac{d_{b_0,j^\star}^{\alpha}I_{e,j^\star}}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2}\right)\right\} + \log_2\left(\frac{1-\phi}{M-K}\right), \qquad (130)
\end{aligned}$$

where

$$\mathbb{E}\left\{\log_2\left(\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2\right)\right\} = \int_0^\infty \log_2 x \frac{e^{-x}x^{M-K-1}}{\Gamma(M-K)} = \frac{\psi(M-K)}{\ln 2},$$

$$\mathbb{E}\left\{\log_2\left(1+\frac{d_{b_0,j^\star}^\alpha I_{e,j^\star}}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2}\right)\right\} = \frac{1}{\ln 2}\int_0^\infty \frac{1-\mathbb{E}\left\{e^{-zd_{b_0,j^\star}^\alpha I_{e,j^\star}}\right\}}{z}\mathbb{E}\left\{e^{-z\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2}\right\}\mathrm{d}z.$$

Using the results in (86), it can be obtained that $\mathbb{E}\left\{e^{-zd_{b_0,j^\star}^\alpha I_{e,j^\star}}\right\} = \left[1+\frac{\lambda_b}{\lambda_e}A_3(\phi z)^{\frac{2}{\alpha}}\right]^{-1}$ and $\mathbb{E}\left\{e^{-z\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2}\right\} = \left(1+\frac{1-\phi}{M-K}z\right)^{-(M-K)}$. The proof is completed by combining the above results and noticing that R_e^U in (40) is obtained as $R_e^U = \frac{R_{e,sum}^U}{K} \geq \frac{R_{e,sum}}{K}$.

H. *Proof of Lemma 4*

We consider $\check{R}_{\text{Net}}^L|_{B=B_{ax}^*}$ as a composite function of $\delta^* \triangleq 2^{-\frac{B_{ax}^*(\phi)}{M-1}}$ and ϕ, i.e., $\check{R}_{\text{Net}}^L[\delta^*,\phi]$, where δ^* is also a functions of ϕ. Then, we can obtain that

$$\frac{\mathrm{d}\check{R}_{\text{Net}}^L[\delta^*,\phi]}{\mathrm{d}\phi}\times\ln 2 = \left(\frac{\partial\check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\phi}+\frac{\partial\check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\delta^*}\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)\times\ln 2. \tag{131}$$

According to (114), we have

$$\begin{aligned}\tilde{R}_u^L &= \mathbb{E}\left\{\log_2\left(1+\frac{A_2[X+(1-\delta)Y]}{A_1\delta Y+r_{b_0,k}^\alpha\tilde{I}_{u,k}}\right)\right\}\\ &= \mathbb{E}\left\{\log_2\left(A_2X+[A_1\delta+A_2(1-\delta)]Y+r_{b_0,k}^\alpha\tilde{I}_{u,k}\right)\right\}-\mathbb{E}\left\{\log_2\left(A_1\delta Y+r_{b_0,k}^\alpha\tilde{I}_{u,k}\right)\right\}.\end{aligned}\tag{132}$$

With (132) and (122), each term in (131) can be obtained as

$$\begin{aligned}\frac{\partial\check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\phi}\times\ln 2 &= \left\{\frac{\partial\hat{R}_u^L[\delta^*,\phi]}{\partial\phi}-\frac{\partial R_e^U(\phi)}{\partial\phi}-\frac{1}{T_c}\frac{\partial B_{ax}^*(\delta^*)}{\partial\phi}\right\}\times\ln 2\\ &= \mathbb{E}\left\{\frac{\frac{M-K}{K(M-1)}(X+Y)-\frac{M-K+1}{K(M-1)}\delta^*Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^\alpha\sum_{i=1,i\neq b_0}^\infty r_{i,k}^{-\alpha}}\right.\\ &\quad+\frac{\frac{1}{K(M-1)}\delta^*Y}{\frac{K-\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^\alpha\sum_{i=1,i\neq b_0}^\infty r_{i,k}^{-\alpha}}\\ &\quad\left.+\frac{1}{K}\frac{-\frac{1}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2+d_{b_0,j^\star}^\alpha\sum_{i=1,i\neq b_0}^\infty d_{i,j^\star}^{-\alpha}\left[\frac{1}{K}\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2-\frac{1}{M-K}\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\right]}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2+d_{b_0,j^\star}^\alpha I_{e,j^\star}}\right\},\end{aligned}\tag{133}$$

$$\begin{aligned}\frac{\partial\check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\delta^*}\times\ln 2 &= \left\{\frac{\partial\hat{R}_u^L[\delta^*,\phi]}{\partial\delta^*}-\frac{\partial R_e^U(\phi)}{\partial\delta^*}-\frac{1}{T_c}\frac{\mathrm{d}B_{ax}^*(\delta^*)}{\mathrm{d}\delta^*}\right\}\times\ln 2\\ &= \mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^\alpha\sum_{i=1,i\neq b_0}^\infty r_{i,k}^{-\alpha}}\right.\\ &\quad\left.-\frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^\alpha\sum_{i=1,i\neq b_0}^\infty r_{i,k}^{-\alpha}}+\frac{M-1}{T_c\delta^*}\right\},\end{aligned}\tag{134}$$

where $B_{ax}^* = -(M-1)\log_2 \delta^*$ and $\frac{\mathrm{d}B_{ax}^*}{\mathrm{d}\delta^*} = -\frac{M-1}{\delta^* \ln 2}$ are employed, and $I_{e,j}$ is given by (8).

