

Problem 4.12 Prove that $(1 - 3x) = \sqrt{5x - 1}$ implies $x = 2/9$

Proof. Suppose x satisfies $(1 - 3x) = \sqrt{5x - 1}$. Squaring both sides yields

$$\begin{aligned} 5x - 1 &= (1 - 3x)^2 \\ 5x - 1 &= 1 - 6x + 9x^2 \\ 0 &= 2 - 11x + 9x^2. \end{aligned}$$

Using the quadratic formula gives

$$x = \frac{11 \pm \sqrt{11^2 - 4 \cdot 2 \cdot 9}}{2 \cdot 9} = \frac{11 \pm 7}{18} = 1, \frac{2}{9}.$$

However, the case of $x = 1$ must be thrown out since

$$(1 - 3 \cdot 1) = -2 \neq 2 = \sqrt{5 \cdot 1 - 1}.$$

Hence $x = \frac{2}{9}$. □