

# 1 EQUILIBRIUM SYSTEM

## 1.1 ART MODEL WITH CAPITAL Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t - hc_{t-1}^a) - \chi n_t^{1+\eta}/(1+\eta)]$$

Budget Constraint:

$$c_t + b_t/(i_t s_t) = w_t n_t + \bar{r}^k k_{t-1} + b_{t-1}/\pi_t + d_t$$

Equilibrium system (11 equations):

$$c_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] y_t \quad (1)$$

$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{i}(\pi_t^{gap})^{\phi_\pi} (c_t/(\bar{g}c_{t-1}))^{\phi_c})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \quad (2)$$

$$i_t = \max\{1, i_t^*\} \quad (3)$$

$$\lambda_t = c_t - hc_{t-1}^a \quad (4)$$

$$w_t = \chi n_t^\eta \lambda_t \quad (5)$$

$$1 = \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t/(\bar{\pi} \pi_{t+1}^{gap}))] \quad (6)$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta w_t/z_t + \beta \varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \quad (7)$$

$$y_t = k_{t-1}^\alpha (z_t n_t)^{1-\alpha} \quad (8)$$

$$k_t = (1 - \delta)k_{t-1} \quad (9)$$

$$g_t = (1 - \rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \quad (10)$$

$$s_t = (1 - \rho_s)\bar{s} + \rho_s s_{t-1} + \sigma_v v_t \quad (11)$$

$$z_t = g_t z_{t-1} \quad (12)$$

Variables:  $\{c, i^*, i, \lambda, w, \pi^{gap}, y, n, k, g, s, z\}$

De-trended Equilibrium System (10 equations):

$$\tilde{c}_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] \tilde{y}_t \quad (1)$$

$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{i}(\pi_t^{gap})^{\phi_\pi} (g_t \tilde{c}_t/(\bar{g} \tilde{c}_{t-1}))^{\phi_c})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \quad (2)$$

$$i_t = \max\{1, i_t^*\} \quad (3)$$

$$\tilde{\lambda}_t = \tilde{c}_t - h \tilde{c}_{t-1}/g_t \quad (4)$$

$$\tilde{w}_t = \chi n_t^\eta \tilde{\lambda}_t \quad (5)$$

$$1 = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(s_t i_t/(\bar{\pi} \pi_{t+1}^{gap} g_{t+1}))] \quad (6)$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta \tilde{w}_t + \beta \varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(\tilde{y}_{t+1}/\tilde{y}_t)] \quad (7)$$

$$\tilde{y}_t = (\tilde{k}_{t-1}/g_t)^\alpha n_t^{1-\alpha} \quad (8)$$

$$\tilde{k}_t = (1 - \delta)(\tilde{k}_{t-1}/g_t) \quad (9)$$

$$g_t = (1 - \rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \quad (10)$$

$$s_t = (1 - \rho_s)s_t + \rho_s s_{t-1} + \sigma_v v_t \quad (11)$$

Variables:  $\{\tilde{c}, i^*, i, \tilde{\lambda}, \tilde{w}, \pi^{gap}, \tilde{y}, n, k, g, s\}$

Log-linear Equilibrium System:

$$\hat{c}_t = \hat{y}_t \quad (1)$$

$$\hat{i}_t^n = \rho_i \hat{i}_{t-1}^n + (1 - \rho_i) \phi_\pi \hat{\pi}_t + (1 - \rho_i) \phi_c (\hat{g} + \hat{c} - \hat{c}_{t-1}) + \sigma_\nu \nu_t \quad (2)$$

$$\hat{i}_t = \hat{i}_t^n \quad (3)$$

$$(1 - h/g) \hat{\lambda}_t = \hat{c}_t + (h/g) (\hat{g} - \hat{c}_{t-1}) \quad (4)$$

$$\hat{w}_t = \eta \hat{n}_t + \hat{\lambda}_t \quad (5)$$

$$\hat{\lambda}_t + \hat{i}_t + s_t = E_t \hat{\lambda}_{t+1} + E_t \hat{\pi}_{t+1} \quad (6)$$

$$\varphi \hat{\pi}_t = (\theta - 1) \hat{w}_t + \beta \varphi E_t \hat{\pi}_{t+1} \quad (7)$$

$$\hat{k}_t ((1 - \delta)/\bar{g}) [\hat{k}_{t-1} - (\bar{k}/\bar{g}) \bar{g}_t] \quad (8)$$

$$\hat{y}_t/\bar{y} = \alpha (\bar{k}_{t-1}/\bar{k} - \hat{g}_t/\bar{g}) + (1 - \alpha) \hat{n}_t/\bar{n} \quad (9)$$

