EE 433 Introduction to DSP

Computer HW Part #2

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5. High-pass Filter

$$cutoff\ frequency \Rightarrow \omega_p = 0.7\pi$$

A. System Function H(z)

$$Highpass \qquad Z^{-1} = -\frac{z^{-1} + \alpha}{1 + \alpha z^{-1}}, \qquad \alpha = -\frac{\cos\left(\frac{\theta_p + \omega_p}{2}\right)}{\cos\left(\frac{\theta_p - \omega_p}{2}\right)}, \qquad \theta_p = 0.28808\pi$$

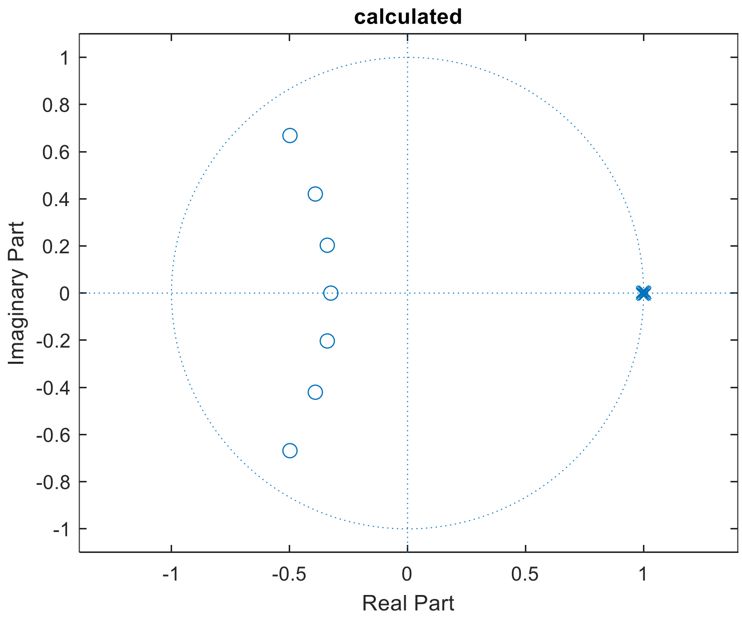
$$\alpha = 0.02346$$

Zeros of $H_{lp}(Z)$ =	Poles of $H_{lp}(Z)$ =
-1.0067 + 0.0033i	0.5257 + 0.6525i
-1.0067 - 0.0033i	0.5257 - 0.6525i
-1.0015 + 0.0073i	0.4144 + 0.4126i
-1.0015 - 0.0073i	0.4144 - 0.4126i
-0.9954 + 0.0057i	0.3615 + 0.1997i
-0.9954 - 0.0057i	0.3615 - 0.1997i
-0.9927 + 0.0000i	0.3457 + 0.0000i

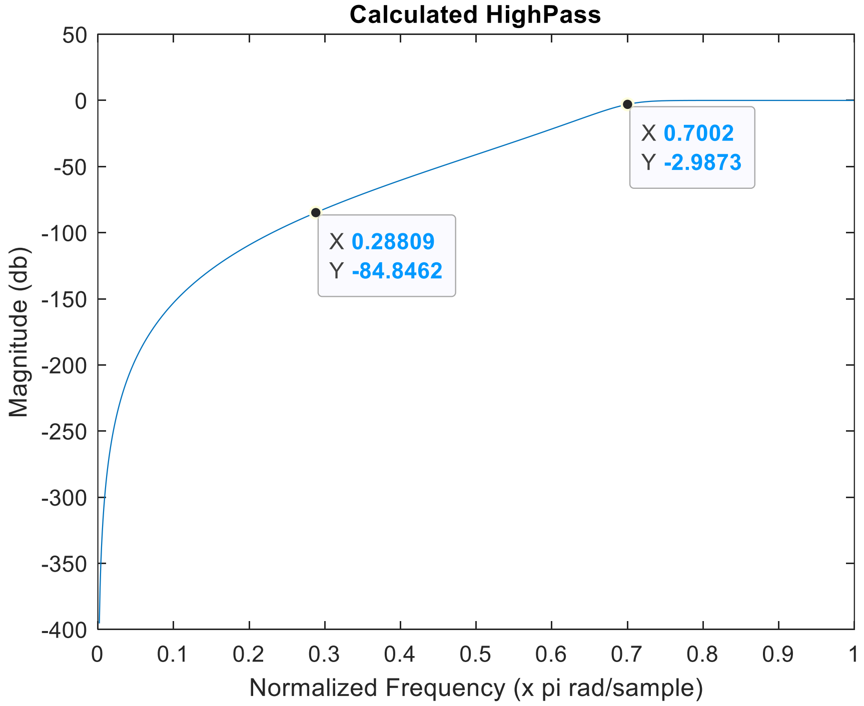
$$H(z) = H_{lp}(Z)\big|_{Z^{-1} = -[(z^{-1} - 0.02346)/(1 - 0.02346z^{-1})]}$$

