

Math 280 Problems for October 9

Pythagoras Level

1. The increasing sequence $S = \{2, 3, 5, 6, 7, 10, 11, \dots\}$ consists of all positive integers which are neither a perfect square nor a perfect cube. What is the 500th term of S ?

2. Solve for x in terms of c :

$$2 \log_x c - \log_{cx} c - 3 \log_{c^2x} c = 0$$

Newton Level

3. Alice, Bob, and Carol repeatedly take turns tossing a fair regular six-sided die. Alice begins; Bob always follows Alice; Carol always follows Bob; and Alice always follows Carol. Find the probability that Carol will be the first to toss a six.

4. Find the exact value of

$$\lim_{x \rightarrow 3} \frac{x}{x-3} \int_3^x \sin t \, dt.$$

Wiles Level

5. An object moves 8 cm in a straight line from A to B , turns at an angle α , measured in radians and chosen at random from the interval $(0, \pi)$, and moves 5 cm in a straight line to C . What is the probability that $AC < 7$?
6. Prove that there exist infinitely many integers n such that $n, n+1, n+2$ are each the sum of the squares of two integers. [Example: $0 = 0^2 + 0^2$, $1 = 0^2 + 1^2$, $2 = 1^2 + 1^2$.]