Further, the second-order derivative can be obtained as

$$\frac{\mathrm{d}^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\mathrm{d}\phi^2}\times\ln 2 = \left\{\frac{\partial^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\phi^2} + 2\frac{\partial^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\phi\partial\delta^*}\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi} + \frac{\partial^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial(\delta^*)^2}\left(\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)^2 \right.$$
$$\left. + \frac{\partial \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\delta^*}\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}\right\}\times\ln 2 \tag{135}$$

Each term of (135) can be obtained individually as follows.

$$\frac{\partial^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\phi^2}\times\ln 2 = \left\{\frac{\partial^2 \hat{R}_u^L[\delta^*,\phi]}{\partial\phi^2} - \frac{\partial^2 R_e^U(\phi)}{\partial\phi^2}\right\}\times\ln 2$$
$$= -\mathbb{E}\left\{\frac{\left[\frac{M-K}{K(M-1)}(X+Y) - \frac{M-K+1}{K(M-1)}\delta^* Y\right]^2}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y) + \frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\right)^2}\right\}$$
$$+\left(\frac{\delta^*}{K(M-1)}\right)^2\mathbb{E}\left\{\frac{Y^2}{\left(\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\right)^2}\right\}$$
$$-\frac{1}{K}\mathbb{E}\left\{\left(\frac{-\frac{1}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j^\star}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} d_{i,j^\star}^{-\alpha}\left[\frac{1}{K}\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2 - \frac{1}{M-K}\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\right]}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j^\star}^{\alpha} I_{e,j^\star}}\right)^2\right\} \tag{136}$$

$$\frac{\partial^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\phi\partial\delta^*}\times\ln 2 = \frac{\partial^2 \hat{R}_u^L[\delta^*,\phi]}{\partial\phi\partial\delta^*}\times\ln 2$$
$$= \mathbb{E}\left\{\frac{\frac{1}{K(M-1)}Y r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}}{\left(\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\right)^2}\right\}$$
$$-\mathbb{E}\left\{\frac{\frac{M-K}{K(M-1)^2}(X+Y)Y + \frac{M-K+1}{K(M-1)}Y r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y) + \frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\right)^2}\right\}, \tag{137}$$

$$\frac{\partial^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial(\delta^*)^2}\times\ln 2 = \left\{\frac{\partial^2 \hat{R}_u^L[\delta^*,\phi]}{\partial(\delta^*)^2} - \frac{1}{T_c}\frac{\mathrm{d}^2 B_{ax}^*(\delta^*)}{\mathrm{d}(\delta^*)^2}\right\}\times\ln 2$$
$$= -\mathbb{E}\left\{\frac{\left[\frac{K-(M-K+1)\phi}{K(M-1)}Y\right]^2}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y) + \frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\right)^2}\right\}$$
$$+\mathbb{E}\left\{\frac{\left(\frac{K-\phi}{K(M-1)}Y\right)^2}{\left(\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\right)^2}\right\} - \frac{M-1}{T_c[\delta^*]^2}, \tag{138}$$

$$\frac{\partial \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\partial\delta^*} = \mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y) + \frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}}\right\}$$
$$-\mathbb{E}\left\{\frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}}\right\} + \frac{M-1}{T_c\delta^*}. \tag{139}$$

It follows that

$$\frac{\mathrm{d}^2\check{R}^L_{\mathrm{Net}}\left[\delta^*,\phi\right]}{\mathrm{d}\phi^2}\times\ln 2=\left\{\frac{\partial^2\check{R}^L_{\mathrm{Net}}\left[\delta^*,\phi\right]}{\partial\phi^2}+2\frac{\partial^2\check{R}^L_{\mathrm{Net}}\left[\delta^*,\phi\right]}{\partial\phi\partial\delta^*}\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}+\frac{\partial^2\check{R}^L_{\mathrm{Net}}\left[\delta^*,\phi\right]}{\partial\left(\delta^*\right)^2}\left(\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)^2\right.$$
$$\left.+\frac{\partial\check{R}^L_{\mathrm{Net}}\left[\delta^*,\phi\right]}{\partial\delta^*}\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}\right\}\times\ln 2 \tag{140}$$