Calculation are done with MATLAB.

Zeros of $H(Z)$ =	Poles of $H(Z)$ =
1.0084 + 0.0000i	-0.4981 + 0.6684i
1.0052 + 0.0066i	-0.4981 - 0.6684i
1.0052 - 0.0066i	-0.3907 + 0.4204i
0.9981 + 0.0081i	-0.3907 - 0.4204i
0.9981 - 0.0081i	-0.3400 + 0.2030i
0.9925 + 0.0036i	-0.3400 - 0.2030i
0.9925 - 0.0036i	-0.3249 + 0.0000i



B. Plot



6. Band-pass Filter

$$cutoff\ frequencies => \omega_{p1} = 0.3\pi,\ \ \omega_{p2} = 0.5\pi$$

A. System Function H(z)

$$Bandpass \qquad Z^{-1} = -\frac{z^{-2} - \frac{2\alpha k}{k+1}z^{-1} + \frac{k-1}{k+1}}{\frac{k-1}{k+1}z^{-2} - \frac{2\alpha k}{k+1}z^{-1} + 1}, \qquad \alpha = -\frac{\cos\left(\frac{\omega_{p2}+\omega_{p1}}{2}\right)}{\cos\left(\frac{\omega_{p2}-\omega_{p1}}{2}\right)},$$

$$k = \cot\left(\frac{\omega_{p2}-\omega_{p1}}{2}\right)\tan\left(\frac{\theta_p}{2}\right), \qquad \theta_p = 0.28808\pi$$

$$\alpha = 0.02346, \qquad k = 1.49625$$

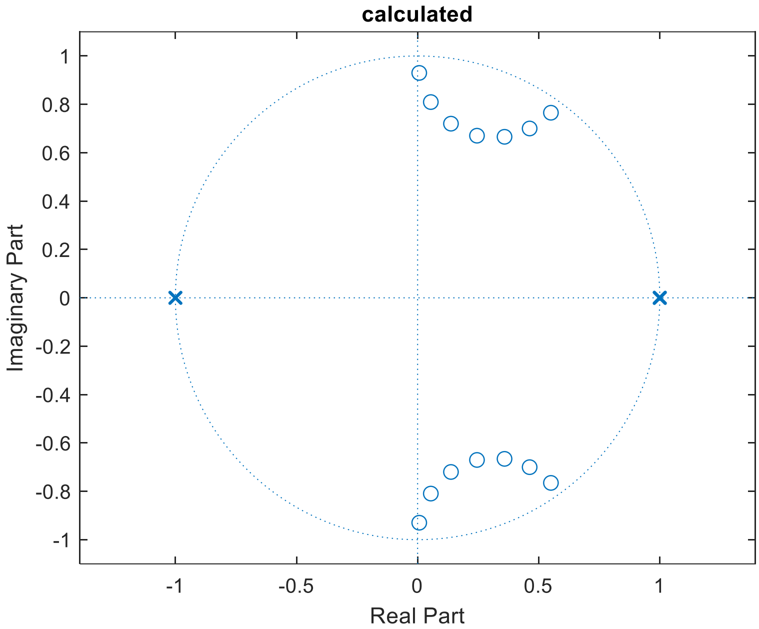
Zeros of $H_{lp}(Z)$ are the same as question 5.

