Up to speed in Statics?

Exercises for the revision of Statique I throughout the Civil Engineering curriculum

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Version 1.0

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Contents

Imprint

Please cite as: Beyer K, Vincens B, Tomić I (2021). *Up to speed in Statics? Exercises for the revision of Statique I throughout the Civil Engineering curriculum*, Exercise booklet and solutions, Version 1.0, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland. https://github.com/eesd-epfl/Statique_I.git.

Foreword

The first course of structural mechanics in any civil engineering curriculum introduces important principles, such as the free body diagram, static equilibrium and the internal forces in structural members, and then applies these principles to statically determinate systems. At EPFL, these concepts are taught in the course Statique I in the second semester of the Bachelor curriculum (BA2). This introductory course lays the foundation to many other civil engineering courses, and engineers in practice or research who design or analyse any type of structure need to have these techniques and concepts at their fingertips.

The idea behind this booklet is twofold: First, we want to reinforce these first principles taught in Statique I throughout the curriculum by applying them to systems treated in the BA3-6 courses that build on Statique I. The presented exercises can serve as a basis for self-study when preparing for a new course or as a homework assignment in week 1 of this course. Second, this booklet is also intended to give Statique I students an idea of how the principles they are learning will be used in later courses and in engineering practice. For these purposes, we have put together a small set of exercises for each course that repeat important concepts and introduce others. In the future, we plan to expand this booklet with exercises on statically indeterminate systems, covered in Statique II.

We would like to thank Prof. Dr Alain Nussbaumer for the exercises on steel structures, Dr Olivier Burdet for his input in general and for the exercises on concrete structures and bridges in particular, and Dr Giovanni De Cesare for his help with the hydraulics exercises.

This booklet is accompanied by a solution manual containing example solutions for all problems. To facilitate re-usage of the material, all source files (Latex files and images) are shared. All material including the pdf-file of this document can be downloaded from the following GitHub repository: https://github.com/eesd-epfl/Statique_I.git.

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Lausanne, June 9, 2021

1 Structural mechanics (BA3)

1.1 Differential equations of equilibrium for beams

Derive the following differential equation of equilibrium for a straight beam in 2D:

$$\frac{dV}{dx} = -q(x)$$

Begin by expressing the equilibrium of the vertical forces for a fragment of the beam of length dx.

1.2 Internal force diagrams of simply supported and cantilever beams

For the four systems shown in Figure 1, sketch first the bending moment diagrams and then the shear force diagrams. Compute the characteristic values for both diagrams (local maximum values, values at the extremities).

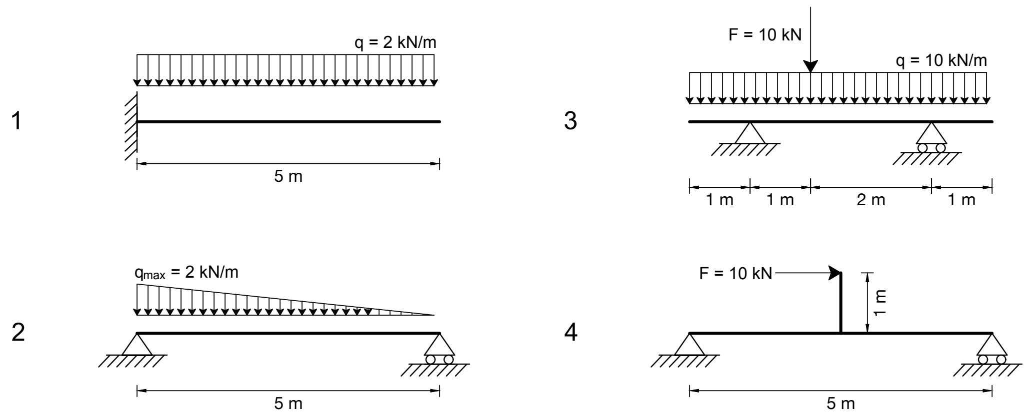


Figure 1: Simply supported and cantilever beams.

1.3 Portal frames

The same portal frame is supported in three different ways.

1. Compute the reaction forces for the three systems in Figure 2 as a function of F and L;

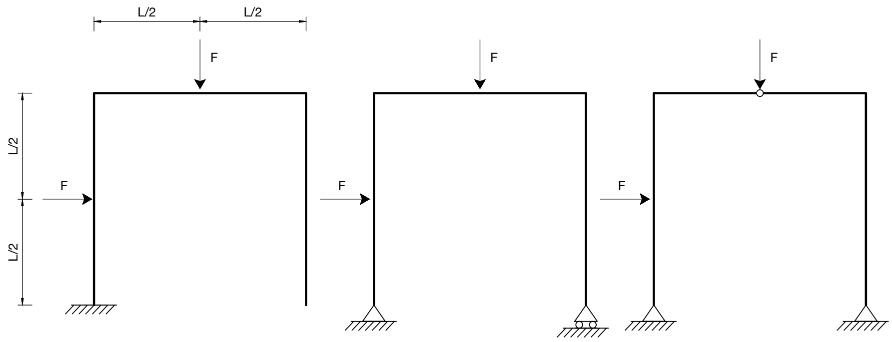


Figure 2: Portal frame with different supports.

2. Sketch the bending moment diagrams, and compute the characteristic values;
3. Sketch the deformed shapes of the portal frames;
4. Using the results from the previous questions, which support system seems most appropriate and economical?

1.4 Inverse problem

In Figure 3, a beam was analysed under a set of loads to obtain the given bending moment diagram (the normal stress is null).

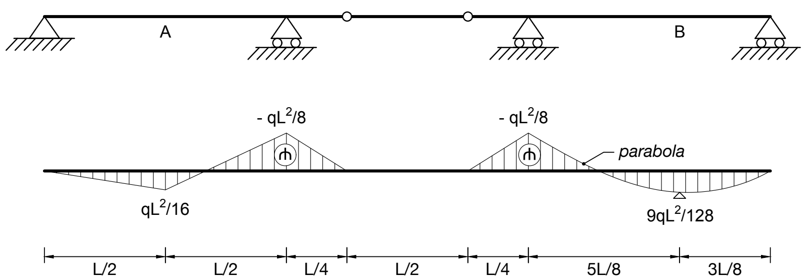


Figure 3: Cantilever beam and bending moment diagram.

1. Draw the shear force diagram, and compute the peak values;
2. Using the shear force diagram, determine the reaction forces and external loads;

3. Sketch the deformed shapes;
4. Using the virtual displacement theorem, check the bending moment values at points A and B.

2 Statique II (BA4)

2.1 Mechanisms, statically determinate and statically indeterminate systems

For each structure in Figures 4 and 5, draw the static system, and determine if it is statically determinate, indeterminate or a mechanism. If the system is statically inderterminate, indicate the degree of hyperstaticity. If the system is a mechanism, sketch the rigid body motion that the mechanism can undergo.

