Joint UL-DL Power Allocations for Massive MIMO URLLC IoT Networks: A Comparative Study of Different Pilot Patterns

Xinyu Tian, Liang Sun, Yuanwei Liu, and Lin Chen

Abstract

In this paper, we employ massive multiple-input and multiple-output (MIMO) system cellular network to support multiple Internet-of-Things (IoT) devices with ultra-reliability and low-latency communication (URLLC) which is required by some critical industrial applications. Specifically, we first derive lower bounds (LBs) on the achievable uplink (UL) and downlink (DL) data rates under the finite blocklength (FBL) and pilot contamination, where each base station (BS) employs maximum-ratio transmission (MRT) in the DL and maximum-ratio combining (MRC) in the UL detection with estimated channel state information. In addition, the LB rates are derived for two types of pilot of the regular pilot (RP) and superimposed pilot (SP). We study joint UL-DL power allocation optimization where the objective is to maximize the UL-DL overall average weighted sum rate (WSR) for the systems individually with RP and SP schemes. The objective functions are non-convex and the constraints are complicated rendering the considered optimization problems non-convex. We propose to employ successive convex approximation to approximately transform the original problems into a series of geometric program problems. Then, iterative algorithm is proposed for each problem to jointly optimize the UL and DL pilot and data payload power allocation. Simulation results are shown to compare the performances of the systems with RP and SP schemes for different settings. Simulation results also verify that the derived LB rates tightly match the corresponding ergodic rates and confirm the rapid convergence speed of the proposed iterative algorithms.

Index Terms

Ultra reliable low latency communications (URLLC), massive MIMO, superimposed pilot, finite blocklength, short-packet communications.

I. Introduction

The fifth generation (5G) wireless system and beyond are being studied and designed to support the various services of Internet of Things (IoT) in addition to the traditional broadband services

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[1]. Massive machine type communications (mMTC) and ultra-reliable and low latency communications (URLLC) are two application scenarios in 5G that distinguish 5G from all previous generations. Among them, enabling URLLC services is the most challenging task as two conflicting requirements of high reliability and low latency need to be satisfied simultaneously by employing short-packet communications. URLLC has a potential to support a vast set of mission critical communication applications like factory automation, intelligent transportation systems, and tactile Internet. The third generation partnership project (3GPP) has specified strict standards that require extremely high reliability (e.g., 99.999%) as well as end-to-end low latency (e.g., less than 1 ms) of particular IoT applications [1–3].

To achieve such stringent latency requirements, it is crucial to use short packets with finite blocklength (FBL) codes [3]. Comparing to the conventional infinite blocklength (IBL) codes, this FBL codes cause two major changes: Firstly, the decoding error probability at a receiver cannot approach zero which cannot be ignored in the system design; secondly, the data rate with FBL codes has a backoff factor determined by the decoding error probability and the blocklength [4]. Recently, there are fast growing interests in the research on the short-packet transmission design based FBL codes. Specifically, the achievable rate under the FBL regime has been approximated by Polyanskiy *et al.* in [4], which characterises a complicated relationship among decoding error probability, blocklength, and the received signal-to-noise ratio (SNR) or signal-to-interference-plus-noise ratio (SINR). The previous studies in [5–8] mainly focused on the performance analysis of the systems with FBL transmission. In order to design a practical URLLC system, it is necessary to intelligently optimize the resource allocation including power, blocklength and bandwidth under the given decoding error probability and delay requirements. Unfortunately, as a result of the complex expression of achievable rate, the resource allocation optimization problem (or beamforming design problem in multiple-antenna systems) for URLLC is highly non-convex and much more computationally challenging than that for the traditional communications with IBL codes. Particularly, the previous works in [9–16] studied the optimizations of transmission design, resource allocation and frame structure for the systems employing various transmission and access technologies, such as non-orthogonal multiple access (NOMA), multiple-antenna or relay transmission.

As we can see from the existing literature on URLLC that there are huge number of the previous works that assumed *perfect instantaneous CSI at receiver or even perfect instantaneous CSI at transmitter (CSIT)* (see e.g., [6–9, 12–14, 16]). [10, 11, 15] are the very few exceptions. Particularly, assuming outdated CSI at the source, the FBL throughput of a two-hop relaying system was maximized in [10] under a reliability constraint. Based on the derived analytical results, [11] optimized the pilot length and the rate adaptation scheme with respect to (w.r.t.) the delay performance. The similar problems in [11] were re-studied for a two-user NOMA uplink system in [15]. It is very natural that the CSI accuracy has a great impact on the reliability and latency performance in URLLC [2, 3]. Unfortunately, it is impossible in practice for any system to obtain perfect instantaneous CSI at either side of communication link. To obtain high-quality CSI is especially difficult for URLLC as very limited overhead can be used. Therefore, it is questionable that the URLLC schemes in the previous literature that were designed based on the perfect CSI at either side of link can work effectively in practical systems.

Moreover, one challenge in mMTC is the scalable and efficient connectivity for a massive number of devices with limited radio resource. To tackle this challenge, different types of radio access technologies are investigated. Particularly, massive multiple-input multiple-output (MIMO) [17–22] for URLLC has attracted fast increasing research interests due to its appealing feature of a large number of spatial degrees of freedom and providing superior spectral efficiency. Particularly, the work in [23] analyzed the performance of the linear minimum error probability detection for massive MIMO system with FBL codes and minimized the decoding error probability. The average data rate performance was studied in [24] with severe shadow fading under the stringent requirements of URLLC. The work in [14] investigated the resource allocation problem in the FBL regime for cellular networks considering both single-antenna base station (BS) and multi-antenna BS. The joint power allocation of pilot and payload transmission for the UL massive MIMO-enabled URLLC was studied in [12], where the best local approximation and geometric program (GP) were employed to solve the optimization problem. However, these works on massive MIMO-enabled URLLC in [12] [23] [24] all concentrated on the single-cell scenario, where a single BS communicates with multiple users by transmitting URLLC packets in DL or UL. It is known that the inter-cell interference has great impacts on the link capacity and reliability. Therefore, it is necessary to consider the design and optimization for URLLC in the multi-cell scenario. To the best of our knowledge, only the very recent the work in [25] has investigated the similar problem as that in [12] in the UL of the user-centric cell-free massive MIMO systems. Moreover, there exist two types of pilot signals for massive MIMO system, i.e., regular pilot (RP) [18, 19] and superimposed pilot (SP) [20–22]. Considering the requirements of URLLC, SP has the advantages over RP that SP is superior for mitigating pilot contamination in multi-cell system [20] and also saves the channel uses (CUs) of the training signals when the ratio of the number of devices over that of antennas are large (e.g. $> 1/10$), and yet SP introduces interference to the payload signals than RP. It is still unknown which type of pilot signal performs better for massive MIMO-enabled URLLC in multi-cell scenario.

In addition, the overwhelming majority of the previous works focused on the URLLC at a single direction of each communication link. Particularly, the works in [12] [24] [25] all focused on the uplink transmission design and resource allocation optimization for URLLC in Massive MIMO systems, and implicitly assumed that the downlink communications can be well accomplished with guaranteed reliability and delay. However, it is known that the results of the uplink transmission design and the power allocation to the pilot and data signals can affect the accuracy of the CSI obtained through UL channel estimation, and further affects the performance of both the UL and DL, and also the DL resource allocation for URLLC. Therefore, the UL and DL resource allocation should be jointly optimized to maximize system performance with the guaranteed reliability and latency requirements for URLLC. However, to the best of our knowledge, the previous works considering this joint UL-DL design for URLLC are very few. [26] is an exception, where the authors considered joint optimization of UL and DL resource configuration to minimize the total bandwidth for the support of URLLC in a local single-cell scenario.

Motivated by the above limitations with the previous works, in this paper we aim to propose an analytical framework to effectively overcome the aforementioned challenges and provide some answers for massive MIMO-enabled URLLC. To the best of our knowledge, our work is the first to investigate the joint UL-DL resource allocation for URLLC enabled by massive MIMO in multi-cell scenario and also the first to provide the performance comparisons between the systems with the different pilot patterns (i.e., RP and SP, and SP with Interference Elimination (SPe)). The main contributions of this paper are summarized as follows:

- By considering the impact of pilot contamination, the closed-form lower bound (LB) approximations on the achievable data rates under FBL codes are individually derived for both the UL and the DL of the cellular massive MIMO systems by considering both the RP and SP signals with linear minimum mean squared error (LMMSE) channel estimations, where each BS employs maximum-ratio transmission (MRT) in the DL and maximum-ratio combining (MRC) in the uplink detection. Considering that each user terminal (UT) in the DL and each BS in the UL both experience interference from the signals intended for multiple UTs, we employ the channel dispersion factor due to short packet transmission in [27] which can be achievable for the interference channels. Simulation results confirm that the obtained LB approximations on the UL and DL achievable data rates with different pilot schemes can be very close to the corresponding ergodic rates, which provides tractable expressions for the joint UL-DL power allocation.
- Given the required UL and DL delay budgets, we formulate the joint UL-DL power allocation optimization problems with the objective to maximize the UL-DL overall average weighted sum rate (WSR) for the systems that individually employ RP and SP schemes. For both RP and SP, the formulated optimization problems are non-convex due to the LB expressions of WSR, and it is challenging to obtain the optimal solutions. The log-function LB approximation is applied to the objective function in an iterative manner. In addition, for the optimization

problem with RP, this method can transform the original optimization problem to a series of geometric programs (GPs). Finally, an iterative algorithm is proposed to jointly optimize the UL and DL pilot and payload power allocation for system with RP.

- Moreover, for the optimization problems with SP and SPe, the expressions of the UL and DL received SINR are more complicated than that with RP. Specifically, the denominators of the equivalent form of the UL and DL received SINRs are posynomial functions that contains all UTs' UL and DL power allocation variables, and thus it is impossible to directly transform the original optimization problems to GPs. To handle this issue, we approximate the posynomial functions with their best local monomial approximations, and the original optimization problem can be approximately addressed by solving a series of GP problems. Finally, an iterative algorithm is proposed to jointly optimize the pilot and payload power allocation for each problem.
- We also provide the convergence analysis for the proposed iterative algorithms that individually solve the optimization problems for systems with RP, SP and SPe schemes. The numerical results demonstrate that our proposed algorithms can converge rapidly. Moreover, when the parameters other than the UL and DL blocklengths are fixed, SPe scheme performs the best and the RP performs the worst in the system DL in general; whereas, which pilot scheme performs better than the others in the system UL depends on the concrete blocklength. We find that the DL average WSR dominates the overall performance, and thus SPe has the best overall performance and RP has the worst overall performance in general. We also exam the varying of average WSRs with the other system parameters, such as the number of BS antennas and the number of served UTs.

II. System and Signal Models

We consider a multi-cell massive MIMO cellular system consisting of N_B cells operating in time-division duplexing (TDD) mode, where the UL and DL communications occur on the same frequency band. Considering the low-cost devices in MTC and the URLLC services, pilot training and channel estimation in the DL and CSI feedback can not be implemented. There is one BS in each cell with M antennas and serves K accessing IoT UTs that are equipped with a single antenna each. We denote one set of the accessing IoT UTs in a cell as $\mathcal{K} = \{1, 2, \cdots, K\}$, where $K \ll M$. In this paper, we consider the system where the IoT UTs need to send and receive information that has stringent latency and reliability requirements such as measured data or their current operation states. This service is achieved through URLLC. Let Φ represent the set containing all N_B BSs indexed by $0, 1, \cdots, N_B - 1$, i.e., the cardinality $|\Phi| = N_B$. Without loss of generality (w.l.g.), we select BS 0 from Φ as the target BS. Accordingly, we define $\Psi = \Phi \backslash \{0\}$ as the set containing all

BSs other than BS_0.

The UL or DL communication channels are modeled as block fading where channel fading is considered to be constant over a coherent time-frequency block of duration T_c and bandwidth B_c. Moreover, channel reciprocity can be assumed for the UL and DL for TDD system, i.e., the propagation factor is same for both links [18–22]. Each coherence block contains $L_c = T_c B_c$ CUs and the entire duration is divided into τ_u and τ_d in UL and DL respectively. The divisions can be selected according to the requirements in practice, subject to the constraint that $T_c = \tau_u + \tau_d$. Accordingly, the numbers of CUs in UL and DL time slots within each block are respectively given by $L_u = \tau_u B_c$ and $L_d = \tau_d B_c$. We assume L_u and L_d are both integers w.l.g..

A. UL and DL Signals

We denote $\mathbf{h}_{ll'k} \in \mathbb{C}^{M\times 1}$ as the UL channel fading vector between the l-th BS's (denoted as BS_l) M antennas and the k-th UT in cell l' (denoted as $\mathrm{UT}_{l'k}$) which is assumed to be uncorrelated Rayleigh-faded, i.e., $\mathbf{h}_{ll'k} \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \beta_{ll'k}\mathbf{I}_M)$, where $\beta_{ll'k}$ denotes the large-scale fading coefficient. For UL, the received signal vector $\mathbf{y}_0 \in \mathbb{C}^{M\times 1}$ at BS_0 can be expressed as

$$\mathbf{y}_0 = \sum_{l'\in\Phi}\sum_{k=1}^{K} \mathbf{h}_{0l'k}\xi_{l'k} + \mathbf{n}_0, \tag{1}$$

where $\xi_{l'k}$ represents the transmitted signal from $\mathrm{UT}_{l'k}$ at one arbitrary CU within the coherence block and $\mathbf{n}_0 \in \mathbb{C}^{M\times 1}$ is the additive white Gaussian noise (AWGN) vector distributed as $\mathbf{n}_0 \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \sigma^2\mathbf{I}_M)$ with σ^2 denoting the variance. The concrete contents of $\xi_{l'k}$ depend on the type of pilot employed. Specifically, $\xi_{l'k}$ contains the pilot signal in CSI training phase and information-bearing signal in data phase for RP, whereas $\xi_{l'k}$ contains the superposition of both pilot signal and information-bearing signal in each CU for SP.

With the UL/DL channel reciprocity in calibrated TDD systems, the DL received signal z_{0k} at UT_{0k} in the target cell can be expressed as

$$z_{0k} = \sum_{l\in\Phi}\sum_{i=1}^{K} \sqrt{\lambda_{li}}\mathbf{h}_{l0k}^T \mathbf{w}_{li} x_{li} + \eta_{0k}, \tag{2}$$

where $x_{l'i}$ is the data symbol intended for UT_{li}, η_{0k} is the AWGN distributed as $\eta_{0k} \sim \mathcal{CN}(0, \sigma^2)$, $\mathbf{w}_{li}$ is the corresponding normalized precoding vector for UT_{li} with $\|\mathbf{w}_{li}\| = 1$, and λ_{li} is the data payload transmit power for UT_{li}. Then, the DL total transmit power at each BS is $\sum_{i=1}^{K} \lambda_{li} = P_{DL}$ $\forall l \in \Phi$. The previous broad studies have shown that low-complexity linear processing techniques are capable to achieve the asymptotically optimal performance in massive MIMO [17, 28]. Therefore, we adopt the low-complexity MRC in the UL detection and MRT in the DL transmission at each BS.

B. Channel Estimations in Cellular Massive MIMO Systems

With the channel hardening effect, the fluctuation of the fast fading in massive MIMO system can be much smaller than that in the corresponding traditional MIMO system [17], and thus can provide higher reliable services for UTs. However, to utilize this benefit with massive MIMO system, the CSI should be available at the massive-antenna side. The TDD protocol is adopted in this paper, where the downlink instantaneous CSI is obtained through channel estimation based on channel reciprocity. Then, the BSs can process both UL and DL signals using the UL channel measurements. There mainly exist two types of UL pilot training in massive MIMO systems, i.e., RP and SP [18–22, 29–31]. We will analyze the ergodic achievable rate and optimize the WSR performance for the massive MIMO-enabled URLLC IoT systems employing both types of UL pilot training, and then compare the performance of both. We adopt standard LMMSE techniques [32] to estimate the channels with both RP and SP cases.

1) Regular Pilots: For RP, a blocklength of L_u^p out of L_u UL CUs in each coherent time block is reserved for UL pilot sequences and the remaining L_u^d CUs are for data transmissions, which satisfy $L_u^d + L_u^p = L_u$. Then, we have a set of L_u^p orthogonal pilot sequences with length L_u^p. Each BS selects K out of all L_u^p distinct pilot sequences uniformly at random in each coherent block independently and allocates them to the served K UTs [21, 33]. We denote the pilot sequence assigned to UT$_{l'k}$ by $\boldsymbol{\phi}_{l'k} \in \mathbb{C}^{L_u^p \times 1}$ with $\left| [\boldsymbol{\phi}_{l'k}]_j \right| = 1, \forall l' \in \Phi$ and $k \in \{1, 2, \cdots, K\}$. For an arbitrary pilot sequence $\boldsymbol{\phi}_{l'k}$ in cell l', we have $\boldsymbol{\phi}_{l'k}^H \boldsymbol{\phi}_{l'i} = L_u^p$ if and only if $k = i$. Otherwise, we have $\boldsymbol{\phi}_{l'k}^H \boldsymbol{\phi}_{l'i} = 0$. For the pilot sequences in the cells other than the target cell, i.e., for cell $l' \neq 0$, we have

$$\boldsymbol{\phi}_{0k}^H \boldsymbol{\phi}_{l'i} = \begin{cases} L_u^p, & \text{with probability } \frac{1}{L_u^p} \\ 0, & \text{with probability } 1 - \frac{1}{L_u^p} \end{cases}. \tag{3}$$

Here, we define the binary random variable (RV) $\chi_{l'i} \triangleq \frac{\boldsymbol{\phi}_{0k}^H \boldsymbol{\phi}_{l'i}}{L_u^p} \in \{0, 1\}$ that indicates whether UT$_{0k}$ is assigned the same pilot sequence as that of the UT$_{l'i}$. According to (3), it can be obtained that $\mathbb{E}[\chi_{l'i}] = \frac{1}{L_u^p}$.

According to (1), the cumulative received pilot signal matrix $\mathbf{Y}_0^{\mathrm{RP}} \in \mathbb{C}^{M \times L_u^p}$ at BS$_0$ can be expressed as

$$\mathbf{Y}_0^{\mathrm{RP}} = \sum_{l' \in \Phi} \sum_{i=1}^{K} \sqrt{q_{l'i}}\, \mathbf{h}_{0l'i} \boldsymbol{\phi}_{l'i}^T + \mathbf{N}_0^p, \tag{4}$$

where $q_{l'i}$ is the allocated transmit power of the pilot symbols of UT$_{l'i}$, and $\mathbf{N}_0^p \sim \mathcal{CN}(\mathbf{0}_{M \times L_u^p}, \sigma^2 \mathbf{I}_M \otimes \mathbf{I}_{L_u^p})$ is the additive noise matrix. By right multiplying $\mathbf{Y}_0^{\mathrm{RP}}$ by $\frac{\boldsymbol{\phi}_{0k}^*}{\sqrt{L_u^p}}$, the received signals after this operation can be obtained as

$$\mathbf{y}_{0k}^{\mathrm{RP}} = \mathbf{Y}_0^{\mathrm{RP}} \frac{\boldsymbol{\phi}_{0k}^*}{\sqrt{L_u^p}} = \sqrt{q_{0k} L_u^p}\, \mathbf{h}_{00k} + \sum_{l' \in \Psi} \sum_{i=1}^{K} \chi_{l'i} \sqrt{q_{l'i} L_u^p}\, \mathbf{h}_{0l'i} + \bar{\mathbf{n}}_0, \tag{5}$$

where $\bar{\mathbf{n}}_0 = \mathbf{N}_0^p \boldsymbol{\phi}_{0k}^* / \sqrt{L_u^p} \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \sigma^2 \mathbf{I}_M)$ is the equivalent noise vector. Notice that $\mathbf{y}_{0k}^{\mathrm{RP}}$ is a sufficient statistic for the estimation of $\mathbf{h}_{00k}$, since any signal in the orthogonal complement of $\boldsymbol{\phi}_{0k}$ is independent of $\mathbf{y}_{0k}^{\mathrm{RP}}$ [32]. Then, we employ LMMSE to estimate $\mathbf{h}_{00k}$ which is given by the following lemma.

Lemma 1: The channel estimation for RP is obtained as

$$\hat{\mathbf{h}}_{00k} = \frac{\sqrt{q_{0k} L_u^p} \beta_{00k}}{q_{0k} L_u^p \beta_{00k} + \sum\limits_{l' \in \Psi} \sum\limits_{i=1}^{K} \chi_{l'i} L_u^p q_{l'i} \beta_{0l'i} + \sigma^2} \mathbf{y}_{0k}^{\mathrm{RP}}. \tag{6}$$

Proof: It follows by applying the standard LMMSE technique in [32, Ch.12] to the problem at hand based on the signal model in (5). □

Since the target signal (i.e., $\mathbf{h}_{00k}$), interference and noise terms within $\mathbf{y}_{0k}^{\mathrm{RP}}$ in (5) are with independent Gaussian distributions, the LMMSE estimator is equivalent to MMSE estimator. It can be concluded that channel estimation $\hat{\mathbf{h}}_{00k}$ and estimation error $\boldsymbol{\varepsilon}_{00k} = \mathbf{h}_{00k} - \hat{\mathbf{h}}_{00k}$ are independent of each other [32].

For the convenience of analysis, we would like to follow the way in [21] to reform the estimation $\hat{\mathbf{h}}_{00k}$ as

$$\hat{\mathbf{h}}_{00k} = \frac{\bar{\gamma}_{0k}^{\mathrm{RP}}}{\sqrt{q_{0k} L_u^p}} \mathbf{y}_{0k}^{\mathrm{RP}}, \tag{7}$$

where $\bar{\gamma}_{0k}^{\mathrm{RP}} \in [0, 1]$ measures the quality of channel estimation which is given by

$$\bar{\gamma}_{0k}^{\mathrm{RP}} = \frac{L_u^p q_{0k} \beta_{00k}}{L_u^p q_{0k} \beta_{00k} + \sum\limits_{l' \in \Psi} \sum\limits_{i=1}^{K} \chi_{l'i} L_u^p q_{l'i} \beta_{0l'i} + \sigma^2}. \tag{8}$$

The covariance matrix of $\hat{\mathbf{h}}_{00k}$ is given by $\mathbb{E}\left\{\hat{\mathbf{h}}_{00k} \hat{\mathbf{h}}_{00k}^H\right\} = \beta_{00k} \bar{\gamma}_{0k}^{\mathrm{RP}} \mathbf{I}_M$. In addition, we have for estimation error that $\mathbb{E}\left\{\boldsymbol{\varepsilon}_{00k} \boldsymbol{\varepsilon}_{00k}^H\right\} = \beta_{00k}(1 - \bar{\gamma}_{0k}^{\mathrm{RP}}) \mathbf{I}_M$. It can be seen that, as L_u^p increases, the estimation errors vanish and the variance of the channel estimation approaches the variance of the true channels. However, in a practical system the pilot length is limited as $L_u^p < L_u$.

2) Superimposed Pilots: With SP, we consider that all the L_u CUs within the coherence time block are used for simultaneously transmitting the superimposed symbols of signal and pilot sequences. In a practical system, the interfering cells considered and the accessing UTs in the same time-frequency block are generally not more than UL blocklength [20, Footnote 6], i.e., $KN_B \leq L_u$. Thus, for simplicity of analysis, we consider in this paper that the pilot sequences of all UTs with the multiple cells are orthogonal to each other, which is the same as the pilot assignment in [20] (but different from that in [21]). Then, there does not exist pilot contamination for the system employing superimposed pilot.

Again, according to (1), the cumulative received signal matrix of one coherent block at BS_0 can be written as

$$\mathbf{Y}_0^{\text{SP}} = \sum_{l' \in \Phi} \sum_{i=1}^{K} \sqrt{q_{l'i}} \mathbf{h}_{0l'i} \boldsymbol{\varphi}_{l'i}^T + \sum_{l' \in \Phi} \sum_{i=1}^{K} \sqrt{p_{l'i}} \mathbf{h}_{0l'i} \mathbf{s}_{l'i}^T + \mathbf{N}_0, \tag{9}$$

where $\mathbf{s}_{l'i} \in \mathbb{C}^{L_u \times 1}$ contains the UL data symbols in the whole coherence block of $\text{UT}_{l'i}$ that is distributed as $\mathbf{s}_{l'i} \sim \mathcal{CN}\left(\mathbf{0}_{L_u \times 1}, \mathbf{I}_{L_u}\right)$, and $\boldsymbol{\varphi}_{l'i} \in \mathbb{C}^{L_u \times 1}$ is the pilot sequence assigned to $\text{UT}_{l'i}$ with $\left| \left[\boldsymbol{\varphi}_{l'i}\right]_j \right| = 1, \forall j \in \{1, \ldots, L_u\}$. $q_{l'i}$ and $p_{l'i}$ is the allocated transmission power of the pilot symbol and data symbol for $\text{UT}_{l'i}$ respectively. $\mathbf{N}_0 \in \mathbb{C}^{M \times L_u} \sim \mathcal{CN}(\mathbf{0}_{M \times L_u}, \sigma^2 \mathbf{I}_M \otimes \mathbf{I}_{L_u})$ is the additive noise matrix. For arbitrary pilot sequence $\boldsymbol{\varphi}_{lk}$ assigned in cell l, we have $\boldsymbol{\varphi}_{lk}^H \boldsymbol{\varphi}_{l'i} = L_u$ if and only if $k = i$ and $l = l'$. Otherwise, $\boldsymbol{\varphi}_{lk}^H \boldsymbol{\varphi}_{l'i} = 0$.

Similar to RP, by right multiplying $\mathbf{Y}_0^{\text{SP}}$ by $\frac{\boldsymbol{\varphi}_{0k}^*}{\sqrt{L_u^p}}$, the received signals after this operation can be obtained as

$$\mathbf{y}_{0k}^{\text{SP}} = \mathbf{Y}_0^{\text{SP}} \frac{\boldsymbol{\varphi}_{0k}^*}{\sqrt{L_u}} = \sqrt{q_{0k} L_u}\, \mathbf{h}_{00k} + \sum_{l' \in \Phi} \sum_{i=1}^{K} \sqrt{\frac{p_{l'i}}{L_u}}\, \mathbf{h}_{0l'i} \mathbf{s}_{l'i}^T \boldsymbol{\varphi}_{0k}^* + \frac{1}{\sqrt{L_u}} \mathbf{N}_0 \boldsymbol{\varphi}_{0k}^*. \tag{10}$$

which is then used to obtain the LMMSE channel estimation of $\mathbf{h}_{00k}$. Similar as *Lemma* 1, we can obtain the following estimation of $\mathbf{h}_{00k}$ with SP using LMMSE estimator.

Lemma 2: The channel estimation for SP is given by

$$\hat{\mathbf{h}}_{00k} = \frac{\sqrt{q_{0k} L_u} \beta_{00k}}{q_{0k} L_u \beta_{00k} + \sum\limits_{l' \in \Phi} \sum\limits_{i=1}^{K} p_{l'i} \beta_{0l'i} + \sigma^2} \mathbf{y}_{0k}^{\text{SP}}. \tag{11}$$

We note that, different from the estimation $\hat{\mathbf{h}}_{00k}$ for RP in (6), since interference term of $\mathbf{y}_{0k}^{\text{RP}}$ (i.e., the second term in (10)) is *not* Gaussian distributed, the LMMSE estimator is *not* equivalent to MMSE estimator [21]. Therefore, estimation $\hat{\mathbf{h}}_{00k}$ and estimation error $\boldsymbol{\varepsilon}_{00k} = \mathbf{h}_{00k} - \hat{\mathbf{h}}_{00k}$ are *only uncorrelated but not independent of each other*. Moreover, the interference term contains UTs' UL data symbols which creates the correlation between the channel estimation and the data symbols from all UTs [21], and further makes the ergodic rate analysis in the UL and DL of the system employing SP much more complex than those in the system employing RP. More detailed discussions on the key difference between the channel estimations with RP and SP can be found with *Remark 1* in [21].

We reform the estimation $\hat{\mathbf{h}}_{00k}$ following the way as in [21]. Define $\gamma_{0k}^{\text{SP}} \in [0, 1)$ as a parameter to measure the quality of channel estimation which is given by

$$\gamma_{0k}^{\text{SP}} = \frac{q_{0k} L_u \beta_{00k}}{q_{0k} L_u \beta_{00k} + \sum\limits_{l' \in \Phi} \sum\limits_{i=1}^{K} p_{l'i} \beta_{0l'i} + \sigma^2}. \tag{12}$$

Then, $\hat{\mathbf{h}}_{00k}$ in (11) can be re-expressed as

$$\hat{\mathbf{h}}_{00k} = \frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}L_u}}\mathbf{y}_{0k}^{\mathrm{SP}}, \tag{13}$$

whose covariance matrix is given by $\mathbb{E}\left\{\hat{\mathbf{h}}_{00k}\hat{\mathbf{h}}_{00k}^H\right\} = \beta_{00k}\gamma_{0k}^{\mathrm{SP}}\mathbf{I}_M$. In addition, the covariance matrix of estimation error is $\mathbb{E}\left\{\boldsymbol{\varepsilon}_{00k}\boldsymbol{\varepsilon}_{00k}^H\right\} = \beta_{00k}(1-\gamma_{0k}^{\mathrm{SP}})\mathbf{I}_M$.

C. Achievable Data Rate with Finite Blocklength Codes

When the channel coding length of each UT's information bits is asymptotically large, the maximum achievable rate at each UT is well known as the Shannon capacity. For URLLC with FBL codes, the achievable rate at each UT depends on the finite blocklength coding theory. New performance metric is necessary to properly optimize the mission-critical IoT systems considered in this paper. Using Gaussian approximation, given the maximum allowable decoding block error rate (BLER) constraint of ε_k for UT_{0k} in target cell, the maximum achievable data rate R_k of UT_{0k} can be accurately approximated as [3, 4]

$$R_k \approx \log_2(1+\gamma_k) - \sqrt{\frac{V(\gamma_k)}{L}}Q^{-1}(\varepsilon_k) \qquad [BPCU], \tag{14}$$

where γ_k is the received signal-to-interference-plus-noise ratio (SINR) of UT_{0k}, L is the block-length of FBL code, $Q^{-1}(x)$ is the inverse of Gaussian Q-function $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}dt$. $V(\gamma_k)$ is the channel dispersion factor whose form depends on the concrete type of channel. Specifically, for AWGN channel, $V(\gamma_k)$ is given by [4]

$$V(\gamma_k) = V_{\mathrm{AWGN}}(\gamma_k) = (\log_2 e)^2\left(1 - \frac{1}{(1+\gamma_k)^2}\right). \tag{15}$$

However, for the system considered in this paper, each UT in the DL and each BS in the UL both experience interference from the signals intended for multiple UTs. For both UL and DL, in order to achieve the rate of (14) for any UT_{0k}, the transmitter would need to use a non-Gaussian codebook [4]. Then, the other UTs would be subject to non-Gaussian interference, and thus the rate in (14) would not hold for the other UTs [16, 27]. Therefore, the rate of (14) is infeasible in system we consider. As an alternative, the authors in [27] considered an interference channel where each transmitter uses an independent and identically distributed (i.i.d.) Gaussian codebook and employs nearest-neighbor decoding. The corresponding channel dispersion factor is characterized as

$$V(\gamma_k) = V_{\mathrm{NE}}(\gamma_k) \triangleq (\log_2 e)^2\frac{2\gamma_k}{1+\gamma_k}. \tag{16}$$

This result can be directly applied to our scenario, since there is no difference between the interference channel considered in [27] and that of ours in the UL or the DL. Although (16) is not the optimal, it is more representative of the actual channel dispersion than (15). Therefore, we will adopt (16) for the system performance analysis and the optimization in this paper.

III. Performance Analysis for UL/DL Cellular Massive MIMO URLLC Systems

In this section, we derive the results of the output SINRs γ_k and the ergodic achievable rate of each UT_{0k} in the target cell 0 with different pilot schemes.

A. Uplink Effective SINR Analysis with MRC Detection

1) UL with Regular Pilots: According to (1), the received signal vector at BS_0 corresponding to an arbitrary data symbol of all UT in one coherent block is given by

$$[\mathbf{Z}_0^{\mathrm{RP}}]_j = \sum_{i=1}^{K} \sqrt{p_{0i}} \mathbf{h}_{00i}[\mathbf{s}_{0i}]_j + \sum_{l' \in \Psi} \sum_{i=1}^{K} \sqrt{p_{l'i}} \mathbf{h}_{0l'i}[\mathbf{s}_{l'i}]_j + [\mathbf{N}_0^d]_j, \tag{17}$$

where $p_{l'i}$ denotes the allocated transmission power of the UL data symbol for $\mathrm{UT}_{l'i}$ again. $[\mathbf{Z}_0^{\mathrm{RP}}]_j$ and $[\mathbf{N}_0^d]_j$ respectively represent the j-th column of the cumulative received signal matrix $\mathbf{Z}_0^{\mathrm{RP}} \in \mathbb{C}^{M \times L_u^d}$ at BS_0 and the AWGN $\mathbf{N}_0^d \in \mathbb{C}^{M \times L_u^d}$. To detect the data symbol of UT_{0k} by MRC, the received signal is combined with vector $\mathbf{v}_{00k} = \hat{\mathbf{h}}_{00k}$ as

$$\begin{aligned} & [\hat{\mathbf{s}}_{0k}]_j = \mathbf{v}_{00k}^H [\mathbf{Z}_0^{\mathrm{RP}}]_j = \hat{\mathbf{h}}_{00k}^H [\mathbf{Z}_0^{\mathrm{RP}}]_j \\ = \; & \sqrt{p_{0k}} \hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00k} [\mathbf{s}_{0k}]_j + \sum_{i \neq k}^{K} \sqrt{p_{0i}} \hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00i} [\mathbf{s}_{0i}]_j + \sum_{l' \in \Psi} \sum_{i=1}^{K} \sqrt{p_{l'i}} \hat{\mathbf{h}}_{00k}^H \, \mathbf{h}_{0l'i} [\mathbf{s}_{l'i}]_j + \hat{\mathbf{h}}_{00k}^H [\mathbf{N}_0^d]_j \\ = \; & \sqrt{p_{0k}} \mathbb{E}\{\hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00k}\} [\mathbf{s}_{0k}]_j + \sqrt{p_{0k}} \left(\hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00k} - \mathbb{E}\{\hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00k}\} \right) [\mathbf{s}_{0k}]_j \qquad (18) \\ & + \sum_{i \neq k}^{K} \sqrt{p_{0i}} \hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00i} [\mathbf{s}_{0i}]_j + \sum_{l' \in \Psi} \sum_{i=1}^{K} \sqrt{p_{l'i}} \hat{\mathbf{h}}_{00k}^H \mathbf{h}_{0l'i} [\mathbf{s}_{l'i}]_j + \hat{\mathbf{h}}_{00k}^H [\mathbf{N}_0^d]_j. \qquad (19) \end{aligned}$$

We note that the second term in (18) and the term in (19) are both non-Gaussian distributed. By using the result in [34, 2.3.4], the effective channel associated with each UT_{0k} given by (18) and (19) can be identified as a scalar point-to-point channel with a deterministic channel fading $\mathbb{E}\{\hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00k}\}$ and non-Gaussian effective noise. Moreover, it is easy to check that the target signal $[\mathbf{s}_{0k}]_j$ is *uncorrelated* with additive non-Gaussian effective noise terms in (18) and (19), and independent with the equivalent channel $\hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00k}$. In addition, since the data symbols and the AWGN noise are independent, the individual terms in non-Gaussian effective noise are uncorrelated. Moreover, non-Gaussian effective noise in (18) and (19) can be rewritten as

$$\sum_{l' \in \Phi} \sum_{i=1}^{K} \sqrt{p_{l'i}} \hat{\mathbf{h}}_{00k}^H \, \mathbf{h}_{0l'i} [\mathbf{s}_{l'i}]_j - \sqrt{p_{0k}} \mathbb{E}\{\hat{\mathbf{h}}_{00k}^H \mathbf{h}_{00k}\} [\mathbf{s}_{0k}]_j + \hat{\mathbf{h}}_{00k}^H [\mathbf{N}_0^d]_j. \tag{20}$$

Then, *given the random pilot allocation*, we can obtain the *effective SINR*[1] of UT_{0k} for the system with RP in the following lemma by using the same method used in [21, *Lemma* 3].