$$H(z) = H_{lp}(Z)\Big|_{Z^{-1}=-\left[\frac{z^{-2}-\frac{2*0.02346*1.49625}{1.49625+1}z^{-1}+\frac{1.49625-1}{1.49625+1}}{\frac{1.49625-1}{1.49625+1}z^{-2}-\frac{2*0.02346*1.49625}{1.49625+1}z^{-1}+1}\right]}$$

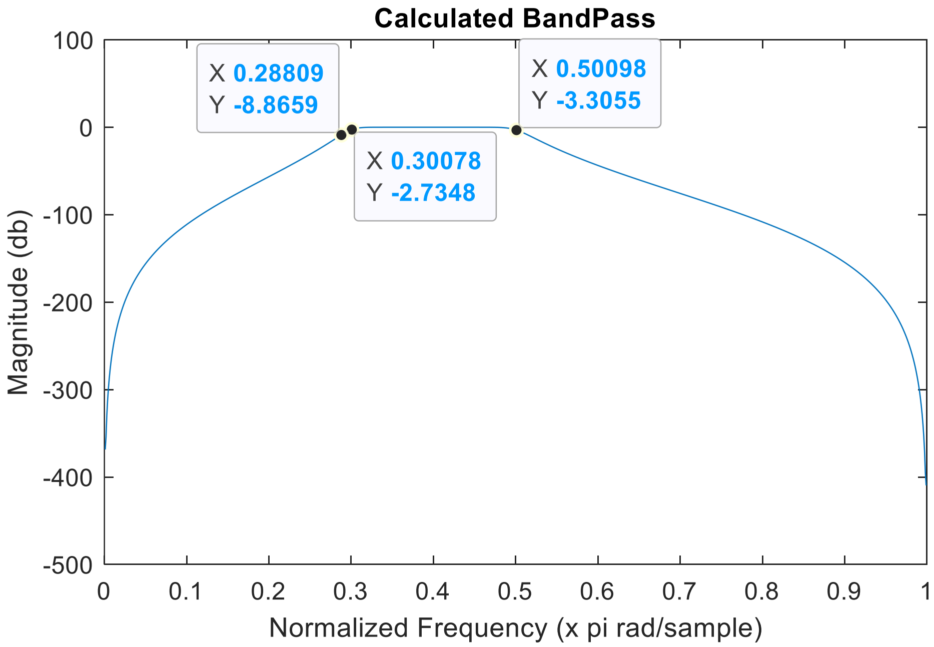
Calculation are done with MATLAB.

Zeros of $H(Z)$ =	Poles of $H(Z)$ =
-1.0036 + 0.0018i	0.0068 + 0.9300i
-1.0036 - 0.0018i	0.0068 - 0.9300i
-1.0008 + 0.0039i	0.0546 + 0.8095i
-1.0008 - 0.0039i	0.0546 - 0.8095i
-0.9975 + 0.0031i	0.5506 + 0.7653i
-0.9975 - 0.0031i	0.5506 - 0.7653i
-0.9961 + 0.0000i	0.1378 + 0.7200i
1.0040 + 0.0019i	0.1378 - 0.7200i
1.0040 - 0.0019i	0.4627 + 0.7002i
1.0010 + 0.0044i	0.4627 - 0.7002i

1.0010 - 0.0044i	0.3590 + 0.6659i
0.9972 + 0.0035i	0.3590 - 0.6659i
0.9972 - 0.0035i	0.2452 + 0.6704i
0.9956 + 0.0000i	0.2452 - 0.6704i



