

1 MODEL

1.1 FIRMS The production sector consists of a continuum of monopolistically competitive intermediate goods firms and a final goods firm. Intermediate firm $f \in [0, 1]$ produces a differentiated good, $y(f)$, according to $y_t(f) = (k_{t-1}(f))^\alpha (z_t n_t(f))^{1-\alpha}$, where $n(f)$ is the labor hired by firm f and $k(f)$ is the capital rented by firm f . $z_t = g_t z_{t-1}$ is technology, which is common across firms. Deviations from the steady-state growth rate, \bar{g} , follow

$$g_t = \bar{g} + \sigma_g \varepsilon_{g,t}, \quad \varepsilon_g \sim \mathcal{N}(0, 1). \quad (1)$$

The final goods firm purchases output from each intermediate firm to produce the final good, $y_t \equiv [\int_0^1 y_t(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$, where $\theta > 1$ is the elasticity of substitution. Dividend maximization determines the demand for intermediate good f , $y_t(f) = (p_t(f)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(f)^{1-\theta} df]^{1/(1-\theta)}$ is the price level. Following Rotemberg (1982), intermediate firms pay a price adjustment cost, $adj_t^p(f) \equiv \varphi(p_t(f)/(\bar{\pi} p_{t-1}(f)) - 1)^2 y_t/2$, where $\varphi > 0$ scales the cost and $\bar{\pi}$ is the steady-state gross inflation rate. Given this cost, firm f chooses $n_t(f)$, $k_{t-1}(f)$, and $p_t(f)$ to maximize the expected discounted present value of future dividends, $E_t \sum_{k=t}^{\infty} q_{t,k} d_k(f)$, subject to its production function and the demand for its product, where $q_{t,t} \equiv 1$, $q_{t,t+1} \equiv \beta(\lambda_t/\lambda_{t+1})$ is the pricing kernel between periods t and $t+1$, $q_{t,k} \equiv \prod_{j=t+1}^{k>t} q_{j-1,j}$, and $d_t(f) = p_t(f)y_t(f)/p_t - w_t n_t(f) - adj_t^p(f)$. In symmetric equilibrium, the optimality conditions reduce to

$$y_t = (k_{t-1})^\alpha (z_t n_t)^{1-\alpha}, \quad (2)$$

$$w_t = (1 - \alpha) m c_t y_t / n_t, \quad (3)$$

$$r_t^k = \alpha m c_t y_t / k_{t-1}, \quad (4)$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta m c_t + \beta \varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)], \quad (5)$$

where $\pi_t^{gap} = \pi_t/\bar{\pi}_t$ and $\pi_t = p_t/p_{t-1}$ is the gross inflation rate. If $\varphi = 0$, the real marginal cost of producing a unit of output ($m c_t$) equals $(\theta - 1)/\theta$, which is the inverse of the markup of price over marginal cost.

1.2 HOUSEHOLDS The households choose $\{c_t, n_t, b_t, x_t, k_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility given by $E_0 \sum_{t=0}^{\infty} \beta [\log(c_t - h c_{t-1}^a) - \chi n_t^{1+\eta}/(1+\eta)]$, where β is the discount factor, $\chi > 0$ determines steady-state labor, $1/\eta$ is the Frisch elasticity of labor supply, c is consumption, c^a is aggregate consumption, h is the degree of external habit persistence, b is the real value of a privately-issued 1-period nominal bond, x is investment, and E_0 is an expectation operator conditional on information available in period 0. The household's budget constraint is given by

$$c_t + x_t + b_t/(i_t s_t) = w_t n_t + r_t^k k_{t-1} + b_{t-1}/\pi_t + d_t$$

where i is the gross nominal interest rate, r^k is the capital rental rate, and d is a real dividend from ownership of intermediate firms. The nominal bond, b is subject to a risk premium, s , that follows

$$s_t = (1 - \rho_s) \bar{s} + \rho_s s_{t-1} + \sigma_s \varepsilon_{s,t}, \quad 0 \leq \rho_s < 1, \quad \varepsilon_s \sim \mathcal{N}(0, 1), \quad (6)$$

where \bar{s} is the steady-state value. An increase in s_t boosts saving, which lowers period- t demand.

Households also face an investment adjustment cost, so the law of motion for capital is given by

$$k_t = (1 - \delta)k_{t-1} + x_t(1 - \nu(x_t^g - 1)^2/2), \quad 0 \leq \delta \leq 1, \quad (7)$$

where $x_t^g = x_t/(\bar{g}x_{t-1})$ is investment growth relative to its steady-state and $\nu \geq 0$ scales the cost.

