

- 2.** Your first LaTeX assignment is to use LaTeX to produce a document that replicates this one as exactly as possible, with just two differences: First, replace the name above with your own. Second, make the following letter substitutions so that I know that you did not just photocopy this document: in Problems 5 and 8, change each m to n ; in Problem 13, change each c to b . Your grade on this assignment will be based on how much your paper looks exactly like this one (including these instructions).

Note: In a regular assignment, for questions with a short answer, you may just respond in a complete sentence (like in 3 below). For questions asking for a proof, restate the assumptions and the statement that you are trying to prove (but you can leave out definitions). For questions where you grade a proof, you can simply give your grade and explanation (as in 21).

3.

- a) $A \cap B = \{1, 4, 5\}$.
- d) False, because $e > 2$.

- 5.** Prove that every integer that is divisible by 6 is even.

Proof. Suppose $n \in \mathbb{Z}$. Then there is some $k \in \mathbb{Z}$ such that $n = 6k$. Therefore $n = 2(3k)$, and since $3k$ is also in \mathbb{Z} , this means that n is divisible by 2 and therefore that n is even. \square

- 8.** Prove that if $n \in A$ then $n = -2, 0$, or 3 .

Proof. Note that

$$\begin{aligned} n^3 - n^2 - 6n &= n(n^2 - n - 6) && \text{(factor out an } n\text{)} \\ &= n(n+2)(n-3). && \text{(factor the quadratic)} \end{aligned}$$

Therefore since $n \in A = \{n \in \mathbb{Z} \mid n^3 - n^2 - 6n = 0\}$ then $n(n+2)(n-3) = 0$. Thus n must be equal to one of $-2, 0$, or 3 . \square

- 13.** Prove that if $a, b \in \mathbb{R}$ with $a \leq b$ then $[b, \infty) \subseteq [a, \infty)$.

Proof. Suppose $a \leq b$ in \mathbb{R} . For all $x \in \mathbb{R}$,

$$\begin{aligned} x \in [b, \infty) &\Rightarrow x \geq b \\ &\Rightarrow x \geq b \geq a && (b \geq a \text{ by hypothesis}) \\ &\Rightarrow x \geq a && (\text{transitivity}) \\ &\Rightarrow x \in [a, \infty). \end{aligned}$$

Therefore we have $[b, \infty) \subseteq [a, \infty)$. \square

- 21.** Proofs to grade

- h) Grade: C. This only shows that $A \subseteq B$. It also needs to show that $B \subseteq A$ to establish that $A = B$. Also they should state “because $f^{-1}(x)$ is onto” after the third step.