## 1.2 ART MODEL PRE-CAPITAL Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t - hc_{t-1}^a) - \chi n_t^{1+\eta}/(1+\eta)]$$

Budget Constraint:

$$c_t + b_t/(i_t s_t) = w_t n_t + b_{t-1}/\pi_t + d_t$$

Equilibrium system (11 equations):

$$c_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] y_t \quad (1)$$

$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{i}(\pi_t^{gap})^{\phi_\pi} (c_t/(\bar{g}c_{t-1}))^{\phi_c})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \quad (2)$$

$$i_t = \max\{1, i_t^*\} \quad (3)$$

$$\lambda_t = c_t - hc_{t-1}^a \quad (4)$$

$$w_t = \chi n_t^\eta \lambda_t \quad (5)$$

$$1 = \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t/(\bar{\pi} \pi_{t+1}^{gap}))] \quad (6)$$

$$\varphi(\pi_t^{gap} - 1) \pi_t^{gap} = 1 - \theta + \theta w_t/z_t + \beta \varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1) \pi_{t+1}^{gap} (y_{t+1}/y_t)] \quad (7)$$

$$y_t = z_t n_t \quad (8)$$

$$g_t = (1 - \rho_g) \bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \quad (9)$$

$$s_t = (1 - \rho_s) \bar{s} + \rho_s s_{t-1} + \sigma_v \nu_t \quad (10)$$

$$z_t = g_t z_{t-1} \quad (11)$$

Variables:  $\{c, i^*, i, \lambda, w, \pi^{gap}, y, n, g, s, z\}$

De-trended Equilibrium System (10 equations):

$$\tilde{c}_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] \tilde{y}_t \quad (1)$$

$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{i}(\pi_t^{gap})^{\phi_\pi} (g_t \tilde{c}_t/(\bar{g} \tilde{c}_{t-1}))^{\phi_c})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \quad (2)$$

$$i_t = \max\{1, i_t^*\} \quad (3)$$

$$\tilde{\lambda}_t = \tilde{c}_t - h \tilde{c}_{t-1}/g_t \quad (4)$$

$$\tilde{w}_t = \chi n_t^\eta \tilde{\lambda}_t \quad (5)$$

$$1 = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(s_t i_t/(\bar{\pi} \pi_{t+1}^{gap} g_{t+1}))] \quad (6)$$

$$\varphi(\pi_t^{gap} - 1) \pi_t^{gap} = 1 - \theta + \theta \tilde{w}_t + \beta \varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap} - 1) \pi_{t+1}^{gap} (\tilde{y}_{t+1}/\tilde{y}_t)] \quad (7)$$

$$\tilde{y}_t = n_t \quad (8)$$

$$g_t = (1 - \rho_g) \bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \quad (9)$$

$$s_t = (1 - \rho_s) \bar{s} + \rho_s s_{t-1} + \sigma_v \nu_t \quad (10)$$

Variables:  $\{\tilde{c}, i^*, i, \tilde{\lambda}, \tilde{w}, \pi^{gap}, \tilde{y}, n, k, g, s\}$

Log-linear Equilibrium System:

$$\hat{c}_t = \hat{y}_t \quad (1)$$

$$\hat{i}_t^n = \rho_i \hat{i}_{t-1}^n + (1 - \rho_i) \phi_\pi \hat{\pi}_t + (1 - \rho_i) \phi_c (\hat{g} + \hat{c} - \hat{c}_{t-1}) + \sigma_\nu \nu_t \quad (2)$$

$$\hat{i}_t = \hat{i}_t^n \quad (3)$$

$$(1 - h/g) \hat{\lambda}_t = \hat{c}_t + (h/g) (\hat{g} - \hat{c}_{t-1}) \quad (4)$$

$$\hat{w}_t = \eta \hat{n}_t + \hat{\lambda}_t \quad (5)$$

$$\hat{\lambda}_t + \hat{i}_t + s_t = E_t \hat{\lambda}_{t+1} + E_t \hat{\pi}_{t+1} \quad (6)$$

$$\varphi \hat{\pi}_t = (\theta - 1) \hat{w}_t + \beta \varphi E_t \hat{\pi}_{t+1} \quad (7)$$

$$\hat{y}_t = \hat{n}_t \quad (8)$$

### 1.3 GUST ET AL MODEL Equilibrium system (13 equations):

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta w_t/z_t + \beta\varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \quad (1)$$