The first order conditions to each household's constrained optimization problem are given by

$$\lambda_t = c_t - hc_{t-1}^a, \quad (8)$$

$$w_t = \chi n_t^\eta \lambda_t, \quad (9)$$

$$1 = \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t/(\bar{\pi} \pi_t^{gap}))], \quad (10)$$

$$q_t = \beta E_t[(\lambda_t/\lambda_{t+1})(r_{t+1}^k + (1 - \delta)q_{t+1})], \quad (11)$$

$$1 = q_t[1 - \nu(x_t^g - 1)^2/2 - \nu(x_t^g - 1)x_t^g] + \nu\beta\bar{g}E_t[q_{t+1}(\lambda_t/\lambda_{t+1})(x_{t+1}^g)^2(x_{t+1}^g - 1)], \quad (12)$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta mc_t + \beta\varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)], \quad (13)$$

where $1/\lambda$ is the marginal utility of consumption and q is Tobin's q .

Monetary Policy The central bank sets the gross nominal interest rate, i , according to

$$i_t = \max\{1, i_t^n\} \quad (14)$$

$$i_t^n = (i_{t-1}^n)^{\rho_i} (\bar{i}(\pi_t^{gap})^{\phi_\pi} (y_t^g)^{\phi_y})^{1-\rho_i} \exp(\sigma_i \varepsilon_{i,t}), \quad 0 \leq \rho_i < 1, \varepsilon_i \sim \mathbb{N}(0, 1), \quad (15)$$

where y^{gdp} is real GDP (i.e., output, y , minus the resources lost due to adjustment costs, adj^p), i^n is the gross notional interest rate, \bar{i} and $\bar{\pi}$ are the target values of the inflation and nominal interest rates, and ϕ_π and ϕ_y are the responses to the inflation and output growth gaps. A more negative net notional rate indicates that the central bank is more constrained.

Competitive Equilibrium The aggregate resource constraint and real GDP definition are given by

$$c_t + x_t = y_t^{gdp} \quad (16)$$

$$y_t^{gdp} = [1 - \varphi(\pi_t^{gap} - 1)^2/2]y_t \quad (17)$$

The model does not have a steady-state due to the unit root in technology, z_t . Therefore, we define the variables with a trend in terms of technology (i.e., $\tilde{x}_t \equiv x_t/z_t$). The detrended equilibrium system is provided in [Appendix A](#). A competitive equilibrium consists of sequences of quantities, $\{\tilde{c}_t, \tilde{y}_t, \tilde{y}_t^{gdp}, x_t^g, y_t^g, n_t, \tilde{k}_t, \tilde{x}_t\}_{t=0}^\infty$, prices, $\{\tilde{w}_t, i_t, i_t^n, \pi_t, \tilde{\lambda}_t, q_t, r_t^k, mc_t\}_{t=0}^\infty$, and exogenous variables, $\{s_t, g_t\}_{t=0}^\infty$, that satisfy the detrended equilibrium system, given the initial conditions, $\{\tilde{c}_{-1}, i_{-1}^n, \tilde{k}_{-1}, \tilde{x}_{-1}, \tilde{w}_{-1}, s_0, g_0, \varepsilon_{i,0}\}$, and three sequences of shocks, $\{\varepsilon_{g,t}, \varepsilon_{s,t}, \varepsilon_{i,t}\}_{t=1}^\infty$.

Subjective Discount Factor	β	0.9949	Rotemberg Price Adjustment Cost	φ	100
Frisch Labor Supply Elasticity	$1/\eta$	3	Inflation Gap Response	ϕ_π	2.0
Price Elasticity of Substitution	θ	6	Output Growth Gap Response	ϕ_y	0.5
Steady-State Labor Hours	\bar{n}	1/3	Habit Persistence	h	0.80
Steady-State Risk Premium	\bar{s}	1.0058	Risk Premium Persistence	ρ_s	0.80
Steady-State Growth Rate	\bar{g}	1.0034	Notional Rate Persistence	ρ_i	0.80
Steady-State Inflation Rate	$\bar{\pi}$	1.0053	Technology Growth Shock SD	σ_g	0.005
Capital Share of Income	α	0.35	Risk Premium Shock SD	σ_s	0.005
Capital Depreciation Rate	δ	0.025	Notional Interest Rate Shock SD	σ_i	0.0035
Investment Adjustment Cost	ν	4			

Table 1: Parameter values

1.3 PARAMETER VALUES Table 1 shows the model parameters. Parameters are from Atkinson et al. (2019), and were chosen to be characteristic of U.S. data.