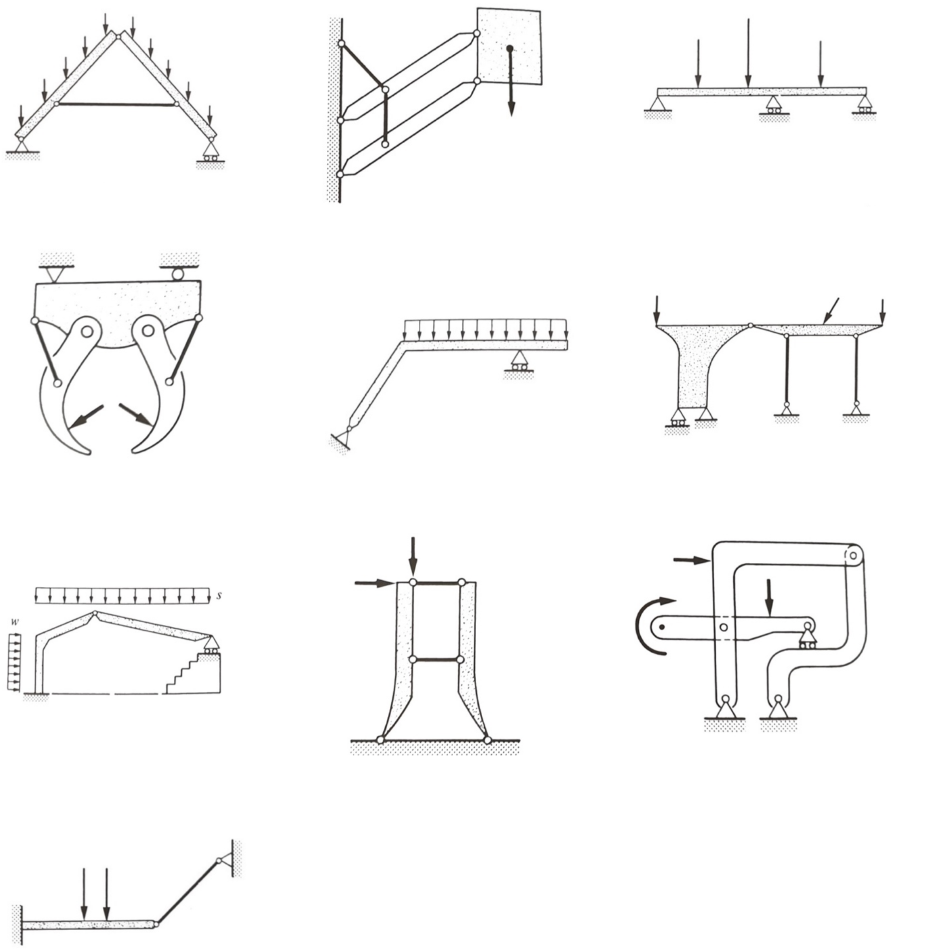
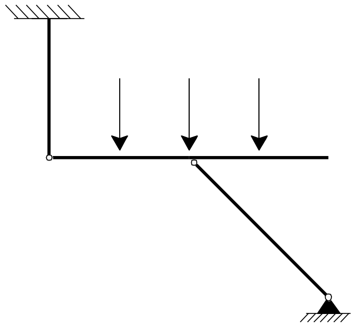
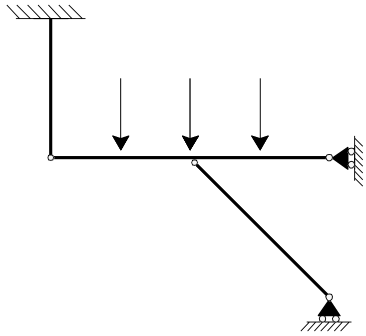
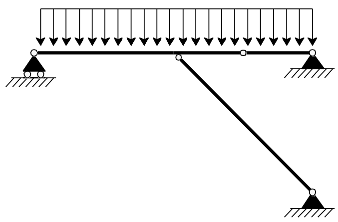
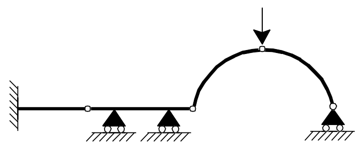
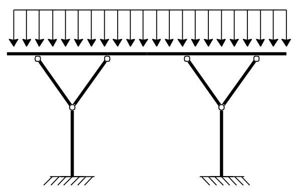
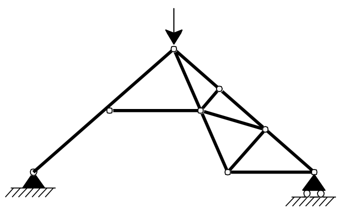
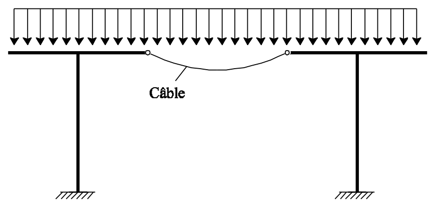


Figure 4: Set of mechanisms, statically determinate and statically indeterminate systems[1]

[1]From Frey, F. (2005). Statique appliquee (TGC vol. 1). EPFL Press.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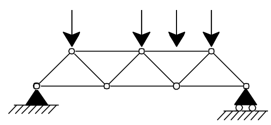
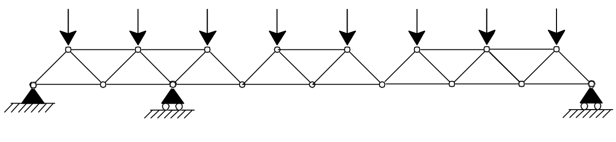
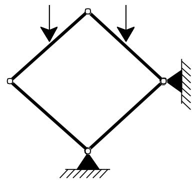

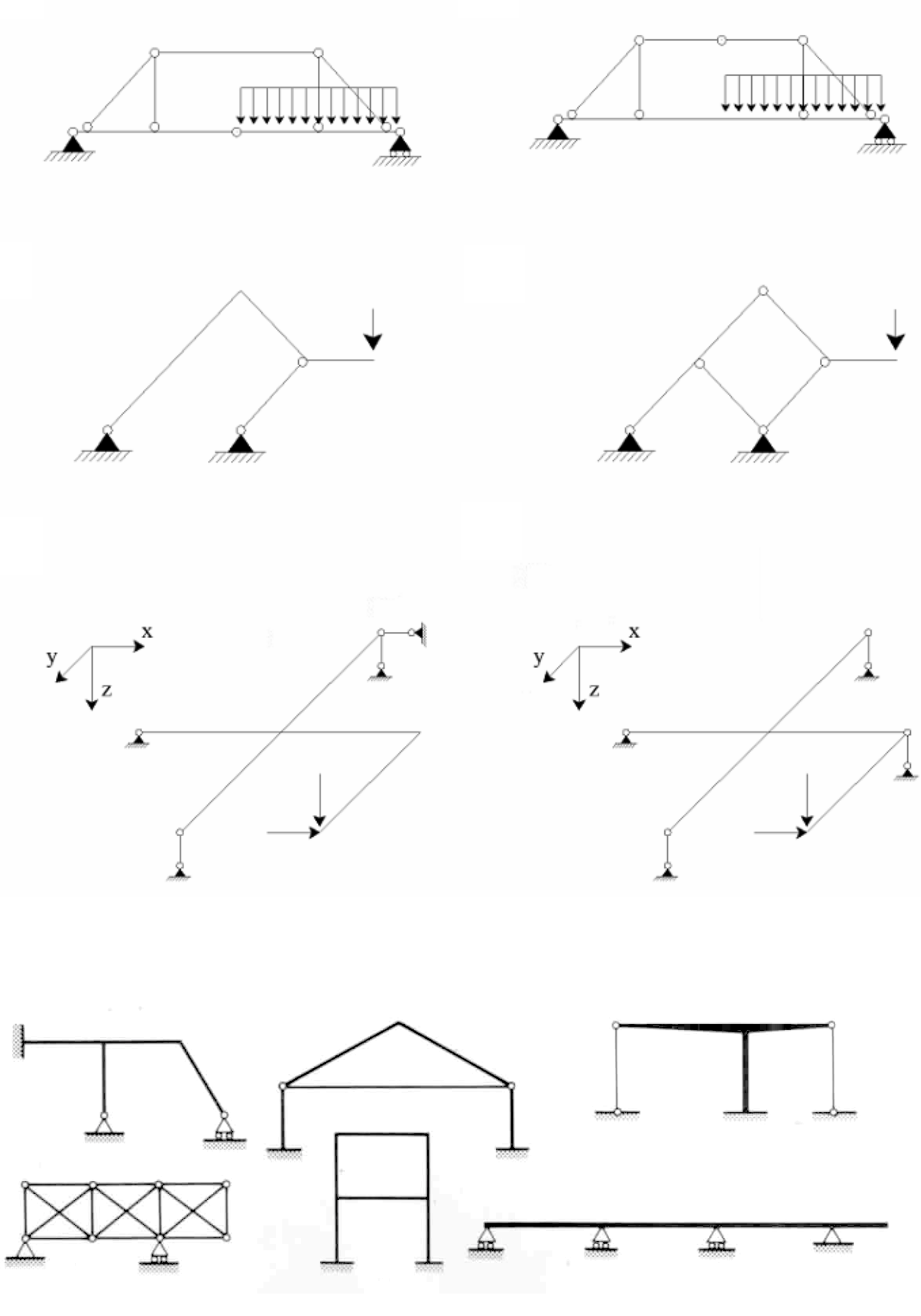

Figure 5: Set of mechanisms, statically determinate and statically indeterminate systems

2.2 Truss structures

Figure 6 shows three truss structures.

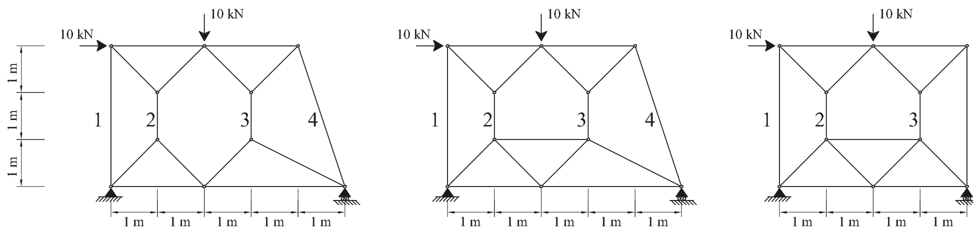

Figure 6: Three truss structures

1. Determine which of the three structures is isostatic.
2. For the isostatic structure, compute the axial force in bars 1 to 4.