$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{i}(\pi_t^{gap})^{\phi_\pi})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \quad (2)$$

$$i_t = \max\{1, i_t^*\} \quad (3)$$

$$1/\lambda_t = \beta E_t[(1/\lambda_{t+1})(s_t i_t / (\bar{\pi} \pi_{t+1}^{gap} g_{t+1}))] \quad (4)$$

$$\lambda_t = c_t \quad (5)$$

$$c_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] y_t \quad (6)$$

$$w_t = \chi n_t^\eta \lambda_t \quad (7)$$

$$y_t = z_t n_t \quad (8)$$

$$g_t = (1 - \rho_g) \bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \quad (9)$$

$$s_t = (1 - \rho_s) \bar{s} + \rho_s s_{t-1} + \sigma_v v_t \quad (10)$$

$$z_t = g_t z_{t-1} \quad (11)$$

Variables:  $\{c, i^*, i, \lambda, w, \pi^{gap}, V_\lambda, y, V_\pi, n, g, s, z\}$

De-trended Equilibrium System (12 equations):

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta \tilde{w}_t + \beta\varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(\tilde{y}_{t+1}/\tilde{y}_t)] \quad (1)$$

$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{i}(\pi_t^{gap})^{\phi_\pi})^{1-\rho_i} \exp(\sigma_\nu \nu_t) \quad (2)$$

$$i_t = \max\{1, i_t^*\} \quad (3)$$

$$1/\tilde{\lambda}_t = \beta E_t[(1/\tilde{\lambda}_{t+1})(s_t i_t / (\bar{\pi} \pi_{t+1}^{gap} g_{t+1}))] \quad (4)$$

$$\tilde{\lambda}_t = \tilde{c}_t \quad (5)$$

$$\tilde{c}_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] \tilde{y}_t \quad (6)$$

$$\tilde{w}_t = \chi n_t^\eta \tilde{\lambda}_t \quad (7)$$

$$\tilde{y}_t = n_t \quad (8)$$

$$g_t = (1 - \rho_g) \bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t \quad (9)$$

$$s_t = (1 - \rho_s) \bar{s} + \rho_s s_{t-1} + \sigma_v v_t \quad (10)$$

Variables:  $\{\tilde{c}, i^*, i, \tilde{\lambda}, \tilde{w}, \pi^{gap}, \tilde{V}_\lambda, \tilde{y}, \tilde{V}_\pi, n, g, s\}$

Log-linear Equilibrium System:

$$\varphi \hat{\pi}_t = (\theta - 1) \hat{w}_t + \beta \varphi E_t \hat{\pi}_{t+1} \quad (1)$$

$$\hat{i}_t^n = \rho_i \hat{i}_{t-1}^n + (1 - \rho_i) \phi_\pi \hat{\pi}_t + \sigma_\nu \nu_t \quad (2)$$

$$\hat{i}_t = \hat{i}_t^n \quad (3)$$

$$-\hat{\lambda}_t = \hat{i}_t + s_t - E_t \hat{\lambda}_{t+1} - E_t \hat{\pi}_{t+1} \quad (4)$$

$$\hat{\lambda}_t = \hat{c}_t \quad (5)$$

$$\hat{c}_t = \hat{y}_t \quad (6)$$

$$\hat{w}_t = \eta \hat{n}_t + \hat{\lambda}_t \quad (7)$$

$$\hat{y}_t = \hat{n}_t \quad (8)$$

$$c_{t+1,1} = \beta E_t[\lambda_t(s_t i_t / (\bar{\pi} \pi_{t+1}^{gap} g_{t+1}))] \quad (1)$$

$$c_{t+1,2} = \beta E_t[\lambda_t(s_t / (\bar{\pi} \pi_{t+1}^{gap} g_{t+1}))] \quad (2)$$

$$(3)$$

for  $k = 1, 2$  where  $k = 1$  corresponds to the non-ZLB regime and  $k = 2$  corresponds to the ZLB regime.

$$c_t = c_{t,1}I + c_{t,2}(1 - I) \quad (4)$$

for  $j = 1, 2$  and where  $I$  is defined by:

$$\begin{cases} I = 1 & \text{if } i > 1 \\ I = 0 & \text{otherwise.} \end{cases}$$