Paper Summaries

Some Joke About VAEs

Written @ Corti

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Abstract

This was written for me to understand papers in my thesis better. Don't be alarmed if you don't understand it 100%, I probably don't either.

0 Auto Encoding Variational Bayes (?)

1 WaveNet

The WaveNet paper presents a CNN-based approach to generating audio samples. [2] Instead of using RNNs as a recurrent architecture, the generative model only conditions on past samples, and as such does not include any hidden "state".

The probability of a waveform $\mathbf{x} \in \mathbb{R}^T$ is expressed purely as:

$$p(\mathbf{x}) = \prod_{t=1}^{T} p(x_t | x_1, ..., x_t) \tag{1}$$

where $p(x_t|x_1, ..., x_t)$ is parametrized only by the weights in the network.

1.1 Architecture and design

The WaveNet Architecture draws advantage from three developments: quantized output spaces (as shown in PixelRNN), dilated causal convolutions and gated activation units,

Quantized Output Space with μ law companding transformation Given an audio waveform $\mathbf{x} \in [-1, 1]^T$, transform the audio according to :

$$f(x_t) = \text{sign}(x_t) \frac{\ln(1 - \mu|x_t|)}{\ln(1 + \mu)} \tag{2}$$

with $\mu = 255$.

Dilated Causal Convolutions A Causal Convolution is a fancy way of saying that audio convolutions only work forward in time, not backward. This is to enforce the forward dependency in eq. (1).

A Dilated Convolution is a convolution where the convolution kernel skips over a dimension, increasing the receptive field and observing more of the surrounding environment. For an image the simplest dilated convolutional is illustrated in fig. 1

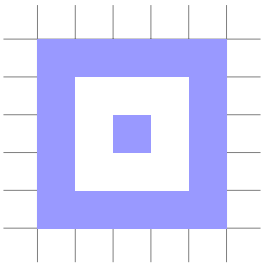

Figure 1: A Simple Pixel Dilated Convolution

Accordingly, for an audio signal, it would look like what we see in fig. 2

Gated Activation Units Each Convolution layer, Insteadof just having a filter weight, also has a **gating weight**. Hence the weights $\mathbf{W} \in \mathbb{R}^{K \times 2}$, with K as the number of layers. The operation of layer $k \in [0, K]$, is parametrized as:

$$\mathbf{z} = \tanh(\mathbf{x} * W_{k,f}) \odot \sigma(\mathbf{x} * W_{k,g}) \tag{3}$$

