Present value

Simple models

Introduction

Geometric series

Geometric series

Geometric series = Numerical series with a constant ratio between consecutive terms

Example : $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = a + ab + ab^2 + ab^3 + \cdots$

with $a = \frac{1}{2}$ and $b = \frac{1}{2}$

Geometric series

What is the sum of the geometric series up to the Nth term?

$$S_N = a + ab + ab^2 + ab^3 + \cdots + ab^{N-1}$$

$$bS_N = b(a + ab + ab^2 + ab^3 + \cdots + ab^{N-1})$$
$$bS_N = ab + ab^2 + ab^3 + ab^4 + \cdots + ab^N$$
$$S_N - bS_N = (a + ab + ab^2 + ab^3 + \cdots + ab^{N-1}) - (ab + ab^2 + ab^3 + ab^4 + \cdots + ab^N)$$
$$(1-b)S_N = a - ab^N$$

$$S_N = \frac{a(1-b^N)}{1-b} \text{ if } b \neq 1$$

Geometric series

What is the sum of the geometric series up to infinitely many terms?

$$\lim_{N \to \infty} S_N = \lim_{N \to \infty} \frac{a\left(1-b^N\right)}{1-b} = \frac{a}{1-b} \text{ if } -1 < b < 1$$

Example : $\lim_{N \to \infty} S_N = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

Present value

General model

General model

$$V_T = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots + \frac{C_T}{(1+r)^T}$$

V_T = Present value

C_t = Cash flow in period t

r = Constant discount rate

Present value

Simple models

Constant cash flows

$$C_1 = C_2 = C_3 = \cdots = C_T \equiv C$$

$$V_T = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^T}$$

$$V_T = \frac{C}{1+r} + \frac{C}{1+r}\left(\frac{1}{1+r}\right) + \frac{C}{1+r}\left(\frac{1}{1+r}\right)^2 + \cdots + \frac{C}{1+r}\left(\frac{1}{1+r}\right)^{T-1}$$

V_T is a geometric series with $a = \frac{C}{1+r}$, $b = \frac{1}{1+r}$ and $N = T$

$$V_T = \frac{\frac{C}{1+r}\left(1-\left(\frac{1}{1+r}\right)^T\right)}{1-\frac{1}{1+r}} = \frac{C\left(1-\left(\frac{1}{1+r}\right)^T\right)}{r} \text{ if } r \neq 0$$

Constant cash flows

$$\lim_{T\to\infty} V_T = \frac{\frac{C}{1+r}}{1-\frac{1}{1+r}} = \frac{C}{r} \text{ if } r > 0$$

Constant cash flow growth rate

$C_1 = C$
$C_t = C_{t-1}(1+g)$ for $t = 2,3,\dots,T$

$$V_T = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots + \frac{C(1+g)^{T-1}}{(1+r)^T}$$
$$V_T = \frac{C}{1+r} + \frac{C}{1+r}\left(\frac{1+g}{1+r}\right) + \frac{C}{1+r}\left(\frac{1+g}{1+r}\right)^2 + \cdots + \frac{C}{1+r}\left(\frac{1+g}{1+r}\right)^{T-1}$$

V_T is a geometric series with $a = \frac{C}{1+r}$, $b = \frac{1+g}{1+r}$ and $N = T$

$$V_T = \frac{\frac{C}{1+r}\left(1-\left(\frac{1+g}{1+r}\right)^T\right)}{1-\frac{1+g}{1+r}} = \frac{C\left(1-\left(\frac{1+g}{1+r}\right)^T\right)}{r-g} \text{ if } \mathrm{r} \neq g$$

Constant cash flow growth rate

$$\lim_{T\to\infty} V_T = \frac{\frac{C}{1+r}}{1-\frac{1+g}{1+r}} = \frac{C}{r-g} \text{ if } r > g$$