2 SOLUTION METHODS

2.1 RICHTER ET AL. (2014) The Richter et al. (2014) solution method is policy function iteration with time iteration and linear interpolation. We first construct an evenly-spaced state space by discretizing the endogenous state variables and using the Rouwenhorst (1995) method to approximate the exogenous state variables s_t , g_t , and $\varepsilon_{i,t}$. As in Richter et al. (2014), we use the Rouwenhorst method so that we do not have to interpolate along the dimensions of the exogenous state variables, providing a faster and more accurate than quadrature methods. To construct the initial policy functions, we solve the log-linear analog of our model with the ZLB not imposed using Sims’s (2002) gensys algorithms. We update the policy functions using a fixed point iteration scheme on each node in the discretized state space and calculate the maximum distance between the updated policy functions and the previous guess. Finally, we update the policy functions with our new guess and iterate until the maximum distance is below a convergence criterion. [Appendix B](#) provides a more detailed discussion of the solution method. The baseline code for this algorithm is provided as a companion to Richter et al (2014).

2.2 GUST ET AL. (2014) The Gust et al. (2014) solution method similarly uses time iteration on a fixed point solution with linear interpolations. Following this solution method, instead of directly computing the policy functions, we estimate smoother functions at and away from the ZLB following Gust, Løgpez-Salido, and Smith (2012) which builds on Christiano and Fisher (2000). Since the policy functions depend directly on the nominal interest rate, they have a kink or non-differentiability associated with the ZLB. On the other hand, the regime-indexed policy functions do not depend on the current indicator function and are thus more likely to be smooth. The smoother functions are approximated by specifying:

$$\mathbf{pf}_t(d) = \mathbf{pf}_{t,1}(d)\mathbb{I}_t(d) + \mathbf{pf}_{t,2}(d)(1 - \mathbb{I}_t(d)) \quad (18)$$

where $\mathbb{I}_t(d)$ is defined by

$$\mathbb{I}_t(d) = \begin{cases} 1 & \text{if } i_t > 1 \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

The variable $i_t = \max\{1, i_t^n\}$ represents the value of the notional rate derived from evaluating the functions $\mathbf{p}_{t,j}(d)$ for $j \in \{1, 2\}$ and using equation (15). (For each variable, $j = 1$ denotes the function associated with the regime with a positive nominal rate and $j = 2$ denotes the function associated with the ZLB regime.) The functions $\mathbf{p}_{t,j}$ satisfy the residual functions $R_{t,l,j}$ for and $l \in \{1, 2, 3, 4\}$ which correspond to the household FOC bond, FOC capital, FOC investment, and the price Phillips curve, respectively, and $j = 1, 2$.

$$R_{t,1,1} = 1 - s_t i_t \beta E_t[(\lambda_t / \lambda_{t+1})(1 / (\bar{\pi} \pi_{t+1}^{gap}))] \quad (20)$$

$$R_{t,1,2} = 1 - s_t \beta E_t[(\lambda_t / \lambda_{t+1})(1 / (\bar{\pi} \pi_{t+1}^{gap}))] \quad (21)$$

$$R_{t,2,j} = q_t - \beta E_t[(\lambda_t / \lambda_{t+1})(r_{t+1}^k + (1 - \delta)q_{t+1})] \quad (22)$$

$$R_{t,3,j} = 1 - q_t[1 - \nu(x_t^g - 1)^2/2 - \nu(x_t^g - 1)x_t^g] - \nu\beta\bar{g}E_t[q_{t+1}(\lambda_t / \lambda_{t+1})(x_{t+1}^g)^2(x_{t+1}^g - 1)] \quad (23)$$

$$R_{t,4,j} = \varphi(\pi_t^{gap} - 1)\pi_t^{gap} - (1 - \theta) - \theta mc_t - \beta\varphi E_t[(\lambda_t / \lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \quad (24)$$

2.3 EULER EQUATION ERRORS We use Euler equation errors to measure the accuracy of our solutions. To measure errors between nodes, we use Gauss-Hermite quadrature instead of the Rouwenhorst method following Richter et al. (2014) to allow exogenous variables to have realizations between grid points. The Euler equation errors are represented in absolute value of the errors in base 10 logarithms. An Euler equation error of -3 means the household makes an error equivalent to one per 1,000 consumption goods.