$$=-\mathbb{E}\left\{\frac{\left[\frac{M-K}{K(M-1)}(X+Y)-\frac{M-K+1}{K(M-1)}\delta^*Y\right]^2}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}\right)^2}\right\}$$
$$+\left(\frac{\delta^*}{K(M-1)}\right)^2\mathbb{E}\left\{\frac{Y^2}{\left(\frac{K-\phi}{K(M-1)}\delta^*Y+r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}\right)^2}\right\}$$
$$-\frac{1}{K}\mathbb{E}\left\{\left(\frac{-\frac{1}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2+d^{\alpha}_{b_0,j^\star}\sum_{i=1,i\neq b_0}^{\infty}d^{-\alpha}_{i,j^\star}\left[\frac{1}{K}\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2-\frac{1}{M-K}\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\right]}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2+d^{\alpha}_{b_0,j^\star}I_{e,j^\star}}\right)^2\right\}$$
$$+2\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\mathbb{E}\left\{\frac{\frac{1}{K(M-1)}Yr^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}}{\left(\frac{K-\phi}{K(M-1)}\delta^*Y+r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}\right)^2}\right\}$$
$$-2\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\mathbb{E}\left\{\frac{\frac{M-K}{K(M-1)^2}(X+Y)Y+\frac{M-K+1}{K(M-1)}Yr^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}\right)^2}\right\}$$
$$-\left(\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)^2\mathbb{E}\left\{\frac{\left[\frac{K-(M-K+1)\phi}{K(M-1)}Y\right]^2}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}\right)^2}\right\}$$
$$+\left(\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)^2\mathbb{E}\left\{\frac{\left(\frac{K-\phi}{K(M-1)}Y\right)^2}{\left(\frac{K-\phi}{K(M-1)}\delta^*Y+r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}\right)^2}\right\}-\left(\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)^2\frac{M-1}{T_c\left[\delta^*\right]^2}$$
$$+\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}\mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}}\right\}$$
$$-\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}\mathbb{E}\left\{\frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^*Y+r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty}r^{-\alpha}_{i,k}}\right\}+\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}\times\frac{M-1}{T_c\delta^*}, \tag{141}$$

Moreover, by using the result in *Theorem* 4, δ^* can be expressed as

$$\delta^* \;=\; \left(\frac{1}{B\left(1-\frac{2}{\alpha},M-1+\frac{2}{\alpha}\right)T_c}\right)^{\frac{\alpha}{2}}\frac{\phi A_3^{\frac{\alpha}{2}}}{A_1}=c\frac{\phi A_3^{\frac{\alpha}{2}}}{K-\phi}, \tag{142}$$

where $c\triangleq K(M-1)\left(\frac{1}{B\left(1-\frac{2}{\alpha},M-1+\frac{2}{\alpha}\right)T_c}\right)^{\frac{\alpha}{2}}$. Then, we have

$$\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}=c\,\frac{(K-\phi)\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\phi A_3^{\frac{\alpha}{2}}\right)+\phi A_3^{\frac{\alpha}{2}}}{(K-\phi)^2}=c\,\frac{(K-\phi)\left[A_3^{\frac{\alpha}{2}}+\frac{\alpha}{2}\phi A_3^{\frac{\alpha}{2}-1}\frac{\mathrm{d}A_3}{\mathrm{d}\phi}\right]+\phi A_3^{\frac{\alpha}{2}}}{(K-\phi)^2}\triangleq c\varsigma, \tag{143}$$

where $\varsigma \triangleq \frac{K}{(K-\phi)^2} A_3^{\frac{\alpha}{2}} + \frac{\alpha}{2} \frac{\phi}{K-\phi} A_3^{\frac{\alpha}{2}-1} \frac{\mathrm{d}A_3}{\mathrm{d}\phi}$. According to the definition of $A_3 \triangleq \Gamma\left(1-\frac{2}{\alpha}\right) \mathbb{E}_{X_i}\left[X_i^{\frac{2}{\alpha}}\right]$, we can obtain

$$\frac{\mathrm{d}A_3}{\mathrm{d}\phi} = -\Gamma\left(1-\frac{2}{\alpha}\right) \frac{2}{\alpha} \frac{1}{\phi^2(M-K)} \mathbb{E}\left[X_i^{\frac{2}{\alpha}-1} g_{M-K,1}\right] < 0. \tag{144}$$

In addition, it follows from *Corollary* 4 that $\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi} > 0$ and thus $\varsigma > 0$.

Moreover, $\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}$ can be obtained as

$$\begin{aligned} \frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2} &= c\, \frac{(K-\phi)^2 \frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right) + 2\left[(K-\phi)\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\phi A_3^{\frac{\alpha}{2}}\right) + \phi A_3^{\frac{\alpha}{2}}\right]}{(K-\phi)^3} \\ &= c\left(\frac{\frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right)}{K-\phi} + \frac{2}{K-\phi}\varsigma\right), \end{aligned} \tag{145}$$

where

$$\frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right) = \alpha A_3^{\frac{\alpha}{2}-1} \frac{\mathrm{d}A_3}{\mathrm{d}\phi} + \frac{\alpha}{2}\left(\frac{\alpha}{2}-1\right) \phi A_3^{\frac{\alpha}{2}-2} \left(\frac{\mathrm{d}A_3}{\mathrm{d}\phi}\right)^2 + \frac{\alpha}{2} \phi A_3^{\frac{\alpha}{2}-1} \frac{\mathrm{d}^2 A_3}{\mathrm{d}\phi^2} \tag{146}$$

with

$$\frac{\mathrm{d}^2 A_3}{\mathrm{d}\phi^2} = \Gamma\left(1-\frac{2}{\alpha}\right) \frac{2}{\alpha} \mathbb{E}\left[\left(\frac{2}{\alpha}-1\right) X_i^{\frac{2}{\alpha}-2} \left(\frac{g_{M-K,1}}{\phi^2(M-K)}\right)^2 + \frac{2X_i^{\frac{2}{\alpha}-1}}{\phi^3(M-K)} g_{M-K,1}\right]. \tag{147}$$

We can observe from the simulation results of $B_{ax}^*(\phi)$ that the convexity/cancavity of $B_{ax}^*(\phi)$ is not determined. Actually, the sign of $\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}$ is not determined.

