

# Publish or Perish in an Intro to Proofs Course

Eric Errthum

Department of Mathematics and Statistics  
Winona State University  
Winona, MN

# Background

- “Foundations of Mathematics”
- Topics: Logic, Set Theory, Field Properties of  $\mathbb{R}$ , Functions, Relations, Modular Arithmetic, Sequences, Groups Intro, ...
- Required for: Abstract Algebra, Advanced Calculus, Number Theory, etc.
- 10-15 students, 50 min periods, 4 days a week

## Typical Issues

- Lecture on content rather than proof writing
- Lots of grading (by instructor) and feedback (from the instructor)
- Quantity vs. Quality
- Copying, copying, copynig, cpyng, ...

# Issues

## Typical Issues

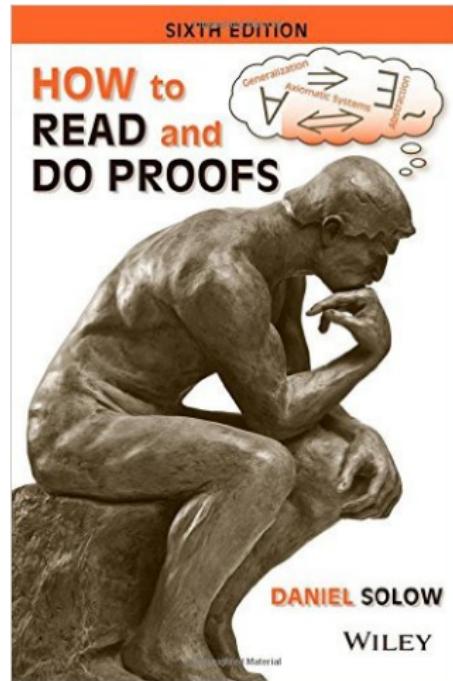
- Lecture on content rather than proof writing
- Lots of grading (by instructor) and feedback (from the instructor)
- Quantity vs. Quality
- Copying, copying, copynig, cpyng, ...

## Larger Issues

- Proving before believing
- Few constructions of examples
- Early struggles hamper the grade

# Support

- Text on proof writing with class lecture on each of 12 chapters
- “Lexicon” of problems with short concept intros and short lecture on each topic
- Rubric outlining what makes a good proof
  - Math Logic
  - Exposition / Communication
  - Grammar
  - Typesetting, spelling, punctuation, etc.



# A New Homework Scheme: Publish

- Each lexicon content section contains 20-25 questions
  - Most "Prove or Disprove"
  - Some "Prove that..." or "Provide an example of..."
  - Many standard proofs usually in the book (but not ours!)
- Students sign up anonymously for a problem when they have a solution
  - Once reserved, a problem cannot be submitted by any other student
  - Most students didn't sign up for a problem until they felt confident of the major steps, but before nailing down the details
- Submit a LaTeX version electronically, due within 24 hours of signing up

## A New Homework Scheme: Publish

- Problem is critiqued in front of whole class / by the whole class (or handed out as a critiquing assignment)
- Problem is deemed "Accepted", "Conditionally Accepted", "Revise & Resubmit", or "Rejected"
  - Revise & Resubmit = two class period extension (or relinquish)
  - Only Examples were ever rejected

# A Walk-Through

## 3.2 Extended Set Theory

**Definition 3.13.** A set of sets is called a **family** or a **collection** of sets.

**Class Example 3.8.** The power set  $\mathcal{P}(A)$  is a family of sets.

**Definition 3.14.** Given a family of sets  $\mathcal{A}$ , the **union over  $\mathcal{A}$**  is

$$\bigcup_{A \in \mathcal{A}} A = \{a \mid (\exists A \in \mathcal{A})(a \in A)\}.$$

The **intersection over  $\mathcal{A}$**  is

$$\bigcap_{A \in \mathcal{A}} A = \{a \mid (\forall A \in \mathcal{A})(a \in A)\}.$$

**Example 3.7.** Give an example of a family of three sets and the union and intersection over the family.

**Problem 3.22.** Prove for every  $B \in \mathcal{A}$ ,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$ .

**Problem 3.23.** Prove for every  $B \in \mathcal{A}$ ,  $B \subseteq \bigcup_{A \in \mathcal{A}} A$ .

**Problem 3.24.** The statement  $\bigcap_{A \in \mathcal{A}} A \subseteq \bigcup_{A \in \mathcal{A}} A$  is false in one special case. Determine that case, why it's false in that case, and then prove it's true in all other cases.

**Definition 3.15.** Let  $\Delta$  be a nonempty set such that for each  $\alpha \in \Delta$  there is a set  $A_\alpha$ . The family  $\{A_\alpha \mid \alpha \in \Delta\}$  is called an **indexed family of sets**. The set  $\Delta$  is called the **indexing set** and  $\alpha \in \Delta$  is called an **index**.

*(Close Example 2.9) Let  $\Delta = \mathbb{N}$  and let each index  $n \in \mathbb{N}$  map to the interval  $I_n = [1/n, 1/(n+1)]$ . Then the*

# A Walk-Through

Doodle

★ Features

皇冠 Pricing

Sign up

Log in

## Mutually agree on a choice

Enter your name in the input field below and select the options of your choice.

## Section 3.2 (Extended Set Theory) Problems

Poll initiated by Eric Errthum | 1 | 0 | less than a minute ago



### Hidden poll

This is a hidden poll. The participants and the result are only shown to the poll initiator.