We simulate 10,000 periods of the model at the nonlinear solution with random shocks. For each period, we follow one iteration of step 2 of the algorithm outlined in Appendix B and then transform the residuals to Euler equation errors.

We considered adding a second measure of accuracy, a statistical measure from Den Haan and Marcet (1994) based on a transformation of the Euler equation residuals to a chi-square distribution. However, it proved uninformative, as several runs of the test statistic were all in the extreme tails of the distribution.

3 RESULTS

	Model without capital		Model with capital	
	Iterations	Total Time	Iterations	Total time
Richter et al. (2013)	67	17.9s	65	0h 15m 37.1s
Gust et al. (2017)	66	46s	1170	5h 32m 48.6s

Table 2: Solution times

3.1 SOLUTION TIMES Table 2 reports the solution times for the Richter et al. (2013) and Gust et al. (2017) solution methods for the models with and without capital. The solution times were computed with multi-core processing using the Parallel Computing Toolbox. With MATLAB executable functions (MEX) provided by the companion Toolbox to Richter et al. (2013), we integrate Fortran in the interpolation steps of the algorithm. The solution times represent one run of each algorithm. The machine used to compute the solutions has two CPUs (at 2.30GHz), each with 16 cores.

- Gust et al. method slower, particularly on the model with capital. The differential blows up for the model with capital.
- The speed and speed differential is influenced by the parameterization, and appears to take more time on the boundary of convergence for each algorithm.

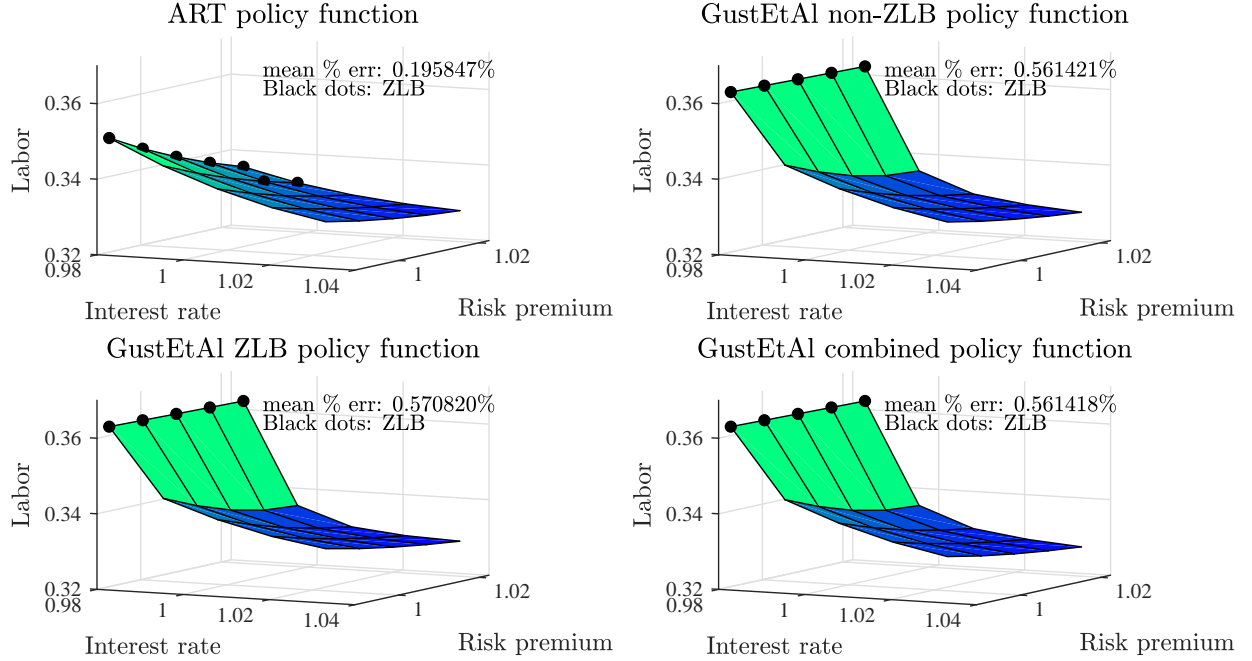


Figure 1: Labor policy function for model with capital

3.2 POLICY FUNCTIONS Figure 1 shows the cross-section of the labor policy functions with the interest rate and risk premium for the model with capital. The black dots on the nodes of the graph represent labor values corresponding to the ZLB.