B. Plot



7. Kaiser Window

A. Formula h[n]

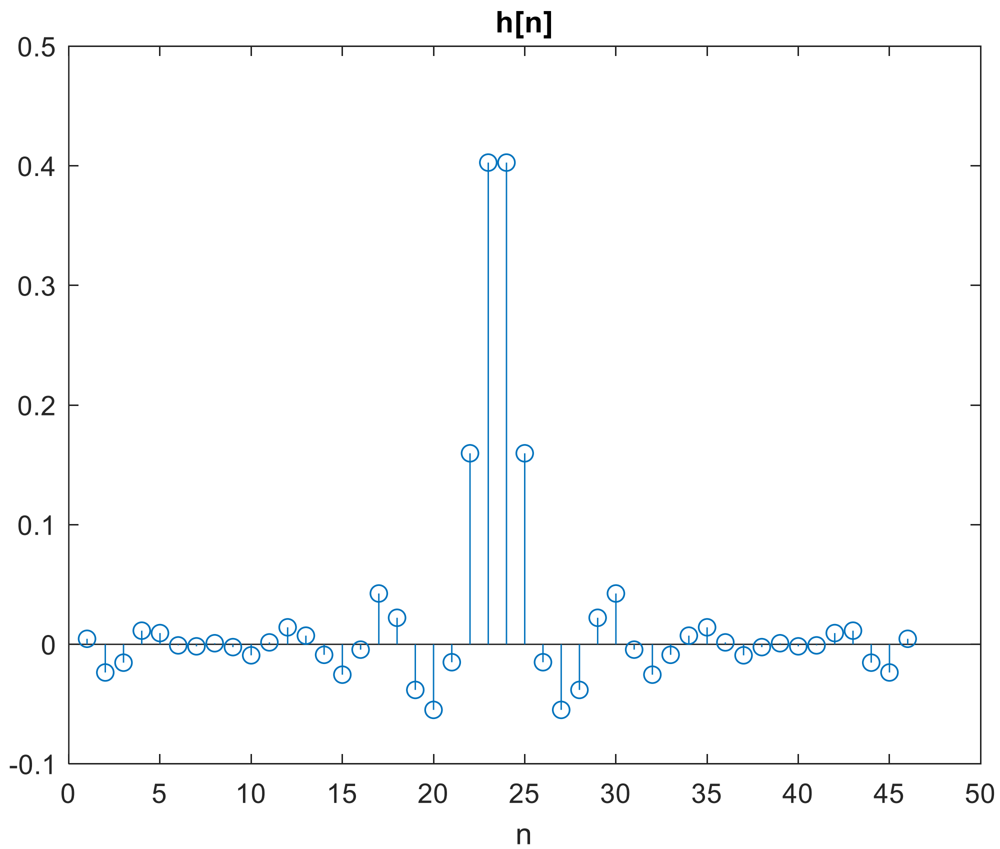
$$A_1 = 1 = 0dB$$
$$A_2 = 0.178 = -15dB$$
$$A_3 = 0.01 = -40dB = \delta$$

$$\Delta\omega \leq 0.1\pi$$

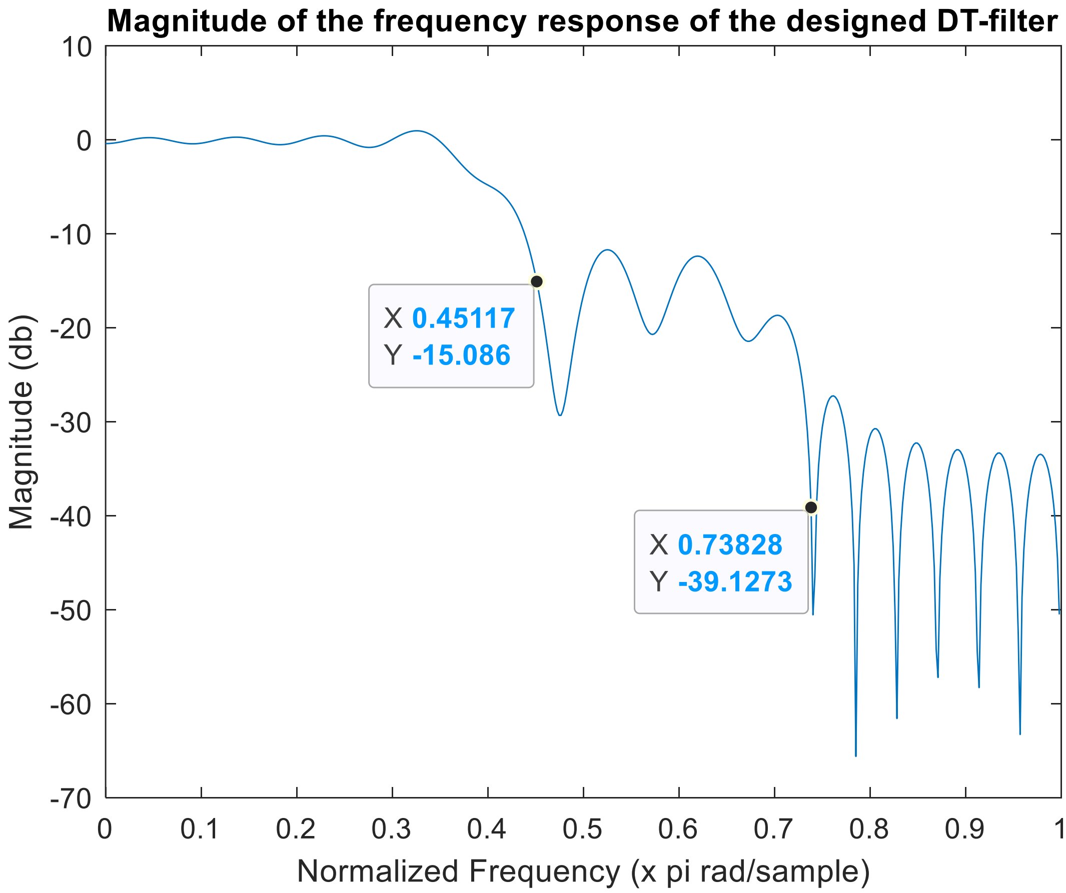
$$A = -20\log\left(10^{\delta}\right) = 40$$
$$\beta = 3.395 \quad (21 \leq A \leq 50)$$
$$M = \frac{A-8}{2.285\Delta\omega} = 44.5 \approx 45$$