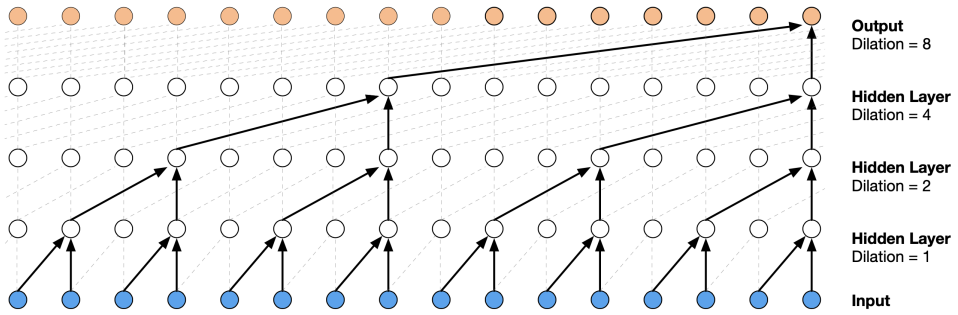

Figure 2: The Dilated Causal Convolution in WaveNet

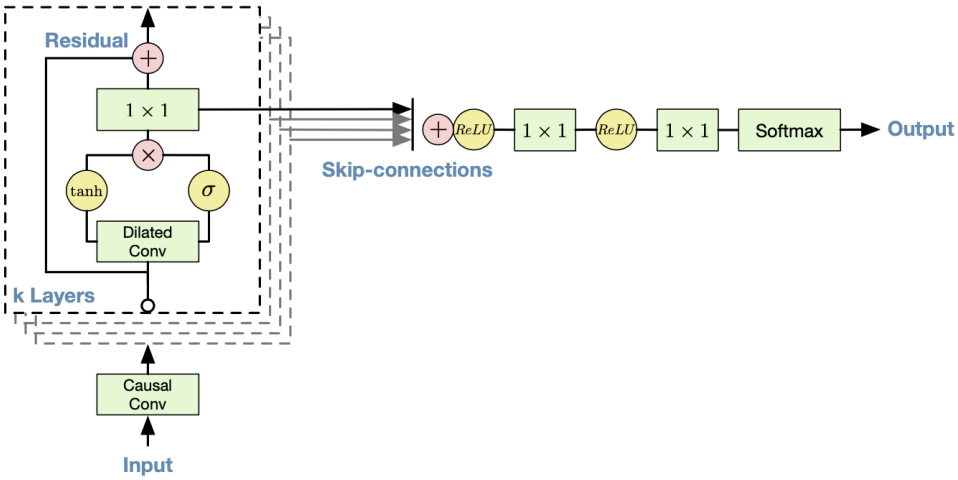

Figure 3: Overall Residual Architecture of WaveNet. Skip connections happen from every Convolutional Layer to the final softmax.

Summary of architechture The architecture is summed up in fig. 3. It's important to note that the Causal Convolution setup as described in fig. 2 only is applied once, as the first layer. This makes the entire rest of the network a simple convolutional network with dilation, as the **first (causal) convolutional stack ensures that the rest of the network will only see samples from the past.** In all other respects we can consider this a standard CNN architecture.

1.1.1 Extending architecture to include latent representations of speaker

It's possible to add a latent representation $\mathbf{h}$, extending eq. (1) to:

$$p(\mathbf{x}|\mathbf{h}) = \prod_{t=1}^{T} p(x_t|x_1, \ldots, x_t, \mathbf{h}) \tag{4}$$

There are two ways to represent this:

Global Conditioning (Speaker, Accent, Noise level) Here we set $\mathbf{h}$ to a single global latent, representing a constant over the entire sequence. The activation from eq. (3) then becomes

$$\mathbf{z} = \tanh(\mathbf{x} * W_{k,f} + V_{k,f}^T \mathbf{h}) \odot \sigma(\mathbf{x} * W_{k,g} + V_{k,g}^T \mathbf{h}) \tag{5}$$

With $V_{k,*}$ is a linear projection , and the resulting vector $V_{k,f}^T \mathbf{h}$ is broadcast over time T.

Local Conditioning (Tone of voice, changing noise levels over the call Here we define h_t, and use a ConvNet to upsample h_t to $\mathbf{y} = f(\mathbf{h})$, so eq. (3) becomes:

$$\mathbf{z} = \tanh(\mathbf{x} * W_{k,f} + V_{k,f} * \mathbf{y}) \odot \sigma(\mathbf{x} * W_{k,g} + V_{k,g} * \mathbf{y}) \tag{6}$$

1.2 Results

The main results here are evaluated on "subjective naturalness" by human evaluators. As such, WaveNets have outperformed previos TTS methods. That's not really that interesting but it makes for a cool listen: here

2 Vector Quantized VAE (VQ-VAE)

Title: Neural Discrete Representation Learning [3].

Main points:

- Avoiding Posterior Collapse
- Discrete Latent
- With the right prior, generates speech/audio well
- Language Learning through raw speech
- Speaker Conversion

The main point of the VQ-VAE lies in that it uses a discrete (i.e. categorical) embedding space as its latent space.

2.1 Model Components and Architecture

The model is described in fig. 4. What's very important to realize is that **the VQ-VAE is deterministic, not stochastic!**

2.2 Discrete Latent Embedding Space

The VQ VAE uses a D-dimensional latent space with K embedding vectors. This means that the latent space **does not sample from a latent space** (so it's not a real VAE) but instead **does a nearest-neighbor embedding lookup**. We define the embedding vectors as

$$e_i \in \mathbb{R}^D, i \in \{1..K\}, \therefore e \in \mathbb{R}^{K \times D}$$

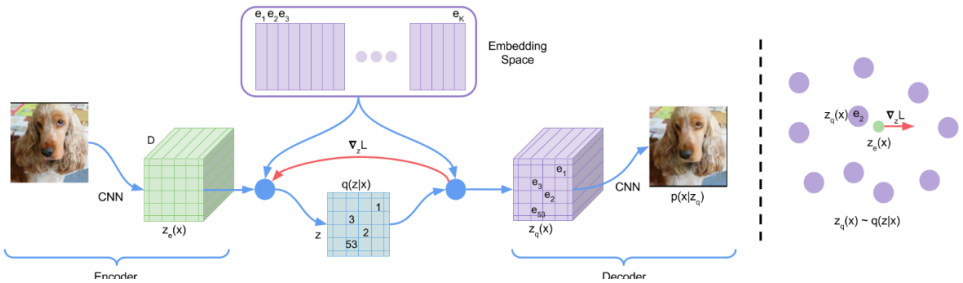

Figure 4: VQ-VAE Architecture. Right: Embedding space. Left: Overall Architecture. Note that the CNN works as a down/upsampling CNN, so as to avoid identity operations all the way through.

