

Math 280 Problems for September 3

Pythagoras Level

#1. In popular culture, many of us are familiar with the stereotype of the mad scientist. In this case, a mad veterinarian invents an animal transmogrifying machine. The machine can transmogrify:

- Two cats into one cat, or vice-versa
- One cat and one dog into one dog, or vice-versa
- Two dogs into one cat, or vice-versa

Beginning with three cats and one dog, is it possible to end up with

- (a) one dog and no cats?
(b) one cat and no dogs?

Be sure to justify your answers.

#2. If the p th term of an arithmetic progression is q and the q th term is p , where $p \neq q$, find the $(p+q)$ th term.

Newton Level

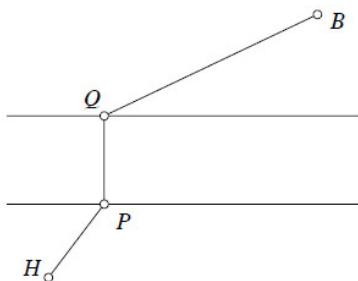
#3. Evaluate the integral

$$I = \int_{1/2}^2 \frac{\ln x}{1+x^2} dx.$$

#4. A smooth function $f(x)$ has $f''(x) > 0$ for all x in $[0, 1]$. For each point a in $[0, 1]$, draw the tangent line to $y = f(x)$ at the point where $x = a$. Let $A(a)$ be the area bounded by the curve $y = f(x)$, the tangent line at a , $x = 0$, and $x = 1$. For what value of a is the area minimized?

Wiles Level

#5. A farmer lives in a farmhouse H on one side of a stream bounded by two parallel lines. He often has to walk to his barn B on the other side of the stream. Since he is tired of getting wet, he wants to build a bridge PQ perpendicular to the stream, with P on the same side of the stream as H. He also wants the total walking distance HP+PQ+QB to be as short as possible. How should he determine where to place the bridge?



#6. How many rearrangements of the string of letters $aabcde$ have exactly two letters in their original places? The two a s are indistinguishable, so an a in either the first or second position is considered to be in its original place.