Lemma 3: The effective SINR of UT_{0k} in the system UL with RP is given by

$$\overline{\mathrm{SINR}}_{0k}^{\mathrm{RP,UL}} = \frac{p_{0k}\left|\mathbb{E}\left\{\mathbf{v}_{00k}^H\mathbf{h}_{00k}\right\}\right|^2}{\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K} p_{l'i}\mathbb{E}\left\{\left|\mathbf{v}_{00k}^H\mathbf{h}_{0l'}\right|^2\right\} - p_{0k}\left|\mathbb{E}\left\{\mathbf{v}_{00k}^H\mathbf{h}_{00k}\right\}\right|^2 + \mathbb{E}\left\{\left|\mathbf{v}_{00k}^H[\mathbf{N}_0^d]_j\right|^2\right\}} \tag{21}$$

$$= \frac{ML_u^d p_{0k} q_{0k}\beta_{00k}^2}{M\sum\limits_{l'\in\Psi}\sum\limits_{l=1}^{K} p_{l'i}q_{l'i}\beta_{0l'i}^2 + \left(q_{0k}L_u^p\beta_{00k} + \sum\limits_{l'\in\Psi}\sum\limits_{i=1}^{K}\chi_{l'i}L_u^p q_{l'i}\beta_{0l'i} + \sigma^2\right)\left(\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2\right)} \tag{22}$$

where $\mathbf{v}_{00k} = \hat{\mathbf{h}}_{00k}$ in (21).

Note that the ergodic achievable rate with effective SINR given by (21) holds not only for $\mathbf{v}_{00k} = \hat{\mathbf{h}}_{00k}$ but for any other selection of combing vector $\mathbf{v}_{00k}$ and also any channel fading distribution. Moreover, we note that the result in (21) are in the same form as that in [21, Eq. (17)], but the expectations in (21) are taken w.r.t. the fast channel fading but not w.r.t. the random pilot allocation at this stage. We would like to leave the expectations w.r.t. the random pilot allocation later while deriving the UTs' ergodic achievable rates.

2) UL with Superimposed Pilots: In this subsection, we derive the effective SINR of UT_{0k} in UL of the system with SP following the same method above. Firstly, based on (10) and (13), we can rewrite the estimation $\hat{\mathbf{h}}_{00k}$ for SP as

$$\begin{aligned}\hat{\mathbf{h}}_{00k} &= \frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}L_u}}\left(\sqrt{q_{0k}L_u}\,\mathbf{h}_{00k} + \sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}\sqrt{\frac{p_{l'i}}{L_u}}\,\mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T\boldsymbol{\varphi}_{0k}^* + \frac{1}{\sqrt{L_u}}\mathbf{N}_0\boldsymbol{\varphi}_{0k}^*\right)\\ &\quad + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^* \qquad (23)\\ &= \bar{\mathbf{h}}_{00k} + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*, \qquad (24)\end{aligned}$$

where we let $\bar{\mathbf{h}}_{00k} \triangleq \frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}L_u}}\left(\sqrt{q_{0k}L_u}\,\mathbf{h}_{00k} + \sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}\sqrt{\frac{p_{l'i}}{L_u}}\mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T\boldsymbol{\varphi}_{0k}^* + \frac{1}{\sqrt{L_u}}\mathbf{N}_0\boldsymbol{\varphi}_{0k}^*\right)$ with $\gamma_{0k}^{\mathrm{SP}}$ given by (12), and define $\zeta_{l'i} = 0$ if $\{l', i\} = \{0, k\}$ and otherwise $\zeta_{l'i} = 1$. Similar to system with RP, the received signal is combined with vector $\mathbf{v}_{00k} = \hat{\mathbf{h}}_{00k}$ again. According to (9) and (24), the detected data symbol $[\mathbf{s}_{0k}]_j$ of UT_{0k} is given by

$$\begin{aligned}[\hat{\mathbf{s}}_{0k}]_j &= \mathbf{v}_{00k}^H[\mathbf{Y}_0^{\mathrm{SP}}]_j = \hat{\mathbf{h}}_{00k}^H[\mathbf{Y}_0^{\mathrm{SP}}]_j = \hat{\mathbf{h}}_{00k}^H\Bigg(\sqrt{p_{0k}}\,\mathbf{h}_{00k}[\mathbf{s}_{0k}]_j + \sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}\sqrt{p_{l'i}}\,\mathbf{h}_{0l'i}[\mathbf{s}_{l'i}]_j\\ &\quad + \sum_{l'\in\Phi}\sum_{i=1}^{K}\sqrt{q_{l'i}}\,\mathbf{h}_{0l'i}[\boldsymbol{\varphi}_{l'i}]_j + [\mathbf{N}_0]_j\Bigg) \qquad (25)\end{aligned}$$

[1]Here, the term effective SINR is the same as defined in [21] and [34, 2.3.4] corresponding to the effective channel given by (18) and (19) as described above.

$$= \left(\bar{\mathbf{h}}_{00k}^H + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}^H \boldsymbol{\varphi}_{0k}^T \mathbf{s}_{0k}^*\right)\sqrt{p_{0k}}\,\mathbf{h}_{00k}[\mathbf{s}_{0k}]_j + \hat{\mathbf{h}}_{00k}^H\Bigg(\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}\sqrt{p_{l'i}}\,\mathbf{h}_{0l'i}[\mathbf{s}_{l'i}]_j$$

$$+\sum_{l'\in\Phi}\sum_{i=1}^{K}\sqrt{q_{l'i}}\,\mathbf{h}_{0l'i}[\boldsymbol{\varphi}_{l'i}]_j + [\mathbf{N}_0]_j\Bigg) \tag{26}$$

$$= \sqrt{p_{0k}}\bar{\mathbf{h}}_{00k}^H\,\mathbf{h}_{00k}[\mathbf{s}_{0k}]_j + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\frac{p_{0k}}{\sqrt{q_{0k}}}\,\mathbf{h}_{00k}^H\,\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j$$

$$+\hat{\mathbf{h}}_{00k}^H\left(\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}\sqrt{p_{l'i}}\,\mathbf{h}_{0l'i}[\mathbf{s}_{l'i}]_j + \sum_{l'\in\Phi}\sum_{i=1}^{K}\sqrt{q_{l'i}}\,\mathbf{h}_{0l'i}[\boldsymbol{\varphi}_{l'i}]_j + [\mathbf{N}_0]_j\right) \tag{27}$$

$$= \sqrt{p_{0k}}\mathbb{E}\{\bar{\mathbf{h}}_{00k}^H\mathbf{h}_{00k}\}[\mathbf{s}_{0k}]_j + \underbrace{\sqrt{p_{0k}}\left(\bar{\mathbf{h}}_{00k}^H\mathbf{h}_{00k} - \mathbb{E}\{\bar{\mathbf{h}}_{00k}^H\mathbf{h}_{00k}\}\right)[\mathbf{s}_{0k}]_j}_{\text{non-Gaussian effective noise: } I_1}$$

$$\underbrace{+\frac{\gamma_{0k}^{\mathrm{SP}}p_{0k}}{L_u\sqrt{q_{0k}}}\,\mathbf{h}_{00k}^H\,\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j + \sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\hat{\mathbf{h}}_{00k}^H\,\mathbf{h}_{0l'i}}_{\text{non-Gaussian effective noise: } I_2}$$

$$+\hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j. \tag{28}$$

Similar to the derivation for the system with RP, we treat $\sqrt{p_{0k}}\mathbb{E}\{\bar{\mathbf{h}}_{00k}^H\mathbf{h}_{00k}\}[\mathbf{s}_{0k}]_j$ as the desired signal and the other terms in (28) as additive non-Gaussian effective noise. With the decomposition of the received signal after detection in (28), it can be observed that the target signal $[\mathbf{s}_{0k}]_j$ is independent with all fast channel fading, and also *uncorrelated* with the non-Gaussian effective noise terms I_1, I_2 and $\hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j$. In addition, we define $n_{kj} \triangleq I_2 + \hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j$. Using the same method used in [21, *Lemma* 3] again, we can derive the effective UL SINR for the system with SP in the following lemma.

Lemma 4: The effective SINR of UT_{0k} in UL of the system with SP is given by

$$\mathrm{SINR}_{0k}^{\mathrm{SP,UL}} = \frac{p_{0k}\left|\mathbb{E}\left\{\bar{\mathbf{h}}_{00k}^H\mathbf{h}_{00k}\right\}\right|^2}{p_{0k}\left(\mathbb{E}\left\{\left|\bar{\mathbf{h}}_{00k}^H\mathbf{h}_{00k}\right|^2\right\} - \left|\mathbb{E}\left\{\bar{\mathbf{h}}_{00k}^H\mathbf{h}_{00k}\right\}\right|^2\right) + \mathbb{E}\left\{\left|n_{kj} - \mathbb{E}\{n_{kj}\}\right|^2\right\}} \tag{29}$$

$$= ML_u p_{0k}q_{0k}\beta_{00k}^2 \Bigg/ \Bigg\{2p_{0k}q_{0k}\beta_{00k}^2 + M\sum_{l'\in\Phi}\sum_{i=1}^{K}(p_{l'i}+q_{l'i})p_{l'i}\beta_{0l'i}^2 + \frac{1}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}p_{l'i}^2\beta_{0l'i}^2$$

$$+\left(q_{0k}L_u\beta_{00k} + \sum_{l'\in\Phi}\sum_{i=1}^{K}p_{l'i}\beta_{0l'i} + \sigma^2\right)\left(\sum_{l'\in\Phi}\sum_{i=1}^{K}(p_{l'i}+q_{l'i})p_{l'i}\beta_{0l'i}^2 + \sigma^2\right)\Bigg\}. \tag{30}$$

Proof: See Appendix A. □

B. Downlink Effective SINR Analysis with MRT Precoding

We employ MRT precoding at each BS for both RP and SP in the DL of the system. The received signal at each UT_{0k} is consistent with the general signal model in (2). z_{0k} in (2) can be rewritten as

$$\begin{aligned}
z_{0k} &= \sqrt{\lambda_{0k}}\mathbf{h}_{00k}^T\mathbf{w}_{0k}\mathrm{x}_{0k} + \sum_{i=1,i\neq k}^{K}\sqrt{\lambda_{0i}}\mathbf{h}_{00k}^T\mathbf{w}_{0i}\mathrm{x}_{0i} + \sum_{l\in\Psi}\sum_{i=1}^{K}\sqrt{\lambda_{li}}\mathbf{h}_{l0k}^T\mathbf{w}_{li}\mathrm{x}_{li} + \eta_{0k} \\
&= \sqrt{\lambda_{0k}}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{w}_{0k}\right\}\mathrm{x}_{0k} + \sqrt{\lambda_{0k}}\left(\mathbf{h}_{00k}^T\mathbf{w}_{0k} - \mathbb{E}\{\mathbf{h}_{00k}^T\mathbf{w}_{0k}\}\right)\mathrm{x}_{0k} \\
&\quad + \sum_{i=1,i\neq k}^{K}\sqrt{\lambda_{0i}}\mathbf{h}_{00k}^T\mathbf{w}_{0i}\mathrm{x}_{0i} + \sum_{l\in\Psi}\sum_{i=1}^{K}\sqrt{\lambda_{li}}\mathbf{h}_{l0k}^T\mathbf{w}_{li}\mathrm{x}_{li} + \eta_{0k}, \qquad (31)
\end{aligned}$$

where λ_{lk} denotes the allocated transmission power of the DL data symbol for UT_{lk}. $\mathbf{w}_{li}$ with $\|\mathbf{w}_{li}\| = 1, \forall l \in \Phi$ and $k \in \{1, 2, \cdots, K\}$, is the MRT precoding vector for UT_{li} employed at BS_l. With SP but without interference elimination from the estimated channel, the precoding vector is given by $\mathbf{w}_{li} \triangleq \frac{\hat{\mathbf{h}}_{lli}^*}{\|\hat{\mathbf{h}}_{lli}^*\|}$, whilst the precoding vector for the system with both SP and interference signal elimination from the estimated channel will be shown at the corresponding subsection below.

We let $\tilde{\eta}_{0k} \triangleq \sqrt{\lambda_{0k}}\left(\mathbf{h}_{00k}^T\mathbf{w}_{0k} - \mathbb{E}\{\mathbf{h}_{00k}^T\mathbf{w}_{0k}\}\right)\mathrm{x}_{0k} + \sum_{i=1,i\neq k}^{K}\sqrt{\lambda_{0i}}\mathbf{h}_{00k}^T\mathbf{w}_{0i}\mathrm{x}_{0i} + \sum_{l\in\Psi}\sum_{i=1}^{K}\sqrt{\lambda_{li}}\mathbf{h}_{l0k}^T\mathbf{w}_{li}\mathrm{x}_{li} + \eta_{0k}$ denote the non-Gaussian effective noise in (31). It can be seen that the first term of (31) is uncorrelated with the other terms. Moreover, all terms of the effective noise $\tilde{\eta}_{0k}$ are also uncorrelated with each other, since the data symbols x_{0i} of all UTs and η_{0k} are independent RVs with zero mean. Again using the result in [34, 2.3.4], we can obtain the DL effective SINR of UT_{0k} as

$$\begin{aligned}
\mathrm{SINR}_{0k}^{\mathrm{DL}} &= \frac{\lambda_{0k}\left|\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{w}_{0k}\right\}\right|^2}{\mathbb{E}\left\{|\tilde{\eta}_{0k}|^2\right\}} \\
&= \lambda_{0k}\underbrace{\left|\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{w}_{0k}\right\}\right|^2}_{S}\Bigg/\Bigg\{\lambda_{0k}\Bigg(\underbrace{\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\mathbf{w}_{0k}\right|^2\right\}}_{I_3} - \underbrace{\left|\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{w}_{0k}\right\}\right|^2}_{S}\Bigg) \\
&\quad + \sum_{i=1,i\neq k}^{K}\lambda_{0i}\underbrace{\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\mathbf{w}_{0i}\right|^2\right\}}_{I_4} + \sum_{l\in\Psi}\sum_{i=1}^{K}\lambda_{li}\underbrace{\mathbb{E}\left\{\left|\mathbf{h}_{l0k}^T\mathbf{w}_{li}\right|^2\right\}}_{I_5} + \sigma^2\Bigg\}, \qquad (32)
\end{aligned}$$

where we have defined the terms S, I_3, I_4, I_5 as indicated in (32). We note that, since the result in (32) is developed based on the general signal model in (2), *it holds for both RP and SP schemes*. Generally, it can be obtained that

$$\begin{aligned}
\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{w}_{0k}\right\} &= \mathbb{E}\left\{\mathbf{h}_{00k}^T\frac{\hat{\mathbf{h}}_{00k}^*}{\|\hat{\mathbf{h}}_{00k}^*\|}\right\} = \mathbb{E}\left\{\left(\hat{\mathbf{h}}_{00k}^T + \boldsymbol{\varepsilon}_{00k}^T\right)\frac{\hat{\mathbf{h}}_{00k}^*}{\|\hat{\mathbf{h}}_{00k}^*\|}\right\} \\
&= \mathbb{E}\left\{\|\hat{\mathbf{h}}_{00k}\|\right\} + \mathbb{E}\left\{\boldsymbol{\varepsilon}_{00k}^T\frac{\hat{\mathbf{h}}_{00k}^*}{\|\hat{\mathbf{h}}_{00k}^*\|}\right\}. \qquad (33)
\end{aligned}$$

We next derive the analytical results of $\mathrm{SINR}_{0k}^{\mathrm{RP,DL}}$ and $\mathrm{SINR}_{0k}^{\mathrm{SP,DL}}$ respectively.

1) DL Effective SINR with Regular Pilots: First, we derive the result for the system with RP. *Given the random pilot allocation*, the DL effective SINR is given by the next lemma.

Lemma 5: The DL effective SINR of UT_{0k} for RP is give by

$$\overline{\mathrm{SINR}}_{0k}^{\mathrm{RP,DL}} = \frac{\lambda_{0k}\beta_{00k}\bar{\gamma}_{0k}^{\mathrm{RP}}\bar{M}}{(M-\bar{M}-1)\lambda_{0k}\beta_{00k}\bar{\gamma}_{0k}^{\mathrm{RP}} + \sum\limits_{i=1}^{K}\lambda_{0i}\beta_{00k} + \sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{0lk} + \sigma^2}, \tag{34}$$

where $\bar{M} \triangleq \left(\frac{\Gamma(M+1/2)}{\Gamma(M)}\right)^2$ and $\bar{\gamma}_{0k}^{\mathrm{RP}}$ is given by (8).

Proof: See Appendix B. □

Similar to the result of UL with RP, the expectations in (34) are taken w.r.t. the fast channel fading but not w.r.t. the random pilot allocation at this stage.

2) DL Effective SINR with Superimposed Pilots: The DL effective SINR with SP can be obtained in the next lemma.

Lemma 6: The DL effective SINR of UT_{0k} with SP is given by

$$\mathrm{SINR}_{0k}^{\mathrm{SP,DL}} = \frac{\lambda_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}}\bar{M}}{\lambda_{0k}\beta_{00k}\left(\alpha_k - \bar{M}\gamma_{0k}^{\mathrm{SP}}\right) + \sum\limits_{i=1,i\neq k}^{K}\lambda_{0i}\beta_{00k}\alpha_{ki} + \sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{0lk} + \sigma^2}, \tag{35}$$

where $\alpha_k = \frac{\left(1+M\gamma_{0k}^{\mathrm{SP}}+\frac{Mp_{0k}\gamma_{0k}^{\mathrm{SP}}}{q_{0k}L_u}\right)}{\left(1+\frac{\gamma_{0k}^{\mathrm{SP}}}{M}+\frac{p_{0k}\gamma_{0k}^{\mathrm{SP}}}{Mq_{0k}L_u}\right)}$, $\alpha_{ki} = \frac{\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{q_{0i}\beta_{00i}L_u}\right)}{\left(1+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{Mq_{0i}\beta_{00i}L_u}\right)}$ for $i\neq k$, and $\gamma_{0k}^{\mathrm{SP}}$ is given by (12).

Proof: See Appendix C. □

3) DL SINR with Superimposed Pilots and Interference Elimination (SPe): In this subsection, we consider further improving the DL rate performance of each UT k through improving the quality of the channel estimation in UL. The block error happens at very low rate in URLLC systems. Therefore, we can assume that the UT_{0i}'s data symbols in $\mathbf{s}_{0i}$ $\forall i \in \{1,2,\cdots,K\}$ are correctly obtained by BS_0. Then, based on the channel estimation of SP in (13) and the detected UL data symbols of all served UTs associated with cell 0, BS_0 can first re-construct the interference signals containing the UL data symbols of the UTs, i.e., $\mathbf{h}_{00i}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*$ $\forall i$, and then remove the interference terms from the channel estimation in (13). Specifically, using (13) we can obtain that

$$\begin{aligned}\check{\mathbf{h}}_{00k} &\triangleq \hat{\mathbf{h}}_{00k} - \frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}L_u}}\sum_{i=1}^{K}\sqrt{\frac{p_{0i}}{L_u}}\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^* = \frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}L_u}}\left(\mathbf{y}_{0k}^{\mathrm{SP}} - \sum_{i=1}^{K}\sqrt{\frac{p_{0i}}{L_u}}\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right) &(36)\\ &\triangleq \frac{\gamma_{0k}^{\mathrm{SPe}}}{\sqrt{q_{0k}L_u}}\check{\mathbf{y}}_{0k}^{\mathrm{SP}}, &(37)\end{aligned}$$

where $\check{\mathbf{y}}_{0k}^{\mathrm{SP}}$ in (37) is given by $\check{\mathbf{y}}_{0k}^{\mathrm{SP}} \triangleq \sqrt{q_{0k}L_u}\,\mathbf{h}_{00k} + \sum_{i=1}^{K}\sqrt{\frac{p_{0i}}{L_u}}(\mathbf{h}_{00i}-\hat{\mathbf{h}}_{00i})\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*$ $+ \sum_{l'\in\Psi}\sum_{i=1}^{K}\sqrt{\frac{p_{l'i}}{L_u}}\,\mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T\boldsymbol{\varphi}_{0k}^* + \frac{1}{\sqrt{L_u}}\mathbf{N}_0\boldsymbol{\varphi}_{0k}^*$ which is obtained by substituting (10) into (36). And the term $\sqrt{q_{0k}L_u}\,\mathbf{h}_{00k}$ is the part we are interested in. Being similar to $\gamma_{0k}^{\mathrm{SP}}$, here we have defined $\gamma_{0k}^{\mathrm{SPe}} \in [0,1)$ as a parameter to measure the quality of the new estimation of $\check{\mathbf{h}}_{00k}$ which can be obtained as

$$\gamma_{0k}^{\mathrm{SPe}} = \frac{q_{0k}L_u \mathbb{E}\left\{\mathbf{h}_{00k}^H \mathbf{h}_{00k}\right\}}{\mathbb{E}\left\{(\check{\mathbf{y}}_{0k}^{\mathrm{SP}})^H \check{\mathbf{y}}_{0k}^{\mathrm{SP}}\right\}} \tag{38}$$

$$= \frac{q_{0k}L_u\beta_{00k}}{q_{0k}L_u\beta_{00k} + \frac{p_{0k}^2 {\gamma_{0k}^{\mathrm{SP}}}^2}{q_{0k}L_u}\beta_{00k} + \sum_{i=1}^{K}(1-\gamma_{0i}^{\mathrm{SP}})p_{0i}\beta_{00i} + \sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2}, \tag{39}$$

where $\gamma_{0i}^{\mathrm{SP}}$ is given by (12).

The numerator of (38) is related to $\mathbf{h}_{00k}$, which is the term concerned. In addition, the covariance matrix of $\check{\mathbf{h}}_{00k}$ can be derived as $\mathbb{E}\left\{\check{\mathbf{h}}_{00k}\check{\mathbf{h}}_{00k}^H\right\} = \frac{(\gamma_{0k}^{\mathrm{SP}})^2}{\gamma_{0k}^{\mathrm{SPe}}}\beta_{00k}\mathbf{I}_M$. In order to maintain consistent form of the channel estimations for the systems with RP, SP and SPe, we let $\hat{\mathbf{g}}_{00k}$ denote the channel estimation with the covariance matrix of $\mathbb{E}\left\{\hat{\mathbf{g}}_{00k}\hat{\mathbf{g}}_{00k}^H\right\} = \gamma_{0k}^{\mathrm{SPe}}\beta_{00k}\mathbf{I}_M$. Then, it follows from (37) and (38) that $\hat{\mathbf{g}}_{00k} = \frac{\gamma_{0k}^{\mathrm{SP}}}{\gamma_{0k}^{\mathrm{SPe}}}\check{\mathbf{h}}_{00k}$. According to (37), $\hat{\mathbf{g}}_{00k}$ is given by

$$\hat{\mathbf{g}}_{00k} = \frac{\gamma_{0k}^{\mathrm{SPe}}}{\gamma_{0k}^{\mathrm{SP}}}\check{\mathbf{h}}_{00k} = \frac{\gamma_{0k}^{\mathrm{SPe}}}{\sqrt{q_{0k}L_u}}\Bigg(\sqrt{q_{0k}L_u}\,\mathbf{h}_{00k} + \sum_{i=1}^{K}\sqrt{\frac{p_{0i}}{L_u}}(\mathbf{h}_{00i} - \hat{\mathbf{h}}_{00i})\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^* + \sum_{l'\in\Psi}\sum_{i=1}^{K}\sqrt{\frac{p_{l'i}}{L_u}}\,\mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T\boldsymbol{\varphi}_{0k}^* + \frac{1}{\sqrt{L_u}}\mathbf{N}_0\boldsymbol{\varphi}_{0k}^*\Bigg). \tag{40}$$

In addition, it can be checked that the new estimation $\hat{\mathbf{g}}_{00k}$ and the corresponding estimation error $\boldsymbol{\varepsilon}_{00k} = \mathbf{h}_{00k} - \hat{\mathbf{g}}_{00k}$ are also uncorrelated according to LMMSE. The corresponding precoding vector is given by $\mathbf{w}_{li} \triangleq \frac{\hat{\mathbf{g}}_{lli}^*}{\|\hat{\mathbf{g}}_{lli}^*\|}$, $\forall l \in \Phi$ and $k \in \{1, 2, \cdots, K\}$, where the estimation $\hat{\mathbf{g}}_{lli}$ in each cell is as obtained in the same way as $\hat{\mathbf{g}}_{00k}$ described above. Then, according to (32), we can derive the DL SINR of SPe in the following lemma.

Lemma 7: The DL SINR of UT_{0k} for SPe is given by

$$\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}} = \frac{\lambda_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SPe}}\bar{M}}{\lambda_{0k}\beta_{00k}\left(\rho_k - \bar{M}\gamma_{0k}^{\mathrm{SPe}}\right) + \sum_{i=1,i\neq k}^{K}\lambda_{0i}\beta_{00k}\rho_{ki} + \sum_{l\in\Psi}\sum_{i=1}^{K}\lambda_{li}\beta_{0lk} + \sigma^2}, \tag{41}$$

where $\rho_k = \frac{1+M\gamma_{0k}^{\mathrm{SPe}} + \frac{Mp_{0k}\gamma_{0k}^{\mathrm{SPe}}}{q_{0k}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \frac{p_{0k}{\gamma_{0k}^{\mathrm{SP}}}^2}{q_{0k}L_u} + \sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\mathrm{SP}}}^2}{q_{0i}L_u}\right)}{1+\frac{\gamma_{0k}^{\mathrm{SPe}}}{M} + \frac{p_{0k}\gamma_{0k}^{\mathrm{SPe}}}{Mq_{0k}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \frac{p_{0k}{\gamma_{0k}^{\mathrm{SP}}}^2}{q_{0k}L_u} + \sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\mathrm{SP}}}^2}{q_{0i}L_u}\right)}$, $\rho_{ki} = \frac{1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{q_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\mathrm{SP}}}^2}{q_{0i}L_u}\right)}{1+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{Mq_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\mathrm{SP}}}^2}{q_{0i}L_u}\right)}$, for $i \neq k$, and $\gamma_{0k}^{\mathrm{SPe}}$ is given by (39).

Proof: See Appendix D. □

C. UL and DL Ergodic Achievable Rates Analysis

In the previous subsection, we have obtained the analytical expressions of the effective SINRs for each UT's UL and DL data symbols in cell 0. For URLLC systems, we should adopt the achievable data rate of FBL codes given by (14) and (16). Then, the ergodic achievable rate for UT_{0k} can be approximated as

$$\begin{aligned} R_k^{\text{sym}} &\approx \log_2(1+\Gamma_k) - \sqrt{\frac{V_{\text{NE}}(\Gamma_k)}{L}} Q^{-1}(\varepsilon_k) \\ &= \log_2(1+\Gamma_k) - \log_2 e \, Q^{-1}(\varepsilon_k) \sqrt{\frac{2}{L}} \sqrt{\frac{\Gamma_k}{1+\Gamma_k}} \qquad (42) \\ &= \log_2 e \left(\ln(1+\Gamma_k) - \theta_k \sqrt{\frac{\Gamma_k}{1+\Gamma_k}} \right) \triangleq f_k\left(\frac{1}{\Gamma_k}\right) \quad [BPCU], \qquad (43) \end{aligned}$$

where Γ_k denotes the received effective SINR of a (UL or DL) communication link of UT_{0k} with a certain type of pilot scheme (i.e., RP or SP or SPe). $\theta_k \triangleq Q^{-1}(\varepsilon_k)\sqrt{\frac{2}{L}} > 0$ in (43) and L denotes the blocklength of data symbols. The functions $f_k(\omega)$ are defined as $f_k(\omega) \triangleq \log_2(e)\big[\ln\big(1+\frac{1}{\omega}\big) - \theta_k\sqrt{\frac{1}{1+\omega}}\big] > 0$ for $k \in \{1, 2, \cdots, K\}$ with $\omega > 0$. The subscript "sym" indicates that the result is for UL/DL with which pilot scheme, i.e., $\text{sym} \in \{\{\text{RP}, \text{UL}\}, \{\text{RP}, \text{DL}\}, \{\text{SP}, \text{UL}\}, \{\text{SP}, \text{DL}\}, \{\text{SPe}, \text{DL}\}\}$.

Recall that we have introduced the binary RV $\chi_{l'i}$ according to the random allocation of orthogonal pilots with RP scheme in (3). The results of (22) and (34) both still contain $\chi_{l'i}$ and thus are not the final effective SINR results. To obtain the final effective SINRs, we need to study the property of the ergodic achievable rate as a function of the received effective SINR. We first introduce the following function that

$$f(\omega) \triangleq \ln\left(1+\frac{1}{\omega}\right) - \theta\sqrt{\frac{1}{1+\omega}} \geq 0, \omega > 0, \qquad (44)$$

where θ is a positive constant value. We note that $f_k \; \forall k$ in (43) has *the same format* as that of f. We first need to derive the feasible region for $f(\omega)$. We can obtain according to (44) that, the sufficient and necessary condition (SNC) for $f(\omega) \geq 0$ is

$$\theta \leq \sqrt{1+\omega}\ln\left(1+\frac{1}{\omega}\right) \triangleq \psi(\omega). \qquad (45)$$

Moreover, the first-order derivative of $\psi(\omega)$ w.r.t. ω is given by

$$\psi'(\omega) = \sqrt{\frac{1}{1+\omega}}\left[\frac{1}{2}\ln\left(1+\frac{1}{\omega}\right) - \frac{1}{\omega}\right] < 0, \quad \omega > 0, \qquad (46)$$

where the inequality follows by using $\ln\left(1+\frac{1}{\omega}\right) - \frac{1}{\omega} < 0$ for $\omega > 0$. Thus, $\psi(\omega)$ is a monotonically decreasing function w.r.t. ω for $\omega > 0$. Then, we have $\psi(\omega) \geq \theta$ when $\omega \in (0, \psi^{-1}(\theta)]$. It follows that the feasible region of function $f(\omega)$ is

$$\Omega = \left\{\omega | 0 < \omega \leq \psi^{-1}(\theta)\right\}. \qquad (47)$$

Lemma 8: $f(\omega)$ is a monotonic decreasing convex function in the feasible region Ω.

Proof: First, the first-order derivative of $f(\omega)$ is given by

$$f'(\omega) = \frac{1}{2\omega(\omega+1)}\left(\theta\omega\sqrt{\frac{1}{1+\omega}}-2\right) < \frac{1}{\omega+1}\left[\frac{1}{2}\ln\left(1+\frac{1}{\omega}\right)-\frac{1}{\omega}\right] < 0, \omega\in\Omega, \quad (48)$$

where the first inequality of (48) follows from (45) and the second follows by using $\ln\left(1+\frac{1}{\omega}\right)-\frac{1}{\omega} < 0$ for $\omega > 0$ again. In addition, the second-order derivative of $f(\omega)$ is given by

$$\begin{aligned} f''(\omega) &= \frac{8\omega - 3\theta\omega^2\sqrt{\frac{1}{1+\omega}}+4}{4\omega^2(\omega+1)^2} = \frac{5\omega+4}{4\omega^2(\omega+1)^2}+\frac{3}{4(\omega+1)^2}\left(\frac{1}{\omega}-\theta\sqrt{\frac{1}{1+\omega}}\right) \\ &> \frac{5\omega+4}{4\omega^2(\omega+1)^2}+\frac{3}{4(\omega+1)^2}\left(\frac{1}{\omega}-\ln\left(1+\frac{1}{\omega}\right)\right) > 0, \quad \omega\in\Omega, \end{aligned} \quad (49)$$

where the first inequality in (49) follows from (45) again and the second inequality in (49) follows by using $\ln\left(1+\frac{1}{\omega}\right)-\frac{1}{\omega} < 0$ for $\omega > 0$ again. The proof is complete. □

In the following theorem, we derive the ergodic achievable rate of UT_{0k} UL or DL with a certain type of pilot signal.

Theorem 1: With FBL coding, an ergodic achievable rate of UT_{0k} in UL/DL with SP or SPe is given by

$$R_k^{\mathrm{sym}} = f_k\left(\frac{1}{\mathrm{SINR}_{0k}^{\mathrm{sym}}}\right), \quad (50)$$

where the effective SINRs $\mathrm{SINR}_{0k}^{\mathrm{sym}}$ ($\mathrm{sym}\in\{\{\mathrm{SP},\mathrm{UL}\},\{\mathrm{SP},\mathrm{DL}\},\{\mathrm{SPe},\mathrm{DL}\}\}$) are given by (30) and (35) for the UL and DL with SP respectively, and (41) for the DL with SPe. When $\mathrm{sym}\in\{\{\mathrm{RP},\mathrm{UL}\},\{\mathrm{RP},\mathrm{DL}\}\}$, a lower bound (LB) on R_k^{sym} can be obtained by averaging over the random pilot allocation which is given by

$$R_k^{\mathrm{sym}} \geq \tilde{R}_k^{\mathrm{sym}} = f_k\left(\frac{1}{\mathrm{SINR}_{0k}^{\mathrm{sym}}}\right), \quad (51)$$

where the final effective SINRs $\mathrm{SINR}_{0k}^{\mathrm{sym}}$ for UL/DL with RP are respectively given by

$$\begin{aligned} &\mathrm{SINR}_{0k}^{\mathrm{RP,UL}} \\ &= \frac{ML_u^d p_{0k}q_{0k}\beta_{00k}^2}{M\sum\limits_{l'\in\Psi}\sum\limits_{l=1}^{K}p_{l'i}q_{l'i}\beta_{0l'i}^2+\left(q_{0k}L_u^p\beta_{00k}+\sum\limits_{l'\in\Psi}\sum\limits_{i=1}^{K}q_{l'i}\beta_{0l'i}+\sigma^2\right)\left(\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2\right)}, \end{aligned} \quad (52)$$

$$\mathrm{SINR}_{0k}^{\mathrm{RP,DL}} = \frac{\lambda_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{RP}}\bar{M}}{(M-\bar{M}-1)\lambda_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{RP}}+\sum\limits_{i=1}^{K}\lambda_{0i}\beta_{00k}+\sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{0lk}+\sigma^2} \quad (53)$$

with

$$\gamma_{0k}^{\mathrm{RP}} = \mathbb{E}\left\{\frac{1}{\overline{\gamma}_{0k}^{\mathrm{RP}}}\right\}^{-1} = \frac{L_u^p q_{0k}\beta_{00k}}{L_u^p q_{0k}\beta_{00k} + \sum\limits_{l'\in\Psi}\sum\limits_{i=1}^{K} q_{l'i}\beta_{0l'i} + \sigma^2}. \tag{54}$$

Proof: Recall that we have introduced the binary random variable $\chi_{l'i}$ according to random allocation of orthogonal pilots with RP scheme in (3). Then, the results of (22) and (34) still contains the random variable $\chi_{l'i}$. We only need to prove the results with RP. With the definition of f_k, the relation in (43) and according to *Lemma* 8, a LB on the ergodic achievable rate R_k^{sym} can be obtained by using Jensen's inequality as

$$R_k^{\mathrm{sym}} = \mathbb{E}\left[f_k\left(\frac{1}{\overline{\mathrm{SINR}}_{0k}^{\mathrm{sym}}}\right)\right] \geq f_k\left(\mathbb{E}\left[\frac{1}{\overline{\mathrm{SINR}}_{0k}^{\mathrm{sym}}}\right]\right) \triangleq \tilde{R}_k^{\mathrm{sym}}. \tag{55}$$

for $\overline{\mathrm{SINR}}_{0k}^{\mathrm{sym}} > \frac{1}{\psi^{-1}(\theta_k)}$, where $\overline{\mathrm{SINR}}_{0k}^{\mathrm{sym}}$ denotes the UL/DL effective SINR for RP given by *Lemma* 3 and 5. Therefore, with $\mathbb{E}[\chi_{l'i}] = \frac{1}{L_u^p}$ we can easily obtain the final effective SINRs as $\mathrm{SINR}_{0k}^{\mathrm{RP,UL}} = 1/\mathbb{E}\left\{\frac{1}{\overline{\mathrm{SINR}}_k^{\mathrm{RP,UL}}}\right\}$ and $\mathrm{SINR}_{0k}^{\mathrm{RP,DL}} = 1/\mathbb{E}\left\{\frac{1}{\overline{\mathrm{SINR}}_k^{\mathrm{RP,DL}}}\right\}$ according to (22) and (34) respectively. The proof is complete. □

Our numerical results in Section V can verify the tightness of the derived LBs on the ergodic achievable rates. Moreover, the gap between the ergodic achievable rate and the corresponding LB is reduced when the number of BS antennas increases large, which is the case with a massive MIMO system. These LBs are commonly used in the literature of massive MIMO systems [20–22, 34]. Hence, we follow the previous literature to use these LBs for RP throughout this paper.

