ENG 180 - Project 9

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1 Introduction

The purpose of this project was primarily about partial differential equations of mixed initial boundary value problems. For the first problem, 2-dimensional compressible inviscid flow over thin airfoils at subsonic and supersonic regimes was explored. For the second problem, incompressible viscous boundary layers over flat plates were explored. The minimum initial conditions were given along with the airfoil function and boundary conditions. For the first problem, contours of the pressure coefficient for each case needed to be plotted. For the second problem, the velocity contours, shear stress on the surface of the plate, and u(x,y) vs y at specific x's needed to be plotted.

2 Methods

2.1 2D compressible inviscid flow over a thin airfoil

$$(1 - M_\infty^2)u_{xx} + u_{yy} = 0, \quad M_\infty = 0.8, 1.4, 1.8 \tag{1}$$

$$u_y = v_x \tag{2}$$

For this problem, equations 1 and 2 needed to be solved in order to obtain the velocity grids needed to calculate the pressure coefficients. There were two cases for solving this problem. The first one, is if $M_\infty < 1$. If it is this case, then the equation is elliptic, and can be solved via line relaxation as done in Project 8. The other case is if $M_\infty > 1$. In this case, the equation is hyperbolic, and a new solver is needed. This new solver is then determined by setting $\beta^2 = (1 - M_\infty^2)$, and using a backwards difference scheme:

$$\beta^2 \frac{u_{i,j} - 2u_{i-1,j} + u_{i-2,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} = 0 \tag{3}$$

These values can then be moved around and turned into the tridiagonal system:

$$\left(\frac{1}{\Delta y^2}\right)u_{i,j-1} + \left(\frac{\beta^2}{\Delta x^2} - \frac{2}{\Delta y^2}\right)u_{i,j} + \left(\frac{1}{\Delta y^2}\right)u_{i,j+1} = \beta^2 \frac{2u_{i-1,j} - u_{i-2,j}}{\Delta x^2} \tag{4}$$

And at the bottom boundary (j=1):

$$\beta^2 \frac{u_{i,1} - 2u_{i-1,1} + u_{i-2,1}}{\Delta x^2} + \frac{2u_{i,2} - u_{i,1}}{\Delta y^2} - \frac{2\tilde{y}_B''(x_i)}{\Delta y} = 0 \tag{5}$$

We can then have our two known B.C.s at $i = 1, 2$, and start our scheme and $i = 3$. c_p can then be calculated as shown below:

$$c_p = -2(u - 1) \tag{6}$$

2.2 Incompressible viscous boundary layer flow over a flat plate

$$uu_x + vu_y = \frac{1}{Re}u_{yy} \tag{7}$$

$$u_y + v_x = 0 \tag{8}$$

For this problem, equations 1 and 2 needed to be solved in order to obtain the velocity grids needed for the rest of the problem. Similarly to problem 1, we can use a central difference scheme in y and a backwards difference scheme in x:

$$u_{i,j}^{old}\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + v_{i,j}^{old}\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y} = \frac{1}{Re}\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} \tag{9}$$

We then use backwards difference in x and y for equation 10. The two derivatives for x are averaged:

$$\frac{1}{2}\left(\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + \frac{u_{i,j-1} - u_{i-1,j-1}}{\Delta y}\right) + \frac{v_{i,j} - v_{i,j-1}}{\Delta y} = 0 \tag{10}$$

c_p can then be calculated as shown below:

$$c_p = -2(u-1) \tag{11}$$

The boundary layer thickness is then approximated by:

$$\frac{\delta}{x} \approx \frac{5.0}{\sqrt{Re_x}}, \qquad Re_x = \frac{\rho U_\infty x}{\mu} \tag{12}$$

The shear stress on the surface of the plate is then calculated using:

$$\tau_w = \frac{2}{Re_L}u_y \tag{13}$$

u_y is then calculated using a 2^{nd} order accurate finite difference scheme as shown below:

