1 Equilibrium System

1.1 ART Model with capital

Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t - hc_{t-1}^a) - \chi n_t^{1+\eta}/(1+\eta)]$$

Budget Constraint:

$$c_t + x_t + b_t/(i_t s_t) = w_t n_t + r_t^k k_{t-1} + b_{t-1}/\pi_t + d_t$$
$$k_t = (1-\delta)k_{t-1} + x_t(1 - \nu(x_t^g - 1)^2/2)$$

Equilibrium system:

$$y_t = k_{t-1}^{\alpha}(z_t n_t)^{1-\alpha} \tag{1}$$
$$r_t^k = \alpha mc_t y_t / k_{t-1} \tag{2}$$
$$w_t = (1-\alpha) mc_t y_t / n_t \tag{3}$$
$$y_t^{gdp} = [1 - \varphi(\pi_t^{gap} - 1)^2/2] y_t \tag{4}$$
$$y_t^g = y_t^{gdp}/(\bar{g} y_{t-1}^{gdp}) \tag{5}$$
$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{\imath}(\pi_t^{gap})^{\phi_\pi} (y_t^g)^{\phi_y})^{1-\rho_i} \exp(mp_t) \tag{6}$$
$$i_t = \max\{1, i_t^*\} \tag{7}$$
$$\lambda_t = c_t - hc_{t-1}^a \tag{8}$$
$$w_t = \chi n_t^{\eta} \lambda_t \tag{9}$$
$$c_t + x_t = y_t^{gdp} \tag{10}$$
$$x_t^g = x_t/(\bar{g} x_{t-1}) \tag{11}$$
$$k_t = (1-\delta)k_{t-1} + x_t(1 - \nu(x_t^g - 1)^2/2) \tag{12}$$
$$1 = \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap}))] \tag{13}$$
$$q_t = \beta E_t[(\lambda_t/\lambda_{t+1})(r_{t+1}^k + (1-\delta)q_{t+1})] \tag{14}$$
$$1 = q_t[1 - \nu(x_t^g - 1)^2/2 - \nu(x_t^g - 1)x_t^g] + \nu\beta\bar{g} E_t[q_{t+1}(\lambda_t/\lambda_{t+1})(x_{t+1}^g)^2(x_{t+1}^g - 1)] \tag{15}$$
$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta mc_t + \beta\varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \tag{16}$$
$$g_t = \bar{g} + \sigma_g \varepsilon_{g,t} \tag{17}$$
$$s_t = (1-\rho_s)\bar{s} + \rho_s s_{t-1} + \sigma_s \varepsilon_{s,t} \tag{18}$$
$$mp_t = \sigma_i \varepsilon_{i,t} \tag{19}$$
$$z_t = g_t z_{t-1} \tag{20}$$

Variables: $\{c, n, x, k, y^{gdp}, y, x^g, y^g, w, r^k, \pi, i, i^n, q, mc, \lambda, g, s, mp, z\}$

De-trended Equilibrium System:

$$\tilde{y}_t = (\tilde{k}_{t-1}/g_t)^{\alpha} n_t^{1-\alpha} \tag{1}$$
$$r_t^k = \alpha mc_t g_t \tilde{y}_t / \tilde{k}_{t-1} \tag{2}$$
$$\tilde{w}_t = (1-\alpha) mc_t \tilde{y}_t / n_t \tag{3}$$
$$\tilde{y}_t^{gdp} = [1 - \varphi(\pi_t^{gap} - 1)^2/2]\tilde{y}_t \tag{4}$$
$$y_t^g = g_t \tilde{y}_t^{gdp} / (\bar{g}\tilde{y}_{t-1}^{gdp}) \tag{5}$$
$$i_t^* = (i_{t-1}^*)^{\rho_i} (\bar{\imath}(\pi_t^{gap})^{\phi_\pi} (y_t^g)^{\phi_y})^{1-\rho_i} \exp(\sigma_i \varepsilon_{i,t}) \tag{6}$$
$$i_t = \max\{1, i_t^*\} \tag{7}$$
$$\tilde{\lambda}_t = \tilde{c}_t \ - h\tilde{c}_{t-1}/g_t \tag{8}$$
$$\tilde{w}_t = \chi n_t^{\eta} \tilde{\lambda}_t \tag{9}$$
$$\tilde{c}_t + \tilde{x}_t = \tilde{y}_t^{gdp} \tag{10}$$
$$x_t^g = g_t \tilde{x}_t / (\bar{g}\tilde{x}_{t-1}) \tag{11}$$
$$\tilde{k}_t = (1-\delta)(\tilde{k}_{t-1}/g_t) + \tilde{x}_t(1 - \nu(x_t^g - 1)^2/2) \tag{12}$$
$$1 = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(s_t i_t / (\bar{\pi}\pi_{t+1}^{gap} g_{t+1}))] \tag{13}$$
$$q_t = \beta E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(r_{t+1}^k + (1-\delta)q_{t+1})/g_{t+1}] \tag{14}$$
$$1 = q_t[1 - \nu(x_t^g - 1)^2/2 - \nu(x_t^g - 1)x_t^g] + \nu\beta\bar{g}E_t[q_{t+1}(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(x_{t+1}^g)^2(x_{t+1}^g - 1)/g_{t+1}] \tag{15}$$
$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta mc_t + \beta\varphi E_t[(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(\tilde{y}_{t+1}/\tilde{y}_t)] \tag{16}$$
$$g_t = \bar{g} + \sigma_g \varepsilon_{g,t} \tag{17}$$
$$s_t = (1-\rho_s)s_t + \rho_s s_{t-1} + \sigma_s \varepsilon_{s,t} \tag{18}$$
$$mp_t = \sigma_i \varepsilon_{i,t} \tag{19}$$

