Higher Degree Total Variation for 3-D Image Recovery

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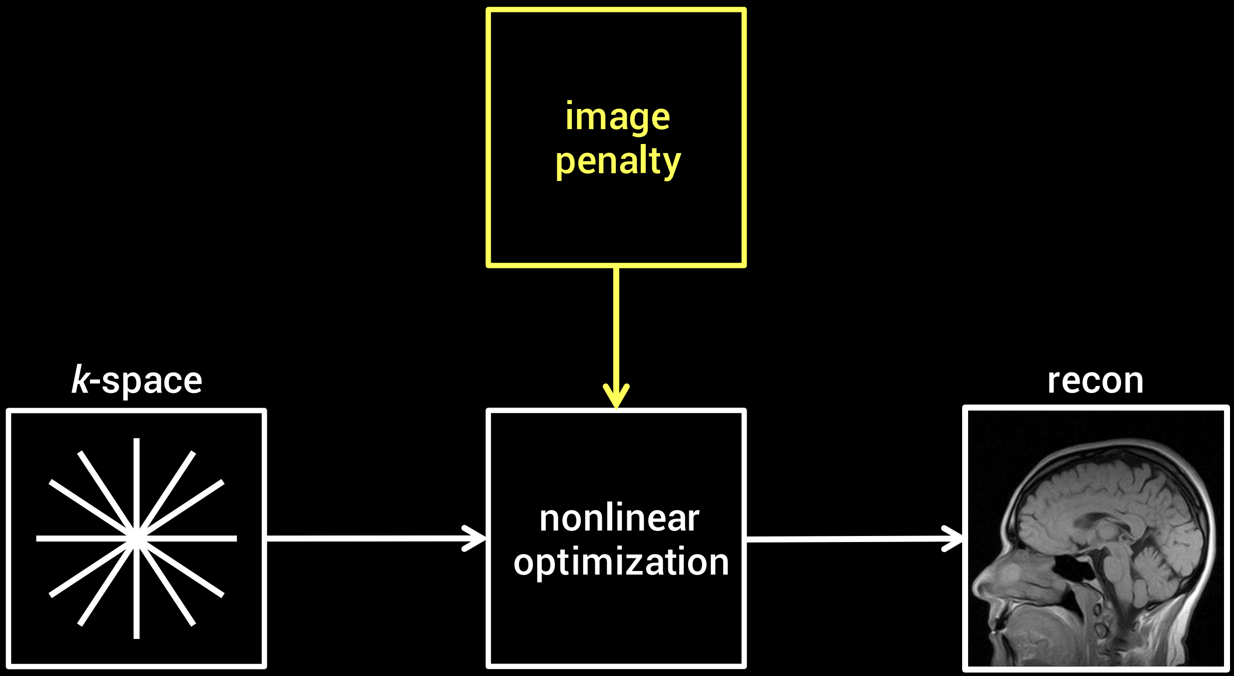
Motivation: Compressed sensing MRI recovery

- Highly undersampled k-space
- Use image penalty to enforce sparsity
- Recon is minimizer of cost function

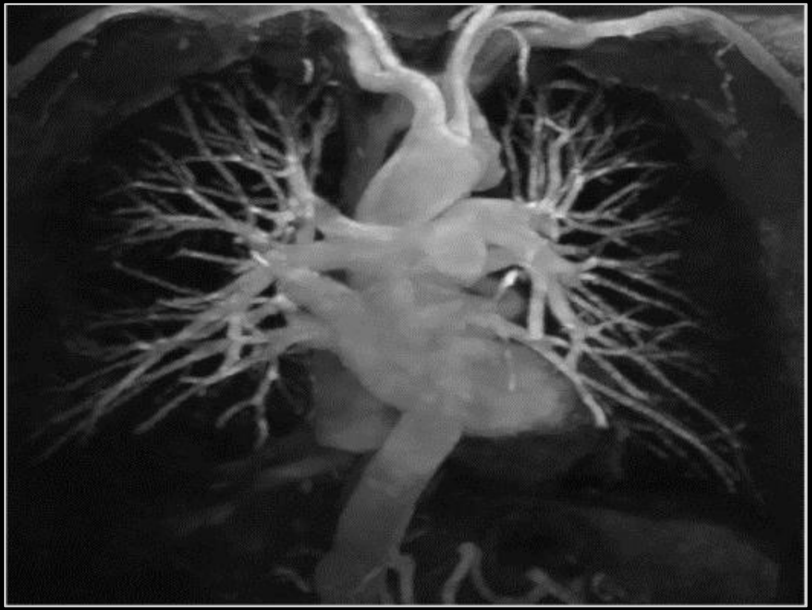


Total Variation (TV) penalty for CS-MRI

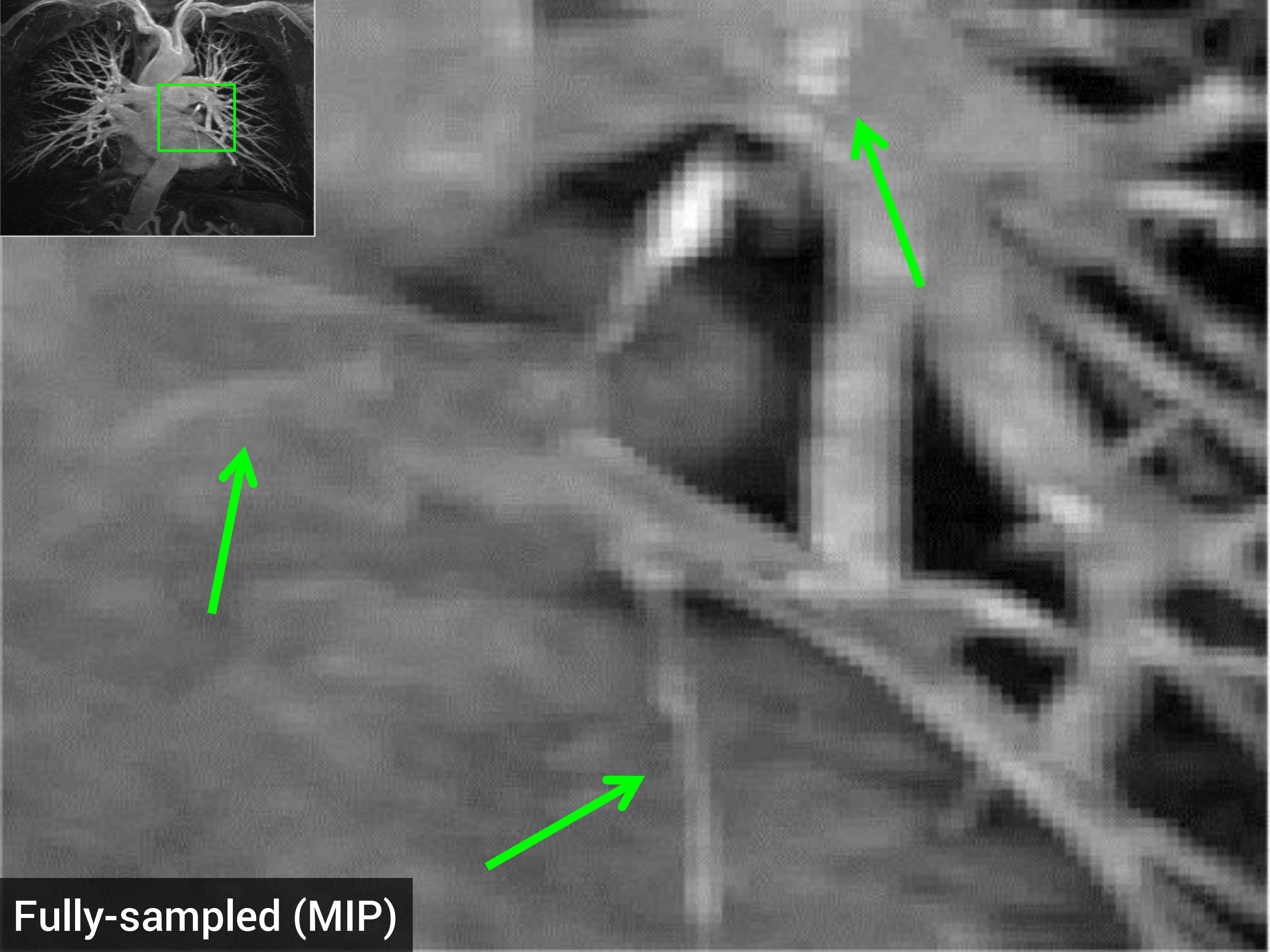
- Promotes recons with sparse gradient <-> piecewise constant regions
- Advantages: fast algorithms, easy to implement
- Disadvantages: loss of detail at high accelerations

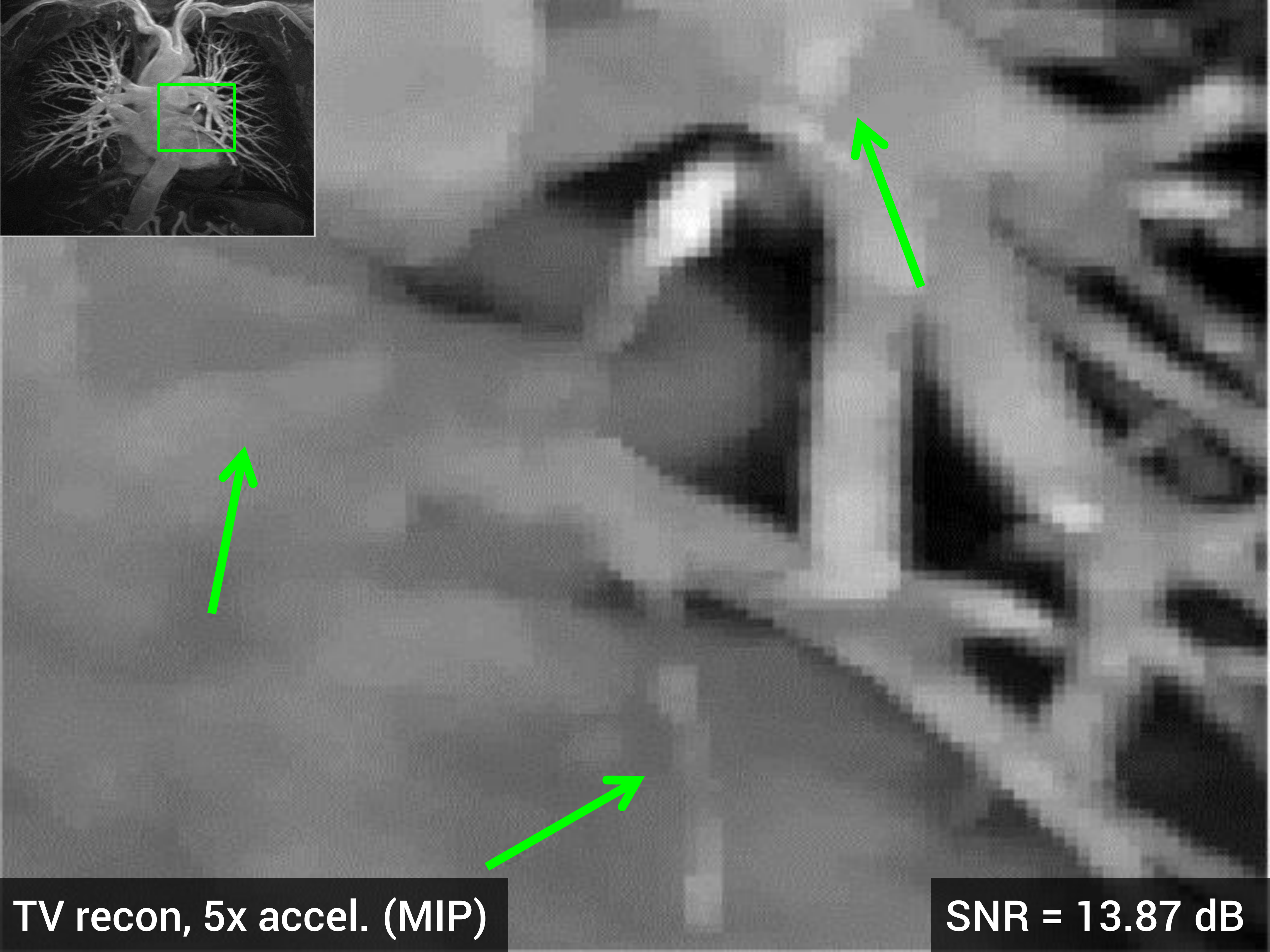
- Ex: 3-D MRA dataset, 5-fold acceleration, random *k*-space samples

Fully-sampled (MIP)

