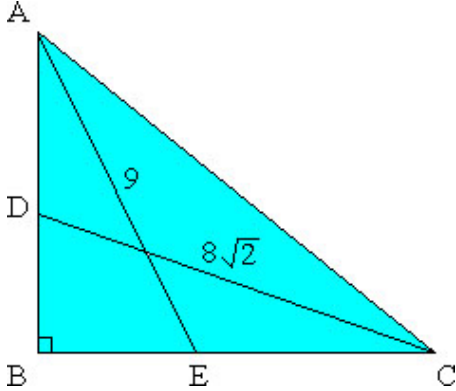


Math 280 Problems for October 3

Pythagoras Level

Problem 1: Triangle ABC is right-angled at B . D is a point on AB such that $\angle BCD = \angle DCA$. E is a point on BC such that $\angle BAE = \angle EAC$. If $AE = 9$ inches and $CD = 8\sqrt{2}$ inches, find AC .



Problem 2: Given that a and b are real numbers satisfying $a^3 - 3ab^2 = 39$ and $b^3 - 3a^2b = \sqrt{487}$, determine $a^2 + b^2$. (Hint: Think complex numbers!)

Newton Level

Problem 3: Find all pairs of real numbers (x, y) satisfying the system of equations

$$\begin{aligned}\frac{1}{x} + \frac{1}{2y} &= (x^2 + 3y^2)(3x^2 + y^2) \\ \frac{1}{x} - \frac{1}{2y} &= 2(y^4 - x^4).\end{aligned}$$

Problem 4: Let f be a nonconstant polynomial with positive integer coefficients. Prove that if n is a positive integer, then $f(n)$ divides $f(f(n) + 1)$ if and only if $n = 1$.

Wiles Level

Problem 5: The sequence (a_n) is defined recursively by $a_1 = a_2 = a_3 = 1$ and for $n \geq 1$,

$$a_{n+3} = a_n + a_{n+1}a_{n+2}$$

Prove that for every positive integer r there is a term a_s divisible by r .

Problem 6: Let $y = y(t)$ be a solution to the differential equation $y' + 2ty = t^2$. Evaluate

$$\lim_{t \rightarrow \infty} \frac{y}{t}.$$