1. This sample document provides a template for writing a LaTeX document suitable for homework assignments in Pf. Errthum's Math210 class. The first few "problems" will explain how the document works. Then there will be some "problems" that illustrate various notations and formatting environments. Compare what is in the typeset version of this document to the file `latexsample.tex`. Note in particular that anything typed after a percent sign in the text file is treated as a comment and is ignored by the compiler. Comments in the text file refer both to LaTeX and to hints about writing good solutions and proofs.

4. Each of your problem solutions should be contained in a "problem" environment. Problems with parts will also need "subprob" environments. These are special environments made up for the purposes of this class and will *not* be found in any other documentation online (see the "preamble" of the text file); see the comment in the text file for the general usage format for these environments. Please use these environments so your homework will be in the standard format for the class. Also please put the correct section and your name at the top of the page as they are in this sample document.

5. The very basics of LaTeX (compare the typeset document and the text file):

a) Extra spaces in the text file do not appear in the typeset document.
Except for a double carriage-return, that makes a new line. Single carriage returns don't do anything.

b) Mathematical expressions are typed between dollar signs like this: $y = x^2 + 1$. To make a centered equation on its own line, use double dollar signs, like this:

$$y = x^2 + 1.$$

The percent sign above the equation in the text file just makes it so that extra space is not added between the centered equation and the main paragraph above.

c) Many LaTeXcommands and math symbols start with a backslash symbol. For example, $\sin x$ and $\{x \in \mathbb{R} \mid x \geq 0\}$. Notice that the set-notation parentheses need to have backslashes before them (while regular parentheses do not). This is because in LaTeX, those squiggly parentheses often have other uses.

d) If you need to put something in italics you do it *like this*. Or maybe you need to have something in **boldface**. Or maybe ***both***.

e) Notice that to have quotes appear "correctly" in the typeset document you may have to type them yourself using the ' and ' keys instead of the " key.

f) Don't forget to end each of your environments. In other words, don't forget your `\end{problem}` or `\end{subprob}`, or you will get a compiling error. Also make sure to end dollar sign environments and parentheses.

8. Here is some random notation you might need:

x_2, x_{25}, x^2, x^{25}, ± 4, $x \neq 17$, $x > 5$, $x < 5$, $x \geq 5$, $x \leq 5$, $\{1, 2, 3\}$, $\{x \mid \sqrt{x} > 2\}$, $\sqrt[5]{x+7}$, ∞, $\overline{x}$, $\mathbb{R}$.

$A \subset B$, $A \subseteq B$, $A \not\subset B$, $A \not\subseteq B$, $A \setminus B$, $\widetilde{A}$, $A \cap B$, $A \cup B$, $x \in A$, $x \notin A$, $|A|$, $\mathcal{P}(A)$, $\emptyset$, $\overline{\overline{A}}$.

$\frac{5}{1+x}$, $\frac{5}{1+x}$, $\bigcap_{i=1}^{n} S_i$, $\bigcap_{i=1}^{n} S_i$, $\bigcup_{i=1}^{n} S_i$, $\bigcup_{i=1}^{n} S_i$, $\sum_{k=1}^{10} a_k$, $\sum_{k=1}^{10} a_k$, $\prod_{k=1}^{10} a_k$, $\prod_{k=1}^{10} a_k$, $\bigcap_{\delta \in \Delta} A_\delta$, $\bigcup_{A \in \mathcal{A}} A$.

$\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{N}$, $\rightarrow$, $\leftarrow$, $\leftrightarrow$, $\longrightarrow$, $\longleftarrow$, $\longleftrightarrow$, $\xrightarrow{1\text{-}1}$, $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$, $\implies$, $\impliedby$, $\iff$, $\mapsto$, $\longmapsto$.

$\mathcal{P}$, $\mathcal{S}$, $\mathcal{F}$, $\forall$, $\exists$, $\vee$, $\wedge$, $\sim$, $\approx$, $\equiv$, $\times$, $*$, $\star$, $|$, $a|b$, $|x|$, $\|x\|$, $\lceil x \rceil$, $\lfloor x \rfloor$, $\{x \in \mathbb{Z} \mid x \text{ is prime}\}$.

gcd, lcm, $\binom{n}{k}$, $\binom{n+1}{k}$, $a = \binom{n+1}{k}$, $\prec$, $\preceq$, $\succ$, $\succeq$, $f \colon [0, \infty) \to \mathbb{R}$, $f \circ g$ $\{$, $\}$, \$, %, &, $\neg$, #, $\setminus$.

$$f(x) = \begin{cases} x^2 & \text{if } x > 2 \\ 3x - 1 & \text{if } x \leq 2 \end{cases}$$

9. Suppose 43 students take algebra, 32 take Spanish, 7 take both.

The number taking algebra or Spanish (or both, of course) is:

$$43 + 32 - 7 = 68.$$

(We have to subtract 7 because those students are counted twice.)

