MATH 247: Finite Differences Oral Homework

1. Use the finite difference method to give the explicit formula for the sequence (starting at $n = 0$):

   (a)
   $$9, 5, 7, 15, 29, 49, 75, 107, 145, 189, \ldots$$

   (b)
   $$-75, -54, -35, -18, -3, 10, 21, 30, 37, 42, \ldots$$

   (c)
   $$-25, -37, -25, 35, 167, 395, 743, 1235, 1895, 2747, \ldots$$

   (d)
   $$-100, -109, -124, -109, -4, 275, 836, 1811, 3356, 5651, \ldots$$

   (e)
   $$0, 1, 17, 98, 354, 979, 2275, 4676, 8772, 15333, \ldots$$

2. Use the finite difference method on log of the sequence to determine an explicit formula (starting at $n = 0$):
   $$64, 32, 48, 216, 2916, 118098, 14348907, \ldots$$

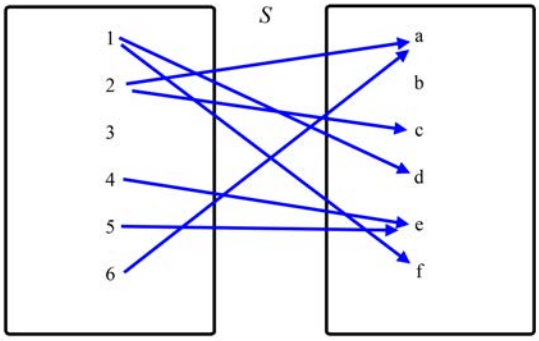
MATH 247: Homework #5

1. Solve the following linear recurrences.
   (a) $b_0 = 0$, $b_1 = 1$, $b_2 = 3$, $b_n + 2b_{n-1} + b_{n-2} - 4b_{n-3} = 0$
   (b) $c_0 = 0$, $c_1 = 1$, $c_n - 2c_{n-1} - 3c_{n-2} = 0$
   (c) $d_0 = 0$, $d_1 = 2$, $d_n - 10d_{n-1} + 25d_{n-2} = 0$
2. Use Mathematical Induction to verify that $f_n = (3+n)5^n$ is the solution to the recurrence $f_0 = 3$, $f_n - 5f_{n-1} = 5^n$.
3. Draw the Hasse diagram for divisibility on the set $\{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 36, 54, 81\}$.
4. Draw the number theoretic Venn diagram for $a = 210$, $b = 2145$, and $c = 7735$

MATH 247: Homework #6

1. For $A = \{a, b, c, d, e, f\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, consider the two relations $R : A \to B$ and $S : B \to A$ given below:

$$R = \{(a, 1),\ (a, 3),\ (b, 6),\ (d, 2),\ (d, 5),\ (e, 4),\ (f, 4)\}$$



   (a) Give $R \circ S$ as a set of ordered pairs.

   (b) Draw the digraph for $S \circ R$.

2. Consider the following relation on $\mathbb{Z}/8\mathbb{Z}$:

$$xMy \Leftrightarrow x^2 \equiv y^2 \mod 8$$

   Use the digraph to determine if the relation has the following properties. Explain how you know.

   (a) Reflexive

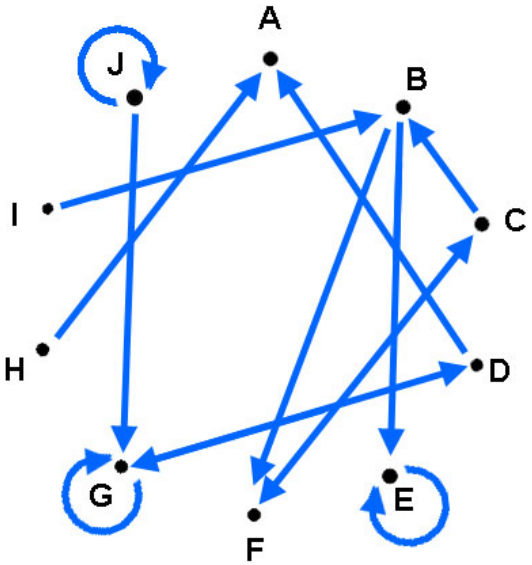
   (b) Symmetric

   (c) Antisymmetric

   (d) Transitive

   (e) Equivalence relation

3. Consider the following graph:



(a) Give the reflexive-closure of the graph.

(b) Give the symmetric-closure of the graph.

(c) Give the transitive-closure of the graph.

(d) Give the anti-symmetric reduction of the graph.

(e) Give the Hasse diagram associated to the transitive closure of the anti-symmetric reduction of the graph.

(f) Give the equivalence classes of the minimal equivalence relation of the graph.

MATH 247: Exam I (64 points possible out of 50 points to be scaled to 128 points possible out of 100 points)

NAME:

Note: Show your work to receive full credit.

