1.1: Functions

A Function is a way of assigning values in a domain to values in a range:

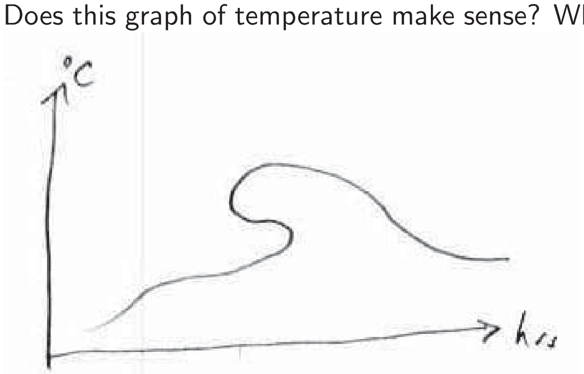
$$\text{Domain} \xrightarrow{\text{Function}} \text{Range}$$

Ex: Domain is all real numbers, function is $f(x) = x^2$. Range is ...

Ex: Domain is day of the year (i.e. $1 \sim$ Jan 1), the function P is the price of JD (John Deere) stock at close of that day. Range is ...

What does $P(32) = 37.5$ mean?

Does this graph of temperature make sense? Why or why not?



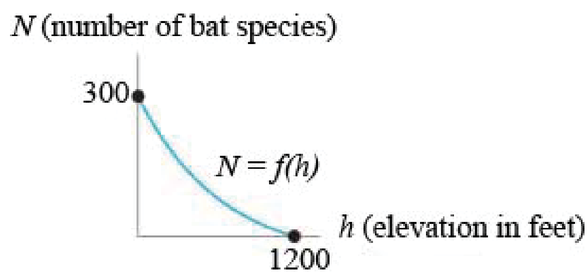
Example: $y = f(x) = x^2 - 4$.

- Find y when $x = 0$.
- What is $f(3)$?
- Find x when $y = 0$.
- When in $f(x) = 21$?
- What is the range?

Typically represent a function in 4 different ways:

- In words
- Equation
- Table of Data
- Graph

Example: $N =$ the number of bat species in Peru. $h =$ the altitude (in feet) above sea level and $N = f(h)$.



? Interpret $f(500) = 100$

? What is the meaning of the intercepts of the graph?

1.2 Linear Functions

Ex: The height of a tree is given by:

year	2000	2002	2003	2006
height (ft)	6	10	12	18

Ex: A car manufactor gives the amount of gas, g, in your tank as a function of the miles you've driven:

$$g(m) = 15 - 0.1m$$

- Sketch a graph
- What are and what are the meanings of the intercept and slope?
- What is the car's gas mileage?

Ex: Given the table of data:

t	0	3	6	9
$f(t)$	2	4	7	11

- Estimate $f(8)$.

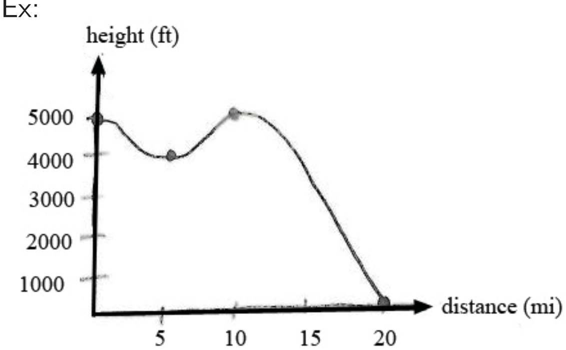
1.3 Rate of Change

Ex: Crop production on a farm is given by:

year	2000	2001	2002	2003	2004	2006
tons	110	119	126	131	134	137

- Which is a function of the other?
- What was the average rate of growth from 2000 to 2002?
- What was the average rate of growth from 2003 to 2006?
- What is happening to the rate of growth?

Ex:

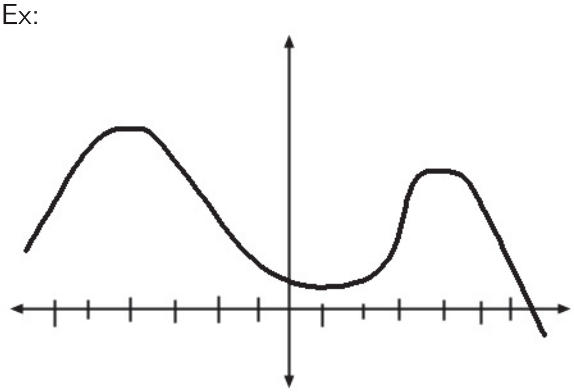


- What is the average rate of change between 0 and 20 miles?
- What is the average rate of change between 5 and 10 miles?

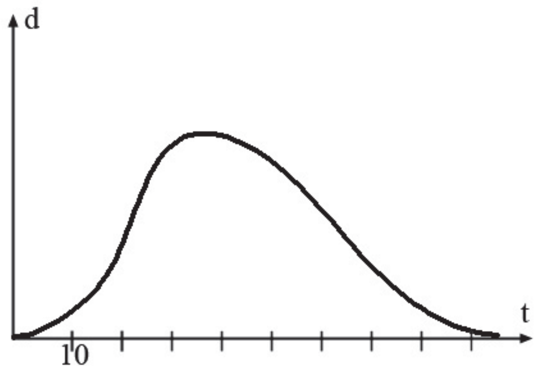
Increasing/Decreasing

Concave Up/Concave Down

Ex:



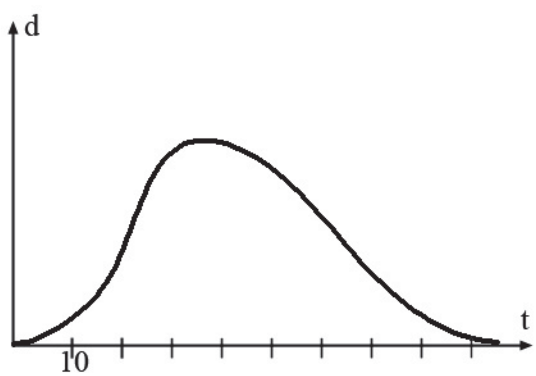
- Where is f increasing? Decreasing?
- Where is f concave up? Concave down?

Ex: 

? When was she driving the fastest?

? When was she speeding up?

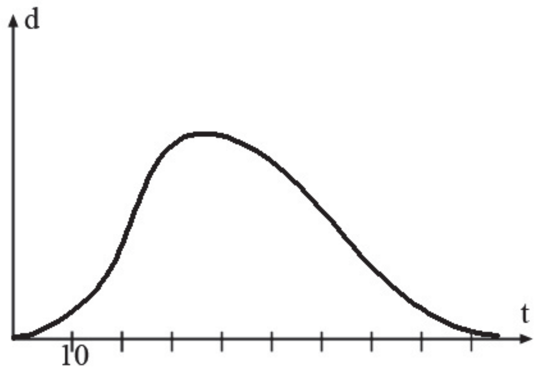
? When was she slowing down?

Ex: 

? What does a negative slope mean?

? What's happening when the slope is negative but concavity is positive?

? What's happening when the slope is negative but concavity is negative?

Ex: 

- How can we find average speed from 20 to 30 minutes?
- How can we find instantaneous speed?

1.4 Functions in Economics

Cost Function

Revenue Function

Profit Function

Break-even Point

Ex: A factory makes boxes. It costs \$3 in materials to make a box. The fixed costs are \$900. The factory charges \$7 per box.

- Give and graph the cost function.
- Give and graph the revenue function.
- Determine the break-even point.
- Give the profit function.

Ex:

$$C(q) = 0.02q + 10 \qquad R(q) = -\frac{q^2}{40000} + 0.105q$$

- What is profit at 1900 units?
- Should the company raise production from 1900 units to 2000 units?
- Should the company raise production to anything else?