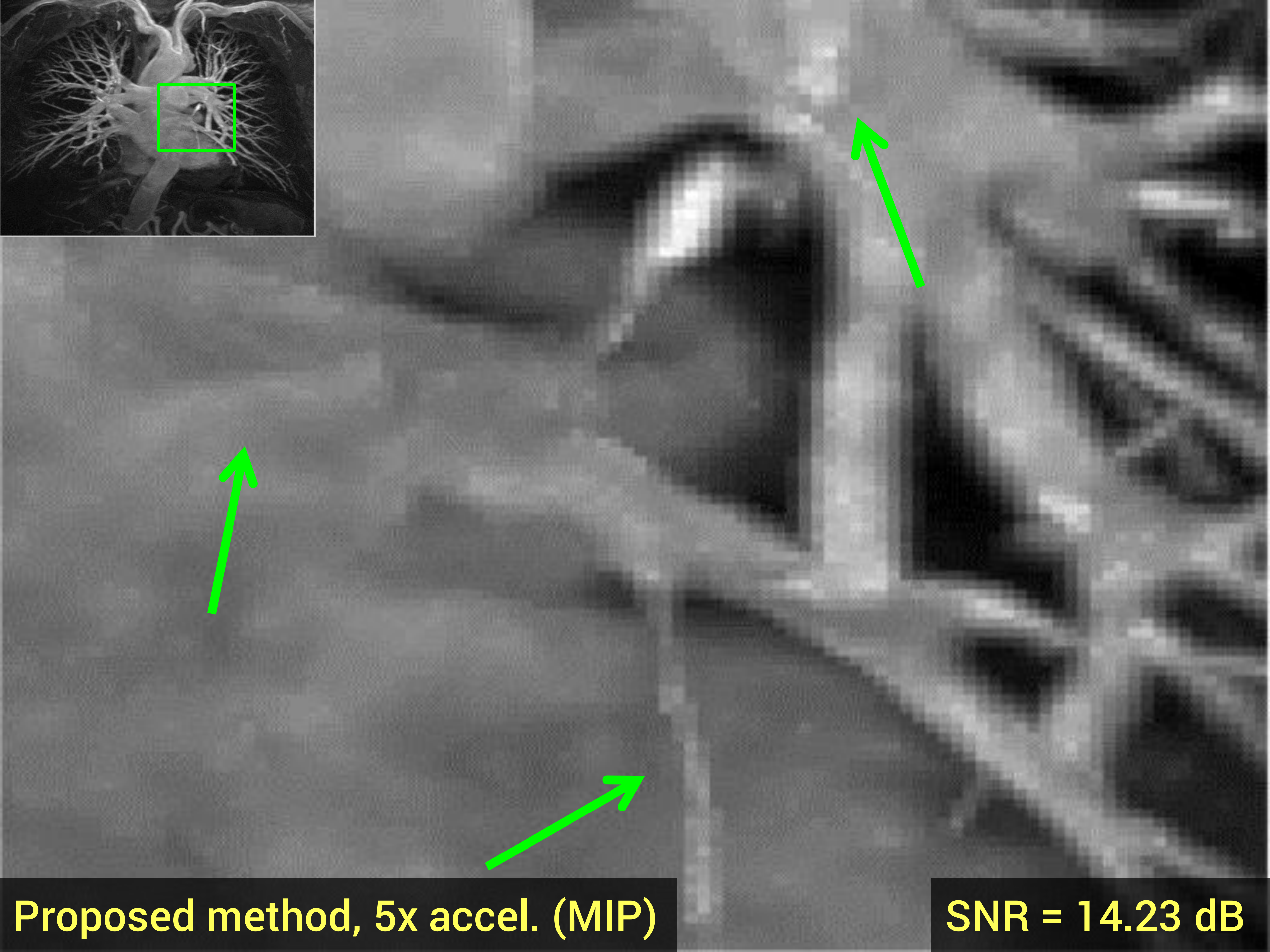


TV recon, 5x accel., SNR = 13.87 dB


Fully-sampled (MIP)



TV recon, 5x accel. (MIP)
SNR = 13.87 dB



Proposed method, 5x accel. (MIP)
SNR = 14.23 dB

Higher Degree Total Variation (HDTV) in 2-D

Higher Degree Total Variation (HDTV) penalties in 2-D

- Family of penalties for general inverse problems
- HDTV generalizes TV to higher degree derivatives

$$\mathrm{TV}(\mathbf{f}) = \frac{1}{4}\int_0^{2\pi} \|\boxed{\partial_\theta \mathbf{f}}\|_1 \, \mathbf{d}\theta$$

directional derivatives

$$\Rightarrow \quad \mathrm{HDTV}\mathbf{n}(\mathbf{f}) = \int_0^{2\pi} \|\partial_\theta^{n} \mathbf{f}\|_1 \, \mathbf{d}\theta$$

L^1-norm of all n^{th} degree directional derivatives

$$\text{ex: } \mathrm{HDTV}\mathbf{2}(\mathbf{f}) = \int_0^{2\pi} \|\cos^2(\theta)\cdot \mathbf{f}_{xx} + \sin(2\theta)\cdot \mathbf{f}_{xy} + \sin^2(\theta)\cdot \mathbf{f}_{yy}\|_1 \, \mathbf{d}\theta$$

- Promotes sparse higher degree directional derivatives
- Rotation- and translation-invariant, preserves edges, convex

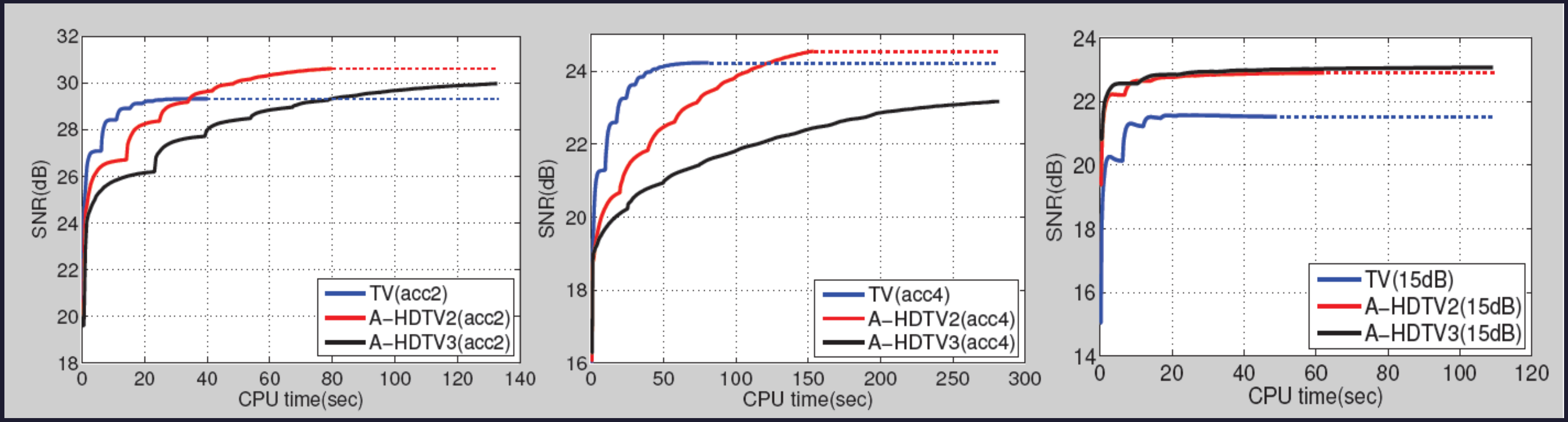
Comparison of HDTV and TV in 2-D

- HDTV routinely outperforms TV for many image recovery problems
- Modest increases in computation time (~2-4 fold)

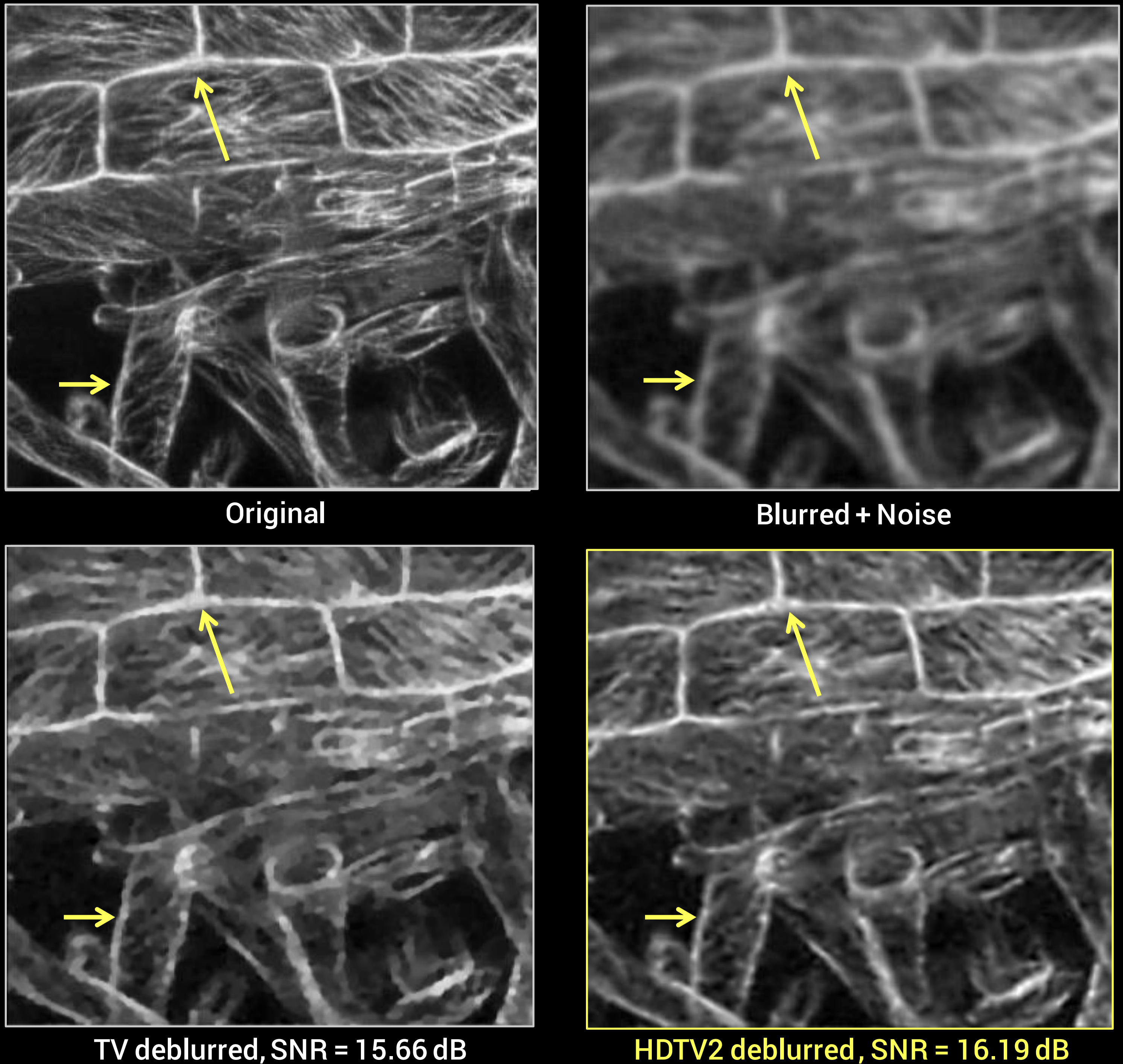
2-D TV Comparison. SNR (in dB) of recovered images with optimal reg. param. (Hu et al, 2014)

	Denoising		Deblurring		CS-MRI	
	Lena	Brain	Cell1	Cell2	Brain	Wrist
TV	27.35	27.60	15.66	16.67	22.77	20.96
HDTV2	**27.65**	28.05	**16.19**	**17.21**	**22.82**	**21.20**
HDTV3	27.45	**28.30**	16.17	17.20	22.53	21.02

SNR vs. CPU time of HDTV and TV. (Hu, Y., & Jacob, M. , 2012)

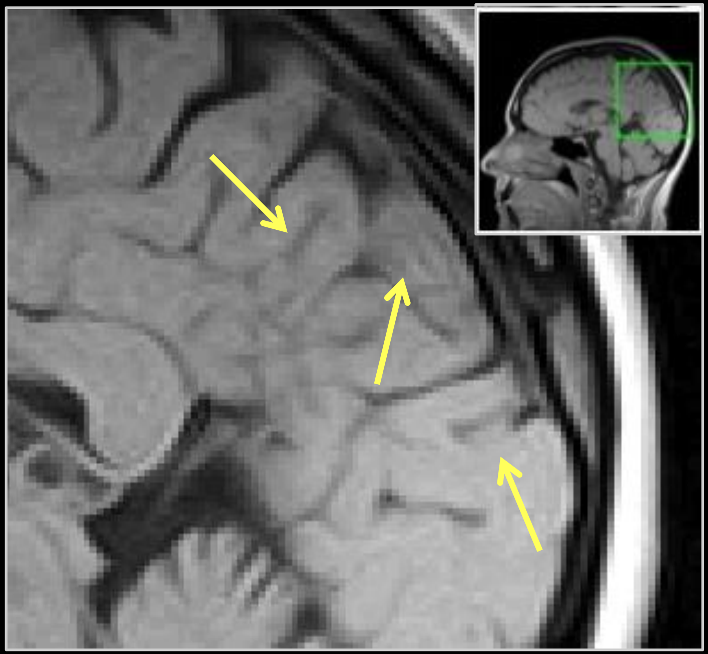


CS-MRI, 2x accel. **CS-MRI, 4x accel.** **Denoising, SNR=15dB**

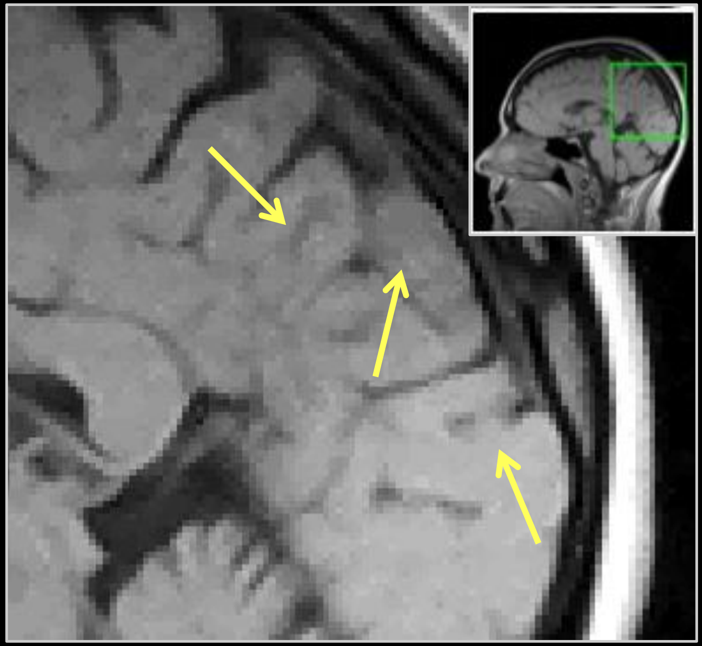

Original
Blurred + Noise
TV deblurred, SNR = 15.66 dB
HDTV2 deblurred, SNR = 16.19 dB

2-D CS-MRI

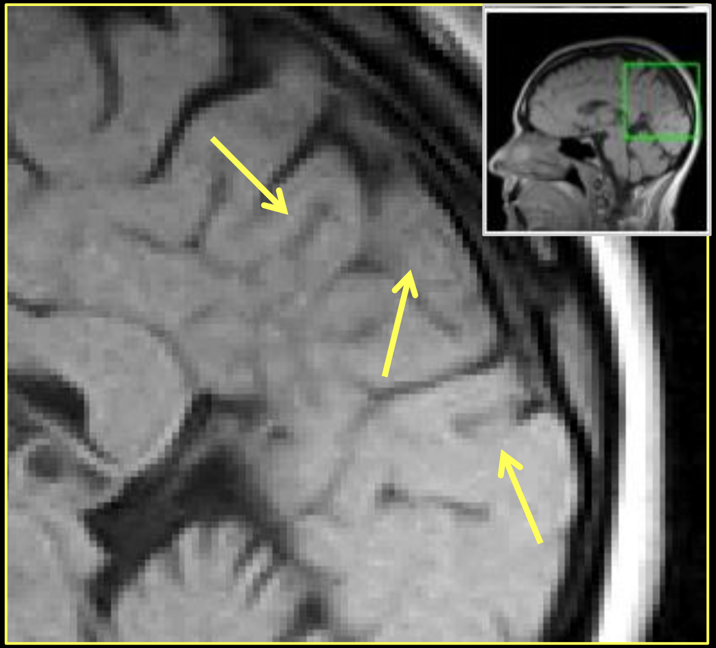
- 1.5x acceleration
- random Gaussian k-space samples



Fully-sampled



TV recon, SNR = 22.77 dB



HDTV2 recon , SNR = 22.82 dB

HDTV2 and Hessian-Schatten Norms, (Lefkimmiatis et al., 2013)

- HS_p = sum of L^p-norm of Hessian eigenvalues over all pixels
- HS_1 "equivalent to" HDTV2 for real-valued images in 2-D:

$$\mathbf{0.63} \cdot \mathrm{HS}_1(\mathbf{f}) \leq \mathrm{HDTV2}(\mathbf{f}) \leq \mathrm{HS}_1(\mathbf{f})$$

  - Inequality only where Hessian eigenvalues have mixed sign
  - No equivalence when image is complex-valued, e.g. CS-MRI.