1. (7 points) For $S = \{1, 2, 3, 6\}$, $E = \{1, 2, 4, 8\}$ in the universe $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, compute the following:
   (a) $S \cap E$

   (b) $S \cup E$

   (c) $S - E$

   (d) $S \oplus E$

   (e) S^C

   (f) Give 4 elements of $S \times E$

   (g) Give 6 elements of $\mathcal{P}(S)$

2. (3 points) Given the collection of sets $A_n = \{m \in \mathbb{Z} \mid 2n - 7 \leq m \leq 10 + n\}$
   (a) List all the elements in A_9.

   (b) List all the elements in $\bigcup_{n=13}^{17} A_n$

   (c) List all the elements in $\bigcap_{n=10}^{14} A_n$

3. (4 points) Using a standard Venn Diagram of three sets A, B, and C in a universe U, shade the region corresponding to

$$(A \oplus B)^C \cap (C - B).$$

4. (5 points) Prove with a Venn diagram or Table:

$$(A \cup B) \cap (A \cup C) = A \cup (B \cap C).$$

5. (3 points) Give the degenerate Venn diagram that represents the assumptions: "If $C \subseteq A^C \cap B^C$, then..."

6. (4 points) Consider the statements

$$p = \text{"Six is a prime number."}$$
$$q = \text{"Six is an even number"}$$

Translate the following into elegant English sentences and then label as either TRUE or FALSE.

(a) $p \wedge \neg q$

(b) $p \rightarrow q$

7. (3 points) Use a truth table to determine all situations where $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee r$. (Circle the appropriate rows)

8. (7 points) The following proposition is false: "If $A \subseteq B$, then $A \oplus C \subseteq B - (A \cap C)$."

   (a) Give sets A, B and C that are a counterexample to this statement.
   (Remember: Make sure to not only give the sets, but also show that they make the statement false.)

   (b) Fill in the blank with an additional minimal assumption that makes the statement true:

   "If $A \subseteq B$ and _____, then $A \oplus C \subseteq B - (A \cap C)$."

(Work space for the problem above)

9. (6 points) Consider the statement:

$$(\forall x \in \mathbb{N})((x \text{ prime}) \to (x \text{ odd}))$$

(a) Translate the statement into elegant English.

(b) Give the simplified symbolic negation of the statement. (I.e. simplify $\neg(\forall x \in \mathbb{N})((x \text{ prime}) \to (x \text{ odd}))$)

(c) Which is true: the original statement or its negation? Why?

10. (a) (2 points) Give the decimal number 247 in binary.

(b) (2 points) Give the binary number 1011011_2 in decimal.

11. (6 points) Consider

$$f = \neg(p \wedge (q \vee \neg(p \wedge r)))$$

(a) Construct the digital circuit diagram for f.

(b) Algebraically simplify f. (You do NOT have to give the digital circuit for the simplification.)

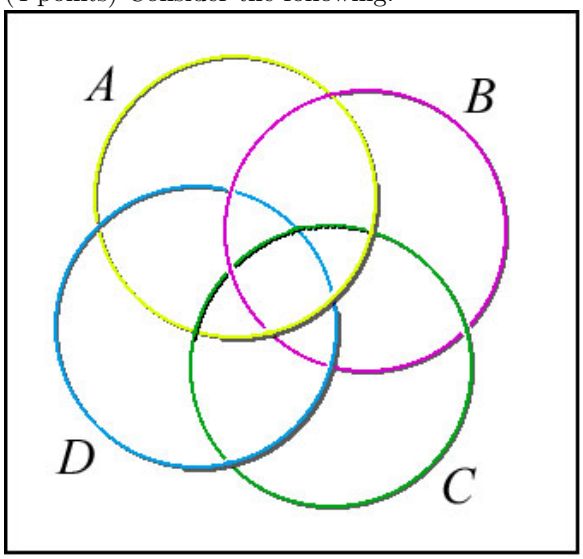
12. (3 points) Let $Gl(x) =$ "x glitters" and $Au(x) =$ "x is gold" and T the universe of things. Give the a symbolic statement equivalent to Shakespeare's famous saying: "All that glitters is not gold."

13. (4 points) There are two programs on the WSU laptops, call them foo.exe and kass.exe. The program kass.exe checks the network to see if you should be allowed to run foo.exe and either returns a "Yes" or "No" answer. So your computer will only allow you to start foo.exe when the following logic statement has truth value TRUE:

    If kass.exe is running and kass.exe returns "No", then foo.exe is not running.

    You really want to run foo.exe when the output of kass.exe is "No". What should you do so that you can run foo.exe? Explain.
    (Note: You cannot change the output of kass.exe since this is a value it pulls from the network. You also cannot disable/change the checking of the logical statement since this is built into the foo.exe program.)

14. (4 points) Consider the following:



    (a) Why is this <u>not</u> a general Venn diagram of 4 sets?

    (b) Give the minimal assumptions on sets A, B, C, and D so that this degenerate Venn diagram is the correct one to use.