Super-resolution MRI Using Finite Rate of Innovation Curves

Greg Ongie*, Mathews Jacob

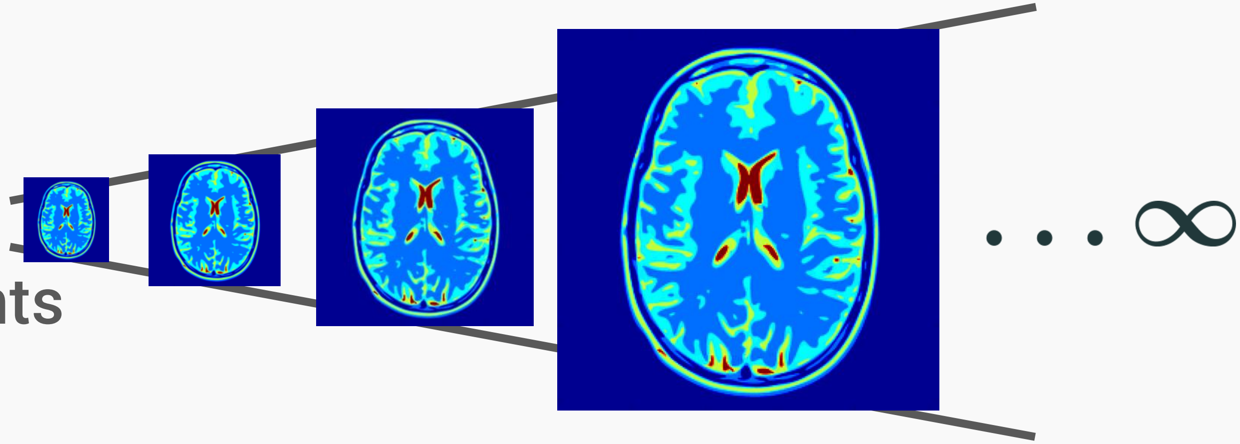
Computational Biomedical Imaging Group (CBIG)
University of Iowa

ISBI 2015
New York City

1. Introduction
2. Off-the-Grid Image Recovery: New Framework
3. Algorithms
4. Experiments
5. Discussion & Conclusion

Our goal is to develop theory and algorithms for off-the-grid imaging

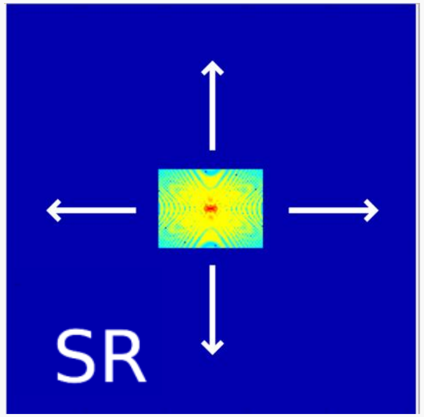
Few measurements →



- Off-the-grid = Continuous domain representation
- Avoids discretization errors
- Continuous domain sparsity $\neq$ Discrete domain sparsity

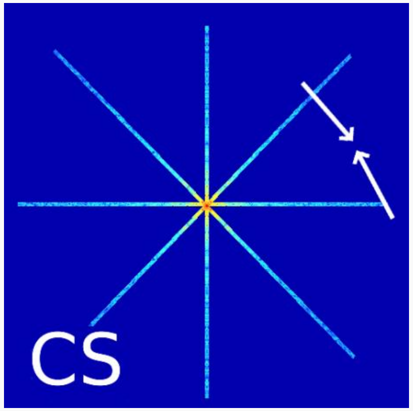
Wide-range of applications

- **SR MRI:** k-space undersampling approach



extrapolation

vs.



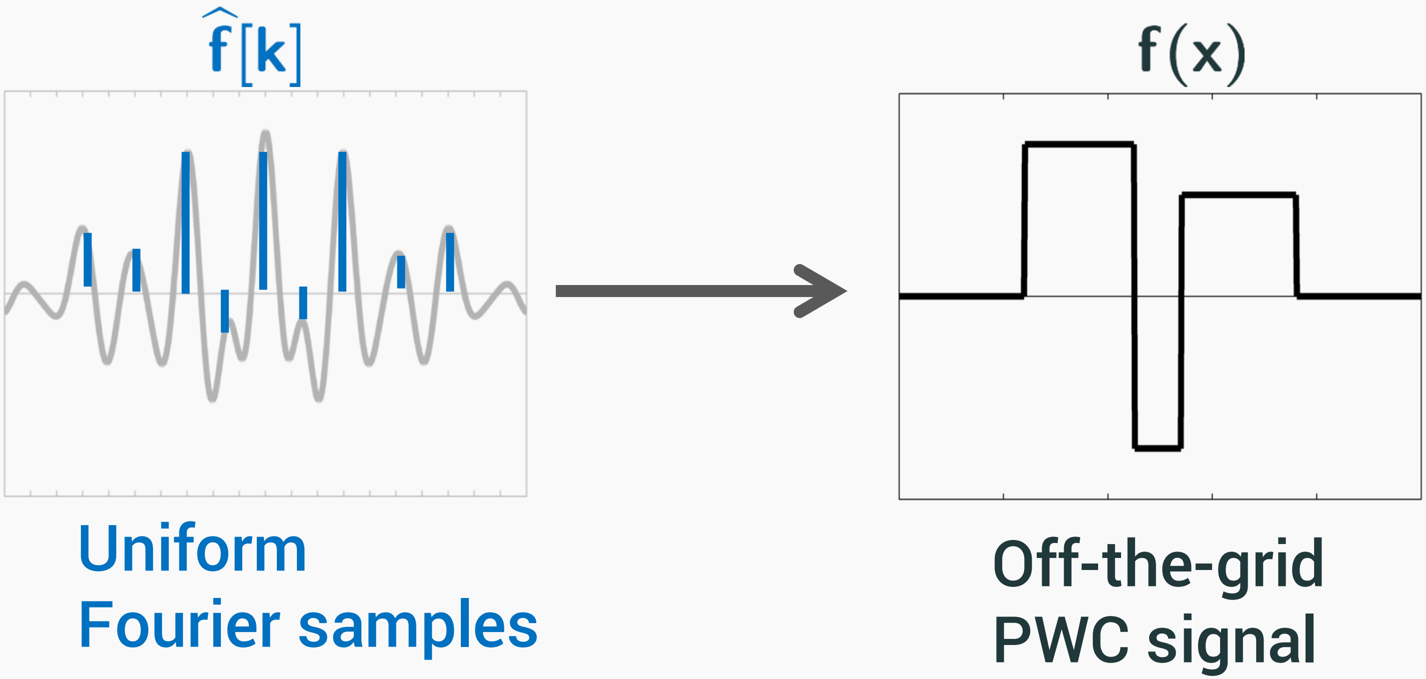
interpolation

  - Modalites: Multi-slice, Dynamic, MRSI

- **CS MRI**

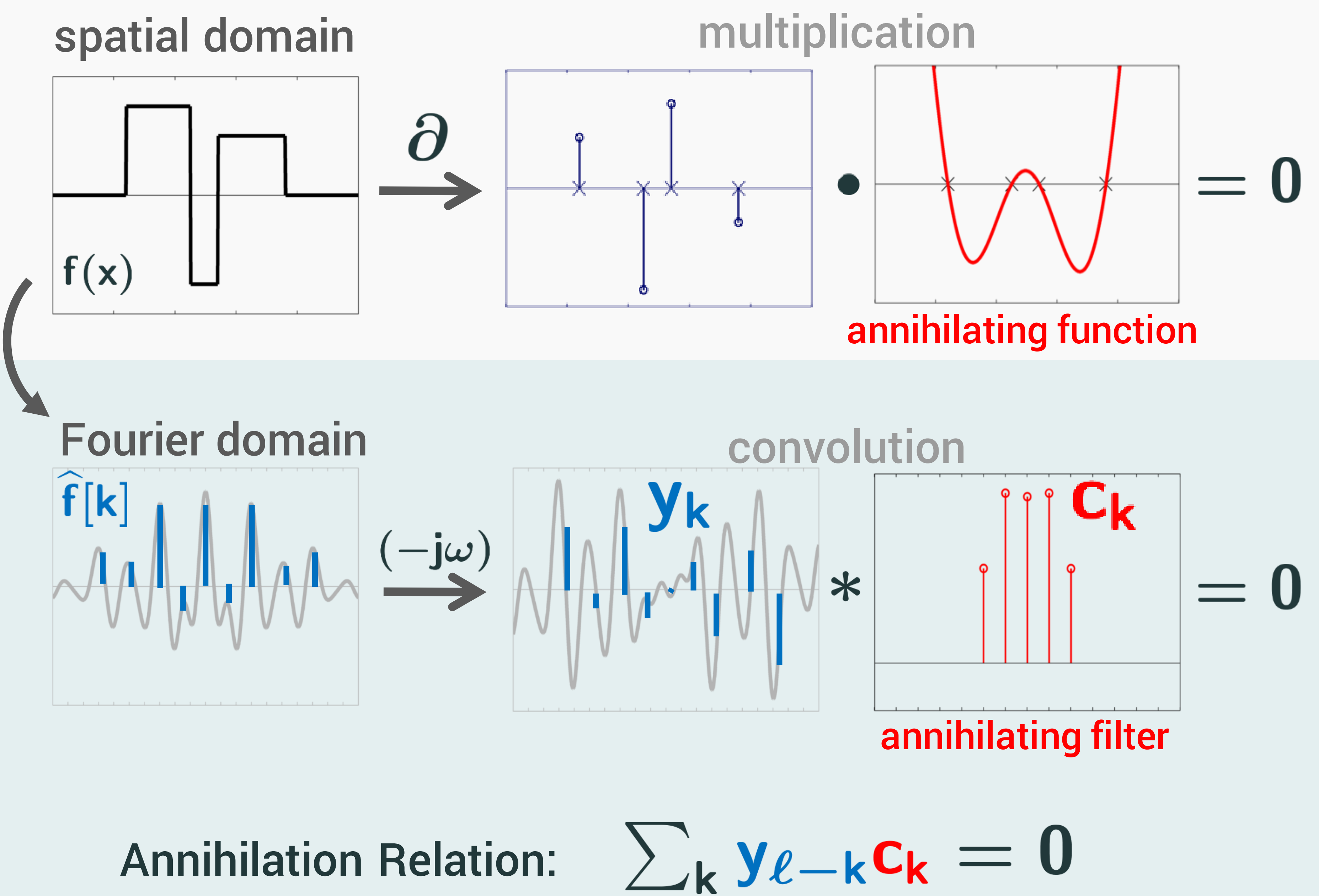
- **Outside MRI:** Deconvolution Microscopy, Denoising, etc.