Table 2: **HDTV2 vs. HS_1. SNR (in dB) of recovered images with optimal reg. param.**

	Denoising		Deblurring		CS-MRI	
	Lena	Brain	Cell1	Cell2	Brain	Wrist
HDTV2	27.65	28.05	16.19	17.21	22.82	21.20
HS1	27.51	28.08	16.17	17.13	22.50	20.51

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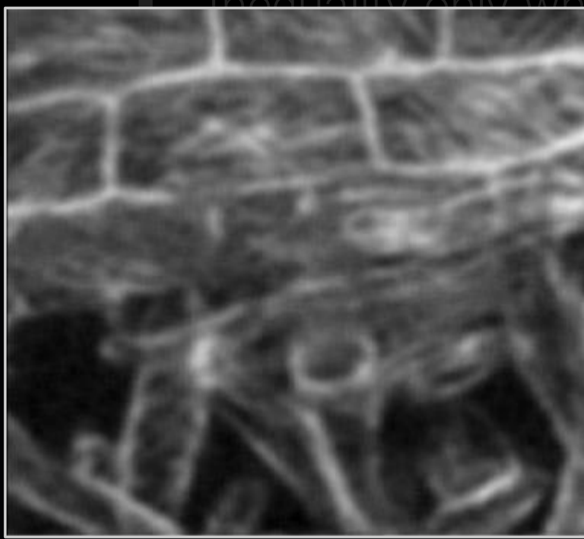
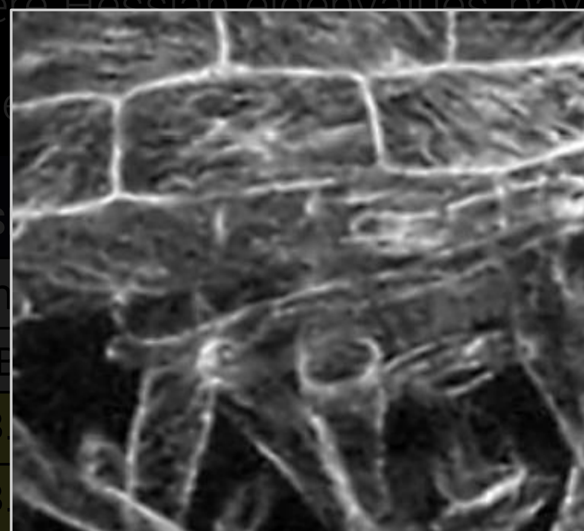
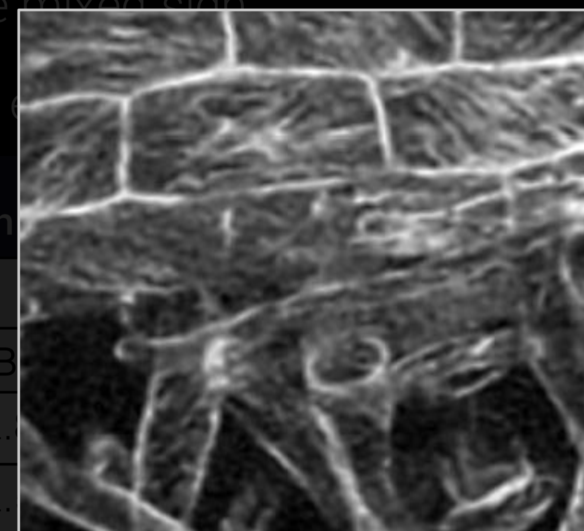
Deblurring of 2-D Cell Florescence Microscopy Image



Blurred + Noise



HDTV2, SNR =16.19 dB



HS1, SNR =16.17 dB

HDTV2 and Hessian-Schatten Norms, (Lefkimmiatis et al., 2013)

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- HS_1 "equivalent to" HDTV2 for real-valued images in 2-D:

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HDTV2	27.65	28.05	16.19	17.21	22.82	21.20
HS1	27.51	28.08	16.17	17.13	22.50	20.51

- Why use HDTV?
  - Easily adaptable to complex-valued images
  - Extends to higher degree deriatives (n > 2)

Extension of HDTV to 3-D

Extension of HDTV to 3-D

- L^1−norm of all n^{th} degree directional derivatives in 3-D

$$\text{HDTV}\mathbf{n}(\mathbf{f}) = \int_{|\mathbf{u}|=1} \|\partial_{\mathbf{u}}^{\mathbf{n}} \mathbf{f}\|_1 \, \mathbf{d}\boldsymbol{\sigma}(\mathbf{u})$$

surface integral over unit sphere

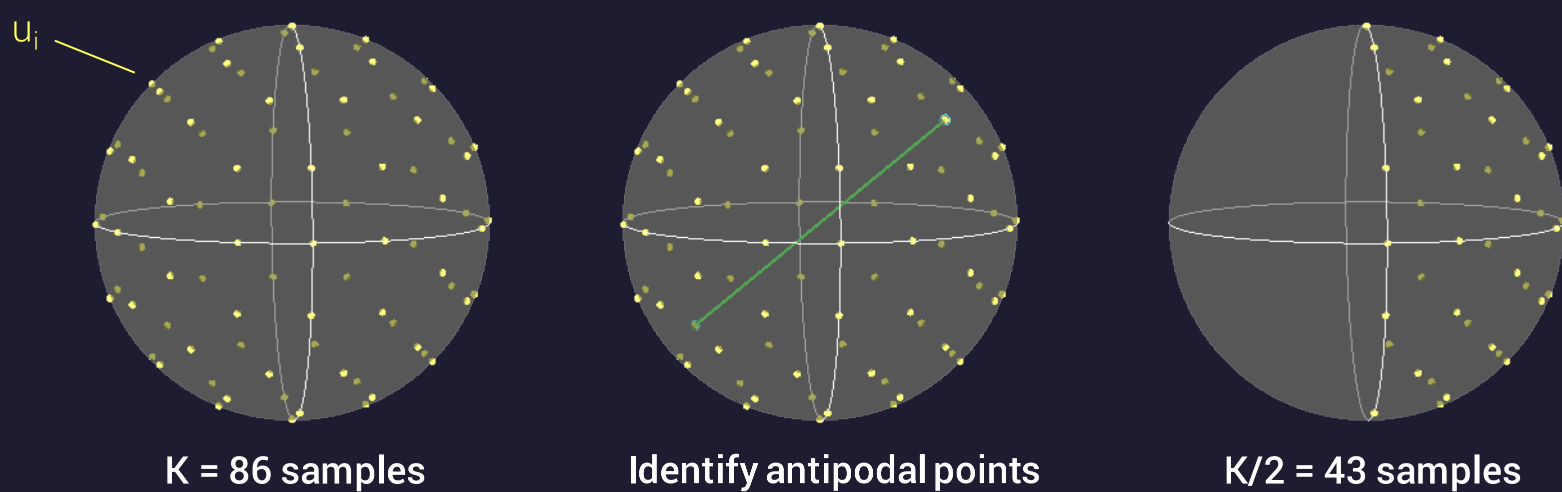
- **Problem:** How to implement this efficiently for inverse problems?
  1. Discretize integral using an efficient quadrature
  2. Exploit steerability of directional derivatives
  3. Employ a fast alternating minimization algorithm

1. Discretize integral using an efficient quadrature

- Quadrature of sphere: {unit-directions u_i and weights w_i, $i=1,\ldots,K$ }

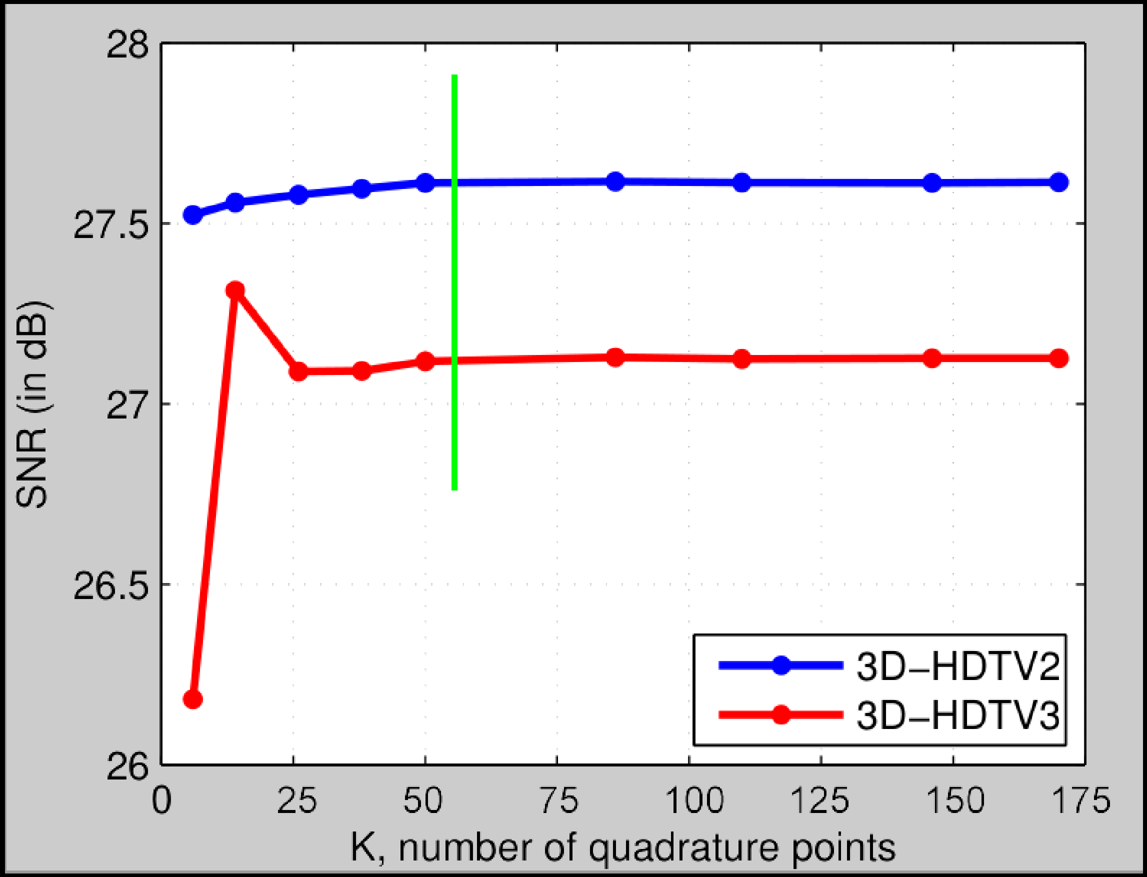
$$\mathrm{HDTV}\mathbf{n}(\mathbf{f}) \approx \sum_{i=1}^{K} w_i \|\partial^{\mathbf{n}}_{u_i} \mathbf{f}\|_1$$

- Lebedev quadrature: efficient, symmetric (Lebedev & Laikov, 1999)
- Exploits symmetry of directional derivatives: $|\partial^{\mathbf{n}}_{\mathbf{u}}\mathbf{f}| = |\partial^{\mathbf{n}}_{-\mathbf{u}}\mathbf{f}|$



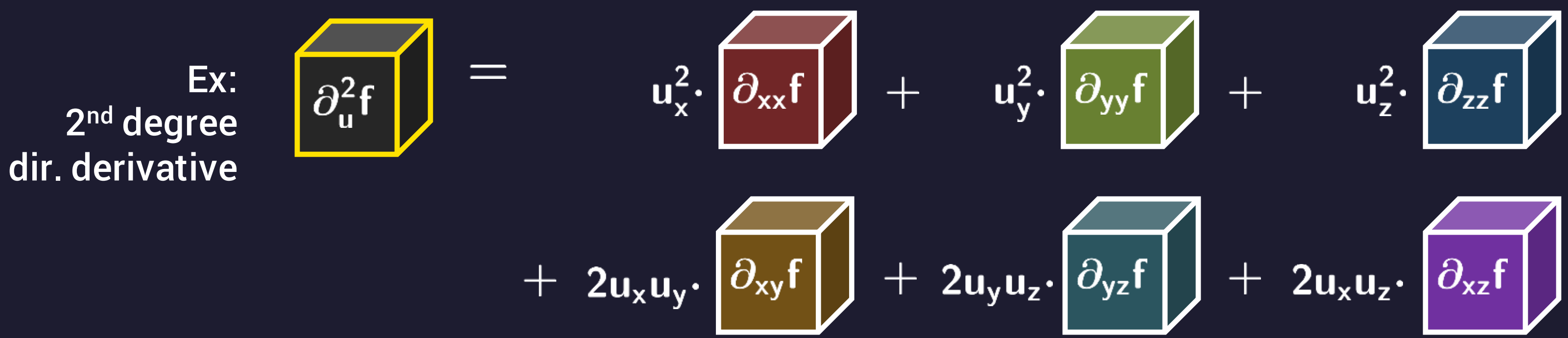
- Numerical experiments show that 30-50 samples are sufficient

SNR vs. #quadrature points in a denoising experiment



2. Exploit **steerability** of directional derivatives

- HD directional derivatives are weighted sum of partial derivatives
- Significantly reduces # filtering operations: 6 for HDTV2, 10 for HDTV3



- Compute discrete partial derivatives with finite differences
- Obtain all K dir. derivatives with matrix multiplication O(KN), N= # voxels.