We can see that the expression of $\frac{\mathrm{d}^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\mathrm{d}\phi^2}$ given by (141) is very complicated. Thus, it is very difficult to strictly prove the sign of $\frac{\mathrm{d}^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\mathrm{d}\phi^2}$. However, we can observe that as $T_c \to +\infty$, c converges to zero at the seed of $\mathrm{O}\left(T_c^{-\frac{\alpha}{2}}\right)$, and $c^2 = \mathrm{O}\left(T_c^{-\alpha}\right)$ converges to zero even faster. Thus, both $(\delta^*)^2 = \mathrm{O}\left(c^2\right) = \mathrm{O}\left(T_c^{-\alpha}\right)$ and $\left(\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)^2 = \mathrm{O}\left(T_c^{-\alpha}\right)$ converge to zero as $T_c \to +\infty$. Moreover, it can be seen that all coefficients of the terms $(\delta^*)^2$ and $\left(\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)^2$ in (141) are finite for any ϕ. Thus, the terms $(\delta^*)^2$ and $\left(\frac{\mathrm{d}\delta^*}{\mathrm{d}\phi}\right)^2$ with $\frac{\mathrm{d}^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\mathrm{d}\phi^2}$ can be approximated zero with the sufficiently large T_c. Then, $\frac{\mathrm{d}^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\mathrm{d}\phi^2}$ can be approximated as

$$\begin{aligned} \frac{\mathrm{d}^2 \check{R}_{\text{Net}}^L[\delta^*,\phi]}{\mathrm{d}\phi^2} \times \ln 2 \approx -\mathbb{E}&\left\{\frac{\left[\frac{M-K}{K(M-1)}(X+Y) - \frac{M-K+1}{K(M-1)}\delta^* Y\right]^2}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y) + \frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha} \sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\right)^2}\right\} \\ -\frac{1}{K}\mathbb{E}&\left\{\left(\frac{-\frac{1}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j^\star}^{\alpha} \sum_{i=1,i\neq b_0}^{\infty} d_{i,j^\star}^{-\alpha}\left[\frac{1}{K}\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2 - \frac{1}{M-K}\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\right]}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d_{b_0,j^\star}^{\alpha} I_{e,j^\star}}\right)^2\right\} \\ -\frac{2c\varsigma}{K(M-1)}\mathbb{E}&\left\{\frac{\frac{M-K}{M-1}(X+Y)Y + (M-K+1)Y r_{b_0,k}^{\alpha} \sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y) + \frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha} \sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}\right)^2}\right\} \end{aligned}$$

$$+\frac{2c\varsigma}{K(M-1)}\mathbb{E}\left\{\frac{Yr_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}}{\left(\frac{K-\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\right)^2}\right\}-\frac{(M-1)(K-\phi)^2\varsigma^2}{T_c\phi^2A_3^{\alpha}}$$
$$+\mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}}\right.$$
$$\left.-\frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}}+\frac{M-1}{T_c\delta^*}\right\}\times\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2},\qquad(148)$$

where we have substituted into (142) and (143) into (141).

Recall the properties of $\tilde{R}_{\text{Net}}^L$ obtained in Section IV. When T_c is sufficiently large to satisfy the condition $\frac{\partial\tilde{R}_{\text{Net}}^L}{\partial B}\big|_{B=0}>0$ or equivalently $T_c>\frac{1}{\epsilon(\phi)}$, $B_{\text{real}}^*=b^*$ is the unique root of the equation $\frac{\partial\tilde{R}_{\text{Net}}^L}{\partial B}=0$ and at the neighborhood of B_{real}^* is a monotonic decreasing function of B. Then, since B_{ax}^* is an asymptotic upper bound on B_{real}^*, according to *Theorem* 4 it follows that $\frac{\partial\tilde{R}_{\text{Net}}^L}{\partial B}\Big|_{B=B_{ax}^*}=\left(\frac{\partial\tilde{R}_{\text{Net}}^L}{\partial\delta}\frac{\mathrm{d}\delta}{\mathrm{d}B}\right)\Big|_{B=B_{ax}^*}<0$. Moreover, since $\frac{\mathrm{d}\delta}{\mathrm{d}B}=-\frac{\ln 2}{(M-1)}\delta<0$, $\frac{\partial\tilde{R}_{\text{Net}}^L}{\partial\delta}\Big|_{\delta=\delta^*}>0$, i.e.,

$$\frac{\partial\tilde{R}_{\text{Net}}^L}{\partial\delta}\bigg|_{\delta=\delta^*}\times\ln 2=\left(\frac{\partial R_u^L}{\partial\delta}+\frac{M-1}{T_c\delta\ln 2}\right)\bigg|_{\delta=\delta^*}\times\ln 2$$
$$=\mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}I_{u,k}}-\frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}I_{u,k}}\right\}+\frac{M-1}{T_c\delta^*}>0.$$

Let $f(x)\triangleq\mathbb{E}_{X,Y}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+x}-\frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^*Y+x}\right\}$. Then, we can obtain

