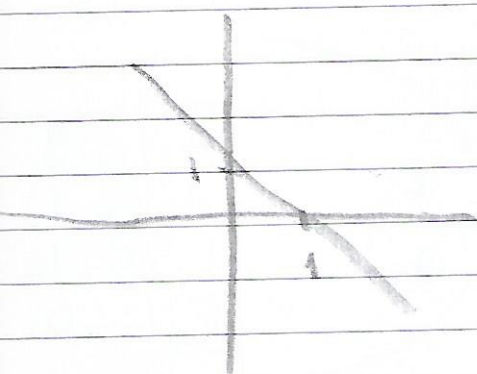$$y = -x + 1$$

$$d((x,y),(0,1)) + d((x,y),(1,0)) = 2^2$$

$$\sqrt{x^2 + (y-1)^2} + \sqrt{(x-1)^2 + y^2} = 3$$

$$\sqrt{x^2 + (y-1)^2} = 3 - \sqrt{(x-1)^2 + y^2}$$

$$x^2 + y^2 - 2y + 1 = 9 - 6\sqrt{(x-1)^2 + y^2} + x^2 - 2x + 1 + y^2$$

$$2(x-y) - 9 = -6\sqrt{x^2 - 2x + 1 + y^2}$$

$$4(x-y)^2 - 36(x-y) + 81 = 36(x^2 - 2x + 1 + y^2)$$

$$4x^2 - 8xy + 4y^2 - 36x + 36y - 36x^2 + 72x - 36y^2 - 36 + 81 = 0$$

$$\boxed{-32x^2 - 32y^2 - 8xy + 36x + 36y + 45 = 0}$$