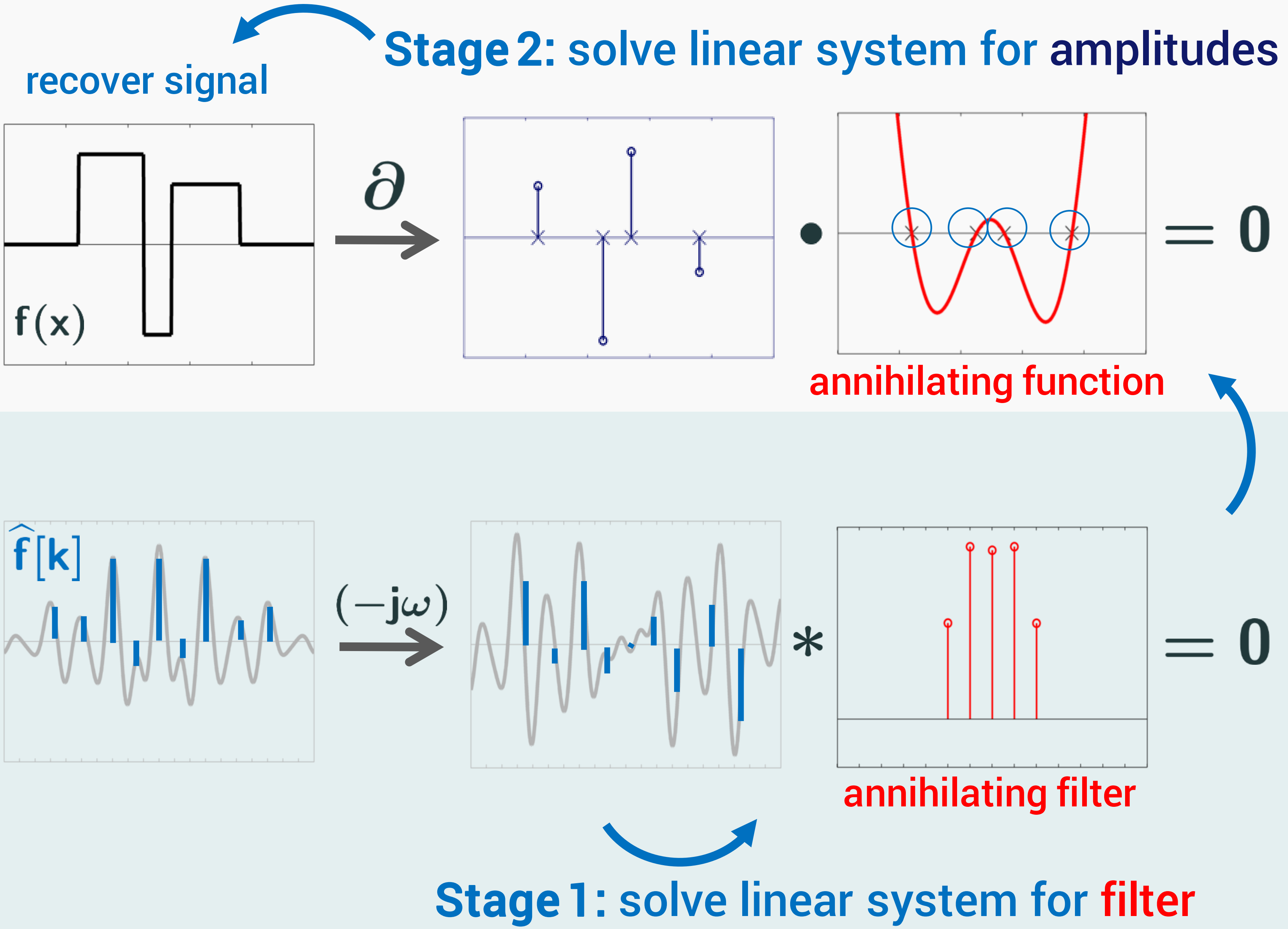
Main inspiration: Finite-Rate-of-Innovation (FRI)



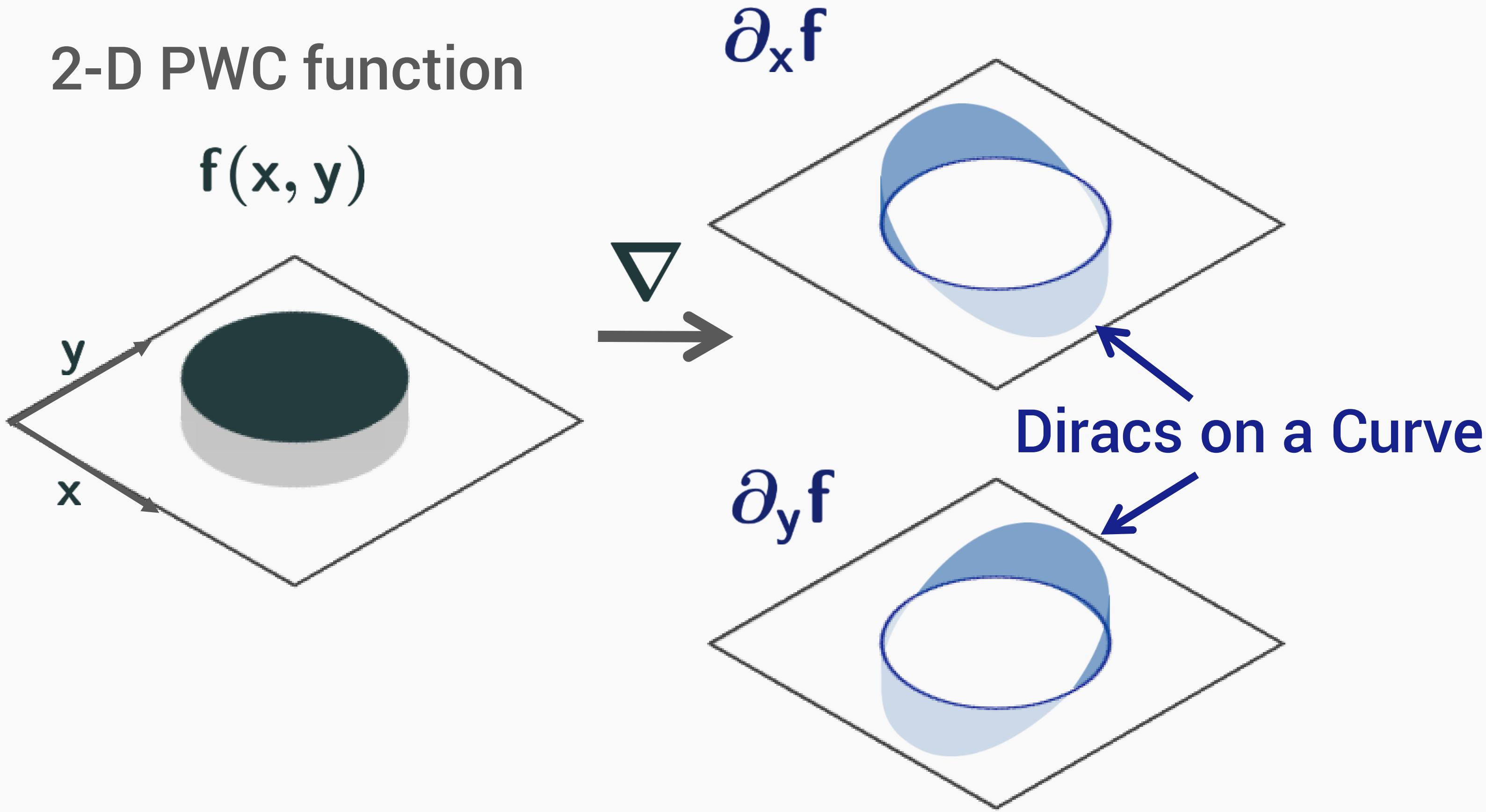
- Recent extension to 2-D images:

  Pan et al. (2014), "Sampling Curves with FRI"


spatial domain
f(x)
∂
multiplication
•
= 0
annihilating function
Fourier domain
f̂[k]
(−jω)
convolution
y_k
*
c_k
= 0
annihilating filter
Annihilation Relation: ∑_k y_{ℓ−k} c_k = 0