IV. Joint Maximization of the UL-DL Overall Weighted Sum Rates

It can be easily observed that, for the considered massive MIMO cellular systems, the power allocation in the UL phase not only affects the UL received SINR of each UT's data but also affects the received SINR of each UT's data in the DL phase. With the aid of the ergodic achievable rates in *Theorem* 1, the overall UL-DL weighted ergodic sum rate of all accessed UTs is optimized in this section, subject to the decoding BLER, the minimum data rate of each UT, and the energy constraints of each UT and the BS. Specifically, we consider the joint optimization of the UL and DL power allocation of the pilot and data payload transmission of each UT in the target cell for the systems with the three pilot schemes individually (i.e., RP, SP and SPe). The weights are introduced to provide fairness between different users in the UL and DL communications. The UL-DL balance is satisfied by the allocated UL and DL blocklengths. The large-scale fading parameters are assumed to be known at the target BS as they change very slowly (e.g., with the scale of thousands of coherent time intervals). The proposed resource optimization scheme in this work only depends on the large-scale fading which makes it feasible to operate on online. Also, the optimization is only required when any large-scale fading parameter changes. Hence, the optimization solutions are applicable for URLLC applications.

A. Problem Formulation

Therefore, we take the UL-DL overall average WSR as the objective function. In addition, we optimize the system performance with RP, SP and SPe separately and then compare among them.

For the different types of pilot signals, we consider the energy constraint at each UT for UL transmission, which results in the following two energy constraints of each UT:

$$L_u^p q_{0k} + L_u^d p_{0k} \leq E_{0k}^u, \ \ \forall k, \qquad \text{for RP}, \tag{56}$$

$$L_u(q_{0k} + p_{0k}) \leq E_{0k}^u, \ \ \forall k, \qquad \text{for SP}, \tag{57}$$

where E_{0k}^u is the upper limit of the available energy of UT_{0k}. Moreover, we have the following energy constraint at BS 0 for DL transmission:

$$L_d \sum_{k=1}^{K} \lambda_{0k} \leq E_0^d, \tag{58}$$

where E_0^d is the total available energy of the BS of cell 0. According to (43), the overall WSR maximization problem for RP can be formulated as

$$\max_{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\}} \quad \sum_{k=1}^{K} \left\{ \left(1 - \frac{L_u^p}{L_u}\right) w_k^u \tilde{R}_k^{\text{RP,UL}} + w_k^d \tilde{R}_k^{\text{RP,DL}} \right\} \tag{59a}$$

$$\text{s.t.} \quad \left(1 - \frac{L_u^p}{L_u}\right) \tilde{R}_k^{\text{RP,UL}} \geq R_k^{\text{u, req}}, \quad \forall k \tag{59b}$$

$$\tilde{R}_k^{\text{RP,DL}} \geq R_k^{\text{d, req}}, \quad \forall k \tag{59c}$$

$$\text{(56), (58)}, \tag{59d}$$

where $\tilde{R}_k^{\text{RP,UL}}$, $\tilde{R}_k^{\text{RP,DL}}$ are respectively the obtained LBs on the UL and DL ergodic achievable rates of UT_{0k} for RP given in *Theorem* 1. $R_k^{\text{u, req}}$ and $R_k^{\text{d, req}}$ are respectively the minimum required effective data rate of UT_{0k} in the UL and DL. $w_k^u \triangleq L_u/(L_u + L_d)\tilde{w}_k^u$ and $w_k^d \triangleq L_d/(L_u + L_d)\tilde{w}_k^d$ respectively denote the weights of UT_{0k} in the DL and UL to guarantee the fairness among the UTs and also to take into account the asymmetric traffics in the UL and DL in general, where $\tilde{w}_k^d$ and $\tilde{w}_k^u$ are respectively the weights of UT_{0k} used to guarantee the fairness among the devices.

Similarly, the corresponding optimization problems for SP and SPe can be respectively formulated as

$$\max_{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\}} \sum_{k=1}^{K} \left\{ w_k^u R_k^{\text{SP,UL}} + w_k^d R_k^{\text{SP,DL}} \right\} \tag{60a}$$

$$\text{s.t.} \quad R_k^{\text{SP,UL}} \geq R_k^{\text{u, req}}, \quad \forall k \tag{60b}$$

$$R_k^{\text{SP,DL}} \geq R_k^{\text{d, req}}, \quad \forall k \tag{60c}$$

$$(57), (58), \tag{60d}$$

and

$$\max_{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\}} \sum_{k=1}^{K} \left\{ w_k^u R_k^{\mathrm{SP,UL}} + w_k^d R_k^{\mathrm{SPe,DL}} \right\} \tag{61a}$$

$$\text{s.t.} \quad R_k^{\mathrm{SP,UL}} \geq R_k^{\text{u, req}}, \quad \forall k \tag{61b}$$

$$R_k^{\mathrm{SPe,DL}} \geq R_k^{\text{d, req}}, \quad \forall k \tag{61c}$$

$$(57), (58), \tag{61d}$$

where $R_k^{\mathrm{SP,UL}}$, $R_k^{\mathrm{SP,DL}}$ and $R_k^{\mathrm{SPe,UL}}$ are given in *Theorem* 1. The only difference between the two optimization problems of (60) and (61) is the ergodic achievable rate with and without interference elimination. Therefore, we can obtain the solution/algorithm to Problem (61) by replacing $R_k^{\mathrm{SP,DL}}$ by $R_k^{\mathrm{SPe,UL}}$ in Problem (60).

It is well known that the power allocation to maximize the WSR with interference is generally an NP-hard problem [35]. It becomes even harder for the cases with the imperfect CSI and the finite blocklength. In this paper, we aim for designing efficient algorithms within polynomial-time complexity to solve the above problems.

B. Simplification of Problem Formulation

Since each of the constraints (59b), (59c), (60b), (60c) and (61c) guarantees that $R_k^{\mathrm{sym}} > 0$ or $\tilde{R}_k^{\mathrm{sym}} > 0$, the feasible effective SINR Γ_k must lies in the region $\Omega_k = \{\Gamma_k | 0 < 1/\Gamma_k \leq \psi^{-1}(\theta_k)\}$. Thus, *Lemma* 8 and $\partial R_k^{\mathrm{sym}} / \partial \Gamma_k > 0$ or $\partial \tilde{R}_k^{\mathrm{sym}} / \partial \Gamma_k > 0$ hold. The constraints (59b), (59c), (60b), (60c) and (61c) can be respectively transformed as

$$\mathrm{SINR}_{0k}^{\mathrm{RP,UL}} \geq 1 \Big/ f^{-1}\left(\frac{R_k^{u,\mathrm{req}} \ln 2}{1 - \frac{L_u^p}{L_u}} \right), \tag{62}$$

$$\mathrm{SINR}_{0k}^{\mathrm{RP,DL}} \geq 1 \Big/ f^{-1}\left(R_k^{d,\mathrm{req}} \ln 2 \right), \tag{63}$$

$$\mathrm{SINR}_{0k}^{\mathrm{SP,UL}} \geq 1 \Big/ f^{-1}\left(R_k^{u,\mathrm{req}} \ln 2 \right), \tag{64}$$

$$\mathrm{SINR}_{0k}^{\mathrm{SP,DL}} \geq 1 \Big/ f^{-1}\left(R_k^{d,\mathrm{req}} \ln 2 \right), \tag{65}$$

$$\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}} \geq 1 \Big/ f^{-1}\left(R_k^{d,\mathrm{req}} \ln 2 \right). \tag{66}$$

Moreover, the objective functions of these optimization problems can also be simplified. We take Problem (59) as a typical example for illustrative purpose. The other two can be solved by following the similar procedures. To further simplify Problem (59), we introduce the auxiliary variables $\upsilon_k^u, \upsilon_k^d, \forall k$. Then, Problem (59) can be equivalently transformed as

$$\max_{\substack{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\},\\ \{\upsilon_k^u\},\{\upsilon_k^d\}}} \sum_{k=1}^{K}\left\{\left(1-\frac{L_u^p}{L_u}\right)\frac{w_k^u}{\ln 2}\left[\ln(1+\upsilon_k^u)-\theta_k^{u,\mathrm{RP}}\sqrt{\frac{\upsilon_k^u}{1+\upsilon_k^u}}\right]\right\}$$

$$+\sum_{k=1}^{K}\left\{\frac{w_k^d}{\ln 2}\left[\ln(1+\upsilon_k^d)-\theta_k^{d,\mathrm{RP}}\sqrt{\frac{\upsilon_k^d}{1+\upsilon_k^d}}\right]\right\} \tag{67a}$$

$$\text{s.t.}\quad \mathrm{SINR}_{0k}^{\mathrm{RP,UL}} \geq \upsilon_k^u, \quad \forall k \tag{67b}$$

$$\mathrm{SINR}_{0k}^{\mathrm{RP,DL}} \geq \upsilon_k^d, \quad \forall k \tag{67c}$$

$$\upsilon_k^u \geq 1\Big/ f^{-1}\left(\frac{R_k^{u,\mathrm{req}}\ln 2}{1-\frac{L_u^p}{L_u}}\right), \quad \forall k \tag{67d}$$

$$\upsilon_k^d \geq 1\Big/ f^{-1}\left(R_k^{d,\mathrm{req}}\ln 2\right), \quad \forall k \tag{67e}$$

$$(56), (58), \tag{67f}$$

where $\theta_k^{u,\mathrm{RP}} = Q^{-1}(\varepsilon_k)\sqrt{\frac{2}{L_u^d}}$, $\theta_k^{d,\mathrm{RP}} = Q^{-1}(\varepsilon_k)\sqrt{\frac{2}{L_d}}$. Problem (59) and Problem (67) are equivalent in the sense that they have the same power allocation solutions and also the same value of objective functions, which can be readily proved by contradiction method.

Similarly, the joint UL-DL optimization problem of SP in (60) can be equivalently transformed as

$$\max_{\substack{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\},\\ \{\upsilon_k^u\},\{\upsilon_k^d\}}} \sum_{k=1}^{K}\left\{\frac{w_k^u}{\ln 2}\left[\ln(1+\upsilon_k^u)-\theta_k^{u,\mathrm{SP}}\sqrt{\frac{\upsilon_k^u}{1+\upsilon_k^u}}\right]\right\}$$

$$+\sum_{k=1}^{K}\left\{\frac{w_k^d}{\ln 2}\left[\ln(1+\upsilon_k^d)-\theta_k^{d,\mathrm{SP}}\sqrt{\frac{\upsilon_k^d}{1+\upsilon_k^d}}\right]\right\} \tag{68a}$$

$$\text{s.t.}\quad \mathrm{SINR}_{0k}^{\mathrm{SP,UL}} \geq \upsilon_k^u, \quad \forall k \tag{68b}$$

$$\mathrm{SINR}_{0k}^{\mathrm{SP,DL}} \geq \upsilon_k^d, \quad \forall k \tag{68c}$$

$$\upsilon_k^u \geq 1\Big/ f^{-1}\left(R_k^{u,\mathrm{req}}\ln 2\right), \quad \forall k \tag{68d}$$

$$\upsilon_k^d \geq 1\Big/ f^{-1}\left(R_k^{d,\mathrm{req}}\ln 2\right), \quad \forall k \tag{68e}$$

$$(57), (58), \tag{68f}$$

where $\theta_k^{u,\mathrm{SP}} = Q^{-1}(\varepsilon_k)\sqrt{\frac{2}{L_u}}$, $\theta_k^{d,\mathrm{SP}} = \theta_k^{d,\mathrm{RP}} = Q^{-1}(\varepsilon_k)\sqrt{\frac{2}{L_d}}$. In addition, the joint UL-DL optimization problem of SPe in (61) can be equivalently transformed as

$$\max_{\substack{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\},\\ \{\upsilon_k^u\},\{\upsilon_k^d\}}} \sum_{k=1}^{K}\left\{\frac{w_k^u}{\ln 2}\left[\ln(1+\upsilon_k^u)-\theta_k^{u,\mathrm{SP}}\sqrt{\frac{\upsilon_k^u}{1+\upsilon_k^u}}\right]\right\}$$

$$+\sum_{k=1}^{K}\left\{\frac{w_k^d}{\ln 2}\left[\ln(1+\upsilon_k^d)-\theta_k^{d,\mathrm{SP}}\sqrt{\frac{\upsilon_k^d}{1+\upsilon_k^d}}\right]\right\} \tag{69a}$$

$$\text{s.t.}\quad \mathrm{SINR}_{0k}^{\mathrm{SPe,DL}} \geq \upsilon_k^d, \quad \forall k \tag{69b}$$

$$\upsilon_k^d \geq 1\Big/ f^{-1}\left(R_k^{d,\mathrm{req}}\ln 2\right), \quad \forall k \tag{69c}$$

$$(68b), (68d), (57), (58). \tag{69d}$$

In the following, we solve these problems.

C. Algorithm Design

We observe that it is difficult to solve these problems due to the complicated form of the objective functions. To resolve this issue, we first study the property of the term in the objective functions that contains the optimization variables which is in the form of $\ln(1+\upsilon)-\theta\sqrt{\frac{\upsilon}{1+\upsilon}}$. Firstly, we have the following result given by [36, *Lemma* 4].

Lemma 9: For any given $\tilde{\upsilon}$, the function $\ln(1+\upsilon)$ has a LB of

$$\ln(1+\upsilon) \geq \kappa\ln\upsilon+\nu,\ \forall\upsilon\geq 0, \tag{70}$$

where κ and ν are given by

$$\kappa=\frac{\tilde{\upsilon}}{1+\tilde{\upsilon}}, \nu=\ln(1+\tilde{\upsilon})-\frac{\tilde{\upsilon}}{1+\tilde{\upsilon}}\ln\tilde{\upsilon}, \tag{71}$$

and $\ln(1+\upsilon)=\kappa\ln\upsilon+\nu$, when $\upsilon=\tilde{\upsilon}$.

Moreover, we have the following lemma.

Lemma 10: For any given $\tilde{\upsilon}\geq\frac{1}{2}$, the function $G(\upsilon)\triangleq\sqrt{\frac{\upsilon}{1+\upsilon}}$ is upper-bounded as

$$G(\upsilon)\leq\hat{\kappa}\ln\upsilon+\hat{\nu}\triangleq F(\upsilon),\ \forall\upsilon\geq\frac{1}{2}, \tag{72}$$

where $\hat{\kappa}$ and $\hat{\nu}$ are given by

$$\hat{\kappa}=\frac{1}{2(1+\tilde{\upsilon})}\sqrt{\frac{\tilde{\upsilon}}{1+\tilde{\upsilon}}}, \hat{\nu}=\sqrt{\frac{\tilde{\upsilon}}{1+\tilde{\upsilon}}}-\hat{\kappa}\ln\tilde{\upsilon}. \tag{73}$$

In addition, we have

$$G(\tilde{\upsilon})=F(\tilde{\upsilon}),\ G'(\tilde{\upsilon})=F'(\tilde{\upsilon}). \tag{74}$$

Proof: It is easy to check that (74) holds by substituting $\hat{\kappa}$ and $\hat{\nu}$ in (73) into (74). We next prove the inequality in (72). Let $U(\upsilon) \triangleq F(\upsilon) - G(\upsilon)$. Then, it holds that $U(\tilde{\upsilon}) = 0$. The first-order derivative of function $U(\upsilon)$ w.r.t. υ can be obtained as

$$U'(\upsilon) = \frac{\frac{(1+\upsilon)^{\frac{3}{2}}}{\sqrt{\upsilon}} - \frac{(1+\tilde{\upsilon})^{\frac{3}{2}}}{\sqrt{\tilde{\upsilon}}}}{2\sqrt{\frac{\upsilon}{\tilde{\upsilon}}}(1+\tilde{\upsilon})^{\frac{3}{2}}(1+\upsilon)^{\frac{3}{2}}}. \tag{75}$$

Obviously, the sign of $U'(\upsilon)$ is determined by the numerator in (75). Moreover, we let $D(\upsilon) \triangleq \frac{(1+\upsilon)^{\frac{3}{2}}}{\sqrt{\upsilon}}$ with $D'(\upsilon) = \frac{(2\upsilon-1)\sqrt{1+\upsilon}}{2\upsilon^{\frac{3}{2}}}$. It is easy to check that $D(\upsilon) \leq D(\tilde{\upsilon})$ when $\frac{1}{2} \leq \upsilon \leq \tilde{\upsilon}$ and $U'(\upsilon) \leq 0$. Thus, $U(\upsilon) \geq U(\tilde{\upsilon}) = 0$ when $\frac{1}{2} \leq \upsilon \leq \tilde{\upsilon}$, or equivalently $F(\upsilon) \geq G(\upsilon)$ when $\frac{1}{2} \leq \upsilon \leq \tilde{\upsilon}$. For the same reason, we have $F(\upsilon) \geq G(\upsilon)$ when $\frac{1}{2} \leq \tilde{\upsilon} \leq \upsilon$. Then, we can conclude that $F(\upsilon) \geq G(\upsilon)$ when $\upsilon \geq \frac{1}{2}$. The proof is complete. □

1) The Algorithm for the System with RP: We can approximate the objective function of Problem (67) by a LB on it with *Lemma* 9 and *Lemma* 10, and then solve the approximate problem in an *iterative manner* which is described as follows.

Firstly, we denote $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \lambda_{0k}^{(t)}, \forall k\right\}$ as the UL and DL power allocation at the t-th iteration. In addition, the corresponding auxiliary variables $\left\{\upsilon_k^u, \upsilon_k^d, \forall k\right\}$ at the t-th iteration are denoted by $\left\{\upsilon_k^{u(t)}, \upsilon_k^{d(t)}, \forall k\right\}$. Using the approximation results in *Lemma* 9 and *Lemma* 10, at the $(t+1)$-th iteration, we get a LB on the objective function in (67a) as

$$\begin{aligned}
&\sum_{k=1}^{K}\left\{\left(1-\frac{L_u^p}{L_u}\right)\frac{w_k^u}{\ln 2}\left[\ln(1+\upsilon_k^u) - \theta_k^{u,\mathrm{RP}}\sqrt{\frac{\upsilon_k^u}{1+\upsilon_k^u}}\right]\right\} \\
&\quad + \sum_{k=1}^{K}\left\{\frac{w_k^d}{\ln 2}\left[\ln(1+\upsilon_k^d) - \theta_k^{d,\mathrm{RP}}\sqrt{\frac{\upsilon_k^d}{1+\upsilon_k^d}}\right]\right\} \\
&\geq \sum_{k=1}^{K}\left\{\left(1-\frac{L_u^p}{L_u}\right)\frac{w_k^u}{\ln 2}\left[\kappa_k^{u(t)}\ln \upsilon_k^u + \nu_k^{u(t)} - \theta_k^{u,\mathrm{RP}}\left(\hat{\kappa}_k^{u(t)}\ln \upsilon_k^u + \hat{\nu}_k^{u(t)}\right)\ \right]\right\} \\
&\quad + \sum_{k=1}^{K}\left\{\frac{w_k^d}{\ln 2}\left[\kappa_k^{d(t)}\ln \upsilon_k^d + \nu_k^{d(t)} - \theta_k^{d,\mathrm{RP}}\left(\hat{\kappa}_k^{d(t)}\ln \upsilon_k^d + \hat{\nu}_k^{d(t)}\right)\ \right]\right\},
\end{aligned} \tag{76}$$

where $\kappa_k^{u(t)} = \frac{\upsilon_k^{u(t)}}{1+\upsilon_k^{u(t)}}$, $\nu_k^{u(t)} = \ln\left(1+\upsilon_k^{u(t)}\right) - \frac{\upsilon_k^{u(t)}}{1+\upsilon_k^{u(t)}}\ln \upsilon_k^{u(t)}$, $\hat{\kappa}_k^{u(t)} = \frac{1}{2\left(1+\upsilon_k^{u(t)}\right)}\sqrt{\frac{\upsilon_k^{u(t)}}{1+\upsilon_k^{u(t)}}}$, $\hat{\nu}_k^{d(t)} = \sqrt{\frac{\upsilon_k^{u(t)}}{1+\upsilon_k^{u(t)}}} - \hat{\kappa}_k^{u(t)}\ln \upsilon_k^{u(t)}$, $\kappa_k^{d(t)} = \frac{\upsilon_k^{d(t)}}{1+\upsilon_k^{d(t)}}$, $\nu_k^{d(t)} = \ln\left(1+\upsilon_k^{d(t)}\right) - \frac{\upsilon_k^{d(t)}}{1+\upsilon_k^{d(t)}}\ln \upsilon_k^{d(t)}$, $\hat{\kappa}_k^{d(t)} = \frac{1}{2\left(1+\upsilon_k^{d(t)}\right)}\sqrt{\frac{\upsilon_k^{d(t)}}{1+\upsilon_k^{d(t)}}}$, $\hat{\nu}_k^{d(t)} = \sqrt{\frac{\upsilon_k^{d(t)}}{1+\upsilon_k^{d(t)}}} - \hat{\kappa}_k^{d(t)}\ln \upsilon_k^{d(t)}$, and the equality holds only when $\upsilon_k^u = \upsilon_k^{u(t)}$ and $\upsilon_k^d = \upsilon_k^{d(t)}$.

Next, we employ the LB at the right hand side (RHS) of (76) as an alternative objective function to optimize the power allocation for Problem (67). At the $(t+1)$-th iteration, the LB maximization problem to be solved can be formulated as

$$\max_{\substack{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\},\\ \{\upsilon_k^u\},\{\upsilon_k^d\}}} \sum_{k=1}^{K} \tilde{w}_k^{u,\mathrm{RP}(t)} \ln \upsilon_k^u + \sum_{k=1}^{K} \tilde{w}_k^{d,\mathrm{RP}(t)} \ln \upsilon_k^d \tag{77a}$$

$$\text{s.t.} \quad (56),(58),(62),(63),(67b),(67c), \tag{77b}$$

where $\tilde{w}_k^{u,\mathrm{RP}(t)} = \left(1 - \frac{L_u^p}{L_u}\right) \frac{w_k^u}{\ln 2} \kappa_k^{u(t)} - \theta_k^{u,\mathrm{RP}} \hat{\kappa}_k^{u(t)}$, $\tilde{w}_k^{d,\mathrm{RP}(t)} = \frac{w_k^d}{\ln 2} \kappa_k^{d(t)} - \theta_k^{d,\mathrm{RP}} \hat{\kappa}_k^{d(t)}$, and the constant terms with the LB on the objective function in (76) is omitted. Further, we can equivalently transform Problem (77) into a GP problem as follows:

$$\max_{\substack{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\},\\ \{\upsilon_k^u\},\{\upsilon_k^d\}}} \prod_{k=1}^{K} {\upsilon_k^u}^{\tilde{w}_k^{u,\mathrm{RP}(t)}} \prod_{k=1}^{K} {\upsilon_k^d}^{\tilde{w}_k^{d,\mathrm{RP}(t)}} \tag{78a}$$

$$\begin{aligned} \text{s.t.} \quad & \upsilon_k^u \left(q_{0k} L_u^p \beta_{00k} + \sum_{l' \in \Psi} \sum_{i=1}^{K} q_{l'i} \beta_{0l'i} + \sigma^2 \right) \left(\sum_{l' \in \Phi} \sum_{i=1}^{K} p_{l'i} \beta_{0l'i} + \sigma^2 \right) \\ & \quad + M \upsilon_k^u \sum_{l' \in \Psi} \sum_{l=1}^{K} p_{l'i} q_{l'i} \beta_{0l'i}^2 \le M L_u^d p_{0k} q_{0k} \beta_{00k}^2, \quad \forall k \end{aligned} \tag{78b}$$

$$\begin{aligned} & \upsilon_k^d \left(\sum_{i=1, i \neq k}^{K} \lambda_{0i} \beta_{00k} + \sum_{l \in \Psi} \sum_{i=1}^{K} \lambda_{li} \beta_{0lk} + \sigma^2 \right) \left(L_u^p q_{0k} \beta_{00k} + \sum_{l' \in \Psi} \sum_{i=1}^{K} q_{l'i} \beta_{0l'i} + \sigma^2 \right) \\ & \quad + \upsilon_k^d \lambda_{0k} \beta_{00k} \left((M - \bar{M}) L_u^p q_{0k} \beta_{00k} + \sum_{l' \in \Psi} \sum_{i=1}^{K} q_{l'i} \beta_{0l'i} + \sigma^2 \right) \\ & \quad \le \bar{M} L_u^p \lambda_{0k} q_{0k} \beta_{00k}^2, \quad \forall k \end{aligned} \tag{78c}$$

$$(56),(58),(62),(63). \tag{78d}$$

It is known that a GP problem can be equivalently transformed into a convex optimization problem by applying a logarithmic change of variables [37]. The globally optimal solution of Problem (78) can be obtained using the interior-point method [37]. Some software packages such as MOSEK and CVX can efficiently solve GP problems [38]. The iterative algorithm to solve Problem (78) is given by **Algorithm 1** below.

Moreover, feasible initial values $\left\{p_{0k}^{(0)}, q_{0k}^{(0)}, \lambda_{0k}^{(0)}, \forall k\right\}$ are required to initialize the iterative algorithm. However, a set of randomly selected power allocation values that satisfies the UL/DL energy constraints may not satisfy their minimum SINR requirements. Hence, the initial power allocation values have to be carefully chosen. We determine $\left\{p_{0k}^{(0)}, q_{0k}^{(0)}, \lambda_{0k}^{(0)}, \forall k\right\}$ by following the method in

Algorithm 1: Iterative Algorithm to Solve Problem (78) for the Systems with RP

1 Initialize iteration number index $t=1$, error tolerances $\varsigma>0$, and a feasible power allocation $\left\{p_{0k}^{(0)}, q_{0k}^{(0)}, \lambda_{0k}^{(0)}, \forall k\right\}$. Calculate $\left\{\upsilon_k^{u(0)}, \upsilon_k^{d(0)}, \kappa_k^{u(0)}, \hat{\kappa}_k^{u(0)}, \kappa_k^{d(0)}, \hat{\kappa}_k^{d(0)},\right.$ $\left.\tilde{w}_k^{u,\mathrm{RP}(0)}, \tilde{w}_k^{d,\mathrm{RP}(0)}, \forall k\right\}$ and the objective function in (67a) denoted as $\mathrm{Obj}^{(0)}$;

2 With given $\left\{\upsilon_k^{u(t-1)}, \upsilon_k^{d(t-1)}, \kappa_k^{u(t-1)}, \hat{\kappa}_k^{u(t-1)}, \kappa_k^{d(t-1)}, \hat{\kappa}_k^{d(t-1)}, \tilde{w}_k^{u,\mathrm{RP}(t-1)}, \tilde{w}_k^{d,\mathrm{RP}(t-1)}, \forall k\right\}$, solve Problem (78) by using the CVX package to obtain $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \lambda_{0k}^{(t)}, \upsilon_k^{u(t)}, \upsilon_k^{d(t)}, \forall k\right\}$;

3 Update $\left\{\upsilon_k^{u(t)}, \upsilon_k^{d(t)}, \kappa_k^{u(t)}, \hat{\kappa}_k^{u(t)}, \kappa_k^{d(t)}, \hat{\kappa}_k^{d(t)}, \tilde{w}_k^{u,\mathrm{RP}(t)}, \tilde{w}_k^{d,\mathrm{RP}(t)}, \forall k\right\}$. Calculate the new objective function in (67a) denoted as $\mathrm{Obj}^{(t)}$;

4 **while** $\mid \mathrm{Obj}^{(t)} - \mathrm{Obj}^{(t-1)} \mid / \mathrm{Obj}^{(t)} \geq \varsigma$ **do**

5 | $t \leftarrow t+1$, go to Step 2 and Step 3;

6 **end**

7 **return** $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \lambda_{0k}^{(t)}, \forall k\right\}$, $\mathrm{Obj}^{(t)}$.

[36, 39] to solve the following optimization problem for each k:

$$\max_{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\},r} \quad r \tag{79a}$$

$$\text{s.t.} \quad \mathrm{SINR}_{0k}^{\mathrm{RP,UL}} \geq r \Big/ f^{-1}\left(\frac{R_k^{u,\mathrm{req}} \ln 2}{1-\frac{L_u^p}{L_u}}\right), \forall k \tag{79b}$$

$$\mathrm{SINR}_{0k}^{\mathrm{RP,DL}} \geq r \Big/ f^{-1}\left(R_k^{d,\mathrm{req}} \ln 2\right), \forall k \tag{79c}$$

$$(56), (58). \tag{79d}$$

where we have introduced an auxiliary variable r. Firstly, Problem (79) is always feasible since at least $\{p_{0k}=0, q_{0k}=0, \lambda_{0k}=0, r=0\}$ is a feasible solution. Moreover, it can be readily verified that the original problem of (59) is feasible if the optimal $r \geq 1$, and the output power allocation can be adopted as the initial inputs $\left\{p_{0k}^{(0)}, q_{0k}^{(0)}, \lambda_{0k}^{(0)}, \forall k\right\}$. Problem (79) can also be equivalently transformed into a GP problem, whose globally optimal solution can be obtained.

For the convergence of **Algorithm 1**, using the similar method in [36] it can be proved that the objective function value of Problem (67) at each iteration satisfies $\mathrm{Obj}^{(t+1)} \geq \mathrm{Obj}^{(t)}$. The details are omitted here due to the limited space. In addition, since each UT and each BS both have its own energy constraint, the objective function value of Problem (67) has an upper bound. As a result, **Algorithm 1** is guaranteed to converge. However, since the original problem of (67) is non-convex, the globally optimal solution generally cannot be obtained. The obtained converged solution only depends on the initial inputs of **Algorithm 1**. Our simulations illustrate that **Algorithm 1** almost converges to the same solution with different initial inputs.

2) The Algorithms for the Systems with SP and SPe: Similar to the above procedure to solve Problem (78), we can develop the GP problems corresponding to Problem (68) and (69). Specifically, we can first transform the objective functions of Problem (68) and (69) to a similar form as that of (78a). For Problem (68), we first obtain a LB on the objective function in (68a) at the $(t+1)$-th iteration as

$$\begin{aligned}&\sum_{k=1}^{K}\left\{\frac{w_k^u}{\ln 2}\left[\ln(1+\upsilon_k^u)-\theta_k^{u,\mathrm{SP}}\sqrt{\frac{\upsilon_k^u}{1+\upsilon_k^u}}\right]\right\}+\sum_{k=1}^{K}\left\{\frac{w_k^d}{\ln 2}\left[\ln(1+\upsilon_k^d)-\theta_k^{d,\mathrm{SP}}\sqrt{\frac{\upsilon_k^d}{1+\upsilon_k^d}}\right]\right\}\\ &\geq\sum_{k=1}^{K}\left\{\frac{w_k^u}{\ln 2}\left[\kappa_k^{u(t)}\ln\upsilon_k^u+\nu_k^{u(t)}-\theta_k^{u,\mathrm{SP}}\left(\hat{\kappa}_k^{u(t)}\ln\upsilon_k^u+\hat{\nu}_k^{u(t)}\right)\right]\right\}\\ &\quad+\sum_{k=1}^{K}\left\{\frac{w_k^d}{\ln 2}\left[\kappa_k^{d(t)}\ln\upsilon_k^d+\nu_k^{d(t)}-\theta_k^{d,\mathrm{SP}}\left(\hat{\kappa}_k^{d(t)}\ln\upsilon_k^d+\hat{\nu}_k^{d(t)}\right)\right]\right\},\end{aligned} \tag{80}$$

where $\{\upsilon_k^u,\upsilon_k^d,\forall k\}$ are the introduced auxiliary variables, the variables $\kappa_k^{u(t)}$, $\nu_k^{u(t)}$, $\hat{\kappa}_k^{u(t)}$, $\hat{\nu}_k^{d(t)}$, $\kappa_k^{d(t)}$, $\hat{\kappa}_k^{d(t)}$, $\hat{\nu}_k^{d(t)}$ are the same as those defined in (76), and the equality holds only when $\upsilon_k^u=\upsilon_k^{u(t)}$ and $\upsilon_k^d=\upsilon_k^{d(t)}$.

However, note that the expression of SINR at UT k with SP or SPe schemes is much more complicated than that with RP scheme, and thus the iterative approximation methods can not be directly adopted. We first need to reform the constraints on each UT's SINR in a more tractable form as follows. By respectively dividing the numerator and the denominator of (35) and (41) by $\gamma_{0k}^{\mathrm{SP}}$ and $\gamma_{0k}^{\mathrm{SPe}}$, the DL SINR expressions $\mathrm{SINR}_{0k}^{\mathrm{SP,DL}}$ and $\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}}$ can be rewritten as

$$\mathrm{SINR}_{0k}^{\mathrm{SP,DL}}=\frac{\lambda_{0k}\beta_{00k}\bar{M}}{\lambda_{0k}\beta_{00k}\left(\frac{\alpha_k}{\gamma_{0k}^{\mathrm{SP}}}-\bar{M}\right)+\sum\limits_{i=1,i\neq k}^{K}\frac{\alpha_{ki}}{\gamma_{0k}^{\mathrm{SP}}}\lambda_{0i}\beta_{00k}+\frac{1}{\gamma_{0k}^{\mathrm{SP}}}\left(\sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{0lk}+\sigma^2\right)}, \tag{81}$$

$$\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}}=\frac{\lambda_{0k}\beta_{00k}\bar{M}}{\lambda_{0k}\beta_{00k}\left(\frac{\rho_k}{\gamma_{0k}^{\mathrm{SPe}}}-\bar{M}\right)+\sum\limits_{i=1,i\neq k}^{K}\frac{\rho_{ki}}{\gamma_{0k}^{\mathrm{SPe}}}\lambda_{0i}\beta_{00k}+\frac{1}{\gamma_{0k}^{\mathrm{SPe}}}\left(\sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{0lk}+\sigma^2\right)}. \tag{82}$$

Unfortunately, after substituting the expressions of the terms $\gamma_{0k}^{\mathrm{SP}}$ and $\gamma_{0k}^{\mathrm{SPe}}$ and some others into the denominators of (81) and (82), we notice that the constraints in (68c) and (69b) cannot be simply transformed into the same form as those of the constraints (67b) and (67c) in (78b) and (78c), where one side is a monomial function and the other side is a posynomial function. Hence, the problems cannot be directly transformed into a GP problem. To deal with this problem, we first need the following result.

Lemma 11: For any given vectors $\mathbf{z}=\{z_1,z_2,...,z_N\}$ and $\tilde{\mathbf{z}}=\{\tilde{z}_1,\tilde{z}_2,...,\tilde{z}_N\}$ with $z_i>0,\tilde{z}_i>0,\forall i$, we have

$$W(\mathbf{z})\triangleq\frac{1}{\sum\limits_{i=1}^{N}z_i}\leq\Upsilon\prod_{i=1}^{N}{z_i}^{c_i}\triangleq V(\mathbf{z}), \tag{83}$$

where Υ and $c_i, \forall i$ are respectively given by

$$\Upsilon = \frac{1}{\sum\limits_{i=1}^{N} \tilde{z}_i \prod\limits_{i=1}^{N} \tilde{z}_i^{c_i}}, \quad c_i = -\tilde{z}_i \Big/ \sum_{i=1}^{N} \tilde{z}_i. \tag{84}$$

In addition, we have

$$W(b\tilde{\mathbf{z}}) = V(b\tilde{\mathbf{z}}), \quad \nabla W(b\tilde{\mathbf{z}}) = \nabla V(b\tilde{\mathbf{z}}), \tag{85}$$

where $b > 0$ is a constant and $\nabla W(\mathbf{z})$ and $\nabla V(\mathbf{z})$ denote the gradient of function $W(\cdot)$ and $Y(\cdot)$ w.r.t. $\mathbf{z}$, respectively.

Proof: See Appendix E. □

Using *Lemma* 11, we would like to approximate the terms in the denominators of (81) and (82) with posynomial functions by their best local monomial approximations. The resulting alternative constraints of (81) and (82) that satisfy the form of a GP problem can be obtained in the two following lemmas.