Marginal Cost and Marginal Revenue

Supply and Demand Curves

Example: For a certain product:

$$E_1 : p = 240 - \frac{q}{500} \qquad E_2 : p = \frac{q}{1000}$$

- Which is supply and which is demand?

- At \$100, what quantity is going to be produced? demanded? What will/should happen?

- What is equilibrium price and quantity?

Relationship between Revenue and Demand

- If revenue is given by $R = 32q - 7q^2$, give the demand equation.

- If demand is modeled by $4p + 7q = 100$, give the Revenue equation.

Ex: A company produces and sells shirts. Fixed costs are \$7000, variable costs are \$5 per shirt and demand is modeled by

$$D : q + 40p = 2000$$

- What is the cost function, $C(q)$?
- What is the revenue function, $R(q)$?
- What is the profit function, $\pi(q)$?
- What is the profit function, $\pi(p)$?
- What profit is realized at a price of \$12?

1.5 Exponential Functions

Ex: Consider the following data of bacteria growth

Time	# of Bact. L	Incr. in L.	# of Bact. E	Incr. in E
0	3		3	
1	6		6	
2	9		12	
3	12		24	
4	15		48	

- Give a function for each.

Exponential Functions

Ex: You have 5 kilograms of nitrogen isotope.

- Give an equation for the amount A of nitrogen after t minutes if each minute .07kg of it radioactively decays.
- Give an equation for the amount A of nitrogen after t minutes if each minute 7% of it radioactively decays.
- Draw a rough graph of each.

Example #21: Cliff Notes started in 1958 with \$4,000 and it sold in 1998 for \$14,000,000.

- Assuming exponential growth, find the annual percentage increase over the 40 years.

- When was the company worth \$1,000,000?

Example #28: Niki invested \$10,000 in the stock market and lost 10% a year for a period of 10 years. She then switched strategies and started to gain 10% a year.

- How much is her investment worth at the end of the first 10 years?

- After switching, how long until she attains a value of \$10,000 again?

Continuous Growth

Ex: A city's population is 1000 and growing at a 5% rate.

- Give functions for the population if the rate is annual vs. continuous.
- In both cases, what is the population after 18 years.
- Give the continuous rate that corresponds to a 5% annual rate.

1.6 The Natural Logarithm

? Solve

$$7 \cdot 3^x = 5 \cdot 2^x$$

Hierarchy of Funtions

Ex: In a given scenario,

$$C(q) = \ln(q+1) + q^{800} + 4000000$$

and

$$R(q) = \frac{q}{q^7+1} + \sqrt{q} + (1.000001)^q.$$

When producing a bazillion items, is there positive or negative profit?

1.7 Exponential Growth and Decay

Ex: Caffeine which leaves the body at a continuous rate of 17% per hour. An individual drinks a cup of coffee at noon. At 2pm a blood test shows they have approximately 50mg of caffeine in their body.

- Find a formula for A, the amount of caffeine in the body.

- How much caffeine will be in the body at 4pm?

- How much caffeine was in the cup of coffee?

Ex: Suppose an investment fund earns 7%.

- How long will it take for a \$1000 investment to grow to \$10,000?
- If you invest \$8000 now, how much will it be worth in 4 years?
- If you need \$11000 for your child's college in four years, how much should you invest now?

Present Value and Future Value

Ex: If the present value of a bond is \$100 and it earns at a rate of 5%, what will be its future value in 10 years?

Ex: If you have a bond that matures at \$160 in ten years, how much is it worth right now (i.e. how much should you sell it for now) if you have access to a 5% rate.

Ex: A business partner in debt to you says he'll give you

Option A. \$5000 right now, or

Option B. \$6000 in three years.

You can invest at 7%.

- What is the future value of option A?
- What is the future value of option B?
- What is the present value of option A?
- What is the present value of option B?
- Which option is worth more?

1.8 New Functions from Old

Ex: Suppose

$$f(x) = x^2 + x \qquad\qquad g(x) = 3\sqrt{x} - 1$$

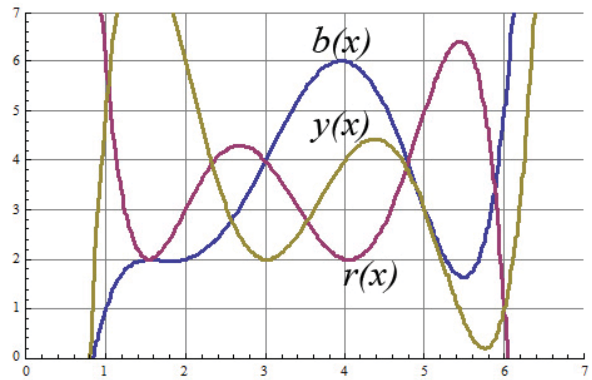
- $g(blarg) = ?$
- $f(x+4) = ?$
- $(g(x)+1)^2 = ?$
- $f(g(p)) = ?$
- $g(f(p)) = ?$
- $g(g(y)) = ?$

Consider the functions:

x	1	2	3	4	5	6
$f(x)$	3	5	2	1	6	4
$g(x)$	6	2	4	3	1	5
$h(x)$	6	4	1	2	5	3

Determine:

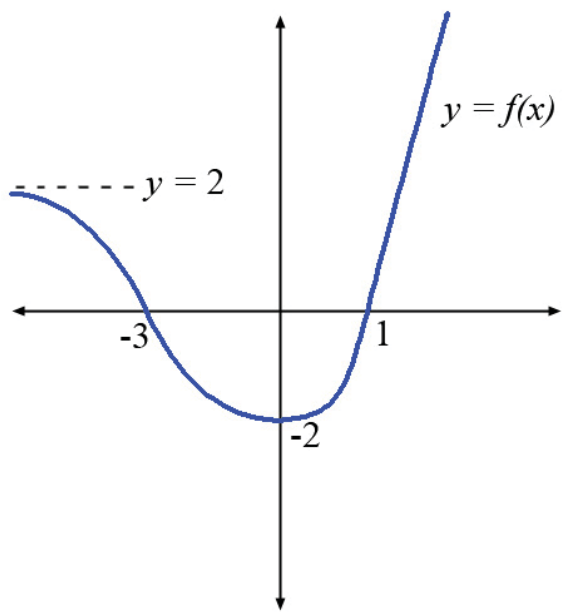
- $f(g(4)) =$
- $h(g(f(2))) =$
- $g(h(4)^2 - 1) + 2 =$
- $h(h(h(3))) =$



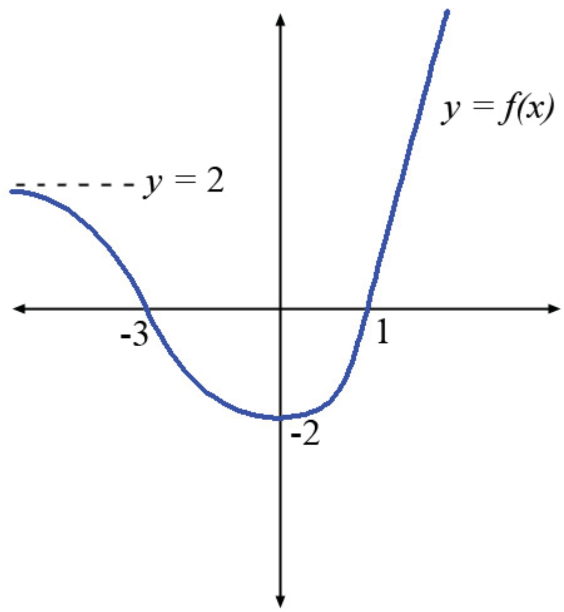
Determine:

- $r(b(4)) =$
- $b(y(5)^2 - 5) + 2 =$
- $y(b(r(2))) =$
- $r(r(4)) =$

Transformations of Graphs : Changing Output



Transformations of Graphs : Changing Input



1.9 Proportionality and Polynomials

Proportional:

Ex: The miles you drive are proportional to the gas you use to do so.