$$h[n] = \sum_{k=1}^{N_{mb}=2} (A_k - A_{k+1}) \frac{sin\omega_c(n-\alpha)}{\pi(n-\alpha)} . \frac{I_0[\beta(1-[(n-\alpha)/\alpha]^2)^{\frac{1}{2}}]}{I_0(\beta)}$$

B. Plot h[n]

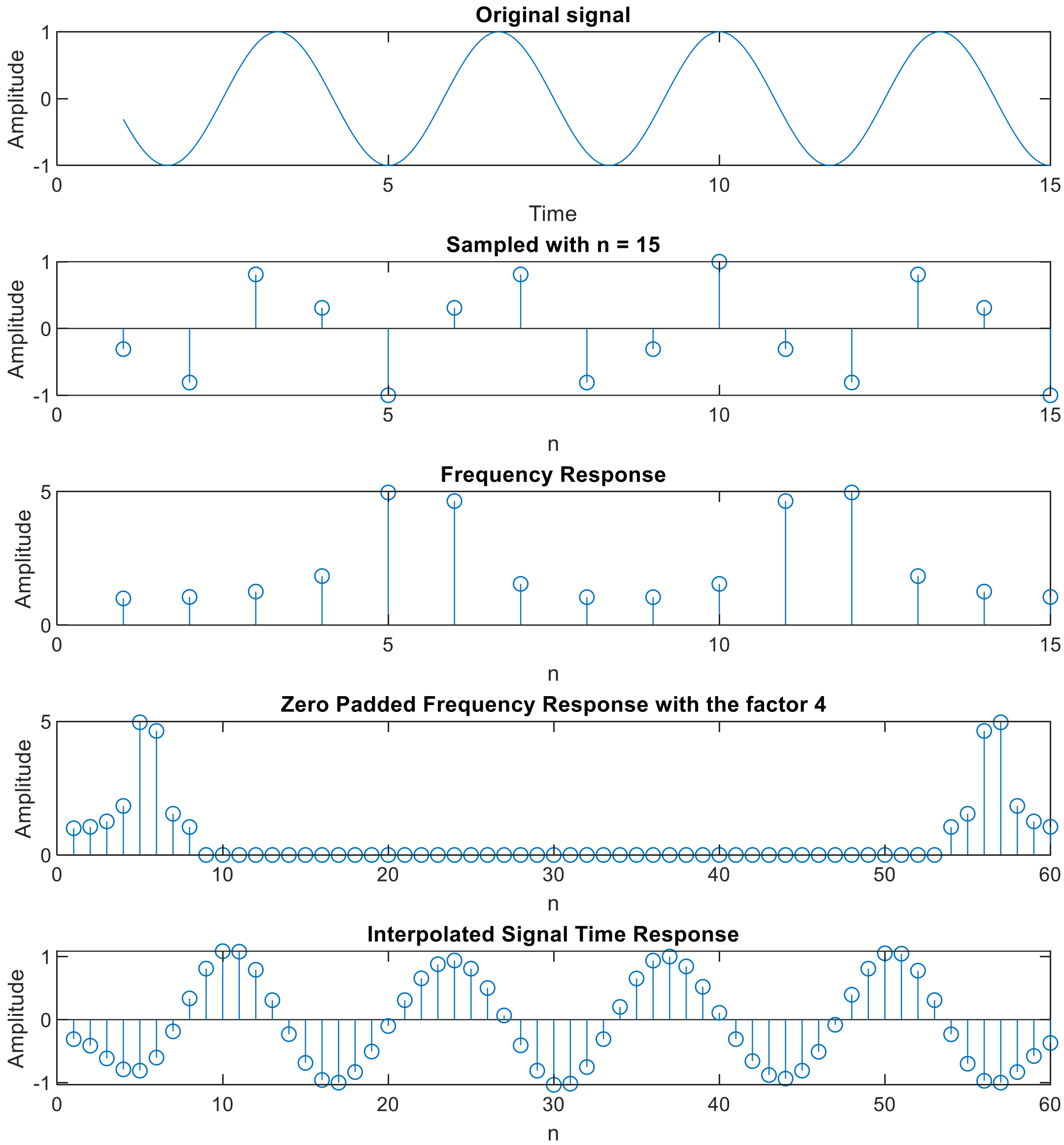


C. Plot the magnitude of the frequency response



8. Interpolation

Graph 1. The Original Signal (Time Response)
Graph 2. The Signal sampled with n = 15 (Time Response)
Graph 3. Frequency Response of the Signal
Graph 4. Zero Padded Signal by a Factor of 4 at the Center (Frequency Response)
Graph 5. Time Response of the Interpolated Signals



MATLAB CODE

Question 5

```matlab
%% 1
fs = 10000;
N = 1024;
theta = 0.28808*pi;
n = 7;
[b1, a1] = butter(n, theta/pi);
[h1, w1] = freqz(b1, a1, N);

roots(b1); %zeros of lowpass
roots(a1); %polse of lowpass

%% 5
wp = 0.7*pi;
alfa = -(cos((theta + wp)/2)/cos((theta -
wp)/2));

syms Z(z)
Z(z) = -(z.^-1 + alfa) / (1 + alfa*z.^-1);

%Nominator
pol = 0;
count = 0;
for i = b1
    pol = pol + i*Z(z).^count;