3. Employ a fast alternating minimization algorithm

- Adapt a new fast algorithm for 2-D HDTV
- Based on variable splitting and quadratic penalty method

Linear Inverse Problem with 3-D HDTV regularization

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}-\mathbf{b}\|_2^2 + \lambda \sum_{i=1}^{K} \|\mathbf{D}_i\mathbf{x}\|_1$$

$$\min_{\mathbf{x},\mathbf{z}} \|\mathbf{A}\mathbf{x}-\mathbf{b}\|_2^2 + \lambda \sum_{i=1}^{K} \|\mathbf{z}_i\|_1 \quad \text{subj. to} \quad \mathbf{D}_i\mathbf{x} = \mathbf{z}_i$$

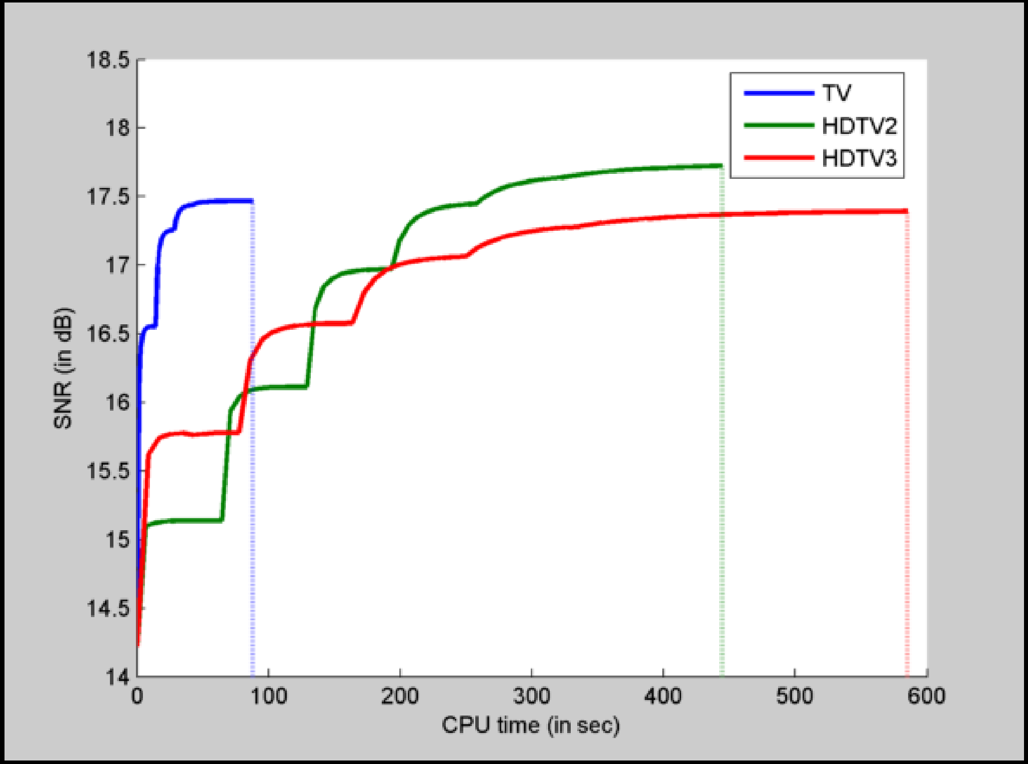
$$\min_{\mathbf{x},\mathbf{z}} \boxed{\|\mathbf{A}\mathbf{x}-\mathbf{b}\|_2^2} + \boxed{\lambda \sum_{i=1}^{K} \|\mathbf{z}_i\|_1 + \frac{\lambda\beta}{2}\|\mathbf{D}_i\mathbf{x}-\mathbf{z}_i\|_2^2;} \quad \beta \to \infty$$

$\mathbf{x} =$ image
$\mathbf{A} =$ linear measurement operator
$\mathbf{b} =$ noisy measurement vector
$\mathbf{D}_i =$ directional derivative operators
$\lambda =$ regularization parameter
$\mathbf{z} =$ auxiliary variable
$\beta =$ continuation parameter

- z-subproblem: shrinkage of directional derivatives
- x-subproblem: invert linear system → FFTs or CG

Estimated computation time

- CS-MRI recovery experiment @1.6x acceleration
- 256x256x76 dataset
- MATLAB implementation running on CPU (Intel Xeon 3.6 GHz, 4 cores)
- Running time:
  - **TV**: 1.5 minutes
  - **HDTV2**: 7.5 minutes
  - **HDTV3**: 10 minutes



Results

3-D Quantitative Results

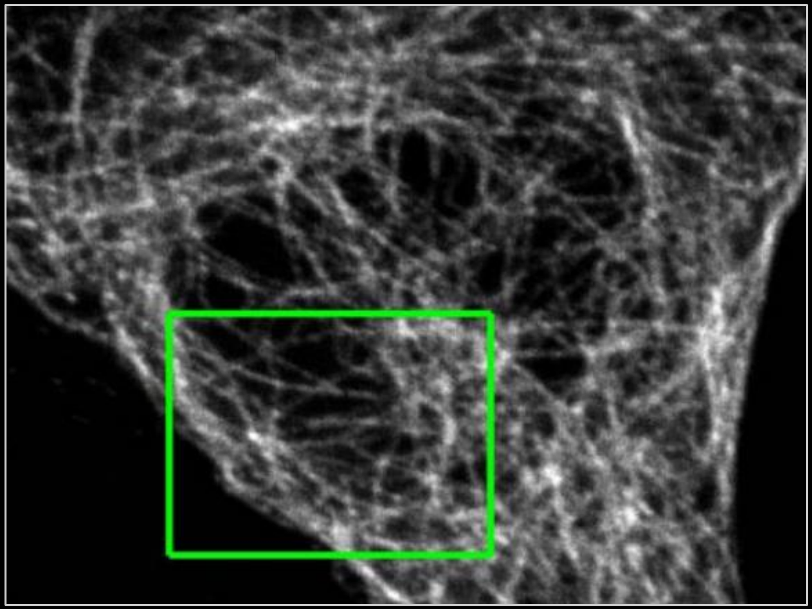
Table 3: **3-D Comparisons**. SNR (in dB) of recovered images with optimal reg. param.

	Denoising		Deblurring			CS-MRI		
	Cell1	Cell2	Cell1	Cell2	Cell3	Angio, acc=5	Angio, acc=1.5	Cardiac
TV	17.12	16.25	19.02	16.43	14.50	13.87	14.53	18.37
HDTV2	17.25	16.70	19.15	16.60	14.87	**14.23**	**15.11**	**18.56**
HDTV3	**17.68**	**17.14**	**19.73**	**17.43**	**15.23**	14.01	14.70	18.50

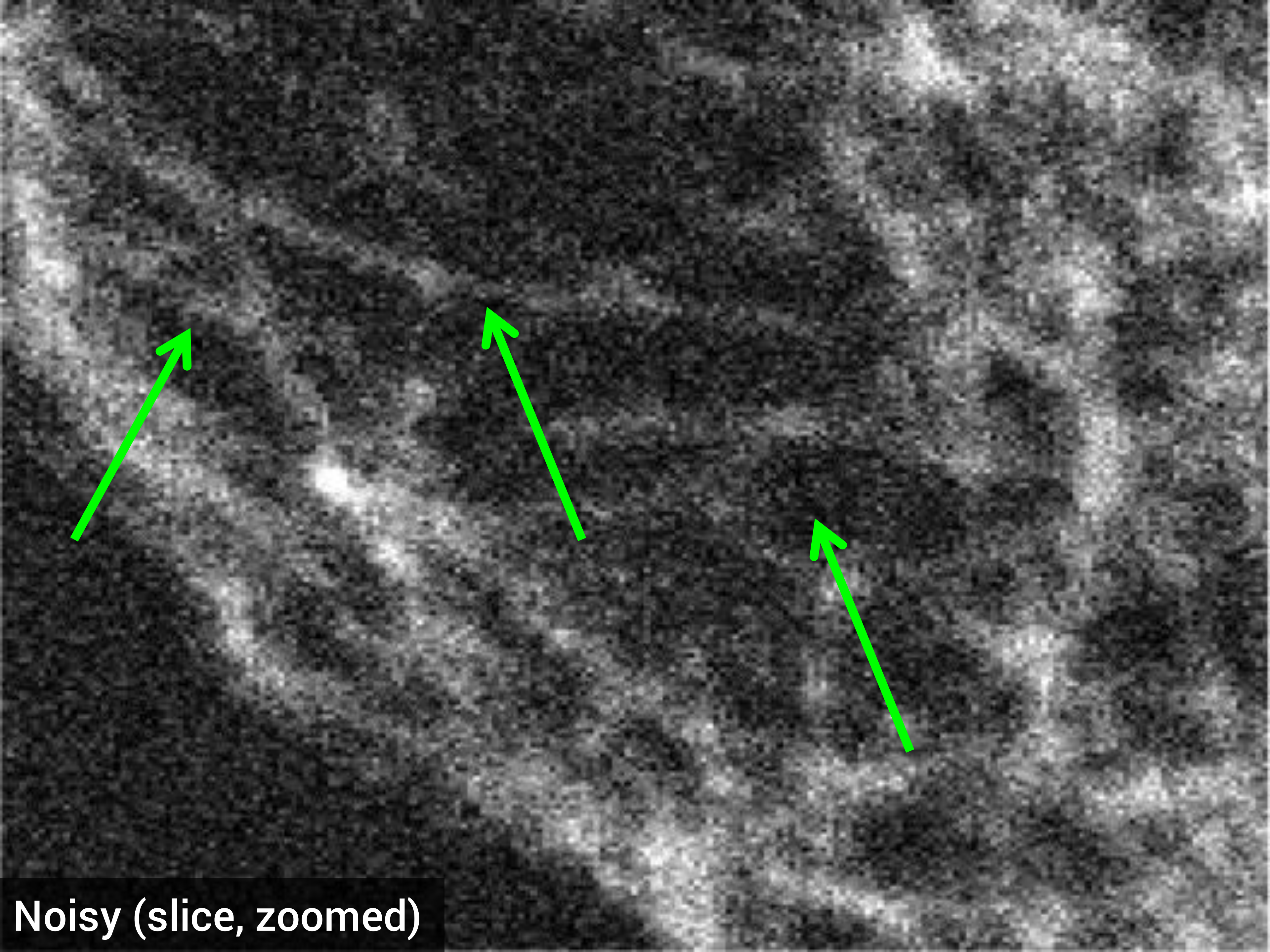
- HDTV outperforms TV in all experiments
- HDTV3 better for denoising and deblurring
- HDTV2 better for CS-MRI

