1 Equilibrium System

1.1 ART Model with capital

Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t - hc^a_{t-1}) - \chi n_t^{1+\eta}/(1+\eta)]$$

Budget Constraint:

$$c_t + b_t/(i_t s_t) = w_t n_t + \bar{r}^k k_{t-1} + b_{t-1}/\pi_t + d_t$$

Equilibrium system (11 equations):

$$c_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] y_t \tag{1}$$
$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{\imath}(\pi_t^{gap})^{\phi_\pi} (c_t/(\bar{g} c_{t-1}))^{\phi_c})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \tag{2}$$
$$i_t = \max\{1, i_t^*\} \tag{3}$$
$$\lambda_t = c_t - hc^a_{t-1} \tag{4}$$
$$w_t = \chi n_t^\eta \lambda_t \tag{5}$$
$$1 = \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap}))] \tag{6}$$
$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta w_t/z_t + \beta\varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \tag{7}$$
$$y_t = k_{t-1}^\alpha (z_t n_t)^{1-\alpha} \tag{8}$$
$$k_t = (1-\delta)k_{t-1} \tag{9}$$
$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \tag{10}$$
$$s_t = (1-\rho_s)\bar{s} + \rho_s s_{t-1} + \sigma_\upsilon \upsilon_t \tag{11}$$
$$z_t = g_t z_{t-1} \tag{12}$$

Variables: $\{c, i^*, i, \lambda, w, \pi^{gap}, y, n, k, g, s, z\}$

De-trended Equilibrium System (10 equations):

$$\tilde{c}_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] \tilde{y}_t \tag{1}$$
$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{\imath}(\pi_t^{gap})^{\phi_\pi} (g_t \tilde{c}_t/(\bar{g} \tilde{c}_{t-1})^{\phi_c})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \tag{2}$$
$$i_t = \max\{1, i_t^*\} \tag{3}$$
$$\tilde{\lambda}_t = \tilde{c}_t - h\tilde{c}_{t-1}/g_t \tag{4}$$
$$\tilde{w}_t = \chi n_t^\eta \tilde{\lambda}_t \tag{5}$$
$$1 = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap} g_{t+1}))] \tag{6}$$
$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta \tilde{w}_t + \beta\varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(\tilde{y}_{t+1}/\tilde{y}_t)] \tag{7}$$
$$\tilde{y}_t = (\tilde{k}_{t-1}/g_t)^\alpha n_t^{1-\alpha} \tag{8}$$
$$\tilde{k}_t = (1-\delta)(\tilde{k}_{t-1}/g_t) \tag{9}$$
$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \tag{10}$$
$$s_t = (1-\rho_s)s_t + \rho_s s_{t-1} + \sigma_\upsilon \upsilon_t \tag{11}$$

Variables: $\{\tilde{c}, i^*, i, \tilde{\lambda}, \tilde{w}, \pi^{gap}, \tilde{y}, n, k, g, s\}$

Log-linear Equilibrium System:

$$\hat{c}_t = \hat{y}_t \tag{1}$$
$$\hat{i}_t^n = \rho_i \hat{i}_{t-1}^n + (1-\rho_i)\phi_\pi \hat{\pi}_t + (1-\rho_i)\phi_c(\hat{g} + \hat{c} - \hat{c}_{t-1}) + \sigma_\nu \nu_t \tag{2}$$
$$\hat{i}_t = \hat{i}_t^n \tag{3}$$
$$(1 - h/g)\hat{\lambda}_t = \hat{c}_t + (h/g)(\hat{g} - \hat{c}_{t-1}) \tag{4}$$
$$\hat{w}_t = \eta \hat{n}_t + \hat{\lambda}_t \tag{5}$$
$$\hat{\lambda}_t + \hat{\imath}_t + s_t = E_t \hat{\lambda}_{t+1} + E_t \hat{\pi}_{t+1} \tag{6}$$
$$\varphi \hat{\pi}_t = (\theta - 1)\hat{w}_t + \beta \varphi E_t \hat{\pi}_{t+1} \tag{7}$$
$$\hat{k}_t((1-\delta)/\bar{g})[\hat{k}_{t-1} - (\bar{k}/\bar{g})\bar{g}_t] \tag{8}$$
$$\hat{y}_t/\bar{y} = \alpha(\bar{k}_{t-1}/\bar{k} - \hat{g}_t/\bar{g}) + (1-\alpha)\hat{n}_t/\bar{n} \tag{9}$$

