

# Math 280 Problems for October 8 17

## Pythagoras Level

**Problem 1:** A cloth bag contains a pool ball, which is known to be a solid ball. A second pool ball is chosen at random in such a way that it is equally likely to be a solid or a stripe ball. The ball is added to the bag, the bag is shaken, and a ball is drawn at random. This ball proves to be a solid. What is the probability that the ball remaining in the bag is also a solid?

**Problem 2:** Consider a trapezoid  $ABCD$  such that the sides  $AB$  and  $CD$  are parallel, and the diagonal  $BD$  is equal in length to side  $AD$ . Given that angle  $CBD$  measures  $30^\circ$  and angle  $DCB$  measures  $110^\circ$ , find the measure of angle  $ADB$ .

## Newton Level

**Problem 3:** Find the volume of the region of points  $(x, y, z)$  such that

$$(x^2 + y^2 + z^2 + 8)^2 \leq 36(x^2 + y^2).$$

**Problem 4:** Evaluate

$$\int_0^1 \frac{\ln(x+1)}{x^2+1} dx.$$

## Wiles Level

**Problem 5:** Let  $f(n) = 25^n - 72n - 1$ . Determine, with proof, the largest integer  $M$  such that  $f(n)$  is divisible by  $M$  for every positive integer  $n$ .

**Problem 6:** It is well known that the harmonic series,  $1/1 + 1/2 + 1/3 + 1/4 + \dots$ , diverges. Consider the depleted harmonic series (see below) which contains only terms whose denominator does not contain a 9 (in decimal representation). Does this series diverge or converge?

$$S = 1/1 + 1/2 + \dots + 1/8 + 1/10 + \dots + 1/18 + 1/20 + \dots + 1/88 + 1/100 + 1/101 + \dots$$