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x}=\mathbb{E}_{X,Y}\left\{-\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+x\right)^2}+\frac{\frac{K-\phi}{K(M-1)}Y}{\left(\frac{K-\phi}{K(M-1)}\delta^*Y+x\right)^2}\right\},\qquad(149)$$

and the second-order derivative of $f(x)$ w.r.t. x is

$$\frac{\mathrm{d}^2f(x)}{\mathrm{d}x^2}=\mathbb{E}_{X,Y}\left\{\frac{2\times\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+x\right)^3}-\frac{2\times\frac{K-\phi}{K(M-1)}Y}{\left(\frac{K-\phi}{K(M-1)}\delta^*Y+x\right)^3}\right\}.\qquad(150)$$

It can be easily observed from (150) that $\frac{\mathrm{d}^2f(x)}{\mathrm{d}x^2}<0$. Thus, $f(x)$ is a concave function of x. Then, it follows from Jensen's inequality that

$$\mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}I_{u,k}}-\frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}I_{u,k}}\right\}$$
$$<\mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}}\right.$$
$$\left.-\frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^*Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}}\right\},\qquad(151)$$

which, according to the definitions of $\tilde{R}_{\text{Net}}^L$ and $\check{R}_{\text{Net}}^L$, implies that $\left.\frac{\partial \check{R}_{\text{Net}}^L}{\partial \delta}\right|_{\delta=\delta^*} > \left.\frac{\partial \tilde{R}_{\text{Net}}^L}{\partial \delta}\right|_{\delta=\delta^*} > 0$, i.e.,

$$\left.\frac{\partial \check{R}_{\text{Net}}^L}{\partial \delta}\right|_{\delta=\delta^*} = \mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} - \frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} + \frac{M-1}{T_c\delta^*}\right\} > 0. \tag{152}$$

Moreover, it can be easily obtained by comparing the numerators and denominators of the two terms in $\frac{\partial \hat{R}_u^L}{\partial \delta}$ that

$$\left.\frac{\partial \hat{R}_u^L}{\partial \delta}\right|_{\delta=\delta^*} = \mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} - \frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}}\right\} < 0. \tag{153}$$

In the following, we first develop the upper bounds of the last term of (148) which we denote as

$$\Upsilon(\phi) \triangleq \mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} - \frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} + \frac{M-1}{T_c\delta^*}\right\} \times \frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}. \tag{154}$$

As we have mentioned above that the sign of $\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\phi^2}$ in (145) is not determined, we will study the upper bound of $\Upsilon(\phi)$ for two cases as follows.

(1) When $\frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right) \geq 0$, $\Upsilon(\phi)$ can be upper-bounded as

$$\Upsilon(\phi) < \mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} - \frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}}\right\} \times \frac{2}{K-\phi}c\varsigma + \frac{M-1}{T_c\delta^*} \times c\left(\frac{\frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right)}{K-\phi} + \frac{2}{K-\phi}\varsigma\right). \tag{155}$$

(2) When $\frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right) < 0$, $\Upsilon(\phi)$ can be upper-bounded as

$$\Upsilon(\phi) < \mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi}{K(M-1)}Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} - \frac{\frac{K-\phi}{K(M-1)}Y}{\frac{K-\phi}{K(M-1)}\delta^* Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} + \frac{M-1}{T_c\delta^*}\right\} \times \frac{2}{K-\phi}c\varsigma. \tag{156}$$

Substituting (155) and (156) into (148), we can obtain

$$
\begin{aligned}
&\frac{\mathrm{d}^2 \check{R}^L_{\text{Net}}\left[\delta^*,\phi\right]}{\mathrm{d}\phi^2}\times\ln 2 < -\mathbb{E}\left\{\frac{\left[\frac{M-K}{K(M-1)}(X+Y)-\frac{M-K+1}{K(M-1)}\delta^* Y\right]^2}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty} r^{-\alpha}_{i,k}\right)^2}\right\} \\
&-\frac{1}{K}\mathbb{E}\left\{\left(\frac{-\frac{1}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d^{\alpha}_{b_0,j^\star}\sum_{i=1,i\neq b_0}^{\infty} d^{-\alpha}_{i,j^\star}\left[\frac{1}{K}\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2-\frac{1}{M-K}\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\right]}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d^{\alpha}_{b_0,j^\star} I_{e,j^\star}}\right)^2\right\} \\
&\underbrace{-\frac{2c\varsigma}{K(M-1)}\mathbb{E}\left\{\frac{\frac{M-K}{M-1}(X+Y)Y+(M-K+1)Y r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty} r^{-\alpha}_{i,k}}{\left(\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty} r^{-\alpha}_{i,k}\right)^2}\right\}}_{(a)} \\
&\underbrace{+\frac{2c\varsigma}{K(M-1)}\mathbb{E}\left\{\frac{Y r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty} r^{-\alpha}_{i,k}}{\left(\frac{K-\phi}{K(M-1)}\delta^* Y + r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty} r^{-\alpha}_{i,k}\right)^2}\right\}}_{(b)} \\
&\underbrace{+\frac{2c\varsigma}{(K-\phi)K(M-1)}\mathbb{E}\left\{\frac{[K-(M-K+1)\phi]Y}{\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^* Y + r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty} r^{-\alpha}_{i,k}}\right\}}_{(c)} \\
&\underbrace{-\frac{2c\varsigma}{K(M-1)}\mathbb{E}\left\{\frac{Y}{\frac{K-\phi}{K(M-1)}\delta^* Y + r^{\alpha}_{b_0,k}\sum_{i=1,i\neq b_0}^{\infty} r^{-\alpha}_{i,k}}\right\}}_{(d)} + \Omega(\phi),
\end{aligned}
\tag{157}
$$