Lemma 12: We can focus on the following constraint instead of the original SINR constraint on $\mathrm{SINR}_{0k}^{\mathrm{SP,DL}}$ in (68c) as

$$v_k^d \Bigg\{ \left(\tilde{\alpha}_k^{(t)} + M - \bar{M}\right) L_u \lambda_{0k} q_{0k} \beta_{00k}^2 + \sum_{i=1, i\neq k}^{K} \tilde{\alpha}_{ki}^{(t)} L_u \lambda_{0i} q_{0k} \beta_{00k}^2 + \left(q_{0k} L_u \beta_{00k} + \sum_{l' \in \Phi} \sum_{i=1}^{K} p_{l'i} \beta_{0l'i} + \sigma^2\right) \left(\sum_{l \in \Psi} \sum_{i=1}^{K} \lambda_{li} \beta_{0lk} + \sigma^2\right) \Bigg\} \leq \bar{M} L_u \lambda_{0k} q_{0k} \beta_{00k}^2 \ \forall k, \tag{86}$$

where $\tilde{\alpha}_k^{(t)} \triangleq \tilde{A}_k^{\mathrm{SP}(t)} \left[\left(\frac{M-1}{M} + \frac{(M^2+M-1)p_{0k}}{M q_{0k} L_u} \right) \left(\sum\limits_{l' \in \Phi} \sum\limits_{i=1}^{K} p_{l'i} \beta_{0l'i} + \sigma^2 \right) + M p_{0k} \beta_{00k} \right] + \frac{\sum\limits_{l' \in \Phi} \sum\limits_{i=1}^{K} p_{l'i} \beta_{0l'i} + \sigma^2}{q_{0k} L_u \beta_{00k}}$,

$$\tilde{\alpha}_{ki}^{(t)} \triangleq \left(1 + \frac{\sum\limits_{l' \in \Phi} \sum\limits_{i=1}^{K} p_{l'i} \beta_{0l'i} + \sigma^2}{q_{0k} L_u \beta_{00k}} \right) \left[1 + \tilde{A}_{ki}^{\mathrm{SP}(t)} \frac{(M^2-1)}{M} p_{0k} \beta_{00k} \right], \ \forall i \neq k \tag{87}$$

are polynomials w.r.t. the power allocation $\left\{ p_{0k}^{(t)}, q_{0k}^{(t)}, \forall k \right\}$ at the t-th iteration with

$$\begin{aligned} \tilde{A}_k^{\mathrm{SP}(t)} &= \Upsilon_{A_k^{\mathrm{SP}}} \left(\frac{M+1}{M} q_{0k} L_u \beta_{00k} \right)^{c_{0k,1}^{(t)}} \left(\frac{1}{M} p_{0k} \beta_{00k} \right)^{c_{0k,2}^{(t)}} \\ &\quad \times \prod_{j=1}^{K} (p_{0j} \beta_{00j})^{c_{0j,4}^{(t)}} \left(\sum_{l' \in \Psi} \sum_{j=1}^{K} p_{l'j} \beta_{0l'j} + \sigma^2 \right)^{c_{\sigma,1}^{(t)}}, \end{aligned} \tag{88}$$

$$\begin{aligned}\tilde{A}_{ki}^{\mathrm{SP}(t)} &= \Upsilon_{A_{ki}^{\mathrm{SP}}} \left(q_{0i} L_u \beta_{00i}\right)^{c_{0i}^{(t)}} \left(\frac{1}{M} p_{0k} \beta_{00k}\right)^{c_{0k,3}^{(t)}} \\ &\times \prod_{j=1}^{K} \left(p_{0j}\beta_{00j}\right)^{c_{0j,5}^{(t)}} \left(\sum_{l'\in\Psi}\sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right)^{c_{\sigma,2}^{(t)}}, \ \forall i \neq k, \end{aligned} \tag{89}$$

where the constants $c_{0k,1}^{(t)}$, $c_{0k,2}^{(t)}$, $c_{0k,3}^{(t)}$, $c_{0i}^{(t)}$ for $i \neq k$, $c_{0j,4}^{(t)}$ $c_{0j,5}^{(t)}$ $\forall j$, $c_{\sigma,1}^{(t)}$, $c_{\sigma,2}^{(t)}$ and $\Upsilon_{A_k^{\mathrm{SP}}}$, $\Upsilon_{A_{ki}^{\mathrm{SP}}}$ are respectively given by

$$c_{0k,1}^{(t)} = \frac{M+1}{M} q_{0k}^{(t)} L_u \beta_{00k} A_k^{\mathrm{SP}(t)},\ c_{0k,2}^{(t)} = \frac{1}{M} p_{0k}^{(t)} \beta_{00k} A_k^{\mathrm{SP}(t)}, c_{0k,3}^{(t)} = \frac{1}{M} p_{0k}^{(t)} \beta_{00k} A_{ki}^{\mathrm{SP}(t)}, i \neq k; \tag{90}$$

$$c_{0j,4}^{(t)} = p_{0j}^{(t)} \beta_{00j} A_k^{\mathrm{SP}(t)}\ \forall j, c_{0j,5}^{(t)} = p_{0j}^{(t)} \beta_{00j} A_{ki}^{\mathrm{SP}(t)}, \forall j; c_{0i}^{(t)} = q_{0i}^{(t)} L_u \beta_{00i} A_{ki}^{\mathrm{SP}(t)}, i \neq k; \tag{91}$$

$$c_{\sigma,1}^{(t)} = \left(\sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2\right) A_k^{\mathrm{SP}(t)},\ c_{\sigma,2}^{(t)} = \left(\sum_{l'\in\Psi}\sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right) A_{ki}^{\mathrm{SP}(t)}, i \neq k. \tag{92}$$

$$\Upsilon_{A_k^{\mathrm{SP}}} \triangleq \frac{A_k^{\mathrm{SP}(t)}}{\left(\frac{M+1}{M} q_{0k}^{(t)} L_u \beta_{00k}\right)^{c_{0k,1}^{(t)}} \left(\frac{1}{M} p_{0k}^{(t)} \beta_{00k}\right)^{c_{0k,2}^{(t)}} \prod\limits_{i=1}^{K} \left(p_{0i}^{(t)} \beta_{00i}\right)^{c_{0j,4}^{(t)}} \left(\sum\limits_{l'\in\Psi}\sum\limits_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2\right)^{c_{\sigma,1}^{(t)}}}, \tag{93}$$

$$\Upsilon_{A_{ki}^{\mathrm{SP}}} \triangleq \frac{A_{ki}^{\mathrm{SP}(t)}}{\left(q_{0i}^{(t)} L_u \beta_{00i}\right)^{c_{0i}^{(t)}} \left(\frac{1}{M} p_{0k}^{(t)} \beta_{00k}\right)^{c_{0k,3}^{(t)}} \prod\limits_{j=1}^{K} \left(p_{0j}^{(t)} \beta_{00j}\right)^{c_{0j,5}^{(t)}} \left(\sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right)^{c_{\sigma,2}^{(t)}}}, \forall i \neq k, \tag{94}$$

with

$$A_k^{\mathrm{SP}(t)} \triangleq \frac{1}{\frac{M+1}{M} q_{0k}^{(t)} L_u \beta_{00k} + \frac{1}{M} p_{0k}^{(t)} \beta_{00k} + \sum\limits_{i=1}^{K} p_{0i}^{(t)} \beta_{00i} + \sum\limits_{l'\in\Psi}\sum\limits_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2}, \tag{95}$$

$$A_{ki}^{\mathrm{SP}(t)} \triangleq \frac{1}{q_{0i}^{(t)} L_u \beta_{00i} + \frac{1}{M} p_{0k}^{(t)} \beta_{00k} + \sum\limits_{j=1}^{K} p_{0j}^{(t)} \beta_{00j} + \sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2}, \forall i \neq k. \tag{96}$$

Proof: See Appendix F. □

Lemma 13: We can focus on the following constraint instead of the original $\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}}$ constraint in (69b) as

$$\begin{aligned} \upsilon_k^d &\Bigg\{ \lambda_{0k}\beta_{00k}\left(\tilde{\rho}_k^{(t)} + M - \bar{M}\right) + \sum_{i=1, i\neq k}^{K} \tilde{\rho}_{ki}^{(t)} \lambda_{0i}\beta_{00k} + \varrho_k^{(t)} \left(\sum_{l\in\Psi}\sum_{i=1}^{K} \lambda_{li}\beta_{0lk} + \sigma^2\right)\Bigg\} \\ &\leq \bar{M}\lambda_{0k}\beta_{00k}, \forall k \end{aligned} \tag{97}$$

where $\tilde{\rho}_k^{(t)}$, $\tilde{\rho}_{ki}^{(t)}$ for $i \neq k$ and $\tilde{\varrho}_k^{(t)}$ are polynomials w.r.t. the power allocation $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \forall k\right\}$ at the t-th iteration that are given by

$$\tilde{\rho}_k^{(t)} \triangleq \Bigg[L_u q_{0k} p_{0k}^2 \beta_{00k}^3 \big(\tilde{B}_k^{(t)}\big)^2 + \left(\sum_{l' \in \Phi} \sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right) \sum_{i=1}^{K} \tilde{B}_i^{(t)} p_{0i}\beta_{00i} + \sum_{l' \in \Psi} \sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2\Bigg] \times \left(\frac{1}{q_{0k}L_u\beta_{00k}} + \tilde{A}_k^{\mathrm{SPe}(t)}\right) + \frac{(M^2-1)p_{0k}\mu_k^{(t)}}{Mq_{0k}L_u}, \quad (98)$$

$$\tilde{\rho}_{ki}^{(t)} \triangleq \varrho_k^{(t)} \Bigg\{ \frac{M^2-1}{M} \tilde{A}_i^{\mathrm{SPe}(t)} p_{0k}\beta_{00k} \Bigg[\Bigg[\tilde{B}_k^{(t)} \left(\sum_{l' \in \Phi} \sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right)\Bigg]^2 + \sum_{i=1}^{K} L_u p_{0i} q_{0i} \beta_{00i}^2 (\tilde{B}_i^{(t)})^2\Bigg]\Bigg\}, \ i \neq k \quad (99)$$

with

$$\mu_k^{(t)} \triangleq \left[\tilde{B}_k^{(t)} \left(\sum_{l' \in \Phi} \sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right)\right]^2 + L_u p_{0k} q_{0k} \beta_{00k}^2 (\tilde{B}_k^{(t)})^2 + \sum_{i=1}^{K} L_u p_{0i} q_{0i} \beta_{00i}^2 (\tilde{B}_i^{(t)})^2, \quad (100)$$

$$\varrho_k^{(t)} \triangleq 1 + p_{0k}^2 \beta_{00k}^2 \big(\tilde{B}_k^{(t)}\big)^2 + \frac{\left(\sum\limits_{l' \in \Phi} \sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right) \sum\limits_{i=1}^{K} \tilde{B}_i^{(t)} p_{0i}\beta_{00i} + \sum\limits_{l' \in \Psi} \sum\limits_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2}{q_{0k}L_u\beta_{00k}}, \quad (101)$$

$$\tilde{A}_i^{\mathrm{SPe}(t)} \triangleq \Upsilon_{A_i^{\mathrm{SPe}}} (q_{0i}L_u\beta_{00i})^{e_{0i,1}^{(t)}} \left(\sum_{l' \in \Psi} \sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right)^{e_{\sigma,1}^{(t)}} \forall i, \quad (102)$$

$$\tilde{B}_i^{(t)} \triangleq \Upsilon_{B_i} (q_{0i}L_u\beta_{00i})^{e_{0i,2}^{(t)}} \prod_{j=1}^{K} (p_{0j}\beta_{00j})^{e_{0j}^{(t)}} \left(\sum_{l' \in \Psi} \sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right)^{e_{\sigma,2}^{(t)}} \forall i, \quad (103)$$

which are monomials w.r.t. $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \forall k\right\}$. The constants $e_{0i,1}^{(t)}, e_{0i,2}^{(t)}$, $e_{0j}^{(t)} (\forall i, j)$, $e_{\sigma,1}^{(t)}, e_{\sigma,2}^{(t)}$ and $\Upsilon_{A_i^{\mathrm{SPe}}}$ and Υ_{B_i} at the $(t+1)$-th iteration are respectively given by

$$e_{0i,1}^{(t)} = q_{0i}^{(t)} L_u \beta_{00i} A_i^{\mathrm{SPe}(t)}, e_{0i,2}^{(t)} = q_{0i}^{(t)} L_u \beta_{00i} B_i^{(t)}, e_{0j}^{(t)} = p_{0j}^{(t)} \beta_{00j} B_i^{(t)}, \quad \forall i, j; \quad (104)$$

$$e_{\sigma,1}^{(t)} = \left(\sum_{l' \in \Psi} \sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right) A_i^{\mathrm{SPe}(t)}, e_{\sigma,2}^{(t)} = \left(\sum_{l' \in \Psi} \sum_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right) B_i^{(t)}, \ \forall i. \quad (105)$$

$$\Upsilon_{A_i^{\mathrm{SPe}}} = \frac{A_i^{\mathrm{SPe}(t)}}{\left(q_{0i}^{(t)} L_u \beta_{00i}\right)^{e_{0i,1}^{(t)}} \left(\sum\limits_{l' \in \Psi} \sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right)^{e_{\sigma,1}^{(t)}}}, \forall i \quad (106)$$

$$\Upsilon_{B_i} = \frac{B_i^{(t)}}{\left(q_{0i}^{(t)} L_u \beta_{00i}\right)^{e_{0i,2}^{(t)}} \prod\limits_{j=1}^{K} \left(p_{0j}^{(t)} \beta_{00j}\right)^{e_{0j}^{(t)}} \left(\sum\limits_{l' \in \Psi} \sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2\right)^{e_{\sigma,2}^{(t)}}}, \forall i, \quad (107)$$

where

$$A_i^{\mathrm{SPe}(t)} = \frac{1}{q_{0i}^{(t)} L_u \beta_{00i} + \sum\limits_{l' \in \Psi} \sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2}, \quad (108)$$

$$B_i^{(t)} = \frac{1}{q_{0i}^{(t)} L_u \beta_{00i} + \sum\limits_{j=1}^{K} p_{0j}^{(t)} \beta_{00j} + \sum\limits_{l' \in \Psi} \sum\limits_{j=1}^{K} p_{l'j} \beta_{0l'j} + \sigma^2}. \tag{109}$$

Proof: See Appendix G. □

Similar to the solution to Problem (78), by using the LB at the RHS of (80) as an alternative objective function and replacing the original constraint in (68c) by the new constraint given by *Lemma* 12, Problem (68) can be transformed into a GP problem as follows:

$$\max_{\substack{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\},\\ \left\{\upsilon_k^u\right\},\left\{\upsilon_k^d\right\}}} \prod_{k=1}^{K} \upsilon_k^{u\ \tilde{w}_k^{u,\mathrm{SP}(t)}} \prod_{k=1}^{K} \upsilon_k^{d\ \tilde{w}_k^{d,\mathrm{SP}(t)}} \tag{110a}$$

$$\begin{aligned} \text{s.t.} \quad & \upsilon_k^u \Bigg\{ 2p_{0k} q_{0k} \beta_{00k}^2 + M \sum_{l' \in \Phi} \sum_{i=1}^{K} (p_{l'i} + q_{l'i}) p_{l'i} \beta_{0l'i}^2 + \frac{1}{L_u} \sum_{l' \in \Phi} \sum_{i=1}^{K} p_{l'i}^2 \beta_{0l'i}^2 \\ & + \left(q_{0k} L_u \beta_{00k} + \sum_{l' \in \Phi} \sum_{i=1}^{K} p_{l'i} \beta_{0l'i} + \sigma^2 \right) \left(\sum_{l' \in \Phi} \sum_{i=1}^{K} (p_{l'i} + q_{l'i}) p_{l'i} \beta_{0l'i}^2 + \sigma^2 \right) \Bigg\} \\ & \leq M L_u p_{0k} q_{0k} \beta_{00k}^2, \end{aligned} \tag{110b}$$

$$(68d), (68e), (86), (57), (58), \tag{110c}$$

where $\tilde{w}_k^{u,\mathrm{SP}(t)} = \frac{w_k^u}{\ln 2} \kappa_k^{u(t)} - \theta_k^{u,\mathrm{SP}} \hat{\kappa}_k^{u(t)}$, $\tilde{w}_k^{d,\mathrm{SP}(t)} = \frac{w_k^d}{\ln 2} \kappa_k^{d(t)} - \theta_k^{d,\mathrm{SP}} \hat{\kappa}_k^{d(t)}$. Again, the constant terms with the LB on the objective function in (80) is omitted. Similarly, we finally transform Problem (69) into a GP problem as follows:

$$\max_{\substack{\{p_{0k}\},\{q_{0k}\},\{\lambda_{0k}\},\\ \left\{\upsilon_k^u\right\},\left\{\upsilon_k^d\right\}}} \prod_{k=1}^{K} \upsilon_k^{u\ \tilde{w}_k^{u,\mathrm{SPe}(t)}} \prod_{k=1}^{K} \upsilon_k^{d\ \tilde{w}_k^{d,\mathrm{SPe}(t)}} \tag{111a}$$

$$\text{s.t.} \quad (68d), (69c), (97), (110b), (57), (58), \tag{111b}$$

where $\tilde{w}_k^{u,\mathrm{SPe}(t)} = \frac{w_k^u}{\ln 2} \kappa_k^{u(t)} - \theta_k^{u,\mathrm{SP}} \hat{\kappa}_k^{u(t)} = \tilde{w}_k^{u,\mathrm{SP}(t)}$, $\tilde{w}_k^{d,\mathrm{SPe}(t)} = \frac{w_k^d}{\ln 2} \kappa_k^{d(t)} - \theta_k^{d,\mathrm{SP}} \hat{\kappa}_k^{d(t)} = \tilde{w}_k^{d,\mathrm{SP}(t)}$.

Then, the iterative algorithms to solve Problems (110) and (111) are proposed, which are respectively shown in **Algorithm 2** and **Algorithm 3**. In addition, the initialization scheme similar to the problem with RP can be adopted to obtain the initial input $\left\{p_{0k}^{(0)}, q_{0k}^{(0)}, \lambda_{0k}^{(0)}, \forall k\right\}$. The convergence of **Algorithm 2** and **Algorithm 3** can be readily proved by following the same method as that of **Algorithm 1**, and thus is not shown here for simplicity.

Moreover, the complexity for each of **Algorithm 1** **Algorithm** 2 and **Algorithm** 3 is mainly determined by the complexity of solving a GP problem at each iteration and the number of iterations. For the complexity of each iteration, it was shown in [40] that a GP problem can be efficiently solved by using the standard interior-point methods with a worst-case complexity of polynomial-time. In addition, we find through simulation results that **Algorithm 1** converges can rapidly, which means that **Algorithm 1** can converge to a local optimal solution with a polynomial

Algorithm 2: Iterative Algorithm for Solving Problem (110) for the Systems with SP

1 Initialize iteration number index $t = 1$, error tolerances $\varsigma > 0$, and a feasible power allocation $\left\{p_{0k}^{(0)}, q_{0k}^{(0)}, \lambda_{0k}^{(0)}, \forall k\right\}$. Calculate $\left\{\upsilon_k^{u(0)}, \upsilon_k^{d(0)}, \kappa_k^{u(0)}, \hat{\kappa}_k^{u(0)}, \kappa_k^{d(0)}, \hat{\kappa}_k^{d(0)}, \tilde{w}_k^{u,\mathrm{SP}(0)}, \tilde{w}_k^{d,\mathrm{SP}(0)}, \tilde{\alpha}_k^{(0)}, \tilde{\alpha}_{ki}^{(0)}, \forall k, \forall i\right\}$ and the objective function of (68a) denoted as $\mathrm{Obj}^{(0)}$;

2 With given $\left\{\upsilon_k^{u(t-1)}, \upsilon_k^{d(t-1)}, \kappa_k^{u(t-1)}, \hat{\kappa}_k^{u(t-1)}, \kappa_k^{d(t-1)}, \hat{\kappa}_k^{d(t-1)}, \tilde{w}_k^{u,\mathrm{SP}(t-1)}, \tilde{w}_k^{d,\mathrm{SP}(t-1)}, \tilde{\alpha}_k^{(t-1)}, \tilde{\alpha}_{ki}^{(t-1)}, \forall k\right\}$, solve Problem (110) by using the CVX package to obtain $\left\{\upsilon_k^{u(t)}, \upsilon_k^{d(t)}, p_{0k}^{(t)}, q_{0k}^{(t)}, \lambda_{0k}^{(t)}, \forall k\right\}$;

3 Update $\left\{\upsilon_k^{u(t)}, \upsilon_k^{d(t)}, \kappa_k^{u(t)}, \hat{\kappa}_k^{u(t)}, \kappa_k^{d(t)}, \hat{\kappa}_k^{d(t)}, \tilde{w}_k^{u,\mathrm{SP}(t)}, \tilde{w}_k^{d,\mathrm{SP}(t)}, \tilde{\alpha}_k^{(t)}, \tilde{\alpha}_{ki}^{(t)}, \forall k\right\}$. Calculate the new objective function in (68a) denoted as $\mathrm{Obj}^{(t)}$;

4 **while** $\mid \mathrm{Obj}^{(t)} - \mathrm{Obj}^{(t-1)} \mid / \mathrm{Obj}^{(t)} > \varsigma$ **do**

5 $\quad t \leftarrow t + 1$, go to Step 2 and Step 3;

6 **end**

7 **return** $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \lambda_{0k}^{(t)}, \forall k\right\}$, $\mathrm{Obj}^{(t)}$.

Algorithm 3: Iterative Algorithm for Solving (111) for the Systems with SPe

1 Initialize iteration number index $t = 1$, error tolerances $\varsigma > 0$, and a feasible power allocation $\left\{p_{0k}^{(0)}, q_{0k}^{(0)}, \lambda_{0k}^{(0)}, \forall k\right\}$. Calculate $\left\{\upsilon_k^{u(0)}, \upsilon_k^{d(0)}, \kappa_k^{u(0)}, \hat{\kappa}_k^{u(0)}, \kappa_k^{d(0)}, \hat{\kappa}_k^{d(0)}, \tilde{w}_k^{u,\mathrm{SPe}(0)}, \tilde{w}_k^{d,\mathrm{SPe}(0)}, \tilde{\rho}_k^{(0)}, \tilde{\rho}_{ki}^{(0)}, \tilde{\varrho}_k^{(0)}, \forall k, i \neq k\right\}$, then calculate objective function of (111a) denoted as $\mathrm{Obj}^{(0)}$;

2 With given $\left\{\upsilon_k^{u(t-1)}, \upsilon_k^{d(t-1)}, \kappa_k^{u(t-1)}, \hat{\kappa}_k^{u(t-1)}, \kappa_k^{d(t-1)}, \hat{\kappa}_k^{d(t-1)}, \tilde{w}_k^{u,\mathrm{SPe}(t-1)}, \tilde{w}_k^{d,\mathrm{SPe}(t-1)}, \tilde{\rho}_k^{(t-1)}, \tilde{\rho}_{ki}^{(t-1)}, \tilde{\varrho}_k^{(t-1)}, \forall k, i \neq k\right\}$, solve Problem (111) by using the CVX package to obtain $\left\{\upsilon_k^{u(t)}, \upsilon_k^{d(t)}, p_{0k}^{(t)}, q_{0k}^{(t)}, \lambda_{0k}^{(t)}, \forall k\right\}$;

3 Update $\left\{\upsilon_k^{u(t)}, \upsilon_k^{d(t)}, \kappa_k^{u(t)}, \hat{\kappa}_k^{u(t)}, \kappa_k^{d(t)}, \hat{\kappa}_k^{d(t)}, \tilde{w}_k^{u,\mathrm{SPe}(t)}, \tilde{w}_k^{d,\mathrm{SPe}(t)}, \tilde{\rho}_k^{(t)}, \tilde{\rho}_{ki}^{(t)}, \tilde{\varrho}_k^{(t)}, \forall k, i \neq k\right\}$. Calculate the new objective function in (111a) denoted as $\mathrm{Obj}^{(t)}$;

4 **while** $\mid \mathrm{Obj}^{(t)} - \mathrm{Obj}^{(t-1)} \mid / \mathrm{Obj}^{(t)} > \varsigma$ **do**

5 $\quad t \leftarrow t + 1$, go to Step 2 and Step 3;

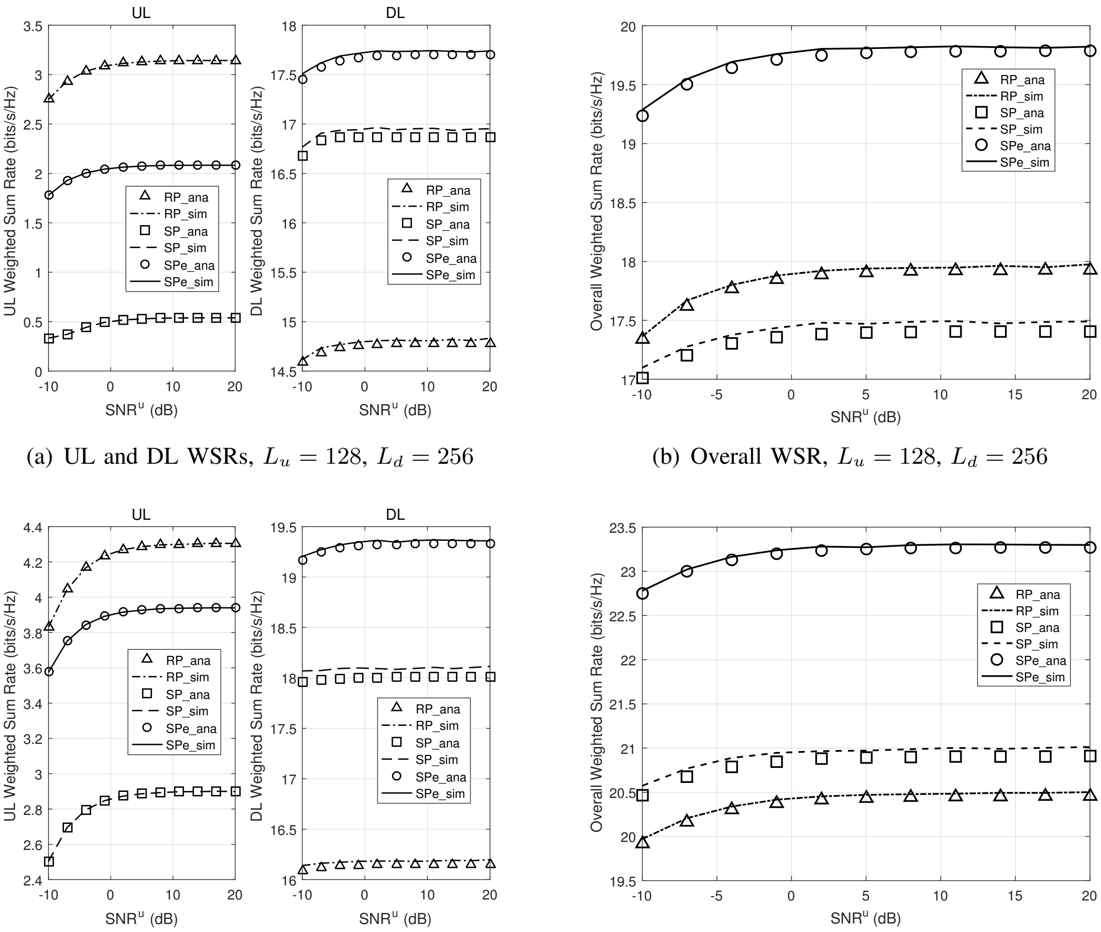
6 **end**

7 **return** $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \lambda_{0k}^{(t)}, \forall k\right\}$, $\mathrm{Obj}^{(t)}$.

time complexity.

V. NUMERICAL RESULTS

In this section, some numerical results are presented to verify the theoretical performance results obtained in the previous section, and compare the WSRs of the systems respectively with RP, SP and SPe in the FBL regime for different sets of system parameters. For the numerical results, we consider a multi-cell network consisting of 7 hexagonal cells with a radius of 50 meters and K UTs in each cell. The UTs are located uniformly at random in the region of each cell that is more



(a) UL and DL WSRs, $L_u = 128$, $L_d = 256$

(b) Overall WSR, $L_u = 128$, $L_d = 256$

(c) UL and DL WSRs, $L_u = 256$, $L_d = 512$

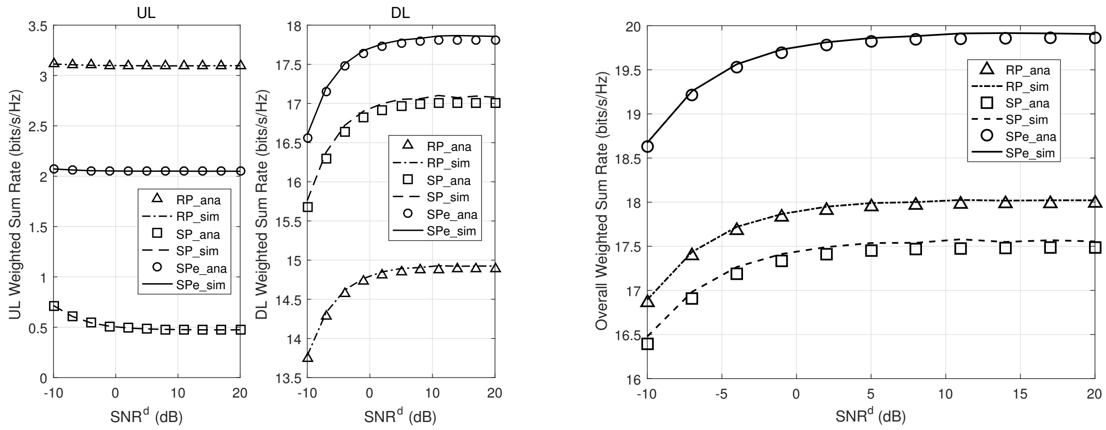
(d) Overall WSR, $L_u = 256$, $L_d = 512$

Fig. 1. The average UL and DL WSRs and the corresponding overall average WSR versus SNR^u for various pilot schemes.

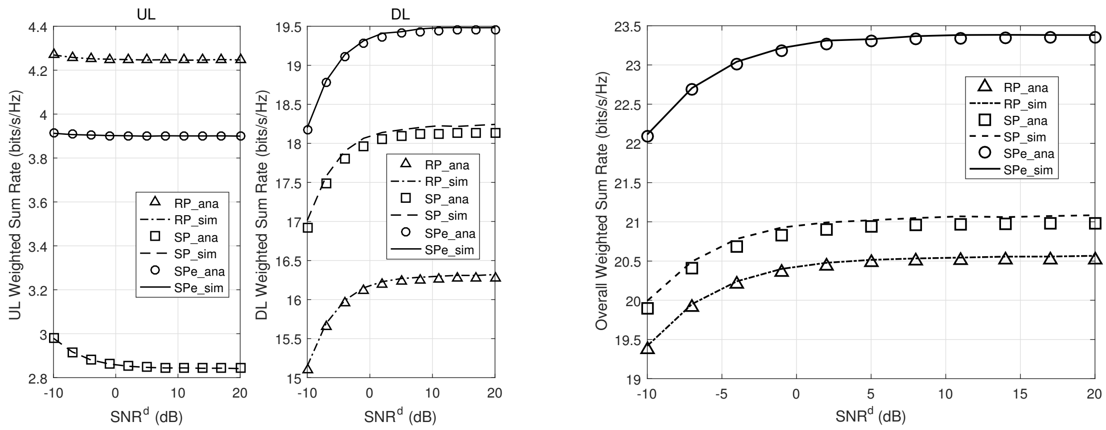
than 20 meters to the associated BS. The path loss is modeled as $PL = 35.3 + 37.6 \log_{10} d$ [41], and the small-scale fading is modeled as Rayleigh fading with zero mean and unit variance. Unless otherwise specified, the other parameters are set as follows: Channel bandwidth of $B_c = 0.18$ MHz, noise power spectral density of $-131\mathrm{dBm/Hz}$, decoding error probability of $\varepsilon_k = 10^{-9}$, $\forall k$, the number of transmit antennas of $M = 128$, the number of devices of $K = 10$, and the pilot blocklength $L_u^p = K$ for RP. The UL energy constraint of each UT is assumed to be equal, i.e., $E_{0k}^u = E^u$, $\forall k$. The UL average received SNR (i.e., SNR^u) and DL average received SNR (i.e., SNR^d) (in dB) for the signals of the UT at the cell vertex (cell edge) and the corresponding UL and DL energy limits at BS and UT are related as follows:

$$\mathrm{SNR}^u = 10 \lg \frac{1000 E^u}{L_u} - PL_e + 131, \tag{112}$$

$$\mathrm{SNR}^d = 10 \lg \frac{1000 E_0^d}{L_d K} - PL_e + 131, \tag{113}$$



(a) The average UL and DL WSRs, $L_u = 128$, $L_d = 256$ (b) The overall WSR, $L_u = 128$, $L_d = 256$

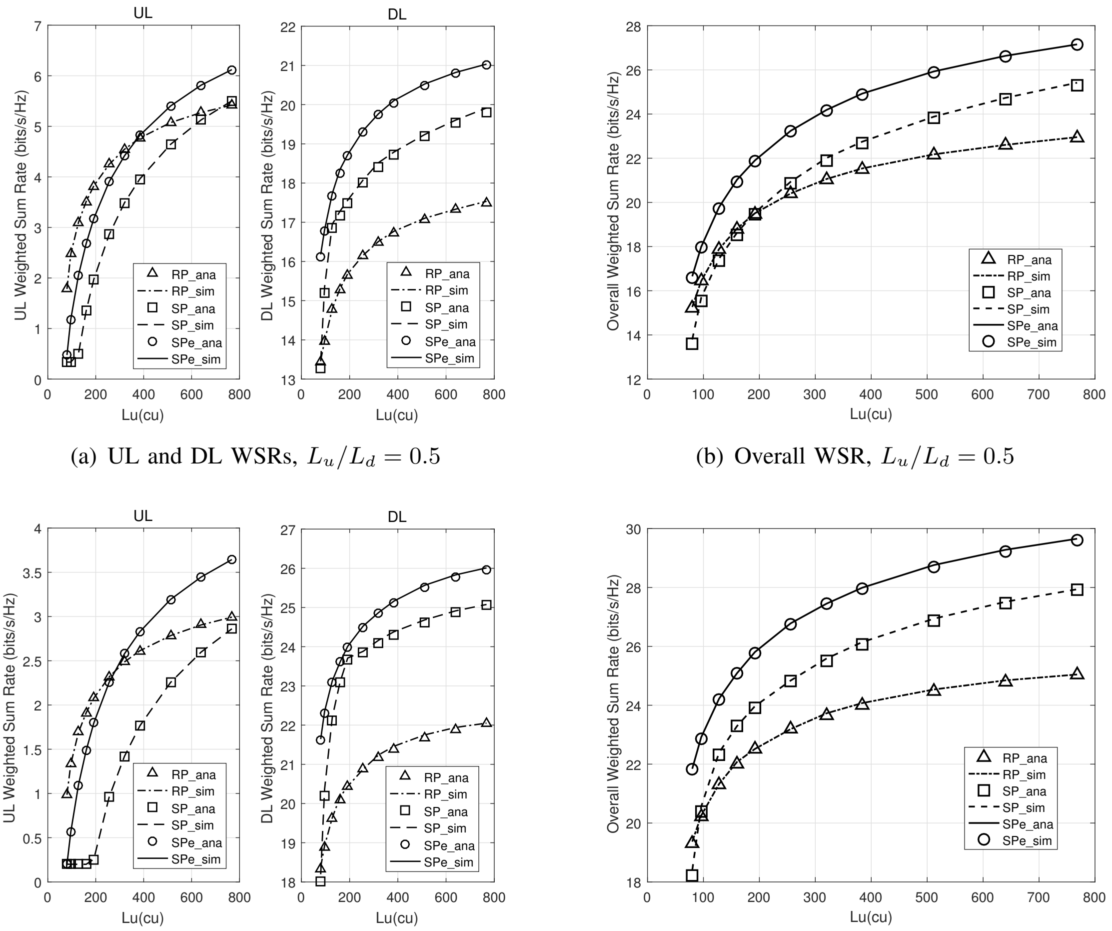


(c) The UL and DL WSRs, $L_u = 256$, $L_d = 512$ (d) The overall WSR, $L_u = 256$, $L_d = 512$

Fig. 2. The average UL and DL WSRs and the corresponding overall average WSR versus SNR^d for various pilot schemes.

where PL_e denote the path loss at the cell vertex. Accordingly, given SNR^u and SNR^d, we can easily obtain E^u and E_0^d. Moreover, each UT has the same minimum required UL or DL data rate targets, $R_k^{\text{u,req}} = R^{\text{u,req}} = 0.1$ bit/s/Hz, $R_k^{\text{d,req}} = R^{\text{d,req}} = 0.3$ bit/s/Hz, $\forall k$. The transmission durations of UL and DL, i.e., τ_u and τ_d, are both generally less than 5 ms. The fairness weights $\tilde{w}_k^d$ and $\tilde{w}_k^u$ for each UT k are set to be 1. All numerical results are obtained through the Monte-Carlo (MC) simulation by averaging over 10^4 channel uses. For the legends of each figure in the following, "sim" denotes the curve obtained from MC simulation and "ana" denotes the corresponding curve of the analytical result. The other parameters unspecified here will be mentioned in each of the following figures.