3 Hydraulics (BA6)

Pour rappel, la forme de la poussée hydrostatique contre une paroi est donnée à la figure 7. Note aussi que, pour une profondeur donnée, la pousée hydrostatique est la même dans toutes les directions.

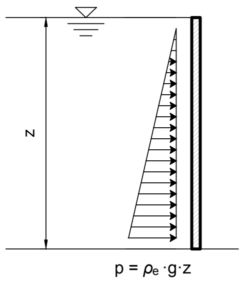

Figure 7: Poussee hydrostatique

3.1 Trappe séparant deux réservoirs[2]

La trappe de la figure 8 sépare deux réservoirs contenant deux liquides différents.

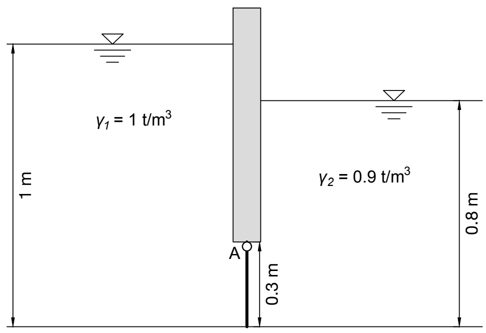

Figure 8: Deux réservoirs séparés par une trappe

1. Déterminer la poussée qui s'applique de chaque côté de la trappe.
2. Calculer l'emplacement et la valeur de la résultante des poussées exercées directement sur la trappe.
3. Calculer le moment exercé en tête de la trappe (point A).

3.2 Tunnel immergé[2]

Un tunnel rectangulaire et un tunnel cylindrique en béton sont immergés dans l'eau (eau: $\gamma_e = 10\ kN/m^3$, béton: $\gamma_b = 25\ kN/m^3$). Ils sont maintenus par des paires de câbles disposés tous les 6 m. La surcharge admise dans les tunnels est de 10 kN/m.

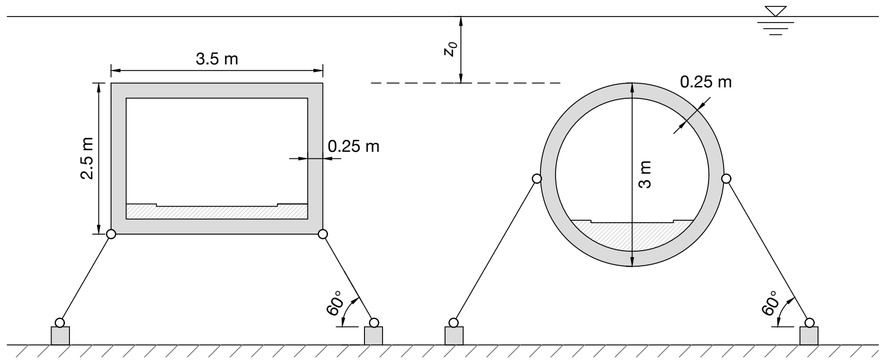

Figure 9: Tunnels rectangulaire et cylindrique immergés

Pour chacun de ces deux tunnels:

1. Dessiner la distribution de la poussée hydrostatique appliquée sur le tunnel.
2. Déterminer l'orientation et la valeur de la résultante de la poussée hydrostatique qui s'applique sur le tunnel. Cette force est aussi appelée "Poussée d'Archimède".
3. Representer dans un schéma toutes les forces qui agissent sur le tunnel (schéma de corps libre). Calculer la force reprise par chaque câble.

3.3 Barrage de la Grande-Dixence

Un barrage poids résiste à la pression exercée par l'eau grâce à son poids propre. Celui-ci doit être suffisant pour empêcher le renversement de l'ouvrage. Le barrage de la Grande-Dixence, situé dans le Val des Dix dans le Valais et construit dans les années 1950 est le plus haut barrage poids du monde.

[2] adapté de De Cesare, G. (2013). Support de cours: Hydraulique I et II. HEIG-VD.

Supposons que la section du barrage soit un triangle tel que représenté à la figure 10 et que le glissement à la base du barrage est entravé.

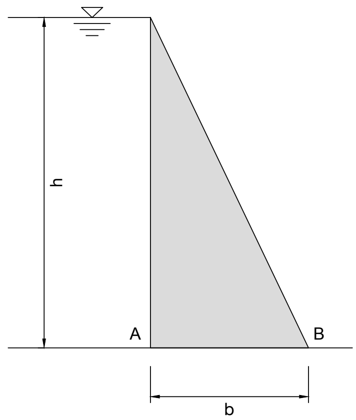

Figure 10: Section simplifiee du barrage poids

1. Déterminer la pression hydraulique qui est exercée sur le barrage ($\gamma_e = 10\ kN/m^2$)

On fait l'hypothèse que la pression de l'eau dans les pores du sol sous le barrage suit une distribution triangulaire entre le point A (pression hydrostatique) et le point B.

2. Déterminer le poids du barrage par mètre de longueur et l'emplacement de sa résultante ($\gamma_b = 25\ kN/m^2$)

3. Calculer le moment au point B et déterminer l'épaisseur minimale b en fonction de h nécessaire pour éviter le renversement du barrage

4 Structures en métal (BA4)

4.1 Cadre avec ferme sous-tendue[3]

Le système statique de la figure 11 est un exemple typique de cadre en acier avec ferme sous-tendue par un tirant. Le tirant est généralement constitué d'un câble en acier ne reprenant donc que des efforts en traction.

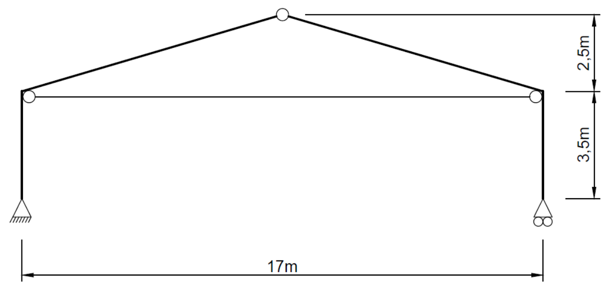

Figure 11: Cadre en acier

1. Démontrer que ce système est isostatique. Que se passe-t-il si on supprime le tirant?

Une charge de neige répartie de 2 kN/m est appliquée uniformément sur la toiture.

2. Calculer les réactions d'appuis
3. Calculer les diagrammes MVN pour chaque élément de la structure
4. Quelles remarques peut-on faire sur ce type de système avec tirant?

Lors d'un évènement on décide d'accrocher un puissant spot lumineux de 100 kg au milieu du tirant. La flèche mesurée est alors de 5 cm. Pour cet exemple le système statique est légèrement modifié (voir figure 12) : le cadre est simplement appuyé sur ses deux côtés, ce qui est plus représentatif de la réalité.

5. Montrer que le système est hyperstatique. Quel est le degré d'hyperstaticité?

[3]The exercices and illustrations were developed by and used with the authorisation of Prof. Dr Alain Nussbaumer

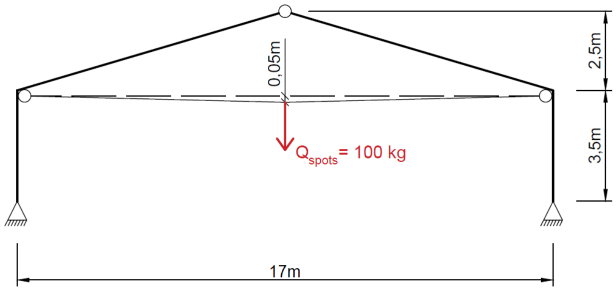

Figure 12: Cadre avec ajout d'une charge au milieu de la travée

6. Calculer les efforts dans le tirant
7. Pourquoi sommes-nous obligés de considérer deux appuis fixes?
8. Peut-on tout de même calculer les efforts dans la structure? Si oui, expliquer pourquoi et effectuer le calcul
9. Que se passe-t-il si la déformation tend à être nulle au point d'application de la charge?
10. Quelles conclusions peut-on tirer de ces résultats?