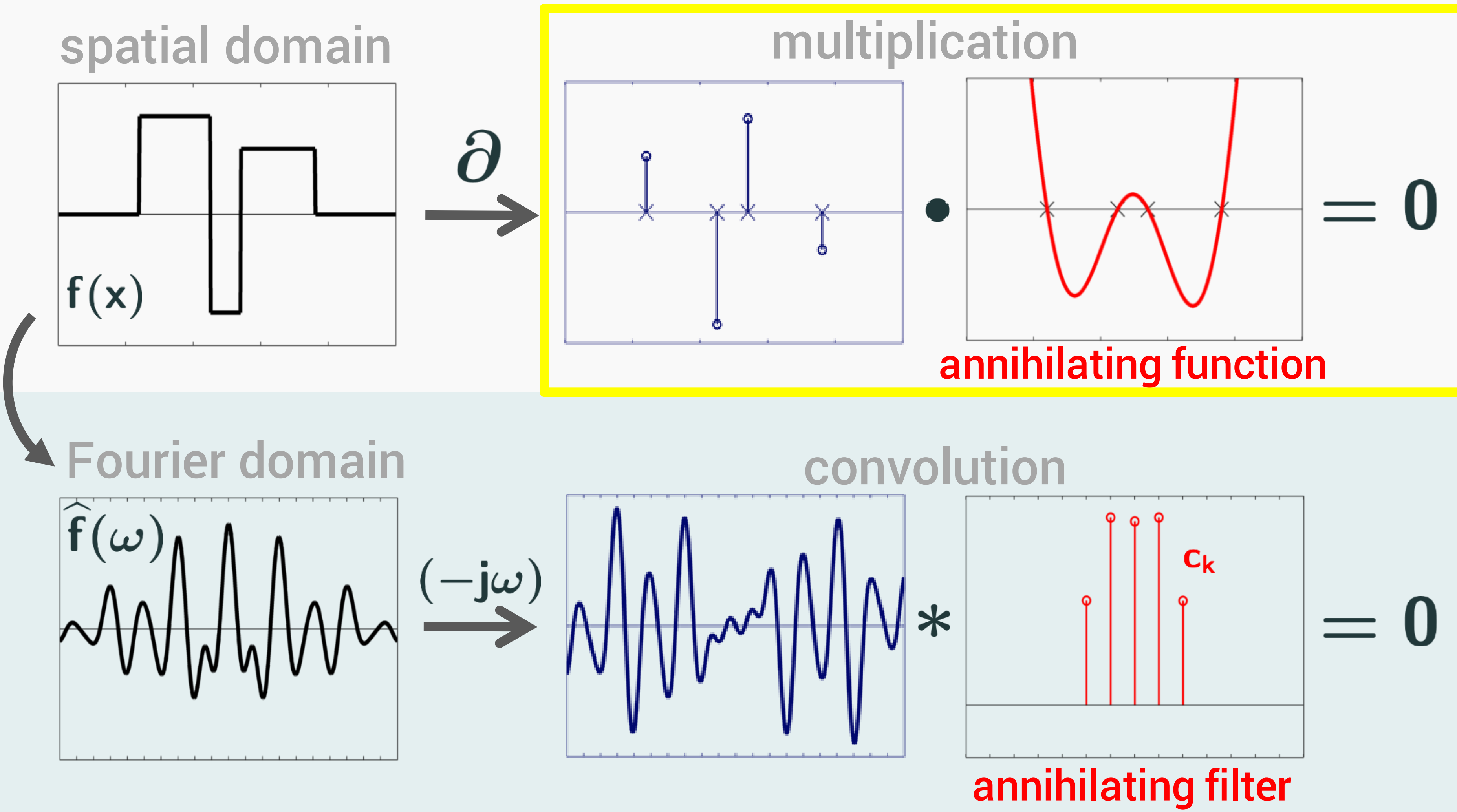

Stage 2: solve linear system for amplitudes
recover signal
f(x)
∂
annihilating function
= 0
f̂[k]
(−jω)
*
annihilating filter
= 0
Stage 1: solve linear system for filter

Extensions of FRI recovery to higher dimensions is non-trivial!

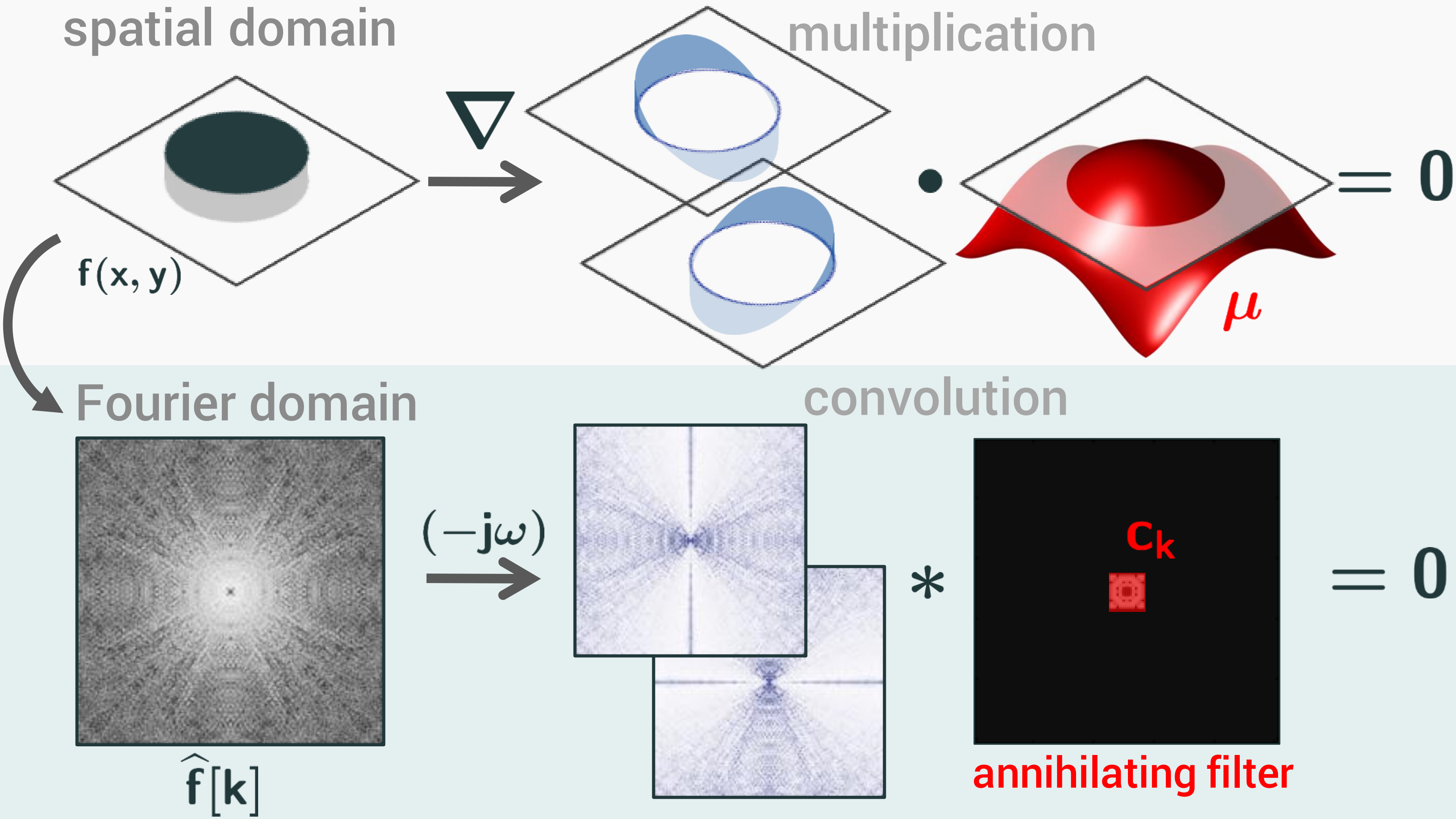


1. Introduction
2. **Off-the-Grid Image Recovery: New Framework**
3. Algorithms
4. Experiments
5. Discussion & Conclusion

Recall 1-D Case...

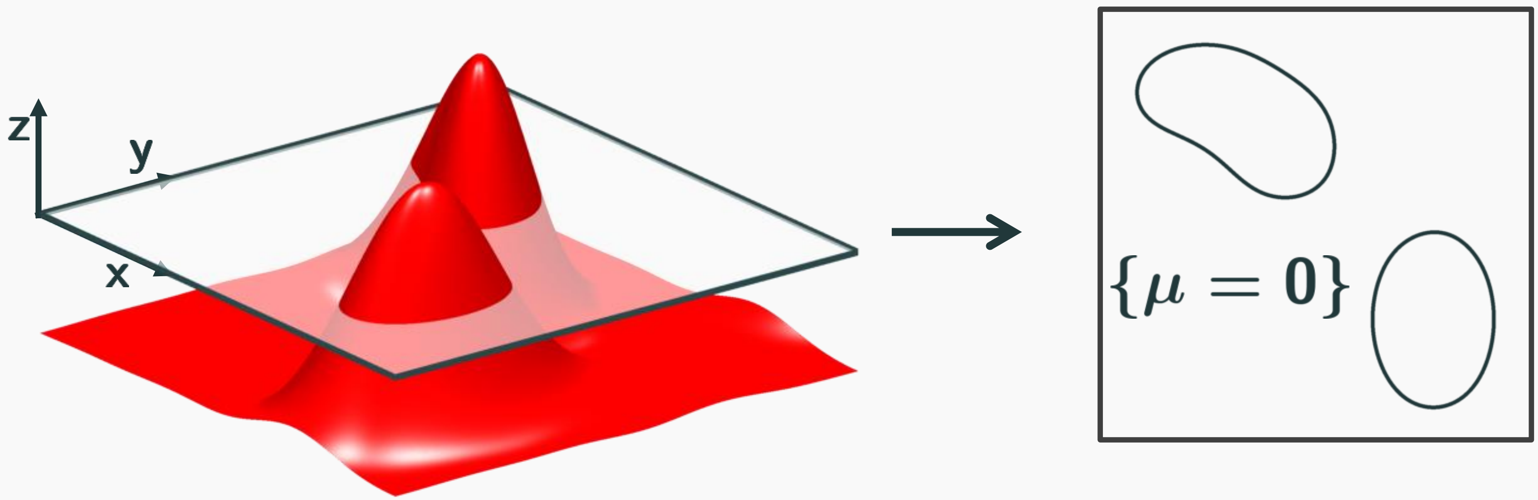


2-D PWC functions satisfy an annihilation relation



Annihilation relation: $\sum_k \widehat{\nabla f}[\ell - k]\, c_k = 0$

Can recover edge set when it is the
level-set of a 2-D bandlimited function
(Pan et al., 2014)