$$u_y = \frac{-u_{i,3} + 4u_{i,2} - 3u_{i,1}}{2\Delta x} \tag{14}$$

3 Results

3.1 2D compressible inviscid flow over a thin airfoil

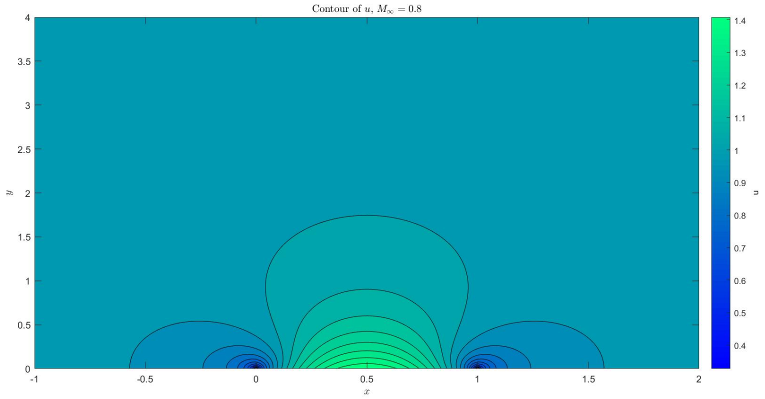

Figure 1: Contour of u, $M_\infty = 0.8$

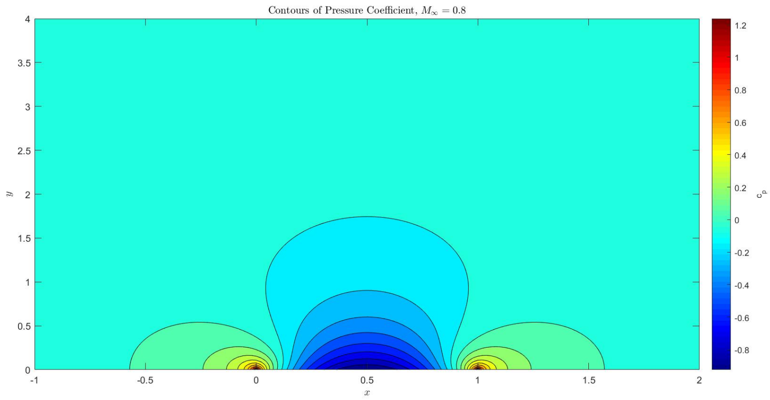

Figure 2: Contours of Pressure Coefficient, $M_\infty = 0.8$

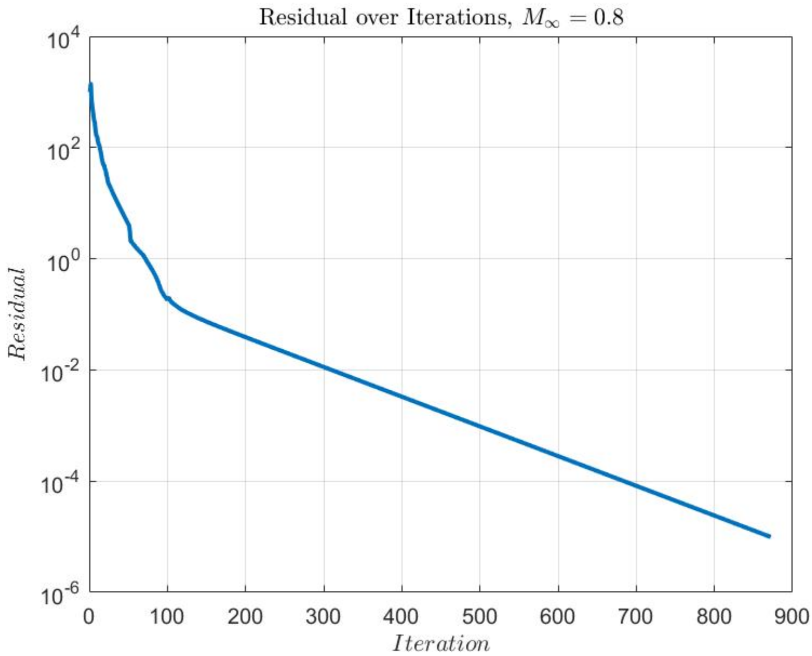

Figure 3: Residual over Iterations, $M_\infty = 0.8$

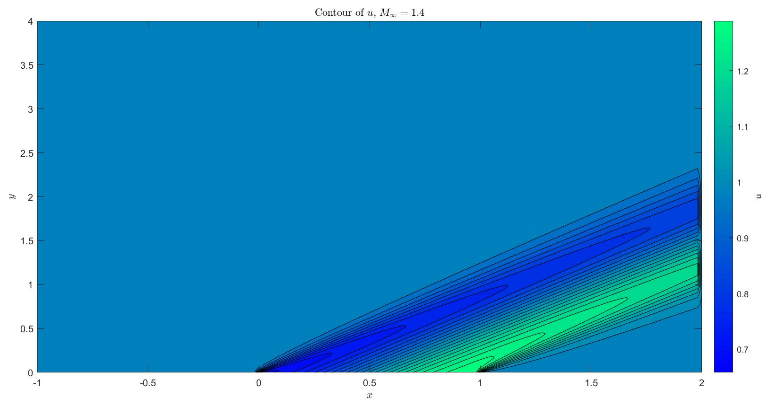

Figure 4: Contour of u, $M_\infty = 1.4$

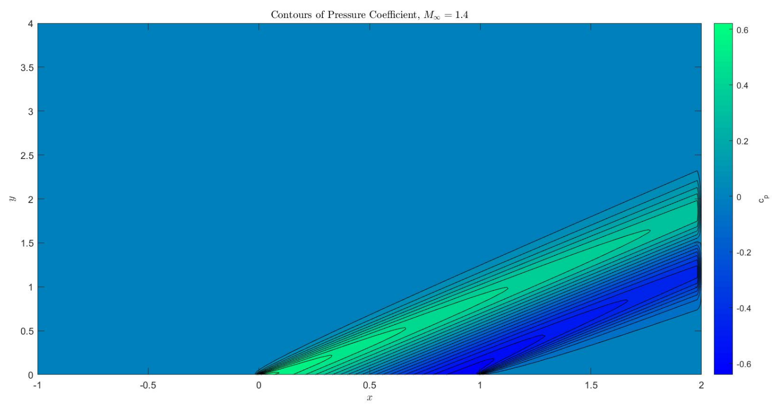

Figure 5: Contours of Pressure Coefficient, $M_\infty = 1.4$

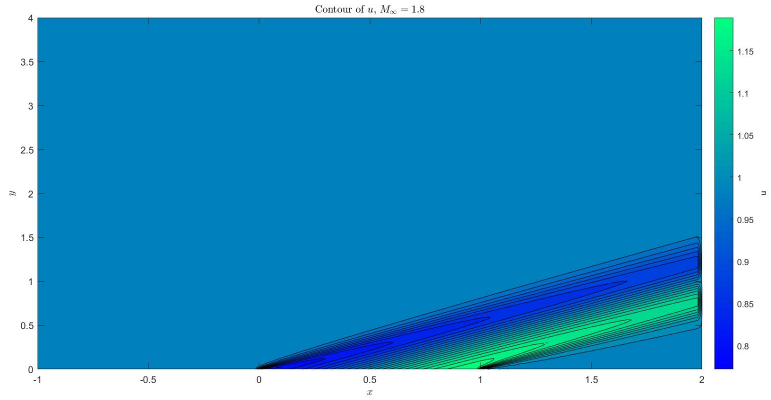

Figure 6: Contour of u, $M_\infty = 1.8$

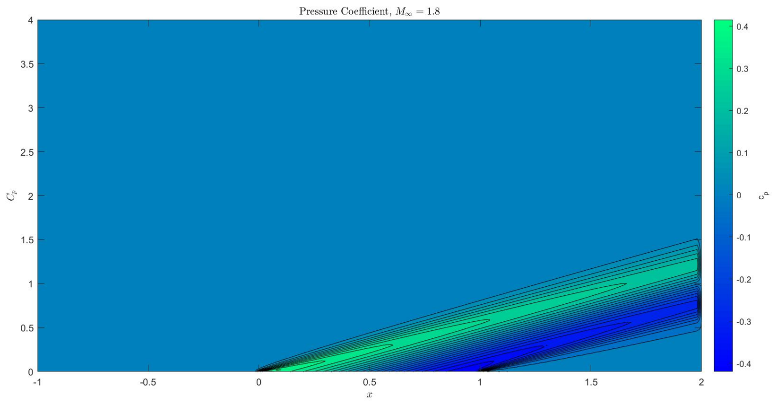

Figure 7: Contours of Pressure Coefficient, $M_\infty = 1.8$

3.2 Incompressible viscous boundary layer flow over a flat plate

