

1 EQUILIBRIUM SYSTEM

1.1 ART MODEL WITH CAPITAL Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t - hc_{t-1}^a) - \chi n_t^{1+\eta}/(1+\eta)]$$

Budget Constraint:

$$c_t + x_t + b_t/(i_t s_t) = w_t n_t + r_t^k k_{t-1} + b_{t-1}/\pi_t + d_t$$

Equilibrium system:

$$y_t = k_{t-1}^\alpha (z_t n_t)^{1-\alpha} \quad (1)$$

$$r_t^k = \alpha m c_t y_t / k_{t-1} \quad (2)$$

$$w_t = (1 - \alpha) m c_t y_t / n_t \quad (3)$$

$$y_t^{gdp} = [1 - \varphi(\pi_t^{gap} - 1)^2/2] y_t \quad (4)$$

$$y_t^g = y_t^{gdp} / (\bar{g} y_{t-1}^{gdp}) \quad (5)$$

$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{i}(\pi_t^{gap})^{\phi_\pi} (y_t^g)^{\phi_y})^{1-\rho_i} \exp(mp_t) \quad (6)$$

$$i_t = \max\{1, i_t^*\} \quad (7)$$

$$\lambda_t = c_t - hc_{t-1}^a \quad (8)$$

$$w_t = \chi n_t^\eta \lambda_t \quad (9)$$

$$c_t + x_t = y_t^{gdp} \quad (10)$$

$$x_t^g = x_t / (\bar{g} x_{t-1}) \quad (11)$$

$$k_t = (1 - \delta) k_{t-1} + x_t (1 - \nu(x_t^g - 1)^2/2) \quad (12)$$

$$1 = \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t / (\bar{\pi} \pi_{t+1}^{gap}))] \quad (13)$$

$$q_t = \beta E_t[(\lambda_t/\lambda_{t+1})(r_{t+1}^k + (1 - \delta) q_{t+1})] \quad (14)$$

$$1 = q_t [1 - (x_t^g - 1)^2/2 - (x_t^g - 1)x_t^g] + \beta \bar{g} E_t[q_{t+1}(\lambda_t/\lambda_{t+1})(x_{t+1}^g)^2(x_{t+1}^g - 1)] \quad (15)$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta m c_t + \beta \varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \quad (16)$$

$$g_t = \bar{g} + \sigma_g \varepsilon_{g,t} \quad (17)$$

$$s_t = (1 - \rho_s) \bar{s} + \rho_s s_{t-1} + \sigma_s \varepsilon_{s,t} \quad (18)$$

$$mp_t = \sigma_i \varepsilon_{i,t} \quad (19)$$

$$z_t = g_t z_{t-1} \quad (20)$$

Variables: $\{c, n, x, k, y^{gdp}, y, x^g, y^g, w, r^k, \pi, i, i^n, q, mc, \lambda, g, s, mp, z\}$

De-trended Equilibrium System:

$$\tilde{y}_t = (\tilde{k}_{t-1}/g_t)^\alpha n_t^{1-\alpha} \quad (1)$$

$$r_t^k = \alpha m c_t g_t \tilde{y}_t / \tilde{k}_{t-1} \quad (2)$$

$$\tilde{w}_t = (1 - \alpha) m c_t \tilde{y}_t / n_t \quad (3)$$

$$\tilde{y}_t^{gdp} = [1 - \varphi(\pi_t^{gap} - 1)^2/2] \tilde{y}_t \quad (4)$$

$$y_t^g = g_t \tilde{y}_t^{gdp} / (\bar{g} \tilde{y}_{t-1}^{gdp}) \quad (5)$$

$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{l}(\pi_t^{gap})^{\phi_\pi} (y_t^g)^{\phi_y})^{1-\rho_i} \exp(\sigma_i \varepsilon_{i,t}) \quad (6)$$

$$i_t = \max\{1, i_t^*\} \quad (7)$$

$$\tilde{\lambda}_t = \tilde{c}_t - h \tilde{c}_{t-1} / g_t \quad (8)$$

$$\tilde{w}_t = \chi n_t^\eta \tilde{\lambda}_t \quad (9)$$

$$\tilde{c}_t + \tilde{x}_t = \tilde{y}_t^{gdp} \quad (10)$$

$$x_t^g = g_t \tilde{x}_t / (\bar{g} \tilde{x}_{t-1}) \quad (11)$$

$$\tilde{k}_t = (1 - \delta)(\tilde{k}_{t-1}/g_t) + \tilde{x}_t(1 - \nu(x_t^g - 1)^2/2) \quad (12)$$

$$1 = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(s_t i_t / (\bar{\pi} \pi_{t+1}^{gap} g_{t+1}))] \quad (13)$$

$$q_t = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(r_{t+1}^k + (1 - \delta)q_{t+1})/g_{t+1}] \quad (14)$$

