

## Written Exercises

### Question 2:

We have the following functions for the given graph.

$$\hat{y}_0 = 0$$

$$\hat{y}_1 = 2x - 2$$

$$\hat{y}_2 = \frac{x}{3} + \frac{4}{3}$$

$$\hat{y}_3 = 2x - 7$$

$$\hat{y}_4 = -\frac{5x}{3} + 15$$

$$\hat{y}_5 = 0$$

Where each layer is in the form

$$y_i = \sigma(w_i y_{i-1}^T + \beta_i)$$

and

$$\sigma(x) \triangleq \begin{cases} x & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

ReLU functions:

$$y_1 = \delta(\hat{y}_1 - \hat{y}_0) = \delta(2x - 2)$$

$$\begin{aligned} y_2 &= \delta(\hat{y}_2 - \hat{y}_1) = \delta\left(\frac{x}{3} + \frac{4}{3} - 2x + 2\right) \\ &= \delta\left(-\frac{5x}{3} + \frac{10}{3}\right) \end{aligned}$$

$$y_3 = \delta(\hat{y}_3 - \hat{y}_1) = \delta\left(x - 7 - \frac{x}{3} - \frac{4}{3}\right) \\ = \delta\left(\frac{5x}{3} - \frac{25}{3}\right)$$

$$y_4 = \delta(\hat{y}_4 - \hat{y}_3) \\ = \delta\left(-\frac{5x}{3} + 15 - 2x + 7\right) \\ = \delta\left(-\frac{11x}{3} + 22\right)$$

$$y_5 = \delta(\hat{y}_5 - \hat{y}_4) \\ = \delta\left(0 - \left(-\frac{5x}{3} + 15\right)\right) \\ = \delta\left(\frac{5x}{3} - 15\right)$$

We have  $Y = \delta(2x-2) + \delta\left(-\frac{5}{3}x + \frac{10}{3}\right) \\ + \delta\left(-\frac{11}{3}x + 22\right) + \delta\left(\frac{5}{3}x - 15\right)$

$$Y = 2 \delta\left(\frac{1}{2}(2x-2)\right) - \frac{10}{3} \delta\left(\left(-\frac{5}{3}x + \frac{10}{3}\right) - \frac{3}{10}\right) \\ + \frac{25}{3} \delta\left(\frac{3}{25}\left(\frac{5x}{3} - \frac{25}{3}\right)\right) \\ - 22 \delta\left(-\frac{1}{22}\left(-\frac{11x}{3} + 22\right)\right) \\ + 15 \delta\left(\frac{1}{15}\left(\frac{5x}{3} - 15\right)\right)$$



Therefore

$$y = \delta(2y_1) + \delta\left(-\frac{10}{3}y_2\right)$$

$$+ \delta\left(\frac{25}{3}y_3\right) + \delta(-22y_4) + \delta(15y_5)$$

$$\therefore w_2 = \begin{bmatrix} 2 \\ -10/3 \\ 25/3 \\ -22 \\ 15 \end{bmatrix}$$

$$\text{and } \beta_2 = 0$$

$$\text{For } \beta_1 = -1;$$

$$y_1 = \delta(x-1)$$

$$y_2 = \delta\left(\frac{1}{2}x-1\right)$$

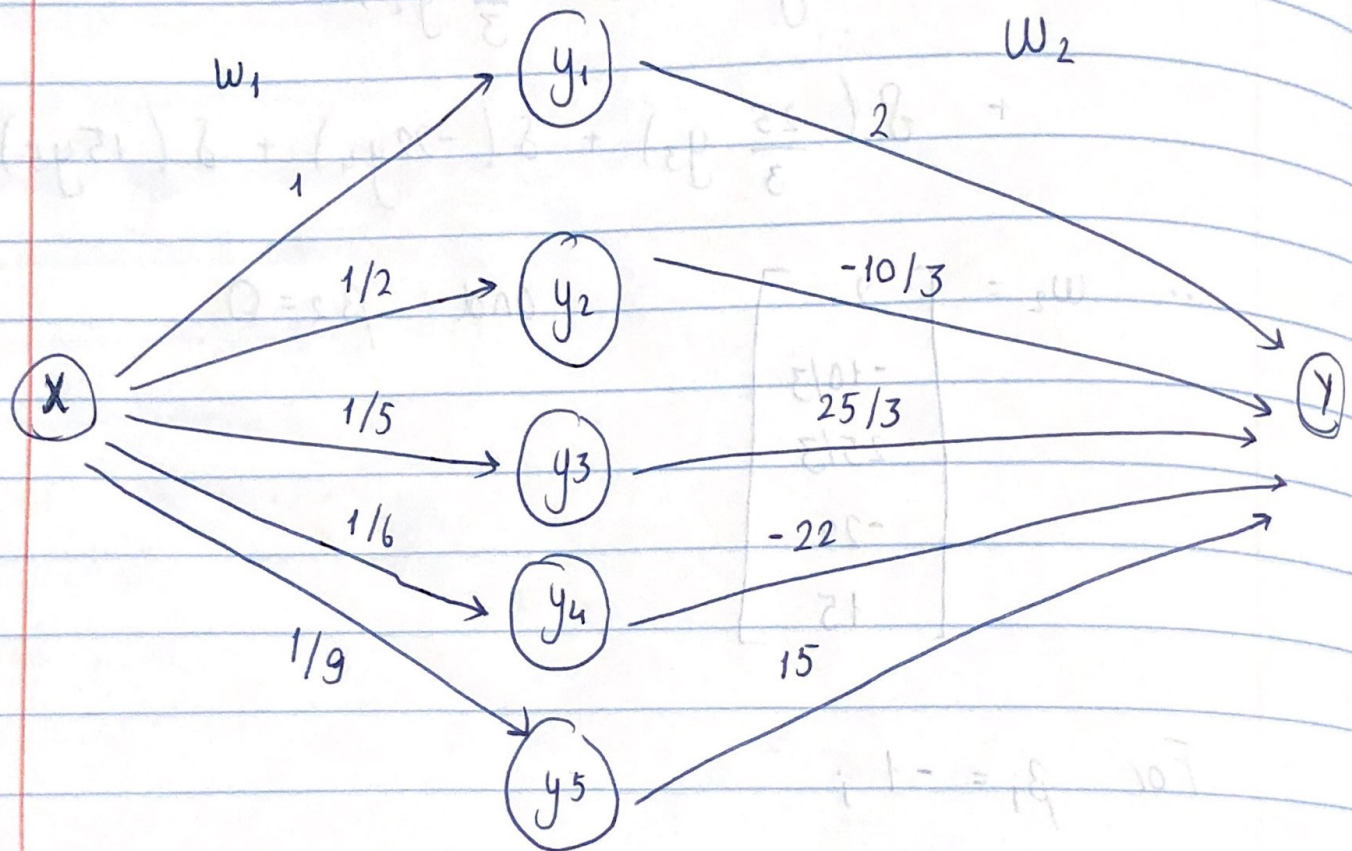
$$y_3 = \delta\left(\frac{1}{5}x-1\right)$$

$$y_4 = \delta\left(\frac{1}{6}x-1\right)$$

$$y_5 = \delta\left(\frac{1}{9}x-1\right)$$

$$\text{and } w_1 = \begin{bmatrix} 1 \\ 1/2 \\ 1/5 \\ 1/6 \\ 1/9 \end{bmatrix}$$

The Neural Network:



Hidden  
Layer

We need 5 neurons in 1 hidden layer.