

Name: Ans Key.

#1. (15 points) Find all local maximums and minimums and inflection points for the function

$$f(x) = x^5 - 5x^4 - 17$$

$$f'(x) = 5x^4 - 20x^3$$

$$\underline{\text{C.P.}} \quad f'(x) = 5x^4 - 20x^3 = 0$$

$$5x^3(x-4) = 0$$

$$\Rightarrow x=0 \text{ or } x=4$$

$$\& \quad f''(x) = 20x^3 - 60x^2$$

$$\underline{\text{I.P.}} \quad f''(x) = 20x^3 - 60x^2 = 0$$

$$20x^2(x-3) = 0$$

$$x=0 \text{ or } x=3$$

$x=0$ is not since it is a local max.

A little below 0: $f'(x) = 5(-)(-) = + \quad \Rightarrow 0 \text{ local max}$

A little above 0: $f'(x) = 5(+)(-) = -$

A little below 4: $f'(x) = 5(+)(-) = - \quad \Rightarrow 4 \text{ local min}$

A little above 4: $f'(x) = 5(+)(+) = +$

A little below 3: $f'' = 20(+)(-) = -$

A little above 3: $f'' = 20(+)(+) = +$

$\Rightarrow 3$ is inflection point

\Rightarrow ① $x=0$ local max, $x=4$ local min & $x=3$ is inflection point.

#2. (5 points) A company determines its cost function and revenue functions to be

$$C(q) = \ln(q+4)$$

$$R(q) = q^2 - 8q + 16$$

At what quantity q is the profit maximized?

$$MC = MR$$

$$\Rightarrow C' = R'$$

$$\Rightarrow \frac{1}{q+4} = 2q - 8$$

$$\Rightarrow 1 = (q+4)(2q-8) = 2q^2 - 32$$

$$\Rightarrow 0 = 2q^2 - 33$$

$$\Rightarrow \frac{33}{2} = q^2 \Rightarrow q = \sqrt{\frac{33}{2}} \approx 4.06.$$