

Math 280 Problems for October 31

Pythagoras Level

Problem 1: There are thirty zombies in a house, ten have the number A etched into their forehead, ten have the number B etched on them, and ten have C etched on them. It is the case that whenever five zombies come outside, there are five other zombies still in the house such that the total of the numbers on the ten zombies is 0. The zombie that is about to eat you has a 2008 etched on his forehead. What are the numbers on the other zombies? Assume that A , B , and C are all different and that zombies are stupid.

Problem 2: The towns of Alpha, Beta, and Gamma are equidistant from each other. A horde of zombies is three miles from Alpha and four miles from Beta, what is the maximum possible distance of the horde from Gamma? Assume the land is flat and zombies are stupid.

Newton Level

Problem 3: A point (p, q) is chosen at random from the square

$$S = \{(p, q) : -1 \leq p, q \leq 1\}.$$

What is the probability that the quadratic equation $x^2 + px + q = 0$ has two distinct real roots? Justify your answer.

Problem 4: Let g be the function defined by

$$g(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Let h be the function defined by $h(x) = \int_x^\pi g(t)dt$. Find the area of the region bounded by the curve $y = h(x)$, $x = 0$, $x = \pi$ and the x -axis.

Wiles Level

Problem 5: Find all triples (x, y, z) of real numbers such that

$$x^3 - y^2 = y^3 - z^2 = z^3 - x^2 = 100.$$

Problem 6: Evaluate

$$\sqrt[8]{2207 - \frac{1}{2207 - \frac{1}{2207 - \dots}}}.$$

Express your answer in the form $\frac{a+b\sqrt{c}}{d}$, where a, b, c, d are integers.