Publish or Perish in an Intro to Proofs Course

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Background

- "Foundations of Mathematics"
- Topics: Logic, Set Theory, Field Properties of $\mathbb{R}$, Functions, Relations, Modular Arithmetic, Sequences, Groups Intro, ...
- Required for: Abstract Algebra, Advanced Calculus, Number Theory, etc.
- 10-15 students, 50 min periods, 4 days a week

Issues

Typical Issues

- Lecture on content rather than proof writing
- Lots of grading (by instructor) and feedback (from the instructor)
- Quantity vs. Quality
- Copying, copying, copynig, cpyng, ...

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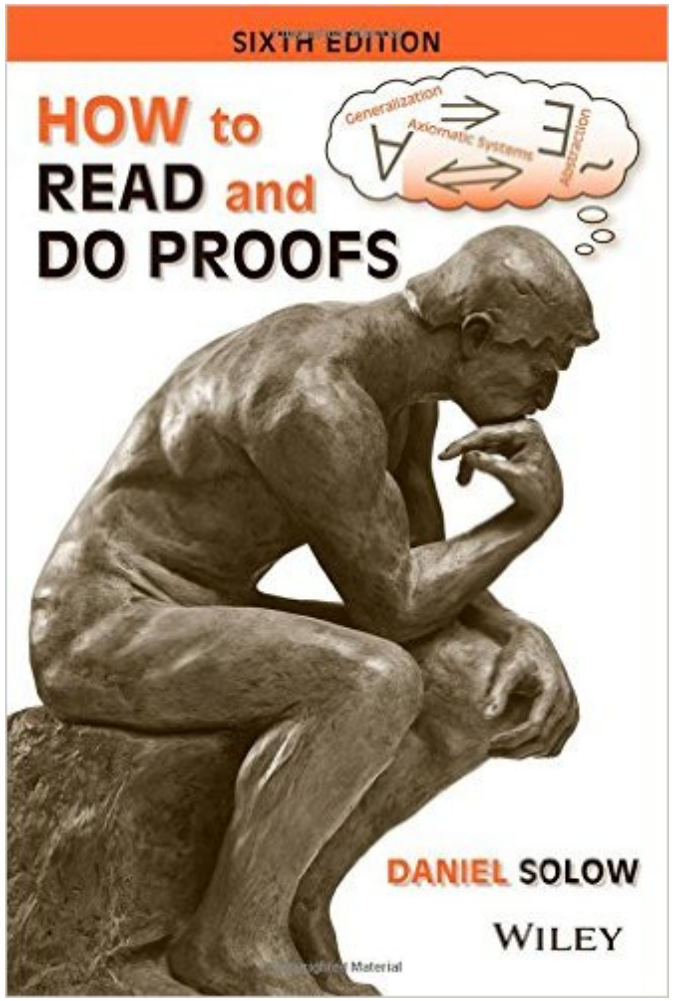
Larger Issues

- Proving before believing
- Few constructions of examples
- Early struggles hamper the grade

Support

- Text on proof writing with class lecture on each of 12 chapters
- "Lexicon" of problems with short concept intros and short lecture on each topic
- Rubric outlining what makes a good proof
  - Math Logic
  - Exposition / Communication
  - Grammar
  - Typesetting, spelling, punctuation, etc.



A New Homework Scheme: Publish

- Each lexicon content section contains 20-25 questions
  - Most "Prove or Disprove"
  - Some "Prove that..." or "Provide an example of..."
  - Many standard proofs usually in the book (but not ours!)
- Students sign up anonymously for a problem when they have a solution
  - Once reserved, a problem cannot be submitted by any other student
  - Most students didn't sign up for a problem until they felt confident of the major steps, but before nailing down the details
- Submit a LaTeX version electronically, due within 24 hours of signing up

A New Homework Scheme: Publish

- Problem is critiqued in front of whole class / by the whole class (or handed out as a critiquing assignment)
- Problem is deemed "Accepted", "Conditionally Accepted", "Revise & Resubmit", or "Rejected"
  - Revise & Resubmit = two class period extension (or relinquish)
  - Only Examples were ever rejected

3.2 Extended Set Theory

Definition 3.13. A set of sets is called a **family** or a **collection** of sets.

Class Example 3.8. The power set $\mathcal{P}(A)$ is a family of sets.