Ex: The area of a circle is proportional to the square of the radius.

Ex: The force on a spring is proportional to the length it is stretched beyond rest. Suppose a 5in spring exerts 12N when stretched to 7in.

- What is the constant of proportionality, also called the spring constant?

- What is the force when stretched to 10in.?

Inversely Proportional

Ex: Your weight is inversely proportional to the square of your distance to the Earth's center.

Power Function

Polynomial

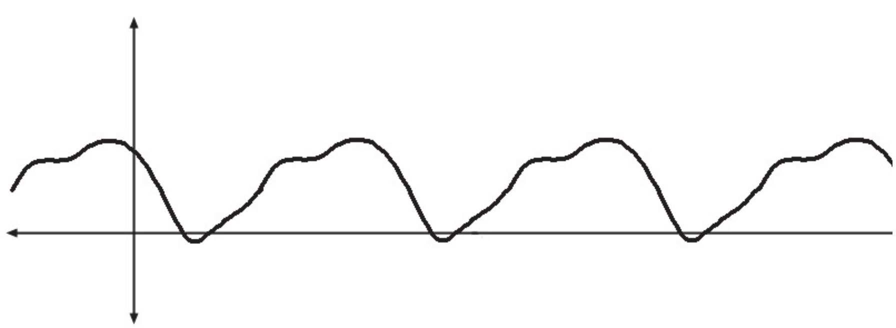
Ex: $p(x) = 5x^2 + 3x - 1$ give:

- degree:
- coefficients:
- leading term:
- constant term:

Ex: $q(x) = 4 + 9x - 7x^5 - 3x^2$ give:

- degree:
- coefficients:
- leading term:
- constant term:

1.10 Periodic Functions



Trig Functions

$\sin(x)$ and $\cos(x)$

- Don't "know" wrong stuff
  - $\sin x$ is NOT $\sin \cdot x$.
  - $\cos(3x)$ is NOT $3\cos(x)$.
  - $\sin(x+3)$ is NOT $\sin x + \sin 3$.
- Only need to know conventions
  - $\cos^2(x) = (\cos(x))^2$
  - Always work in Radians
  - Otherwise, treat $\sin(x)$ and $\cos(x)$ like you do $f(x)$... as if you know nothing.

2.1 Instantaneous Rates of Change

Average Rate of Change

Instantaneous Rate of Change

Graphical View

IROC for equations

Ex: Find instantaneous rate of change in $f(x) = 2x^3 + x$ near $x = 1$.

Derivative at a point

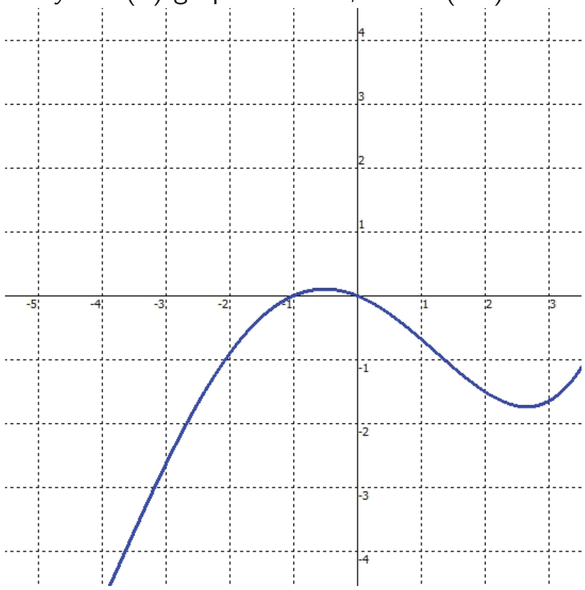
Ex: For $g(t) = 6 \cdot 3^t$, approximate $\frac{dg}{dt}|_{t=0}$ to 3 decimal places.

Ex: Consider the function $k(t) = t \cdot \ln(t)$.

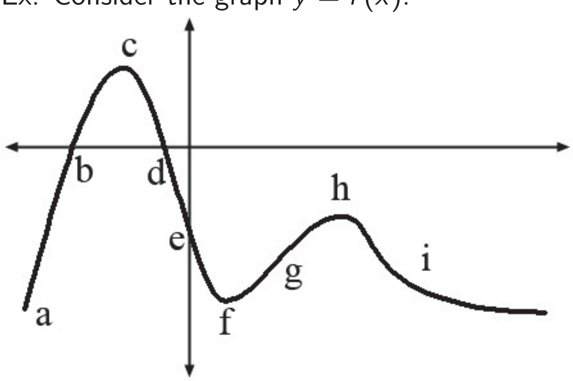
- Determine the instantaneous rate of change at $t = 1$.
- Determine the instantaneous rate of change at $t = 2$.
- Sketch a rough graph of $k(t)$ around the points $t = 1$ and $t = 2$.
- what does this imply about the concavity on the interval $1 < t < 2$?

Derivatives for Graphs

For $y = f(x)$ graphed below, find $f'(-1)$.



Ex: Consider the graph $y = f(x)$:



Determine the positions where $f'(x)$ is

- positive
- negative
- zero

Derivatives for Tables

Ex: Use the following table of data:

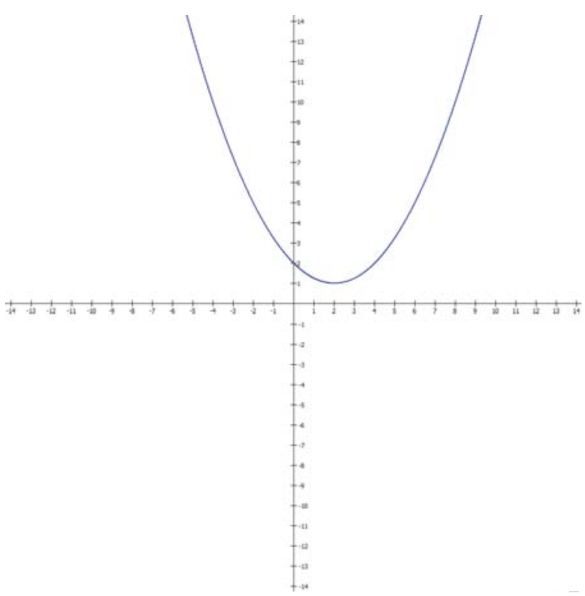
x	0	6	10	17	20
$f(x)$	100	70	55	46	40

? Approximate $f'(5)$.

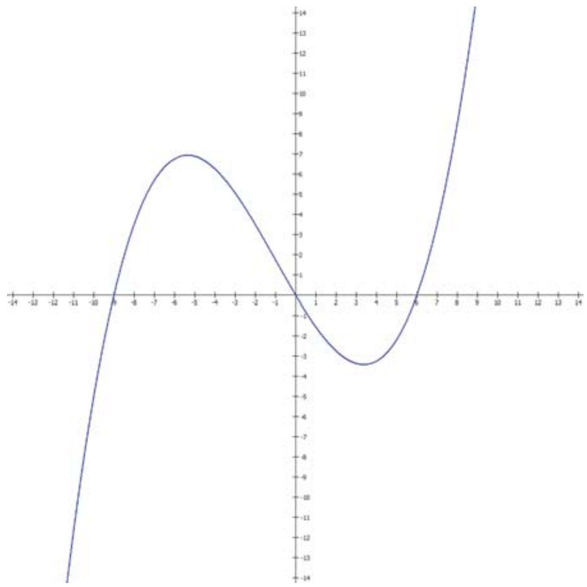
? Approximate $f'(10)$.

2.2 The Derivative as a Function

? Given the following graph of $f(x)$, sketch the graph of $f'(x)$.

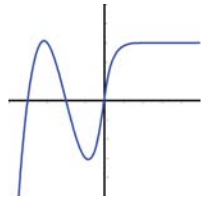


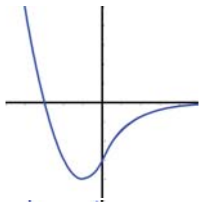
? Given the following graph of $f(x)$, sketch the graph of $f'(x)$.

