

# Math 280 Problems for September 13

## Pythagoras Level

**Problem 1:** The sum of the reciprocals of two real numbers is  $-1$ , and the sum of their cubes is  $4$ . What are the numbers?

**Problem 2:** Two students play a game based on the total roll of two standard dice. Student A says that a  $12$  will be rolled first. Student B says that two consecutive  $7$ s will be rolled first. The students keep rolling until one of them wins. What is the probability that A will win?

## Newton Level

**Problem 3:** Evaluate

$$\int_1^{2008} \frac{dx}{x + \lfloor \log_{10} x \rfloor}.$$

(For a real number  $u$ ,  $\lfloor u \rfloor$  denotes the greatest integer less than or equal to  $u$ .)

**Problem 4:** Express the product

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \cdots \left(1 - \frac{1}{2008^2}\right)$$

as simply as you can as a rational fraction in lowest terms. Justify your answer.

## Wiles Level

**Problem 5:** Prove that

$$\frac{\gcd(m, n)}{n} \binom{n}{m}$$

is an integer for all pairs of integers  $n \geq m \geq 1$ .

**Problem 6:** For any positive integer  $n$ , let  $\langle n \rangle$  denote the closest integer to  $\sqrt{n}$ . Evaluate

$$\sum_{n=1}^{\infty} \frac{2^{\langle n \rangle} + 2^{-\langle n \rangle}}{2^n}.$$