Firstly, in Fig. 1 we show and compare the optimized UL and DL WSR performances versus the UL average received SNR of each UT located at the cell edge for the systems individually employing SP, RP and SPe with the different UL and DL blocklengths. We fix the DL average received SNR of each UT that is located at cell edge as $\mathrm{SNR}^d = 0$dB. Then, the corresponding total the energy limits at each UT E^u and at BS E_0^d can be easily obtained according to (112)



(a) UL and DL WSRs, $L_u/L_d = 0.5$

(b) Overall WSR, $L_u/L_d = 0.5$

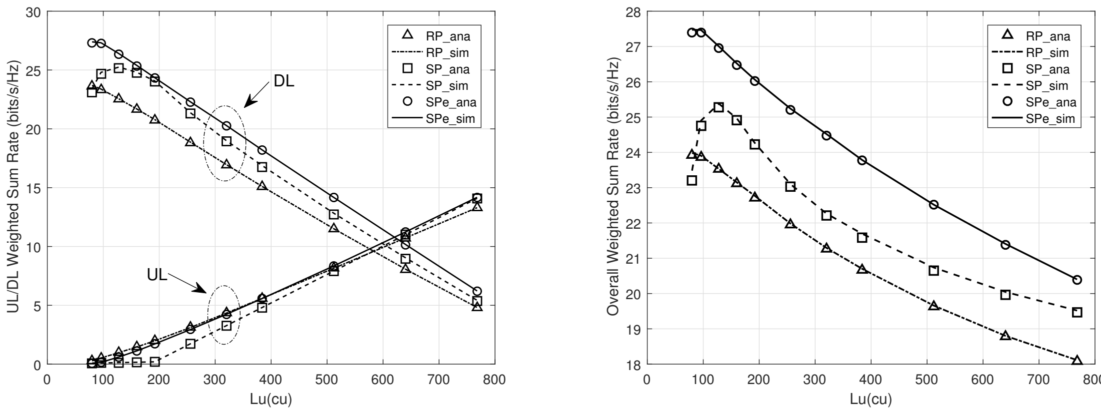
(c) UL and DL WSRs, $L_u/L_d = 0.25$

(d) Overall WSR, $L_u/L_d = 0.25$

Fig. 3. The average UL and DL WSRs and the corresponding overall average WSR versus L_u whilst keeping L_u/L_d as a constant (i.e., $L_u/L_d = 0.25, 0.5$).

and (113). Specifically, in Fig. 1(a) we show respectively the UL and DL WSRs of the systems versus SNR^u for the systems with $L_u = 128$ and $L_d = 256$. And in Fig. 1(b) we show the corresponding overall WSRs of UL and DL versus SNR^u. Additionally, in Fig. 1(c) and 1(d) we show the corresponding results for the systems with $L_u = 256$ and $L_d = 512$. From the figures, it is observed that the optimized UL and DL WSRs and also the overall WSR all improve as the average UL received SNR SNR^u (or equivalently the available energy at each UT) increases for all pilot schemes. Moreover, the RP scheme performs the best in the UL and the SP scheme performs the worst in the UL. In the contrast, the RP scheme performs the worst in the DL and the SPe scheme performs the best in the DL. The DL average WSR dominate the overall performance. Considering the UL and DL overall average WSRs, it would be the best choice to employ the SPe pilot scheme in spite of the higher computation cost.

Similar to Fig. 1, we show in Fig. 2 the optimized UL and DL WSR performances versus the DL average received SNR of each UT located at the cell edge (or equivalently the available energy at the BS) for the different pilot schemes and the different UL and DL blocklengths. The UL average



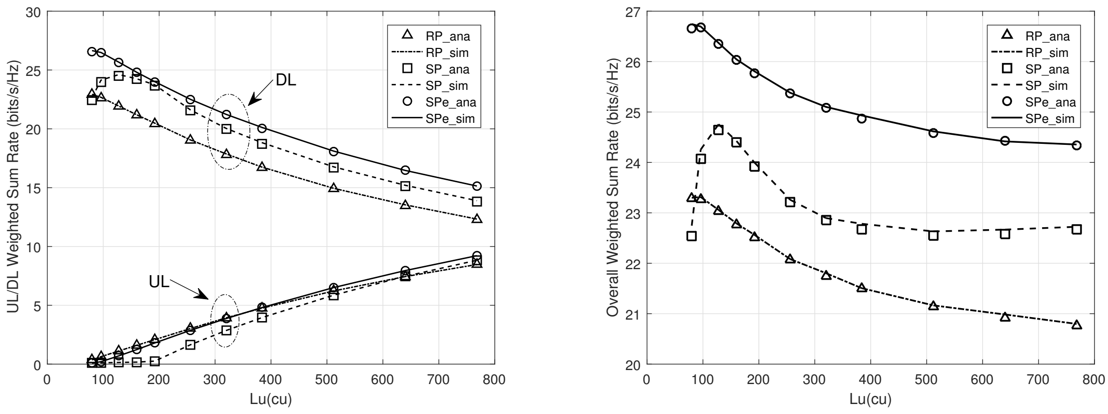
(a) UL and DL WSRs (b) Overall WSR

Fig. 4. The average UL and DL average WSRs and the corresponding overall average WSRs versus L_u whilst keeping $L_d + L_u = 1000$.

received SNR of each UT at cell edge is fixed as $\mathrm{SNR}^u = 0$dB. It is observed that the optimized DL WSR and the overall WSR still both increase rapidly as the average DL received SNR SNR^d increases for all pilot schemes. However, with the UL transmit power optimization at each UT, the UL WSRs of both RP and SPe pilots are slightly reduced and the UL WSR of SP pilot significant drops as the average DL received SNR SNR^d increases. Still, the SPe pilot scheme achieves the best overall average WSR in spite of the higher computation cost.

In Fig. 3, we show and compare the optimized UL and DL WSR performances as well as the corresponding overall average WSR versus L_u for the systems individually employing SP, RP and SPe whilst keeping L_u/L_d fixed. We set $\mathrm{SNR}^u = 0$dB and $\mathrm{SNR}^d = 0$dB. It can be observed from the results in the figures that, with the transmit power optimized in both UL and DL, in the system DL the SPe scheme can always outperform the other two pilot schemes for all blocklengths and with fixed ratios of L_u/L_d. However, in the system UL the RP scheme outperforms the other two pilot schemes with very short blocklength L_u, and in the contrast performs worse than the other two pilot schemes with long enough blocklength L_u. Which scheme performs the best depends on the concrete values of L_u. Moreover, the SPe scheme performs the best and the RP scheme performs the worst among the three pilot schemes in the system DL *in general*. Except when the UL/DL blocklength is very short, the RP scheme can slightly outperform the SP scheme. When we look at the overall average WSRs of both UL and DL, we observe the similar result as that of the DL: The SPe scheme performs the best and the SP scheme performs better than the RP scheme for the general UL/DL blocklength, except when the UL/DL blocklength is very short.

In Fig. 4, we show the optimized UL and DL average WSR performances as well as the corresponding overall average WSRs versus blocklength L_u for the systems individually employing SP,



(a) UL and DL WSRs

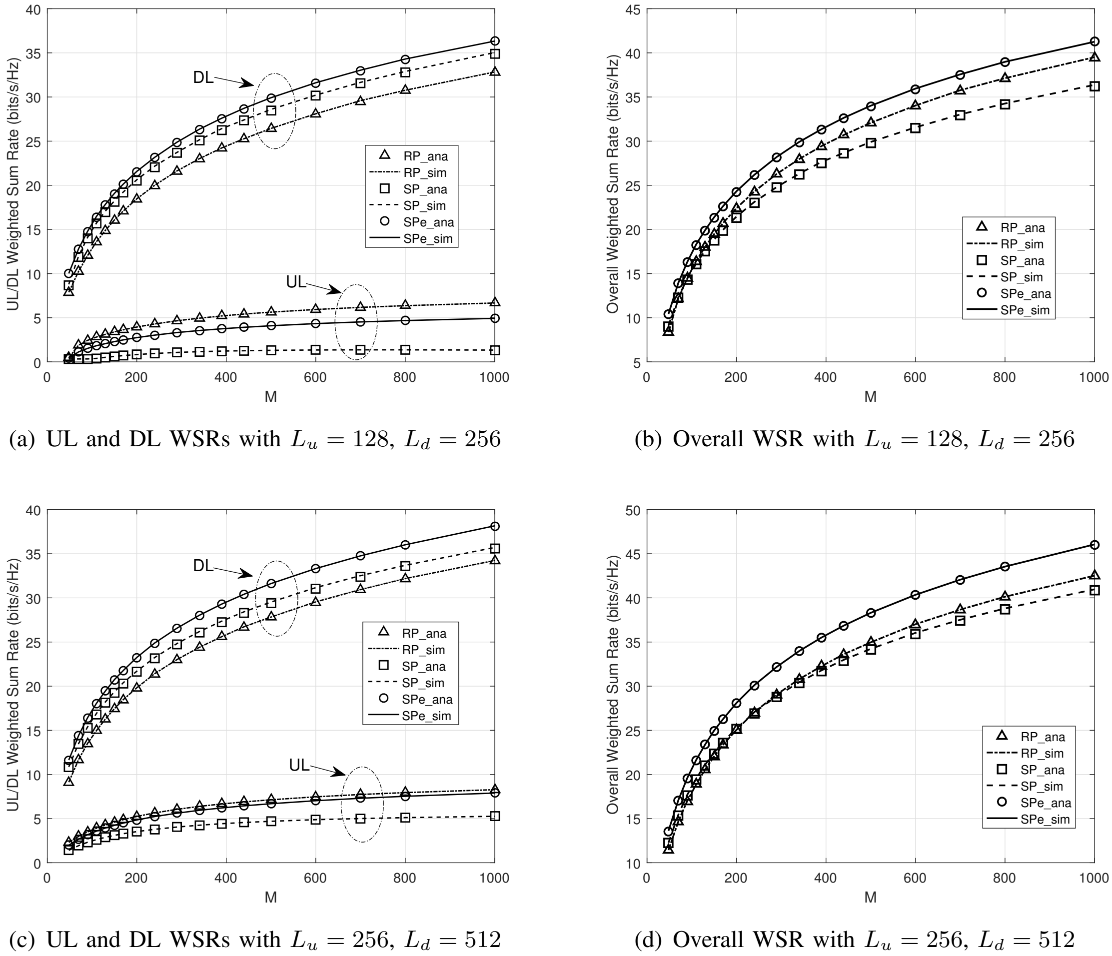
(b) Overall WSR

Fig. 5. The average UL and DL average WSRs and the corresponding overall average WSR versus L_u whilst keeping $L_d = 768$.

RP and SPe whilst keeping the total UL and DL blocklength as a constant, i.e., $L_u + L_d = 1000$. Still, we set $\mathrm{SNR}^u = 0$dB and $\mathrm{SNR}^d = 0$dB. It is rather intuitive and can be seen from the figures that, as L_u increases, the UL average WSR increases and the DL average WSR reduces. Moreover, since the DL average WSR dominates the overall performance, the overall average WSR also reduces as L_u increases in general. Further in Fig. 5, we show the optimized UL and DL WSRs as well as the corresponding overall average WSR versus L_u for the systems individually employing SP, RP and SPe whilst keeping L_d as a constant (i.e., $L_d = 768$). We can observe the similar performance results as those in Fig. 4, where $L_d + L_u = 1000$ holds.

We show in Fig. 6, the optimized UL and DL average WSR performances as well as the corresponding overall average WSR versus the number of the BS antennas M for the systems with different UL and DL blocklengths L_u, L_d. We can observe that the average WSRs of all pilot schemes increase with M without bound. Again, we observe the three pilot schemes have the different performance advantage in the system UL and DL that is similar to the results in Fig. 1 – 3.

In Fig. 7, we show the optimized UL and DL average WSR performances as well as the corresponding overall average WSR versus the number of UTs K for the systems with the different UL and DL blocklengths L_u, L_d. Moreover, we have shown in Fig. 8 the WSRs corresponding to those in Fig. 7 averaged by K. We can observe that although both the DL and the system overall average WSRs of all pilot schemes increase as K increases, the corresponding WSRs averaged by K for system UL, DL and the overall WSRs all reduce as K increases. This is because the interuser interference of useful signals and the pilot signals increase. In addition, we can see the interesting results that the average WSRs of all pilot schemes with UL do not always increase as K increases



(a) UL and DL WSRs with $L_u = 128$, $L_d = 256$

(b) Overall WSR with $L_u = 128$, $L_d = 256$

(c) UL and DL WSRs with $L_u = 256$, $L_d = 512$

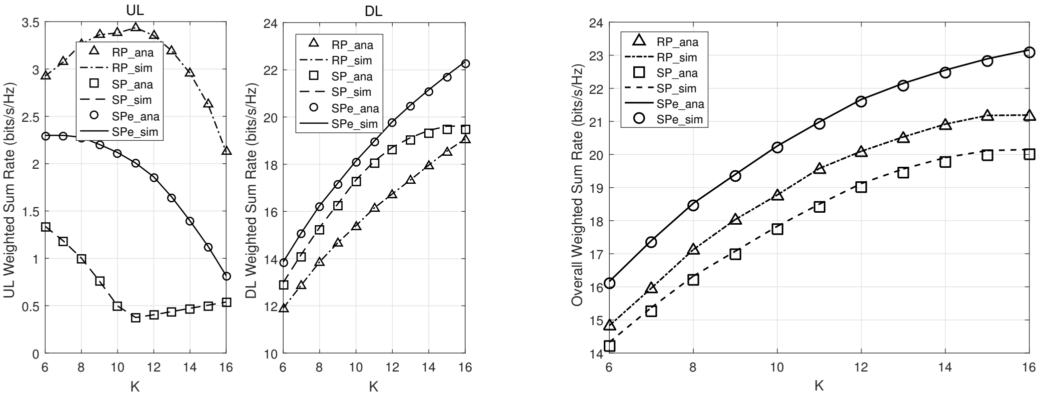
(d) Overall WSR with $L_u = 256$, $L_d = 512$

Fig. 6. The average WSRs versus M with the different combinations of L_u and L_d.

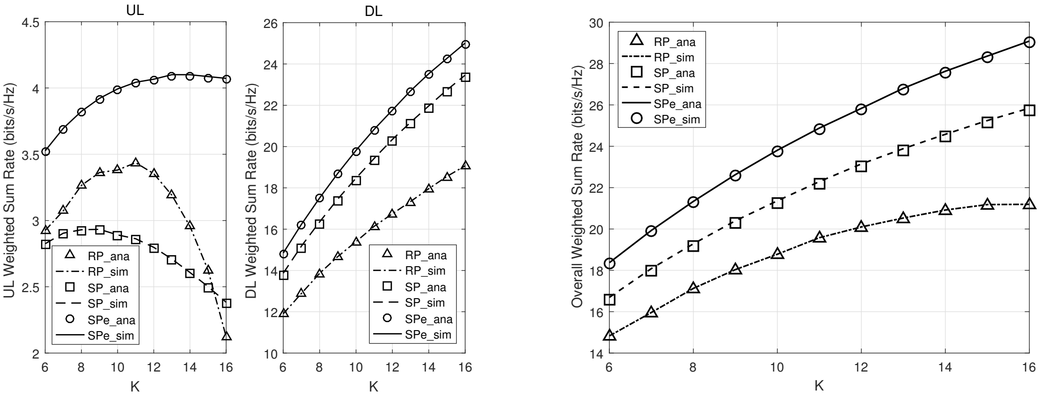
and which pilot scheme performs better than the other depends on the concrete numbers of K and UL/DL blocklengths.

In Fig. 9, we show the convergence behaviours of the proposed **Algorithm** 1 for the RP scheme, the proposed **Algorithm** 2 for SP scheme and the proposed **Algorithm** 3 for SPe scheme for the system with $L_u = L_d = 256$. From the figures, we can observe that the algorithms converge rapidly for each pilot scheme, and at most 4 iterations are sufficient for the algorithms to converge.

We also investigate in the above figures the tightness of the corresponding analytical results of data rates for the three pilot schemes. It is observed that the UL and DL WSRs are very tight for the system employing any of the three pilot schemes and with all combination of system parameters, including the number of transmit antennas and the number of the UTs. This verifies that the analytical data rate derived in this paper are suitable for system optimization. Note that when the power allocation solution is infeasible, the average WSR achieved by the proposed algorithm can approach zero.



(a) UL and DL WSRs, $L_u = 128$, $L_d = 256$ (b) Overall WSRs, $L_u = 128$, $L_d = 256$



(c) UL and DL WSRs, $L_u = 256$, $L_d = 512$ (d) Overall WSRs, $L_u = 256$, $L_d = 512$

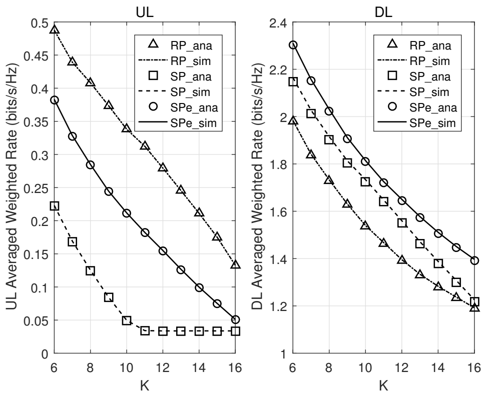
Fig. 7. The WSRs versus K with the different combinations of L_u and L_d.

APPENDIX

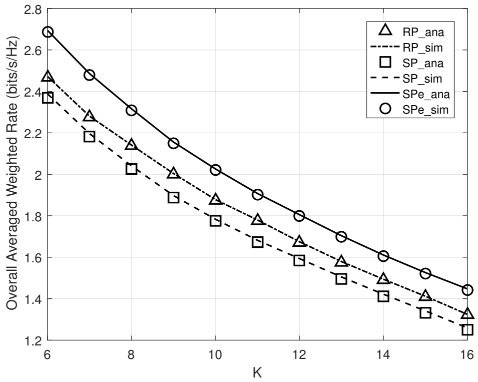
A. *Proof of Lemma 4*

We prove the lemma by using the result in [34, 2.3.4] based on the equivalent decomposition of the received signal in (25). Firstly, $\mathbb{E}\left\{\left|\bar{\mathbf{h}}_{00k}^H \mathbf{h}_{00k}\right|^2\right\}$ can be obtained by using $\bar{\mathbf{h}}_{00k}$ in (23) as

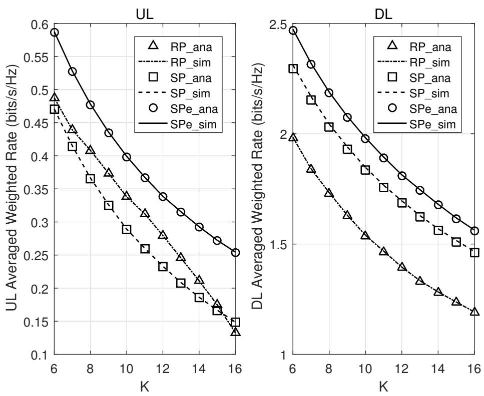
$$
\begin{aligned}
&\mathbb{E}\left\{\left|\bar{\mathbf{h}}_{00k}^H \mathbf{h}_{00k}\right|^2\right\} = \mathbb{E}\left\{\bar{\mathbf{h}}_{00k}^H \mathbf{h}_{00k} \mathbf{h}_{00k}^H \bar{\mathbf{h}}_{00k}\right\} \\
= \; & \frac{\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k} L_u} \mathbb{E}\Bigg\{ q_{0k} L_u \mathbf{h}_{00k}^H \mathbf{h}_{00k} \mathbf{h}_{00k}^H \mathbf{h}_{00k} + \sum_{l' \in \Phi} \sum_{i=1}^{K} \zeta_{l'i} \frac{p_{l'i}}{L_u} \mathbf{h}_{0l'i}^H \mathbf{h}_{00k} \mathbf{h}_{00k}^H \mathbf{h}_{0l'i} \boldsymbol{\varphi}_{0k}^T \mathbf{s}_{l'i}^* \mathbf{s}_{l'i}^T \boldsymbol{\varphi}_{0k}^* \\
& + \frac{1}{L_u} \boldsymbol{\varphi}_{0k}^T \mathbf{N}_0^H \mathbf{h}_{00k} \mathbf{h}_{00k}^H \mathbf{N}_0 \boldsymbol{\varphi}_{0k}^* \Bigg\} \\
= \; & M(M+1)\beta_{00k}^2 \gamma_{0k}^{\mathrm{SP}\,2} + \frac{M \beta_{00k} \gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k} L_u} \left(\sum_{l' \in \Phi} \sum_{i=1}^{K} \zeta_{l'i} p_{l'i} \beta_{0l'i} + \sigma^2 \right).
\end{aligned} \tag{114}
$$



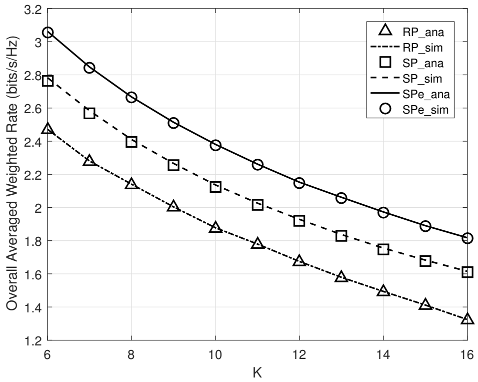
(a) UL and DL WSRs, $L_u = 128$, $L_d = 256$



(b) Overall WSR, $L_u = 128$, $L_d = 256$

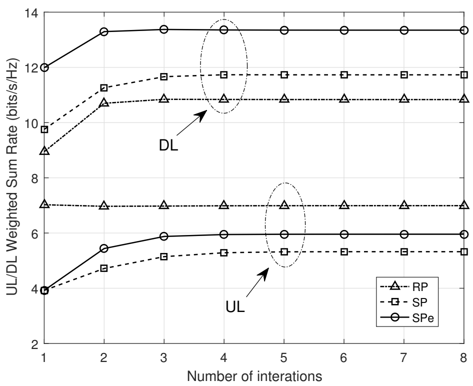


(c) UL and DL WSRs, $L_u = 256$, $L_d = 512$

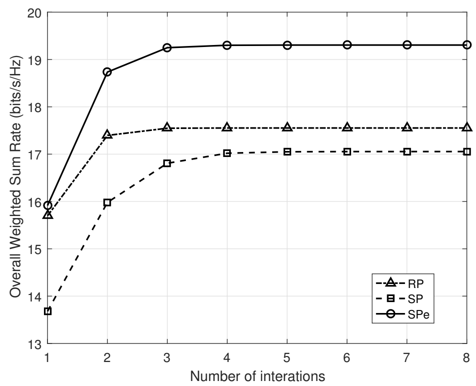


(d) Overall WSR, $L_u = 256$, $L_d = 512$

Fig. 8. The WSRs averaged over K versus K with the different combinations of L_u and L_d.



(a) UL and DL WSRs



(b) Overall WSR

Fig. 9. Convergence behaviour of the proposed algorithms for the different pilot schemes.

Moreover, according to the definition of $\bar{\mathbf{h}}_{00k}^{H}$ in (24), we can obtain that $\mathbb{E}\left\{\bar{\mathbf{h}}_{00k}^{H}\mathbf{h}_{00k}\right\} = M\beta_{00k}\gamma_{0k}^{\mathrm{SP}}$. Then, together with (114), we can obtain that

$$\begin{aligned}
& p_{0k}\left(\mathbb{E}\left\{\left|\bar{\mathbf{h}}_{00k}^{H}\mathbf{h}_{00k}\right|^{2}\right\} - \left|\mathbb{E}\left\{\bar{\mathbf{h}}_{00k}^{H}\mathbf{h}_{00k}\right\}\right|^{2}\right) \\
= \; & M(M+1)p_{0k}\beta_{00k}^{2}\gamma_{0k}^{\mathrm{SP}\,2} + \frac{Mp_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k}L_u}\left(\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i} + \sigma^2\right) - \left(Mp_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}}\right)^{2} \\
= \; & Mp_{0k}\beta_{00k}^{2}\gamma_{0k}^{\mathrm{SP}\,2} + \frac{Mp_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k}L_u}\left(\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i} + \sigma^2\right) \\
= \; & \frac{Mp_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k}L_u}\left(q_{0k}\beta_{00k}L_u + \sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i} + \sigma^2\right) \\
= \; & \frac{Mp_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k}L_u}\left(q_{0k}\beta_{00k}L_u + \sum_{l'\in\Phi}\sum_{i=1}^{K}p_{l'i}\beta_{0l'i} + \sigma^2\right) - \frac{Mp_{0k}^{2}\beta_{00k}^{2}\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k}L_u} \\
= \; & Mp_{0k}\beta_{00k}^{2}\gamma_{0k}^{\mathrm{SP}} - \frac{Mp_{0k}^{2}\beta_{00k}^{2}\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k}L_u},
\end{aligned} \tag{115}$$

where (115) follows from (12). Moreover, $\mathbb{E}\left\{\left|n_{kj} - \mathbb{E}\{n_{kj}\}\right|^{2}\right\}$ can be expressed as

$$\mathbb{E}\left\{\left|n_{kj} - \mathbb{E}\{n_{kj}\}\right|^{2}\right\} = \mathbb{E}\left\{\left|n_{kj}\right|^{2}\right\} - \left|\mathbb{E}\{n_{kj}\}\right|^{2}. \tag{116}$$

According to the definitions of n_{kj} and I_2 in (28) and substituting (7), we can obtain

$$\begin{aligned}
\mathbb{E}\{n_{kj}\} = \; & \frac{\gamma_{0k}^{\mathrm{SP}}p_{0k}}{L_u\sqrt{q_{0k}}}\mathbb{E}\left\{\,\mathbf{h}_{00k}^{H}\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^{T}\mathbf{s}_{0k}^{*}[\mathbf{s}_{0k}]_j\right\} \\
& + \sum_{l'\in\Phi}\sum_{i=1}^{K}\mathbb{E}\left\{\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\mathbf{h}_{0l'i}\right\} + \mathbb{E}\left\{\hat{\mathbf{h}}_{00k}^{H}[\mathbf{N}_0]_j\right\} \\
= \; & \frac{\gamma_{0k}^{\mathrm{SP}}p_{0k}}{L_u\sqrt{q_{0k}}}\mathbb{E}\left\{\mathbf{h}_{00k}^{H}\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^{T}\mathbf{s}_{0k}^{*}[\mathbf{s}_{0k}]_j\right\} + \sum_{l'\in\Phi}\sum_{i=1}^{K}\mathbb{E}\left\{\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\right. \\
& \left.\times\left(\gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k}^{H} + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}}\mathbf{h}_{0l'i}^{H}\boldsymbol{\varphi}_{0k}^{T}\mathbf{s}_{l'i}^{*}\right)\mathbf{h}_{0l'i}\right\} + \mathbb{E}\left\{\hat{\mathbf{h}}_{00k}^{H}[\mathbf{N}_0]_j\right\} \\
= \; & \frac{\gamma_{0k}^{\mathrm{SP}}p_{0k}}{L_u\sqrt{q_{0k}}}\mathbb{E}\left\{\mathbf{h}_{00k}^{H}\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^{T}\mathbf{s}_{0k}^{*}[\mathbf{s}_{0k}]_j\right\} + \sum_{l'\in\Phi}\sum_{i=1}^{K}\mathbb{E}\left\{\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j\frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}}\right. \\
& \left.\times\mathbf{h}_{0l'i}^{H}\boldsymbol{\varphi}_{0k}^{T}\mathbf{s}_{l'i}^{*}\mathbf{h}_{0l'i}\right\} + \gamma_{0k}^{\mathrm{SP}}\sqrt{q_{0k}}[\boldsymbol{\varphi}_{0k}]_j\mathbb{E}\left\{\mathbf{h}_{00k}^{H}\mathbf{h}_{00k}\right\} + \mathbb{E}\left\{\hat{\mathbf{h}}_{00k}^{H}[\mathbf{N}_0]_j\right\} \\
= \; & \frac{Mp_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}}}{L_u\sqrt{q_{0k}}}[\boldsymbol{\varphi}_{0k}]_j + \frac{M\gamma_{0k}^{\mathrm{SP}}}{L_u\sqrt{q_{0k}}}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{l'i}[\boldsymbol{\varphi}_{0k}]_j \\
& + M\sqrt{q_{0k}}\beta_{00k}\gamma_{0k}^{\mathrm{SP}}[\boldsymbol{\varphi}_{0k}]_j + \frac{M\sigma^2\gamma_{0k}^{\mathrm{SP}}}{L_u\sqrt{q_{0k}}}[\boldsymbol{\varphi}_{0k}]_j \\
= \; & \frac{M\gamma_{0k}^{\mathrm{SP}}}{L_u\sqrt{q_{0k}}}\left(L_u q_{0k}\beta_{00k} + \sum_{l'\in\Phi}\sum_{i=1}^{K}p_{l'i}\beta_{l'i} + \sigma^2\right)[\boldsymbol{\varphi}_{0k}]_j.
\end{aligned} \tag{117}$$

In Appendix H, we have derived $\mathbb{E}\left\{|n_{kj}|^2\right\}$ as

$$\mathbb{E}\left\{|n_{kj}|^2\right\} = \frac{\gamma_{0k}^{\mathrm{SP}^2} p_{0k}^2}{L_u^2 q_{0k}} \mathbb{E}\left\{\left|\ \mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T \mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right|^2\right\}$$
$$+\mathbb{E}\left\{\left|\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\hat{\mathbf{h}}_{00k}^H\ \mathbf{h}_{0l'i}\right|^2\right\} + \mathbb{E}\left\{\left|\hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j\right|^2\right\}$$
$$+2\Re\left[\mathbb{E}\left\{\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^*]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}^*]_j\right)\ \mathbf{h}_{0l'i}^H\hat{\mathbf{h}}_{00k}\mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]$$
$$+2\Re\left[\mathbb{E}\left\{[\mathbf{N}_0]_j^H\hat{\mathbf{h}}_{00k}\ \mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]$$
$$+2\Re\left[\mathbb{E}\left\{\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^*]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}^*]_j\right)\ \mathbf{h}_{0l'i}^H\ \hat{\mathbf{h}}_{00k}\ \hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j\right\}\right] \quad (118)$$
$$= \frac{M(M+1)(L_u+1)p_{0k}^2\beta_{00k}^2\gamma_{0k}^{\mathrm{SP}^2}}{q_{0k}{L_u}^2} + \frac{M^2\gamma_{0k}^{\mathrm{SP}^2}}{q_{0k}}\left(q_{0k}\beta_{00k} + \frac{1}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i}\right)^2$$
$$+\frac{M^2\gamma_{0k}^{\mathrm{SP}^2}}{q_{0k}L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(q_{l'i}+\zeta_{l'i}p_{l'i}\right)p_{l'i}\beta_{0l'i}^2 + \frac{M\gamma_{0k}^{\mathrm{SP}^2}}{{L_u}^2 q_{0k}}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}^2\beta_{0l'i}^2$$
$$+M\beta_{00k}\gamma_{0k}^{\mathrm{SP}}\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(q_{l'i}+\zeta_{l'i}p_{l'i}\right)\beta_{0l'i} + \frac{M^2\sigma^4\gamma_{0k}^{\mathrm{SP}^2}}{q_{0k}{L_u}^2} + M\sigma^2\beta_{00k}\gamma_{0k}^{\mathrm{SP}}$$
$$+\frac{2Mp_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}^2}}{L_u q_{0k}}\left[q_{0k}\beta_{00k} + M\left(q_{0k}\beta_{00k} + \frac{1}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i}\right)\right]$$
$$+\frac{2M^2\sigma^2 p_{0k}\beta_{00k}\gamma_{0k}^{\mathrm{SP}^2}}{q_{0k}{L_u}^2} + \frac{2M^2\sigma^2\gamma_{0k}^{\mathrm{SP}^2}}{L_u q_{0k}}\left(q_{0k}\beta_{00k} + \frac{1}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i}\right). \quad (119)$$

Combining (117) and (119), we can obtain

$$\mathbb{E}\left\{|n_{kj} - \mathbb{E}\{n_{kj}\}|^2\right\} = \frac{Mp_{0k}^2\beta_{00k}^2\gamma_{0k}^{\mathrm{SP}^2}}{q_{0k}L_u} + \frac{2Mp_{0k}\beta_{00k}^2\gamma_{0k}^{\mathrm{SP}^2}}{L_u} - Mp_{0k}\beta_{00k}^2\gamma_{0k}^{\mathrm{SP}}$$
$$+\frac{M^2\gamma_{0k}^{\mathrm{SP}^2}}{q_{0k}L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(q_{l'i}+p_{l'i}\right)p_{l'i}\beta_{0l'i}^2 + \frac{M\gamma_{0k}^{\mathrm{SP}^2}}{{L_u}^2 q_{0k}}\sum_{l'\in\Phi}\sum_{i=1}^{K}p_{l'i}^2\beta_{0l'i}^2$$
$$+M\beta_{00k}\gamma_{0k}^{\mathrm{SP}}\left(\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(q_{l'i}+p_{l'i}\right)\beta_{0l'i} + \sigma^2\right). \quad (120)$$

Then, the lemma is proved by substituting (119) and (120) into (29).

B. Proof of Lemma 5

For RP with LMMSE channel estimation, $\boldsymbol{\varepsilon}_{00k}$ and $\hat{\mathbf{h}}_{00k}$ are independent. Then, we can obtain that $\mathbb{E}\left\{\boldsymbol{\varepsilon}_{00k}^T\frac{\hat{\mathbf{h}}_{00k}^*}{\|\hat{\mathbf{h}}_{00k}^*\|}\right\} = 0$. It follows from (7) and (33) that $S = \left|\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00k}\|\right\}\right|^2 = \beta_{00k}\bar{\gamma}_{0k}^{\mathrm{RP}}\bar{M}$ with $\bar{M} \triangleq \left(\frac{\Gamma(M+1/2)}{\Gamma(M)}\right)^2$, where we have used the fact that $\|\hat{\mathbf{h}}_{00k}\|^2$ has a Chi-square distribution [19].