4.2 Cadre avec pont-roulant[3]

Le système statique ci-après (figure 13) décrit un exemple typique de halle industrielle que l'on observe de nos jours. Celle-ci est constituée d'un cadre simple sur lequel viennent s'encastrer des corbeaux sur chaque colonne, lesquels supportent les rails du pont-roulant.

1. Démontrer que ce système est isostatique. Que se passe-t-il si l'on supprime le pont-roulant?

Une charge ponctuelle de 45 kN correspondant à une charge soulevée par le pont roulant est appliquée comme dans la figure 14.

2. Calculer les réactions d'appuis
3. Calculer les efforts dans chaque élément de la structure (M,V et N) et dessiner les diagrammes correspondants

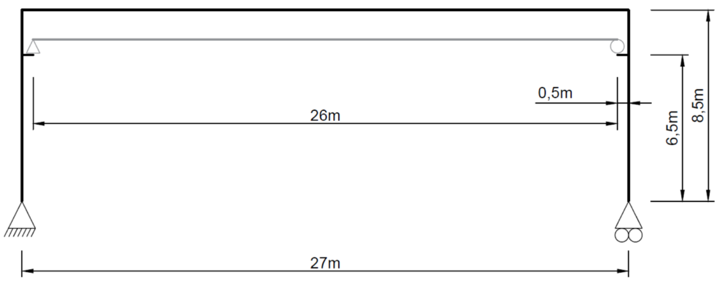

Figure 13: Cadre d'une halle industrielle

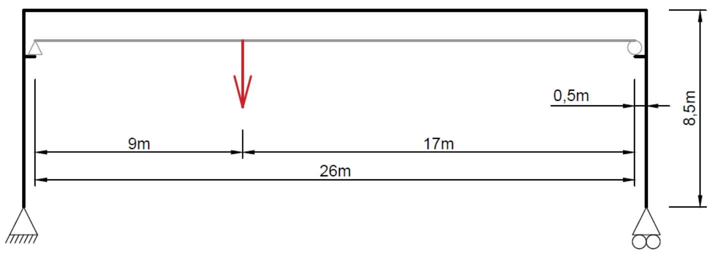

Figure 14: Cadre avec charge soulevee par le pont roulant

Un fort vent créé une charge répartie négative sur toute la structure. Pour cet exercice, on ne considèrera que l'effet sur la toiture, soit une dépression de 1.5 kN/m comme montré dans le schéma de la figure 15.

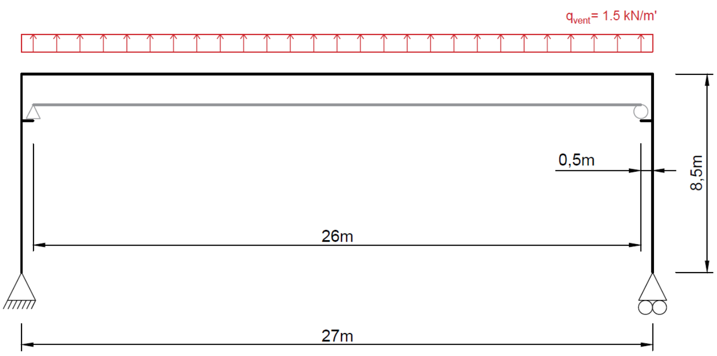

Figure 15: Cadre avec effet du vent

4. Calculer les réactions d'appuis
5. Calculer les efforts dans chaque élément de la structure (M,V et N) et dessiner les diagrammes correspondants
6. Quelles conclusions peut-on tirer de ces résultats?

Suite à une manœuvre avec le pont roulant, un freinage d'urgence de celui-ci est enclenché et engendre une forte décélération. Cette décélération peut être représentée par une charge latérale de 10 kN combinée à la charge verticale de la question 2 (45 kN).

7. Calculer les réactions d'appuis
8. Calculer les efforts dans chaque élément de la structure (M,V et N) et dessiner les diagrammes correspondants
9. Quelles conclusions peut-on tirer de ces résultats?

4.3 Sporthalle Buchholz[3]

La Sporthalle Buchholz est une halle avec trois terrains de sport dont la construction s'est terminée en 1998. La surface au sol du bâtiment est de 2260 m^2. Le poids total de la structure porteuse en acier est de 145 t. La structure est composée de cadres à sections variables. La portée principale de la structure au-dessus des terrains est de 27 m tandis que la portée secondaire qui enjambe le foyer et la promenade mesure 14 m. La plus grande colonne, à droite sur la figure 16, mesure 7.50 m de hauteur. La colonne centrale mesure 3.60 m et la colonne de gauche 4.80 m. Une rotule est située au 1/6 de la portée principale.

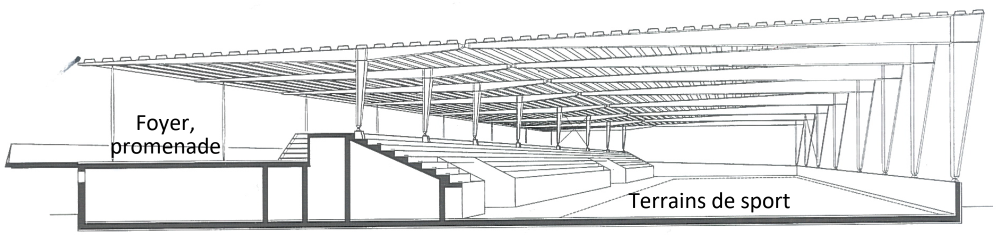

Figure 16: Perspective de la salle de sport

1. Définir le système statique de cette structure. On négligera les sections à inertie variable ainsi que la pente du toit

2. Démontrer que ce système est hyperstatique. Quel est son degré d'hyperstaticité?

La structure est soumise à une charge de neige répartie de 2 kN/m appliquée sur la toiture. La réaction d'appui verticale de la colonne centrale est égale à 45.20 kN.

3. Cette donnée supplémentaire est-elle suffisante pour calculer les efforts? Pourquoi?

4. Calculer les efforts dans la structure en utilisant la réaction d'appui verticale de la colonne centrale et en sachant que la réaction d'appui horizontale de la colonne de gauche est égale à 0 (cette colonne est "pendulaire" ou "bi-articulée")

La charge de neige est cette fois appliquée sur la moitié droite de la toiture. La partie à gauche de la toiture n'est soumise qu'à la moitié de la charge de neige, soit 1 kN/m. Ce chargement est caractéristique des cas de charges donnés dans les normes suisses SIA pour le dimensionnement des structures. Pour simplifier les calculs, on donne cette fois les réactions d'appuis verticales dans les trois colonnes (de gauche à droite): $4,37\ kN$, $30,41\ kN$ et 26.72 kN.

5. Calculer les efforts dans la structure

6. Que peut-on conclure à partir de ces résultats et de ceux obtenus à la question 4?

7. Suite à des problèmes d'étanchéité, des techniciens devront déblayer la neige sur la moitié gauche de la structure pour pouvoir travailler. Suite aux résultats obtenus précédemment, quelles conclusions peut-on déjà faire?

8. Selon vous quelles sont les conséquences de la simplification du système par le choix des inerties constantes? Cette simplification vous semble-t-elle correcte?

5 Structures en béton (BA5)

5.1 Static analysis of a frame

The concrete frame of Figure 17 is part of an industrial building. It supports the loads applied by snow, wind, an overhead crane and machines propped on the roof.

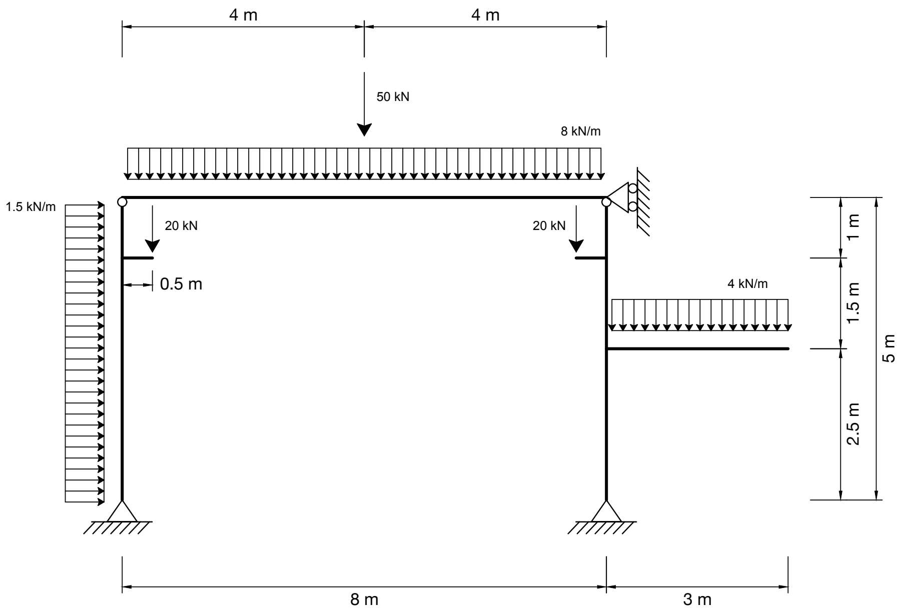

Figure 17: Statically determinate frame with applied loads.

