Log-linearization

$$f(X, Y) = \frac{1+Y}{1+X}$$

$$a = \frac{Y}{X} \rightarrow f(X) = \frac{1+aX}{1+X}$$

$$f(X) = \frac{g(X)}{h(X)} \text{ where } g(X) = 1 + aX \text{ and } h(X) = 1 + X$$

$$\frac{\partial f}{\partial g} = \frac{1}{h(X)}$$

$$\frac{\partial f}{\partial h} = -\frac{g(X)}{h(X)^2}$$

$$df(X) = \frac{\partial f}{\partial g} dg(X) + \frac{\partial f}{\partial h} dh(X)$$

$$\frac{df(X)}{dX} = \frac{\partial f}{\partial g} \frac{dg(X)}{dX} + \frac{\partial f}{\partial h} \frac{dh(X)}{dX}$$

$$f'(X) = \frac{\partial f}{\partial g} g'(X) + \frac{\partial f}{\partial h} h'(X)$$

$$f'(X) = \frac{g'(X)}{h(X)} - \frac{g(X)}{h(X)^2} h'(X)$$

$$g'(X) = a$$

$$h'(X) = 1$$

$$f'(X) = \frac{a}{1+X} - \frac{1+aX}{(1+X)^2} = \frac{a - \frac{1+aX}{1+X}}{1+X} = \frac{a+aX-1-aX}{(1+X)^2} = \frac{a-1}{(1+X)^2}$$

Taylor series expansion (in the neighbourhood of $X_0 = 0$) :

$$\tilde{f}(X) = f(0) + \frac{f'(0)}{1!}(X - 0) + \cdots \text{ where } \ldots \approx 0$$

$$= 1 + (a-1)X$$

$$= 1 + \left(\frac{Y}{X} - 1\right)X$$

$$= 1 + \left(\frac{Y-X}{X}\right)X$$

$$= 1 + Y - X$$