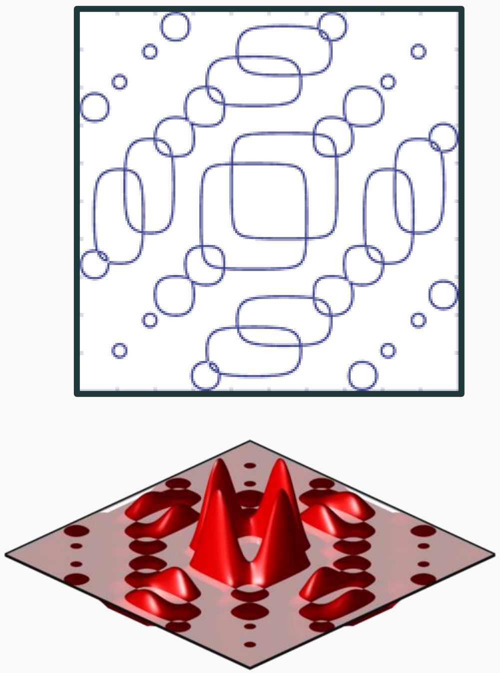


$$\mu(x, y) = \sum_{(k,l)\in\Lambda} c_{k,l}\, e^{j2\pi(kx+ly)}$$

"FRI Curve"

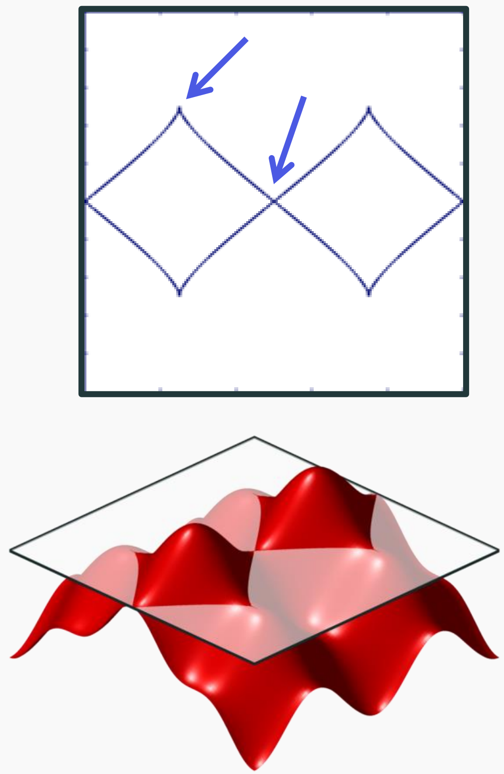
FRI curves can represent complicated edge geometries

Multiple curves & intersections



13x13 coefficients

Non-smooth points



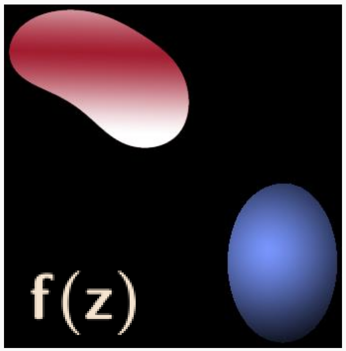
7x9 coefficients

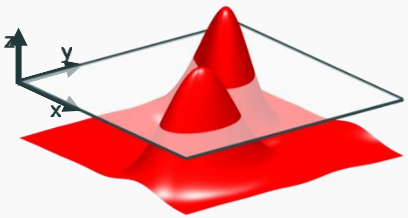
Approximate complex curves

21x21 coefficients

We give an improved theoretical framework for higher dimensional FRI recovery

Pan et al. (2014):

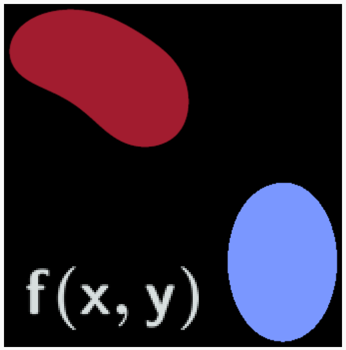


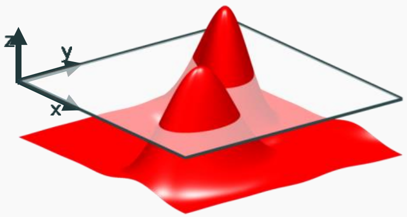


PW complex analytic

- 2-D only
- No uniqueness guarantees

New formulation:





PW constant/polynomial

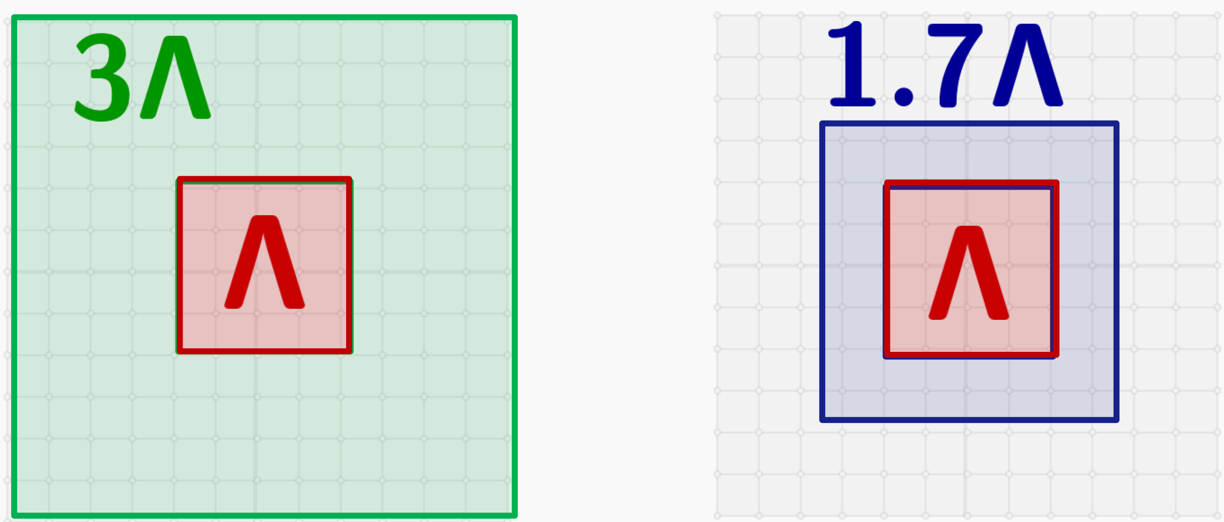
✓ Extends to n-D

✓ Provable uniqueness

✓ Fewer samples

Sampling guarantees for unique edge set recovery

Theorem: If the level-set function is bandlimited to Λ we can recover it uniquely from Fourier samples in 3Λ