1.2 ART MODEL PRE-CAPITAL

Preferences:

$$E_0 \textstyle\sum_{t=0}^{\infty} \beta^t [\log(c_t - h c_{t-1}^a) - \chi n_t^{1+\eta}/(1+\eta)]$$

Budget Constraint:

$$c_t + b_t/(i_t s_t) = w_t n_t + b_{t-1}/\pi_t + d_t$$

Equilibrium system (11 equations):

$$c_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] y_t \tag{1}$$
$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{\imath}(\pi_t^{gap})^{\phi_\pi}(c_t/(\bar{g}c_{t-1}))^{\phi_c})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \tag{2}$$
$$i_t = \max\{1, i_t^*\} \tag{3}$$
$$\lambda_t = c_t - h c_{t-1}^a \tag{4}$$
$$w_t = \chi n_t^\eta \lambda_t \tag{5}$$
$$1 = \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap}))] \tag{6}$$
$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta w_t/z_t + \beta\varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \tag{7}$$
$$y_t = z_t n_t \tag{8}$$
$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \tag{9}$$
$$s_t = (1-\rho_s)\bar{s} + \rho_s s_{t-1} + \sigma_\upsilon \upsilon_t \tag{10}$$
$$z_t = g_t z_{t-1} \tag{11}$$

Variables: $\{c, i^*, i, \lambda, w, \pi^{gap}, y, n, g, s, z\}$

De-trended Equilibrium System (10 equations):

$$\tilde{c}_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2]\tilde{y}_t \tag{1}$$

$$i_t^* = (i_{t-1}^*)^{\rho_i}(\bar{\imath}(\pi_t^{gap})^{\phi_\pi}(g_t\tilde{c}_t/(\bar{g}\tilde{c}_{t-1})^{\phi_c})^{1-\rho_i}\exp(\sigma_\nu\nu_t) \tag{2}$$

$$i_t = \max\{1, i_t^*\} \tag{3}$$

$$\tilde{\lambda}_t = \tilde{c}_t \ - h\tilde{c}_{t-1}/g_t \tag{4}$$

$$\tilde{w}_t = \chi n_t^\eta \tilde{\lambda}_t \tag{5}$$

$$1 = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap}g_{t+1}))] \tag{6}$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta\tilde{w}_t + \beta\varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(\tilde{y}_{t+1}/\tilde{y}_t)] \tag{7}$$

$$\tilde{y}_t = n_t \tag{8}$$

$$g_t = (1 - \rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon\varepsilon_t \tag{9}$$

$$s_t = (1 - \rho_s)s_t + \rho_s s_{t-1} + \sigma_\upsilon\upsilon_t \tag{10}$$

Variables: $\{\tilde{c}, i^*, i, \tilde{\lambda}, \tilde{w}, \pi^{gap}, \tilde{y}, n, g, s\}$

Log-linear Equilibrium System:

$$\hat{c}_t = \hat{y}_t \tag{1}$$

$$\hat{\imath}_t^n = \rho_i\hat{\imath}_{t-1}^n + (1 - \rho_i)\phi_\pi\hat{\pi}_t + (1 - \rho_i)\phi_c(\hat{g} + \hat{c} - \hat{c}_{t-1}) + \sigma_\nu\nu_t \tag{2}$$

$$\hat{\imath}_t = \hat{\imath}_t^n \tag{3}$$

$$(1 - h/g)\hat{\lambda}_t = \hat{c}_t + (h/g)(\hat{g} - \hat{c}_{t-1}) \tag{4}$$

$$\hat{w}_t = \eta\hat{n}_t + \hat{\lambda}_t \tag{5}$$

$$\hat{\lambda}_t + \hat{\imath}_t + s_t = E_t\hat{\lambda}_{t+1} + E_t\hat{\pi}_{t+1} \tag{6}$$

$$\varphi\hat{\pi}_t = (\theta - 1)\hat{w}_t + \beta\varphi E_t\hat{\pi}_{t+1} \tag{7}$$

$$\hat{y}_t = \hat{n}_t \tag{8}$$

1.3 Gust et al Model

Equilibrium system (13 equations):

