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1) $\int_0^1 (3x^2 - e^x)\,dx = \lim_{n\to\infty}\left(\sum_{i=1}^{n} 3i^2\Delta x^3 - \sum_{i=1}^{n} e^{i\Delta x}\cdot\Delta x\right)$

$\lim_{n\to\infty}\left[3\Delta x^3 \cdot \frac{n(n+1)(2n+1)}{6} - \frac{\Delta x \cdot e^{\Delta x}(e^{n\Delta x}-1)}{e^{\Delta x}-1}\right]$

L'Hopital

$\lim_{n\to\infty}\left[\frac{3(n+1)(2n+1)}{6n^2} - \frac{\Delta x\, e^{n\Delta x+\Delta x} - \Delta x}{e^{\Delta x}-1}\right]$

$\lim_{n\to\infty}\left[\frac{2n^2+3n+1}{2n^2} - \frac{(1\cdot e^{n\Delta x+\Delta x} + (n+1)e^{n\Delta x+\Delta x}\cdot\Delta x - 1)}{e^{\Delta x}}\right]$

$= 2 - e$

2) $\Delta x = \frac{8-0}{4} = 2$

a) $\sum_{i=1}^{4} f(x_i)\Delta x = 2\,(f(0)+f(2)+f(4)+f(6)) = 2(2+1+2-2) = 6$

b) $\sum_{i=1}^{4} f(x_i)\Delta x = 2(f(2)+f(4)+f(6)+f(8)) = 2(1+2-2+1) = 4$

c) $\sum_{i=1}^{4} f(x_i)\Delta x = 2(f(1)+f(3)+f(5)+f(7)) = 2(3+2+1-1) = 10$

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3) a) $\int_a^b e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \left(\int_0^a e^{-t^2} dt + \int_0^b e^{-t^2} dt \right)$

$= \frac{2}{\sqrt{\pi}} \left(- \int_a^0 e^{-t^2} dt + \int_0^b e^{-t^2} dt \right)$

$= \frac{2}{\sqrt{\pi}} \left(E(b) - E(a) \right)$

3) b) $d\left(\frac{2}{\sqrt{\pi}} e^{x^2} \int_0^x e^{-t^2} dt \right) / dx = \frac{2}{\sqrt{\pi}} \cdot 2x \cdot e^{x^2} \int_0^x e^{-t^2} dt + \frac{2}{\sqrt{\pi}} \cancel{e^{x^2}} \cdot \cancel{e^{-x^2}}$

$\Rightarrow \quad 2x f(x) + \frac{2}{\sqrt{\pi}}$

4) a) $A = A_1 - A_2$

A_1: $x^2 = 2x + 8$

$x^2 - 2x - 8 = 0 \longrightarrow A_1 = \int_{-2}^{4} -x^2 + 2x + 8 \, dx$

$A_1 = -\frac{x^3}{3} + 8x + x^2 \Big]_{-2}^{4} = 36$

A_2: $x^2 = 2 - x^2 \qquad A_2 = \int_{-1}^{1} 2 - x^2 - x^2 dx = 2x - \frac{2x^3}{3} \Big]_{-1}^{1}$

$x = \pm 1$

$A_2 = \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right) = 2,\overline{66}$

$A = 36 - 2,66 \approx 33,33$

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4)b) $A_{r_1} = \int_{-\pi/4}^{\pi/4} 2\cos\theta \, d\theta = 2\int_0^{\pi/2} \cos\theta \, d\theta = 2\sin\frac{\pi}{2} \cancel{-2\sin 0}$

$A_{r_1} = 2$

$1 = 2\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}$

$A_{c_1} = \int_{-\pi/3}^{\pi/3} 1 \, d\theta = \frac{\pi}{3} - \left(-\frac{\pi}{3}\right) = \frac{2\pi}{3}$

$A_{T_1} = A_{r_1} - A_{c_1} = 2 - \frac{2\pi}{3}$

$A = 4A_{T_1} = 8 - \frac{8\pi}{3}$

5)a) $A = \pi(f(x))^2$

$V = \int_{-1}^{3} \pi(\cancel{\sqrt{1+y}})^{\cancel{2}} dy \rightarrow V = \pi y + \frac{\pi y^2}{2}\Big]_{-1}^{3}$

$V = \left(3\pi + \frac{9\pi}{2}\right) - \left(-\pi + \frac{\pi}{2}\right) \rightarrow V = \frac{15\pi}{2} + \frac{\pi}{2} = 8\pi$

5)b) $A = \pi(R^2 - r^2) = \pi(x^2)^2 - \pi(x^3)^2 = \pi(x^4 - x^6)$

$V = \int_0^1 \pi(x^4 - x^6)\, dx = \frac{\pi x^5}{5} - \frac{\pi x^7}{7}\Big]_0^1 = \frac{\pi}{5} - \frac{\pi}{7} = \frac{2\pi}{35}$

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6) $\frac{d(\ln(\sec x))}{dx} = \frac{1}{\cancel{\sec x}} . \cancel{\sec x} . \tan x = \tan x$

$L = \int_0^{\pi/4} \sqrt{1+\tan^2 x}\, dx = \int_0^{\pi/4} \sec x \, dx$

$L = \ln|\sec x + \tan x| \Big]_0^{\pi/4} = \ln|\sqrt{2}+1| - \cancel{\ln|1|}^{0}$

$L = \ln(\sqrt{2}+1)$

7) a) $A_{\square} = 20 \Rightarrow A_T \cong 26 . 20 = 520$

$V_m \cong \frac{520}{12} \approx 43,\overline{33}$

b) Sim, pelo teorema do valor médio, em um t qualquer, a velocidade instantânea de uma partícula será igual a velocidade media.

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