1

① $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$

② $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i$

③ $\boxed{a} = \frac{1}{N}\sum_{i=1}^{N} a = \frac{1}{\cancel{N}}\cdot\cancel{N}\cdot a$ $\boxed{+}$ $\boxed{b\cdot m(X)} = \frac{1}{N}\sum_{i=1}^{N} bx_i = b\cdot\frac{1}{N}\sum_{i=1}^{N} x_i$

<u>therefore...</u>

④ $m(a+bX) = a + b\cdot m(X)$

2

$$cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - (m(a+bY))$$

$$m(a+bY) = a + b\cdot m(Y)$$

$$(a+by_i) - (a+b\cdot m(Y)) = b(y_i - m(Y))$$

so... $cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$

and <u>therefore:</u> $cov(X, a+bY) = b\cdot cov(X,Y)$

(3)

$$cov(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{N}((a+bx_i)-(a+bm(X)))^2$$

$$cov(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{N}(b(x_i-m(X)))^2$$

$$cov(a+bX, a+bX) = b^2 \cdot \frac{1}{N}\sum_{i=1}^{N}(x_i-m(X))^2$$

$$cov(a+bX, a+bX) = b^2 \cdot cov(X, X)$$

therefore: $cov(X, X) = s^2$

(4)

For a non-decreasing transformation, the median of the transformed variable is the transformed median so:

$m(g(X)) = g(m(X))$

yes, the property above applies to any quantile... (a non dec. function preseves the order of the data)

$$Q_p(g(X)) = g(Q_p(X))$$

yes, this applies to the IRQ: (since g is non-decreasing)

$$IRQ(X) = Q_3(X) - Q_1(X)$$

$$IRQ(g(X)) = g(Q_3(X)) - g(Q_1(X)) = g(Q_3(X) - Q_1(X)) = g(IRQ(X))$$

yes, and the range...

$$Range(X) = max(X) - min(X)$$

$$Range(g(X)) = max(g(X)) - min(g(X)) = g(max(X)) - g(min(X)) = g(Range(X))$$

(5) yes, it is always true that $m(g(X)) = g(m(X))$ for a non decreasing transformation.