**ENG 180** **Engineering Analysis**

**Project 9, Assignment Handout**

Partial Differential Equations: Mixed Initial Boundary Value Problems

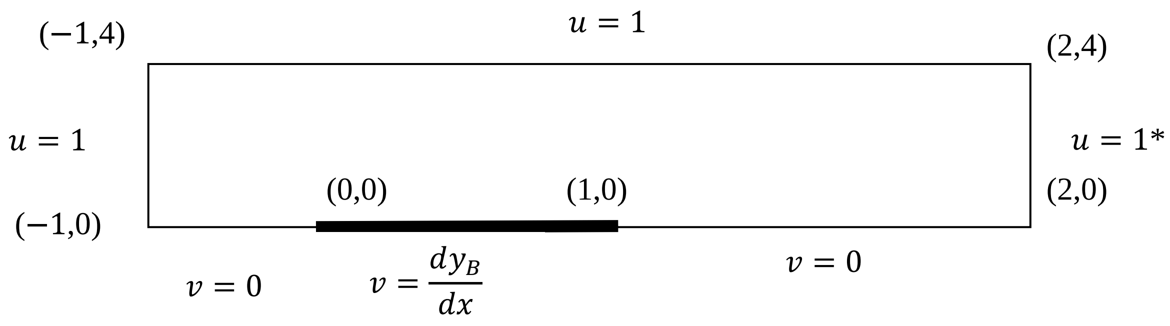
## I. 2D compressible inviscid flow over a thin airfoil

The supersonic flow over a thin airfoil is governed by the following:

$$(1 - M_\infty^2)u_{xx} + u_{yy} = 0 \quad M_\infty = 0.8, 1.4, \text{and } 1.8$$

$$u_y = v_x$$

Where u is the normalized x component of the flow and v is the normalized y component of the flow. Use the following initial and boundary conditions (note * only applies to subsonic case):




$$y_B(x) = \tau \sin(\pi x) \quad \tau = 0.1$$

Use a grid size of $\Delta x = \Delta y = 0.02$

<u>Plot the contours of the pressure coefficient for each case.</u>

$$c_p = -2(u - 1)$$

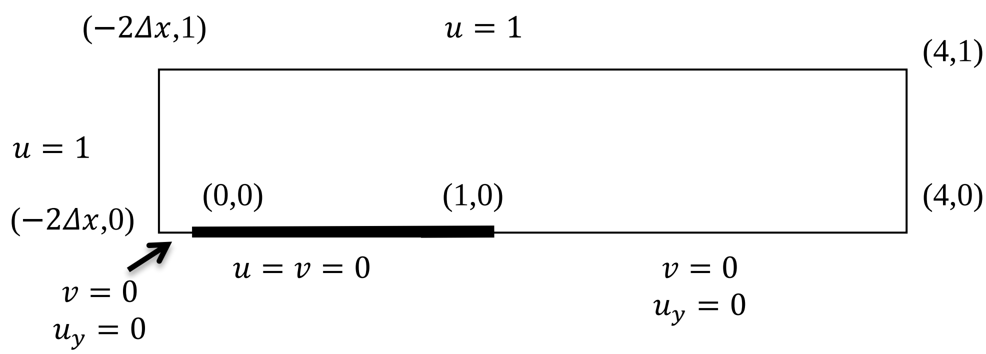
## II. Incompressible viscous boundary layer flow over a flat plate

The viscous flow over a flat plate is governed by the boundary layer equations:

$$u_x + v_y = 0$$

$$uu_x + vu_y = \frac{1}{Re_L} u_{yy}$$

Use the following initial and boundary conditions:




Use a grid size of $\Delta x = \Delta y = 0.01$

Find u and v for $Re_L \in \{100, 1000, 10000\}$

<u>Plot the velocity contours</u>.

Plot u(x,y) vs y (u on the x-axis) at $x \in \{0.0, 0.25, 0.5, 0.75, 1.0, 1+\Delta x, 2.0, 3.0, 4.0\}$
On the same plot, plot the boundary layer thickness over the plate ($0 \leq x \leq 1$).
The boundary layer thickness is approximated by:

$$\frac{\delta}{x} \approx \frac{5.0}{\sqrt{Re_x}} \quad Re_x = \frac{\rho U_\infty x}{\mu}$$

Note, you are given the $Re_L = \frac{\rho U_\infty L}{\mu}$, where $L$ is the plate length.
Plot the shear stress on the surface of the plate using a 2nd order accurate finite difference scheme

$$\tau_w = \frac{2}{Re_L} u_y$$

## III. Wave Equation (Extra credit – 10 points)

Solve the 1D wave equation

$$u_{tt} = c^2 u_{xx}$$

With the following initial and boundary conditions:

$$u(x,0) = \sin(x)$$
$$u_t(x,0) = 0$$
$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

Use $c = 1$ and solve for $0 \leq t \leq 10\pi$ with each of the following methods

a) Explicit (forward difference in time)
b) Backward (backward difference in time)
c) Trapezoidal
d) RK4

## IV. Heat Equation (Extra credit – 10 points)

Solve the 1D heat equation

$$u_t = k u_{xx}$$

With the following initial and boundary conditions:

$$u(x,0) = \sin(x)$$
$$u_t(x,0) = 0$$
$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

Use $k = 1$ and solve for $0 \leq t \leq \pi$ with each of the following methods

a) Explicit (forward difference in time)
b) Backward (backward difference in time)
c) Trapezoidal
d) RK4