	Model without capital		Model with capital	
	Mean % Error	RMSE	Mean % Error	RMSE
Richter et al. policy	0.46545%	0.0020137 consumption units	0.19585%	.0029035 labor units
Gust et al. policy	0.45556%	0.0019792 consumption units	0.56142%	0.0076665 labor units

Table 3: Smoothness measures for labor policy functions (c for model with capital and n for model with capital). Gust et al. combined policy functions are reported.

Table 3 reports the RMSE (root mean square error) and mean percent error from linear policy functions. The mean percent error is calculated as $1/N \sum_i |\mathbf{pf.n}(i) - \hat{\mathbf{n}}(i)|/\bar{n}$ where N is the number of periods, $\mathbf{pf.n}$ is the labor policy function, $\hat{\mathbf{n}}$ is the linear regression model of $\mathbf{pf.n}$ fit to the state variables, and \bar{n} is steady state labor. The corresponding results for the model without capital is reported in Appendix C.

- ART is visually a lot smoother; matched with statistics

- the Gust Et Al combined policy function is very similar to the non-ZLB policy function. All Gust Et Al policy functions have a kink corresponding to the ZLB.
- Percent error and RMSE error wise though, this is not too big of a difference?
- say something about what the mean % error and RMSE is

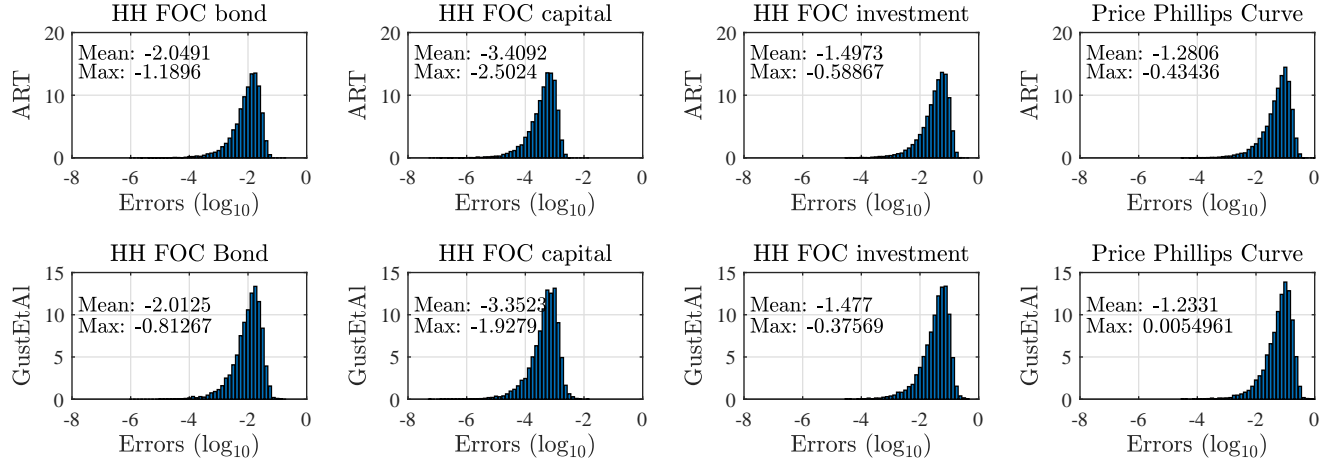


Figure 2: Euler equation errors for model with capital

3.3 EULER EQUATION ERRORS Figure 2 shows the distribution of the absolute value of the Euler equation errors in base 10 logarithms for the household FOC bond, FOC capital, FOC investment, and price Phillips curve in the model with capital. We also report the mean and maximum error. The corresponding results for the model without capital is reported in [Appendix C](#).

- Euler equations very comparable between algorithms
- Gust et al does slightly worse. Means are similar but max is higher.