Variables: $\{\tilde{c}, \tilde{n}, \tilde{x}, \tilde{k}, \tilde{y^{gdp}}, \tilde{y}, x^g, y^g, \tilde{w}, r^k, \pi, i, i^n, q, mc, \tilde{\lambda}, g, s, mp\}$

Log-linear Equilibrium System:

$$\hat{y}_t/\bar{y} = \alpha(\hat{k}_{t-1}/\bar{k} - \hat{g}_t/\bar{g}) + (1-\alpha)\hat{n}_t/\bar{n} \tag{1}$$

$$\hat{r}_t^k/\bar{r}^k = \hat{mc}_t/\bar{mc} + \hat{g}_t/\bar{g} + \hat{y}_t/\bar{y} - \hat{k}_{t-1}/\bar{k} \tag{2}$$

$$\hat{w}_t/\bar{w} = \hat{mc}_t/\bar{mc} + \hat{y}_t/\bar{y} - \hat{n}_t/\bar{n} \tag{3}$$

$$\hat{y}_t^{gdp} = \hat{y}_t \tag{4}$$

$$\hat{y}_t^g = \hat{g}_t/\bar{g} + \hat{y}_t^{gdp}/\bar{y}^{gdp} - \hat{y}_{t-1}^{gdp}/\bar{y}^{gdp} \tag{5}$$

$$\hat{i}_t/\bar{i} = \rho_i \hat{i}_{t-1}/\bar{i} + (1-\rho_i)(\phi_\pi \hat{\pi}_t^{gap} + \phi_y \hat{y}_t^g) + \hat{mp}_t \tag{6}$$

$$\hat{\imath}_t = \hat{\imath}_t^n \tag{7}$$

$$\hat{\lambda}_t = \hat{c}_t - (h/\bar{g})\hat{c}_{t-1} + (h\bar{c}/\bar{g}^2)\hat{g}_t \tag{8}$$

$$\hat{c}_t + \hat{x}_t = \hat{y}_t^{gdp} \tag{9}$$

$$\hat{x}_t^g = \hat{g}_t/\bar{g} + \hat{x}_t/\bar{x} - \hat{x}_{t-1}/\bar{x} \tag{10}$$

$$\hat{k}_t = ((1-\delta)/\bar{g})[\hat{k}_{t-1} - (\bar{k}/\bar{g})\hat{g}_t] + \hat{x}_t \tag{11}$$

$$\hat{\lambda}_t/\bar{\lambda} + \hat{\imath}_t/\bar{\imath} + \hat{s}_t/\bar{s} = E_t\hat{\lambda}_{t+1}/\bar{\lambda} + E_t\hat{g}_{t+1}/\bar{g} + E_t\hat{\pi}_{t+1}/\bar{\pi} \tag{12}$$

$$\hat{q}_t = \hat{\lambda}_t/\bar{\lambda} - E_t\hat{\lambda}_{t+1}/\bar{\lambda} + (\beta/\bar{g})[E_t\hat{r}_{t+1}^k + (1-\delta)E_t\hat{q}_{t+1}] - E_t\hat{g}_{t+1}/\bar{g} \tag{13}$$

$$\hat{x}_t^g = \hat{q}_t + \beta E_t \hat{x}_{t+1}^g \tag{14}$$

$$\varphi\hat{\pi}_t^{gap} = \theta\hat{mc}_t + \beta\varphi E_t\hat{\pi}_{t+1}^{gap} \tag{15}$$

$$\hat{g}_t = \sigma_g \varepsilon_{g,t} \tag{16}$$

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + \sigma_s \varepsilon_{s,t} \tag{17}$$

$$\hat{mp}_t = \sigma_i \varepsilon_{i,t} \tag{18}$$

Variables:$\{\hat{c}, \hat{n}, \hat{x}, \hat{k}, \hat{y^{gdp}}, \hat{y}, \hat{x^g}, \hat{y^g}, \hat{w}, \hat{r^k}, \hat{\pi}, \hat{\imath}, \hat{i^n}, \hat{q}, \hat{\lambda}, \hat{g}, \hat{s}, \hat{mp}\}$