1. Draw the NVM diagrams, and indicate all the characteristic values;
2. Sketch the deformed shape of the frame.

With a concrete structure, it is sometimes simpler to avoid hinges, making the structure monolithic. In this case, the frame becomes statically indeterminate, as shown in Figure 18. The frame is subjected to the same loads as its statically determinate counterpart.

3. Qualitatively, sketch how the moment diagram and the deformed shape would change if we remove the two hinges;
4. Overall, is this choice better than the previous structural system shown in Figure 17?

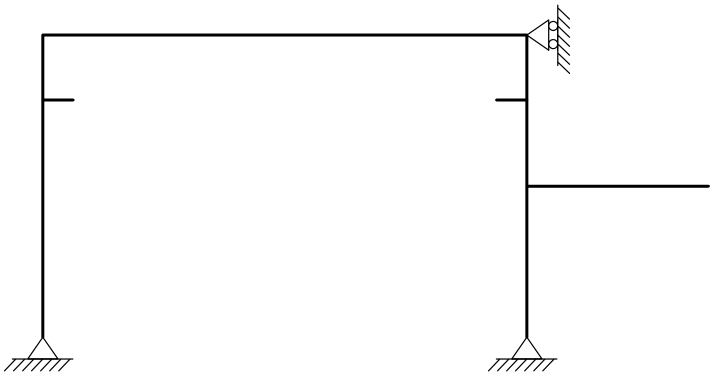

Figure 18: Statically indeterminate frame.

5.2 Simplified analysis of a concrete slab

The following slab is the floor of a multi-storey residential building. At the centre of the floor is an opening to accommodate a staircase. The floor is supported by two walls that leave the slab free to rotate. The floor is subjected to a uniformly distributed load of 4 kN/m^2, which already includes the weight of the floor. To design this concrete slab by hand, it can be modelled as a one-way spanning slab and approximated as a beam. We want to estimate the internal forces in the hatched slice of the slab.

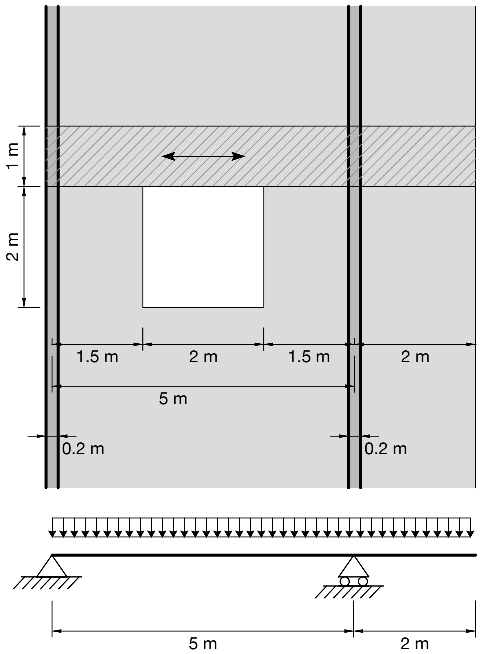

Figure 19: Concrete slab and simplified static system for the hatched strip.

First, ignoring the presence of the opening, the slab can be reduced to a statically determinate beam, as illustrated in Figure 19.

1. Calculate the moment diagram of the slab and the reactions on each wall for a load of 4 kN/m^2.

To have a more realistic approximation that accounts for the presence of the opening, we can use Hillerborg's strip method. This method consists of separating the slab into beams (called strips), which can either lean on the walls or onto other strips. Here, we make a strip on either side of the opening that leans onto two supporting strips that in turn lean on the walls. The part of the slab away from the opening functions as in the previous model.

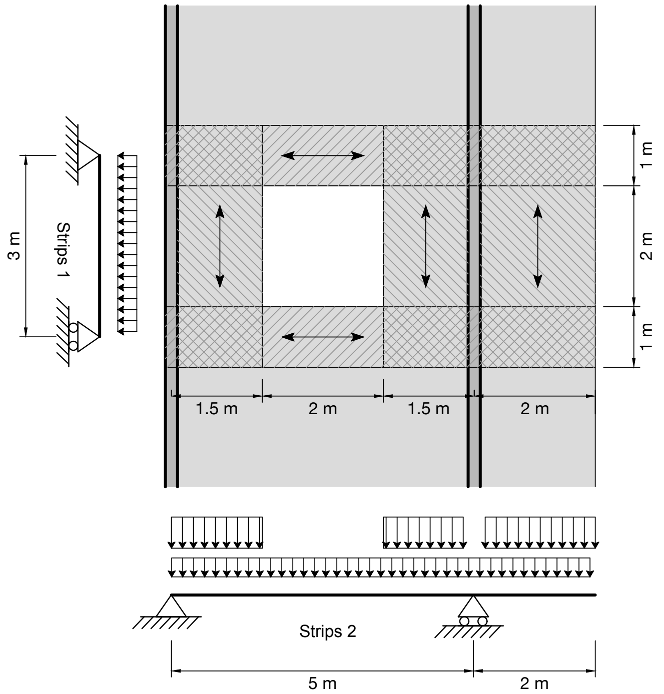

Figure 20: Concrete slab modelled based on Hillerborg's strip method.

2. Calculate the reactions of Strip 1 that are passed to Strip 2. The reaction can be uniformly distributed on the leaning surface.

3. Calculate the moment diagram of Strip 2 and the reactions on each wall.

5.3 Strut-and-tie models

Strut-and-tie models (or truss models) are used to approximate how forces travel through a reinforced concrete structure. Once a truss model is established, the internal forces can be calculated exactly like for a truss system. Struts are truss elements that are in compression; the forces in the struts are carried by the concrete. Ties are truss elements in tension, and the forces in the ties are used to design the reinforcement bars. The strut and tie forces can be calculated efficiently and precisely using graphic statics or by analytically formulating the nodal equilibrium. For the three following models in Figure 21, calculate the internal forces.

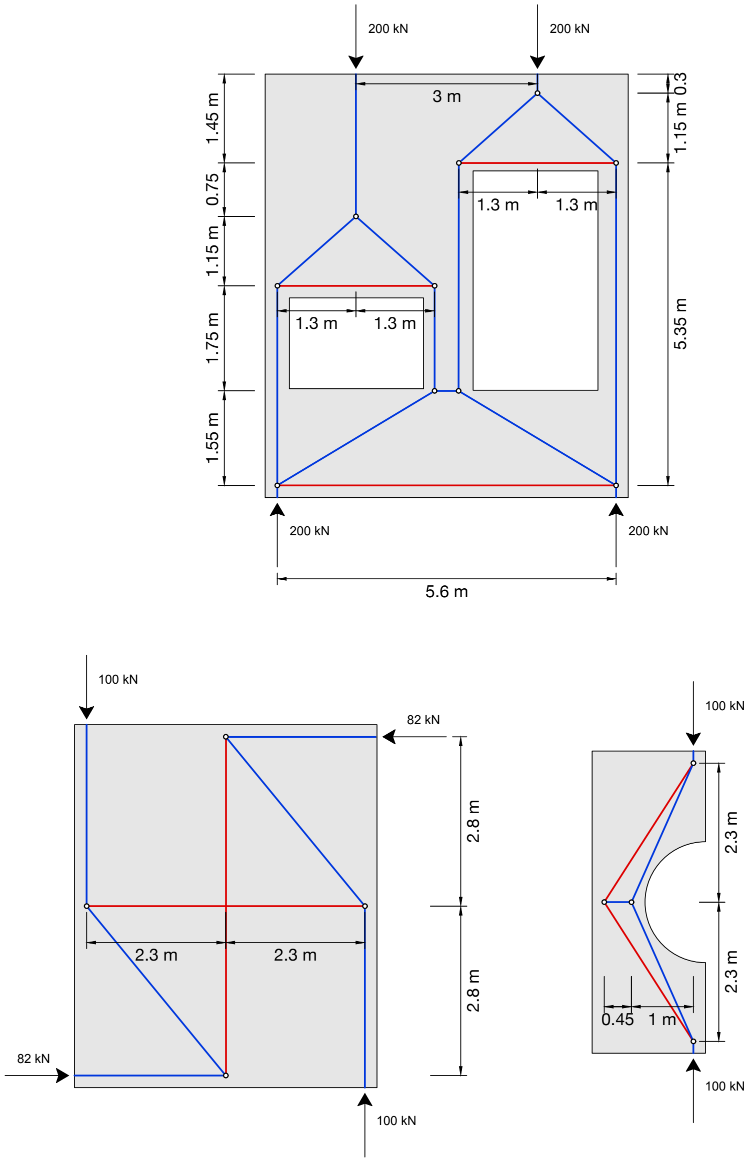

Figure 21: Truss models.