$$\varphi(\pi_t^{gap}-1)\pi_t^{gap} = 1-\theta+\theta w_t/z_t + \beta\varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap}-1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \tag{1}$$
$$i_t^* = (i_{t-1}^*)^{\rho_i}(\bar{\imath}(\pi_t^{gap})^{\phi_\pi})^{1-\rho_i}\exp(\sigma_\nu \nu_t) \tag{2}$$
$$i_t = \max\{1, i_t^*\} \tag{3}$$
$$1/\lambda_t = \beta E_t[(1/\lambda_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap} g_{t+1}))] \tag{4}$$
$$\lambda_t = c_t \tag{5}$$
$$c_t = [1-\varphi(\pi_t^{gap}-1)^2/2]y_t \tag{6}$$
$$w_t = \chi n_t^\eta \lambda_t \tag{7}$$
$$y_t = z_t n_t \tag{8}$$
$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \tag{9}$$
$$s_t = (1-\rho_s)\bar{s} + \rho_s s_{t-1} + \sigma_\upsilon \upsilon_t \tag{10}$$
$$z_t = g_t z_{t-1} \tag{11}$$

Variables: $\{c, i^*, i, \lambda, w, \pi^{gap}, V_\lambda, y, V_\pi, n, g, s, z\}$

De-trended Equilibrium System (12 equations):

$$\varphi(\pi_t^{gap}-1)\pi_t^{gap} = 1-\theta+\theta\tilde{w}_t + \beta\varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap}-1)\pi_{t+1}^{gap}(\tilde{y}_{t+1}/\tilde{y}_t)] \tag{1}$$
$$i_t^* = (i_{t-1}^*)^{\rho_i}(\bar{\imath}(\pi_t^{gap})^{\phi_\pi})^{1-\rho_i}\exp(\sigma_\nu \nu_t) \tag{2}$$
$$i_t = \max\{1, i_t^*\} \tag{3}$$
$$1/\tilde{\lambda}_t = \beta E_t[(1/\tilde{\lambda}_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap} g_{t+1}))] \tag{4}$$
$$\tilde{\lambda}_t = \tilde{c}_t \tag{5}$$
$$\tilde{c}_t = [1-\varphi(\pi_t^{gap}-1)^2/2]\tilde{y}_t \tag{6}$$
$$\tilde{w}_t = \chi n_t^\eta \tilde{\lambda}_t \tag{7}$$
$$\tilde{y}_t = n_t \tag{8}$$
$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \tag{9}$$
$$s_t = (1-\rho_s)s_t + \rho_s s_{t-1} + \sigma_\upsilon \upsilon_t \tag{10}$$

Variables: $\{\tilde{c}, i^*, i, \tilde{\lambda}, \tilde{w}, \pi^{gap}, \tilde{V}_\lambda, \tilde{y}, \tilde{V}_\pi, n, g, s\}$

Log-linear Equilibrium System:

$$\varphi\hat{\pi}_t = (\theta-1)\hat{w}_t + \beta\varphi E_t\hat{\pi}_{t+1} \tag{1}$$
$$\hat{\imath}_t^n = \rho_i \hat{\imath}_{t-1}^n + (1-\rho_i)\phi_\pi\hat{\pi}_t + \sigma_\nu \nu_t \tag{2}$$
$$\hat{\imath}_t = \hat{\imath}_t^n \tag{3}$$
$$-\hat{\lambda}_t = \hat{\imath}_t + s_t - E_t\hat{\lambda}_{t+1} - E_t\hat{\pi}_{t+1} \tag{4}$$
$$\hat{\lambda}_t = \hat{c}_t \tag{5}$$
$$\hat{c}_t = \hat{y}_t \tag{6}$$
$$\hat{w}_t = \eta\hat{n}_t + \hat{\lambda}_t \tag{7}$$
$$\hat{y}_t = \hat{n}_t \tag{8}$$

Gust et al Indicator Functions

$$c_{t+1,1} = \beta E_t[\lambda_t(s_t i_t/(\bar{\pi}\pi^{gap}_{t+1} g_{t+1}))] \tag{1}$$
$$c_{t+1,2} = \beta E_t[\lambda_t(s_t/(\bar{\pi}\pi^{gap}_{t+1} g_{t+1}))] \tag{2}$$
$$\tag{3}$$

for $k = 1, 2$ where $k = 1$ corresponds to the non-ZLB regime and $k = 2$ corresponds to the ZLB regime.

$$c_t = c_{t,1} I + c_{t,2}(1 - I) \tag{4}$$

for $j = 1, 2$ and where I is defined by:

$$\begin{cases} I = 1 & \text{if } i > 1 \\ I = 0 & \text{otherwise.} \end{cases}$$