EE 433 Introduction to DSP

Computer HW Part #1

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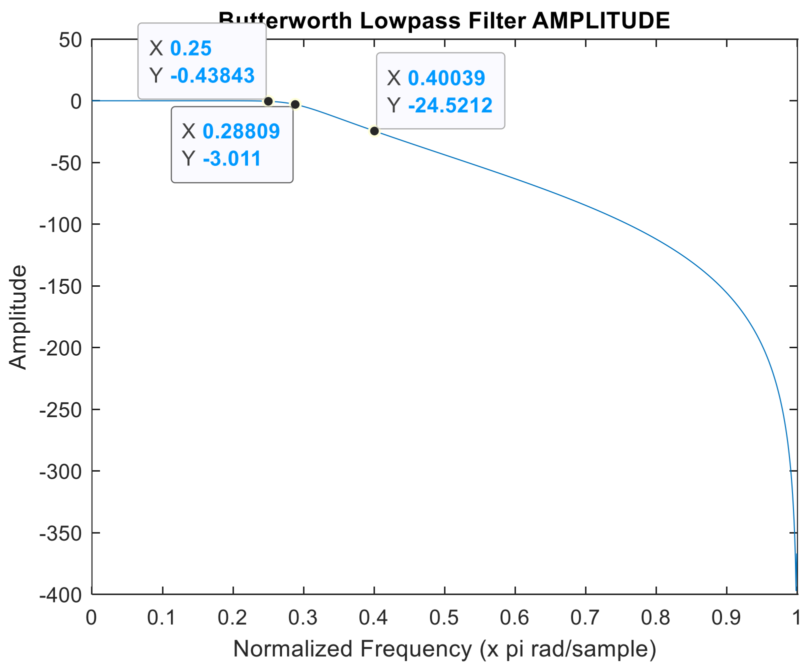
$$0.9 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq \omega \leq 0.25\pi$$
$$|H(e^{j\omega})| \leq 0.1, \quad 0.4\pi \leq \omega \leq \pi$$

