1 Model

A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t) - \chi n_t^{1+\eta}/(1+\eta)]$, where β is the subjective discount factor, χ determines the steady state labor supply, $1/\eta$ is the Frisch elasticity of labor supply, c is consumption, h is the degree of external habit persistence, n is labor hours, b is the real value of a privately-issued 1-period nominal bond, and E_0 is the mathematical expectation operator conditional on information available in period 0. The household's choices are constrained by $c_t + b_t/(i_t s_t) = w_t n_t + b_{t-1}/\pi_t + d_t$, where π is the gross inflation rate, w is the real wage rate, i is the gross nominal interest rate, d is a real dividend from ownership of intermediate firms, and s is a risk premium shock that follows

$$s_t = (1-\rho_s)\bar{s} + \rho_s s_{t-1} + \sigma_\upsilon \upsilon_t, 0 \leq \rho_s < 1, \upsilon \sim \mathbb{N}(0,1). \tag{1}$$

An increase in s_t lowers the marginal cost of saving in the risk-free bond (JDM Fisher).
The first order conditions to the household's constrained optimization problems are given by

$$\begin{aligned} \lambda_t &= c_t \\ w_t &= \chi n_t^{\eta} \lambda_t \\ 1 &= \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap} g_{t+1}))] \end{aligned}$$

where $1/\lambda$ is the marginal utility of wealth (i.e., the Lagrange multiplier on the budget constraint). The production sector consists of a continuum of monopolistically competitive intermediate goods firms and a final goods firm. Intermediate firm $i \in [0,1]$ produces a differentiated good, $y_t^f(i)$, according to $y_t^f = z_t n_t(i)$, where $n(i)$ is the labor hired by firm i and $z_t = g_t z_{t-1}$ is technology, which is common across firms. Deviations from the balanced growth rate, $\bar{g}$, follow

$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_\varepsilon \varepsilon_t, \varepsilon \sim \mathbb{N}(0,1). \tag{2}$$

An increase in g_t acts just like a typical supply shock, lowering inflation and raising output growth. The final goods firm purchases $y_t^f(i)$ units from each intermediate firm to produce the final good, $y_t^f \equiv [\int_0^1 y_t^f(i)^{\epsilon-1} di]^{\epsilon/(\epsilon-1)}$, where $\epsilon > 1$ is the elasticity of substitution. It then maximizes dividends to determine its demand function for intermediate good i, $y_t^f(i) = (p_t(i)/p_t)^{-\epsilon} y_t^f$, where $p_t = [\int_0^1 p_t(i)^{1-\epsilon} di]^{1/(1-\epsilon)}$ is the price level. Following Rotemberg (1982), each intermediate firm pays a price adjustment cost $adj_t(i) \equiv \varphi(p_t(i)/(\bar{\pi} p_{t-1}(i)) - 1)^2 y_t^f/2$, where $\varphi > 0$ scales the cost and $\bar{pi}$ is the gross inflation rate along the balanced growth path. Given the adjustment cost, firm i chooses $n_t(i)$ and $p_t(i)$ to maximize the expected discounted present value of future dividends, $E_t \sum_{k=t}^{\infty} q_{t,k} d_k(i)$, subject to its production function and the demand for its product, where $q_{t,t} \equiv 1$, $q_{t,t+1} \equiv \beta(\lambda_t/\lambda_{t+1})$ is the pricing kernel between periods t and $t+1$, $q_{t,k} \equiv \Pi_{j=t+1}^{k>t} q_{j-1,j}$, and $d_t(i) = p_t(i) y_t^f(i)/p_t - w_t n_t(i) - adj_t(i)$. In symmetric equilibrium, all firms make identical decisions (i.e., $p_t(i) = p_t$, $n_t(i) = n_t$, and $y_t^f(i) = y_t^f$). Therefore, the optimality conditions imply

$$y_t^f = z_t n_t \tag{3}$$

$$\varphi(\pi_t/\bar{\pi} - 1)(\pi_t/\bar{\pi}) = 1 - \epsilon + \epsilon w_t/z_t + \beta\varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}/\bar{\pi} - 1)(\pi_{t+1}/\bar{\pi} - 1)(\pi_{t+1}/\bar{\pi})(y_{t+1}^f/y_t^f)]. \tag{4}$$

Without price adjustment costs (i.e., $\varphi = 0$), the real marginal cost of producing a unit of output (w_t/z_t) equals $(\epsilon - 1)/\epsilon$, which is the inverse of a firm's markup of price over marginal cost (μ). The central bank sets the gross nominal interest rate, i, according to

$$i_t = \max\{1, i_t^n\}, \tag{5}$$

$$i_t^n = (i_{t-1}^n)^{\rho_i}(\bar{\imath}(\pi_t^{gap})^{\phi_\pi})^{1-\rho_i}\exp(\sigma_\nu \nu_t), 0 \leq \rho_i < 1, \varepsilon_i \sim \mathbb{N}(0,1), \tag{6}$$

where y is output (i.e., final goods, y^f, minus the resources lost due to price adjustment costs, adj), i^n is the gross notional interest rate, $\bar{\imath}$ and $\bar{\pi}$ are the steady-state or target values of the inflation and nominal interest rates, and ϕ_π and ϕ_y determine the central bank's responses to deviations of inflation from the target rate and deviations of output growth from the balanced growth rate. When the net notional rate is positive, $i_t = i_t^n$. When it is negative, the ZLB binds and $i_t = 1$. A more negative net notional rate means the central bank is more constrained and hte model is more nonlinear.

The model does not possess a steady-state due to the unit root in technology, z_t. Therefore, we redefine the subset of variables with a trend in terms of technology (i.e., $\tilde{x}_t \equiv x_t/z_t$). The detrended equilibrium system includes the two stochastic processes, (1) and (2), the ZLB constraint, (5), and

$$c_t = [1 - \varphi(\pi_t^{gap} - 1)^2/2]y_t \tag{7}$$

$$i_t^* = (i_{t-1}^*)^{\rho_i}(\bar{\imath}(\pi_t^{gap})^{\phi_\pi})^{1-\rho_i}\exp(\sigma_\nu \nu_t) \tag{8}$$

$$\lambda_t = c_t \tag{9}$$

$$w_t = \chi n_t^\eta \lambda_t \tag{10}$$

$$1 = \beta E_t[(\lambda_t/\lambda_{t+1})(s_t i_t/(\bar{\pi}\pi_{t+1}^{gap} g_{t+1}))] \tag{11}$$

$$\varphi(\pi_t^{gap} - 1)\pi_t^{gap} = 1 - \theta + \theta w_t/z_t + \beta\varphi E_t[(\lambda_t/\lambda_{t+1})(\pi_{t+1}^{gap} - 1)\pi_{t+1}^{gap}(y_{t+1}/y_t)] \tag{12}$$

A competitive equilibrium consists of infinite sequences of quantities, $\{\tilde{c}_t, \tilde{y}_t, \tilde{y}_t^f, y_t^g\}_{t=0}^\infty$, prices $\{\tilde{w}_t, i_t, i_t^n, \pi_t, \tilde{\lambda}_t\}_{t=0}^\infty$, and exogenous variables $\{s_t, g_t\}_{t=0}^\infty$, that satisfy the detrended equilibrium system, given the initial conditions, $\{\tilde{c}_{-1}, i_{-1}^n, s_0, g_0, \varepsilon_{i,0}\}$, and shock sequences, $\{\varepsilon_{g,t}, \varepsilon_{s,t}, \varepsilon_{i,t}\}_{t=1,\infty}$.