Using (32) and $\mathbf{h}_{00k}^T = \hat{\mathbf{h}}_{00k}^T + \boldsymbol{\varepsilon}_{00k}^T$, I_3 in (32) can be obtained as

$$\begin{aligned}
I_3 &= \mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\mathbf{w}_{0k}\right|^2\right\} = \mathbb{E}\left\{\left|\left(\hat{\mathbf{h}}_{00k}^T + \boldsymbol{\varepsilon}_{00k}^T\right)\frac{\hat{\mathbf{h}}_{00k}^*}{\|\hat{\mathbf{h}}_{00k}^*\|}\right|^2\right\} \\
&= \mathbb{E}\left\{\frac{1}{\|\hat{\mathbf{h}}_{00k}\|^2}\left(\hat{\mathbf{h}}_{00k}^T + \boldsymbol{\varepsilon}_{00k}^T\right)\hat{\mathbf{h}}_{00k}^*\hat{\mathbf{h}}_{00k}^T\left(\hat{\mathbf{h}}_{00k}^* + \boldsymbol{\varepsilon}_{00k}^*\right)\right\} \\
&= \mathbb{E}\left\{\frac{1}{\|\hat{\mathbf{h}}_{00k}\|^2}\left(\hat{\mathbf{h}}_{00k}^T\hat{\mathbf{h}}_{00k}^*\hat{\mathbf{h}}_{00k}^T\hat{\mathbf{h}}_{00k}^* + \boldsymbol{\varepsilon}_{00k}^T\hat{\mathbf{h}}_{00k}^*\hat{\mathbf{h}}_{00k}^T\boldsymbol{\varepsilon}_{00k}^*\right.\right. \\
&\qquad \left.\left.+\boldsymbol{\varepsilon}_{00k}^T\hat{\mathbf{h}}_{00k}^*\hat{\mathbf{h}}_{00k}^T\hat{\mathbf{h}}_{00k}^* + \hat{\mathbf{h}}_{00k}^T\hat{\mathbf{h}}_{00k}^*\hat{\mathbf{h}}_{00k}^T\boldsymbol{\varepsilon}_{00k}^*\right)\right\} \\
&= \mathbb{E}\left\{\|\hat{\mathbf{h}}_{00k}\|^2\right\} + \mathbb{E}\left\{\left|\boldsymbol{\varepsilon}_{00k}^T\frac{\hat{\mathbf{h}}_{00k}^*}{\|\hat{\mathbf{h}}_{00k}^*\|}\right|^2\right\} + \mathbb{E}\left\{\boldsymbol{\varepsilon}_{00k}^T\hat{\mathbf{h}}_{00k}^*\right\} + \mathbb{E}\left\{\hat{\mathbf{h}}_{00k}^T\boldsymbol{\varepsilon}_{00k}^*\right\} \\
&= \mathbb{E}\left\{\|\hat{\mathbf{h}}_{00k}\|^2\right\} + \mathbb{E}\left\{\left|\boldsymbol{\varepsilon}_{00k}^T\frac{\hat{\mathbf{h}}_{00k}^*}{\|\hat{\mathbf{h}}_{00k}^*\|}\right|^2\right\} \qquad (121) \\
&= M\beta_{00k}\bar{\gamma}_{0k}^{\mathrm{RP}} + \beta_{00k}(1-\bar{\gamma}_{0k}^{\mathrm{RP}}) \qquad (122) \\
&= (M-1)\beta_{00k}\bar{\gamma}_{0k}^{\mathrm{RP}} + \beta_{00k}, \qquad (123)
\end{aligned}$$

where (121) follows from the fact that $\boldsymbol{\varepsilon}_{00k}$ and $\hat{\mathbf{h}}_{00k}$ are independent and (122) follows from $\hat{\mathbf{h}}_{00k} \sim \mathcal{CN}\left(0, \beta_{00k}\bar{\gamma}_{0k}^{\mathrm{RP}}\mathbf{I}_M\right)$, $\boldsymbol{\varepsilon}_{00k} \sim \mathcal{CN}\left(0, \beta_{00k}\left(1-\bar{\gamma}_{0k}^{\mathrm{RP}}\right)\mathbf{I}_M\right)$ and [19, *Lemma* 9].

Moreover, since $\mathbf{h}_{00k}$ and $\mathbf{w}_{0i} = \frac{\hat{\mathbf{h}}_{00i}^*}{\|\hat{\mathbf{h}}_{00i}^*\|}$ are independent, we can obtain $I_4 = \beta_{00k}$ in (32) using [19, *Lemma* 9] again. For $I_5 = \mathbb{E}\left\{\mathbf{h}_{l0k}^T\frac{\hat{\mathbf{h}}_{lli}^*}{\|\hat{\mathbf{h}}_{lli}^*\|}\right\}$ in (32), the estimation $\hat{\mathbf{h}}_{lli}^*$ with cell l generally contains a weak contamination component caused by $\mathbf{h}_{l0k}$. However, this component in $\hat{\mathbf{h}}_{lli}^*$ that comes out of $\mathbf{h}_{l0k}$ is only a relatively very small portion of $\hat{\mathbf{h}}_{lli}^*$. Then, I_5 can be approximated as $I_5 \approx \beta_{l0k}$ by ignoring this small term. By substituting the results of S, I_3, I_4, I_5 into (32), we can obtain $\overline{\mathrm{SINR}}_{0k}^{\mathrm{RP,UL}}$ given in (34).

C. Proof of Lemma 6

Strictly speaking, since $\hat{\mathbf{h}}_{00k}$ contains the product of Gaussian variables, $\hat{\mathbf{h}}_{00k}$ is no longer the Gaussian distributed. However, with accurate channel estimation, the Gaussian components will dominate in $\hat{\mathbf{h}}_{00k}$. Then, we can approximate $\|\hat{\mathbf{h}}_{00k}\|$ as a Chi RV and $\|\hat{\mathbf{h}}_{00k}^*\| \to \sqrt{\beta_{00k}\gamma_{0k}^{\mathrm{SP}}M}$ as $M \to \infty$. Moreover, the terms $\boldsymbol{\varepsilon}_{00k}$ and $\hat{\mathbf{h}}_{00k}$ for LMMSE estimation of SP are uncorrelated but not independent, i.e., $\mathbb{E}\left\{\boldsymbol{\varepsilon}_{00k}^T\hat{\mathbf{h}}_{00k}^*\right\} = 0$. Then, it follows that $\lim_{M\to\infty}\mathbb{E}\left\{\boldsymbol{\varepsilon}_{00k}^T\frac{\hat{\mathbf{h}}_{00k}^*}{\|\hat{\mathbf{h}}_{00k}^*\|}\right\} = \lim_{M\to\infty}\mathbb{E}\left\{\boldsymbol{\varepsilon}_{00k}^T\hat{\mathbf{h}}_{00k}^*\right\}\frac{1}{\sqrt{\beta_{00k}\bar{\gamma}_{0k}^{\mathrm{RP}}M}} = 0$. Then, according to (33), we can approximate as $S \approx \left|\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00k}\|\right\}\right|^2 \approx \beta_{00k}\gamma_{0k}^{\mathrm{SP}}\bar{M}$ as $M \to \infty$. In addition, $\|\hat{\mathbf{h}}_{00k}\| \to \sqrt{\beta_{00k}\gamma_{0k}^{\mathrm{SP}}M} \approx \sqrt{\beta_{00k}\gamma_{0k}^{\mathrm{SP}}\bar{M}}$ as $M \to \infty$ since $M - \bar{M} \approx 0.25$ [18], which also shows the rationality of the above approximation on S.

In addition, for the derivations of I_3 and I_4 in (32), the authors in [29] and [20] had utilized the approximation that $\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\mathbf{w}_{0i}\right|^2\right\} = \mathbb{E}\left\{\frac{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2}{\|\hat{\mathbf{h}}_{00i}^*\|^2}\right\} \approx \frac{\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2\right\}}{\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00i}^*\|^2\right\}}$ in massive MIMO systems. In this paper, we propose another approximation method, i.e.,

$$\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\mathbf{w}_{0i}\right|^2\right\} = \mathbb{E}\left\{\frac{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2}{\|\hat{\mathbf{h}}_{00i}^*\|^2}\right\} \approx \frac{\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2\right\}\mathbb{E}\left\{\|\mathbf{h}_{00k}\|^2\right\}}{\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00i}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}}, \tag{124}$$

$\forall i,k \in \{1,2,\cdots,K\}$, which can be verified by numerical results to be tighter than that utilized in [29] and [20]. A brief proof is given as follows. Even though $\hat{\mathbf{h}}_{00i}$ $\forall i \in \{k = 1,2,\cdots,K\}$ is not strictly complex Gaussian distributed, it has the complex Gaussian random vectors as the major part. Thus, we can approximate $\hat{\mathbf{h}}_{00i}$ to be a complex Gaussian vector. Particularly, we can approximate that $\hat{\mathbf{h}}_{00i}$ and $\mathbf{h}_{00k}$ for $i \neq k$ are independent, and $\hat{\mathbf{h}}_{00k} \approx \mathbf{h}_{00k}$ in the case with high channel estimation quality. Then, we assume that $\hat{\mathbf{h}}_{00k}$ has the same statistical distribution as that of $\mathbf{h}_{00k}$. Moreover, it is known that for a complex Gaussian random vector $\mathbf{h}$ with zero-mean i.i.d entries, its magnitude $\|\mathbf{h}\|^2$ and direction $\tilde{\mathbf{h}} = \mathbf{h}/\|\mathbf{h}\|^2$ are independent [42]. Based on the above approximations, we can obtain that

$$\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2\right\} \approx \mathbb{E}\left\{\frac{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2}{\|\hat{\mathbf{h}}_{00i}^*\|^2\|\mathbf{h}_{00k}\|^2}\right\}\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00i}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}, \tag{125}$$

$$\mathbb{E}\left\{\frac{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2}{\|\hat{\mathbf{h}}_{00i}^*\|^2}\right\} \approx \mathbb{E}\left\{\frac{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2}{\|\hat{\mathbf{h}}_{00i}^*\|^2\|\mathbf{h}_{00k}\|^2}\right\}\mathbb{E}\left\{\|\mathbf{h}_{00k}\|^2\right\}. \tag{126}$$

Then, (124) follows by combining (125) and (126). The approximations in (125) and (126) become more accurate as M increases large.

According to (13) and (124), I_3 in (32) can be accurately approximated as

$$I_3 \approx \frac{\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00k}^*\right|^2\right\}\mathbb{E}\left\{\|\mathbf{h}_{00k}\|^2\right\}}{\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00k}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}}. \tag{127}$$

First, $\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00k}^*\right|^2\right\}$ can be obtained as

$$\begin{aligned}\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00k}^*\right|^2\right\} = \mathbb{E}\Bigg\{&\mathbf{h}_{00k}^T\left(\hat{\mathbf{h}}_{00k} - \gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k} - \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^* \\ &\times\mathbf{h}_{00k}^H\left(\hat{\mathbf{h}}_{00k} - \gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k} - \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)\Bigg\} \\ +\mathbb{E}\Bigg\{&\mathbf{h}_{00k}^T\left(\gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k} + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^*\mathbf{h}_{00k}^H\left(\gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k} + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)\Bigg\}\end{aligned}$$

$$
\begin{aligned}
&= \frac{M\beta_{00k}(\gamma_{0k}^{\rm SP})^2}{q_{0k}L_u}\left(\sum_{\substack{l'\in\Phi\\(l',i)\neq(0,k)}}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i}+\sigma^2\right)+M(M+1)\beta_{00k}^2(\gamma_{0k}^{\rm SP})^2\left(1+\frac{p_{0k}}{q_{0k}L_u}\right)\\
&= M\beta_{00k}^2\gamma_{0k}^{\rm SP}\left(1+M\gamma_{0k}^{\rm SP}+\frac{Mp_{0k}\gamma_{0k}^{\rm SP}}{q_{0k}L_u}\right), \qquad (128)
\end{aligned}
$$

where we note that $\mathbf{h}_{00k}$ and $\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\rm SP}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*$ are independent of each other, and thus $\mathbb{E}\Big\{\mathbf{h}_{00k}^T\Big(\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\rm SP}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\Big)^*\mathbf{h}_{00k}^H\Big(\gamma_{0k}^{\rm SP}\mathbf{h}_{00k}+\frac{\gamma_{0k}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\Big)\Big\}=0$. Similarly, we can obtain

$$
\begin{aligned}
&\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00k}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}\\
&= \mathbb{E}\left\{\left(\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\rm SP}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^T\right.\\
&\qquad\left.\times\left(\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\rm SP}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^*\mathbf{h}_{00k}^H\mathbf{h}_{00k}\right\}\\
&\quad+\mathbb{E}\left\{\left(\gamma_{0k}^{\rm SP}\mathbf{h}_{00k}+\frac{\gamma_{0k}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^T\times\left(\gamma_{0k}^{\rm SP}\mathbf{h}_{00k}+\frac{\gamma_{0k}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^*\mathbf{h}_{00k}^H\mathbf{h}_{00k}\right\}\\
&= \frac{M^2\beta_{00k}(\gamma_{0k}^{\rm SP})^2}{q_{0k}L_u}\left(\sum_{\substack{l'\in\Phi\\l'i\neq0k}}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i}+\sigma^2\right)+M(M+1)\beta_{00k}^2(\gamma_{0k}^{\rm SP})^2\left(1+\frac{p_{0k}}{q_{0k}L_u}\right)\\
&= M\beta_{00k}^2\gamma_{0k}^{\rm SP}\left(M+\gamma_{0k}^{\rm SP}+\frac{p_{0k}\gamma_{0k}^{\rm SP}}{q_{0k}L_u}\right), \qquad (129)
\end{aligned}
$$

where we have used again the property that $\mathbf{h}_{00k}$ and $\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\rm SP}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*$ are independent of each other.

Substituting (128) and (129) into (127), we can obtain

$$
I_3 = \frac{M\beta_{00k}^2\gamma_{0k}^{\rm SP}\left(1+M\gamma_{0k}^{\rm SP}+\frac{Mp_{0k}\gamma_{0k}^{\rm SP}}{q_{0k}L_u}\right)M\beta_{00k}}{M\beta_{00k}^2\gamma_{0k}^{\rm SP}\left(M+\gamma_{0k}^{\rm SP}+\frac{p_{0k}\gamma_{0k}^{\rm SP}}{q_{0k}L_u}\right)} = \beta_{00k}\frac{\left(1+M\gamma_{0k}^{\rm SP}+\frac{Mp_{0k}\gamma_{0k}^{\rm SP}}{q_{0k}L_u}\right)}{\left(1+\frac{\gamma_{0k}^{\rm SP}}{M}+\frac{p_{0k}\gamma_{0k}^{\rm SP}}{Mq_{0k}L_u}\right)}. \qquad (130)
$$

Similarly, according to (13) and (124), I_4 in (32) can be accurately approximated as

$$
I_4 \approx \frac{\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2\right\}\mathbb{E}\left\{\|\mathbf{h}_{00k}\|^2\right\}}{\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00i}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}},\ i\neq k, \qquad (131)
$$

Firstly, $\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2\right\}, i\neq k$ is obtained as

$$
\begin{aligned}
&\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\right|^2\right\}\\
&= \mathbb{E}\left\{\mathbf{h}_{00k}^T\left(\hat{\mathbf{h}}_{00i}-\frac{\gamma_{0i}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)^*\mathbf{h}_{00k}^H\left(\hat{\mathbf{h}}_{00i}-\frac{\gamma_{0i}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)\right\}\\
&\quad+\mathbb{E}\left\{\mathbf{h}_{00k}^T\left(\frac{\gamma_{0i}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)^*\mathbf{h}_{00k}^H\left(\frac{\gamma_{0i}^{\rm SP}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)\right\}
\end{aligned}
$$

$$= \frac{M\beta_{00k}(\gamma_{0i}^{\mathrm{SP}})^2}{q_{0i}L_u}\left(q_{0i}L_u\beta_{00i}+\sum_{\substack{l'\in\Phi\\ l'i\neq 0k}}\sum_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2\right)+\frac{M(M+1)p_{0k}\beta_{00k}^2(\gamma_{0i}^{\mathrm{SP}})^2}{q_{0i}L_u}$$

$$= M\beta_{00k}\beta_{00i}\gamma_{0i}^{\mathrm{SP}}\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{q_{0i}\beta_{00i}L_u}\right) \tag{132}$$

where we note that $\mathbf{h}_{00k}$ and $\hat{\mathbf{h}}_{00k}-\frac{\gamma_{0i}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*$ are independent of each other. And $\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00i}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}$ is obtained as

$$\begin{aligned}
&\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00i}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}\\
=\;&\mathbb{E}\left\{\left(\hat{\mathbf{h}}_{00i}-\frac{\gamma_{0i}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)^T\left(\hat{\mathbf{h}}_{00i}-\frac{\gamma_{0i}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)^*\mathbf{h}_{00k}^H\mathbf{h}_{00k}\right\}\\
&+\mathbb{E}\left\{\left(\frac{\gamma_{0i}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)^T\left(\frac{\gamma_{0i}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)^*\mathbf{h}_{00k}^H\mathbf{h}_{00k}\right\}\\
=\;&\frac{M^2\beta_{00k}(\gamma_{0i}^{\mathrm{SP}})^2}{q_{0i}L_u}\left(q_{0i}L_u\beta_{00i}+\sum_{\substack{l'\in\Phi\\ l'i\neq 0k}}\sum_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2\right)+\frac{M(M+1)p_{0k}\beta_{00k}^2(\gamma_{0i}^{\mathrm{SP}})^2}{q_{0i}L_u}\\
=\;&M\beta_{00k}\beta_{00i}\gamma_{0i}^{\mathrm{SP}}\left(M+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{q_{0i}\beta_{00i}L_u}\right),
\end{aligned} \tag{133}$$

where we notice that $\mathbf{h}_{00k}$ and $\hat{\mathbf{h}}_{00k}-\frac{\gamma_{0i}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0i}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*$ are independent of each other. Substituting (132) and (133) into (131), we can obtain

$$I_4 \;\approx\; = \frac{M\beta_{00k}\beta_{00i}\gamma_{0i}^{\mathrm{SP}}\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{q_{0i}\beta_{00i}L_u}\right)M\beta_{00k}}{M\beta_{00k}\beta_{00i}\gamma_{0i}^{\mathrm{SP}}\left(M+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{q_{0i}\beta_{00i}L_u}\right)}=\beta_{00k}\frac{1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{q_{0i}\beta_{00i}L_u}}{1+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{Mq_{0i}\beta_{00i}L_u}},\; i\neq k, \tag{134}$$

The derivation of I_5 is similar to that of I_2 with RP. Since the proportion of contamination caused by $\mathbf{h}_{l0k}$ in $\hat{\mathbf{h}}_{lli}^*$ $l\in\Psi$ is very small, the relevant components between $\mathbf{h}_{l0k}$ and $\hat{\mathbf{h}}_{lli}^*$ can be ignored. Then, $\mathbf{h}_{l0k}$ and $\hat{\mathbf{h}}_{lli}^*$ can be identified approximately independent of each other. It follows that $I_5\approx\beta_{l0k}$. Then, the proof is complete by substituting the analytical results of S, I_3, I_4, I_5 above into (32).

D. Proof of Lemma 7

Following the similar derivations as those in *Lemma* 6, we can also obtain the approximation that $S\approx\left|\mathbb{E}\left\{\|\hat{\mathbf{g}}_{00k}\|\right\}\right|^2\approx\beta_{00k}\gamma_{0k}^{\mathrm{SP}}\bar{M}$ as $M\to\infty$ for SPe. In addition, I_3, I_4 and I_5 in (32) can be derived in the similar way as that with SP. According to (40) and (124), I_3 can be obtained as

$$I_3 \;\approx\; \frac{\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{g}}_{00k}^*\right|^2\right\}\mathbb{E}\left\{\|\mathbf{h}_{00k}\|^2\right\}}{\mathbb{E}\left\{\|\hat{\mathbf{g}}_{00k}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}}. \tag{135}$$

First, we denote the terms in (40) as $X_1^k\triangleq\sqrt{q_{0k}L_u}\,\mathbf{h}_{00k}$, $X_2^k\triangleq\sum_{i=1}^K\sqrt{\frac{p_{0i}}{L_u}}(\mathbf{h}_{00i}-\hat{\mathbf{h}}_{00i})\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*$,

$X_3^k \triangleq \sum_{l'\in\Psi}\sum_{i=1}^{K}\sqrt{\frac{p_{l'i}}{L_u}}\,\mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T\boldsymbol{\varphi}_{0k}^*$ and $X_4^k \triangleq \frac{1}{\sqrt{L_u}}\mathbf{N}_0\boldsymbol{\varphi}_{0k}^*, \forall k$. Then, we can obtain

$$\begin{aligned}
&\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{g}}_{00k}^*\right|^2\right\} = \mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{g}}_{00k}^*\mathbf{h}_{00k}^H\hat{\mathbf{g}}_{00k}\right\}\\
=\;& \frac{(\gamma_{0k}^{\text{SPe}})^2}{q_{0k}L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T X_1^{k*}\mathbf{h}_{00k}^H X_1^k + \mathbf{h}_{00k}^T X_2^{k*}\mathbf{h}_{00k}^H X_2^k + \mathbf{h}_{00k}^T X_3^{k*}\mathbf{h}_{00k}^H X_3^k + \mathbf{h}_{00k}^T X_4^{k*}\mathbf{h}_{00k}^H X_4^k\right\} \quad (136)
\end{aligned}$$

In (136) it is easy to obtain that $\mathbb{E}\left\{\mathbf{h}_{00k}^T X_1^{k*}\mathbf{h}_{00k}^H X_1^k\right\} = M(M+1)(\gamma_{0k}^{\text{SPe}})^2\beta_{00k}^2$, $\mathbb{E}\left\{\mathbf{h}_{00k}^T X_3^{k*}\mathbf{h}_{00k}^H X_3^k\right\}$ $= \frac{M\beta_{00k}(\gamma_{0k}^{\text{SPe}})^2}{q_{0k}L_u}\sum_{l'\in\Psi}\sum_{i=1}^{K}p_{l'i}\beta_{0l'i}$ and $\mathbb{E}\left\{\mathbf{h}_{00k}^T X_4^{k*}\mathbf{h}_{00k}^H X_4^k\right\} = \frac{M\beta_{00k}\sigma^2(\gamma_{0k}^{\text{SPe}})^2}{q_{0k}L_u}$. Moreover, we can obtain that

$$\begin{aligned}
&\mathbb{E}\left\{\mathbf{h}_{00k}^T X_2^{k*}\mathbf{h}_{00k}^H X_2^k\right\}\\
=\;& \mathbb{E}\left\{\mathbf{h}_{00k}^T\left[\sum_{i=1}^{K}\sqrt{\frac{p_{0i}}{L_u}}(\mathbf{h}_{00i}-\hat{\mathbf{h}}_{00i})\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right]^*\mathbf{h}_{00k}^H\left[\sum_{i=1}^{K}\sqrt{\frac{p_{0i}}{L_u}}(\mathbf{h}_{00i}-\hat{\mathbf{h}}_{00i})\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right]\right\}\\
=\;& \sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T(\mathbf{h}_{00i}^*-\hat{\mathbf{h}}_{00i}^*)\mathbf{h}_{00k}^H(\mathbf{h}_{00i}-\hat{\mathbf{h}}_{00i})\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\}\\
=\;& \sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{h}_{00i}^*\mathbf{h}_{00k}^H\mathbf{h}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\} + \sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\}\\
&-\sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{h}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\} - \sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\mathbf{h}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\}\\
=\;& M\beta_{00k}\left(\sum_{i=1}^{K}p_{0i}\beta_{00i} + Mp_{0k}\beta_{00k}\right) - 2M\beta_{00k}\left(\sum_{i=1}^{K}\gamma_{0i}^{\text{SP}}p_{0i}\beta_{00i} + M\gamma_{0k}^{\text{SP}}p_{0k}\beta_{00k}\right)\\
&+\sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\}, \quad (137)
\end{aligned}$$

where (137) follows from the results that $\sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{h}_{00i}^*\mathbf{h}_{00k}^H\mathbf{h}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\} = M\beta_{00k}$ $\left(\sum_{i=1}^{K}p_{0i}\beta_{00i} + Mp_{0k}\beta_{00k}\right)$, and $\sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{h}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\} =$ $\sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\mathbf{h}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\} = M\gamma_{0i}^{\text{SP}}\beta_{00k}\left(\sum_{i=1}^{K}p_{0i}\beta_{00i} + Mp_{0k}\beta_{00k}\right)$. The last term in (137) can be obtained as

$$\begin{aligned}
&\sum_{i=1}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\}\\
=\;& \frac{p_{0k}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00k}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00k}\mathbf{s}_{0k}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right\} + \sum_{i=1,i\neq k}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0k}^*\right\}\\
=\;& \frac{p_{0k}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\left(\hat{\mathbf{h}}_{00k} - \gamma_{0k}^{\text{SP}}\mathbf{h}_{00k} - \frac{\gamma_{0k}^{\text{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^*\mathbf{h}_{00k}^H\right.\\
&\times\left.\left(\hat{\mathbf{h}}_{00k} - \gamma_{0k}^{\text{SP}}\mathbf{h}_{00k} - \frac{\gamma_{0k}^{\text{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)\mathbf{s}_{0k}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right\}\\
&+\frac{p_{0k}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\left(\gamma_{0k}^{\text{SP}}\mathbf{h}_{00k} + \frac{\gamma_{0k}^{\text{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^*\mathbf{h}_{00k}^H\right.\\
&\times\left.\left(\gamma_{0k}^{\text{SP}}\mathbf{h}_{00k} + \frac{\gamma_{0k}^{\text{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)\mathbf{s}_{0k}^H\boldsymbol{\varphi}_{0k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right\} + \sum_{i=1,i\neq k}^{K}\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\right\}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{Mp_{0k}\beta_{00k}(\gamma_{0k}^{\text{SP}})^2}{q_{0k}L_u}\left(\sum_{\substack{l'\in\Phi\\ l'i\neq 0k}}\sum_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2\right)+M(M+1)p_{0k}\beta_{00k}^2(\gamma_{0k}^{\text{SP}})^2\left(1+\frac{2p_{0k}}{q_{0k}L_u}\right)\\
&\quad +M\beta_{00k}\sum_{i=1,i\neq k}^{K}p_{0i}\beta_{00i}\gamma_{0i}^{\text{SP}}\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\text{SP}}}{q_{0i}\beta_{00i}L_u}\right)\\
&= Mp_{0k}\beta_{00k}^2(\gamma_{0k}^{\text{SP}})^2\left(M+\frac{(1+M)p_{0k}}{q_{0k}L_u}\right)+M\beta_{00k}\sum_{i=1}^{K}p_{0i}\beta_{00i}\gamma_{0i}^{\text{SP}}\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\text{SP}}}{q_{0i}\beta_{00i}L_u}\right)
\end{aligned}
\tag{138}
$$

where (138) is obtained by using (132) and the independence between $\left(\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\text{SP}}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\text{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)$ and $\mathbf{h}_{00k}$.

Then, combing (137) and (138) with (136), we can obtain that

$$
\begin{aligned}
&\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{g}}_{00k}^*\right|^2\right\}\\
&= \frac{(\gamma_{0k}^{\text{SPe}})^2}{q_{0k}L_u}\Bigg\{M\beta_{00k}\left(\sum_{i=1}^{K}p_{0i}\beta_{00i}+Mp_{0k}\beta_{00k}\right)-2M\beta_{00k}\left(\sum_{i=1}^{K}\gamma_{0i}^{\text{SP}}p_{0i}\beta_{00i}+M\gamma_{0k}^{\text{SP}}p_{0k}\beta_{00k}\right)\\
&\quad +Mp_{0k}\beta_{00k}^2(\gamma_{0k}^{\text{SP}})^2\left(M+\frac{(1+M)p_{0k}}{q_{0k}L_u}\right)+M\beta_{00k}\sum_{i=1}^{K}p_{0i}\beta_{00i}\gamma_{0i}^{\text{SP}}\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\text{SP}}}{q_{0i}\beta_{00i}L_u}\right)\Bigg\}\\
&\quad +\frac{(\gamma_{0k}^{\text{SPe}})^2}{q_{0k}L_u}\left(M(M+1)L_uq_{0k}\beta_{00k}^2+M\beta_{00k}\sum_{l'\in\Psi}\sum_{i=1}^{K}p_{li}\beta_{0l'i}+M\sigma^2\beta_{00k}\right)\\
&= M\beta_{00k}^2\gamma_{0k}^{\text{SPe}}\left[1+M\gamma_{0k}^{\text{SPe}}+\frac{Mp_{0k}\gamma_{0k}^{\text{SPe}}}{q_{0k}L_u}\left((1-\gamma_{0k}^{\text{SP}})^2+\frac{p_{0k}{\gamma_{0k}^{\text{SP}}}^2}{q_{0k}L_u}+\sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\text{SP}}}^2}{q_{0i}L_u}\right)\right].
\end{aligned}
\tag{139}
$$

Following the same procedure, we can obtain that

$$
\mathbb{E}\left\{\|\hat{\mathbf{h}}_{00k}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}=M\beta_{00k}^2\gamma_{0k}^{\text{SPe}}\left[M+\gamma_{0k}^{\text{SPe}}+\frac{p_{0k}\gamma_{0k}^{\text{SPe}}}{q_{0k}L_u}\left((1-\gamma_{0k}^{\text{SP}})^2+\frac{p_{0k}{\gamma_{0k}^{\text{SP}}}^2}{q_{0k}L_u}+\sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\text{SP}}}^2}{q_{0i}L_u}\right)\right]. \tag{140}
$$

Substituting (139) and (140) and into (141), we can obtain

$$
\begin{aligned}
I_3 &\approx \frac{M\beta_{00k}^2\gamma_{0k}^{\text{SPe}}\left[1+M\gamma_{0k}^{\text{SPe}}+\frac{Mp_{0k}\gamma_{0k}^{\text{SPe}}}{q_{0k}L_u}\left((1-\gamma_{0k}^{\text{SP}})^2+\frac{p_{0k}{\gamma_{0k}^{\text{SP}}}^2}{q_{0k}L_u}+\sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\text{SP}}}^2}{q_{0i}L_u}\right)\right]M\beta_{00k}}{M\beta_{00k}^2\gamma_{0k}^{\text{SPe}}\left[M+\gamma_{0k}^{\text{SPe}}+\frac{p_{0k}\gamma_{0k}^{\text{SPe}}}{q_{0k}L_u}\left((1-\gamma_{0k}^{\text{SP}})^2+\frac{p_{0k}{\gamma_{0k}^{\text{SP}}}^2}{q_{0k}L_u}+\sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\text{SP}}}^2}{q_{0i}L_u}\right)\right]}\\
&= \beta_{00k}\frac{1+M\gamma_{0k}^{\text{SPe}}+\frac{Mp_{0k}\gamma_{0k}^{\text{SPe}}}{q_{0k}L_u}\left((1-\gamma_{0k}^{\text{SP}})^2+\frac{p_{0k}{\gamma_{0k}^{\text{SP}}}^2}{q_{0k}L_u}+\sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\text{SP}}}^2}{q_{0i}L_u}\right)}{1+\frac{\gamma_{0k}^{\text{SPe}}}{M}+\frac{p_{0k}\gamma_{0k}^{\text{SPe}}}{Mq_{0k}L_u}\left((1-\gamma_{0k}^{\text{SP}})^2+\frac{p_{0k}{\gamma_{0k}^{\text{SP}}}^2}{q_{0k}L_u}+\sum_{i=1}^{K}\frac{p_{0i}{\gamma_{0i}^{\text{SP}}}^2}{q_{0i}L_u}\right)}.
\end{aligned}
\tag{141}
$$

Similarly, according to (40) and (124), I_4 can be obtained as

$$
I_4 \approx \frac{\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{g}}_{00i}^*\right|^2\right\}\mathbb{E}\left\{\|\mathbf{h}_{00k}\|^2\right\}}{\mathbb{E}\left\{\|\hat{\mathbf{g}}_{00i}^*\|^2\|\mathbf{h}_{00k}\|^2\right\}}\ i\neq k, \tag{142}
$$

We first derive the result of $\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{g}}_{00i}^*\right|^2\right\}$ as follows. According to (40), we can express $\hat{\mathbf{g}}_{00i}$ for

$i \neq k$ as

$$\hat{\mathbf{g}}_{00i} = \frac{\gamma_{0i}^{\mathrm{SPe}}}{\gamma_{0i}^{\mathrm{SP}}}\check{\mathbf{h}}_{00i} = \frac{\gamma_{0i}^{\mathrm{SPe}}}{\sqrt{q_{0i}L_u}}\Bigg(\underbrace{\sqrt{q_{0k}L_u}\,\mathbf{h}_{00i}}_{X_1^i} + \underbrace{\sum_{j=1}^{K}\sqrt{\frac{p_{0j}}{L_u}}(\mathbf{h}_{00j}-\hat{\mathbf{h}}_{00j})\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*}_{X_2^i}$$
$$+\underbrace{\sum_{l'\in\Psi}\sum_{j=1}^{K}\sqrt{\frac{p_{l'j}}{L_u}}\,\mathbf{h}_{0l'i}\mathbf{s}_{l'j}^T\boldsymbol{\varphi}_{0i}^*}_{X_3^i} + \underbrace{\frac{1}{\sqrt{L_u}}\mathbf{N}_0\boldsymbol{\varphi}_{0i}^*}_{X_4^i}\Bigg). \tag{143}$$

Then, we have

$$\mathbb{E}\left\{\left|\mathbf{h}_{00k}^T\hat{\mathbf{g}}_{00i}^*\right|^2\right\} = \mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{g}}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{g}}_{00i}\right\}$$
$$= \frac{(\gamma_{0i}^{\mathrm{SPe}})^2}{q_{0i}L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T X_1^{i*}\mathbf{h}_{00k}^H X_1^i + \mathbf{h}_{00k}^T X_2^{i*}\mathbf{h}_{00k}^H X_2^i + \mathbf{h}_{00k}^T X_3^{i*}\mathbf{h}_{00k}^H X_3^i + \mathbf{h}_{00k}^T X_4^{i*}\mathbf{h}_{00k}^H X_4^i\right\}. \tag{144}$$

In (144), it can be obtained that $\mathbb{E}\left\{\mathbf{h}_{00k}^T X_1^{i*}\mathbf{h}_{00k}^H X_1^i\right\} = M(\gamma_{0i}^{\mathrm{SPe}})^2\beta_{00k}^2$, $\mathbb{E}\left\{\mathbf{h}_{00k}^T X_3^{i*}\mathbf{h}_{00k}^H X_3^i\right\} = \frac{M\beta_{00k}(\gamma_{0i}^{\mathrm{SPe}})^2}{q_{0k}L_u}\sum_{l'\in\Psi}\sum_{j=1}^{K}p_{lj}\beta_{0l'j}$, and $\mathbb{E}\left\{\mathbf{h}_{00k}^T X_4^{i*}\mathbf{h}_{00k}^H X_4^i\right\} = \frac{M\beta_{00k}\sigma^2(\gamma_{0i}^{\mathrm{SPe}})^2}{q_{0k}L_u}$. Moreover, $\mathbb{E}\left\{\mathbf{h}_{00k}^T X_2^{i*}\mathbf{h}_{00k}^H X_2^i\right\}$ can be obtained as

$$\mathbb{E}\left\{\mathbf{h}_{00k}^T X_2^{i*}\mathbf{h}_{00k}^H X_2^i\right\}$$
$$= \mathbb{E}\left\{\mathbf{h}_{00k}^T\left[\sum_{j=1}^{K}\sqrt{\frac{p_{0j}}{L_u}}(\mathbf{h}_{00j}-\hat{\mathbf{h}}_{00j})\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right]^*\mathbf{h}_{00k}^H\left[\sum_{j=1}^{K}\sqrt{\frac{p_{0j}}{L_u}}(\mathbf{h}_{00j}-\hat{\mathbf{h}}_{00j})\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right]\right\}$$
$$= \sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T(\mathbf{h}_{00j}^*-\hat{\mathbf{h}}_{00j}^*)\mathbf{h}_{00k}^H(\mathbf{h}_{00j}-\hat{\mathbf{h}}_{00j})\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\}$$
$$= \sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{h}_{00j}^*\mathbf{h}_{00k}^H\mathbf{h}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\} + \sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00j}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\}$$
$$-\sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{h}_{00j}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\} - \sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00j}^*\mathbf{h}_{00k}^H\mathbf{h}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\}$$
$$= M\beta_{00k}\left(\sum_{j=1}^{K}p_{0j}\beta_{00j} + Mp_{0k}\beta_{00k}\right) - 2M\beta_{00k}\left(\sum_{j=1}^{K}\gamma_{0j}^{\mathrm{SP}}p_{0j}\beta_{00j} + M\gamma_{0k}^{\mathrm{SP}}p_{0k}\beta_{00k}\right)$$
$$+\sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00j}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\}. \tag{145}$$

where (145) follows from $\sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{h}_{00j}^*\mathbf{h}_{00k}^H\mathbf{h}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\} = M\beta_{00k}\Big(\sum_{j=1}^{K}p_{0j}\beta_{00j} + Mp_{0k}\beta_{00k}\Big)$, and $\sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\mathbf{h}_{00j}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\} = \sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\Big\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00j}^*\mathbf{h}_{00k}^H\mathbf{h}_{00j}$ $\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\Big\} = M\gamma_{0j}^{\mathrm{SP}}\beta_{00k}\left(\sum_{j=1}^{K}p_{0j}\beta_{00j} + Mp_{0k}\beta_{00k}\right)$.