    count = count + 1;
end
%Denominator
den = 0;
count = 0;
for i = a1
    den = den + i*Z(z).^count;
    count = count + 1;
end

trfunc = pol/den;

[num, denum] = numden(trfunc);

bhp = sym2poly(num);
ahp = sym2poly(denum);
[hhp, whp] = freqz(bhp, ahp, N);

% Built-in
[bbi, abi] = iirlp2hp(b1, a1, theta/pi, wp/pi);
[hbi, wbi] = freqz(bbi, abi, N);

figure
zplane(ahp,bhp);
title("calculated");

roots(bhp); %zeros of calculated transfer
function
roots(ahp); %poles of calculated transfer
function

figure
subplot(311); plot(w1/pi, 20*log10(abs(h1)));
title('LowPass From Qustion 1');
xlabel('Normalized Frequency (x pi
rad/sample)');
ylabel('Magnitude (db)');
subplot(312);
plot(wbi/pi, 20*log10(abs(hbi)));
title('Builtin HighPass');
xlabel('Normalized Frequency (x pi
rad/sample)');
ylabel('Magnitude (db)');
subplot(313);
plot(whp/pi, 20*log10(abs(hhp)));
title('Calculated HighPass');
xlabel('Normalized Frequency (x pi
rad/sample)');
ylabel('Magnitude (db)');
```

