On thinking fast and slow

A bat and a ball together cost 1.1 dollars. The bat costs 1 dollar more than the ball.

What is a dollar? We all know what a dollar is … because it is denoted by the symbol $. TEACHER, WHAT IS A DOLLAR?

…

$$\begin{cases} 1.1\$ = P_{Bat} + P_{Ball} \\ P_{Bat} - P_{Ball} = 1\$ \end{cases}$$

$$\begin{cases} P_{Bat} + P_{Ball} = 1.1\$ \\ P_{Bat} - P_{Ball} = 1\$ \end{cases}$$

∴

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = \frac{\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}}{1*(-1)-1*1} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1*1-1*(-1) \\ -1*1+1*(-1) \end{pmatrix}$$

$$\begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1+1 \\ -1-1 \end{pmatrix}$$

$$\begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} P_{Bat} \\ P_{Ball} \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$P_{Bat} = -1\$$ and $P_{Ball} = 1\$$ WHAT?

OR

$P_{Bat} = 1\$ + P_{Ball}$

AND

$1.1\$ = 1\$ + P_{Ball} + P_{Ball}$

$0.1\$ = 2P_{Ball}$

$P_{Ball} = 0.05\$$ and $P_{Bat} = 1.05\$$ TEACHER?

On dollars

$P_{Bat} = -1\$$ and $P_{Ball} = 1\$$

AND

$P_{Bat} = (y + x)\$$ and $P_{Ball} = x\$$

Assume that $P_{Bat} + P_{Ball} = P_{Bat} + P_{Ball}$

$\therefore$

$-1\$ + 1\$ = (y + x)\$ + x\$$

$0 = (y + x)\$ + x\$$

$0 = y\$ + x\$ + x\$$

$0 = \$(y + 2x)$

$\therefore$

$y + 2x = 0$

$-y = 2x$

$\frac{x}{y} = -\frac{1}{2}$

OR

$\$ = 0$

In the previous case the values were $x = 0.1$ and $y = 1$ and the solution was $P_{Ball} = 0.05\$$ and $P_{Bat} = 1.05\$$

The condition does not hold because $\frac{0.1}{1} = \frac{\frac{1}{10}}{1} = \frac{1}{10} = 0.1 \neq -\frac{1}{2}$

$\therefore$

$\$ = 0$

Lessons learned

Some statements do not generalize

Think about what x and y could represent

My conclusion is that y could represent the quantity of information already learned while x could represent the quantity of information yet to be learned