The last term in (145) can be obtained as

$$
\begin{aligned}
&\sum_{j=1}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00j}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\}\\
=\;& \frac{p_{0k}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00k}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00k}\mathbf{s}_{0k}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right\}+\sum_{j=1,j\neq k,i}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00j}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\}\\
&+\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0i}^*\right\}\\
=\;& \frac{p_{0k}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\left(\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^*\mathbf{h}_{00k}^H\right.\\
&\left.\times\left(\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)\mathbf{s}_{0k}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right\}\\
&+\frac{p_{0k}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\left(\gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k}+\frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)^*\mathbf{h}_{00k}^H\right.\\
&\left.\times\left(\gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k}+\frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0k}^*\right)\mathbf{s}_{0k}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right\}\\
&+\sum_{j=1,j\neq k,i}^{K}\frac{p_{0j}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00j}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00j}\mathbf{s}_{0j}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0j}^T\boldsymbol{\varphi}_{0i}^*\right\}+\frac{p_{0i}}{L_u}\mathbb{E}\left\{\mathbf{h}_{00k}^T\hat{\mathbf{h}}_{00i}^*\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00i}\mathbf{s}_{0i}^H\boldsymbol{\varphi}_{0i}\mathbf{s}_{0i}^T\boldsymbol{\varphi}_{0i}^*\right\}\\
=\;& \frac{Mp_{0k}\beta_{00k}(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}L_u}\left(\sum_{\substack{l'\in\Phi\\(l',j)\neq(0,k)}}\sum_{j=1}^{K}p_{l'j}\beta_{0l'j}+\sigma^2\right)+M(M+1)p_{0k}\beta_{00k}^2(\gamma_{0k}^{\mathrm{SP}})^2\left(1+\frac{p_{0k}}{q_{0k}L_u}\right)\\
&+M\beta_{00k}\sum_{j=1,j\neq k}^{K}p_{0j}\beta_{00j}\gamma_{0j}^{\mathrm{SP}}\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0j}^{\mathrm{SP}}}{q_{0j}\beta_{00j}L_u}\right)+\frac{Mp_{0i}^2\beta_{00i}\beta_{00k}(\gamma_{0i}^{\mathrm{SP}})^2}{q_{0i}L_u} && (146)\\
=\;& M^2p_{0k}\beta_{00k}^2(\gamma_{0k}^{\mathrm{SP}})^2+M\beta_{00k}\sum_{j=1}^{K}p_{0j}\beta_{00j}\gamma_{0j}^{\mathrm{SP}}\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0j}^{\mathrm{SP}}}{q_{0j}\beta_{00j}L_u}\right)+\frac{Mp_{0i}^2\beta_{00i}\beta_{00k}(\gamma_{0i}^{\mathrm{SP}})^2}{q_{0i}L_u}, && (147)
\end{aligned}
$$

where (146) follows by using (128) and (132), and also the independence between $\left(\hat{\mathbf{h}}_{00k}-\gamma_{0k}^{\mathrm{SP}}\mathbf{h}_{00k}-\frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{0k}}{q_{0k}}}\,\mathbf{h}_{00k}\mathbf{s}_{0k}^T\boldsymbol{\varphi}_{0i}^*\right)$ and $\mathbf{h}_{00k}$.

Combining the results in (145) and (147) with (144), we can obtain

$$
\begin{aligned}
& \mathbb{E}\left\{\left|\mathbf{h}_{00k}^{T}\hat{\mathbf{g}}_{00i}^{*}\right|^{2}\right\} \\
= & \frac{(\gamma_{0i}^{\mathrm{SPe}})^2}{q_{0i}L_u}\Bigg\{M\beta_{00k}\left(\sum_{j=1}^{K}p_{0j}\beta_{00j}+Mp_{0k}\beta_{00k}\right)-2M\beta_{00k}\left(\sum_{j=1}^{K}\gamma_{0j}^{\mathrm{SP}}p_{0j}\beta_{00j}+M\gamma_{0k}^{\mathrm{SP}}p_{0k}\beta_{00k}\right) \\
& M^2p_{0k}\beta_{00k}^2(\gamma_{0k}^{\mathrm{SP}})^2+M\beta_{00k}\sum_{j=1}^{K}p_{0j}\beta_{00j}\gamma_{0j}^{\mathrm{SP}}\left(1+\frac{Mp_{0k}\beta_{00k}\gamma_{0j}^{\mathrm{SP}}}{q_{0j}\beta_{00j}L_u}\right)+\frac{Mp_{0i}^2\beta_{00i}\beta_{00k}(\gamma_{0i}^{\mathrm{SP}})^2}{q_{0i}L_u}\Bigg\} \\
& +\frac{(\gamma_{0i}^{\mathrm{SPe}})^2}{q_{0i}L_u}\left(ML_uq_{0k}\beta_{00k}^2+M\beta_{00k}\sum_{l'\in\Psi}\sum_{j=1}^{K}p_{lj}\beta_{0l'j}+M\sigma^2\beta_{00k}\right) \\
= & \frac{(\gamma_{0i}^{\mathrm{SPe}})^2}{q_{0i}L_u}\Bigg\{M^2p_{0k}\beta_{00k}^2\left((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right) \\
& +M\beta_{00k}\left(\sum_{j=1}^{K}p_{0j}\beta_{00j}(1-\gamma_{0j}^{\mathrm{SP}})+\frac{p_{0i}^2\beta_{00i}(\gamma_{0i}^{\mathrm{SP}})^2}{q_{0i}L_u}\right)\Bigg\} \\
& +\frac{(\gamma_{0i}^{\mathrm{SPe}})^2}{q_{0i}L_u}\left(ML_uq_{0k}\beta_{00k}^2+M\beta_{00k}\sum_{l'\in\Psi}\sum_{j=1}^{K}p_{lj}\beta_{0l'j}+M\sigma^2\beta_{00k}\right) \\
= & M\beta_{00k}\beta_{00i}\gamma_{0i}^{\mathrm{SPe}}\left[1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{q_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)\right].
\end{aligned}
\tag{148}
$$

Similarly, we can obtain

$$
\mathbb{E}\left\{\|\hat{\mathbf{g}}_{00i}^{*}\|^2\|\mathbf{h}_{00k}\|^2\right\}=M\beta_{00k}\beta_{00i}\gamma_{0i}^{\mathrm{SPe}}\left[M+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{q_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)\right]. \tag{149}
$$

Substituting (149) and (148) into (150), we can obtain

$$
\begin{aligned}
I_4 \approx & \frac{M\beta_{00k}\beta_{00i}\gamma_{0i}^{\mathrm{SPe}}\left[1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{q_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)\right]M\beta_{00k}}{M\beta_{00k}\beta_{00i}\gamma_{0i}^{\mathrm{SPe}}\left[M+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{q_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)\right]} \qquad (150) \\
= & \beta_{00k}\frac{1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{q_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)}{1+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{Mq_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)},\ i\neq k, \qquad (151)
\end{aligned}
$$

For the same reason as in the analysis of SP, we consider that $\mathbf{h}_{l0k}$ and $\hat{\mathbf{g}}_{lli}^{*}$ are approximately independent of each other, and we derive that $I_5 \approx \beta_{l0k}$. Then, the proof is complete by substituting the analytical results of S, I_3, I_4, I_5 above into (32).

E. Proof of Lemma 11

According to (85), we have

$$W(b\tilde{\mathbf{z}}) = \frac{1}{b\sum\limits_{i=1}^{N}\tilde{z}_i}, \tag{152}$$

$$V(b\tilde{\mathbf{z}}) = \frac{\prod\limits_{i=1}^{N}(bz_i)^{c_i}}{\sum\limits_{i=1}^{N}\tilde{z}_i\prod\limits_{i=1}^{N}\tilde{z}_i^{c_i}} = \frac{b^{\sum\limits_{i=1}^{N}c_i}}{\sum\limits_{i=1}^{N}\tilde{z}_i^{N}} = \frac{1}{b\sum\limits_{i=1}^{N}\tilde{z}_i}. \tag{153}$$

So we can derive (85). The partial derivative of $W(b\tilde{\mathbf{z}})$ and $V(b\tilde{\mathbf{z}})$ w.r.t. $b\tilde{\mathbf{z}}$ are given by

$$\frac{\partial\, W(b\tilde{\mathbf{z}})}{\partial\, bz_k} = \frac{1}{b^2\left(\sum\limits_{i=1}^{N}\tilde{z}_i\right)^2}, \quad k = 1, 2, ..., N, \tag{154}$$

$$\frac{\partial\, V(b\tilde{\mathbf{z}})}{\partial\, bz_k} = \frac{c_k(bz_k)^{-1}\prod\limits_{i=1}^{N}(bz_i)^{c_i}}{\sum\limits_{i=1}^{N}\tilde{z}_i\prod\limits_{i=1}^{N}\tilde{z}_i^{c_i}} = \frac{1}{b^2\left(\sum\limits_{i=1}^{N}\tilde{z}_i\right)^2}, \quad k = 1, 2, ..., N, \tag{155}$$

where (155) follows $c_i = -\tilde{z}_i\Big/\sum\limits_{i=1}^{N}\tilde{z}_i$ and (153). Next, let's prove (83). According to the inequality in (83), we can obtain an inequivalent form given by

$$J(\mathbf{z}) = \sum_{i=1}^{N} z_i \prod_{i=1}^{N} {z_i}^{c_i} \geq \sum_{i=1}^{N}\tilde{z}_i\prod_{i=1}^{N}\tilde{z}_i^{c_i}. \tag{156}$$

The partial derivative of $J(\mathbf{z})$ w.r.t. $\mathbf{z}$ is given by

$$\begin{aligned}
\frac{\partial\, J(\mathbf{z})}{\partial\, z_k} &= z_k^{-1}\left((1+c_k)z_k + c_k\sum_{i=1, i\neq k}^{N} z_i\right)\prod_{i=1}^{N}{z_i}^{c_i} \\
&= \left(z_k\frac{\sum\limits_{i=1,i\neq k}^{N}\tilde{z}_i}{\sum\limits_{i=1}^{N}\tilde{z}_i} - \frac{\tilde{z}_k\sum\limits_{i=1,i\neq k}^{N} z_i}{\sum\limits_{i=1}^{N}\tilde{z}_i}\right)\prod_{i=1}^{N}{z_i}^{c_i} \qquad (157) \\
&= \frac{\prod\limits_{i=1}^{N}{z_i}^{c_i}\sum\limits_{i=1,i\neq k}^{N}\tilde{z}_i}{\sum\limits_{i=1}^{N}\tilde{z}_i}\left(z_k - \frac{\tilde{z}_k\sum\limits_{i=1,i\neq k}^{N} z_i}{\sum\limits_{i=1,i\neq k}^{N}\tilde{z}_i}\right), \quad k = 1, 2, ..., N, \qquad (158)
\end{aligned}$$

where (157) follows $c_k = -\tilde{z}_k\Big/\sum\limits_{k=1}^{N}\tilde{z}_i$, and $\tilde{z}_k\sum\limits_{i=1,i\neq k}^{N} z_i\Big/\sum\limits_{i=1,i\neq k}^{N}\tilde{z}_i$ is a variable independent of z_k. Obviously, the expression (158) has a zero crossing point w.r.t. z_k. Let $\frac{\partial J(\mathbf{z})}{\partial z_k}\Big|_{z_k=\dot{z}_k} = 0$, we can get zero crossing point that given by $\dot{z}_k = \tilde{z}_k\sum\limits_{i=1,i\neq k}^{N} z_i\Big/\sum\limits_{i=1,i\neq k}^{N}\tilde{z}_i$. So we have a homogeneous system of linear equations given by

$$\begin{cases} \dot{z}_1 = \frac{\tilde{z}_1 \sum\limits_{i=2}^{N} \dot{z}_i}{\sum\limits_{i=2}^{N} \tilde{z}_i} \\ \dot{z}_2 = \frac{\tilde{z}_2 \sum\limits_{i=1,i\neq 2}^{N} \dot{z}_i}{\sum\limits_{i=1,i\neq 2}^{N} \tilde{z}_i} \\ \vdots \\ \dot{z}_N = \frac{\tilde{z}_k \sum\limits_{i=1}^{N-1} \dot{z}_i}{\sum\limits_{i=1}^{N-1} \tilde{z}_i} \end{cases} \Rightarrow \begin{cases} \dot{z}_1 - \frac{\tilde{z}_1}{\tilde{z}_2}\dot{z}_2 = 0 \\ \dot{z}_2 - \frac{\tilde{z}_2}{\tilde{z}_3}\dot{z}_3 = 0 \\ \vdots \\ \dot{z}_N - \frac{\tilde{z}_N}{\tilde{z}_1}\dot{z}_1 = 0 \end{cases} \Rightarrow A\dot{\mathbf{z}} = 0, \tag{159}$$

where A is the coefficient matrix of the system, and the rank of A is $N-1$. Therefore, the system has a non-zero solution that given by $\dot{\mathbf{z}} = b\tilde{\mathbf{z}}$. Next, we will prove that $\dot{\mathbf{z}}$ is the minimum point of $J(\mathbf{z})$. To get the Hessen matrix, we take the second partial of $J(\mathbf{z})$ w.r.t. $\mathbf{z}$ is given by

$$\frac{\partial^2 J(\mathbf{z})}{\partial {z_k}^2} = c_k z_k^{-1}\left(1 + c_k + (c_k - 1) z_k^{-1} \sum_{i=1,i\neq k}^{N} z_i\right) \prod_{i=1}^{N} {z_i}^{c_i}, \tag{160}$$

$$\frac{\partial^2 J(\mathbf{z})}{\partial z_k \partial z_j} = \left(c_k c_j z_k^{-1} z_j^{-1} \sum_{i=1,i\neq k,j}^{N} z_i + c_k(1+c_j) z_k^{-1} + c_j(1+c_k) z_j^{-1}\right) \prod_{i=1}^{N} {z_i}^{c_i}. \tag{161}$$

Let $\mathbf{z} = \dot{\mathbf{z}}$ and (160) and (161) can be rewritten as

$$\left.\frac{\partial^2 J(\mathbf{z})}{\partial {z_k}^2}\right|_{\mathbf{z}=\dot{\mathbf{z}}} = -\frac{\prod\limits_{i=1}^{N} (b\tilde{z}_i)^{c_i}}{b\sum\limits_{i=1}^{N} \tilde{z}_i}, \tag{162}$$

$$\left.\frac{\partial^2 J(\mathbf{z})}{\partial z_k \partial z_j}\right|_{\mathbf{z}=\dot{\mathbf{z}}} = \frac{\prod\limits_{i=1}^{N} (b\tilde{z}_i)^{c_i} \sum\limits_{i=1,i\neq k}^{N} \tilde{z}_i}{b\tilde{z}_k \sum\limits_{i=1}^{N} \tilde{z}_i}. \tag{163}$$

Therefore, we can get Hessen matrix $\mathbf{H}$ that given by

$$\mathbf{H} = \frac{\prod\limits_{i=1}^{N} (b\tilde{z}_i)^{c_i}}{b\sum\limits_{i=1}^{N} \tilde{z}_i} \begin{bmatrix} \frac{\sum\limits_{i=2}^{N} \tilde{z}_i}{\tilde{z}_1} & -1 & -1 & \cdots & -1 \\ -1 & \frac{\sum\limits_{i=1,i\neq 2}^{N} \tilde{z}_i}{\tilde{z}_2} & -1 & \cdots & -1 \\ -1 & -1 & \frac{\sum\limits_{i=1,i\neq 3}^{N} \tilde{z}_i}{\tilde{z}_3} & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & -1 & \frac{\sum\limits_{i=1,i\neq N}^{N} \tilde{z}_i}{\tilde{z}_N} \end{bmatrix}. \tag{164}$$

According to (164), to prove that $\mathbf{H}$ is a positive semidefinite matrix, we can further obtain that

$$
\begin{aligned}
[\mathbf{z}-\dot{\mathbf{z}}]^T\mathbf{H}[\mathbf{z}-\dot{\mathbf{z}}] &= \sum_{j=1}^{N}\left(\frac{z_j}{\tilde{z}_j}\sum_{i=1,i\neq j}^{N}\tilde{z}_i-\sum_{i=1,i\neq j}^{N}z_i\right)(z_j-b\tilde{z}_j)\\
&= \sum_{j=1}^{N}\left(\frac{z_j}{\tilde{z}_j}\sum_{i=1}^{N}\tilde{z}_i-\sum_{i=1}^{N}z_i\right)(z_j-b\tilde{z}_j)\\
&= \sum_{i=1}^{N}\tilde{z}_i\sum_{j=1}^{N}\frac{z_j^2}{\tilde{z}_j}-\sum_{i=1}^{N}z_i^2\\
&= \sum_{i=1}^{N}\tilde{z}_i\sum_{j=1}^{N}z_j^2\left(\frac{1}{\tilde{z}_j}-\frac{1}{\sum\limits_{i=1}^{N}\tilde{z}_i}\right)\geq 0.
\end{aligned}
\tag{165}
$$

Therefore, we can derive that $\mathbf{z}=\dot{\mathbf{z}}$ is the minimum point of the function $J(\mathbf{z})$, and (156) can be proved. The proof is complete.

F. *Proof of Lemma 12*

First, we consider the approximation of the denominator of (81) by polynomials. According to *Lemma* 6 and (12), $\frac{\alpha_k}{\gamma_{0k}^{\mathrm{SP}}}$ in (81) is given by

$$
\begin{aligned}
\frac{\alpha_k}{\gamma_{0k}^{\mathrm{SP}}} &= \frac{1+M\gamma_{0k}^{\mathrm{SP}}+\frac{Mp_{0k}\gamma_{0k}^{\mathrm{SP}}}{q_{0k}L_u}}{\gamma_{0k}^{\mathrm{SP}}\left(1+\frac{\gamma_{0k}^{\mathrm{SP}}}{M}+\frac{p_{0k}\gamma_{0k}^{\mathrm{SP}}}{Mq_{0k}L_u}\right)}=\frac{\frac{1}{\gamma_{0k}^{\mathrm{SP}}}+M+\frac{Mp_{0k}}{q_{0k}L_u}}{1+\frac{\gamma_{0k}^{\mathrm{SP}}}{M}+\frac{p_{0k}\gamma_{0k}^{\mathrm{SP}}}{Mq_{0k}L_u}}\\
&= M-1+\frac{1}{\gamma_{0k}^{\mathrm{SP}}}+\frac{\frac{M-1}{M}\left[\left(1-\gamma_{0k}^{\mathrm{SP}}\right)\left(1+\frac{p_{0k}}{q_{0k}L_u}\right)+\frac{Mp_{0k}}{q_{0k}L_u}\right]}{1+\frac{\gamma_{0k}^{\mathrm{SP}}}{M}+\frac{p_{0k}\gamma_{0k}^{\mathrm{SP}}}{Mq_{0k}L_u}}\\
&= M+\frac{\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2}{q_{0k}L_u\beta_{00k}}\\
&\quad+\frac{\left(\frac{M-1}{M}+\frac{(M^2+M-1)p_{0k}}{Mq_{0k}L_u}\right)\left(\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2\right)+Mp_{0k}\beta_{00k}}{\frac{M+1}{M}q_{0k}L_u\beta_{00k}+\frac{1}{M}p_{0k}\beta_{00k}+\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2} \qquad (166)\\
&= M+\frac{\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2}{q_{0k}L_u\beta_{00k}}\\
&\quad+A_k^{\mathrm{SP}}\left[\left(\frac{M-1}{M}+\frac{(M^2+M-1)p_{0k}}{Mq_{0k}L_u}\right)\left(\sum_{l'\in\Phi}\sum_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2\right)+Mp_{0k}\beta_{00k}\right], \qquad (167)
\end{aligned}
$$

where $A_k^{\mathrm{SP}}\triangleq\frac{1}{\frac{M+1}{M}q_{0k}L_u\beta_{00k}+\frac{1}{M}p_{0k}\beta_{00k}+\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K}p_{l'i}\beta_{0l'i}+\sigma^2}$, and (166) follows by substituting (12) into (166). We can obtain a polynomial approximation on $\frac{\alpha_k}{\gamma_{0k}^{\mathrm{SP}}}$ if we can approximate A_k^{SP} by a monomial function. Firstly, the corresponding A_k^{SP} at the $(t+1)$-th iteration is given by $A_k^{\mathrm{SP}(t)}$

in (95). Then, by using *Lemma* 11 and identifying the monomial terms of $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \forall k\right\}$ and the constant terms in the denominator of $A_k^{\mathrm{SP}(t)}$ in (95) as $\tilde{z}_i$ in *Lemma* 11, the best local monomial approximation [43, Section IV] on A_k^{SP} can be obtained as $A_k^{\mathrm{SP}} \leq \tilde{A}_k^{\mathrm{SP}(t)}$, where $\tilde{A}_k^{\mathrm{SP}(t)}$ is a monomial w.r.t. $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \forall k\right\}$ given by (88). $\Upsilon_{A_k^{\mathrm{SP}}}$ is a constant and given by (93), and the constants $\left\{c_{0k,1}^{(t)}, c_{0k,2}^{(t)}, c_{0j}^{(t)}\ \forall j, c_{\sigma,1}^{(t)}\right\}$ are respectively given by (90) and (92). Moreover, since the term $\sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2$ in A_k^{SP} is a constant over each iteration, we let $b = 1$ in Lemma 11. According to *Lemma* 11, we have $A_k^{\mathrm{SP}} = \tilde{A}_k^{\mathrm{SP}(t)}$ when $p_{0k}^{(t)} = p_{0k}, q_{0k}^{(t)} = q_{0k}, \forall k$. Then, it follows that $\frac{\alpha_k}{\gamma_{0k}^{\mathrm{SP}}} \leq \tilde{\alpha}_k^{(t)} + M$ by substituting $A_k^{\mathrm{SP}} \leq \tilde{A}_k^{\mathrm{SP}(t)}$ into (167), where $\tilde{\alpha}_k^{(t)}$ is defined in *Lemma* 6.

Similarly, we can obtain a polynomial approximation of $\frac{\alpha_{ki}}{\gamma_{0k}^{\mathrm{SP}}}$ $(i \neq k)$ as follows. Firstly,

$$
\begin{aligned}
\frac{\alpha_{ki}}{\gamma_{0k}^{\mathrm{SP}}} &= \frac{1 + \frac{M p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{q_{0i}\beta_{00i}L_u}}{\gamma_{0k}^{\mathrm{SP}}\left(1 + \frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{M q_{0i}\beta_{00i}L_u}\right)} = \frac{1}{\gamma_{0k}^{\mathrm{SP}}}\left[1 + \frac{\frac{(M^2-1)p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{M q_{0i}\beta_{00i}L_u}}{1 + \frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SP}}}{M q_{0i}\beta_{00i}L_u}}\right] \\
&= \left(1 + \frac{\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2}{q_{0k}L_u\beta_{00k}}\right)\left[1 + \frac{\frac{(M^2-1)}{M}p_{0k}\beta_{00k}}{q_{0i}\beta_{00i}L_u + \frac{1}{M}p_{0k}\beta_{00k} + \sum\limits_{l'\in\Phi}\sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2}\right] \\
&= \left(1 + \frac{\sum\limits_{l'\in\Phi}\sum\limits_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2}{q_{0k}L_u\beta_{00k}}\right)\left[1 + A_{ki}^{\mathrm{SP}}\frac{(M^2-1)}{M}p_{0k}\beta_{00k}\right], i \neq k,
\end{aligned}
\tag{168}
$$

where $A_{ki}^{\mathrm{SP}} = \frac{1}{q_{0i}\beta_{00i}L_u + \frac{1}{M}p_{0k}\beta_{00k} + \sum\limits_{l'\in\Phi}\sum\limits_{j=1}^{K} p_{l'j}\beta_{0l'j} + \sigma^2}, i \neq k$. We can obtain a polynomial approximation on $\frac{\alpha_{ki}}{\gamma_{0k}^{\mathrm{SP}}}$ if we can approximate A_{ki}^{SP} by a monomial function. The corresponding A_{ki}^{SP} at the $(t+1)$-th iteration is given by $A_{ki}^{\mathrm{SP}(t)}$ in (96). Then, the best local monomial approximation on A_{ki}^{SP} can be obtained using the similar method of A_k^{SP} as $A_{ki}^{\mathrm{SP}} \leq \tilde{A}_{ki}^{\mathrm{SP}(t)}$, where $\tilde{A}_{ki}^{\mathrm{SP}(t)}$ is a monomial w.r.t. $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \forall k\right\}$ given by (89), and $\Upsilon_{A_i^{\mathrm{SP}}}$ is a constant given by (94). Similarly, we can obtain $A_{ki}^{\mathrm{SP}} = \tilde{A}_{ki}^{\mathrm{SP}(t)}$ when $p_{0k}^{(t)} = p_{0k}, q_{0k}^{(t)} = q_{0k}, \forall k$. It follows from (168) that $\frac{\alpha_{ki}}{\gamma_{0k}^{\mathrm{SP}}} \leq \tilde{\alpha}_{ki}^{(t)}$, where $\tilde{\alpha}_{ki}^{(t)}$ is defined in *Lemma* 6.

By substituting the upper bound approximations of $\frac{\alpha_k}{\gamma_{0k}^{\mathrm{SP}}}$ and $\frac{\alpha_{ki}}{\gamma_{0k}^{\mathrm{SP}}}$ above into (81), at the $(t+1)$-th iteration we can focus on the following constraint instead of the original constraint in (68c) as

$$
v_k^d \geq \frac{\lambda_{0k}\beta_{00k}\bar{M}}{\lambda_{0k}\beta_{00k}\left(\tilde{\alpha}_k^{(t)} + M - \bar{M}\right) + \sum\limits_{i=1,i\neq k}^{K}\tilde{\alpha}_{ki}^{(t)}\lambda_{0i}\beta_{00k} + \frac{1}{\gamma_{0k}^{\mathrm{SP}}}\left(\sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{0lk} + \sigma^2\right)}. \tag{169}
$$

By further simplifications, the constraint of (169) can be equivalently transformed to (86).

G. *Proof of Lemma 13*

First, we consider the approximation of the denominator of (82). Specifically, we approximate the terms $\frac{\rho_k}{\gamma_{0k}^{\text{SPe}}}$, $\frac{\rho_{ki}}{\gamma_{0k}^{\text{SPe}}}$ $i \neq k$, and $\frac{1}{\gamma_{0k}^{\text{SPe}}}$ by polynomials. According to *Lemma* 7, we have

$$\begin{aligned}
\frac{\rho_k}{\gamma_{0k}^{\text{SPe}}} &= \frac{1}{\gamma_{0k}^{\text{SPe}}} \frac{1 + M\gamma_{0k}^{\text{SPe}} + \frac{Mp_{0k}\gamma_{0k}^{\text{SPe}}\mu_k}{q_{0k}L_u}}{1 + \frac{\gamma_{0k}^{\text{SPe}}}{M} + \frac{p_{0k}\gamma_{0k}^{\text{SPe}}\mu_k}{Mq_{0k}L_u}} \lessapprox \frac{1}{\gamma_{0k}^{\text{SPe}}} \frac{1 + M\gamma_{0k}^{\text{SPe}} + \frac{Mp_{0k}\gamma_{0k}^{\text{SPe}}\mu_k}{q_{0k}L_u}}{1 + \frac{\gamma_{0k}^{\text{SPe}}}{M}} && (170)\\
&= \frac{1}{\gamma_{0k}^{\text{SPe}}} \left\{ M\gamma_{0k}^{\text{SPe}} + \frac{1}{1 + \frac{\gamma_{0k}^{\text{SPe}}}{M}} \left[1 - (\gamma_{0k}^{\text{SPe}})^2 + \frac{(M^2-1)p_{0k}\gamma_{0k}^{\text{SPe}}\mu_k}{Mq_{0k}L_u} \right] \right\}\\
&\lessapprox \frac{1}{\gamma_{0k}^{\text{SPe}}} \left\{ M\gamma_{0k}^{\text{SPe}} + 1 - (\gamma_{0k}^{\text{SPe}})^2 + \frac{(M^2-1)p_{0k}\gamma_{0k}^{\text{SPe}}\mu_k}{Mq_{0k}L_u} \right\} && (171)\\
&= M + \frac{1}{\gamma_{0k}^{\text{SPe}}} - \gamma_{0k}^{\text{SPe}} + \frac{(M^2-1)p_{0k}\mu_k}{Mq_{0k}L_u} && (172)\\
&= M + \frac{\frac{p_{0k}^2\gamma_{0k}^{\text{SP}\,2}}{q_{0k}L_u}\beta_{00k} + \sum_{i=1}^{K}(1-\gamma_{0i}^{\text{SP}})p_{0i}\beta_{00i} + \sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2}{q_{0k}L_u\beta_{00k}} + 1\\
&\quad - \frac{q_{0k}L_u\beta_{00k}}{q_{0k}L_u\beta_{00k} + \frac{p_{0k}^2\gamma_{0k}^{\text{SP}\,2}}{q_{0k}L_u}\beta_{00k} + \sum_{i=1}^{K}(1-\gamma_{0i}^{\text{SP}})p_{0i}\beta_{00i} + \sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2}\\
&\quad + \frac{(M^2-1)p_{0k}\mu_k}{Mq_{0k}L_u} && (173)\\
&= M + \left[\frac{p_{0k}^2\gamma_{0k}^{\text{SP}\,2}}{q_{0k}L_u}\beta_{00k} + \sum_{i=1}^{K}(1-\gamma_{0i}^{\text{SP}})p_{0i}\beta_{00i} + \sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2 \right] \left(\frac{1}{q_{0k}L_u\beta_{00k}} \right.\\
&\quad \left. + \frac{1}{q_{0k}L_u\beta_{00k} + \frac{p_{0k}^2\gamma_{0k}^{\text{SP}\,2}}{q_{0k}L_u}\beta_{00k} + \sum_{i=1}^{K}(1-\gamma_{0i}^{\text{SP}})p_{0i}\beta_{00i} + \sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2} \right)\\
&\quad + \frac{(M^2-1)p_{0k}\mu_k}{Mq_{0k}L_u}\\
&\lessapprox M + \left[\frac{p_{0k}^2\gamma_{0k}^{\text{SP}\,2}}{q_{0k}L_u}\beta_{00k} + \sum_{i=1}^{K}(1-\gamma_{0i}^{\text{SP}})p_{0i}\beta_{00i} + \sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2 \right] \left(\frac{1}{q_{0k}L_u\beta_{00k}} \right.\\
&\quad \left. + \frac{1}{q_{0k}L_u\beta_{00k} + \sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2} \right) + \frac{(M^2-1)p_{0k}\mu_k}{Mq_{0k}L_u} = M + \tilde{\rho}_k, && (174)
\end{aligned}$$

where $\tilde{\rho}_k$ is given by

$$\begin{aligned}
\tilde{\rho}_k &\triangleq \left[\frac{p_{0k}^2\gamma_{0k}^{\text{SP}\,2}}{q_{0k}L_u}\beta_{00k} + \sum_{i=1}^{K}(1-\gamma_{0i}^{\text{SP}})p_{0i}\beta_{00i} + \sum_{l'\in\Psi}\sum_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2 \right] \left(\frac{1}{q_{0k}L_u\beta_{00k}} + A_k^{\text{SPe}} \right)\\
&\quad + \frac{(M^2-1)p_{0k}\mu_k}{Mq_{0k}L_u} && (175)
\end{aligned}$$

with

$$\mu_k \triangleq (1-\gamma_{0k}^{\mathrm{SP}})^2 + \frac{p_{0k}(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}L_u} + \sum_{j=1}^{K}\frac{p_{0j}(\gamma_{0j}^{\mathrm{SP}})^2}{q_{0j}L_u}, A_k^{\mathrm{SPe}} \triangleq \frac{1}{q_{0k}L_u\beta_{00k} + \sum\limits_{l'\in\Psi}\sum\limits_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2}. \tag{176}$$

(170) follows from the fact that $1+\frac{\gamma_{0k}^{\mathrm{SPe}}}{M} \gg \frac{p_{0k}\gamma_{0k}^{\mathrm{SPe}}\mu_k}{Mq_{0k}L_u}$, since $ML_u \gg 1$, $\gamma_{0k}^{\mathrm{SPe}} < 1$ and μ_k is generally smaller than 1. (171) follows from $\frac{\gamma_{0k}^{\mathrm{SPe}}}{M} \ll 1$. (173) is obtained by using (39). (174) follows from the observation that $q_{0k}L_u\beta_{00k} + \sum\limits_{l'\in\Psi}\sum\limits_{i=1}^{K} p_{l'i}\beta_{0l'i} + \sigma^2 \gg \frac{p_{0k}^2\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k}L_u}\beta_{00k} + \sum\limits_{i=1}^{K}(1-\gamma_{0i}^{\mathrm{SP}})p_{0i}\beta_{00i}$, since L_u is large and $(1-\gamma_{0i}^{\mathrm{SP}}) \to 0$ as the channel estimation becomes accurate. We further approximate $\tilde{\rho}_k$ as a polynomial w.r.t. $\{p_{0k}, q_{0k}, \forall k\}$ later using the best local monomial approximation.

In addition, we can approximate $\frac{\rho_{ki}}{\gamma_{0k}^{\mathrm{SPe}}}$ in (82) as

$$\begin{aligned}
\frac{\rho_{ki}}{\gamma_{0k}^{\mathrm{SPe}}} &= \frac{1+\frac{Mp_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{q_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum\limits_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)}{\gamma_{0k}^{\mathrm{SPe}}\left[1+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{Mq_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum\limits_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)\right]} \\
&= \frac{1}{\gamma_{0k}^{\mathrm{SPe}}}\left[1+\frac{\frac{(M^2-1)p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{Mq_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum\limits_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)}{1+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{Mq_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum\limits_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)}\right] \\
&\gtrapprox \frac{1}{\gamma_{0k}^{\mathrm{SPe}}}\left[1+\frac{(M^2-1)p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{Mq_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)\right] && (177) \\
&= \frac{1}{\gamma_{0k}^{\mathrm{SPe}}}\left[1+\frac{\frac{M^2-1}{M}p_{0k}\beta_{00k}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum\limits_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)}{q_{0i}L_u\beta_{00i} + \frac{p_{0i}^2\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\beta_{00i} + \sum\limits_{i=1}^{K}(1-\gamma_{0j}^{\mathrm{SP}})p_{0j}\beta_{00j} + \sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2}\right] \\
&\gtrapprox \frac{1}{\gamma_{0k}^{\mathrm{SPe}}}\left[1+\frac{\frac{M^2-1}{M}p_{0k}\beta_{00k}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum\limits_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)}{q_{0i}L_u\beta_{00i} + \sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2}\right] && (178) \\
&= \frac{1}{\gamma_{0k}^{\mathrm{SPe}}}\left[1+\frac{M^2-1}{M}A_i^{\mathrm{SPe}}p_{0k}\beta_{00k}\left((1-\gamma_{0k}^{\mathrm{SP}})^2 + \sum_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right)\right] \triangleq \tilde{\rho}_{ki}, i\neq k, && (179)
\end{aligned}$$

where $A_i^{\mathrm{SPe}} \triangleq \frac{1}{q_{0i}L_u\beta_{00i} + \sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2}, i \neq k$. (177) follows from the fact that $1+\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{Mq_{0i}\beta_{00i}L_u}\Big((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum\limits_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\Big) \gtrapprox 1$ because $\frac{p_{0k}\beta_{00k}\gamma_{0i}^{\mathrm{SPe}}}{Mq_{0i}\beta_{00i}L_u}\left((1-\gamma_{0k}^{\mathrm{SP}})^2+\sum\limits_{i=1}^{K}\frac{p_{0i}\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\right) \ll 1$ with $ML_u \gg 1$. (178) follows from the fact that $q_{0i}L_u\beta_{00i}+\sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K}p_{l'j}\beta_{0l'j}+\sigma^2 \gg \frac{p_{0i}^2\gamma_{0i}^{\mathrm{SP}\,2}}{q_{0i}L_u}\beta_{00i}+\sum\limits_{j=1}^{K}(1-\gamma_{0j}^{\mathrm{SP}})p_{0j}\beta_{00j}, \forall i$, since L_u is large and $(1-\gamma_{0j}^{\mathrm{SP}}) \to 0$ as the channel estimation becomes accurate. *We notice that the expressions of A_k^{SPe} and $A_i^{\mathrm{SPe}}, i \neq k$ are consistent.* Then, using (174) and (179), $\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}}$ in

(82) can be lower approximated as

$$\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}} \gtrapprox \frac{\lambda_{0k}\beta_{00k}\bar{M}}{\lambda_{0k}\beta_{00k}\left(\tilde{\rho}_k + M - \bar{M}\right) + \sum\limits_{i=1,i\neq k}^{K}\tilde{\rho}_{ki}\lambda_{0i}\beta_{00k} + \frac{1}{\gamma_{0k}^{\mathrm{SPe}}}\left(\sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{l0k} + \sigma^2\right)}. \quad (180)$$

In the following, we further develop an approximation on the term $\frac{1}{\gamma_{0k}^{\mathrm{SPe}}}$ in (180). According to (12), we have

$$\gamma_{0i}^{\mathrm{SP}} = \frac{q_{0i}L_u\beta_{00i}}{q_{0i}L_u\beta_{00i} + \sum\limits_{j=1}^{K}p_{0j}\beta_{00j} + \sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2} = B_i q_{0i}L_u\beta_{00i}, \quad (181)$$

$$1-\gamma_{0i}^{\mathrm{SP}} = \frac{\sum\limits_{l'\in\Phi}\sum\limits_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2}{q_{0i}L_u\beta_{00i} + \sum\limits_{j=1}^{K}p_{0j}\beta_{00j} + \sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2} = B_i\left(\sum_{l'\in\Phi}\sum_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2\right), \quad (182)$$

where $B_i \triangleq \frac{1}{q_{0i}L_u\beta_{00i} + \sum\limits_{j=1}^{K}p_{0j}\beta_{00j} + \sum\limits_{l'\in\Psi}\sum\limits_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2}$. Then, we can obtain a polynomial approximation on $\frac{\rho_k}{\gamma_{0k}^{\mathrm{SPe}}}$, $\frac{\rho_{ki}}{\gamma_{0k}^{\mathrm{SPe}}}$ $i \neq k$, and $\frac{1}{\gamma_{0k}^{\mathrm{SPe}}}$ if we can approximate A_i^{SPe} and B_i $\forall i$ by monomial functions. Similar to the derivations of *Lemma* 12, A_i^{SPe} and B_i at the $(t+1)$-th iteration are respectively given by (108) and (109). Then, the best local monomial approximations on A_i^{SPe} and B_i at the $(t+1)$-th iteration can be obtained using *Lemma* 11 as

$$A_i^{\mathrm{SPe}} \le \tilde{A}_i^{\mathrm{SPe}(t)};\ B_i \le \tilde{B}_i^{(t)},\ \forall i, \quad (183)$$

where $\tilde{A}_i^{\mathrm{SPe}(t)}$ and $\tilde{B}_i^{\mathrm{SP}(t)}$ are monomials w.r.t. $\left\{p_{0k}^{(t)}, q_{0k}^{(t)}, \forall k\right\}$ given by (102) and (103), and $\Upsilon_{A_i^{\mathrm{SPe}}}$ and Υ_{B_i} are constants given by (106) and (107). The constants $e_{0i,1}^{(t)}, e_{0i,2}^{(t)}$, $e_{0j}^{(t)}$, $e_{\sigma,1}^{(t)}, e_{\sigma,2}^{(t)}$ at the $(t+1)$-th iteration are respectively given by (104) and (105). Moreover, we have $A_i^{\mathrm{SPe}} = \tilde{A}_i^{\mathrm{SPe}(t)}$ and $B_i = \tilde{B}_i$ when $p_{0i}^{(t)} = p_{0i}, q_{0i}^{(t)} = q_{0i}, \forall i$. According to (181), (182) and (103), we can obtain

$$\gamma_{0i}^{\mathrm{SP}} \le \tilde{B}_i^{(t)}q_{0i}L_u\beta_{00i},\ 1-\gamma_{0i}^{\mathrm{SP}} \le \tilde{B}_i^{(t)}\left(\sum_{l'\in\Phi}\sum_{j=1}^{K}p_{l'j}\beta_{0l'j} + \sigma^2\right), \quad (184)$$

where the RHSs of the two inequalities in (184) are monomials w.r.t $\{p_{0i}, q_{0i}, \forall i\}$. Then, μ_k in (176) can be bounded by substituting (184) into (176) as $\mu_k \le \mu_k^{(t)}$, where $\mu_k^{(t)}$ is given by (100). Moreover, an approximation on $\frac{1}{\gamma_{0k}^{\mathrm{SPe}}}$ can be obtained by substituting (184) into (39) as

$$\frac{1}{\gamma_{0k}^{\mathrm{SPe}}} = 1 + \frac{\frac{p_{0k}^2\gamma_{0k}^{\mathrm{SP}\,2}}{q_{0k}L_u}\beta_{00k} + \sum\limits_{i=1}^{K}(1-\gamma_{0i}^{\mathrm{SP}})p_{0i}\beta_{00i} + \sum\limits_{l'\in\Psi}\sum\limits_{i=1}^{K}p_{l'i}\beta_{0l'i} + \sigma^2}{q_{0k}L_u\beta_{00k}} \le \varrho_k^{(t)}, \quad (185)$$

where $\varrho_k^{(t)}$ in (185) is a polynomial w.r.t $\{p_{0i}, q_{0i}, \forall i\}$ given by (101).