6 Conception des ponts (BA6)

6.1 Static systems of bridges

For the following existing bridges, find a suitable static system, and determine the degree of hyperstaticity of the structure.

Kirchenfeldbrücke, Berne, 1883

This arch bridge spanning the Aar river in the city centre of Bern is one of the most iconic bridges of Switzerland. It is now used only by pedestrians and public transport[4].

This bridge can be separated into several distinct structures. Consider the supports on the concrete piers to be freely sliding.

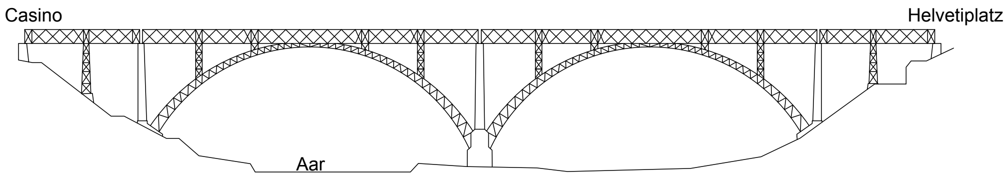

Figure 22: Kirchenfeldbrücke.

Pont de la Luette, Canton du Valais, 1962

This prestressed concrete bridge is located in the Swiss Alps and was constructed as part of new road infrastructure for tourists. It was designed by the well-known Swiss engineer Alexandre Sarrasin.

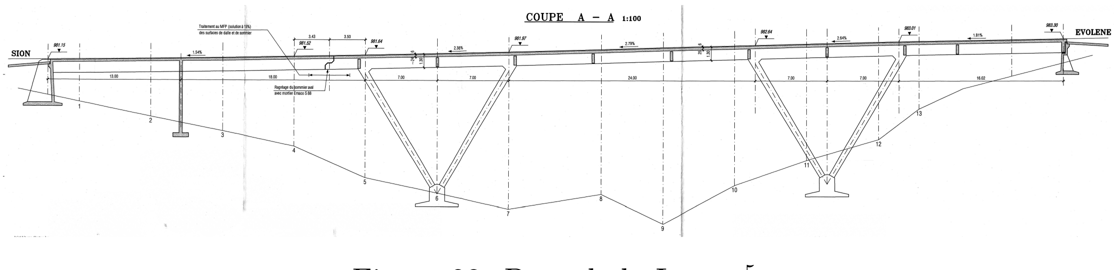

Figure 23: Pont de la Luette[5].

[4]See https://de.wikipedia.org/wiki/Kirchenfeldbr%C3%BCcke.

[5]With the authorisation of Jacques Rudaz © Etat du Valais, DMTE-INFRA cellule ouvrage d'art.

Waldshut–Koblenz Rhine Bridge, Germany, 1859

This bridge supports a single railway track used by passenger trains to this day. It crosses the Rhine and the border between Switzerland and Germany. It is one of the few preserved lattice truss bridges[6].

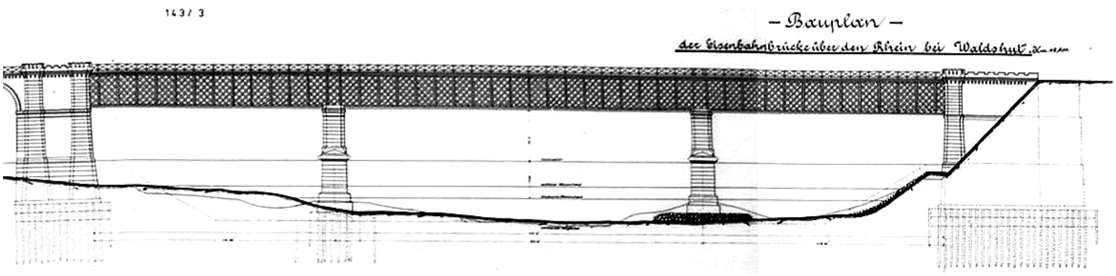

Figure 24: Waldshut–Koblenz Rhine Bridge[7].

Pont Eric-Tabarly, Nantes, France, 2011

This stay-cable bridge was built to cross the river Loire in the city of Nantes. It was prefabricated in its entirety in Belgium[8].

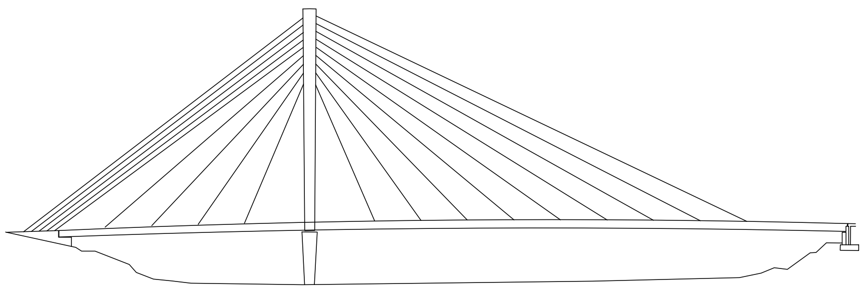

Figure 25: Pont Eric-Tabarly.

Mainbrücke Haßfurt, 1867

This cantilever bridge was designed by the well-known German engineer Heinrich Gerber. It supports rail traffic[9].

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[6] See https://en.wikipedia.org/wiki/Waldshut%E2%80%93Koblenz_Rhine_Bridge.

[7]
Retrieved from https://commons.wikimedia.org/wiki/File:Wp-plan.jpg.

[8] See https://fr.wikipedia.org/wiki/Pont_%C3%89ric-Tabarly.

[9] See https://structurae.net/fr/ouvrages/mainbrucke-hassfurt-1867

The trusses can be modelled as beams.

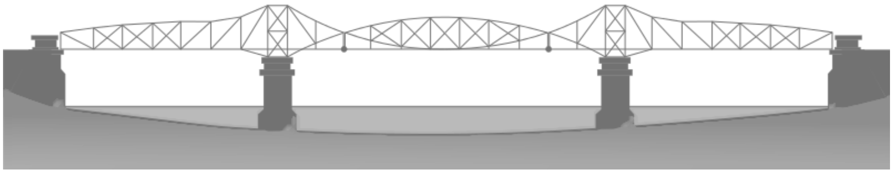

Figure 26: Mainbrücke Haßfurt[10].

Pont de la Dala, Canton du Valais, 1989

This 210–meter-long portal bridge carries road traffic. Its construction was remarkable: the struts were first rotated into place from a vertical position, and the deck was afterwards launched into place on top of the struts[11].

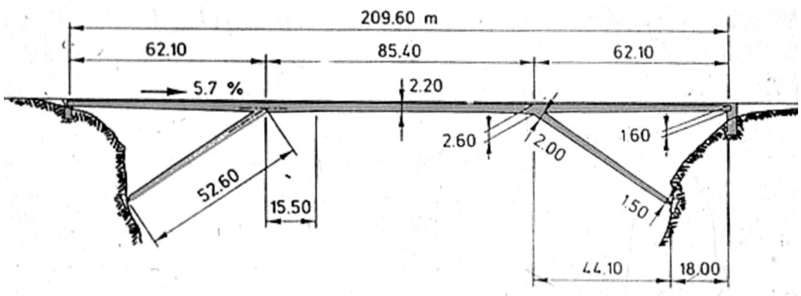

Figure 27: Pont de la Dala[12].

6.2 Longitudinal design

The Pont sur le Main bridge in Figure 28 was designed by the engineer H. Gerber in 1867, and it still supports rail traffic to this day. Truss bridges such as this one can be first analysed using equivalent beams to obtain reduced internal forces as presented below the bridge diagram in

[10] © Pechristener. CC BY-SA 3.0.
Retrieved from https://commons.wikimedia.org/wiki/File:Mainbr%C3%BCcke_Hassfurt.svg.
[11] See https://structurae.net/fr/ouvrages/pont-des-gorges-du-dala.
[12] With the authorisation of Dr Olivier Burdet. © IBETON 2021.
Retrieved from https://ibeton.epfl.ch/photos/PlancheContact.asp?II21i21-22.jpg.