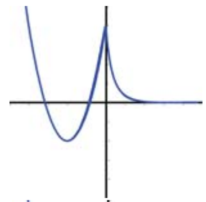


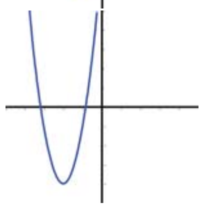
Graphical Information from the Derivative

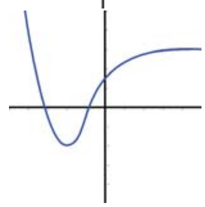
? Give the following graph of $f(x)$, which is the graph of $f'(x)$?



A. 

C. 

B. 

D. 

2.3 Interpretations of the Derivative

The cost C (in dollars) to produce g gallons of purified water is given by

$$C = f(g)$$

- What are the units of $f'(g)$?

- What does $f'(g)$ measure?

The time for a chemical reaction T (in minutes) is a function of the amount, a, of catalyst present (in ml):

$$T = f(a).$$

- Suppose $f(5) = 18$. What are the units on the 5?

- What are the units on the 18?

- Interpret $f(5) = 18$ in the context of the problem.

The time for a chemical reaction T (in minutes) is a function of the amount, a, of catalyst present (in ml):

$$T = f(a).$$

- Suppose $f'(5) = -3$. What are the units on the 5?
- What are the units on the -3?
- Interpret $f'(5) = -3$ in the context of the problem.

The time for a chemical reaction T (in minutes) is a function of the amount, a, of catalyst present (in ml):

$$T = f(a).$$

Suppose $f(5) = 18$ and $f'(5) = -3$.

- Approximate $f(7)$.
- Approximate $f(4.5)$.

The size of a dose, D, in mg is a function of w, the weight in pounds of the patient: $D = f(w)$.

- Interpret $f(140) = 120$ in the context of the problem.

- Interpret $f'(140) = 7$ in the context of the problem.

- Approximate $f(145)$.

The quantity Q in mg of nicotine in the body t minutes after smoking is given by $Q = f(t)$.

- Interpret $f(20) = 0.36$ in the context of the problem.

- Interpret $f'(20) = -0.002$ in the context of the problem.

- Approximate $f(21)$ and $f(30)$.

Local Linear Approximation

Suppose you know that

$$f(2) = 1 \qquad\qquad f'(2) = 3$$

- Give the equation of the tangent line to the graph of $f(x)$ at $x = 2$.

- Approximate $f(2.1)$.

- Approximate $f(1.8)$.

- Approximate $f(x)$ for x near 2.

? Approximate $\sqrt{26}$ by using the fact that

$$\left.\frac{d(\sqrt{x})}{dx}\right|_{x=25} = \frac{1}{10}.$$

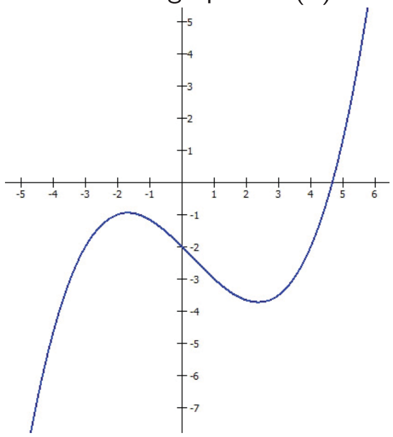
? Approximate $\sqrt{41}$ using the same info.

? Which approximation is truer to the actual value?

? What does the concavity of the graph of $y = \sqrt{x}$ tell us about our approximations?

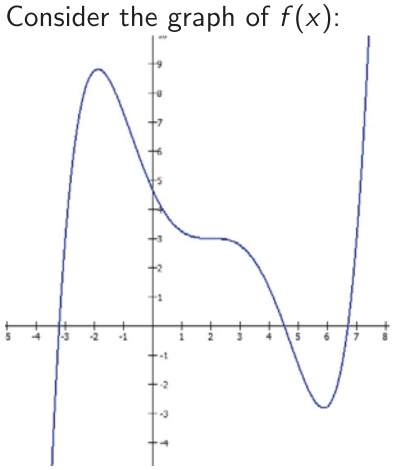
2.4 Second Derivative

Consider the graph of $f(x)$:

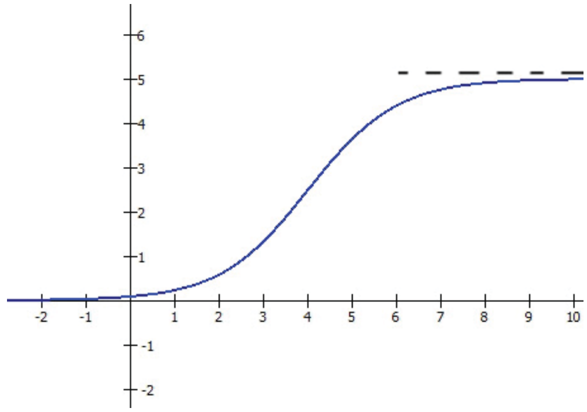


- Where is $f(x) > 0$? $f(x) < 0$?
- Where is $f'(x) > 0$? $f'(x) < 0$?
- Where is $f''(x) > 0$? $f''(x) < 0$?

Consider the graph of $f(x)$:

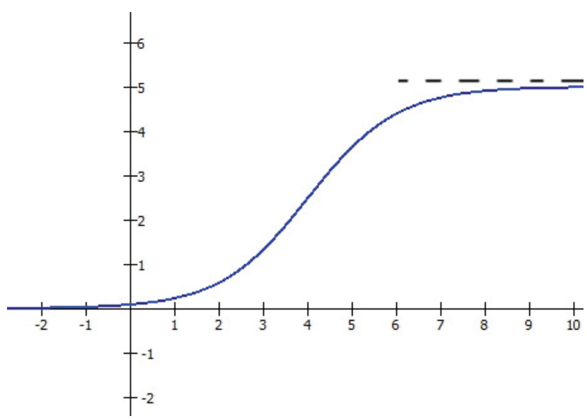


- Where is $f(x) > 0$? $f(x) < 0$?
- Where is $f'(x) > 0$? $f'(x) < 0$?
- Where is $f''(x) > 0$? $f''(x) < 0$?



⑦ Draw a sketch of $f'(x)$.

⑦ Draw a sketch of $f''(x)$.



⑦ What's special about $x = 4$ on the graph of $f(x)$?

⑦ What's special about $x = 4$ on the graph of $f'(x)$?

⑦ What's special about $x = 4$ on the graph of $f''(x)$?

Sketch a graph of a continuous function f such that

- $f'(x) > 0$ of all x
- $f''(x) < 0$ for $x < 2$ and $f''(x) > 0$ for $x > 2$

Sketch a graph of a function f such that

- $f(2) = 5$
- $f'(2) = \frac{1}{2}$, and
- $f''(2) > 0$.

Suppose

- $f(5) = 20$
- $f'(5) = 2$ and
- $f''(x) < 0$ for $x > 5$.

? Which values are possible for $f(7)$?

20 22 24 26 28

Consider the table fo data

t	0	2	4	6	8	10
$p(t)$	-56	-98	-122	-137	-145	-150

? What is the sign of the first derivative?

? What is the sign of the second derivative?

? Approximate the second derivative at $t = 4$.