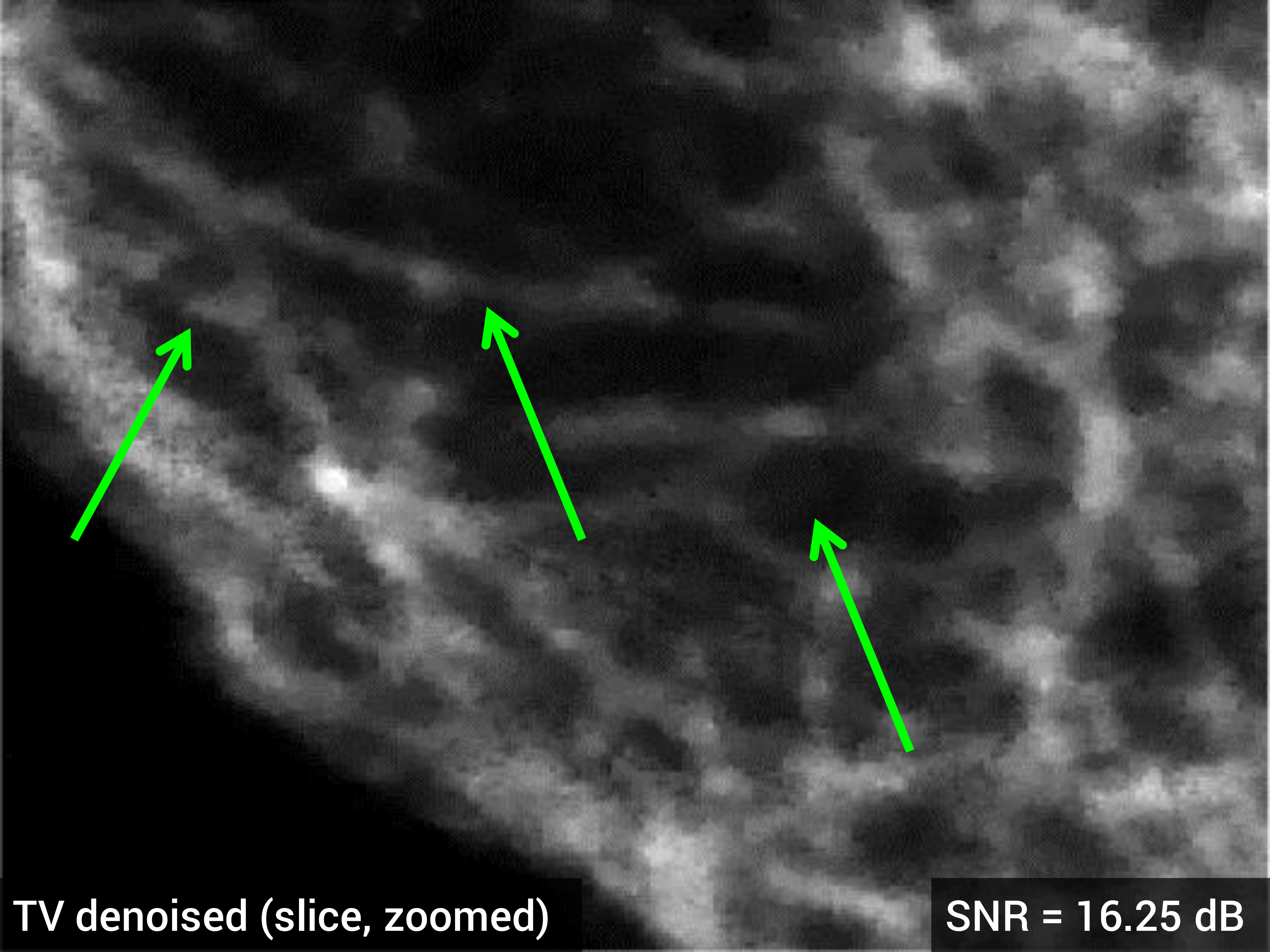
Denoising of 3-D Florescence Microscopy

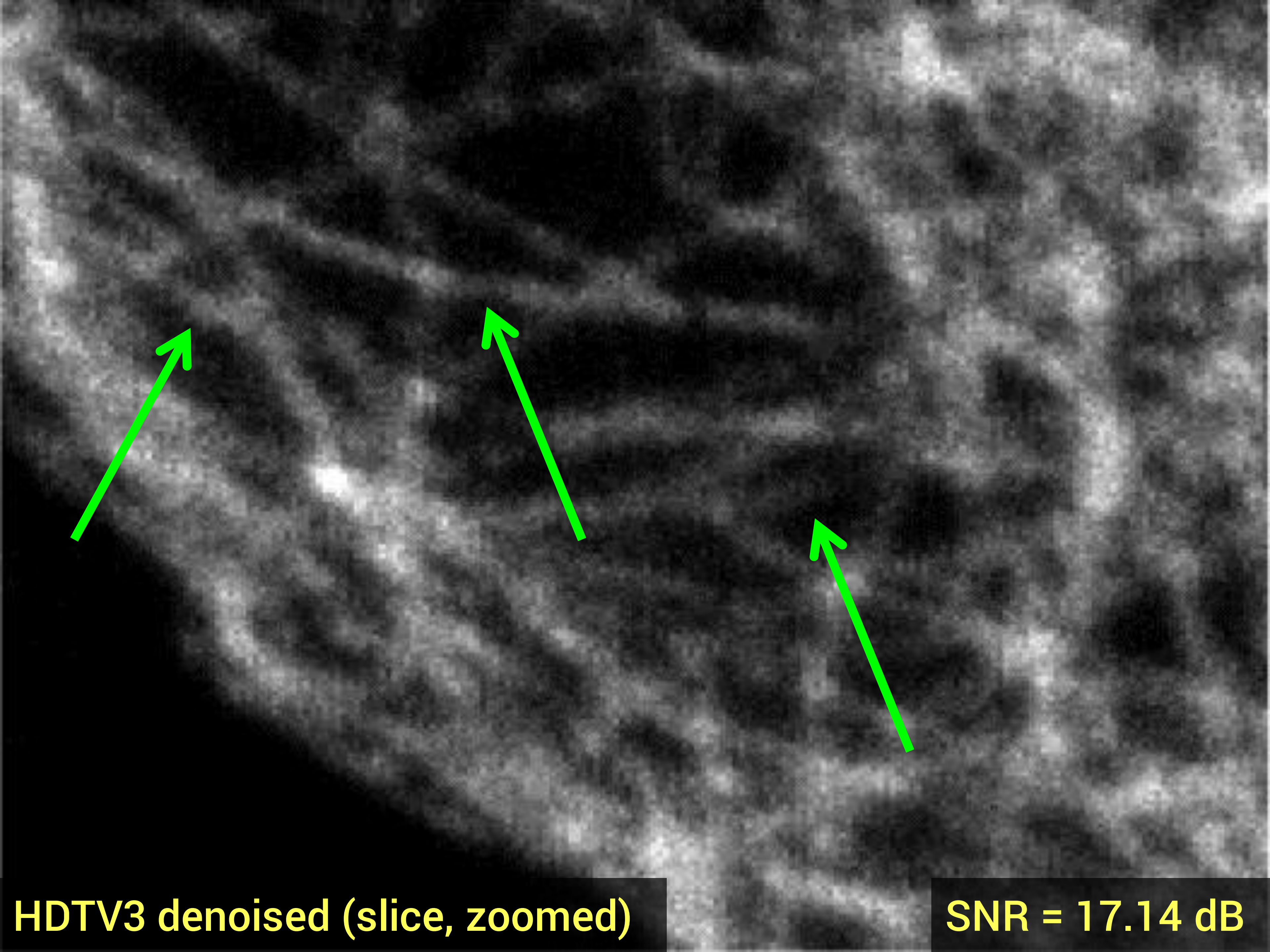
- 1024x1024x17 voxels
- Additive Gaussian noise, mean = 0, std. dev. = 1
- Noisy image has SNR = 15 dB
- Optimized regularization parameter



original dataset (z-slice)

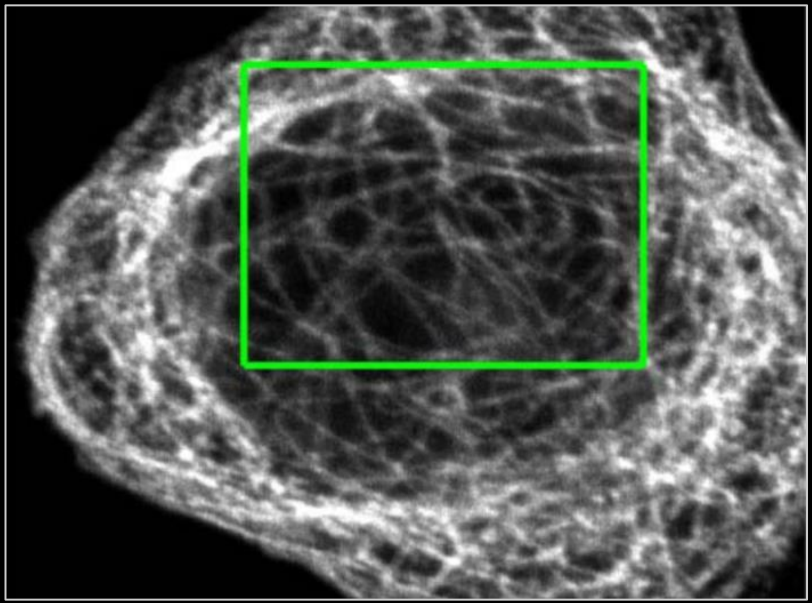

Noisy (slice, zoomed)


TV denoised (slice, zoomed)
SNR = 16.25 dB

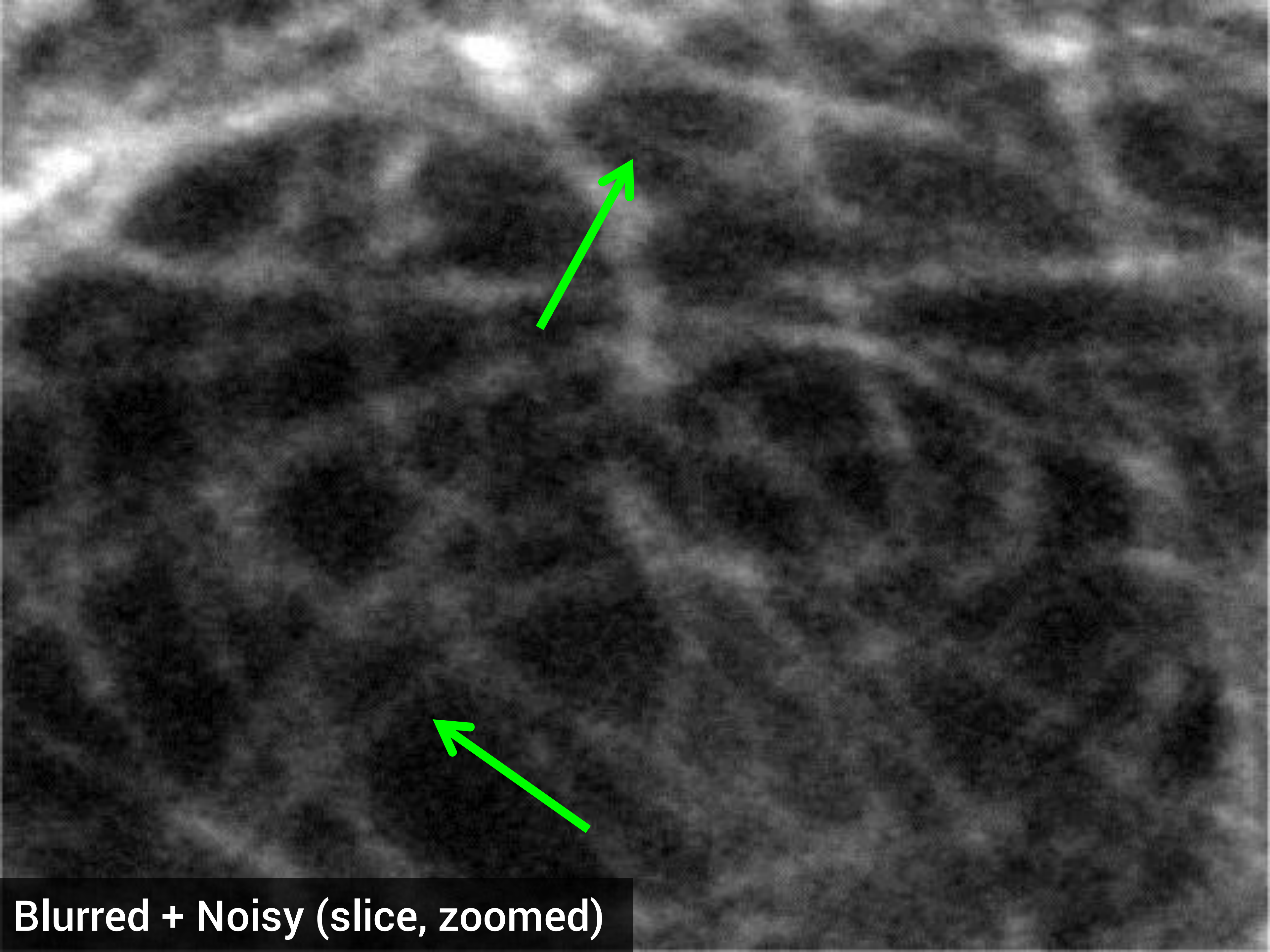

HDTV3 denoised (slice, zoomed)
SNR = 17.14 dB

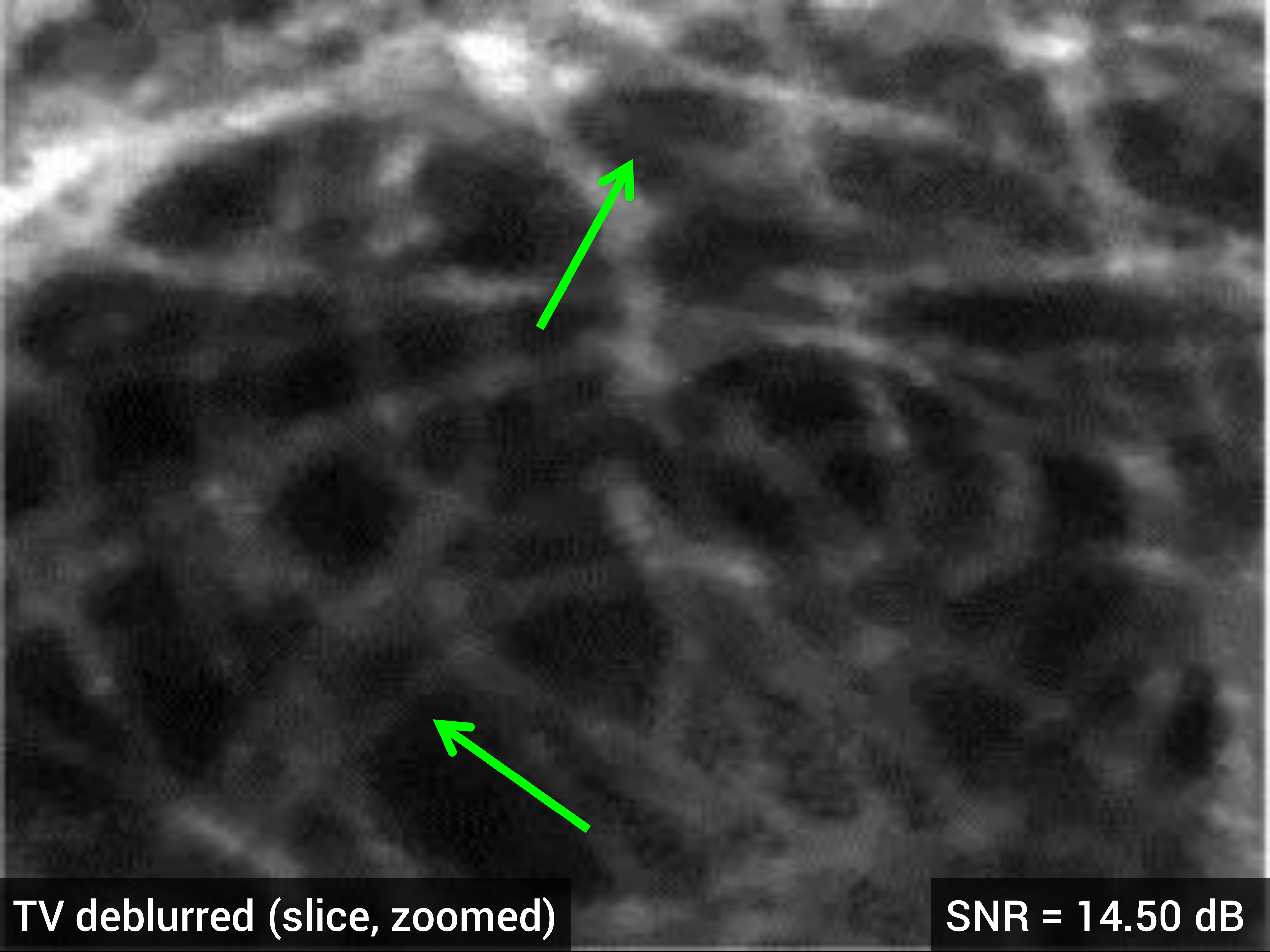
Deblurring of 3-D Florescence Microscopy