where $\Omega(\phi)$, according to (155) and (156), is given by

$$
\begin{aligned}
\Omega(\phi) \;&\triangleq\; \begin{cases} \frac{M-1}{T_c\delta^*}\times c\left(\frac{\frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right)}{K-\phi}+\frac{2}{K-\phi}\varsigma\right) - \frac{(M-1)(K-\phi)^2\varsigma^2}{T_c\phi^2 A_3^{\alpha}}, & \frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right)\geq 0 \\ \frac{M-1}{T_c\delta^*}\frac{2}{K-\phi}c\varsigma - \frac{(M-1)(K-\phi)^2\varsigma^2}{T_c\phi^2 A_3^{\alpha}}, & \text{otherwise} \end{cases} \\
&= \begin{cases} \frac{2(M-1)}{T_c\phi A_3^{\frac{\alpha}{2}}}\varsigma + \frac{(M-1)}{T_c\phi A_3^{\frac{\alpha}{2}}}\frac{\mathrm{d}^2\left(\phi A_3^{\frac{\alpha}{2}}\right)}{\mathrm{d}\phi^2} - \frac{(M-1)(K-\phi)^2\varsigma^2}{T_c\phi^2 A_3^{\alpha}}, & \frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\left(\phi A_3^{\frac{\alpha}{2}}\right)\geq 0 \\ \frac{2(M-1)}{T_c\phi A_3^{\frac{\alpha}{2}}}\varsigma - \frac{(M-1)(K-\phi)^2\varsigma^2}{T_c\phi^2 A_3^{\alpha}}, & \text{otherwise} \end{cases}.
\end{aligned}
\tag{158}
$$

We illustrate the effect of $\Omega(\phi)$ on the sign of $\frac{\mathrm{d}^2 \check{R}^L_{\text{Net}}[\delta^*,\phi]}{\mathrm{d}\phi^2}$ for two different scenarios. By using the result of $\frac{\mathrm{d}A_3}{\mathrm{d}\phi}$ given by (144) and $\frac{\mathrm{d}^2 A_3}{\mathrm{d}\phi^2}$ is given by (147) and the definition of ς above, it is easy to check that when ϕ is finite and not very small, $\Omega(\phi) = O(\frac{1}{T_c})$ tends to be zero as $T_c \to \infty$. Moreover, after some simple manipulations it is easy to see that when $\phi \to 0^+$ together with $T_c \to \infty$, the term $\frac{(M-1)(K-\phi)^2\varsigma^2}{T_c\phi^2 A_3^{\alpha}} = \mathrm{O}(\frac{1}{T_c\phi^2})$ and the other terms is $\mathrm{O}(\frac{1}{T_c\phi})$ in $\Omega(\phi)$. Thus, even though $(T_c\phi) \to 0^+$ as $\phi \to 0^+$ and $T_c \to \infty$, $\Omega(\phi) \to -\infty$, which implies $\Omega(\phi)$ is negative. Therefore, in the following we only focus on the former scenario and assume $\Omega(\phi) \approx 0$.

It can be easily observed that the sign of $(b)+(d)$ in (157) can be determined as

$$-\mathbb{E}\left\{\frac{Y}{\frac{K-\phi}{K(M-1)}\delta^{*}Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}}\right\}+\mathbb{E}\left\{\frac{Yr_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}}{\left(\frac{K-\phi}{K(M-1)}\delta^{*}Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\right)^{2}}\right\}$$
$$=-\mathbb{E}\left\{\frac{\frac{K-\phi}{K(M-1)}\delta^{*}Y^{2}}{\left(\frac{K-\phi}{K(M-1)}\delta^{*}Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\right)}\right\}<0. \tag{159}$$

Moreover, the sign of the combination of $(a)+(c)$ in (157) can be determined in the following for two cases.

(1) When $\frac{K}{M-K+1}\leq\phi\leq 1$, it is easy to see both the terms $(a)<0$ and $(c)<0$, which implies $\frac{\mathrm{d}^2\check{R}_{\text{Net}}^{L}[\delta^{*},\phi]}{\mathrm{d}\phi^{2}}<0$.

(2) When $0<\phi<\frac{K}{M-K+1}$, the problem is more involved. The combination of the terms $(b)+(d)$ in (157) can be written as $(b)+(d)=\frac{2\varsigma}{K(M-1)}\mathbb{E}\left[\frac{N(\phi)}{D(\phi)}\right]$, where the denominator $N(\phi)$ is given by

$$\begin{aligned}N(\phi)=&-c\left[\frac{M-K}{M-1}(X+Y)Y+(M-K+1)Yr_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\right]+\left[K-(M-K+1)\phi\right]Y\\&\times\left[\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^{*}Y+r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\right]\frac{c}{K-\phi}\\=&-c\frac{M-K}{M-1}(X+Y)Y+c\frac{(M-K)\phi}{K(M-1)}\frac{[K-(M-K+1)\phi]}{K-\phi}(X+Y)Y\\&-c(M-K+1)Y\,r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}+c\frac{K-(M-K+1)\phi}{K-\phi}Yr_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\\&+\frac{[K-(M-K+1)\phi]^{2}}{K(M-1)(K-\phi)}c\delta^{*}Y^{2},\end{aligned} \tag{160}$$

and the denominator $D(\phi)$ is given by $D(\phi)=\left(\frac{(M-K)\phi}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi}{K(M-1)}\delta^{*}Y+r_{b_0,k}^{\alpha}\right.$ $\left.\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}\right)^{2}$. The the sign of the first two terms with (160) is determined as

$$-\frac{M-K}{M-1}+\frac{(M-K)\phi}{K(M-1)}\frac{[K-(M-K+1)\phi]}{K-\phi}=-\frac{(K-\phi)^{2}(M-K)+(M-K)^{2}\phi^{2}}{K(M-1)(K-\phi)}<0.$$