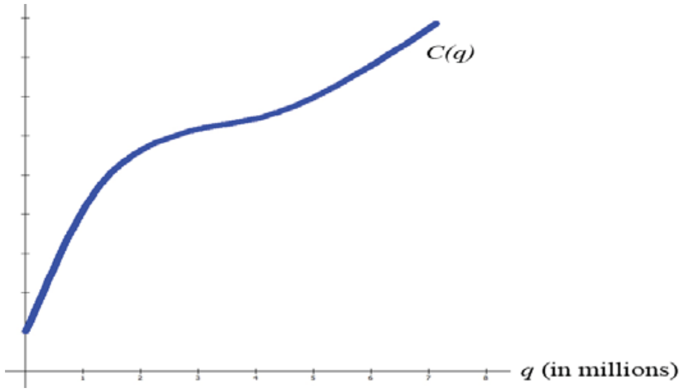
Interpreting Second Derivative

Higher Derivatives

2.5 Marginal Cost and Marginal Revenue

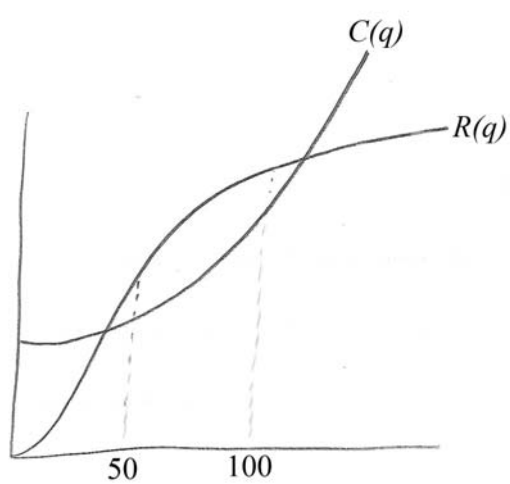
Ex: Suppose the cost of processing T tons of wheat is given by $C(T)$. And the revenue received is $R(T)$.

- Interpret $C(4) = 350$.
- Interpret $MC(4) = 60$.
- Interpret $R(4) = 500$.
- Interpret $MR(4) = 40$.



Ⓘ Which individual item costs the most to produce: the 1 millionth, 3 millionth, or 7 millionth item?

Ⓘ Which costs the least?



Ⓘ If currently producing and selling 50 items, should I try to produce more?

Ⓘ If currently producing and selling 100 items, should I try to produce more?

Ex: A company produces and sells shirts. Fixed costs are $7000, variable costs are $6 per shirt and demand is modeled by

$$D : q + 40p = 2000$$

- What is the marginal cost at 800 shirts? At 900 shirts?
- What is the revenue function?
- What is the marginal revenue at 800 shirts? Should you produce more or less?
- What is the marginal revenue at 900 shirts? Should you produce more or less?
- Where is the optimal production level?

3.X Derivatives of Functions

Compute

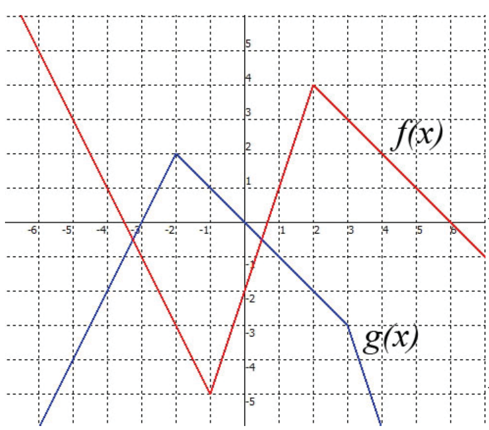
- $x^6 + 4x^3 - 2$
- $\sqrt[3]{x^2+1}$
- $\ln(t)$
- 3^y
- e^x
- $\sin(x)$

Compute the first six derivatives of:

$$f(x) = 4 + x^3 + \frac{1}{x^3} + \sqrt[5]{x} + \ln(x) + \sin x + e^{2x}$$

Linearity Rule

Determine from the graph:



$$\frac{d(f(x)+g(x))}{dx}\Big|_{x=4}$$

$$\frac{d(4g(x))}{dx}\Big|_{x=-3}$$

$$\frac{d(3f(x)-5g(x))}{dx}\Big|_{x=0}$$

Determine from the table:

x	-2	1	4	7	10	13
$f(x)$	10	12	16	24	40	72
$g(x)$	11	8	2	-7	-19	-34

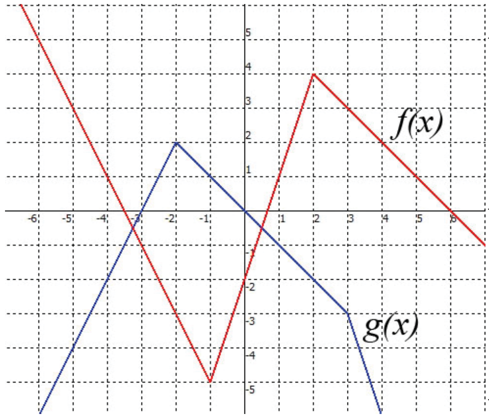
- $\frac{d(f(x)-g(x))}{dx}\big|_{x=0}$

- $\frac{d(3f(x))}{dx}\big|_{x=7}$

- $\frac{d(2g(x)+6f(x))}{dx}\big|_{x=5}$

3.3 Chain Rule

Determine from the graph:



$$\frac{d(f(g(x)))}{dx}\,|_{x=4}$$

$$\frac{d(g(x^4+2f(x)))}{dx}\,|_{x=1}$$

$$\frac{d\left(\sqrt{f(x)}\right)}{dx}\,|_{x=4}$$

Consider the table of data.

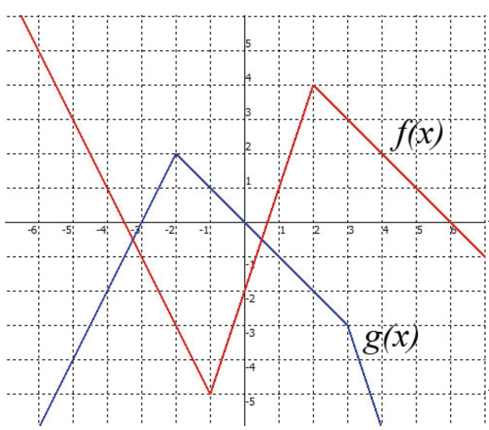
x	0	1	2	3	4	5
$f(x)$	0	3	4	1	5	2
$g(x)$	0	5	1	4	3	2

Compute the following

- $\frac{d(g(f(x)))}{dx}\,|_{x=3}$
- $\frac{d(\ln(g(x)))}{dx}\,|_{x=4}$
- $\frac{d(f(x^3-x^2))}{dx}\,|_{x=2}$

3.4 Product Rule

Determine from the graph:



$$\frac{d(f(x)g(x))}{dx}\Big|_{x=-4}$$

$$\frac{d(x^4 f(x))}{dx}\Big|_{x=1}$$

$$\frac{d\left(f(x)\cos(g(x))\right)}{dx}\Big|_{x=-3}$$

Consider the table of data.

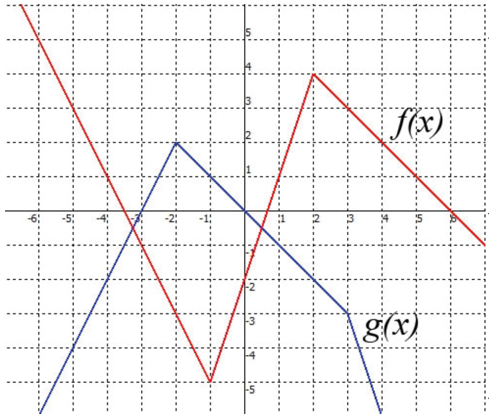
x	0	1	2	3	4	5
$f(x)$	0	3	4	1	5	2
$g(x)$	0	5	1	4	3	2

Compute the following

- $\frac{d(g(x)(f(x))}{dx}\,|_{x=3}$
- $\frac{d(g(x)e^{f(x)})}{dx}\,|_{x=4}$
- $\frac{d(f(x^2)g(\sqrt{x}))}{dx}\,|_{x=2}$

Quotient Rule

Determine from the graph:



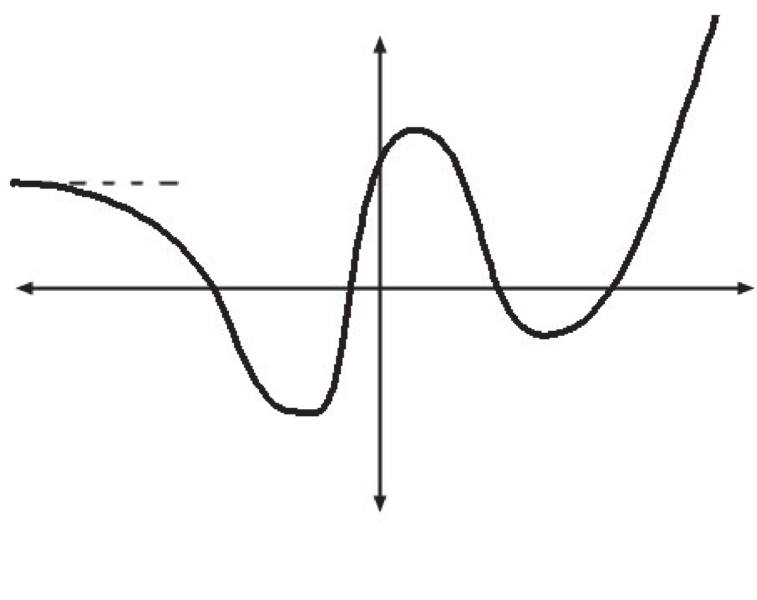
$$\frac{d}{dx}\frac{f(x)}{g(x)}\Big|_{x=-4}$$

$$\frac{d}{dx}\frac{f(x)}{\sqrt[3]{x}}\Big|_{x=-2}$$

$$\frac{d}{dx}\left(\frac{x^3 g(x)}{\sqrt{f(x)}}\right)\Big|_{x=-5}$$

Mins and Maxes (4.1 and 4.3)

x	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	6	7	5	8	12	15	11	4	3	7



Ex: Find the global mins and maxes of

$$f(x) = 2x^3 + 3x^2 - 12x + 4 \qquad \text{on } [-3, 3]$$

Ex: Find the global mins and maxes of

$$f(x) = e^x + e^{-x} \qquad \text{on } [-1, 1]$$

Ex: Find the global mins and maxes of

$$f(x) = \frac{2x+1}{e^x} \qquad \text{on } [0, \infty]$$

Ex: Find the global mins and maxes of

$$f(x) = x \ln x \qquad \text{on } [0, \infty]$$

Ex: Find the global mins and maxes of

$$f(x) = \frac{3x^2 + 6x + 12}{2x^2 + 15} \qquad \text{on } [0, \infty]$$

4.1: Local Mins and Maxes

First Derivative Test for Local Extrema

Ex: Find the local mins and/or maxes of

$$f(x) = xe^x.$$

Ex: Find the local mins and/or maxes of

$$f(x) = \ln(x+1) - x^2.$$

Ex: Find the local mins and/or maxes of

$$f(x) = x^3 - 3x^2 + 2x - 4.$$

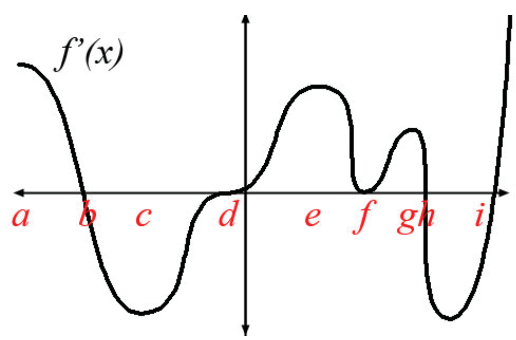
Second Derivative Test for Local Extrema

Ex: Find the local mins and/or maxes of

$$f(x) = 2x^3 - 9x^2 + 12x + 2.$$

Ex: Find the local mins and/or maxes of

$$f(x) = x^6 - 3x^4 + 7.$$



Where are the critical points of $f(x)$?

Local maxes of $f(x)$?

Local mins of $f(x)$?

Places where the 2nd derivative test fails?

Places where the 1st derivative test fails?

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	?	0	?	-	-	0	-	-	0	?	?	+	+
$f'(x)$	-	-	-	0	?	0	?	0	+	0	-	0	+
$f''(x)$	+	+	+	+	0	-	0	+	0	?	0	+	+

Where are the critical points of $f(x)$?

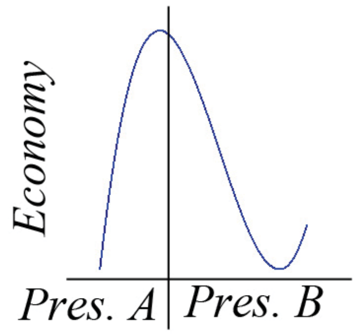
Local maxes of $f(x)$?

Local mins of $f(x)$?

Fill in the ?'s

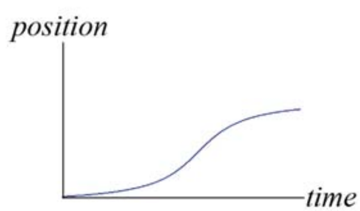
4.2: Inflection Points

Consider the hypothetical economic graph during the period of two presidents from opposing parties.



- What would supporters of Pres. A say?
- What would supporters of Pres. B say?

Consider a drag racing car with the following position vs. time graph.



Locate and describe what happened at the inflection point.

Finding and Testing Inflection Points

Ex: Find the local mins and/or maxes and inflection points of

$$f(x) = x^4 - 2x^2.$$

Ex: Find the local mins and/or maxes and inflection points of

$$f(x) = \frac{1}{12}x^4 - x^3 + \frac{9}{2}x^2 - 1.$$

4.4: Profit, Cost , Revenue

Suppose

$$R(q) = 12q - 0.01q^2 \qquad\qquad C(q) = 10 + 0.9q$$

Maximize profit for $100 \leq q \leq 1000$.

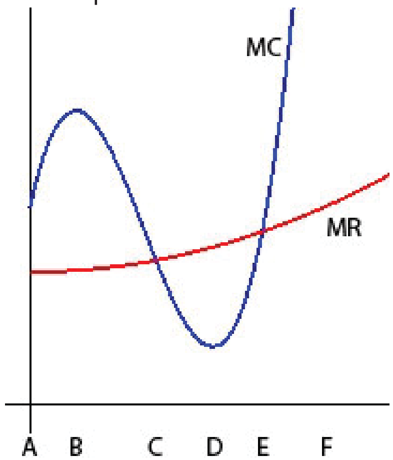
Suppose

$$R(q) = q^3 - 12q^2 + 48q \qquad\qquad C(q) = 4q^2 + 20$$

- Maximize profit on $0 \leq q \leq 10$

- Maximize profit on $0 \leq q$

Which production level maximizes profit?



Marginal revenue and marginal cost are given in the following table. Estimate the production levels that maximize profit.

q	1000	2000	3000	4000	5000	6000
MR	78	76	74	72	70	68
MC	100	80	70	65	75	90

Suppose demand is given by

$$p + 5q = 4000$$

and the cost function is

$$C(q) = 6q + 5$$

Find the quantity that maximizes profit and the profit at that level.

The city bus costs \$200 to run for a day. At a price of \$0.75 per ride, the bus gets 250 people to ride. Every \$0.05 decrease in price results in 20 more riders. What price maximizes profit?

At a price of \$8/ticket the show is just barely sold out at 1500 people. For each additional dollar charged, 75 less people buy. What ticket price maximizes profit?

4.5: Average Cost

A company has cost

$$C(q) = 500 + 30q.$$

- What is the marginal cost at 100 items? At 1000 items?
- What is the average cost at 100 items? At 1000 items?
- As q gets really large, what happens to marginal cost? To average cost?