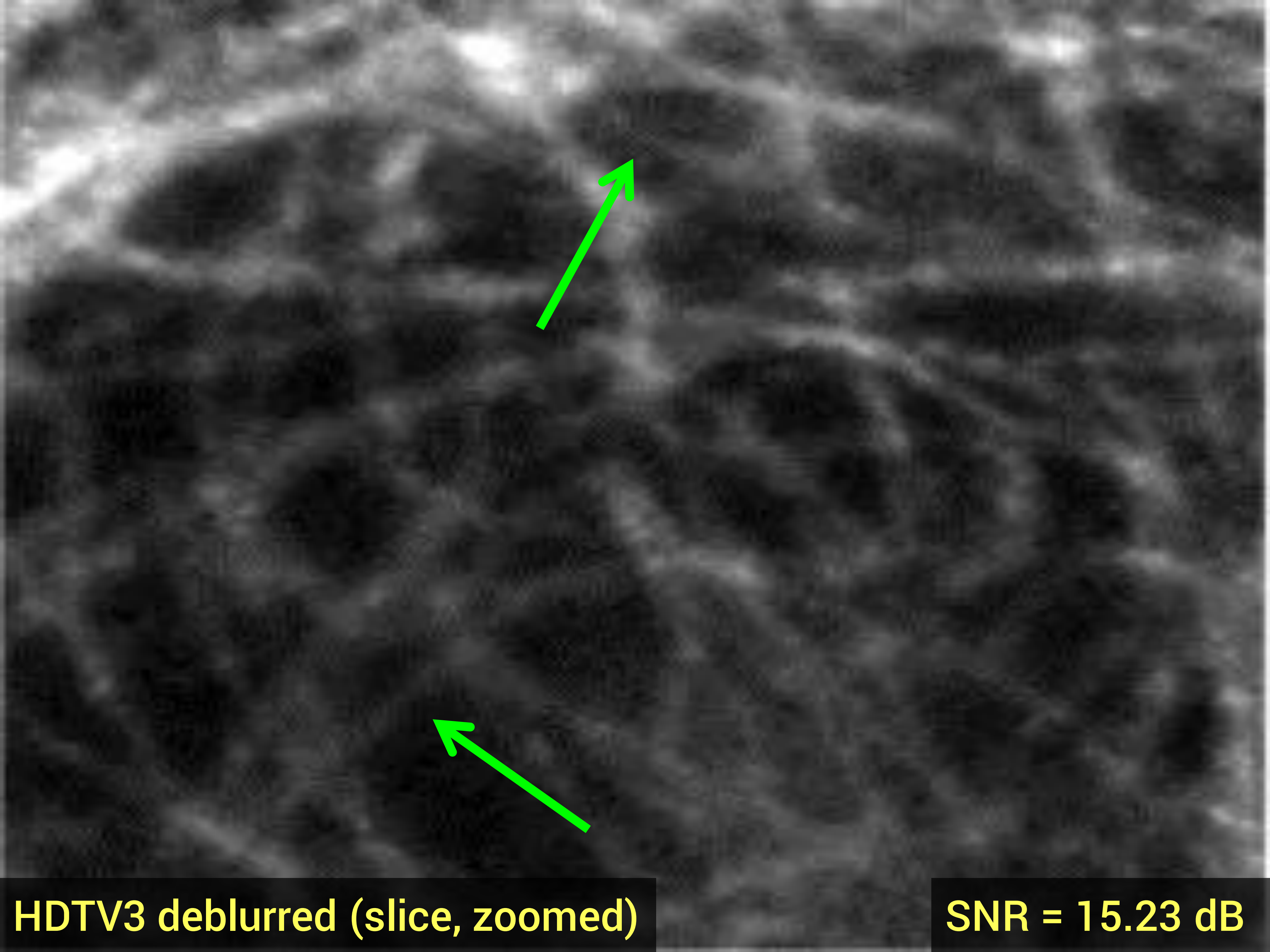
- 1024x1024x17 voxels
- 3x3x3 Gaussian blur kernel, std. dev = 0.05
- 5 dB additive Gaussian noise
- Optimized regularization parameter



original dataset (z-slice)


Blurred + Noisy (slice, zoomed)