The combination of the third of the fourth terms with (160) is given by $-\frac{K(M-K)}{K-\phi}Y\,r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty}r_{i,k}^{-\alpha}$ whose sign is negative. Then, the last term with (160) is given by $\frac{[K-(M-K+1)\phi]^{2}}{K(M-1)(K-\phi)}c\delta^{*}Y^{2}=\mathrm{O}\left(\frac{1}{T_c^{\alpha}}\right)$ which can be approximated as zero. Thus, the sign of the combination of the terms $(b)+(d)$ in (157) can be determined as negative for sufficiently large T_c. Then, the conclusion that $\phi^{\star}$ is monotonic decreasing in T_c follows by substituting all above results into (157). The proof is completed by observing from (131), (133) and (134) that, when $T_c\to\infty$, the root $\phi^{\star}$ of equation $\frac{\mathrm{d}\check{R}_{\text{Net}}^{L}[\delta^{*},\phi]}{\mathrm{d}\phi}=0$ can not be smaller than zero.

I. Proof of Corollary 5

We consider both $\phi^\star$ and $\delta^* = 2^{-\frac{B^*_{ax}(\phi)}{M-1}}$ are functions of T_c, where $B^*_{ax}(\phi)$ is given by (37). Meanwhile, $\delta^\star$ is also a function of $\phi^\star$, i.e., $\delta^\star = \delta^*\left[\phi^\star(T_c), T_c\right]$. $\phi^\star$ is the unique root of the equation $\frac{\mathrm{d}\check{R}^L_{\text{Net}}[\delta^*,\phi]}{\mathrm{d}\phi} \triangleq \mathcal{F}(\delta^*,\phi,T_c) = 0$. It follows that $\mathcal{F}(\delta^\star,\phi^\star,T_c) = 0$. Then, it can be obtained that

$$\begin{aligned}
&\frac{\mathrm{d}\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\mathrm{d}T_c} = \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\delta^\star} \times \left(\frac{\partial\delta^\star}{\partial\phi^\star}\frac{\mathrm{d}\phi^\star}{\mathrm{d}T_c} + \frac{\partial\delta^\star}{\partial T_c}\right) + \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\phi^\star}\frac{\mathrm{d}\phi^\star}{\mathrm{d}T_c} + \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial T_c}\\
&= \left(\frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\delta^\star}\frac{\partial\delta^\star}{\partial\phi^\star} + \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\phi^\star}\right) \times \frac{\mathrm{d}\phi^\star}{\mathrm{d}T_c} + \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\delta^\star}\frac{\partial\delta^\star}{\partial T_c} + \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial T_c} = 0.
\end{aligned}$$

Then, it follows that

$$\frac{\mathrm{d}\phi^\star}{\mathrm{d}T_c} = -\left(\frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\delta^\star}\frac{\partial\delta^\star}{\partial T_c} + \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial T_c}\right) \Big/ \left(\frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\delta^\star}\frac{\partial\delta^\star}{\partial\phi^\star} + \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\phi^\star}\right), \tag{161}$$

where it follows from *Lemma* 4 that $\left(\frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\delta^\star}\frac{\partial\delta^\star}{\partial\phi^\star} + \frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\phi^\star}\right) = \frac{\mathrm{d}^2\check{R}^L_{\text{Net}}[\delta^*,\phi]}{\mathrm{d}\phi^2}\Big|_{\phi=\phi^\star} < 0$. Moreover, it follows from (131), (133) and (134) that