Figure 28. In a second step, each truss bar can be individually modelled to obtain the axial forces used to design them.

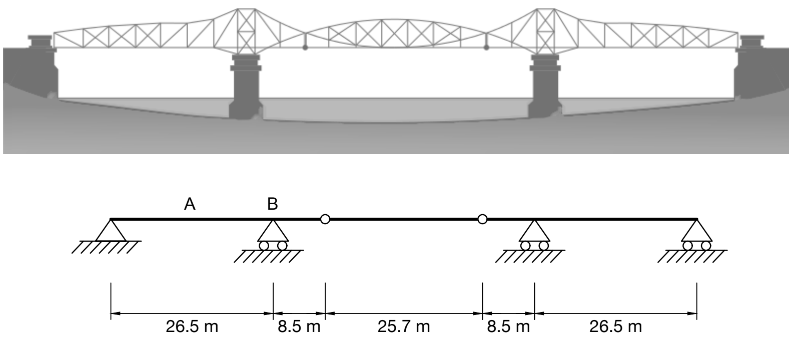

Figure 28: Mainbrücke, Haßfurt and its corresponding model.

1. Using the simplified beam model, draw the influence lines for the moment at points A and B.

For the design of rail bridges, two load models are used, as indicated in Figure 29. To calculate the internal design forces, the most disadvantageous model has to be placed in the most disadvantageous position on the bridge. When the loads are favourable, they should be neglected.

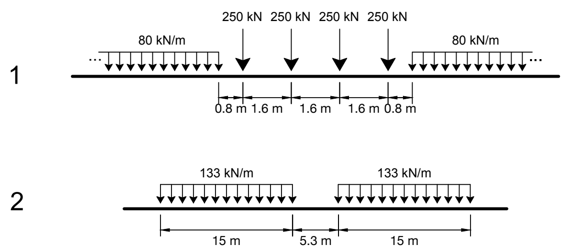

Figure 29: Load models used for the design of rail bridges.

2. Determine the defining load model and its position to maximize the moment at points A and B.

The middle section of the bridge between the two hinges is a simply supported beam that sits on the extremities of the two external beams of the bridge. The truss beam and the applied external loads are shown in

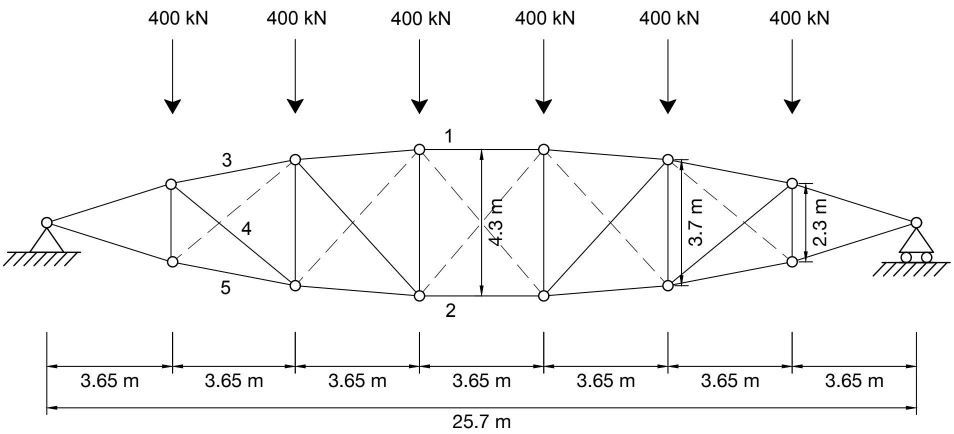

Figure 30: Central truss beam of the bridge.

Figure 30. X-diagonals render the truss statically indeterminate, though we can conservatively simplify the system into a statically determinate system by ignoring the diagonals that are in compression, which is a reasonable assumption because they are prone to buckling." (dotted lines).

3. Calculate the internal forces in members 1 and 2;
4. Using graphic statics or nodal equilibrium, calculate the internal forces in truss bars 3, 4 and 5.

6.3 Design of a cross-section

A typical cross-section for a concrete road bridge is given below. For a preliminary design of the slab of the box-girder, we can use a statically determinate beam model as shown in Figure 31.

1. Draw the influence lines of the mid-span moment, of the moment at the supports and of the shear force at the right-hand side of the left support;
2. Express the weight of the slab by a distributed load, and include this load in the model as a uniformly distributed load. The unit weight of concrete is 25 kN/m^3. An additional load of 3 kN/m^2 accounting for the weight of the asphalt also needs to be applied. Consider the load for a slice of the slab that is 1 m wide.

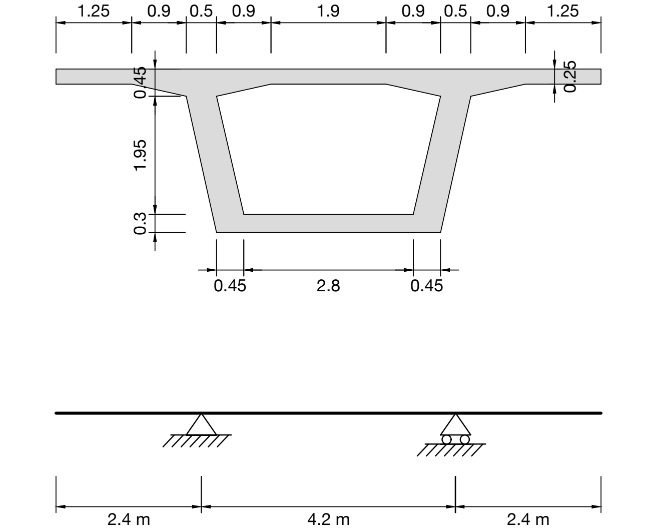

Figure 31: Cross section of the box-girder bridge.

To design the cross-section of the road bridge, traffic loads are applied along the traffic lanes, where each traffic lane has a width of 3 m. On each lane, a uniformly distributed load and two concentrated axle loads of value $Q/2$ are applied. The positions of the lanes are interchangeable, and the concentrated loads can be placed freely within the bounds of the lane, keeping the distance between the two loads of a lane always at 2 m. The loads of a lane can be removed if the load is favourable. An example of load placement is given below in Figure 32.

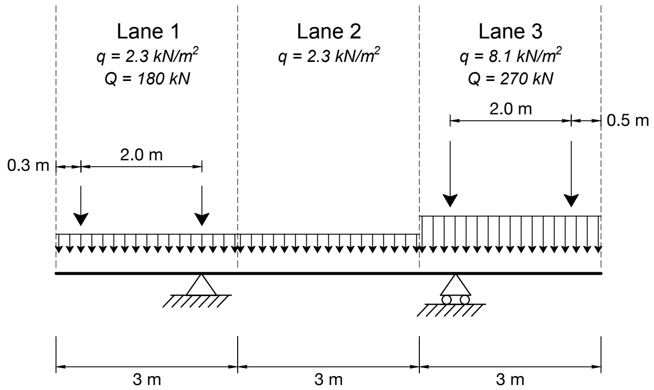

Figure 32: Example of load placement used for the design of deck slabs.

3. Using the influence lines from question 1, determine the placement

of the lanes and the position of the concentrated loads that maximize each internal force.

4. Using the results from questions 2 and 3, compute the values for the maximum mid-span moment.

5. Discuss how realistic and conservative this 1D model seems. In which regard would a 2D model of the slab differ from this 1D model?

6.4 Optimisation of the geometry of a portal bridge

The geometry of a bridge is often optimised under uniformly distributed loads. The portal bridge in Figure 33 will have two Gerber joints that are modelled by hinges in the static system.

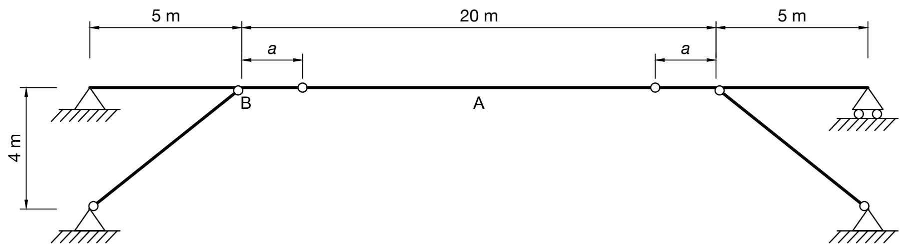

Figure 33: Portal bridge.