1 participant	Example 3.7	Problem 3.22	Problem 3.23	Problem 3.24	Problem 3.25
Participant 1					
Your name	<input type="checkbox"/>				

No option works for me

Save

# A Walk-Through

**Proposition 3.22.** *For every  $B \in \mathcal{A}$ ,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$ .*

*Proof.* Suppose  $B \in \mathcal{A}$ . The intersection over  $\mathcal{A}$  is,

$$\bigcap_{A \in \mathcal{A}} A = \{a \mid (\forall A \in \mathcal{A})(a \in A)\}.$$

Since  $B \in \mathcal{A}$ , then all elements  $a$  of  $\bigcap_{A \in \mathcal{A}} A$  are in  $B$ . So by Definition 3.4,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$ .

□

# A Walk-Through

Let  $\mathcal{A}$  be a family of sets

**Proposition 3.22.** For every  $B \in \mathcal{A}$ ,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$ .

*Proof.* Suppose  $B \in \mathcal{A}$ . The intersection over  $\mathcal{A}$  is,

$$\bigcap_{A \in \mathcal{A}} A = \{a \mid (\forall A \in \mathcal{A})(a \in A)\}.$$

Since  $B \in \mathcal{A}$ , then all elements  $a \in \bigcap_{A \in \mathcal{A}} A$  are in  $B$ . So by Definition 3.4,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$ .

+ the statement  $a \in B$ .

$a \in A$  for all  $A \in \mathcal{A}$ .

QED. Accept.

# A Walk-Through

## 3.2 Extended Set Theory

**Definition 3.13.** A set of sets is called a **family** or a **collection** of sets.

**Class Example 3.8.** The power set  $\mathcal{P}(A)$  is a family of sets.

**Definition 3.14.** Given a family of sets  $\mathcal{A}$ , the **union over  $\mathcal{A}$**  is

$$\bigcup_{A \in \mathcal{A}} A = \{a \mid (\exists A \in \mathcal{A})(a \in A)\}.$$

The **intersection over  $\mathcal{A}$**  is

$$\bigcap_{A \in \mathcal{A}} A = \{a \mid (\forall A \in \mathcal{A})(a \in A)\}.$$

**Example 3.7.** Give an example of a family of three sets and the union and intersection over the family.

**Proposition 3.22.** Prove for every  $B \in \mathcal{A}$ ,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$ . For every  $B \in \mathcal{A}$ ,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$ .

*Proof.* Let  $\mathcal{A}$  be a family of sets, and  $B$  be a set such that  $B \in \mathcal{A}$ . The intersection over  $\mathcal{A}$  is,

$$\bigcap_{A \in \mathcal{A}} A = \{a \mid (\forall A \in \mathcal{A})(a \in A)\}.$$

Let  $a \in \bigcap_{A \in \mathcal{A}} A$  then  $a \in A$  for all  $A \in \mathcal{A}$ . Since  $B \in \mathcal{A}$ ,  $a \in B$ . So by Definition 3.4,  $\bigcap_{A \in \mathcal{A}} A \subseteq B$ . □

**Problem 3.23.** Prove for every  $B \in \mathcal{A}$ ,  $B \subseteq \bigcup_{A \in \mathcal{A}} A$ .

**Problem 3.24.** The statement  $\bigcap_{A \in \mathcal{A}} A \subseteq \bigcup_{A \in \mathcal{A}} A$  is false in one special case. Determine that case, why it's false in

# Experiences

- First submissions were comically bad in exposition and formatting (logic okay)
- But students had few/no critiques
- As time went on, students learned to be critical
- By end of semester, most submissions were of two types
  - Accepted: Well-written and correct
  - Revise & Resubmit: Concept wasn't understood
- Instructor Trade-Off: Less grading, more logistical coordinating and bookkeeping

# Reputation

Action	Result
Having a solution Accepted	+20 points
Having a solution Conditionally Accepted	+15 points
Having a Revised and Resubmitted Solution Accepted	+15 points
Having an example accepted	+7 points
Scoring perfect on a peer evaluation	+5 points
Top score on Vocab/Theorem quiz	+5 points
Answering a reading question in class	+3 points
Finding a typo in the lexicon	+2 points
Having a solution Rejected	-5 points
Having a reservation expire	-5 points

# Grades

Grades were based on the following

- Vocab/Theorem Quizzes  23.44%
- Reputation  10.94%
- Peer Evaluations  10.94%
- Contributed Written Work (10 highest)  18.75%
- Miscellaneous Assignments  4.68%
- Exam on Solow Text  7.81%
- Final Portfolio of Written Work  7.81%
- Final Exam  15.63%

# Reactions

## From Student Surveys:

- Some struggled without the usual rigidity of a course
- Comments
  - “Enjoyed the submission process”
  - “The more submissions I did, the more I learned.”
- Success from
  - “having to struggle through things & getting feedback from peers.”
  - “being able to correct mistakes” and “evaluating other proofs in class.”
  - “all the constructive criticism.”

# Changes for Next Time

- Require more written submissions
- Give some sort of general deadlines
- Focus more on content later in the semester

# Changes for Next Time

- Require more written submissions
- Give some sort of general deadlines
- Focus more on content later in the semester

Thank you.

Contact Info: Eric Errthum, eerrthum@winona.edu

# Internet Usage

From Student Internet Survey:

- Depended on Internet sources about the same as other classes
- Searched for solutions on-line: “Rarely” to “A Few Times”
- When they did search: found the solution “A Few Times”
- When they found the solution:
  - typically used it to know how the proof should start / get a hint
  - at worse they used it to make sense of the proof so they could put it in their own words.
- On average, they reported that 84% of their submitted work was solely their own.
- Main sources of the other 16%
  - Other students
  - The instructor
  - Internet

New sections were available to students once prerequisite sections were mostly complete.

