Math 280 Problems for September 5

Pythagoras Level

#1. In popular culture, many of us are familiar with the stereotype of the mad scientist. In this case, a mad veterinarian invents an animal transmogrifying machine. The machine can transmogrify:

- Two cats into one cat, or vice-versa

- One cat and one dog into one dog, or vice-versa

- Two dogs into one cat, or vice-versa

Beginning with three cats and one dog, is it possible to end up with

(a) one dog and no cats?

(b) one cat and no dogs?

Be sure to justify your answers.

#2. If the pth term of an arithmetic progression is q and the qth term is p, where $p \neq q$, find the $(p+q)$th term.
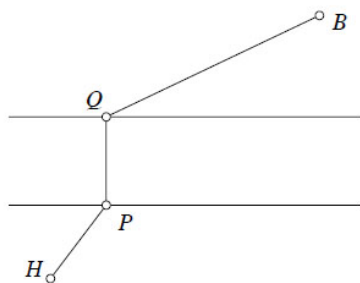
Newton Level

#3. Evaluate the integral

$$I = \int_{1/2}^{2} \frac{\ln x}{1 + x^2} \, dx.$$

#4. A smooth function $f(x)$ has $f''(x) > 0$ for all x in $[0, 1]$. For each point a in $[0, 1]$, draw the tangent line to $y = f(x)$ at the point where $x = a$. Let $A(a)$ be the area bounded by the curve $y = f(x)$, the tangent line at a, $x = 0$, and $x = 1$. For what value of a is the area minimized?

Wiles Level

#5. A farmer lives in a farmhouse H on one side of a stream bounded by two parallel lines. He often has to walk to his barn B on the other side of the stream. Since he is tired of getting wet, he wants to build a bridge PQ perpendicular to the stream, with P on the same side of the stream as H. He also wants the total walking distance HP+PQ+QB to be as short as possible. How should he determine where to place the bridge?

#6. How many rearrangements of the string of letters *aabcde* have exactly two letters in their original places? The two *a*s are indistinguishable, so an *a* in either the first or second position is considered to be in its original place.