2.3 Loss function

The loss function eq. (7) is composed of 3 parts, here covered in detail:

$$L = \log p(x|z_q(x)) + ||\text{sg}[z_e(x)] - e||_2^2 + ||z_e(x) - \text{sg}[e]||_2^2 \tag{7}$$

We denote $z_e(x)$ as the **encoder output** and $z_q(x)$ as the **quantized encoding.** We also use sg to denote the stopgradient operator (identity function forward but 0 partial derivatives).

Reconstruction Loss

$$\log p(x|z_q(x)) +$$

This refers to the probability of the data x given the embedding $z_q(x)$.

The reconstruction loss is optimized by the **encoder** and **decoder**.

Embedding Loss

$$||\text{sg}[z_e(x)] - e||_2^2$$

The embedding loss quantifies how far away from the samples the embeddings are. This loss comes from Vector Quantization (VQ), a dictionary learning algorithm. The VQ objective seen here moves the discretized embedding vectors $e_i \in \mathbb{R}^D$ towards the continuous encoder outputs $z_e(x)$. This means that the model is able to learn embeddings and update them at need.

The embedding loss is optimized by the **embedding**.

Commitment Loss

$$||z_e(x) - \text{sg}[e]||_2^2$$

The commitment loss quantifies how far away from the embeddings a sample is. The embedding space is dimensionless in volume, and therefore the output of the encoder can grow arbitrarily large, while embeddings can't keep up. This equates to the encoder seeing an out-of-distribution sample and encoding it as very far away from the latent space embeddings.

The commitment loss is optimized by the **encoder**.

2.4 Experiments and Results

2.4.1 Images

Downsampling from $128 \times 128 \times 3$ to a $32 \times 32 \times 1$ discretized latent space on the ImageNet ($128 \times 128 \times 3$) dataset.

For images, the encoder/decoder is the PixelCNN.

2.4.2 Audio

For audio, the VQ-VAE is trained on the VCTK dataset, which has 109 different speakers. The encoder is **6 strided convolutions with stride 2 and window-size**

4, corresponding to a downsampling of 64x. The latent space is a single 512-imensional discretized space. In addition to the latent space, the decoder is conditioned on a 1-hot speaker embedding.

In order to make a prior for the latent space distribution, the authors trained a WaveNet model on the latent variables and used it as a prior on the latent space. [2]

Results The VQ-VAE manages to learn to interchange speakers very well. To cite the paper:

> This means that the VQ-VAE has, without any form of linguistic supervision, learned a high-level abstract space that is invariant to low-level features and only encodes the content of the speech.

The authors also ran an experiment where they mapped a 128-dimensional discrete latent space to 41 phonemes. Using this simple mapping they found the accuracy to be 49.3%, without further mappings.

3 Clockwork Variational Autoencoders

The clockwork VAE [5] aims to learn higher-level, abstract prediction timelines without needing to predict the actual images going forward. They term this "Temporally Abstract Latent Dynamics Models".

3.1 CW-VAE Architecture and components

The CW-VAE is composed of a hierarchy of "states" as seen in fig. 5

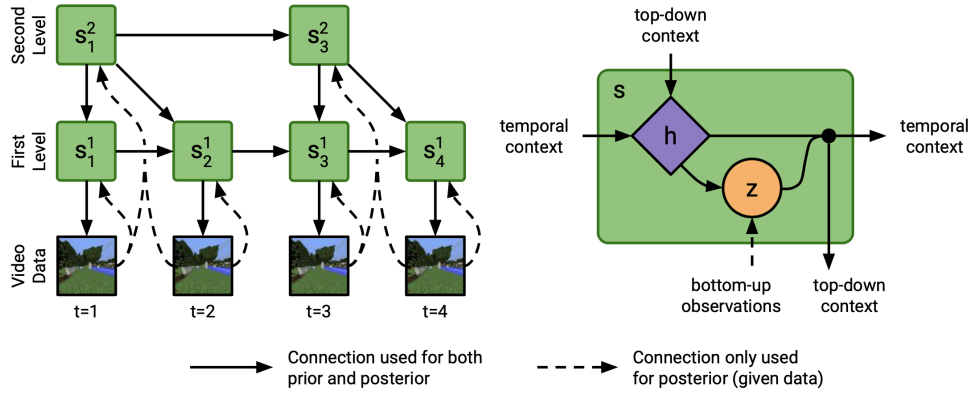

Figure 5: **Arrows:** Solid is the generative model, dashed is the inference model. **Left:** The general setup of the CW-VAE, with a temporal abstraction factor $k = 2$. The upper state only updates every kth timestep, and this would compound in higher hierarchies. As such we see that the video frame with $t = 2$ still will feed into s_1^2, and likewise the frame at $t = 4$ feeds into s_3^2 **Right:** The internals of states s_l^t.