4 CONCLUSION

Opportunities for future work

- Chebyshev polynomials
- Smolyak grids
- This might make solution method faster, so we could potentially increase the number of grid points and make it better than Richter et al

A DETRENDED EQUILIBRIUM SYSTEM

Medium-Scale Model The detrended system includes (1),(6),(14),(15) and

$$\tilde{y}_t = (\tilde{k}_{t-1}/g_t)^\alpha n_t^{1-\alpha}, \quad (25)$$

$$r_t^k = \alpha m c_t g_t \tilde{y}_t / \tilde{k}_{t-1}, \quad (26)$$

$$\tilde{w}_t = (1 - \alpha) m c_t \tilde{y}_t / n_t, \quad (27)$$

$$\tilde{y}_t^{gap} = [1 - \varphi(\pi_t^{gap} - 1)^2/2] \tilde{y}_t, \quad (28)$$

$$y_t^g = g_t \tilde{y}_t^{gap} / (\bar{g} \tilde{y}_{t-1}^{gap}), \quad (29)$$

$$\tilde{\lambda}_t = \tilde{c}_t - h \tilde{c}_{t-1} / g_t, \quad (30)$$

$$\tilde{w}_t = \chi n_t^\eta \tilde{\lambda}_t, \quad (31)$$

$$\tilde{c}_t + \tilde{x}_t = \tilde{y}_t^{gap}, \quad (32)$$

$$x_t^g = g_t \tilde{x}_t / (\bar{g} \tilde{x}_{t-1}), \quad (33)$$

$$\tilde{k}_t = (1 - \delta)(\tilde{k}_{t-1}/g_t) + \tilde{x}_t(1 - \nu(x_t^g - 1)^2/2), \quad (34)$$

$$1 = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(s_t i_t / (\bar{\pi} \pi_{t+1}^{gap} g_{t+1}))], \quad (35)$$

$$q_t = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(r_{t+1}^k + (1 - \delta)q_{t+1})/g_{t+1}], \quad (36)$$

$$1 = q_t[1 - \nu(x_t^g - 1)^2/2 - \nu(x_t^g - 1)x_t^g] + \nu \beta \bar{g} E_t[q_{t+1}(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(x_{t+1}^g)^2(x_{t+1}^g - 1)/g_{t+1}], \quad (37)$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta m c_t + \beta \varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(\tilde{y}_{t+1}/\tilde{y}_t)]. \quad (38)$$

The variables are $\tilde{c}, \tilde{n}, \tilde{x}, \tilde{k}, y^{gap}, \tilde{y}, x^g, y^g, \tilde{w}, r^k, \pi, i, i^n, q, m c, \tilde{\lambda}, g$, and s .

Small-Scale Model The detrended system includes (1),(6),(14),(15)(31),(35),(38), and

$$\tilde{\lambda}_t = \tilde{c}_t, \quad (39)$$

$$\tilde{c}_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2] \tilde{y}_t, \quad (40)$$

$$\tilde{y}_t = n_t. \quad (41)$$

The variables are $\tilde{c}, i^n, i, \tilde{\lambda}, \tilde{w}, \pi^{gap}, \tilde{y}, n, g$, and s .

B NONLINEAR SOLUTION METHOD

The following discussion is based on Atkinson et al. (2019) and Richter and Throckmorton (2014). We express the detrended nonlinear system compactly as

$$E[f(\mathbf{s}_{t+1}, \mathbf{s}_t, \varepsilon_{t+1}) | \mathbf{z}_t, \vartheta] = 0,$$

where f is a vector-valued function, \mathbf{s}_t is a vector of variables, $\varepsilon_t \equiv [\varepsilon_{s,t}, \varepsilon_{g,t}, \varepsilon_{i,t}]'$ is a vector of shocks, \mathbf{z}_t is a vector of states ($\mathbf{z}_t \equiv [\tilde{c}_{t-1}, i_{t-1}^n, \tilde{k}_{t-1}, \tilde{x}_{t-1}, s_t, g_t, \varepsilon_{i,t}]'$ for the model with capital and $\mathbf{z}_t \equiv [\tilde{c}_{t-1}, i_{t-1}^n, s_t, g_t, \varepsilon_{i,t}]'$ for the model without capital), and ϑ is a vector of parameters.

We use the Markov chain method in Rouwenhorst (1995) to discretize the endogenous state variables, s_t, g_t , and $\varepsilon_{i,t}$ following Richter et al (2014). The bounds on $\tilde{c}_{t-1}, i_{t-1}^n, \tilde{k}_{t-1}$, and \tilde{x}_{t-1} are

set to $\pm 2.5\%$, $\pm 6\%$, $\pm 8\%$, $\pm 15\%$, respectively, following Atkinson et al. (2019). We discretize the states into 5 evenly-spaced points for the model with capital and 7 evenly-spaced points for the model without capital. The product of the points in each dimension, D , represents the total nodes in the state space ($D = 78125$ for the model with capital and $D = 2401$ for the model without capital). The realization of \mathbf{z}_t on node d is denoted $\mathbf{z}_t(d)$. The Rouwenhorst method provides integration nodes, $[s_{t+1}(m), g_{t+1}(m), \varepsilon_{i,t+1}(m)]$, with weights, $\phi(m)$, for $m \in \{1, \dots, M\}$.