Definition 3.14. Given a family of sets $\mathcal{A}$, the **union over** $\mathcal{A}$ is

$$\bigcup_{A\in\mathcal{A}} A = \{a \mid (\exists A \in \mathcal{A})(a \in A)\}.$$

The **intersection over** $\mathcal{A}$ is

$$\bigcap_{A\in\mathcal{A}} A = \{a \mid (\forall A \in \mathcal{A})(a \in A)\}.$$

Example 3.7. Give an example of a family of three sets and the union and intersection over the family.

Problem 3.22. *Prove for every* $B \in \mathcal{A}$, $\bigcap_{A\in\mathcal{A}} A \subseteq B$.

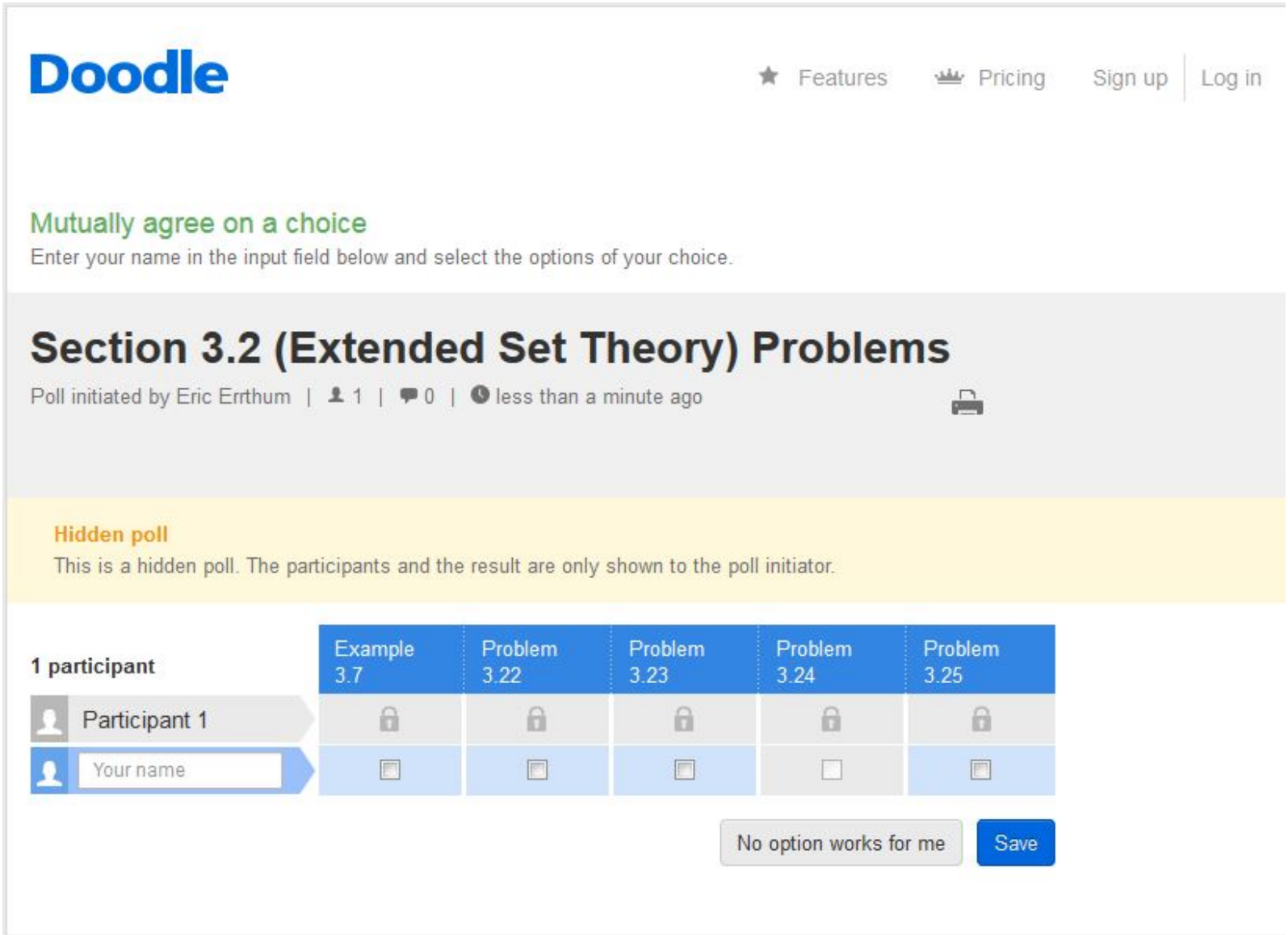
Problem 3.23. *Prove for every* $B \in \mathcal{A}$, $B \subseteq \bigcup_{A\in\mathcal{A}} A$.

