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1) a) $AA^T = 7A$

$(A+B)(A+B)^T = 7(A+B)$

$\underbrace{AA^T}_{7A} + \underbrace{AB^T + BA^T}_{\text{termo sobrando}} + \underbrace{BB^T}_{7B} = 7(A+B) \notin U$

não é um subspaço vetorial de V

b) $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} , \begin{matrix} 7a - 3b + c - 7d = 0 \\ a + 7b - 7d = 0 \end{matrix} \right\} \begin{matrix} \Rightarrow c = 7d - 7a + 3b \\ \Rightarrow a = -7b + 7d \end{matrix}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$

$(A+B) = \begin{bmatrix} a+x & b+y \\ c+z & d+w \end{bmatrix} \in W$

$c+z = 3(b+y) + 7(d+w) - 7(a+x) \Rightarrow$ satizfaz $c = 7d - 7a + 3b$

$a+x = 7(y-b) + 7(d+w) \Rightarrow$ satizfaz $a = 7d - 7b$

$KA = \begin{bmatrix} Ka & Kb \\ Kc & Kd \end{bmatrix}$, $\begin{matrix} Kc = 7Kd - 7Ka + 3Kb \\ Ka = 7Kd - 7Kb \end{matrix} \in W$

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c) base de W feita por

$$\begin{bmatrix} 7d-7b & b \\ 7b-7a+3b & d \end{bmatrix} = a\begin{bmatrix} 0 & 0 \\ -7 & 0 \end{bmatrix} + b\begin{bmatrix} -7 & 1 \\ 7 & 0 \end{bmatrix} + d\begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\beta W = \left\{ \begin{bmatrix} 0 & 0 \\ -7 & 0 \end{bmatrix} ; \begin{bmatrix} -7 & 1 \\ 7 & 0 \end{bmatrix} ; \begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix} \right\} \quad \dim W = 3.$$

2) $V_1 = (-1, 0, 1, 2)$, $V_2 = (1, -7, 14, -1)$, $V_3 = (0, 2, -1, 1)$, $V_4 = (2, -4, -4, -2)$

$$\begin{array}{cccccc} & & & & u & w \\ \end{array}$$
$$\left[\begin{array}{cccc|c|c} -1 & 1 & 0 & 2 & 2 & 1 \\ 0 & -7 & 2 & -4 & -7 & -1 \\ 1 & 14 & -1 & -4 & 13 & 1 \\ 2 & -1 & 1 & -2 & -3 & -1 \end{array}\right] \begin{array}{l} \\ L_3+L_1 \\ L_4+2L_1 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c|c} -1 & 1 & 0 & 2 & 2 & 1 \\ 0 & -7 & 2 & -4 & -7 & -1 \\ 0 & 15 & -1 & -2 & 15 & 2 \\ 0 & 1 & 1 & 2 & 3 & 1 \end{array}\right] \; L_3 - 15L_4$$

$$\left[\begin{array}{cccc|c|c} -1 & 1 & 0 & 2 & 2 & 1 \\ 0 & -7 & 2 & -4 & -7 & -1 \\ 0 & 0 & -16 & 28 & -30 & -13 \\ 0 & 1 & 1 & 2 & 3 & 1 \end{array}\right] L_4 + \frac{L_2}{7} \Rightarrow \left[\begin{array}{cccccc} -1 & 1 & 0 & 2 & 2 & 1 \\ 0 & -7 & 2 & -4 & -7 & -1 \\ 0 & 0 & -16 & 28 & -30 & -13 \\ 0 & 0 & 9 & 10 & 14 & 6 \end{array}\right]$$

$$L_4 + \frac{9}{16} L_3$$

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$-\frac{15 \cdot 9}{8} + 14$ $\quad 14 \cdot 3 = 42$

$$\left[\begin{array}{cccc|c|c} -1 & 1 & 0 & 2 & 2 & 1 \\ 0 & -7 & 2 & -4 & -7 & -1 \\ 0 & 0 & -16 & 28 & -30 & -13 \\ 0 & 0 & 0 & \frac{103}{4} & \frac{23}{8} & \frac{-21}{16} \end{array}\right]$$

$\therefore$ U e W são uma combinação linear de V_1 V_2 V_3 e V_4.

b)
$$\left[\begin{array}{cccc|c} -1 & 1 & 0 & 2 & 0 \\ 0 & -7 & 2 & -4 & 0 \\ 0 & 0 & -16 & 28 & 0 \\ 0 & 0 & 0 & \frac{103}{4} & 0 \end{array}\right]$$

O conjunto A é linearmente dependente pois $-2V_1 + 2V_3 = V_4$.

c) Base $= \{(-1,0,1,2);(1,-7,14,-1);(0,-2,-1,1)\}$

$x = -a + b$
$y = -7b - 2c$
$z = a + 14b - c \Rightarrow z + w = 3a + 13b$
$w = 2a - b + c \quad z + w + 3x = 16b \Rightarrow b = \frac{z + w + 3x}{16}$

$\Rightarrow w + \frac{y}{2} = 2a - \frac{9b}{2} \Rightarrow -\frac{3w}{2} - \frac{3y}{4} = -3a + \frac{27b}{4}$

$z + \frac{w}{2} - \frac{3y}{4} = \frac{27b}{4} \Rightarrow b = \frac{4}{27}\left(z - \frac{w}{2} - \frac{3y}{4}\right)$

$$\frac{4}{27}\left(z - \frac{w}{2} - \frac{3y}{4}\right) = \frac{z + w + 3x}{16}$$

$$64z - 32w - 48y = 27z + 27w + 81x$$

$$37z - 59w - 48y - 81x = 0$$

$$\{(x, y, z, w) \in \mathbb{R}^4 \,/\, 81x + 48y - 37z + 59w = 0\}$$

d) Base $= \{(-1, 0, 1, 2); (1, -7, 14, -1); (0, -2, -1, 1)\}$

Dim $= 3$.