Math 280 Problems for September 19

Pythagoras Level

Problem 1: How many of the integers from 1 to 2008 may be written as the sum of two or more distinct integral powers of 3? (For example, $28 = 3^0 + 3^3$ is such an integer.) Justify your answer.

Problem 2: Find all real solutions (x, y) of the system

$$
\begin{aligned}
|x| + x + y &= 10, \\
x + |y| - y &= 12.
\end{aligned}
$$

Justify your answer.

Newton Level

Problem 3: Find the value of the infinite product

$$P = \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \cdots \times \frac{k^3 - 1}{k^3 + 1} \times \cdots$$
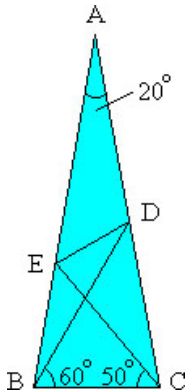
Problem 4: Prove that

$$\frac{\gcd(m, n)}{n} \binom{n}{m}$$

is an integer for all pairs of integers $n \geq m \geq 1$.

Wiles Level

Problem 5: Let ABC be an isosceles triangle ($AB = AC$) with $\angle BAC = 20°$. Point D is on side AC such that $\angle DBC = 60°$. Point E is on side AB such that $\angle ECB = 50°$. Find, with proof, the measure of $\angle EDB$.

Problem 6: For any positive integer n, let $\langle n \rangle$ denote the closest integer to $\sqrt{n}$. Evaluate

$$\sum_{n=1}^{\infty} \frac{2^{\langle n \rangle} + 2^{-\langle n \rangle}}{2^n}.$$