10. In this problem will will prove a theorem two different ways, to illustrate two different formats for writing proofs with steps and reasons clearly written out.

a) Prove that $n^3 + n$ is even for every integer n.

Proof. Suppose n is any integer. We will examine two cases:

If n is even, then $n = 2k$ for some $k \in \mathbb{Z}$, and therefore:

$$
\begin{aligned}
n^3 + n &= (2k)^3 + (2k) &&\text{(since } n = 2k\text{)} \\
&= 8k^3 + 2k \\
&= 2(4k^3 + k). &&\text{(factor out a 2)}
\end{aligned}
$$

Since $4k^3 + k \in \mathbb{Z}$, this means $n^3 + n$ is divisible by 2 and therefore is even.

On the other hand, suppose n is odd. Then $n = 2k + 1$ for some $k \in \mathbb{Z}$, and thus:

$$
\begin{aligned}
n^3 + n &= (2k + 1)^3 + (2k + 1) &&\text{(since } n = 2k + 1\text{)} \\
&= (8k^3 + 12k^2 + 6k + 1) + (2k + 1) &&\text{(multiply out)} \\
&= 8k^3 + 12k^2 + 8k + 2 \\
&= 2(4k^3 + 6k^2 + 4k + 1). &&\text{(factor out a 2)}
\end{aligned}
$$

Once again, $n^3 + n$ is a multiple of 2 and therefore is even. $\square$

b) Prove that $n^3 + n$ is even for every integer n.

Proof. Note that $n^3 + n = n(n^2 + 1)$. It suffices to show that for any $n \in \mathbb{Z}$, either n is even or $n^2 + 1$ is even. (Since then the product of n and $n^2 + 1$ will have to be even.) For any integer n we have:

$$
\begin{aligned}
n \text{ is even} &\Longleftrightarrow n^2 \text{ is even} &&\text{(Theorem 2.1.9)} \\
&\Longleftrightarrow n^2 + 1 \text{ is odd.}
\end{aligned}
$$

Therefore if n is even, $n^2 + 1$ is odd; and if n is odd, then $n^2 + 1$ is even. (The second implication is the contrapositive of the backwards part of the "if-and-only-if" statement: If $n^2 + 1$ is odd, then n is even.) In any case, one of n or $n^2 + 1$ must be even, and therefore $n^3 + n = n(n^2 + 1)$ must be even. $\square$

12. Prove by induction that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for all natural numbers n.

Proof. Proceed by induction.
Base Case: $1 = \frac{2}{2} = \frac{1 \cdot 2}{2} = \frac{1(1+1)}{2}$.
Inductive Case: Suppose for some k that $1 + 2 + \cdots + k = \frac{k(k+1)}{2}$. Then

$$
\begin{aligned}
1 + 2 + \cdots + k + (k+1) &= \frac{k(k+1)}{2} + (k+1) &&\text{(by Inductive Hypothesis)} \\
&= \frac{k^2 + k}{2} + \frac{2k + 2}{2} \\
&= \frac{k^2 + 3k + 2}{2} \\
&= \frac{(k+1)(k+2)}{2}
\end{aligned}
$$

Thus, by the Principle of Mathematical Induction, $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for all natural numbers n. $\square$

13. You don't have to do your truth tables in LaTeX; you can write them in by hand if you like (using a \vspace command to create the extra room). But in case you are interested, this is how to do it:

P	Q	$P \wedge Q$	$\sim (P \wedge Q)$	$\sim Q$	$\sim (P \wedge Q) \wedge \sim Q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	F
F	F	F	T	T	T

Since the truth-values for $\sim Q$ and $\sim (P \wedge Q) \wedge \sim Q$ are the same for all possible truth-values of P and Q, the two statements are logicaly equivalent.