

## A Quick Reminder of Proofs

1. The answer to a “Prove that...” or “Show that...” is never something you can just put a box around at the end of a series of equations. Remember, when you’re trying to prove something, you’re constructing an argument. You should explain why one point leads to the next until you arrive at the desired conclusion (i.e. create a path from your original assumptions to the point you’re arguing).
2. When in doubt, teach it. Pretend you’re explaining it to someone who understands basic math and any technical definitions from the subject. So you should show each step, indicating why you’re doing the next thing and how you’re using the given knowledge and/or assumptions.
3. Always begin with what you know (your assumptions, conditions, etc.) and always end with what you were trying to prove. Stating what you want to prove in the beginning is okay, as long as it doesn’t feed into your argument. I recommend underlining any statements you make in the beginning that are facts you want to prove. This way they’re more like headings or titles for the arguments that follow but don’t contribute to the assumptions.
4. Don’t use what you’re trying to prove, especially when arguing with equalities. Consider the following two examples:

Prove that  $F_1 + F_2 + \dots + F_{n-1} + F_n = F_{n+2} - 1$  given that  $F_1 + F_2 + \dots + F_{n-1} = F_{n+1} - 1$  and  $F_n + F_{n+1} = F_{n+2}$

### WRONG PROOF

$$\begin{aligned}F_1 + F_2 + \dots + F_{n-1} + F_n &= F_{n+2} - 1 \\F_{n+1} - 1 + F_n &= F_{n+2} - 1 \\F_{n+1} + F_n &= F_{n+2} \\F_{n+2} &= F_{n+2} \checkmark\end{aligned}$$

### RIGHT PROOF

$$\begin{aligned}F_1 + F_2 + \dots + F_{n-1} + F_n &= F_{n+1} - 1 + F_n \\&= F_n + F_{n+1} - 1 \\&= F_{n+2} - 1\end{aligned}$$

The wrong proof may look convincing, but that’s because ultimately the statement is true. However, consider this “proof” of something we all know is false:

Prove that  $2 \ln(-1) = 0$  given the law of logs:  $n \ln(a) = \ln(a^n)$ .

Pf:

$$\begin{aligned}2 \ln(-1) &= 0 \\ \ln((-1)^2) &= 0 \text{ by law of logs} \\ \ln(1) &= 0 \\ e^{\ln(1)} &= e^0 \\ 1 &= 1 \checkmark\end{aligned}$$

The point is, yes, an argument like the one on the above left can look very convincing when the thing you’re trying to prove is in fact true (which is often the case with problems out of a book), but it can also look convincing when the statement you’re proving is in fact false. So it is not a valid test of truth, i.e. it’s not a valid proof.

5. The back of the book will sometime give you a hint or a start, but rarely will it give you the full solution. If it did, I wouldn’t assign that problem. I don’t care to grade you on your ability to copy something from the back of the text.
6. You should use complete sentences when appropriate, but also take advantage of the fact that “ $a^2 - b^2 = (a+b)(a-b)$ ” can also be a sentence. (In this case “ $a^2 - b^2$ ” is the noun, “=” is the verb, and “ $(a+b)(a-b)$ ” is the direct object.) Most of the time what really ties a proof together grammatically is the use of transition words like “but”, “if”, “then”, “because”, “suppose”, “given that”, “thus”, “therefore”, etc. When you’re done with a proof, you should read it back to yourself. If you read it aloud and it doesn’t make sense (or doesn’t sound human) without adding some extra words, you should probably write those extra words down.
7. Lastly, my job as a grader is to play Devil’s Advocate. Even though I may know what you mean because I know the solution, I’m grading you on your ability to convince me that you’re right, whether you are or not.