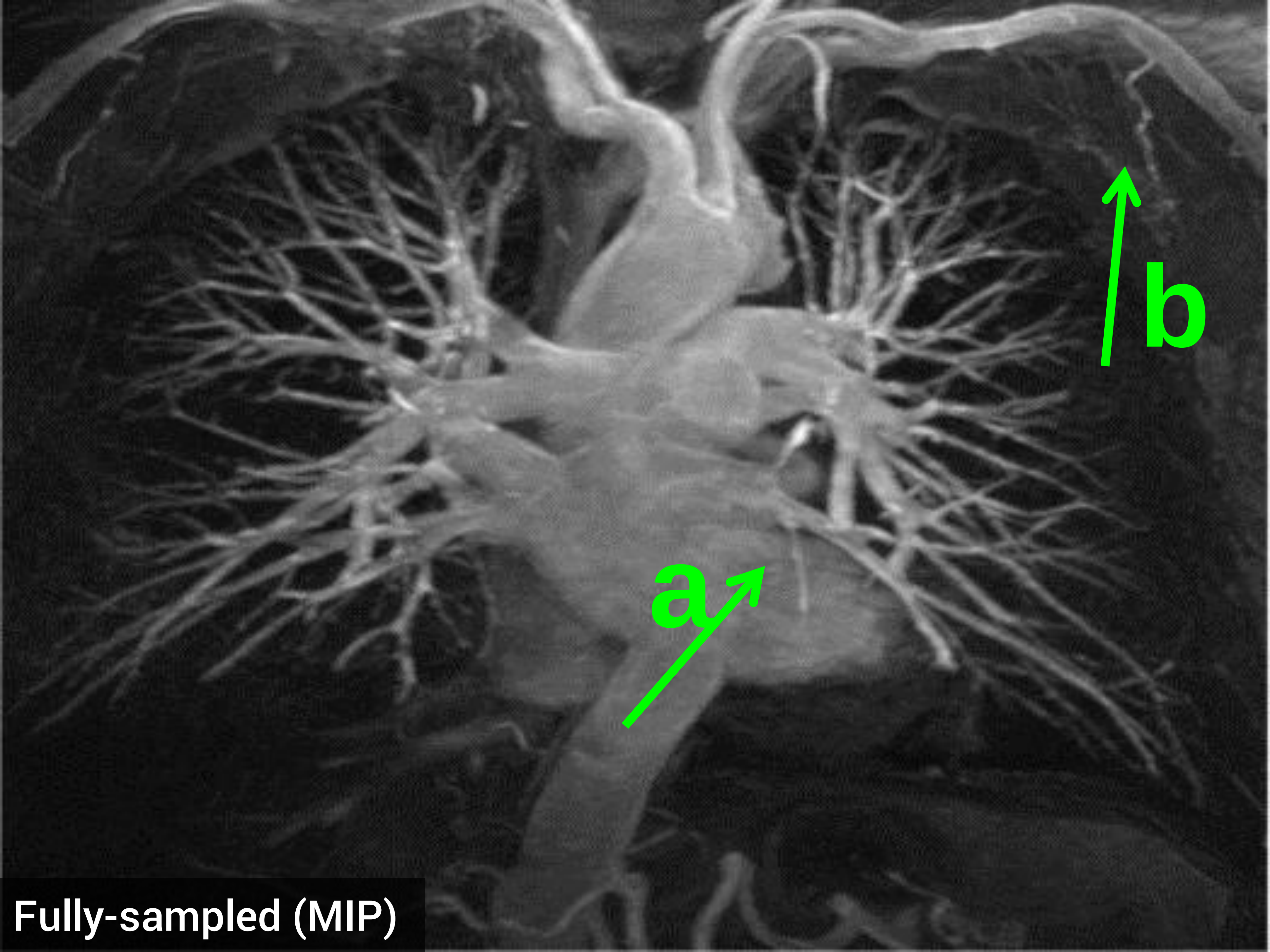

TV deblurred (slice, zoomed)
SNR = 14.50 dB

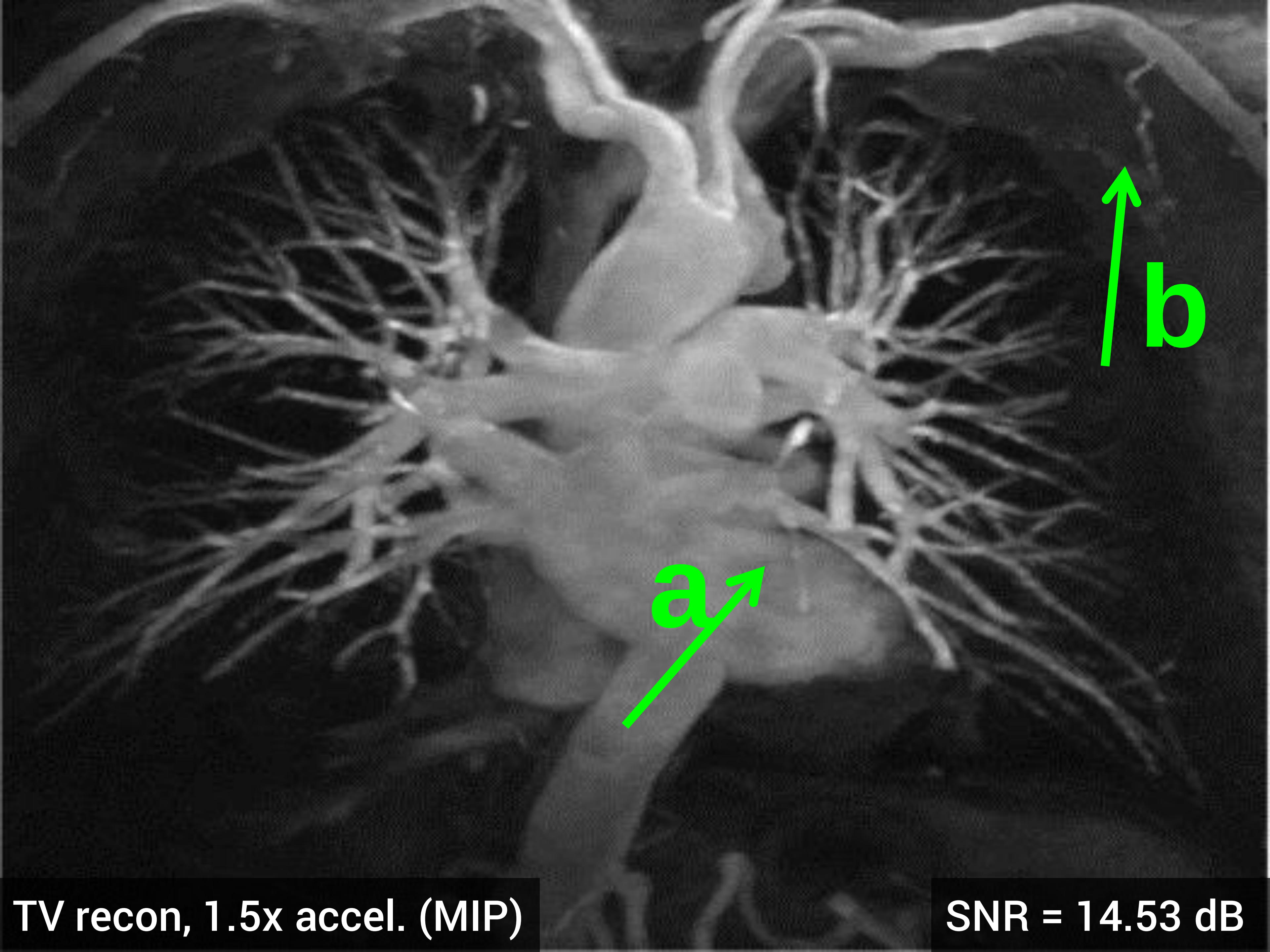

HDTV3 deblurred (slice, zoomed)
SNR = 15.23 dB

3-D Compressed Sensing MRA

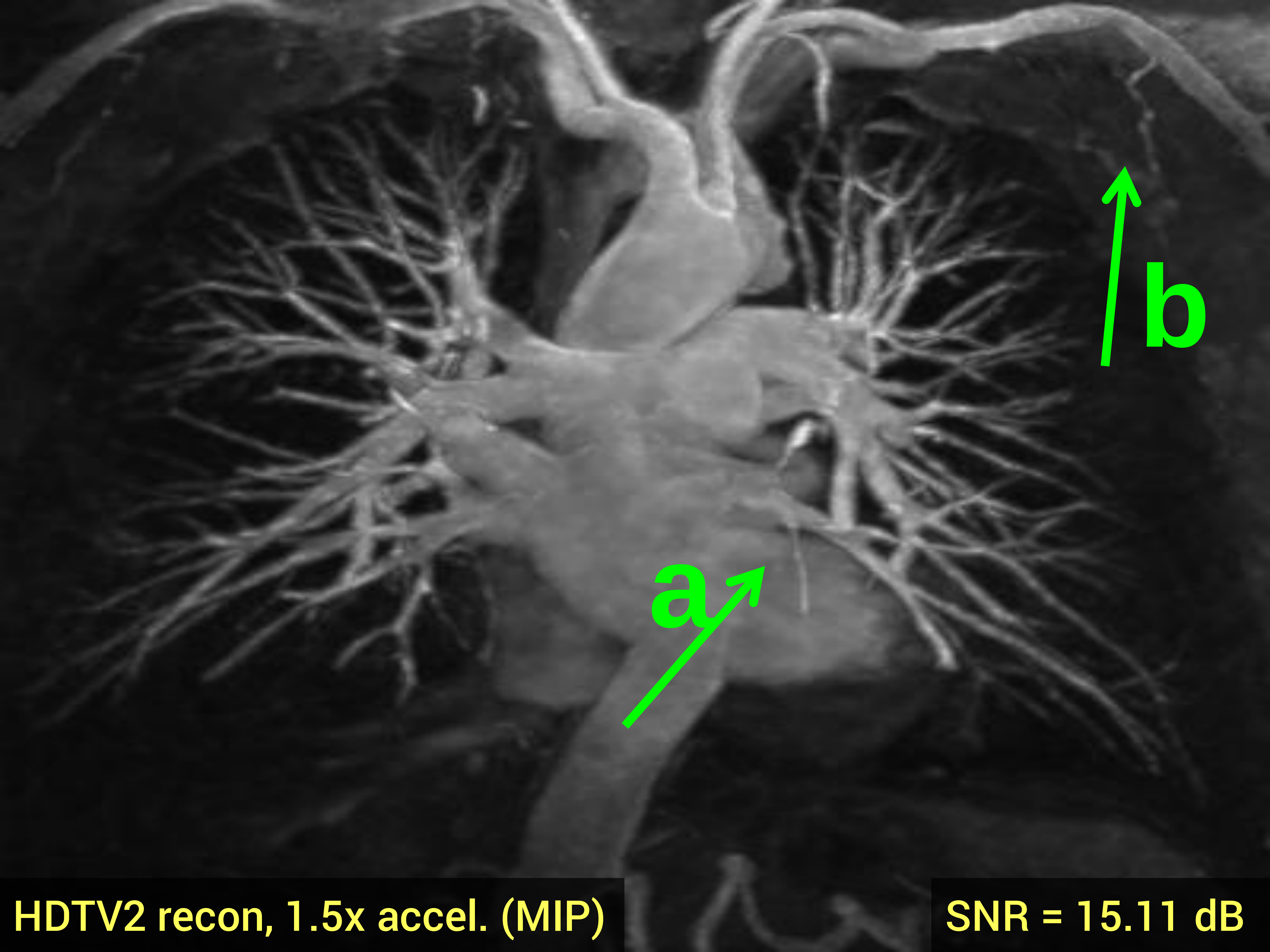
- 512x512x76 voxel MRA dataset obtained from physiobank (see ref. [6])
- Simulated single coil acquisition
- Retroactively undersampled at 1.5-fold acceleration
- Random Gaussian sampling of k-space
- 5 dB additive Gaussian noise
- Optimized regularization parameter

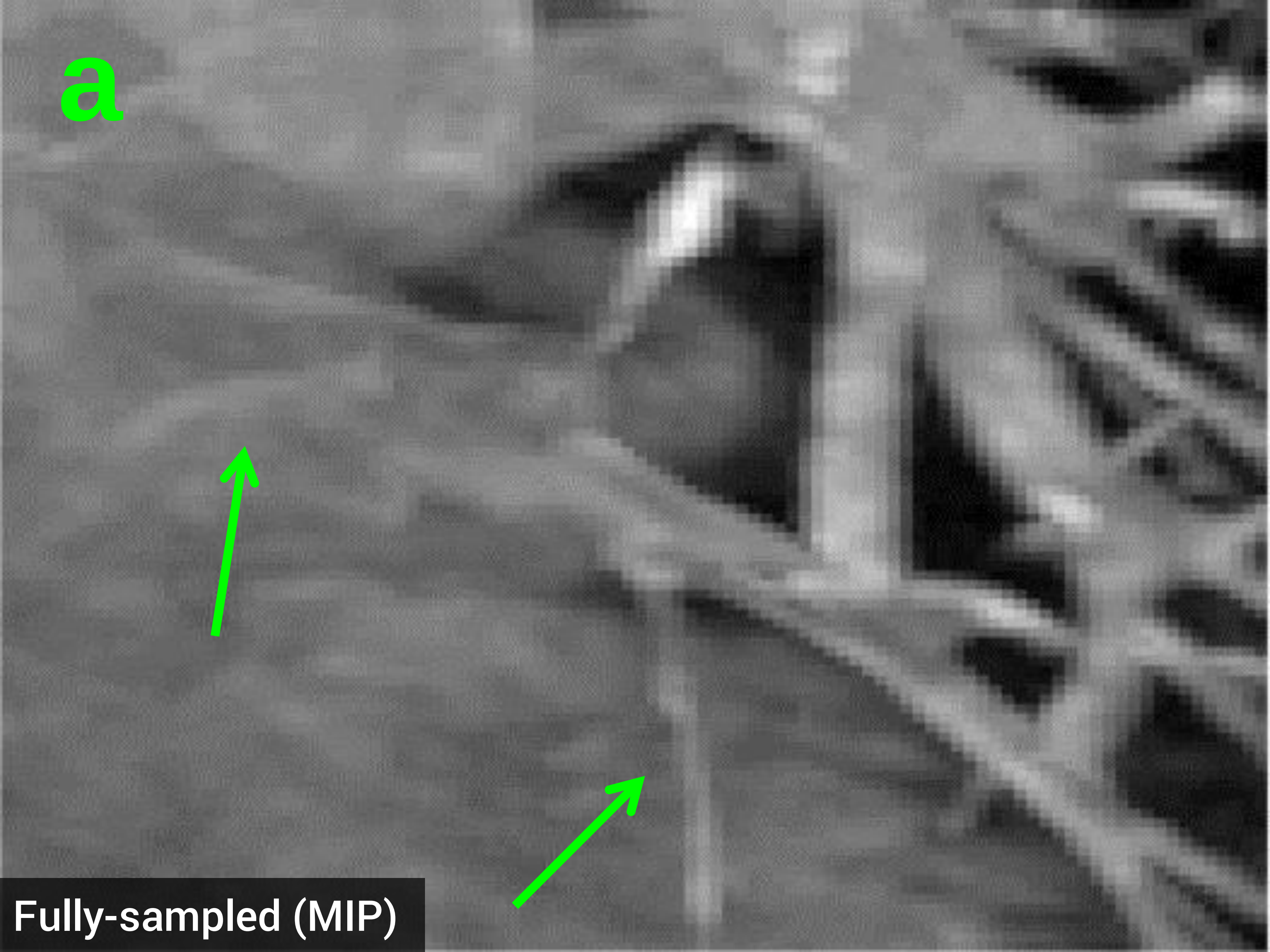
MIP of original MRA dataset

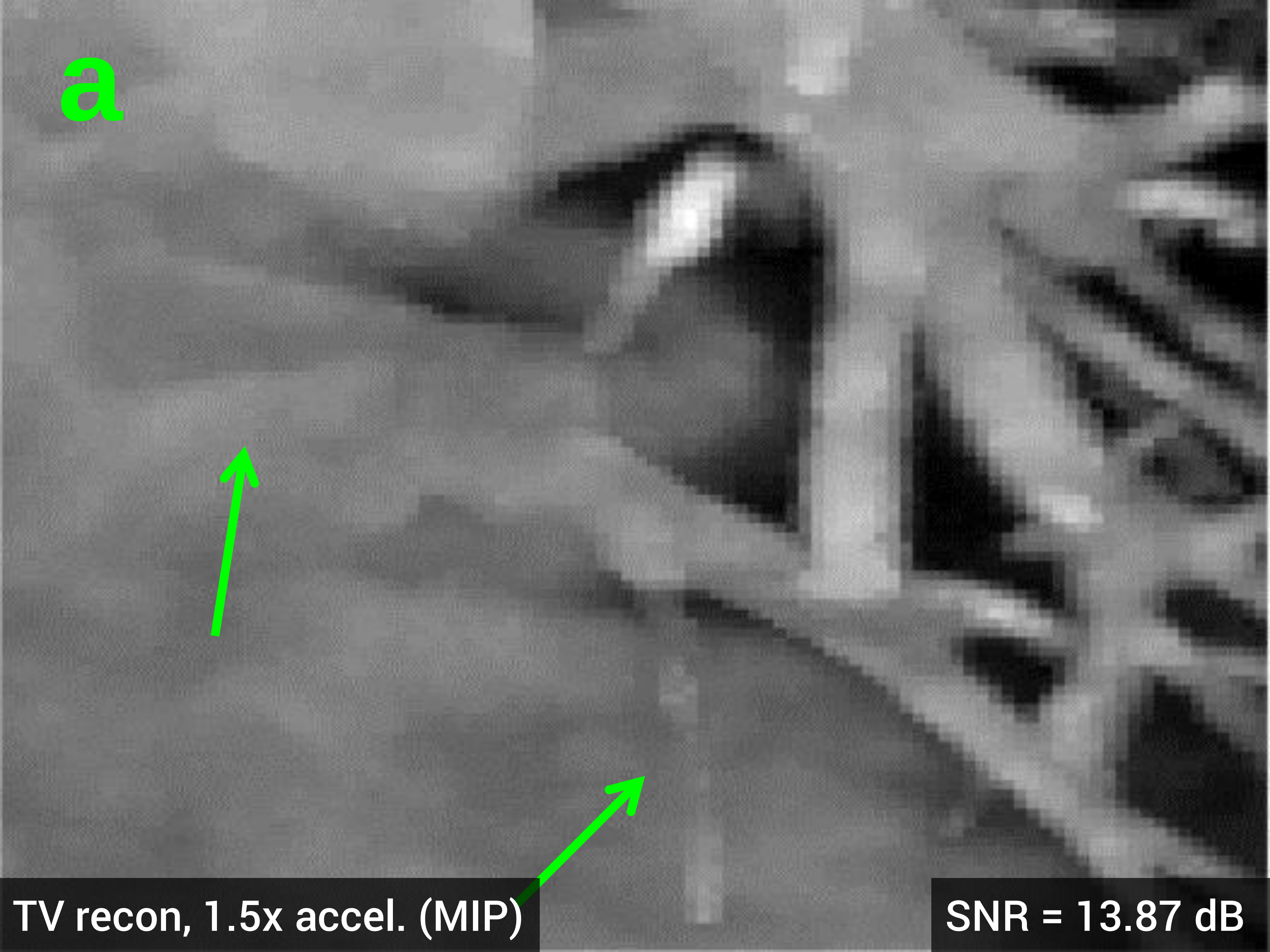

a
b
Fully-sampled (MIP)

