Math 140: Final Exam (150 points total)
Name:

Remember that IF YOU USE A CALCULATOR and wish to receive partial credit for incorrect answers, you must indicate what you had the calculator compute and the output it gave.

#1. (11 points) Choose from the following terms to fill in the blanks.

concave down	increasing	concave up	linear	decreasing	horizontal
$f'(c) < 0$	$f''(c) < 0$	$f'(c) = 0$	$f''(c) = 0$	$f'(c) > 0$	vertical
local minimum		inflection point		local maximum	$f''(c) > 0$

- $f'' > 0$ on an interval means f' is _____, so the graph of f is _____.

- $f'' < 0$ on an interval means f' is _____, so the graph of f is _____.

- If $f(x)$ has a critical point at $x = c$, then _____.

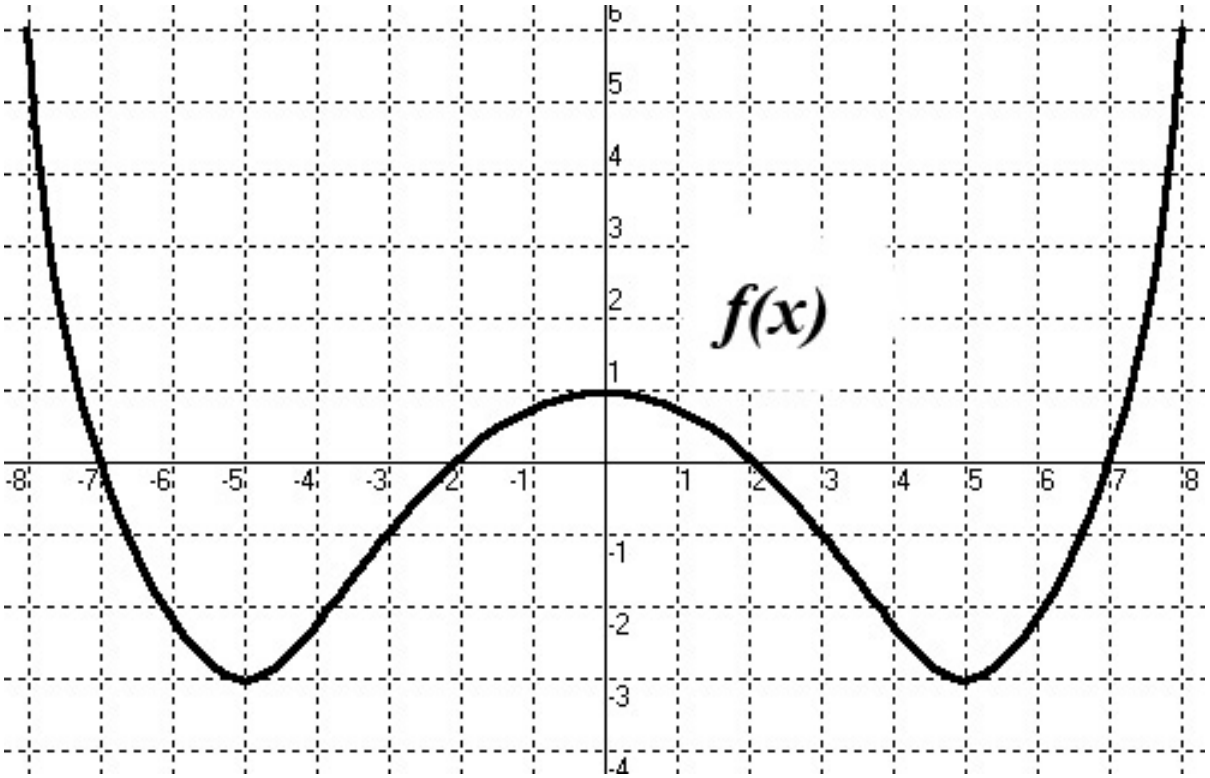
- If $f(x)$ is _____then $f'(x)$ is constant.

- If $x = c$ is a place where $f(x)$ changes concavity, then $(c, f(c))$ is a _____.

- If $x = c$ is a critical point and _____, then $f(x)$ is concave up around $x = c$ and thus $(c, f(c))$ is a _____.

- If $x = c$ is a critical point and _____, then $f(x)$ is concave down around $x = c$ and thus $(c, f(c))$ is a _____.

#2. (31 points) Suppose $y = f(x)$ has the following graph:



- On what interval(s) is f increasing?
- On what interval(s) is f concave down?
- On what interval(s) is $f' < 0$?
- At what values of x is $f'(x) = 0$?
- On what interval(s) is f' increasing?
- On what interval(s) is $f'' > 0$?
- On what interval(s) is $\int f(x)dx$ increasing?
- On what interval(s) is the graph of $\int f(x)dx$ concave down?

#3. (6 points) The town of Whoville has population given by

$$P = 10000(1.01)^t$$

where t is the number of years past 1900. In what year did the town reach a population of 25000?

#4 (10 points) Hooray! You won the lottery! You have two options:

(a) Get a $29,000 lump sum right now and another $29,000 lump sum in 10 years
OR
(b) Get a continuous stream of income over 10 years of $6000/year.

Assume that you can invest at 7% compounded continuously.

(i) Compute EITHER the future value or the present value of option (a). Make sure to clearly label which you've calculated.

(ii) Compute EITHER the future value or the present value of option (b). Make sure to clearly label which you've calculated.

(iii) Which option is better financially and why?

#5. (25 points) Compute the following

(i) $\frac{d}{dx}[4x^3 \ln(x)]$

(ii) $\frac{d}{dx}[\sqrt{5x^7 + 2x}]$

(iii) $\int \left(\frac{1}{\sqrt{x}} + \frac{1}{x} - 3x^2 \right) dx$

(iv) $\int 4xe^{(3x^2)} dx$

(v) $\frac{d}{dx} \left(\int \frac{x^2}{\ln x} dx \right)$

#6. (6 points) The supply and demand curves for a product are given by

$$S : p = \frac{q^2}{\ln(q+2)} \qquad D : p = 428e^{-q/60}$$

Find the total value gained from trade if 30 units were sold.

#7. (9 points) The rate of ad revenue generated by Yahoo! on a certain day was given by

$$A(t) = \ln(t+10) \cdot e^{-.01(t-12)^2} \text{ (dollars/minute)}$$

where t is the number of hours past midnight. What was the average rate of revenue between 8am and 5pm on that day? (*Note: 5pm is 17 hours past midnight.*)

#8. (10 points) Marginal costs to produce q units is given by

$$MC(q) = q^{1/2}.$$

It costs \$30 to produce 9 units. Determine the cost function $C(q)$, i.e. determine the total cost of producing q units.

#9. (10 points) Find all local maximums, local minimums, and inflection points of

$$y = x^3 - 4x^2 + 5.$$

Make sure to clearly label what you find.

#10. (10 points) You run an experiment and compile the following table of data:

t	1	3	5	7	9	11	13	15
$f(t)$	1	1	2	3	5	8	13	21

(i) Approximate $f'(13)$.

(ii) Approximate $\int_5^{11} f(t)\ dt$.

(iii) What can you do differently the next time to make your values in (i) and (ii) better estimates?

#11. (8 points) Find the equation of the tangent line to $y = e^{3x}$ at the point $(0, 1)$.

#12. (14 points) Suppose rate of income $I(t)$ (in dollars per day) for a company is a function of t, the number of days since opening.
(i) What are the units of $I'(t)$?

(ii) What does $I'(10) = 300$ mean in words?

(iii) What are the units of $\int I(t)dt$?

(iv) What does $\int_0^9 I(t)dt = 450$ mean in words?