Visualizing Average Cost and Finding Its Minimum

Suppose a non-profit organization is going to make medicine kits to sell to low-income citizens. The cost function for making the kits is

$$C(q) = .01q^3 - .06q^2 + 13q + 120,$$

for q in thousands of units. What is the production level that allows you to charge the minimum price? What is that price?

Consider the table of data:

q	0	10	20	30	40	50
$C(q)$	40	45	55	70	85	115
MC	0.5	0.75	1.25	1.5	2.25	3
$a(q)$	∞	4.5	2.75	2.333	2.125	2.3

Where is the average cost minimized?

4.6: Elasticity of Demand

Suppose demand curve is given by $q = 14e^{-(p/100)^2}$.

- Find and interpret the elasticity of demand at $p = 10$ and $p = 200$.

- How is revenue affected by raising the price at $p = 10$ and $p = 200$?

Relationship to Revenue

Netflix Example

From October 15, 2014:
"Netflix Says a $1 Price Increase Crushed Its Subscriber Growth"

http://www.slate.com/blogs/moneybox/2014/10/15/netflix_earnings_the_company_says_price_hikes_crushed_its_subscriber_growth.html

Which do you think are inelastic and which are elastic over their typical range of prices?

- Campbell's soup
- Gas
- Honda CRV
- Salt
- Packers vs. Vikings ticket
- Diamonds
- Hershey's chocolate bar
- Airline tickets on Thanksgiving weekend
- Cigarettes
- Star Tribune subscription

5.1: Distance and Accumulated Change

? Travelled 60 mph for 3 hours. How far did you go? Draw a picture to illustrate the concept...

If your velocity is given by

$$v(t) = 3t$$

- How far do you go between $t = 0$ and $t = 4$?
- How far do you go between $t = 0$ and $t = T$?

A water tank accumulates water at a rate of 200 L/s for the first 3 seconds. Then the rate increases linearly to 400 L/s for the last 5 seconds.

- How much water enters the tank?
- How much water is in the tank after the 8 seconds?

A filter slowly loses its ability to filter pollution out of the water.
Consider the table of data for a filter:

Day	0	5	12	19	24	30
kg/day	7	8	10	13	18	35

? Estimate the total amount of pollution that entered the lake.

The velocity of a car is given by:

time	0	2	4	6	8	10
vel.	0	4	16	36	64	100

? Estimate the total distance traveled?

? How can you make your estimate better?

The velocity of a car is given by:

time	0	1	2	3	4	5	6	7	8	9	10
vel.	0	1	4	9	16	25	36	49	64	81	100

? Estimate the total distance traveled?

5.2

Left/Right Handed Sums

Let

$$f(x) = 3x^2 + 2x$$

- ? Find LHS on $[3, 4]$ with $n = 10$.
- ? Find LHS on $[3, 4]$ with $n = 100$.
- ? Find LHS on $[3, 4]$ with $n = 1000$.
- ? Find LHS on $[3, 4]$ with $n = \infty$.

Definite Integral

Compute

$$\int_1^3 e^x \, dx$$

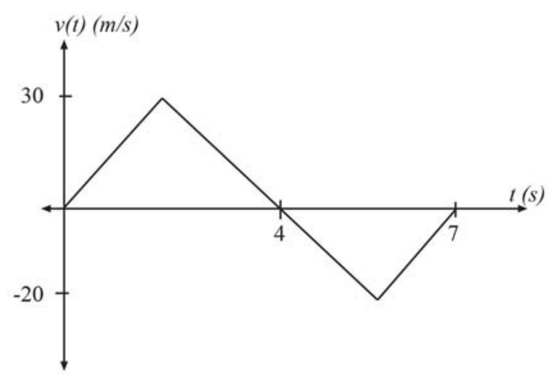
5.3: The Integral as Signed Area

Find the area under the curve

$$f(x) = 1/x$$

on the interval $[\frac{1}{2}, 2]$.

Suppose velocity as a function of time has the following graph:



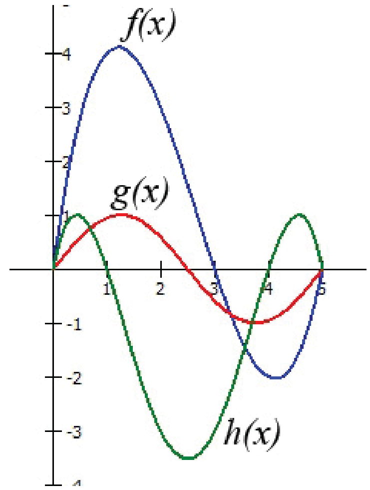
- How many meters did it travel?
- How far from the initial position is the object?

Determine if the following are positive, negative, or approximately zero:

$$\int_0^5 f(x)\ dx$$

$$\int_0^5 g(x)\ dx$$

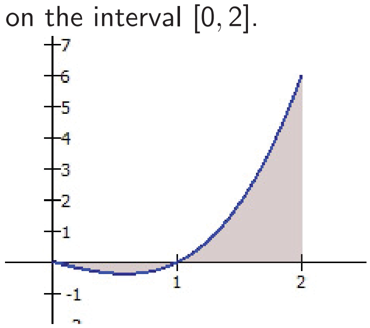
$$\int_0^5 h(x)\ dx$$



Suppose

$$f(x) = x(x^2 - 1)$$

on the interval $[0, 2]$.



- What is the value of $\int_0^2 f(x)\ dx$?
- What is the area of the shaded region?

Find the geometrical area between the curves

$$f(x) = x^2 - 2x + 1 \qquad\qquad g(x) = -x^2 + 4x - 3$$

Find the geometrical area between the curves

$$f(x) = x^3 - 8x^2 + 19x \qquad\qquad g(x) = 4x$$

Two cars start off together at time $t = 0$ minutes. Car A travels with velocity (in meters/minute)

$$A(t) = t^3 - 8t^2 + 19t$$

and Car B travels with velocity

$$B(t) = 4t.$$

How far ahead is Car A after 5 minutes?

5.4: Interpreting the Definite Integral

Suppose $v(t)$ is velocity (in mph) at time t (in hours). Consider the statement

$$\int_3^7 v(t)\ dt = 134.$$

- What are the units on 134?

- What is the meaning of the integral statement?

Suppose a 11-ft wide wall has height $h(x)$ (in ft) at a distance of x feet away from one end. Consider the statement

$$\int_0^{11} h(x)\ dx = 72.$$

- What are the units on 72?

- What is the meaning of the integral statement?

Suppose money is getting deposited into a bank account at a rate of $D(t)$ (in $/min) where t is minutes after midnight. Consider the statement

$$\int_{120}^{480} D(t)\ dt = 157.$$

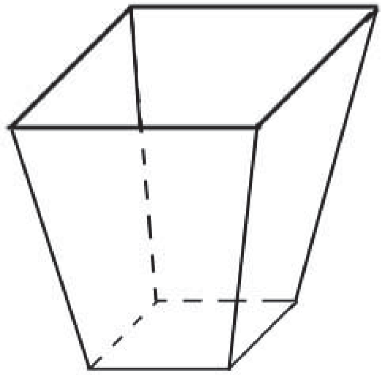
? What are the units on 157?

? What is the meaning of the integral statement?

? A rocket is accelerating at a constant rate of $a = 14$ m/s^2. How fast is it going after 10 seconds?

? A rocket is accelerating at $a(t) = 3t + 14$ m/s^2 at t seconds after launch. How fast is it going after 10 seconds?

A 15-inch tall wastebasket has a 8×8 inch square bottom and a 13×13 inch square opening at the top. What is the volume of the wastebasket?



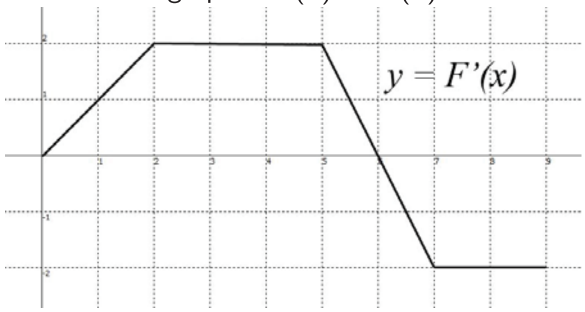
5.5: Fundamental Theorem of Calculus

Marginal Cost and Revenue

Ex: Suppose $MC(q) = 6q^2 - 16q + 70$ and fixed costs are \$500. Find cost of producing at $q = 20$.