$$\begin{aligned}
\mathcal{F}(\delta^\star,\phi^\star,T_c) = &\frac{1}{\ln 2}\mathbb{E}\left\{\frac{\frac{M-K}{K(M-1)}(X+Y) - \frac{M-K+1}{K(M-1)}\delta^\star Y}{\frac{(M-K)\phi^\star}{K(M-1)}(X+Y) + \frac{K-(M-K+1)\phi^\star}{K(M-1)}\delta^\star Y + r^\alpha_{b_0,k}\sum_{i=1,i\neq b_0}^\infty r^{-\alpha}_{i,k}}\right.\\
&+\frac{\frac{1}{K(M-1)}\delta^\star Y}{\frac{K-\phi^\star}{K(M-1)}\delta^\star Y + r^\alpha_{b_0,k}\sum_{i=1,i\neq b_0}^\infty r^{-\alpha}_{i,k}}\\
&\left.+\frac{1}{K}\frac{-\frac{1}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d^\alpha_{b_0,j^\star}\sum_{i=1,i\neq b_0}^\infty d^{-\alpha}_{i,j^\star}\left[\frac{1}{K}\|\mathbf{g}_{i,j^\star}\mathbf{W}_i\|^2 - \frac{1}{M-K}\|\mathbf{g}_{i,j^\star}\mathbf{Z}_i\|^2\right]}{\frac{1-\phi}{M-K}\|\mathbf{g}_{b_0,j^\star}\mathbf{Z}_{b_0}\|^2 + d^\alpha_{b_0,j^\star}I_{e,j^\star}}\right\}\\
&+\frac{1}{\ln 2}\mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi^\star}{K(M-1)}Y}{\frac{(M-K)\phi^\star}{K(M-1)}(X+Y) + \frac{K-(M-K+1)\phi^\star}{K(M-1)}\delta^\star Y + r^\alpha_{b_0,k}\sum_{i=1,i\neq b_0}^\infty r^{-\alpha}_{i,k}}\right.\\
&\left.-\frac{\frac{K-\phi^\star}{K(M-1)}Y}{\frac{K-\phi^\star}{K(M-1)}\delta^\star Y + r^\alpha_{b_0,k}\sum_{i=1,i\neq b_0}^\infty r^{-\alpha}_{i,k}} + \frac{M-1}{T_c\delta^\star}\right\} \times \frac{\partial\delta^\star}{\partial\phi^\star},
\end{aligned}\tag{162}$$

where $\frac{\partial\delta^\star}{\partial\phi^\star} = \frac{\mathrm{d}\delta^\star}{\mathrm{d}\phi}\big|_{\phi=\phi^\star} = K(M-1)\left(\frac{1}{B\left(1-\frac{2}{\alpha},M-1+\frac{2}{\alpha}\right)T_c}\right)^{\frac{\alpha}{2}}\varsigma(\phi^\star)$, which is obtained from (143) and is positive according to *Corollary* 4. It follows from (142) that

$$\frac{\partial\delta^\star}{\partial T_c} = -\frac{\alpha}{2}\frac{K(M-1)\phi A_3^{\frac{\alpha}{2}}}{(K-\phi)\left[B\left(1-\frac{2}{\alpha},M-1+\frac{2}{\alpha}\right)\right]^{\frac{\alpha}{2}}(T_c)^{\frac{\alpha}{2}+1}} < 0. \tag{163}$$

Thus, when T_c is sufficiently large, the term $\frac{\partial\delta^\star}{\partial T_c} \to 0^-$ at the speed of $\mathrm{O}\left(\frac{1}{(T_c)^{\frac{\alpha}{2}+1}}\right)$. It is easy to see that $\frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\delta^\star}$ is finite as T_c goes large. It follows that $\frac{\partial\mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial\delta^\star} \times \frac{\partial\delta^\star}{\partial T_c} = \mathrm{O}\left(\frac{\partial\delta^\star}{\partial T_c}\right) =$

$\mathrm{O}\left(\frac{1}{T_c^{\frac{\alpha}{2}+1}}\right)$. In addition, we can obtain that

$$\frac{\partial \mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial T_c} = \frac{1}{\ln 2}\mathbb{E}\left\{\frac{\frac{K-(M-K+1)\phi^\star}{K(M-1)}Y}{\frac{(M-K)\phi^\star}{K(M-1)}(X+Y)+\frac{K-(M-K+1)\phi^\star}{K(M-1)}\delta^\star Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}}\right.$$
$$\left. -\frac{\frac{K-\phi^\star}{K(M-1)}Y}{\frac{K-\phi^\star}{K(M-1)}\delta^\star Y + r_{b_0,k}^{\alpha}\sum_{i=1,i\neq b_0}^{\infty} r_{i,k}^{-\alpha}} + \frac{M-1}{T_c\delta^\star}\right\}\times\frac{\mathrm{d}\left(\frac{\partial\delta^\star}{\partial\phi^\star}\right)}{\mathrm{d}T_c} - \frac{1}{\ln 2}\frac{M-1}{(T_c)^2\delta^\star}\times\frac{\partial\delta^\star}{\partial\phi^\star}. \quad (164)$$

where

$$\frac{\mathrm{d}\left(\frac{\partial\delta^\star}{\partial\phi^\star}\right)}{\mathrm{d}T_c} = \frac{\left(-\frac{\alpha}{2}\right)K(M-1)\varsigma(\phi^\star)}{\left[B\left(1-\frac{2}{\alpha},M-1+\frac{2}{\alpha}\right)\right]^{\frac{\alpha}{2}}(T_c)^{\frac{\alpha}{2}+1}} < 0. \quad (165)$$

The expectation in (164) has been proved to be positive in (152). Moreover, it is easy to see that the first and the second additive term of $\frac{\partial \mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial T_c}$ both scale as $\mathrm{O}\left(\frac{1}{T_c^2}\right)$. Thus, $\frac{\partial \mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial T_c} = \mathrm{O}\left(\frac{1}{T_c^2}\right) \to 0^-$ as $T_c \to +\infty$. It follows that the numerator of (161) is dominated by the term $\frac{\partial \mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial T_c}$, and thus $\frac{\partial \mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial \delta^\star}\frac{\partial\delta^\star}{\partial T_c} + \frac{\partial \mathcal{F}(\delta^\star,\phi^\star,T_c)}{\partial T_c} = \mathrm{O}\left(\frac{1}{T_c^2}\right) \to 0^-$ as $T_c \to \infty$. The proof is completed by substituting all above results into (161).

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