By substituting (176), (183), (184) and (102) into (175), a polynomial approximation on $\tilde{\rho}_k$ at the $(t+1)$-th iteration is obtained as

$$\tilde{\rho}_k \quad \leq \quad \tilde{\rho}_k^{(t)}, \tag{186}$$

where $\tilde{\rho}_k^{(t)}$ is given by (98). Similarly, by substituting (183), (184) and (102) into (179), a polynomial approximation on $\tilde{\rho}_{ki}$ for $i \neq k$ at the $(t+1)$-th iteration can be obtained as

$$\tilde{\rho}_{ki} \leq \tilde{\rho}_{ki}^{(t)},\ i \neq k, \tag{187}$$

where $\tilde{\rho}_{ki}^{(t)}$ is given by (99).

Finally, substituting the approximations on $\frac{1}{\gamma_{0k}^{\mathrm{SPe}}}$, $\frac{\rho_k}{\gamma_{0k}^{\mathrm{SPe}}}$ and $\frac{\rho_{ki}}{\gamma_{0k}^{\mathrm{SPe}}}$ respectively given by (185), (186) and (187) into (180), an approximation on $\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}}$ in (82) can be obtained as

$$\mathrm{SINR}_{0k}^{\mathrm{SPe,DL}} \geq \frac{\lambda_{0k}\beta_{00k}\bar{M}}{\lambda_{0k}\beta_{00k}\left(\tilde{\rho}_k^{(t)} + M - \bar{M}\right) + \sum\limits_{i=1,i\neq k}^{K} \tilde{\rho}_{ki}^{(t)}\lambda_{0i}\beta_{00k} + \varrho_k^{(t)}\left(\sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{l0k} + \sigma^2\right)} \tag{188}$$

According to (188), at the $(t+1)$-th iteration we can focus on the following constraint instead of the original constraint in (69b) as

$$\upsilon_k^d \geq \frac{\lambda_{0k}\beta_{00k}\bar{M}}{\lambda_{0k}\beta_{00k}\left(\tilde{\rho}_k^{(t)} + M - \bar{M}\right) + \sum\limits_{i=1,i\neq k}^{K} \tilde{\rho}_{ki}^{(t)}\lambda_{0i}\beta_{00k} + \varrho_k^{(t)}\left(\sum\limits_{l\in\Psi}\sum\limits_{i=1}^{K}\lambda_{li}\beta_{0lk} + \sigma^2\right)}. \tag{189}$$

By further simplifications, the constraint (189) can be equivalently transformed to (97).

H. Derivation of $\mathbb{E}\left\{|n_{kj}|^2\right\}$.

Let $D_1 \triangleq \frac{\gamma_{0k}^{\mathrm{SP}} p_{0k}}{L_u\sqrt{q_{0k}}}\,\mathbf{h}_{00k}^H\,\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j$, $D_2 \triangleq \sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)$ $\left(\hat{\mathbf{h}}_{00k}^H\mathbf{h}_{0l'i}\right)$ and $D_3 \triangleq \hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j$. It follows that

$$\begin{aligned}
&\mathbb{E}\left\{|n_e|^2\right\} = \mathbb{E}\left\{|D_1|^2 + |D_2|^2 + |D_3|^2\right\} + 2\Re\left[\mathbb{E}\left\{D_2^*D_1 + D_3^*D_1 + D_2^*D_3\right\}\right] \\
= &\left(\frac{\gamma_{0k}^{\mathrm{SP}}p_{0k}}{L_u\sqrt{q_{0k}}}\right)^2 \underbrace{\mathbb{E}\left\{\left|\,\mathbf{h}_{00k}^H\,\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right|^2\right\}}_{D_{1,1}} \\
&+ \underbrace{\mathbb{E}\left\{\left|\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\hat{\mathbf{h}}_{00k}^H\,\mathbf{h}_{0l'i}\right|^2\right\}}_{D_{2,2}} + \underbrace{\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j\right|^2\right\}}_{D_{3,3}} \\
&+ \frac{2\gamma_{0k}^{\mathrm{SP}}p_{0k}}{L_u\sqrt{q_{0k}}}\underbrace{\Re\left[\mathbb{E}\left\{\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^*]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}^*]_j\right)\mathbf{h}_{0l'i}^H\hat{\mathbf{h}}_{00k}\mathbf{h}_{00k}^H\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]}_{D_{2,1}} \\
&+ \frac{2\gamma_{0k}^{\mathrm{SP}}p_{0k}}{L_u\sqrt{q_{0k}}}\underbrace{\Re\left[\mathbb{E}\left\{[\mathbf{N}_0]_j^H\hat{\mathbf{h}}_{00k}\mathbf{h}_{00k}^H\mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]}_{D_{3,1}}
\end{aligned} \tag{190}$$

$$+2\,\Re\underbrace{\left[\mathbb{E}\left\{\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^{*}]_j+\sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}^{*}]_j\right)\ \mathbf{h}_{0l'i}^{H}\hat{\mathbf{h}}_{00k}\hat{\mathbf{h}}_{00k}^{H}[\mathbf{N}_0]_j\right\}\right]}_{D_{2,3}}. \tag{191}$$

We derive each of the terms in (190) as follows. As for $D_{1,1}$, we can easily obtain $D_{1,1}=M(M+1)(L_u+1)\beta_{00k}^2$ by using

$$\mathbb{E}\left\{\mathbf{h}_{0l'i}^{H}\mathbf{h}_{0l'i}\mathbf{h}_{0l'i}^{H}\mathbf{h}_{0l'i}\right\} = M(M+1){\beta_{0l'i}}^2, \tag{192}$$

$$\mathbb{E}\left\{\boldsymbol{\varphi}_{0k}^{T}\mathbf{s}_{l'i}^{*}\mathbf{s}_{l'i}^{T}\boldsymbol{\varphi}_{0k}^{*}[\mathbf{s}_{l'i}]_j[\mathbf{s}_{l'i}^{*}]_j\right\} = L_u+1. \tag{193}$$

For $D_{2,2}$, we can obtain that

$$\begin{aligned}
D_{2,2} &= \mathbb{E}\left\{\left[\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j+\sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\,\mathbf{h}_{0l'i}\right]\right.\\
&\qquad\left.\times\left[\sum_{\hat{l}'\in\Phi}\sum_{\hat{i}=1}^{K}\left(\zeta_{\hat{l}'\hat{i}}\sqrt{p_{\hat{l}'\hat{i}}}[\mathbf{s}_{\hat{l}'\hat{i}}]_j+\sqrt{q_{\hat{l}'\hat{i}}}[\boldsymbol{\varphi}_{\hat{l}'\hat{i}}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\mathbf{h}_{0\hat{l}'\hat{i}}\right]^{*}\right\}\\
&= \underbrace{\mathbb{E}\left\{\left[\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j+\sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\,\mathbf{h}_{0l'i}\right]\right.}_{D_{2,2,1}}\\
&\qquad\underbrace{\left.\times\left[\sum_{\substack{\hat{l}'\in\Phi\\(\hat{l}',\hat{i})\neq(l',i)}}\sum_{\hat{i}=1}^{K}\left(\zeta_{\hat{l}'\hat{i}}\sqrt{p_{\hat{l}'\hat{i}}}[\mathbf{s}_{\hat{l}'\hat{i}}]_j+\sqrt{q_{\hat{l}'\hat{i}}}[\boldsymbol{\varphi}_{\hat{l}'\hat{i}}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\mathbf{h}_{0\hat{l}'\hat{i}}\right]^{*}\right\}}_{D_{2,2,1}}\\
&\quad+\underbrace{\mathbb{E}\left\{\sum_{\substack{l'\in\Phi\\(l',i)\neq(0,k)}}\sum_{i=1}^{K}\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j+\sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\,\mathbf{h}_{0l'i}\right.}_{D_{2,2,2}}\\
&\qquad\underbrace{\left.\times\left[\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j+\sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\mathbf{h}_{0l'i}\right]^{*}\right\}}_{D_{2,2,2}}\\
&\qquad+\underbrace{\mathbb{E}\left\{\left[\left(\sqrt{q_{0k}}[\boldsymbol{\varphi}_{0k}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\mathbf{h}_{00k}\right]\left[\left(\sqrt{q_{0k}}[\boldsymbol{\varphi}_{0k}]_j\right)\hat{\mathbf{h}}_{00k}^{H}\mathbf{h}_{00k}\right]^{*}\right\}}_{D_{2,2,3}},
\end{aligned} \tag{194}$$

where $D_{2,2,1}$ is obtained as

$$\begin{aligned}
D_{2,2,1} &= \mathbb{E}\Bigg\{ \sum_{l'\in\Phi}\sum_{i=1}^{K}\sum_{\substack{\hat{l}'\in\Phi \\ (\hat{l}',\hat{i})\neq(l',i)}}\sum_{\hat{i}=1}^{K} \left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\left(\zeta_{\hat{l}'\hat{i}}\sqrt{p_{\hat{l}'\hat{i}}}[\mathbf{s}^*_{\hat{l}'\hat{i}}]_j + \sqrt{q_{\hat{l}'\hat{i}}}[\boldsymbol{\varphi}^*_{\hat{l}'\hat{i}}]_j\right) \\
&\qquad \times \hat{\mathbf{h}}^H_{00k}\mathbf{h}_{0l'i}\mathbf{h}^H_{0\hat{l}'\hat{i}}\hat{\mathbf{h}}_{00k} \Bigg\} \\
&= \mathbb{E}\Bigg\{ \sum_{l'\in\Phi}\sum_{i=1}^{K}\sum_{\substack{\hat{l}'\in\Phi \\ (\hat{l}',\hat{i})\neq(l',i)}}\sum_{\hat{i}=1}^{K} \left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right)\left(\zeta_{\hat{l}'\hat{i}}\sqrt{p_{\hat{l}'\hat{i}}}[\mathbf{s}^*_{\hat{l}'\hat{i}}]_j + \sqrt{q_{\hat{l}'\hat{i}}}[\boldsymbol{\varphi}^*_{\hat{l}'\hat{i}}]_j\right) \\
&\quad \times \left(\gamma^{\mathrm{SP}}_{0k}\,\mathbf{h}^H_{00k} + \frac{\gamma^{\mathrm{SP}}_{0k}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}}\mathbf{s}^*_{l'i}\boldsymbol{\varphi}^T_{0k}\,\mathbf{h}^H_{0l'i}\right)\mathbf{h}_{0l'i}\,\mathbf{h}^H_{0\hat{l}'\hat{i}}\left(\gamma^{\mathrm{SP}}_{0k}\,\mathbf{h}_{00k} + \frac{\gamma^{\mathrm{SP}}_{0k}}{L_u}\sqrt{\frac{p_{\hat{l}'\hat{i}}}{q_{0k}}}\,\mathbf{h}_{0\hat{l}'\hat{i}}\mathbf{s}^T_{\hat{l}'\hat{i}}\boldsymbol{\varphi}^*_{0k}\right) \Bigg\} \\
&= \frac{(\gamma^{\mathrm{SP}}_{0k})^2}{{L_u}^2 q_{0k}}\mathbb{E}\Bigg\{ \sum_{l'\in\Phi}\sum_{i=1}^{K}\sum_{\substack{\hat{l}'\in\Phi \\ (\hat{l}',\hat{i})\neq(l',i)}}\sum_{\hat{i}=1}^{K} \zeta_{l'i}p_{l'i}[\mathbf{s}_{l'i}]_j\mathbf{s}^*_{l'i}\boldsymbol{\varphi}^T_{0k}\,\mathbf{h}^H_{0l'i}\mathbf{h}_{0l'i}\,\mathbf{h}^H_{0\hat{l}'\hat{i}}\,\mathbf{h}_{0\hat{l}'\hat{i}}\zeta_{\hat{l}'\hat{i}}p_{\hat{l}'\hat{i}}[\mathbf{s}^*_{\hat{l}'\hat{i}}]_j\mathbf{s}^T_{\hat{l}'\hat{i}}\boldsymbol{\varphi}^*_{0k} \Bigg\} \\
&\qquad + \frac{(\gamma^{\mathrm{SP}}_{0k})^2}{L_u}\mathbb{E}\Bigg\{ \sum_{\hat{l}'\in\Phi}\sum_{\hat{i}=1}^{K}\zeta_{\hat{l}'\hat{i}}p_{\hat{l}'\hat{i}}\,\mathbf{h}^H_{00k}\mathbf{h}_{00k}\,\mathbf{h}^H_{0\hat{l}'\hat{i}}\,\mathbf{h}_{0\hat{l}'\hat{i}}[\mathbf{s}^*_{\hat{l}'\hat{i}}]_j\mathbf{s}^T_{\hat{l}'\hat{i}}\boldsymbol{\varphi}^*_{0k}[\boldsymbol{\varphi}_{0k}]_j \Bigg\} \\
&\quad + \frac{(\gamma^{\mathrm{SP}}_{0k})^2}{L_u}\mathbb{E}\Bigg\{ \sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\,\mathbf{h}^H_{0l'i}\mathbf{h}_{0l'i}\,\mathbf{h}^H_{00k}\,\mathbf{h}_{00k}[\mathbf{s}^T_{l'i}]_j\mathbf{s}^*_{l'i}\boldsymbol{\varphi}^T_{0k}[\boldsymbol{\varphi}^*_{0k}]_j \Bigg\} \\
&= \frac{(\gamma^{\mathrm{SP}}_{0k})^2}{{L_u}^2 q_{0k}}\sum_{l'\in\Phi}\sum_{i=1}^{K}\sum_{\substack{\hat{l}'\in\Phi \\ (\hat{l}',\hat{i})\neq(l',i)}}\sum_{\hat{i}=1}^{K} M^2\zeta_{l'i}\zeta_{\hat{l}'\hat{i}}p_{l'i}p_{\hat{l}'\hat{i}}\beta_{0l'i}\beta_{0\hat{l}'\hat{i}} + \frac{(\gamma^{\mathrm{SP}}_{0k})^2}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}M^2\zeta_{l'i}p_{l'i}\beta_{0li}\beta_{00k} \\
&\quad + \frac{(\gamma^{\mathrm{SP}}_{0k})^2}{L_u}\sum_{\hat{l}'\in\Phi}\sum_{\hat{i}=1}^{K}M^2\zeta_{\hat{l}'\hat{i}}p_{\hat{l}'\hat{i}}\beta_{0\hat{l}\hat{i}}\beta_{00k} \\
&= \frac{(M\gamma^{\mathrm{SP}}_{0k})^2}{q_{0k}}\left[\left(\frac{1}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i}\right)^2 - \frac{1}{{L_u}^2}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p^2_{l'i}\beta^2_{0l'i}\right] \\
&\quad + \frac{2(M\gamma^{\mathrm{SP}}_{0k})^2}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0li}\beta_{00k},
\end{aligned} \tag{195}$$

and $D_{2,2,2}$ is obtained as

$$
\begin{aligned}
D_{2,2,2} &= \mathbb{E}\Bigg\{ \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} \left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right) \hat{\mathbf{h}}_{00k}^H \, \mathbf{h}_{0l'i} \\
&\quad \times \left[\left(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}]_j\right) \hat{\mathbf{h}}_{00k}^H \, \mathbf{h}_{0l'i}\right]^H \Bigg\} \\
&= \mathbb{E}\Bigg\{ \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} \Big(\zeta_{l'i} p_{l'i}[\mathbf{s}_{l'i}]_j[\mathbf{s}_{l'i}^*]_j + q_{l'i}[\boldsymbol{\varphi}_{l'i}]_j[\boldsymbol{\varphi}_{l'i}^*]_j + \zeta_{l'i}\sqrt{p_{l'i}q_{l'i}}[\mathbf{s}_{l'i}]_j[\boldsymbol{\varphi}_{l'i}^*]_j \\
&\quad + \zeta_{l'i}\sqrt{p_{l'i}q_{l'i}}[\mathbf{s}_{l'i}^*]_j[\boldsymbol{\varphi}_{l'i}]_j\Big) \hat{\mathbf{h}}_{00k}^H \, \mathbf{h}_{0l'i} \, \mathbf{h}_{0l'i}^H \hat{\mathbf{h}}_{00k} \Bigg\} \\
&= \mathbb{E}\Bigg\{ \sum_{\substack{l'\in\Phi \\ l'i\neq 0k}} \sum_{i=1}^{K} \Big(\zeta_{l'i} p_{l'i}[\mathbf{s}_{l'i}]_j[\mathbf{s}_{l'i}^*]_j + q_{l'i}[\boldsymbol{\varphi}_{l'i}]_j[\boldsymbol{\varphi}_{l'i}^*]_j + \zeta_{l'i}\sqrt{p_{l'i}q_{l'i}}[\mathbf{s}_{l'i}]_j[\boldsymbol{\varphi}_{l'i}^*]_j \\
&\quad + \zeta_{l'i}\sqrt{p_{l'i}q_{l'i}}[\mathbf{s}_{l'i}^*]_j[\boldsymbol{\varphi}_{l'i}]_j\Big) \left[\left(\hat{\mathbf{h}}_{00k}^H - \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}} \, \mathbf{h}_{0l'i}^H \boldsymbol{\varphi}_{0k}^T \mathbf{s}_{l'i}^*\right) + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}} \, \mathbf{h}_{0l'i}^H \boldsymbol{\varphi}_{0k}^T \mathbf{s}_{l'i}^*\right] \\
&\quad \times \mathbf{h}_{0l'i} \, \mathbf{h}_{0l'i}^H \left[\left(\hat{\mathbf{h}}_{00k} - \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}} \, \mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T \boldsymbol{\varphi}_{0k}^*\right) + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}} \, \mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T \boldsymbol{\varphi}_{0k}^*\right] \Bigg\} \\
&= \mathbb{E}\Bigg\{ \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} \left(\zeta_{l'i} p_{l'i}[\mathbf{s}_{l'i}]_j[\mathbf{s}_{l'i}^*]_j + q_{l'i}[\boldsymbol{\varphi}_{l'i}]_j[\boldsymbol{\varphi}_{l'i}^*]_j\right) \left(\hat{\mathbf{h}}_{00k}^H - \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}} \, \mathbf{h}_{0l'i}^H \boldsymbol{\varphi}_{0k}^T \mathbf{s}_{l'i}^*\right) \\
&\quad \times \mathbf{h}_{0l'i} \, \mathbf{h}_{0l'i}^H \left(\hat{\mathbf{h}}_{00k} - \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}} \, \mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T \boldsymbol{\varphi}_{0k}^*\right) \Bigg\} \\
&\quad + \mathbb{E}\Bigg\{ \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} \left(\zeta_{l'i} p_{l'i}[\mathbf{s}_{l'i}]_j[\mathbf{s}_{l'i}^*]_j + q_{l'i}[\boldsymbol{\varphi}_{l'i}]_j[\boldsymbol{\varphi}_{l'i}^*]_j\right) \\
&\quad \times \left(\frac{(\gamma_{0k}^{\mathrm{SP}})^2 p_{l'i}}{{L_u}^2 q_{0k}} \boldsymbol{\varphi}_{0k}^T \mathbf{s}_{l'i}^* \mathbf{s}_{l'i}^T \boldsymbol{\varphi}_{0k}^* \, \mathbf{h}_{0l'i}^H \, \mathbf{h}_{0l'i} \, \mathbf{h}_{0l'i}^H \, \mathbf{h}_{0l'i}\right) \Bigg\} \\
&= M\gamma_{0k}^{\mathrm{SP}}\beta_{00k} \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} \Big(\zeta_{l'i}p_{l'i} + q_{l'i}\Big)\beta_{0l'i} - \frac{M(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}L_u} \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} p_{l'i}\Big(\zeta_{l'i}p_{l'i} + q_{l'i}\Big){\beta_{0l'i}}^2 \\
&\quad + \frac{M(M+1)(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}{L_u}^2} \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} p_{l'i}\Big(\zeta_{l'i}p_{l'i}(L_u+1) + q_{l'i}L_u\Big){\beta_{0l'i}}^2 \\
&= M\gamma_{0k}^{\mathrm{SP}}\beta_{00k} \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} \Big(\zeta_{l'i}p_{l'i} + q_{l'i}\Big)\beta_{0l'i} + \frac{M(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}{L_u}^2} \sum_{\substack{l'\in\Phi \\ l'i\neq 0k}} \sum_{i=1}^{K} \zeta_{l'i}p_{l'i}^2{\beta_{0l'i}}^2 \\
&\quad + \frac{M^2(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}{L_u}^2} \sum_{\substack{l'\in\Phi \\ (\hat{l}',\hat{i})\neq(0,k)}} \sum_{i=1}^{K} p_{l'i}\Big(\zeta_{l'i}p_{l'i}(L_u+1) + q_{l'i}L_u\Big){\beta_{0l'i}}^2, \qquad (196)
\end{aligned}
$$

where (196) follows by using (192) and (193). Finally, $D_{2,2,3}$ is obtained as

$$\begin{aligned}
D_{2,2,3} &= \mathbb{E}\left\{\left[(\sqrt{q_{0k}}[\boldsymbol{\varphi}_{0k}]_j)\,\hat{\mathbf{h}}_{00k}^H\,\mathbf{h}_{00k}\right]\left[(\sqrt{q_{0k}}[\boldsymbol{\varphi}_{0k}]_j)\,\hat{\mathbf{h}}_{00k}^H\,\mathbf{h}_{00k}\right]^H\right\} \\
&= \mathbb{E}\left\{{q_{0k}}^2[\boldsymbol{\varphi}_{0k}]_j[\boldsymbol{\varphi}_{0k}^*]_j\hat{\mathbf{h}}_{00k}^H\,\mathbf{h}_{00k}\,\mathbf{h}_{00k}^H\hat{\mathbf{h}}_{00k}\right\} \\
&= M\gamma_{0k}^{\mathrm{SP}}q_{0k}\beta_{00k}^2 + M^2(\gamma_{0k}^{\mathrm{SP}})^2\beta_{00k}^2\left(\frac{p_{0k}}{L_u}+q_{0k}\right). \qquad (197)
\end{aligned}$$

Substituting (195), (196) and (197) into (194), we can obtain

$$\begin{aligned}
D_{2,2} &= D_{2,2,1}+D_{2,2,2}+D_{2,2,3} \\
&= \frac{(M\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}}\left[\left(\frac{1}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i}\right)^2-\frac{1}{{L_u}^2}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}^2\beta_{0l'i}^2\right] \\
&\quad+\frac{2(M\gamma_{0k}^{\mathrm{SP}})^2}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0li}\beta_{00k} \\
&\quad+M\gamma_{0k}^{\mathrm{SP}}\beta_{00k}\sum_{\substack{l'\in\Phi\\ l'i\neq 0k}}\sum_{i=1}^{K}\Big(\zeta_{l'i}p_{l'i}+q_{l'i}\Big)\beta_{0l'i}+\frac{M(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}{L_u}^2}\sum_{\substack{l'\in\Phi\\ l'i\neq 0k}}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}^2{\beta_{0l'i}}^2 \\
&\quad+\frac{M^2(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}{L_u}^2}\sum_{\substack{l'\in\Phi\\ l'i\neq 0k}}\sum_{i=1}^{K}p_{l'i}\Big(\zeta_{l'i}p_{l'i}(L_u+1)+q_{l'i}L_u\Big){\beta_{0l'i}}^2 \\
&\quad+M\gamma_{0k}^{\mathrm{SP}}q_{0k}\beta_{00k}^2+M^2(\gamma_{0k}^{\mathrm{SP}})^2\beta_{00k}^2\left(\frac{p_{0k}}{L_u}+q_{0k}\right) \\
&= \frac{M^2{\gamma_{0k}^{\mathrm{SP}}}^2}{q_{0k}}\left(q_{0k}\beta_{00k}+\frac{1}{L_u}\sum_{l'\in\Phi_D}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i}\right)^2 \\
&\quad+\frac{M^2{\gamma_{0k}^{\mathrm{SP}}}^2}{q_{0k}L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\left(q_{l'i}+\zeta_{l'i}p_{l'i}\right)p_{l'i}\beta_{0l'i}^2+\frac{M{\gamma_{0k}^{\mathrm{SP}}}^2}{{L_u}^2q_{0k}}\sum_{l'\in\Phi_D}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}^2\beta_{0l'i}^2 \\
&\quad+M\beta_{00k}\gamma_{0k}^{\mathrm{SP}}\sum_{l'\in\Phi_D}\sum_{i=1}^{K}\left(q_{l'i}+\zeta_{l'i}p_{l'i}\right)\beta_{0l'i}. \qquad (198)
\end{aligned}$$

For $D_{3,3}$, we can obtain that

$$\begin{aligned}
&\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j\right|^2\right\}=\mathbb{E}\left\{\hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j([\mathbf{N}_0]_j)^H\hat{\mathbf{h}}_{00k}\right\} \\
=\ &\mathbb{E}\left\{\left(\hat{\mathbf{h}}_{00k}^H-\frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}L_u}[\boldsymbol{\varphi}_{0k}^T]_j([\mathbf{N}_0]_j)^H\right)[\mathbf{N}_0]_j([\mathbf{N}_0]_j)^H\left(\hat{\mathbf{h}}_{00k}-\frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}L_u}[\mathbf{N}_0]_j[\boldsymbol{\varphi}_{0k}^*]_j\right)\right\} \\
&+\mathbb{E}\left\{\frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}L_u}[\boldsymbol{\varphi}_{0k}^T]_j([\mathbf{N}_0]_j)^H[\mathbf{N}_0]_j([\mathbf{N}_0]_j)^H\frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}L_u}[\mathbf{N}_0]_j[\boldsymbol{\varphi}_{0k}^*]_j\right\} \\
=\ &\frac{M\sigma^2(\gamma_{0k}^{\mathrm{SP}})^2}{q_{0k}L_u}\left(q_{0k}\beta_{00k}L_u+\sum_{l'\in\Phi}\sum_{i=1}^{K}p_{l'i}\beta_{0l'i}+\frac{L_u-1}{L_u}\sigma^2\right)+\frac{M(M+1)\sigma^4{\gamma_{0k}^{\mathrm{SP}}}^2}{q_{0k}{L_u}^2} \\
=\ &M\sigma^2\beta_{00k}\gamma_{0k}^{\mathrm{SP}}+\frac{M^2\sigma^4{\gamma_{0k}^{\mathrm{SP}}}^2}{q_{0k}{L_u}^2}, \qquad (199)
\end{aligned}$$

where we note that $\left(\hat{\mathbf{h}}_{00k} - \frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}L_u}[\mathbf{N}_0]_j[\boldsymbol{\varphi}_{0k}^*]_j\right)$ and $[\mathbf{N}_0]_j$ are independent of each other.

For $D_{2,1}$, we can obtain that

$$\begin{aligned}
&\Re\left[\mathbb{E}\left\{\sum_{l'\in\Phi}\sum_{i=1}^{K}(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^*]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}^*]_j)\ \mathbf{h}_{0l'i}^H\ \hat{\mathbf{h}}_{00k}\ \mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]\\
=\ &\Re\left[\mathbb{E}\left\{\sum_{\substack{l'\in\Phi\\ l'i\neq 0k}}\sum_{i=1}^{K}(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^*]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}^*]_j)\ \mathbf{h}_{0l'i}^H\ \hat{\mathbf{h}}_{00k}\ \mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]\\
&+\Re\left[\mathbb{E}\left\{\sqrt{q_{0k}}[\boldsymbol{\varphi}_{0k}^*]_j\ \mathbf{h}_{00k}^H\ \hat{\mathbf{h}}_{00k}\ \mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]\\
=\ &\Re\left[\mathbb{E}\left\{\sum_{\substack{l'\in\Phi\\ l'i\neq 0k}}\sum_{i=1}^{K}\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^*]_j\ \mathbf{h}_{0l'i}^H\frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}}\ \mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T\boldsymbol{\varphi}_{0k}^*\ \mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]\\
&+\Re\left[\mathbb{E}\left\{\sqrt{q_{0k}}[\boldsymbol{\varphi}_{0k}^*]_j\ \mathbf{h}_{00k}^H\gamma_{0k}^{\mathrm{SP}}\ \mathbf{h}_{00k}\ \mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]\\
=\ &\frac{M^2\beta_{00k}\gamma_{0k}^{\mathrm{SP}}}{L_u\sqrt{q_{0k}}}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i} + M(M+1)\sqrt{q_{0k}}\gamma_{0k}^{\mathrm{SP}}\beta_{00k}^2\\
=\ &\frac{M\beta_{00k}\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}}\left[q_{0k}\beta_{00k} + M\left(q_{0k}\beta_{00k} + \frac{1}{L_u}\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i}\right)\right].
\end{aligned}\tag{200}$$

For $D_{3,1}$, we can obtain that

$$\begin{aligned}
D_{3,1} &= \Re\left[\mathbb{E}\left\{[\mathbf{N}_0]_j^H\hat{\mathbf{h}}_{00k}\ \mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right]\\
&= \Re\left[\mathbb{E}\left\{[\mathbf{N}_0]_j^H\frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}L_u}[\mathbf{N}_0]_j[\boldsymbol{\varphi}_{0k}^*]_j\mathbf{h}_{00k}^H\ \mathbf{h}_{00k}\boldsymbol{\varphi}_{0k}^T\mathbf{s}_{0k}^*[\mathbf{s}_{0k}]_j\right\}\right] = \frac{M^2\sigma^2\beta_{00k}^2\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}L_u}.
\end{aligned}\tag{201}$$

For $D_{2,3}$, we can obtain that

$$\begin{aligned}
D_{2,3} &= \Re\left[\mathbb{E}\left\{\sum_{l'\in\Phi}\sum_{i=1}^{K}(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^*]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}^*]_j)\ \mathbf{h}_{0l'i}^H\ \hat{\mathbf{h}}_{00k}\ \hat{\mathbf{h}}_{00k}^H[\mathbf{N}_0]_j\right\}\right]\\
&= \Re\left[\mathbb{E}\left\{\sum_{l'\in\Phi}\sum_{i=1}^{K}(\zeta_{l'i}\sqrt{p_{l'i}}[\mathbf{s}_{l'i}^*]_j + \sqrt{q_{l'i}}[\boldsymbol{\varphi}_{l'i}^*]_j)\ \mathbf{h}_{0l'i}^H\left(\gamma_{0k}^{\mathrm{SP}}\ \mathbf{h}_{00k} + \frac{\gamma_{0k}^{\mathrm{SP}}}{L_u}\sqrt{\frac{p_{l'i}}{q_{0k}}}\ \mathbf{h}_{0l'i}\mathbf{s}_{l'i}^T\boldsymbol{\varphi}_{0k}^*\right)\right.\right.\\
&\quad\left.\left.\times\frac{\gamma_{0k}^{\mathrm{SP}}}{\sqrt{q_{0k}}L_u}[\boldsymbol{\varphi}_{0k}^T]_j([\mathbf{N}_0]_j)^H[\mathbf{N}_0]_j\right\}\right]\\
&= \frac{M\sigma^2\left(\gamma_{0k}^{\mathrm{SP}}\right)^2}{q_{0k}L_u^2}\Re\left[\mathbb{E}\left\{\sum_{l'\in\Phi}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}[\mathbf{s}_{l'i}^*]_j\mathbf{s}_{l'i}^T\boldsymbol{\varphi}_{0k}^*[\boldsymbol{\varphi}_{0k}^T]_j\ \mathbf{h}_{0l'i}^H\ \mathbf{h}_{0l'i}\right\}\right]\\
&\quad+\frac{M\sigma^2(\gamma_{0k}^{\mathrm{SP}})^2}{\sqrt{q_{0k}}L_u}\Re\left[\mathbb{E}\left\{\sqrt{q_{0k}}[\boldsymbol{\varphi}_{0k}^*]_j\ \mathbf{h}_{00k}^H\ \hat{\mathbf{h}}_{00k}[\boldsymbol{\varphi}_{0k}^T]_j\right\}\right]\\
&= \frac{M^2\sigma^2\gamma_{0k}^{\mathrm{SP}\,2}}{L_u q_{0k}}\left(q_{0k}\beta_{00k} + \frac{1}{L_u}\sum_{l'\in\Phi_D}\sum_{i=1}^{K}\zeta_{l'i}p_{l'i}\beta_{0l'i}\right).
\end{aligned}\tag{202}$$

The final result follows by substituting the results in (198), (199), (200), (201), (202) and $D_{1,1} =$

$M(M+1)(L_u+1)\beta_{00k}^2$ into (190).

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