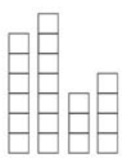
Ex: Suppose $MR(q) = 200 - 12\sqrt{q}$ and the revenue at $q = 16$ is \$2688. Find $R(49)$.

Consider the graph of $f(x) = F'(x)$:



- Determine $F(7) - F(0)$.
- Determine $F(9) - F(5)$.
- If $F(3) = -3$, find $F(0)$.
- If $F(3) = -3$, find $F(8)$.

5.6: Average Value



Ex: Compute the average value of $f(x) = x^2 + 1$ on the interval $[-1, 2]$.

Ex: Compute the average value of $g(t) = \sin t$ on the interval $[\pi, 3\pi/2]$.

Suppose the temperature (in $^\circ$F) over a given day was given by

$$T(t) = \frac{-15}{128}t^2 + \frac{39}{16}t + 52$$

at t hours past midnight.

- What was the high temperature?
- What was the low temperature?
- What was the average temperature?

Consider the function $f(x) = x^2 + 4x - 5$ on the interval $[1, 3]$.
Compute the average rate of change.

6.1: Antiderivatives

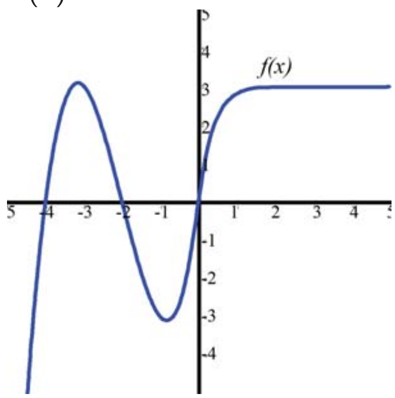
Numerical Antiderivatives

Given the following data about $f(x)$, approximate the values for the antiderivative $F(x)$.

x	0.0	0.4	0.8	1.2	1.6	2.0
$f(x)$	2	3	1	-2	-3	-5
$F(x)$	7					

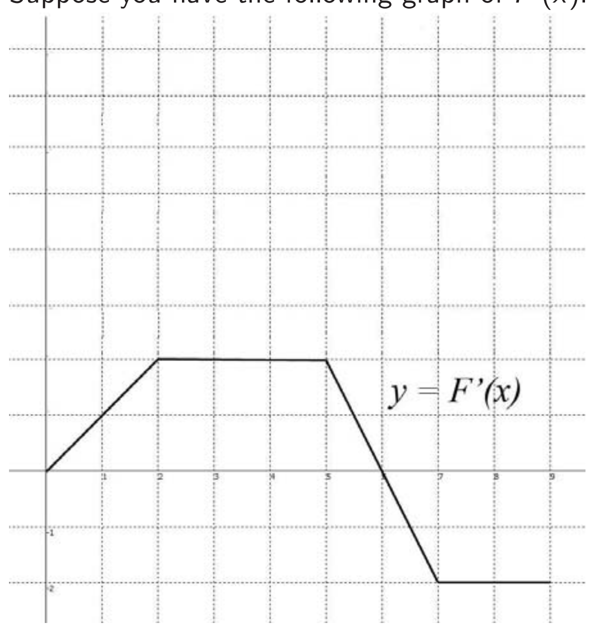
Derivative/Antiderivative Graphical Relationships

Suppose you have the following graph of $f(x)$ with antiderivative $F(x)$.



- Where is $F(x)$ increasing?
- Where is $F(x)$ concave down?
- Where is $F(x)$ positive?
- Describe $F(x)$ as it heads to infinity?

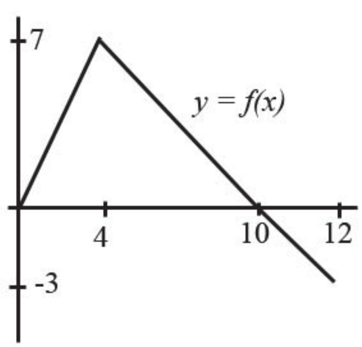
Suppose you have the following graph of $F'(x)$.



Sketch a rough graph of $F(x)$ assuming $F(0) = -2$.

6.2: Antiderivatives and the Indefinite Integral

Given the graph of $f(x)$:



Let $F(x) = \int f(x)\ dx$ with $F(0) = 9$.

- What is $F(4)$?
- $F(10)$?
- $F(12)$?

Definite vs. Indefinite Integral

Linearity Rule of Antiderivatives

Antiderivatives on the TI-89

Suppose $F'(y) = \frac{1}{y} + y + 1$

- Give the general form of $F(y)$.

- Give the specific $F(y)$ such that $F(1) = 2$.

Suppose velocity (in mph) of a car at time t (in hr) is given by

$$v(t) = e^{-t} + 3t^2.$$

If the car starts 4 miles west of home, how far away is the car at time t.

6.3: Using the Fundamental Theorem of Calculus

Ex: Suppose the velocity of a car is

$$v(t) = \sqrt{t}$$

at time t hours past noon. At 4pm the car is at mile marker 25.

- Where was it at noon?
- When does it reach mile marker 50?

Ex: Suppose $MC = \dfrac{q+2}{q^2+4q+8}$ and fixed costs are \$20. Give the cost function, $C(q)$.

Ex: Consider the function $y = 8x^3$. Find a value of b such that the area under the curve from 1 to b is exactly 17.

Ex: Inventory in a warehouse is modeled by

$$I(t) = \frac{400}{t+1}$$

at t days into the year. Determine when one could say that the average amount of inventory for the year to date was 100 units.

Ex: A 100 million barrel oil reserve is discovered and extraction begins at time $t = 0$ years. The rate of extraction is given by

$$r(t) = 10 - \frac{t}{10}$$

in millions of barrels/year. How long until the reserve is empty?

6.4: Consumer and Producer Surplus

Ex: You have an autographed picture of Justin Bieber...

Graphical View

Ex: Suppose

$$D: \ \ln(p/30) + .003q = 0$$

$$S: \ p - .02q = 5.$$

Find the surpluses and total gain from trade.

Ex: Find consumer surplus if demand is given by $p + 4q = 100$ and 10 units are sold.

Effect of Price Controls

Ex: Suppose $D: \;\; p = 400 - 3q$ and $S: p = 50 + 4q$.

? Determine the surpluses and total gain from trade.

? Suppose regulation adds an additional \$5 to the equilibrium price. Determine the surpluses and total gain from trade.

6.5: Present and Future Values of Income Streams

Future Value: Integral Point of View

Ex: Farm income throughout a year varies with the seasons. Suppose a farmer anticipates an income stream given by

$$S(t) = 300 + 150\sin(2\pi t)$$

(t measured in years) for the next 7 years. If he can invest at 4% determine the value accrued after 7 years.

Future Value: Differential Equation Point of View

Ex: Farm income throughout a year varies with the seasons. Suppose a farmer anticipates an income stream given by

$$S(t) = 300 + 150\sin(2\pi t)$$

(t measured in years) for the next 7 years. If he can invest at 4% determine the value accrued after 7 years.

Present Value

Ex: A wealthy investor has access to a 6% rate. A dying company is projected to produce income at $S(t) = 2500 - 100t^2$. Should she buy the company for \$6500?

Ex: Your company will need renovations valued at \$500,000 in 4 years and wants to start saving at a constant rate now in anticipation.

- If the company gets 3% on investments, at what constant rate should they save to achieve the required amount?

- If the company is only able to save at a rate of \$110,000 per year, what rate do they need to get on investments to achieve the required amount?