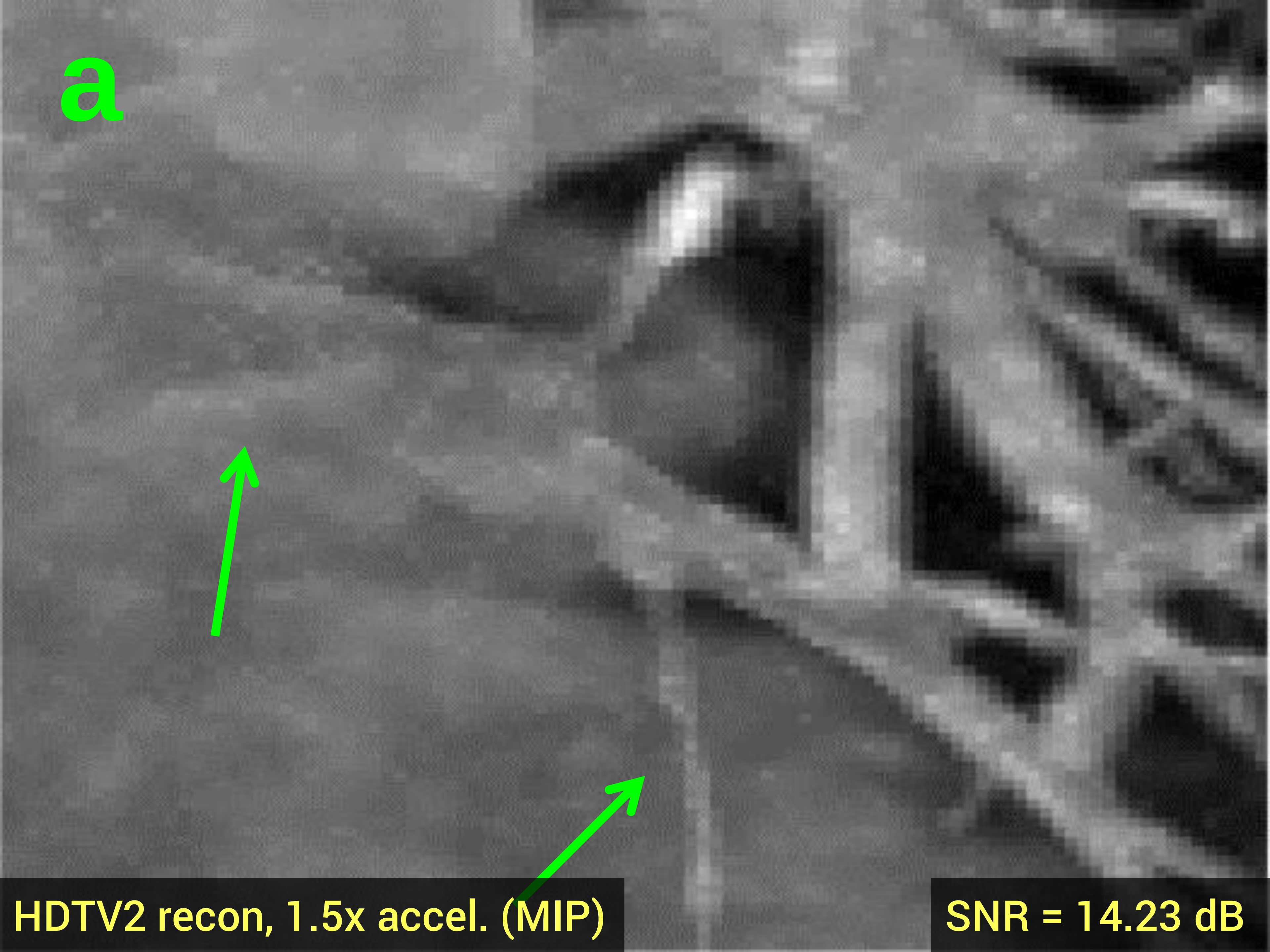


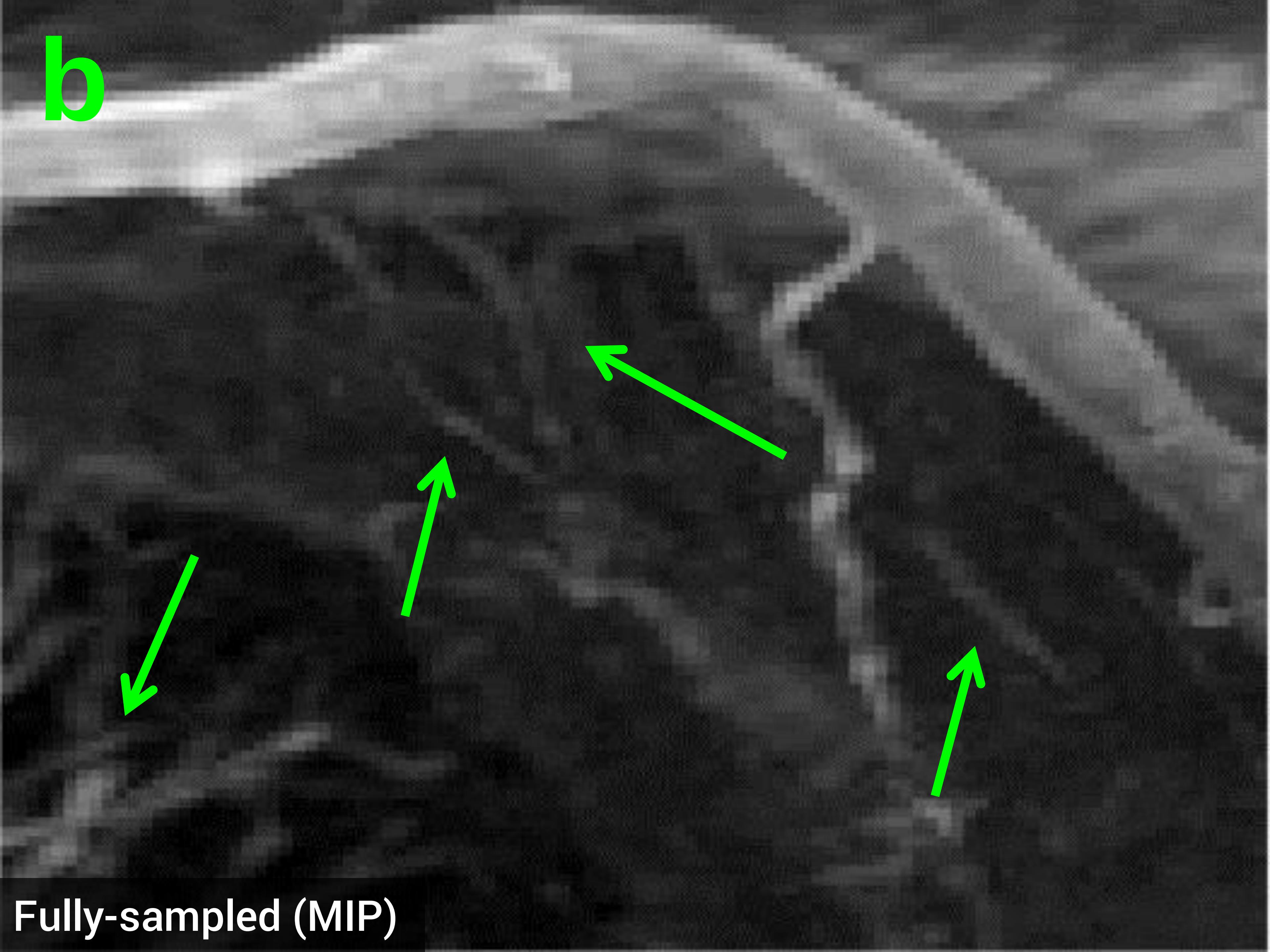
b
a
TV recon, 1.5x accel. (MIP)
SNR = 14.53 dB

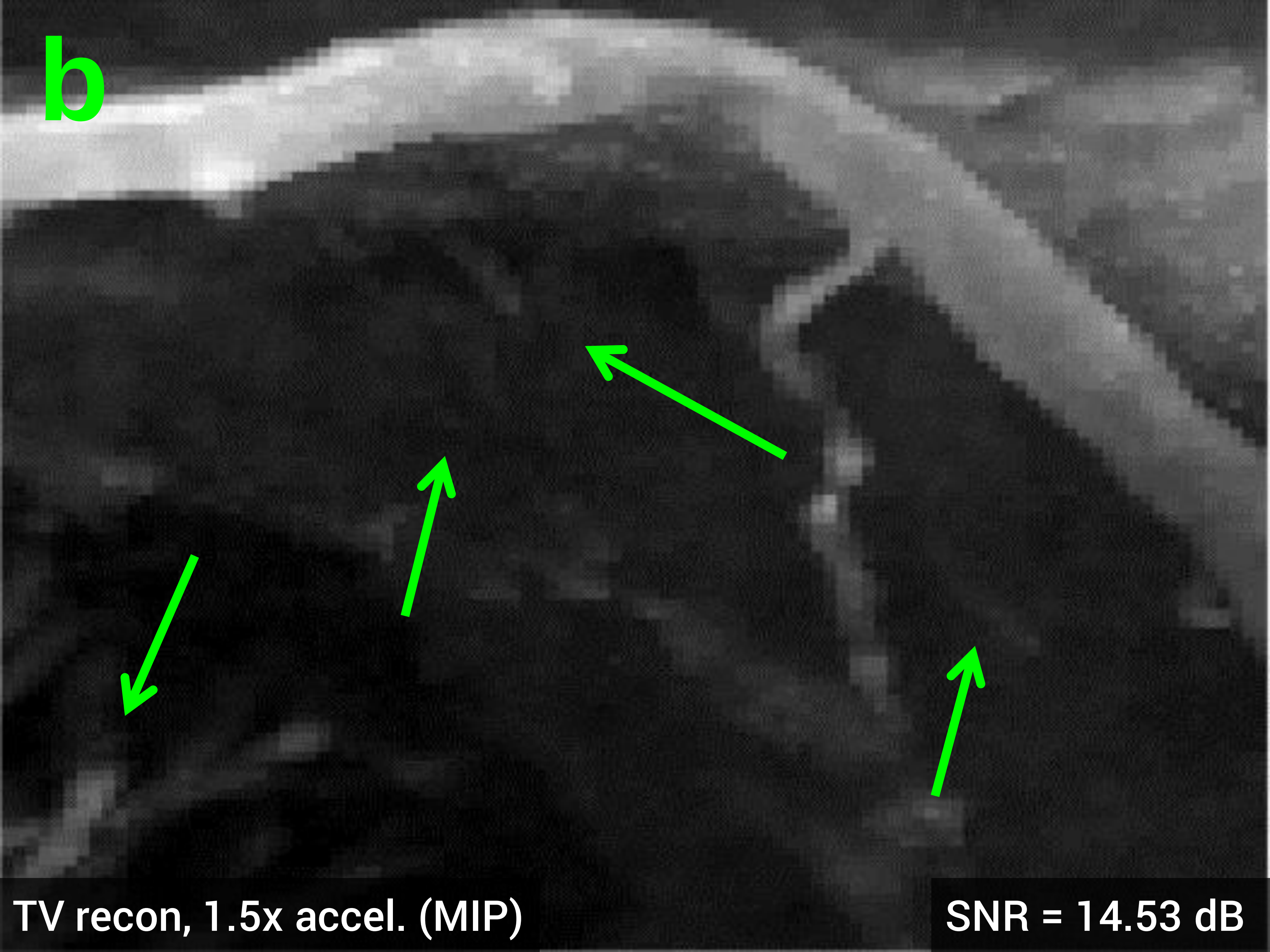

a
b
HDTV2 recon, 1.5x accel. (MIP)
SNR = 15.11 dB

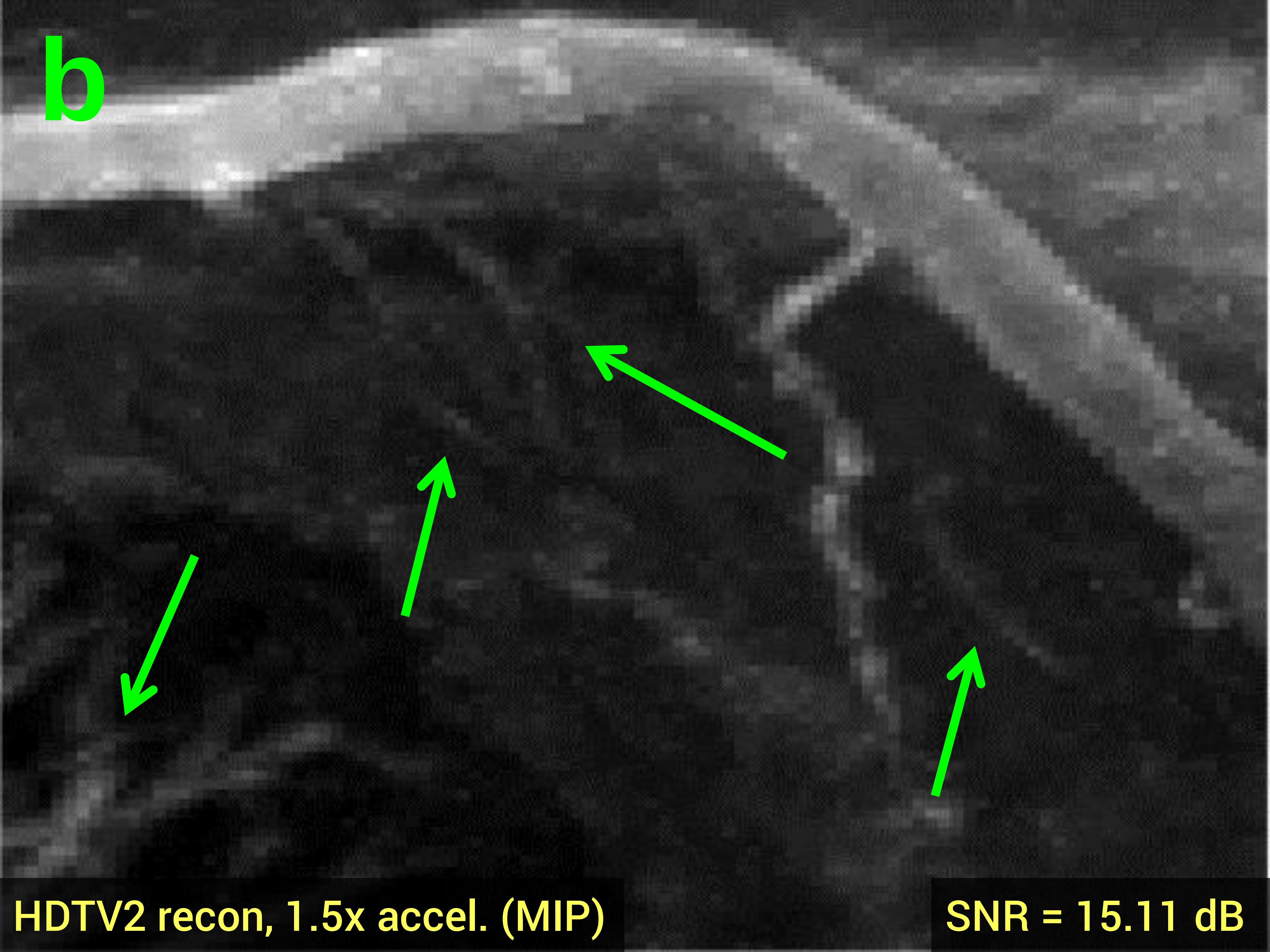

a
Fully-sampled (MIP)


a
TV recon, 1.5x accel. (MIP)
SNR = 13.87 dB


a
HDTV2 recon, 1.5x accel. (MIP)
SNR = 14.23 dB


b
Fully-sampled (MIP)


b
TV recon, 1.5x accel. (MIP)
SNR = 14.53 dB


b
HDTV2 recon, 1.5x accel. (MIP)
SNR = 15.11 dB

Conclusion

Summary

- We extended the HDTV penalties to 3-D
- Implemented efficiently: quadrature, steerabililty, alternating minimize
- HDTV outperformed TV in our 3-D image recovery experiments
- 3-D HDTV2 showed promising application to CS-MRI recovery
- 3-D HDTV3 denoising and deblurring

Code

- MATLAB implementation available at:

  CBIG Website: http://research.engineering.uiowa.edu/cbig

  http://github.com/cbig-iowa/hdtv

- plug-in for 3-D HDTV denoising (in development)

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Thank You!

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[6] Physiobank: http://physionet.org/physiobank/database/images/,

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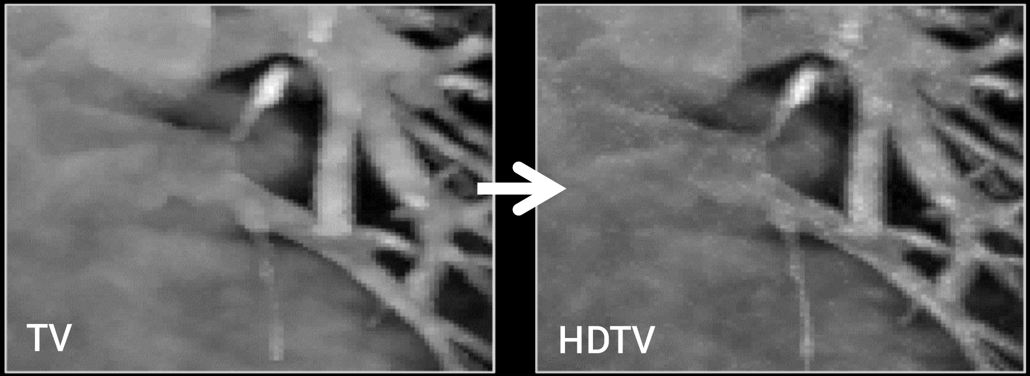

Code:
http://research.engineering.uiowa.edu/cbig
http://github.com/cbig-iowa/hdtv

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