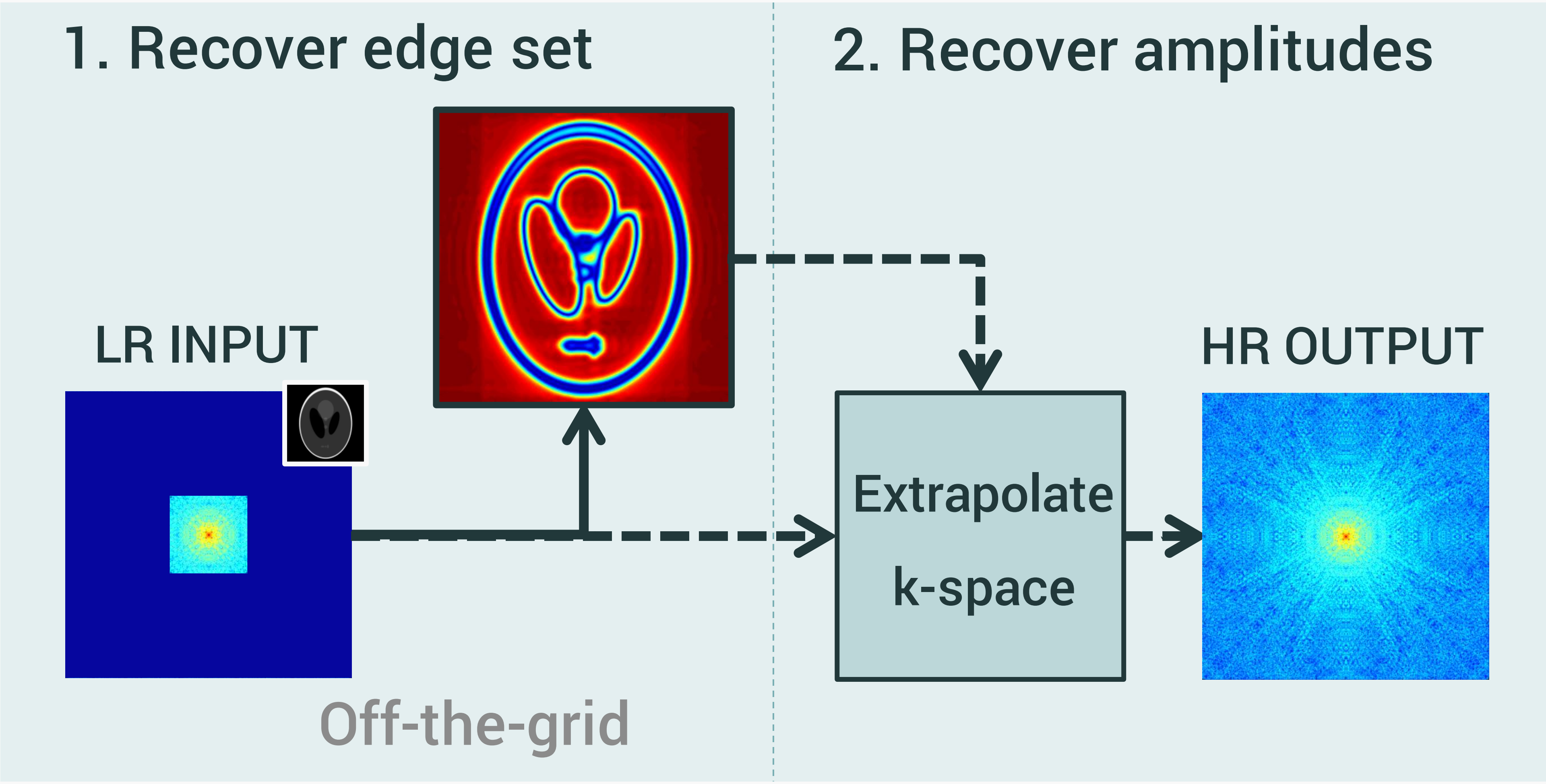


Sufficient

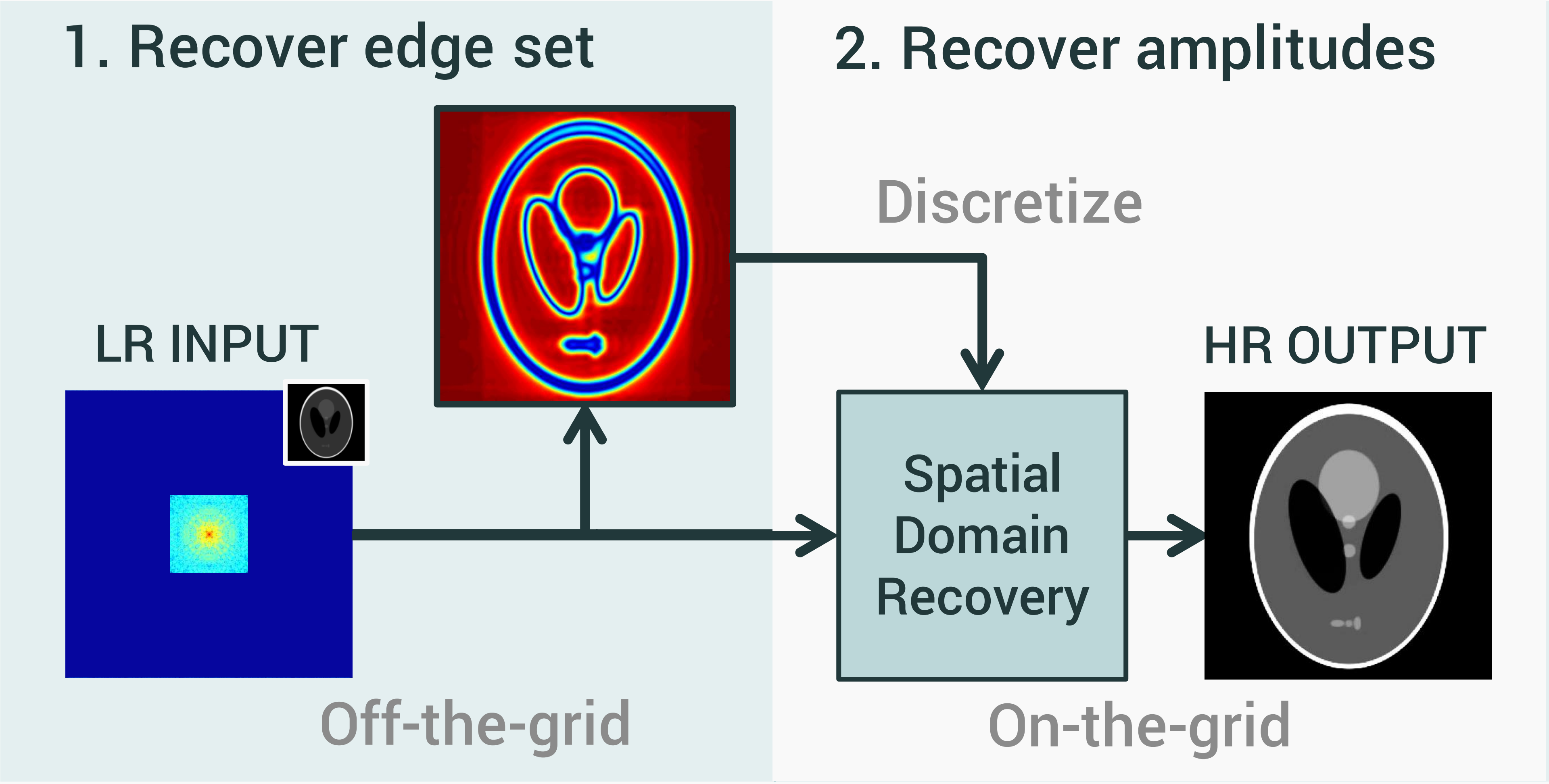
Necessary

1. Introduction
2. Off-the-Grid Image Recovery: New Framework
3. **Algorithms**
4. Experiments
5. Discussion & Conclusion

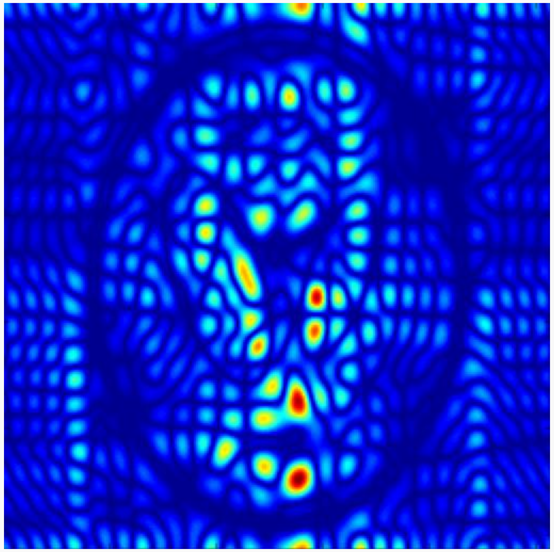
Two-stage SR MRI recovery scheme:



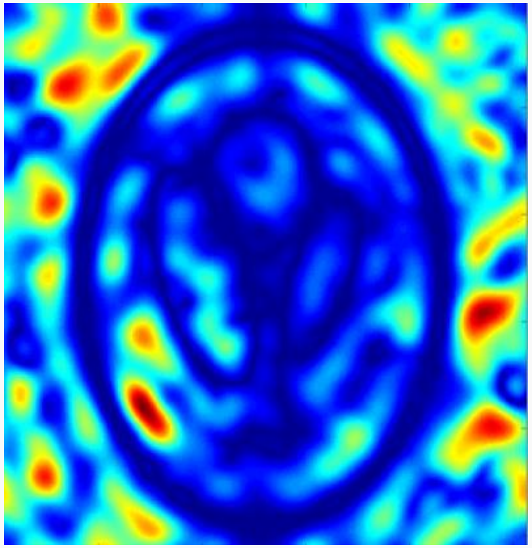
Two-stage SR MRI recovery scheme:



Practical difficulties to recovery of annihilating filter/edge set



1. Spurious zeros / model order selection



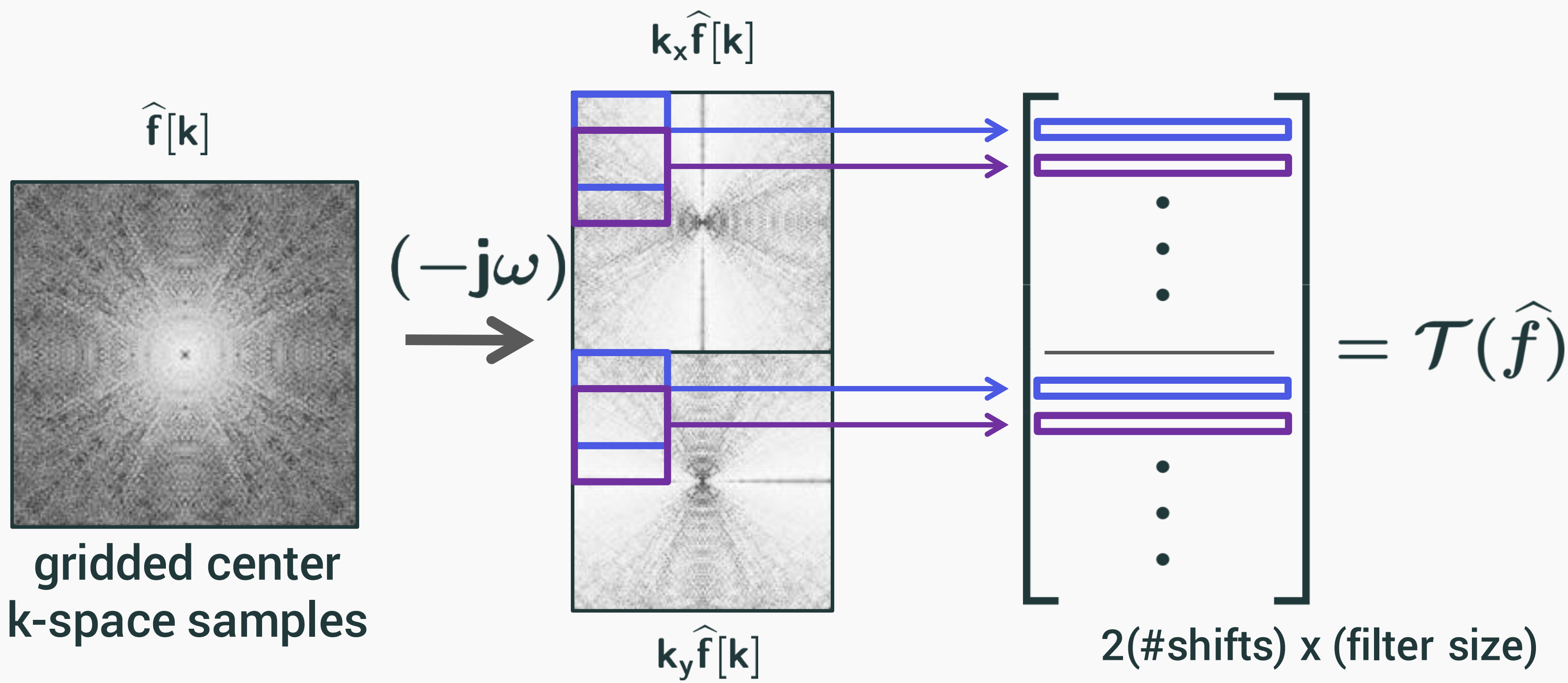
2. Sensitivity to noise / model-mismatch

Matrix representation of annihilation

$$\mathcal{T}(\widehat{f})\boldsymbol{c} = 0$$

2-D convolution matrix
(block Toeplitz)

vector of filter coefficients

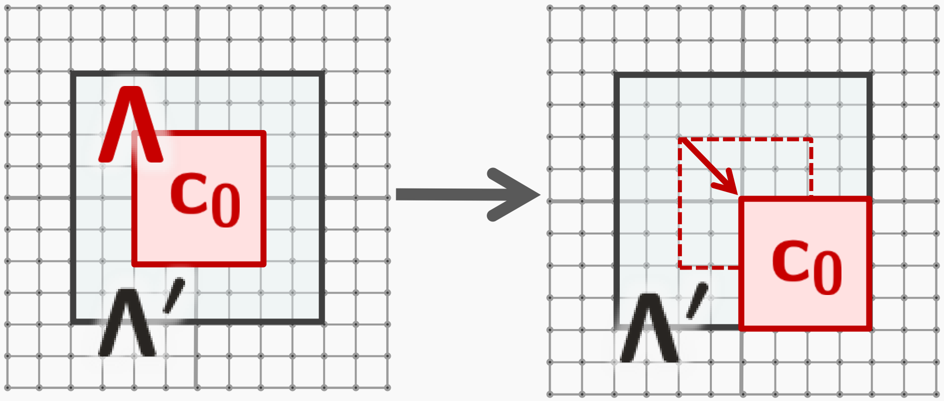


Annihilation matrix is typically low-rank

Prop: If the level-set function is bandlimited to Λ and the assumed filter support $\Lambda' \supset \Lambda$ then

$$\mathrm{rank}[\mathcal{T}(\widehat{\mathbf{f}})] \leq |\Lambda'| - (\#\mathrm{shifts}\ \Lambda\ \mathrm{in}\ \Lambda')$$

Fourier domain



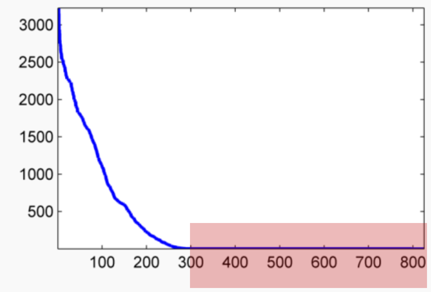
Spatial domain $\mu(x, y) \longrightarrow e^{j2\pi(kx+ly)}\mu(x, y)$

Stage 1: Robust annihilting filter estimation

1. Compute SVD

$$\mathcal{T}(\widehat{\mathbf{f}}) = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{H}}$$

$\sigma(\mathcal{T}(\widehat{\mathbf{f}}))$

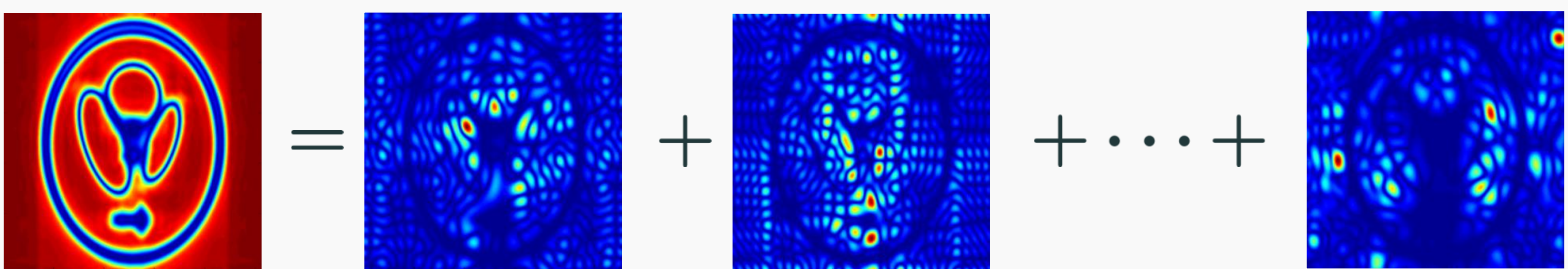


2. Identify null space

$$\mathbf{V} = [\mathbf{V_S}\ \mathbf{V_N}], \qquad \mathbf{V_N} = [\mathbf{c}_1, \ldots, \mathbf{c}_n]$$

3. Compute sum-of-squares average

$$\mu = |\mathcal{F}^{-1}\mathbf{c}_1|^2 + |\mathcal{F}^{-1}\mathbf{c}_2|^2 + \cdots + |\mathcal{F}^{-1}\mathbf{c}_n|^2$$



Recover common zeros

Stage 2: Weighted TV Recovery

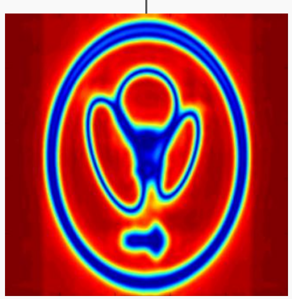
- If f is PWC with edges $\{\mu = 0\}$ then it is a minimizer of:

$$\min_{f} \int \underbrace{|\mu \cdot \nabla f|}_{\text{Annihilation relation}} \, dr \quad \text{s.t.} \quad \hat{f}[k,l] = b[k,l], (k,l) \in \Gamma$$

discretize

relax

$$\min_{x} \sum_{i} w_i \cdot |(Dx)_i| + \lambda ||Ax - b||^2$$



Edge wts.