Problem 3.24. *The statement* $\bigcap_{A\in\mathcal{A}} A \subseteq \bigcup_{A\in\mathcal{A}} A$ *is false in one special case. Determine that case, why it's false in that case, and then prove it's true in all other cases.*

Definition 3.15. Let Δ be a nonempty set such that for each $\alpha \in \Delta$ there is a set A_α. The family $\{A_\alpha \mid \alpha \in \Delta\}$ is called an **indexed family of sets**. The set Δ is called the **indexing set** and $\alpha \in \Delta$ is call an **index**.

Class Example 3.9. Let $\Delta = \mathbb{N}$ and to each index $n \in \mathbb{N}$ associate the interval $I_n = [1 - \frac{1}{n}, 1 + \frac{1}{n})$. Then the

A Walk-Through



A Walk-Through

Proposition 3.22. *For every* $B \in \mathcal{A}$, $\bigcap_{A \in \mathcal{A}} A \subseteq B$.

Proof. Suppose $B \in \mathcal{A}$. The intersection over $\mathcal{A}$ is,

$$\bigcap_{A \in \mathcal{A}} A = \{a \mid (\forall A \in \mathcal{A})(a \in A\}.$$

Since $B \in \mathcal{A}$, then all elements a of $\bigcap_{A \in \mathcal{A}} A$ are in B. So by Definition 3.4, $\bigcap_{A \in \mathcal{A}} A \subseteq B$.

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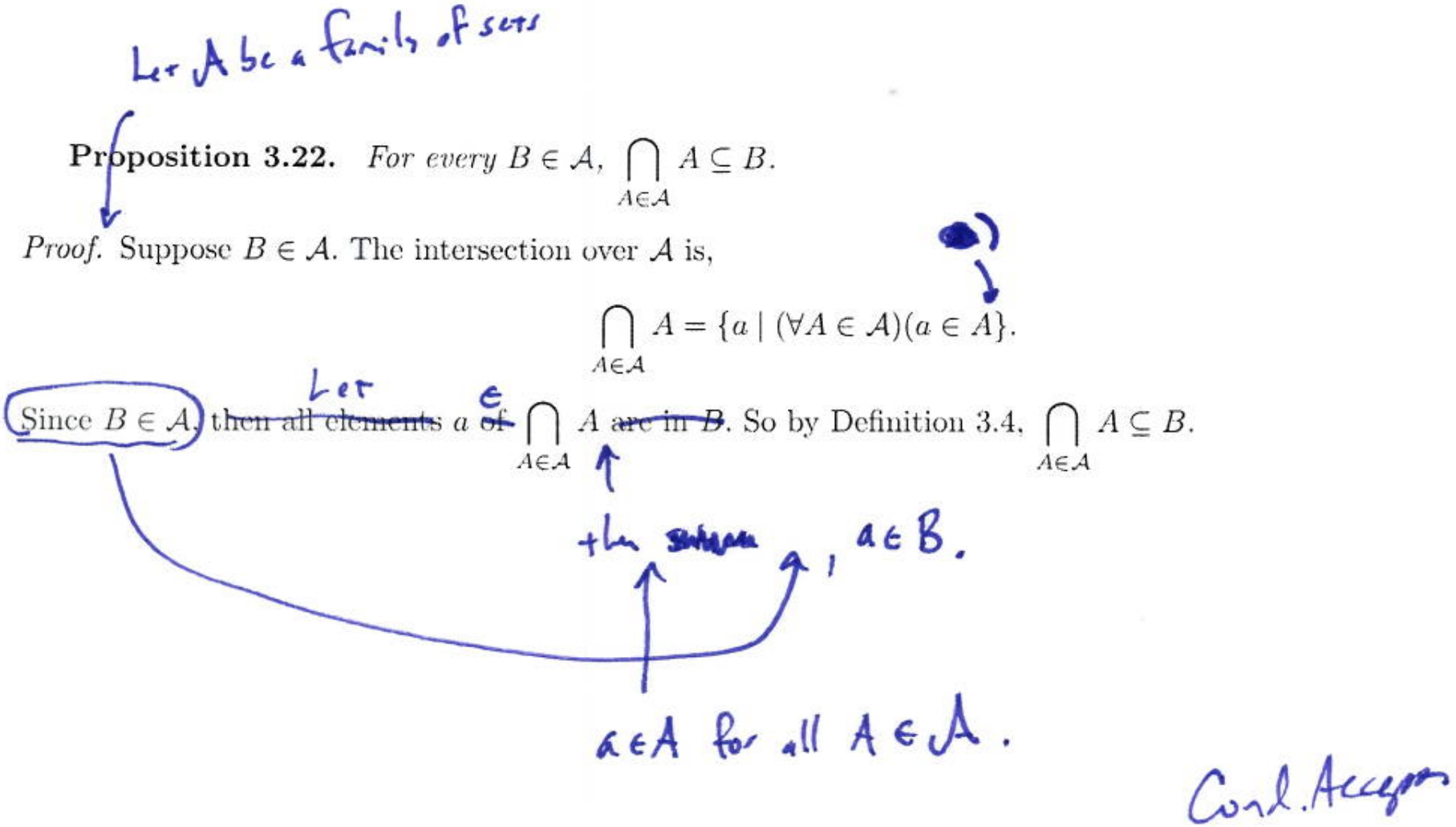