Question 6

```matlab
%% 6
wp1 = 0.3*pi;
wp2 = 0.5*pi;

alfabp = cos((wp2 + wp1)/2) / cos((wp2 -
wp1)/2);
kbp = cot((wp2 - wp1)/2) * tan(theta/2);


syms Z(z)
Zbp(z) = -(z.^-2 - (2*alfabp*kbp*z.^-1)/(kbp+1)
+ (kbp-1)/(kbp+1))/ (((kbp-1)/(kbp+1))*z.^-2 -
(2*alfabp*kbp*z^-1)/(kbp+1) + 1);

%Denominator
polbp = 0;
count = 0;
for i = b1
    polbp = polbp + i*Zbp(z).^count;
    count = count + 1;
end

%Denominator
denbp = 0;
count = 0;
for i = a1
    denbp = denbp + i*Zbp(z).^count;
    count = count + 1;
end


trfuncbp = polbp/denbp;
[numbp, denumbp] = numden(trfuncbp);

bbp = sym2poly(numbp);
abp = sym2poly(denumbp);
[hbp, wbp] = freqz(bbp, abp, N);


% Built-in
[abpbi, bbpbi] = iirlp2bp(b1, a1, theta/pi,
[wp1/pi wp2/pi]);
[hbpbi, wbpbi] = freqz(abpbi, bbpbi, N);

figure
zplane(abp,bbp);
title("calculated");

roots(bbp); %zeros of calculated transfer
function
roots(abp); %poles of calculated transfer
function

figure
subplot(311);
plot(w1/pi, 20*log10(abs(h1)));
title('LowPass From Qustion 1');
xlabel('Normalized Frequency (x pi
rad/sample)');
ylabel('Magnitude (db)');
subplot(312);
plot(wbpbi/pi, 20*log10(abs(hbpbi)));
title('Builtin BandPass');
xlabel('Normalized Frequency (x pi
rad/sample)');
ylabel('Magnitude (db)');
subplot(313);
plot(wbp/pi, 20*log10(abs(hbp)));
title('Calculated BandPass');
xlabel('Normalized Frequency (x pi
rad/sample)');
ylabel('Magnitude (db)');
```

Question 7

```
%% 7
A1 = 1;
A2 = 0.178;
A3 = 10^-2;

w1 = 0.4*pi;
w2 = 0.7*pi;

A = 40;
beta = 3.395;
Nmb = 2;
M = 45;
alfa = M/2;
G = [A1 A2];
w = [w1 w2];

h1 = zeros(M+1,1);
for i=1:M+1
    h1(i)=(G(1)-G(2))*(sin(w(1)*((i-1)-
alfa))/(pi*((i-1)-
alfa)))*(besselj(0,beta*(1-((i-1-
alfa)/alfa)^2)^(1/2)))/besselj(0,beta);
end

h2 = zeros(M,1);
for i=1:M+1
    h2(i)=(G(2)-A3)*(sin(w(2)*((i-1)-
alfa))/(pi*((i-1)-
alfa)))*(besselj(0,beta*(1-((i-1-
alfa)/alfa)^2)^(1/2)))/besselj(0,beta);
end

hn = h1 + h2;
[hfreq, wferq]=freqz(hn);

figure
stem(hn);
title('h[n]');
xlabel('n');

figure
plot(wferq/pi,20*log10(abs(hfreq)));
title('Magnitude of the frequency
response of the designed DT-filter');
xlabel('Normalized Frequency (x pi
rad/sample)');
ylabel('Magnitude (db)');
```

Question 8

```
%% 8
n = 1:15;
n1 = 1:0.01:15;
x = cos(2*pi*0.3*n);
x1 = cos(2*pi*0.3*n1);
factor = 4;

figure
plot(n1,x1);
axis([1 15 -inf inf]);
title("Original signal");
ylabel('Amplitude');
xlabel('Time');

figure
stem(n,x);
title("Sampled with n = 15");
ylabel('Amplitude');
xlabel('n');

pad = length(n)*(factor - 1);
half = ceil((length(x)+1)/2);
z = fft(x);

figure
stem(abs(z));
title("Frequency Response");
ylabel('Amplitude');
xlabel('n');

zeropading = zeros(1, pad);
zeropadded = [z(1:half) zeropading
z(half+1:end)];

figure
stem(abs(zeropadded));
title("Zero Padded Frequency
Response with the factor 4");
ylabel('Amplitude');
xlabel('n');

interpolated =
real(ifft(zeropadded))*factor;

figure
stem(interpolated);
title("Interpolated Signal Time
Response");
ylabel('Amplitude');
xlabel('n');
```