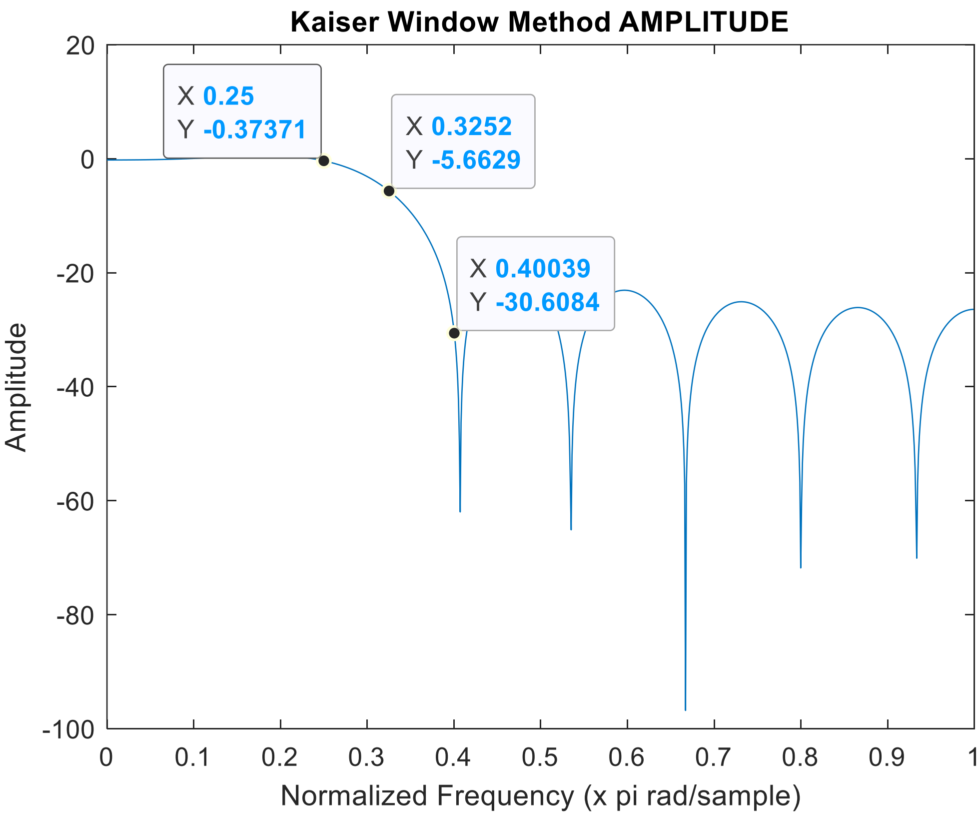
1 Butterworth Lowpass Filter

$$T_d = 1, \omega = \Omega$$
$$0.9 \leq |H_c(j\Omega)| \leq 1, \quad 0 \leq |\Omega| \leq 0.25\pi$$
$$|H_c(j\Omega)| \leq 0.1, \quad 0.4\pi \leq |\Omega| \leq \pi$$
$$Passband => |H_c(j0.25\pi)| \geq 0.9$$
$$Stopband => |H_c(j0.4\pi)| \leq 0.1$$
$$|H_c(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$
$$1 + \left(\frac{0.25\pi}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.9}\right)^2, \quad 1 + \left(\frac{0.4\pi}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.1}\right)^2$$
$$\left(\frac{0.25\pi}{\Omega_c}\right)^{2N} = 0.23456, \quad \left(\frac{0.4\pi}{\Omega_c}\right)^{2N} = 99$$
$$\frac{\left(\frac{0.25\pi}{\Omega_c}\right)^{2N}}{\left(\frac{0.4\pi}{\Omega_c}\right)^{2N}} = \frac{0.23456}{99}$$
$$(0.625)^{2N} = 2.36929 * 10^{-3}$$
$$2N = \log_{0.625} 2.36929 * 10^{-3}$$
$$N = 6.43097 \approx 7, \quad \Omega_c = 0.905033162 = 0.28808\pi$$



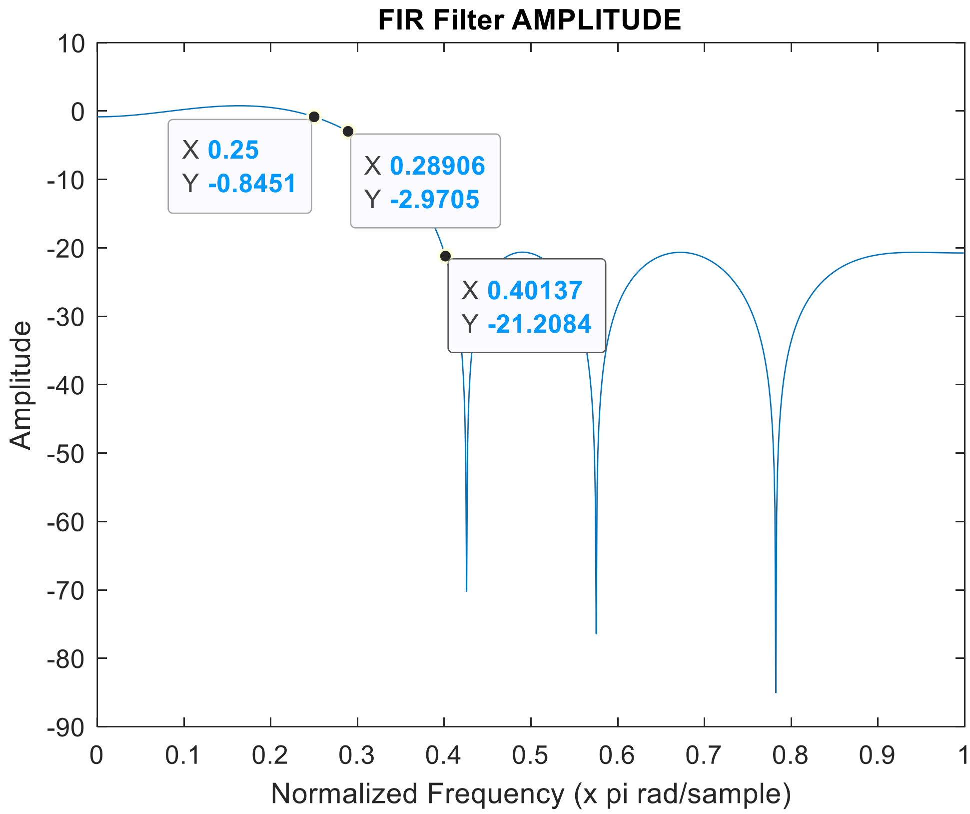
2 Kaiser window method

$$\omega_p = 0.25\pi, \qquad \omega_s = 0.4\pi$$
$$\omega_c = \frac{(\omega_p + \omega_s)}{2} = 0.325\pi$$
$$\Delta\omega = \omega_s - \omega_p = 0.15\pi$$
$$\delta = 0.1$$
$$A = -20\log_{10}\delta = -20\log_{10}0.1 = 20$$
$$A < 21 \;\; => \;\; \beta = 0\ (Rectangular\ Window)$$
$$M = \frac{A-8}{2.285\Delta\omega} = \frac{20-8}{2.285 * 0.15\pi} = 11.144 \approx 12 \pm 2$$
$$\alpha = \frac{M}{2} = 7, \qquad L = M + 1 = 15\ (window\ length)$$