$$1 = q_t[1 - (x_t^g - 1)^2/2 - (x_t^g - 1)x_t^g] + \beta \bar{g} E_t[q_{t+1}(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(x_{t+1}^g)^2(x_{t+1}^g - 1)/g_{t+1}] \quad (15)$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta m c_t + \beta \varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(\tilde{y}_{t+1}/\tilde{y}_t)] \quad (16)$$

$$g_t = \bar{g} + \sigma_g \varepsilon_{g,t} \quad (17)$$

$$s_t = (1 - \rho_s)s_t + \rho_s s_{t-1} + \sigma_s \varepsilon_{s,t} \quad (18)$$

$$m p_t = \sigma_i \varepsilon_{i,t} \quad (19)$$

Variables: $\{\tilde{c}, \tilde{n}, \tilde{x}, \tilde{k}, \tilde{y}^{gdp}, \tilde{y}, x^g, y^g, \tilde{w}, r^k, \pi, i, i^n, q, m c, \tilde{\lambda}, g, s, m p\}$

Log-linear Equilibrium System:

$$\hat{y}_t/\bar{y} = \alpha(\hat{k}_{t-1}/\bar{k} - \hat{g}_t/\bar{g}) + (1 - \alpha)\hat{n}_t/\bar{n} \quad (1)$$

$$\hat{r}_t^k/\bar{r}^k = \hat{m}c_t/\bar{m}c + \hat{g}_t/\bar{g} + \hat{y}_t/\bar{y} - \hat{k}_{t-1}/\bar{k} \quad (2)$$

$$\hat{w}_t/\bar{w} = \hat{m}c_t/\bar{m}c + \hat{y}_t/\bar{y} - \hat{n}_t/\bar{n} \quad (3)$$

$$\hat{y}_t^{gdp} = \hat{y}_t \quad (4)$$

$$\hat{y}_t^g = \hat{g}_t/\bar{g} + \hat{y}_t^{gdp}/\bar{y}^{gdp} - \hat{y}_{t-1}^{gdp}/\bar{y}^{gdp} \quad (5)$$

$$\hat{i}_t/\bar{i} = \rho_i \hat{i}_{t-1}/\bar{i} + (1 - \rho_i)(\phi_\pi \hat{\pi}_t^{gap} + \phi_y \hat{y}_t^g) + \hat{m}p_t \quad (6)$$

$$\hat{i}_t = \hat{i}_t^n \quad (7)$$

$$\hat{\lambda}_t = \hat{c}_t - (h/\bar{g})\hat{c}_{t-1} + (h\bar{c}/\bar{g}^2)\hat{g}_t \quad (8)$$

$$\hat{c}_t + \hat{x}_t = \hat{y}_t^{gdp} \quad (9)$$

$$\hat{x}_t^g = \hat{g}_t/\bar{g} + \hat{x}_t/\bar{x} - \hat{x}_{t-1}/\bar{x} \quad (10)$$

$$\hat{k}_t = ((1 - \delta)/\bar{g})[\hat{k}_{t-1} - (\bar{k}/\bar{g})\hat{g}_t] + \hat{x}_t \quad (11)$$

$$\hat{\lambda}_t/\bar{\lambda} + \hat{i}_t/\bar{i} + \hat{s}_t/\bar{s} = E_t \hat{\lambda}_{t+1}/\bar{\lambda} + E_t \hat{g}_{t+1}/\bar{g} + E_t \hat{\pi}_{t+1}/\bar{\pi} \quad (12)$$

$$\hat{q}_t = \hat{\lambda}_t/\bar{\lambda} - E_t \hat{\lambda}_{t+1}/\bar{\lambda} + (\beta/\bar{g})[E_t \hat{r}_{t+1}^k + (1 - \delta)E_t \hat{q}_{t+1}] - E_t \hat{g}_{t+1}/\bar{g} \quad (13)$$

$$\hat{x}_t^g = \hat{q}_t + \beta E_t \hat{x}_{t+1}^g \quad (14)$$

$$\varphi \hat{\pi}_t^{gap} = \theta \hat{m}c_t + \beta \varphi E_t \hat{\pi}_{t+1}^{gap} \quad (15)$$

$$\hat{g}_t = \sigma_g \varepsilon_{g,t} \quad (16)$$

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + \sigma_s \varepsilon_{s,t} \quad (17)$$

$$\hat{m}p_t = \sigma_i \varepsilon_{i,t} \quad (18)$$

Variables: $\{\hat{c}, \hat{n}, \hat{x}, \hat{k}, \hat{y}^{gdp}, \hat{y}, \hat{x}^g, \hat{y}^g, \hat{w}, \hat{r}^k, \hat{\pi}, \hat{i}, \hat{i}^n, \hat{q}, \hat{\lambda}, \hat{g}, \hat{s}, \hat{m}p\}$