3.1.1 The latent states s_t^l

The latent state is composed of a deterministic variable h_t and a stochastic variable z_t.

Updates to latent states during inference The latent states are updated at the "active" timesteps $\mathcal{T}_l = \{t \in [1, T] | t \text{mod} k^{l-1} = 1\}$, where k is the dilation factor. This means that latent states will only be updated at each k_{l-1} timestep, where l is the "level" of the latent variable. Otherwise, the states are copied from the previous timestep.

During inference, all "active" latents will receive a CNN image embedding (in fig. 5 this is the "bottom-up observations"). The posterior q_t^l for that latent is calculated "as a function (what function?) of the input features, (ASK JAKOB: Is this the arrow from z to the combination node?) the posterior sample at the previous step (temporal context), and the posterior sample above (top-down context)" The posterior / "Gaussian Belief" q_t^l, which is a diagonal Gaussians with means and variances predicted from the deterministic variable.

The deteministic variable is updated with a GRU at every active step.

3.1.2 Embeddings and layers in between.

The authors (Appendix C) claim to have used ”architectures very similar to the DC-GAN”.

The DCGAN paper’s decoder architecture is shown in fig. 6. [4]

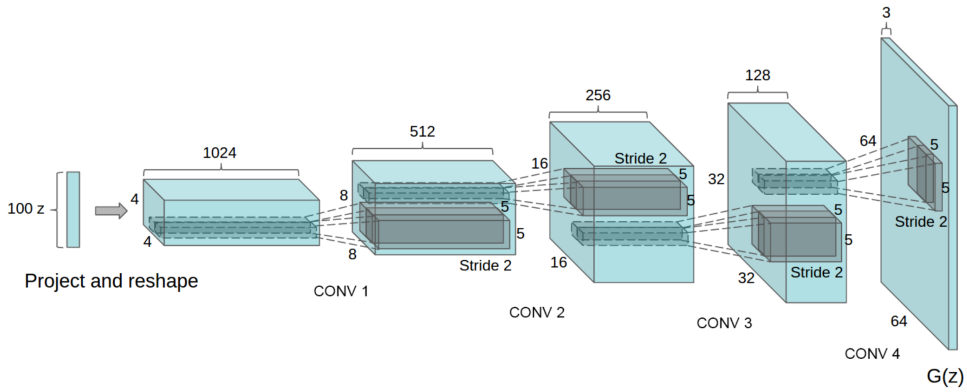

Figure 6: From DCGAN paper: A 100 dimensional uniform distribution Z is projected to a small spatial extent convolutional representation with many feature maps. A series of four fractionally-strided convolutions (in some recent papers, these are wrongly called deconvolutions) then convert this high level representation into a 64 × 64 pixel image. Notably, no fully connected or pooling layers are used.

4 Variational Temporal Abstraction

Authors: Taesup Kim, Sungjin Ahn, Yoshua Bengio [1]

This paper is among the first to introduce a hierarchical recurrent state space model with latent variables.

References

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[2] A. v. d. Oord, S. Dieleman, H. Zen, K. Simonyan, O. Vinyals, A. Graves, N. Kalchbrenner, A. Senior, and K. Kavukcuoglu. WaveNet: A Generative Model for Raw Audio. arXiv:1609.03499 [cs], Sept. 2016. arXiv: 1609.03499.

[3] A. v. d. Oord, O. Vinyals, and K. Kavukcuoglu. Neural Discrete Representation Learning. arXiv:1711.00937 [cs], May 2018. arXiv: 1711.00937.

[4] A. Radford, L. Metz, and S. Chintala. Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks. arXiv:1511.06434 [cs], Jan. 2016. arXiv: 1511.06434.

[5] V. Saxena, J. Ba, and D. Hafner. Clockwork Variational Autoencoders. arXiv:2102.09532 [cs], Feb. 2021. arXiv: 2102.09532.