3 FIR filter

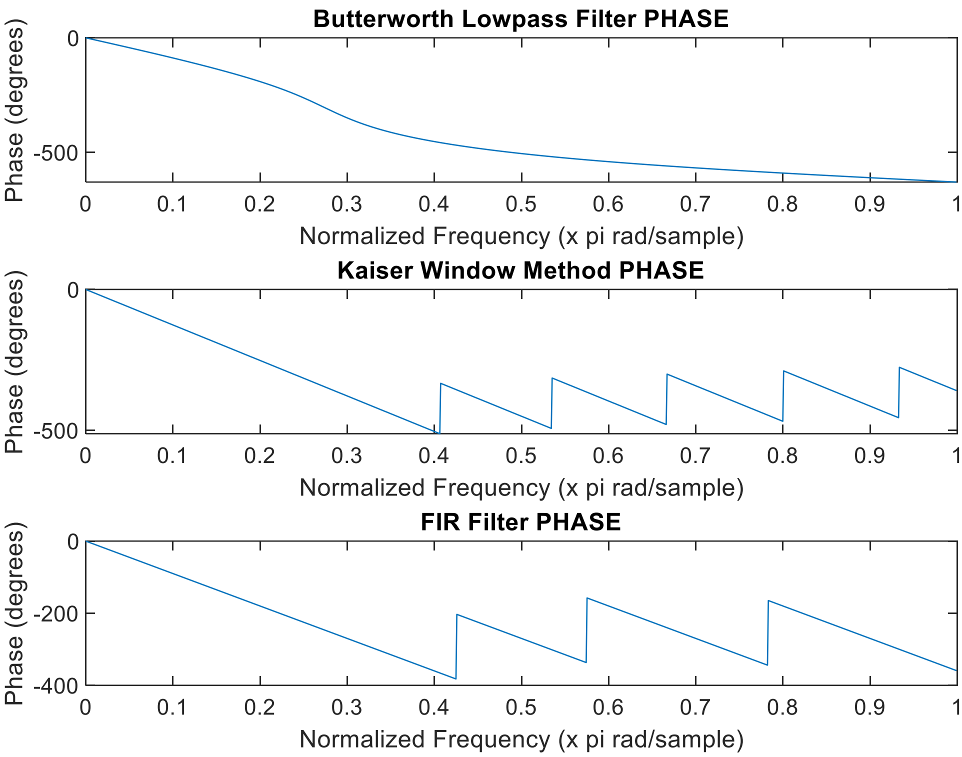
To design FIR filter, I used firpmord and firpm functions in MATLAB.



4(a) Phases of the frequency responses corresponding to the filters

Kaiser Window and FIR filter have linear phase at all frequencies which makes them stable and because of that, they have constant group delay. Even though Butterworth filter is mostly linear, It is nonlinear at the transition band $0.25\pi \leq \omega \leq 0.4\pi$.

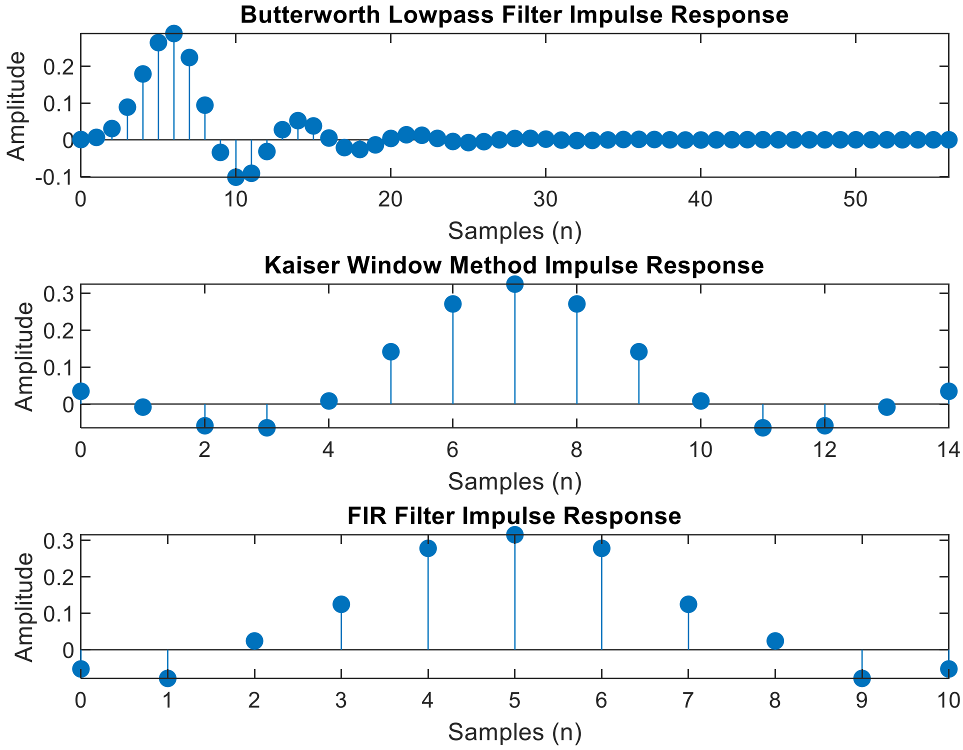
Kaiser Window and FIR filter have phase wraps after the main lobe. Because phase change is greater than Butterworth filter.



