Math 280 Problems for November 7

Pythagoras Level

Problem 1: Let N be the product of all positive divisors of 10^5. Find $\log_{10} N$.

Problem 2: Solve for $0 \leq x \leq 1$:

$$\log_x 4 - \log_2 x = 1.$$

Newton Level

Problem 3: Evaluate

$$\left(\frac{1}{3}\right)^{1/5} \left(\frac{1}{9}\right)^{1/25} \left(\frac{1}{27}\right)^{1/125} \cdots$$

Problem 4: Calculate

$$\lim_{x \to \infty} \int_0^x x e^{t^2 - x^2} \, dt$$

Wiles Level

Problem 5: Sarah intended to multiply a two-digit number and a three-digit number, but she left out the multiplication sign and simply placed the two-digit number to the left of the three-digit number, thereby forming a five-digit number. This number is exactly nine times the product Sarah should have obtained. What is the sum of the two-digit number and the three-digit number?

Problem 6: Suppose a_n and b_n satisfy the recursive definitions:

$$a_{n+1} = 2(a_n + b_n) \quad \text{and} \quad b_{n+1} = 2b_n$$

for all $n \in \mathbb{N}$. Given that there exist constants x, y, z, w, such that

$$a_{n+2008} = xa_n + yb_n \quad \text{and} \quad b_{n+2008} = za_n + wb_n.$$

Find x, y, z, w.