x = discrete spatial domain image

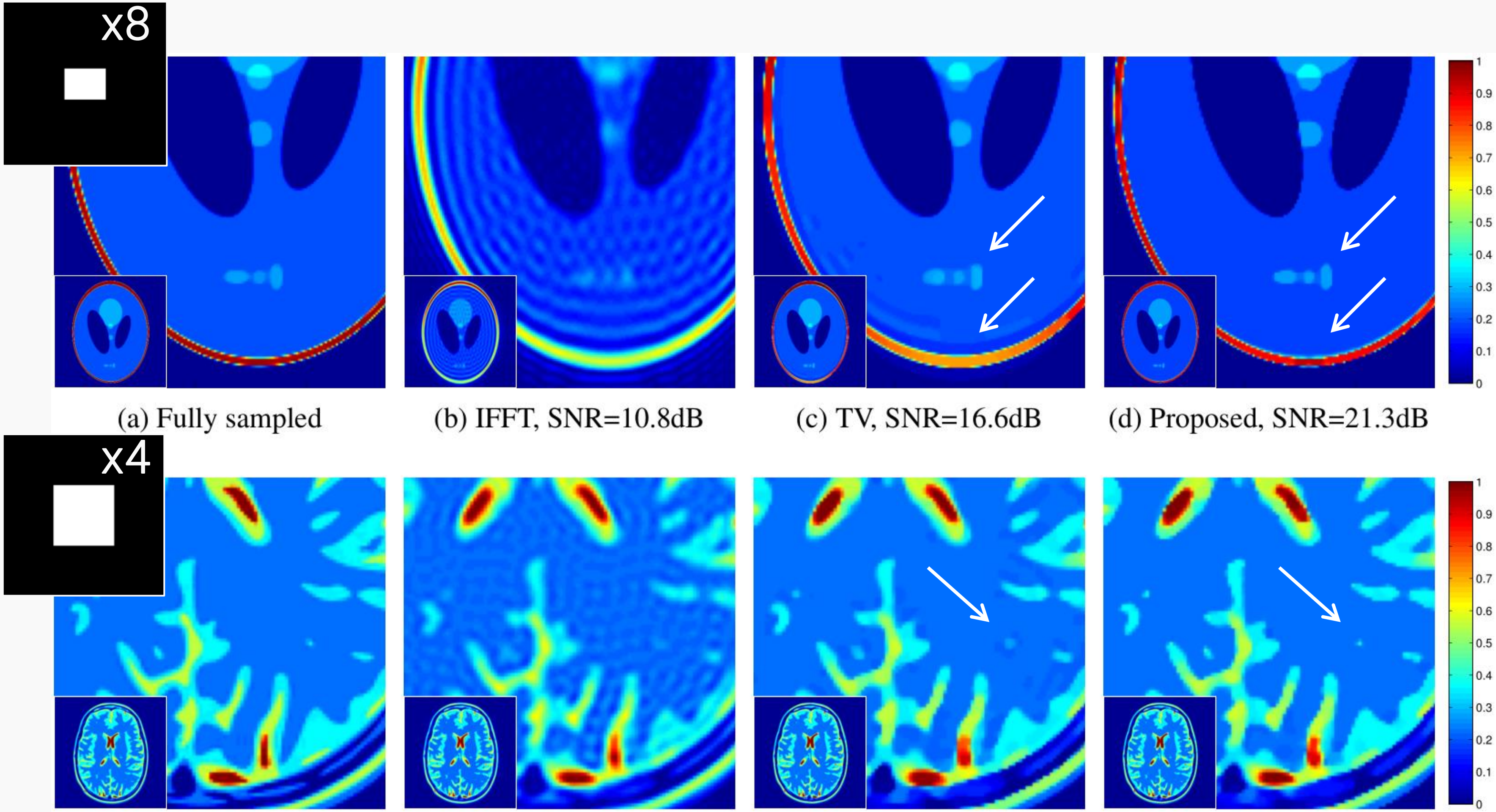
A = Fourier undersampling operator

D = discrete gradient

b = k-space samples

1. Introduction
2. Off-the-Grid Image Recovery: New Framework
3. Algorithms
4. **Experiments**
5. Discussion & Conclusion

Recovery of Phantoms (Noiseless)

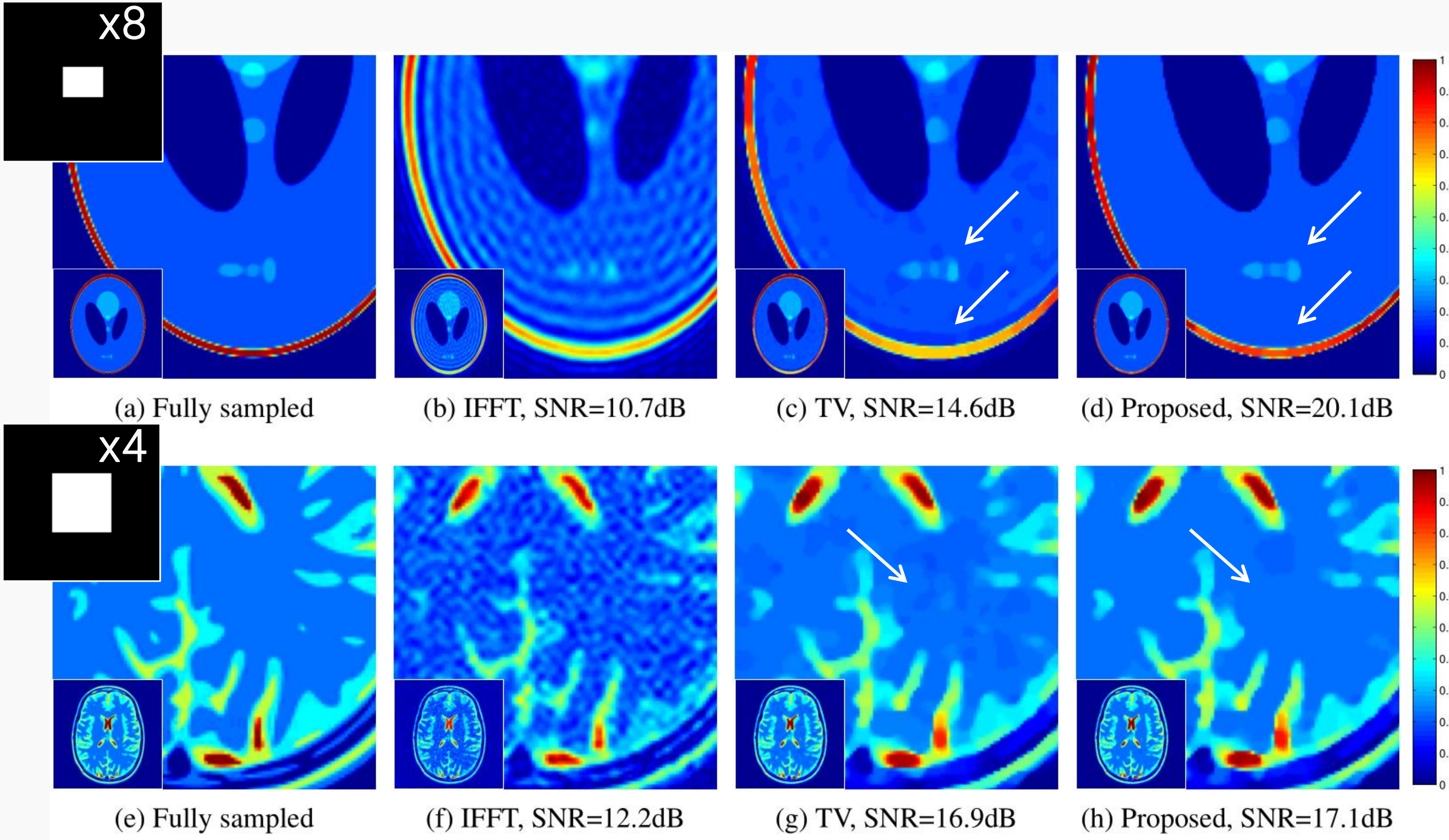


(a) Fully sampled (b) IFFT, SNR=10.8dB (c) TV, SNR=16.6dB (d) Proposed, SNR=21.3dB

(e) Fully sampled (f) IFFT, SNR=19.2dB (g) TV, SNR=19.1dB (h) Proposed, SNR=19.0dB

- Analytical phantoms from (Guerquin-Kern, 2012)

Recovery of Phantoms (Noisy)

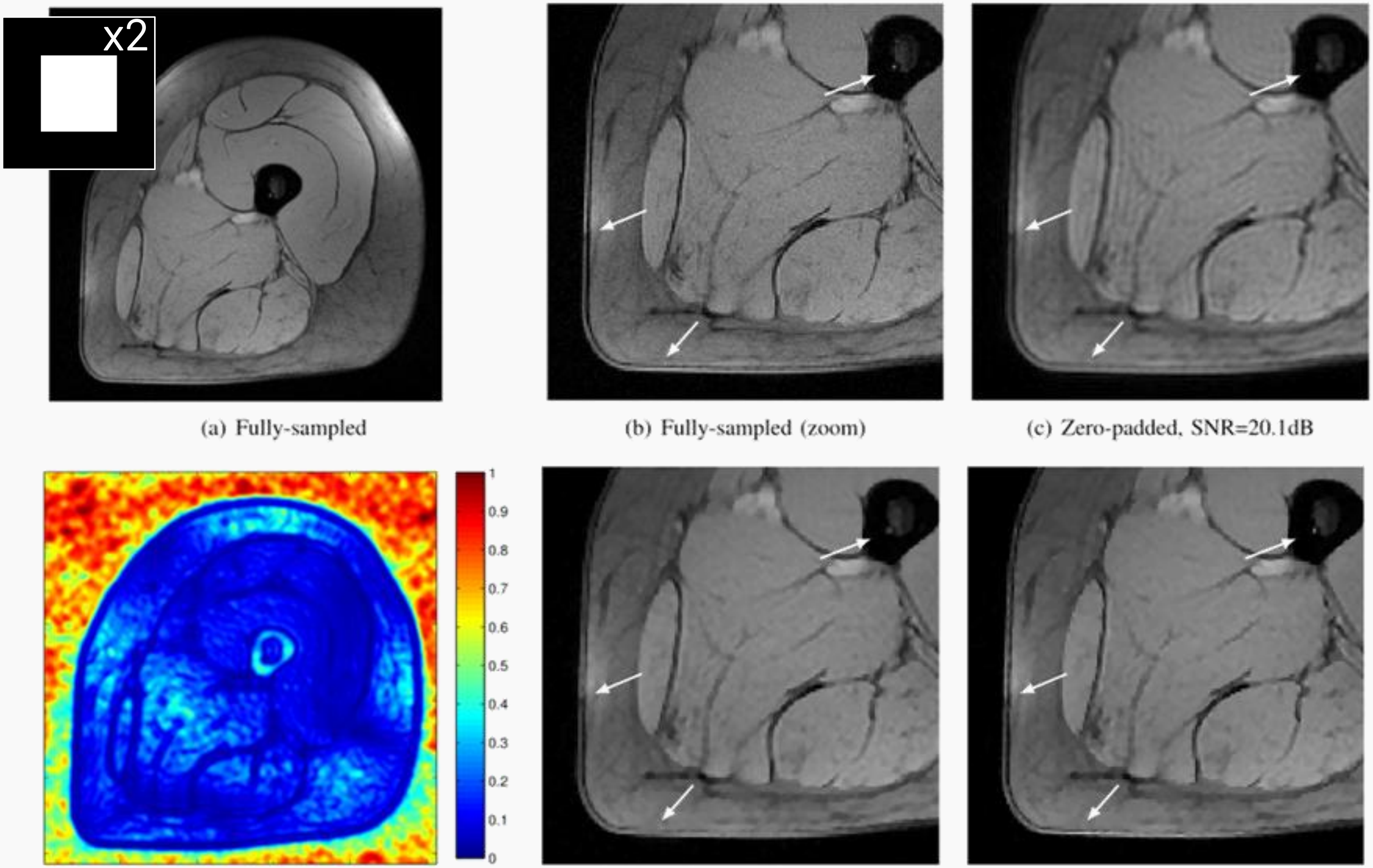


(a) Fully sampled (b) IFFT, SNR=10.7dB (c) TV, SNR=14.6dB (d) Proposed, SNR=20.1dB