The vector of policy functions is denoted $\mathbf{p}\mathbf{f}_t$ and the realization on node d is denoted $\mathbf{p}\mathbf{f}_t(d)$ ($\mathbf{p}\mathbf{f}_t(d) \equiv [\tilde{\pi}_t^{gap}(\mathbf{z}_t), n_t(\mathbf{z}_t), q_t(\mathbf{z}_t), mc_t(\mathbf{z}_t)]$ for the model with capital and $\mathbf{p}\mathbf{f}_t(d) \equiv [\tilde{\pi}_t^{gap}(\mathbf{z}_t), \tilde{c}_t(\mathbf{z}_t)]$). The policy functions are selected so that solving for other variables in the nonlinear system is straightforward.

The following steps outline the global policy function iteration algorithm:

1. Use Sims's (2002) `gensys` algorithm to solve the log-linear model without the ZLB constraint and obtain conjectures for the policy functions $\mathbf{p}\mathbf{f}_0$.
2. On each node $d \in \{1, \dots, D\}$,
 - (a) Solve for all variables dated at time t , given $\mathbf{p}\mathbf{f}_t(d)$ and $\mathbf{z}_t(d)$.
 - (b) Linearly interpolate the policy functions $\mathbf{p}\mathbf{f}_{j-1}$, at the updated state variables $\mathbf{z}_{t+1}(m)$, to obtain $\mathbf{p}\mathbf{f}_{t+1}(m)$ on every integration node, $m \in \{1, \dots, M\}$.
 - (c) Given $\{\mathbf{p}\mathbf{f}_{t+1}(m)\}_{m=1}^M$, solve for the other elements of $\mathbf{s}_{t+1}(m)$ and approximate the expectation operators

$$E[f(\mathbf{s}_{t+1}, \mathbf{s}_t(d), \varepsilon_{t+1}) | \mathbf{z}_t(d), \vartheta] \approx \sum_{m=1}^M \phi(m) f(\mathbf{s}_{t+1}(m), \mathbf{s}_t(d), \varepsilon_{t+1}(m)).$$

- (d) Solve for $\mathbf{p}\mathbf{f}_t(d)$ from the expectation operators and $\mathbf{s}_{t+1}(m)$. Set $\mathbf{p}\mathbf{f}_j(d) = \mathbf{p}\mathbf{f}_t(d)$.
3. Repeat step 2 until $\text{maxdist}_j < 10^{-6}$, where $\text{maxdist}_j \equiv \max\{|\mathbf{p}\mathbf{f}_j - \mathbf{p}\mathbf{f}_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate nonlinear solution.

C RESULTS FOR MODEL WITHOUT CAPITAL

The following reports the corresponding results for the model without capital.