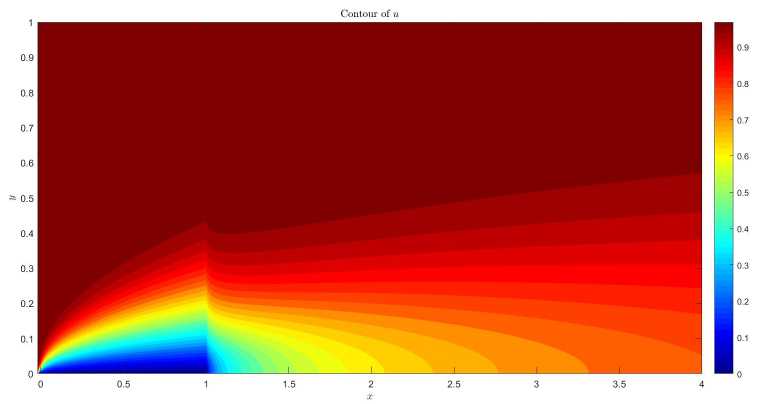

Figure 8: Contour of u, $Re = 100$

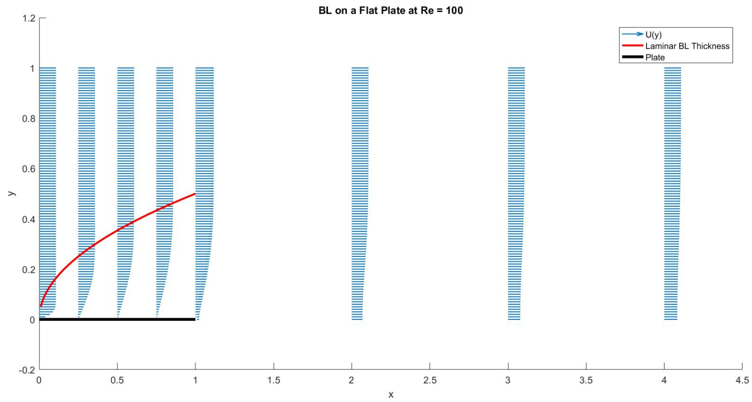

Figure 9: BL on a Flat Plate at $Re = 100$

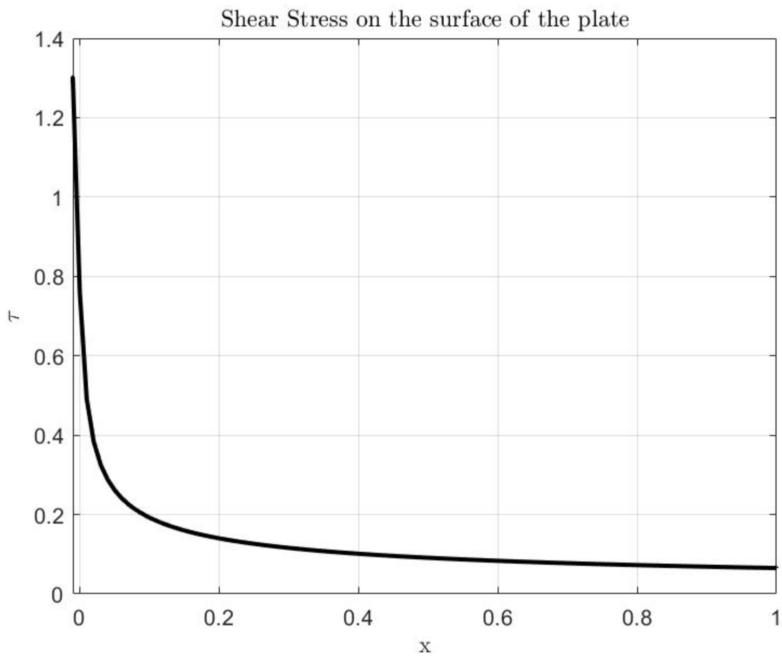

Figure 10: Shear Stress on the surface of the plate, $Re = 100$

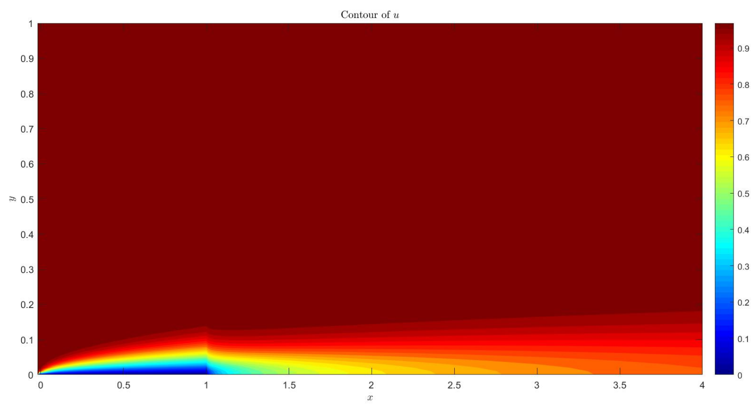

Figure 11: Contour of u, $Re = 1000$

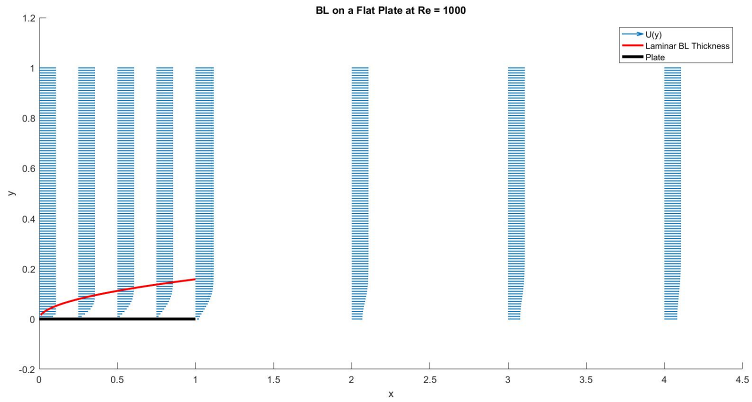

Figure 12: BL on a Flat Plate at $Re = 1000$

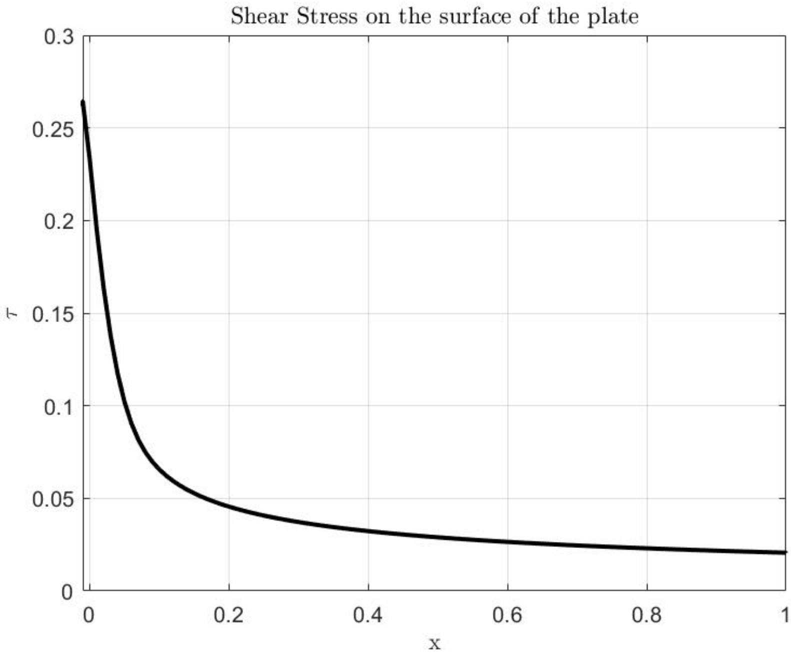

Figure 13: Shear Stress on the surface of the plate, $Re = 1000$

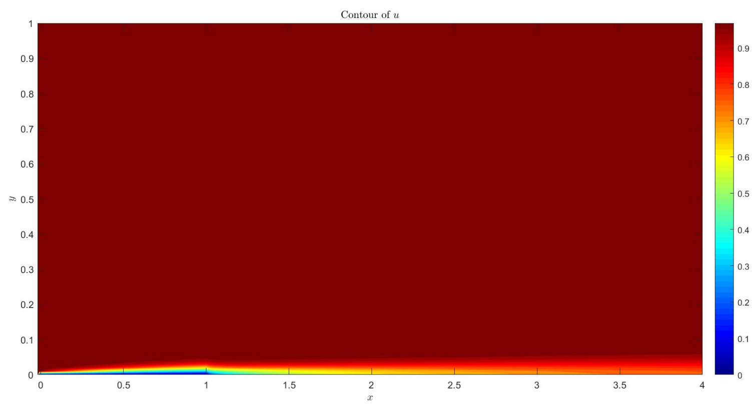

Figure 14: Contour of u, $Re = 10000$

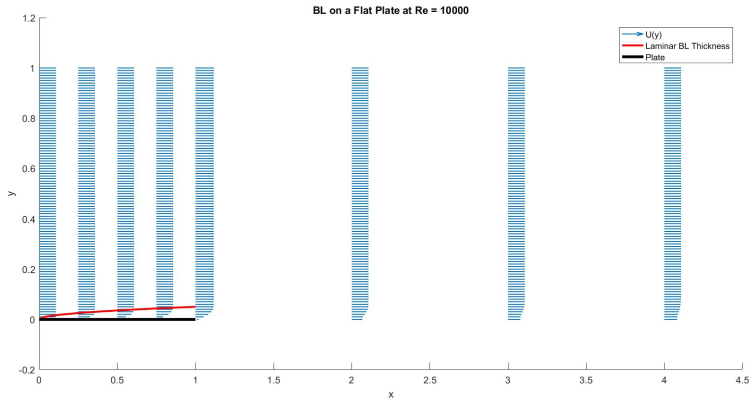

Figure 15: BL on a Flat Plate at $Re = 10000$

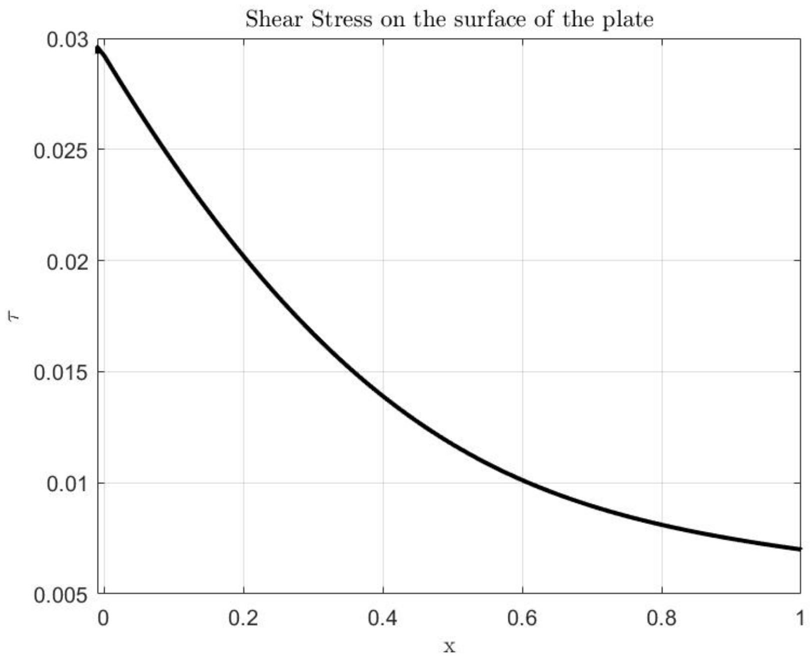

Figure 16: Shear Stress on the surface of the plate, $Re = 10000$

4 Discussion