4(b) Impulse responses of the filters

Butterworth filter has fewer order which makes it faster than Kaiser and FIR. It never stops its oscillation around zero, thus it does not reach exact zero at all. That is why, it is length is longer and less stable than others. Also, because of the feedback system, we observe recursive behavior.

On the other hand, Kaiser Window and FIR filter has symmetric at the center and their orders are even. Thus, they are Type I FIR Linear Phase Systems. Delay of Kaiser is 7, FIR is 5 which are half of their order. Also, their lengths are 15 and 11 which are order + 1. Because they have greater order, they are slower than Butterworth filter. However, they have stable behavior.



MATLAB CODE

```matlab
clear all; clc; close all;
% DSP Homework 1
Fs = 10000;
N = 1024;

% Butterworth Lowpass Filter
[b1,a1] = butter(7,0.28808);
[h1 w1] = freqz(b1,a1,N);

% Kaiser Window Method
beta = 0;
n = 12+2;
Wc = 0.325;
hh =
fir1(n,Wc,'low',kaiser(n+1,beta),'nosc
ale');
[h2 w2] = freqz(hh,1,N);

% FIR Filter
[n1,fo,ao,w] = firpmord([Fs/2*0.25
Fs/2*0.4],[1 0],[0.1 0.1],Fs);
n1 = n1 + 1; %to make it same as
specifications
b2 = firpm(n1,fo,ao,w);
[h3 w3] = freqz(b2,1,1024);

% Amplitude of Filters
figure
subplot(311);
plot(w1/pi,20*log10(abs(h1)));
title('Butterworth Lowpass Filter
AMPLITUDE');
ylabel('Amplitude');
xlabel('Normalized Frequency (x pi
rad/sample)');
subplot(312);
plot(w2/pi,20*log10(abs(h2)));
title("Kaiser Window Method
AMPLITUDE");
ylabel('Amplitude');
xlabel('Normalized Frequency (x pi
rad/sample)');
subplot(313);
plot(w3/pi,20*log10(abs(h3)));
title("FIR Filter AMPLITUDE");
ylabel('Amplitude');
xlabel('Normalized Frequency (x pi
rad/sample)');
```

```matlab
% Phase of Filters
figure
subplot(311);
plot(w1/pi,
360/(2*pi)*unwrap(angle(h1)));
title('Butterworth Lowpass Filter
PHASE');
ylabel('Phase (degrees)');
xlabel('Normalized Frequency (x pi
rad/sample)');
subplot(312);
plot(w2/pi,
360/(2*pi)*unwrap(angle(h2)));
title("Kaiser Window Method PHASE");
ylabel('Phase (degrees)');
xlabel('Normalized Frequency (x pi
rad/sample)');
subplot(313);
plot(w3/pi,
360/(2*pi)*unwrap(angle(h3)));
title("FIR Filter PHASE");
ylabel('Phase (degrees)');
xlabel('Normalized Frequency (x pi
rad/sample)');

% Impulse Response of Filters
figure
subplot(311);
impz(b1,a1);
title("Butterworth Lowpass Filter
Impulse Response");
ylabel('Amplitude');
xlabel('Samples (n)');
subplot(312);
impz(hh,1);
title("Kaiser Window Method Impulse
Response");
ylabel('Amplitude');
xlabel('Samples (n)');
subplot(313);
impz(b2,1);
title("FIR Filter Impulse Response");
ylabel('Amplitude');
xlabel('Samples (n)');
```