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Example 3.7. Give an example of a family of three sets and the union and intersection over the family.

Proposition 3.22. *Prove for every* $B \in \mathcal{A}$, $\bigcap_{A\in\mathcal{A}} A \subseteq B$. *For every* $B \in \mathcal{A}$, $\bigcap_{A\in\mathcal{A}} A \subseteq B$.

Proof. Let $\mathcal{A}$ be a family of sets, and B be a set such that $B \in \mathcal{A}$. The intersection over $\mathcal{A}$ is,

$$\bigcap_{A\in\mathcal{A}} A = \{a \mid (\forall A \in \mathcal{A})(a \in A)\}.$$

Let $a \in \bigcap_{A\in\mathcal{A}} A$ then $a \in A$ for all $A \in \mathcal{A}$. Since $B \in \mathcal{A}, a \in B$. So by Definition 3.4, $\bigcap_{A\in\mathcal{A}} A \subseteq B$.

□

Problem 3.23. *Prove for every* $B \in \mathcal{A}$, $B \subseteq \bigcup_{A\in\mathcal{A}} A$.

Problem 3.24. *The statement* $\bigcap_{A\in\mathcal{A}} A \subseteq \bigcup_{A\in\mathcal{A}} A$ *is false in one special case. Determine that case, why it's false in*

Experiences

- First submissions were comically bad in exposition and formatting (logic okay)
- But students had few/no critiques
- As time went on, students learned to be critical
- By end of semester, most submissions were of two types
  - Accepted: Well-written and correct
  - Revise & Resubmit: Concept wasn't understood
- Instructor Trade-Off: Less grading, more logistical coordinating and bookkeeping

Reputation

Action	Result
Having a solution Accepted	+20 points
Having a solution Conditionally Accepted	+15 points
Having a Revised and Resubmitted Solution Accepted	+15 points
Having an example accepted	+7 points
Scoring perfect on a peer evaluation	+5 points
Top score on Vocab/Theorem quiz	+5 points
Answering a reading question in class	+3 points
Finding a typo in the lexicon	+2 points
Having a solution Rejected	–5 points
Having a reservation expire	–5 points

Grades

Grades were based on the following

- Vocab/Theorem Quizzes — 23.44%
- Reputation — 10.94%
- Peer Evaluations — 10.94%
- Contributed Written Work (10 highest) — 18.75%
- Miscellaneous Assignments — 4.68%
- Exam on Solow Text — 7.81%
- Final Portfolio of Written Work — 7.81%
- Final Exam — 15.63%

Reactions

From Student Surveys:

- Some struggled without the usual structuce of deadlines and linear order
- Comments
  - "Enjoyed the submission process"
  - "The more submissions I did, the more I learned."
- Success from
  - "having to struggle through things & getting feedback from peers."
  - "being able to correct mistakes" and "evaluating other proofs in class."
  - "all the constructive criticism."

Changes for Next Time

- Require more written submissions
- Give some sort of general deadlines
- Focus more on content later in the semester

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Thank you.
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Internet Usage

From Student Internet Survey:

- Depended on Internet sources about the same as other classes
- Searched for solutions on-line: "Rarely" to "A Few Times"
- When they did search: found the solution "A Few Times"
- When they found the solution:
  - typically used it to know how the proof should start / get a hint
  - at worse they used it to make sense of the proof so they could put it in their own words.
- On average, they reported that 84% of their submitted work was solely their own.
- Main sources of the other 16%
  - Other students
  - The instructor
  - Internet

New sections were available to students once prerequisite sections were mostly complete.