4.1 2D compressible inviscid flow over a thin airfoil

For the first case, where $M_\infty = 0.8$, the stagnation points are found on figure 2. They are located at the leading edge and the trailing edge. The C_p is then much lower over the airfoil. This was expected since the case is very similar to the one of project 8. It can be seen in figure 3 that it is accurate enough iteration. This was a fairly straight forward case, no big problems arose from it.

For the second case, where $M_\infty = 1.4$, The C_p is distributed much differently. It can be seen in figure 5 that the stagnation point is now only at the leading edge. This is reasonable since it is in the shape of a shockwave due to the fact that it is supersonic. From figure 7, we can see that the same shock type of shock is observed for the case where $M_\infty = 1.8$, however, it is a steeper shockwave. This makes sense because the same plate is moving faster. A challenge of solving this problem became writing the scheme in a loop in order to solve the backwards difference, specifically the d component in thomas3.

4.2 Incompressible viscous boundary layer flow over a flat plate

For problem 2, the biggest issue became finding a correct order to do the problem. The residual was originally not reset for every i and that started many problems. The u_y also had to be debugged. From figures 8, 11, and 14, we see the contours of u for $Re = 100$ reach a much higher y value than for $Re = 1000$, and the $Re = 1000$ contours are then higher than the $Re = 10000$ ones. This then means that the higher the Re, the less less it affects the further freestream. In other words, it only affects the freestream closer and closer to the plate as Re gets bigger and bigger.

Furthermore, from figures 9, 12, and 15, we see that the $U(y)$ becomes less and less steep as Re gets bigger. This means that the flow separates less for the higher Re due to more momentum near the surface. This can also be seen in the BL thickness plots of each. As Re goes up, the BL thickness gets smaller. From figures 10, 13, and 16, we can see that for all Res the shear stress on the surface of the plate gets smaller as x increases. However, as Re increases, the value for $\tau|_{x=0}$ decreases significantly. This then leads to a much smaller $\tau|_{x=1}$. This is expected because Re is in the denominator of the τ relation. The more important fact is that it also agrees with the $U(y)$ conclusion that the flow separates less for the higher Re due to more momentum near the surface.