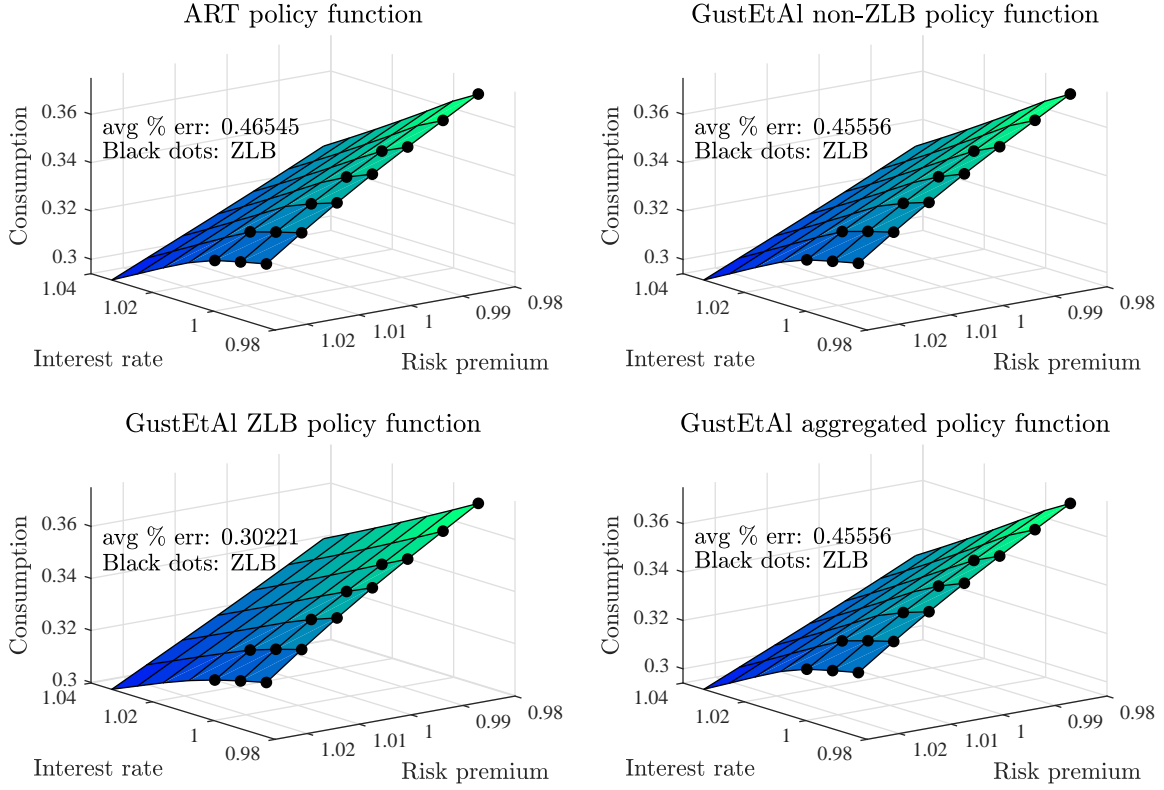


Figure 3: Consumption policy function for model without capital

Figure 3 shows the cross-section of the consumption policy functions with the interest rate and risk premium.

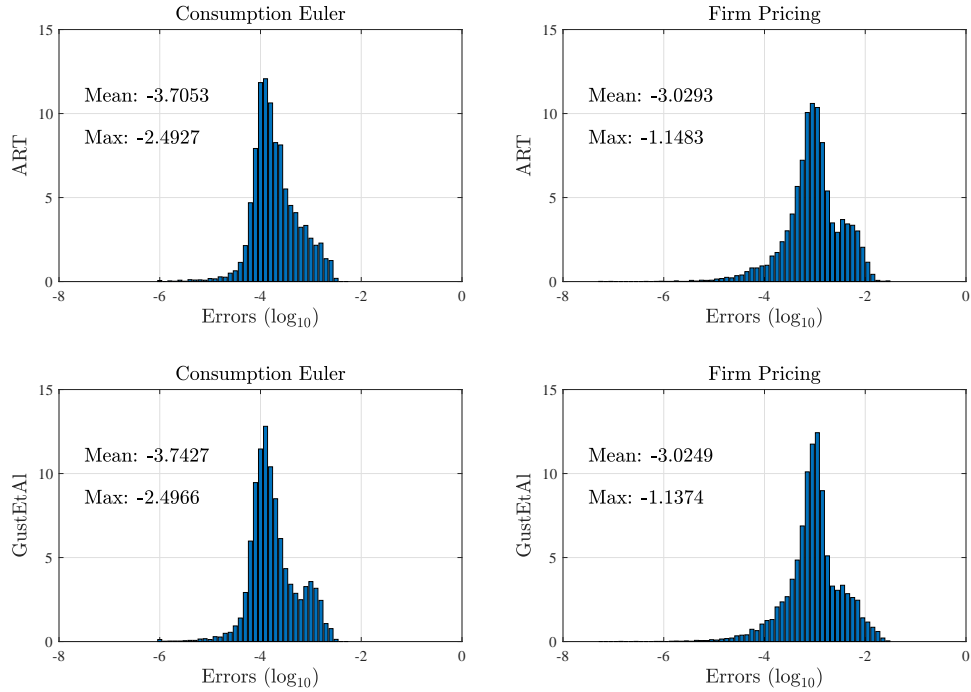


Figure 4: Euler equation errors for model without capital

Figure 4 shows the distribution of the absolute value of the Euler equation errors in base 10 logarithms for the consumption euler Equation and the price Phillips curve.

- Smoother in general
- Very close between ART and GustEtAl
- Bimodality more distinct in Gust et al policy functions
- Very similar with regards to mean and max; Gust et al does slightly better in consumption Euler