(e) Fully sampled (f) IFFT, SNR=12.2dB (g) TV, SNR=16.9dB (h) Proposed, SNR=17.1dB

- 25dB complex AWGN added to k-space

Recovery of Real MR Data



(a) Fully-sampled

(b) Fully-sampled (zoom)

(c) Zero-padded, SNR=20.1dB

(d) Edge mask (65×65 coefficients)

(e) TV regularization, SNR=21.0dB

(f) Proposed, SNR=21.1dB

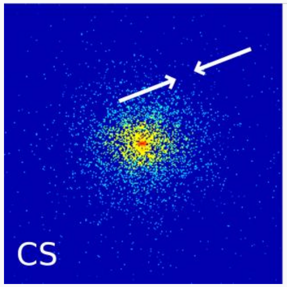
- Simulated Single Coil Acquisition (8 Coil SENSE w/phase)

1. Introduction
2. Off-the-Grid Image Recovery: New Framework
3. Algorithms
4. Experiments
5. **Discussion & Conclusion**

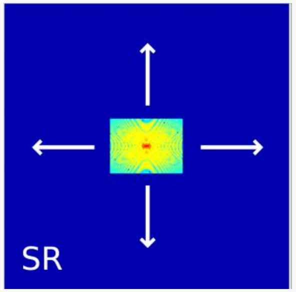
We can pose recovery as a structured low-rank matrix completion problem [*]

$$\min_{\widehat{\mathbf{f}}} \quad \underbrace{\|\mathbf{P}\widehat{\mathbf{f}} - \mathbf{b}\|^2}_{\text{Data Consistency}} + \lambda \underbrace{\|\mathcal{T}(\widehat{\mathbf{f}})\|_*}_{\text{Regularization penalty}}$$

- Entirely off the grid
- Extends to CS paradigm
- Fast algorithm



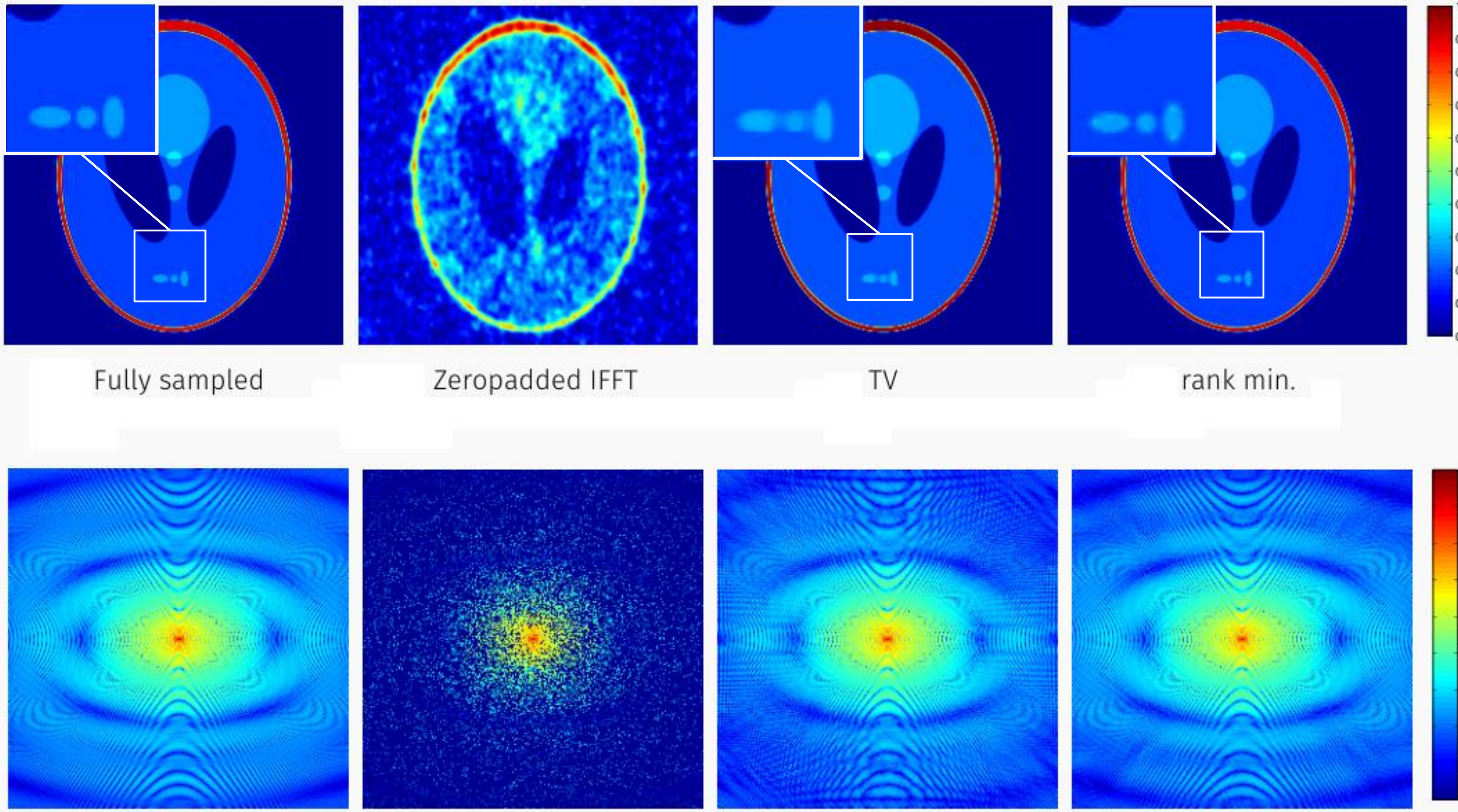
or



- Use regularization penalty for other inverse problems

  →off-the-grid alternative to TV, HDTV, etc

[*]Accepted to SampTA 2015



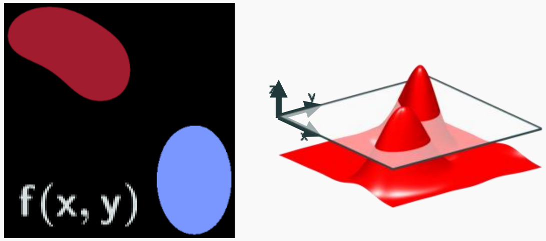
Fully sampled | Zeropadded IFFT | TV | rank min.

Fully sampled | Undersampled | TV k-space | rank min k-sapce

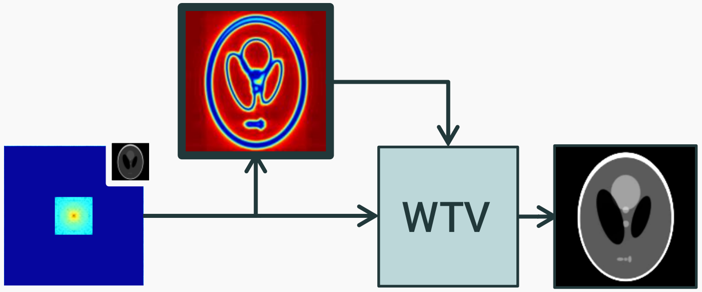
20-fold accel.

Summary

- **New framework for off-the-grid image recovery**
  - Piecewise polynomial signal model
  - Extends easily to n-D
  - Sampling guarantees



- **Two stage recovery scheme for SR MRI**
  - Robust edge mask estimation
  - Fast weighted TV algorithm
  - Better performance than standard TV



- **Novel Fourier domain low-rank prior**
  - Convex, Off-the-Grid, & widely applicable

$$\min_{\hat{\mathbf{f}}} \|\mathcal{T}(\hat{\mathbf{f}})\|_*$$

Thank You!

Acknowledgements

- Supported by grants:
  NSF CCF-0844812, NSF CCF-1116067,
  NIH 1R21HL109710-01A1, ACS RSG-11-267-01-CCE,
  and ONR-N000141310202.

References

- Pan, H., Blu, T., & Dragotti, P. L. (2014). Sampling curves with finite rate of innovation. *Signal Processing, IEEE Transactions on*, *62*(2), 458-471.
- Guerquin-Kern, M., Lejeune, L., Pruessmann, K. P., & Unser, M. (2012). Realistic analytical phantoms for parallel Magnetic Resonance Imaging. *Medical Imaging, IEEE Transactions on*, *31*(3), 626-636
- Ongie, G., & Mathews, J. (2015) Recovery of Piecewise Smooth Images from Few Fourier Samples. SampTA 2015, to appear.