1. Check that the structure is statically determinate.

2. Place the joints so that the magnitude of the moments at points A and B are equal under a uniformly distributed load.

To realise the longitudinal design of a road bridge, the following traffic loads are applied:

- A uniformly distributed load, which depends on the width of the deck. For the given design, this load is applied only where it is disadvantageous, depending on the load case.

- Two concentrated loads placed 1.20 m apart, which simulate axle loads. These are placed at the most disadvantageous location of the bridge to maximise a particular internal force at a given location for the design of the section at this location.

3. For the following internal forces, draw the influence lines, and point out where the uniformly distributed and the concentrated traffic loads should be positioned to maximize these internal forces:
   - the mid-span moment (A);
   - the moment above the strut (B);
   - the axial force in one of the struts;
   - the shear force on the left-hand side of point B.

6.5 3D truss bridge

Figure 34 represents the static system of a typical timber bridge of small span. It consists of truss elements and a central moment-resisting frame.

The applied forces due to gravity load and wind are: $P = 50\ kN$, $T = 10\ kN$ and $B = 5\ kN$.

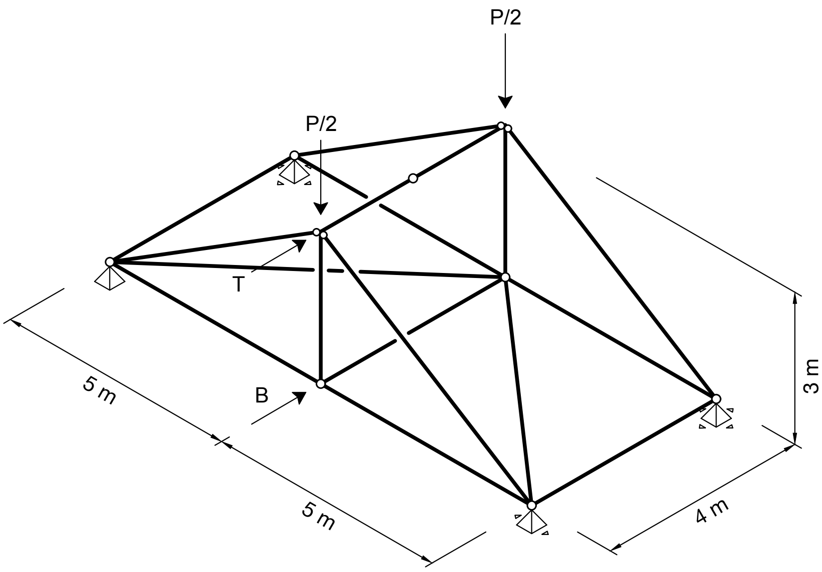

Figure 34: Static system of a truss bridge in axonometric.

1. Check the isostaticity of the structure.
2. Making the necessary simplifications of the static system, compute the internal forces due to P.
3. Making the necessary simplifications of the static system, compute the internal forces due to B.
4. Compute the internal forces due to T.

6.6 3D cable-stayed structure

The static system of a small bridge held by two cables is represented in Figure 35. The length a is a variable.

A uniformly distributed load $q = 10\ kN/m$ is applied on the beam.

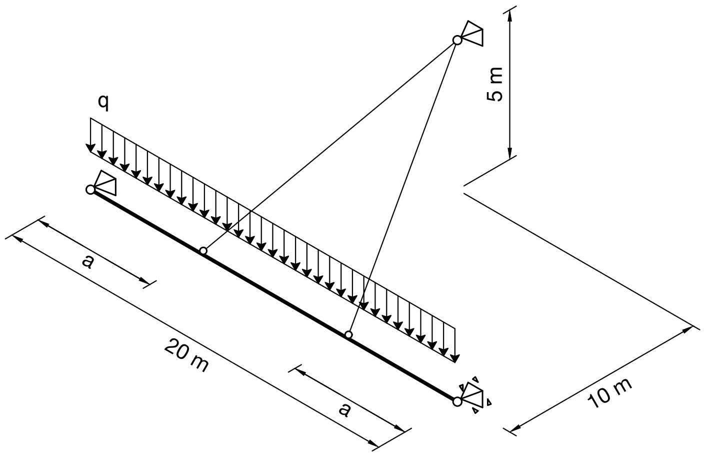

Figure 35: Static system of small cable-stayed bridge in axonometric.

1. Check the isostaticity of the structure.
2. Compute all the internal forces in the beam and the cables as a function of a.
3. Find a so that the maximum and minimum bending moments M_y^+ and M_y^- in the beam are equal.

7 Ouvrages géotechniques (BA6)

7.1 Flat foundation

A flat foundation is subjected to a vertical force F, which is the resultant of all forces applied to the foundation (including its own weight).

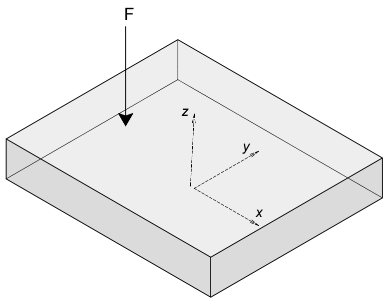

Figure 36: Flat foundation.

One-dimensional case

We first consider the case when the load is applied along a symmetry axis of the foundation (here: $y = 0$). Which values of eccentricity x_P prevent foundation uplift on one side?

Make the following assumptions:

- The soil pressure beneath the foundation is linearly distributed.
- The soil-foundation interface only transfers compression stresses.
- The foundation uplifts where the contact pressure between the foundation and the soil is zero.

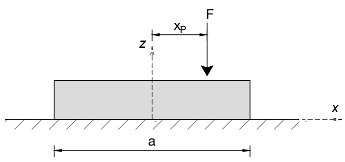

Figure 37: Flat foundation: One-dimensional case.

Two-dimensional case

As a second case, we consider a load applied at any point on the foundation (Figure 38). The load eccentricities along the x and y axes are x_P and y_P, respectively. Which combinations of x_P and y_P prevent foundation uplift? Graphically represent this solution space.

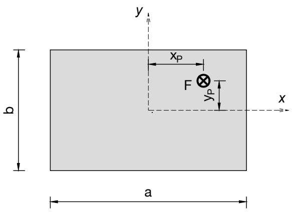

Figure 38: Flat foundation: Two-dimensional case.

7.2 Dimensionnement d'une paroi rectangulaire et de sa foundation

Une paroi de 4 m et de 30 cm d'épaisseur sépare deux réservoirs d'eau. La profondeur de l'eau dans chaque bassin peut varier entre 0 et 4 m. Considérer pour tous les calculs une tranche de mur de profondeur 1 m.

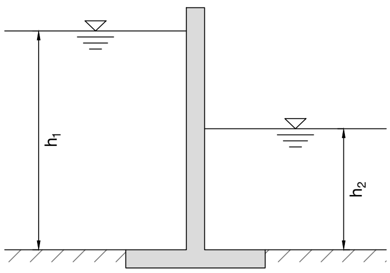

Figure 39: Paroi separant deux bassins

1. Pour des profondeurs $h_1 = 4\ m$ et $h_2 = 2\ m$ tracer les diagrammes NVM de la paroi en béton. Considérer comme charges la pression hydrostatique ($\gamma_e = 10\ kN/m^3$) sur la paroi et le poids propre de la paroi en béton ($\gamma_b = 25\ kN/m^3$)

2. Pour quelle paire de valeurs h_1 et h_2 les moments sont-ils maximaux? Déterminer les charges déterminantes correspondantes à appliquer sur le mur

3. Dimensionner pour la situation de la figure 39 la taille minimale b de la fondation. Les forces de réactions du sol peuvent être représentées par deux forces concentrées F_x et F_y. Ces deux forces agissent au coin de la fondation. Considérer le poids propre de la paroi et celui de la fondation.

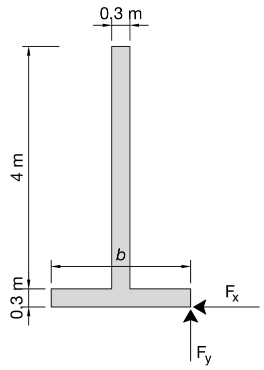

Figure 40: Reactions d'appuis de la parois

7.3 Internal forces of a pile

A pile is subjected to a set of forces at its head (Figure 41). Determine F, H and M that caused the soil reaction forces shown in the figure. Then, calculate the shear force and bending moment diagram for the pile.

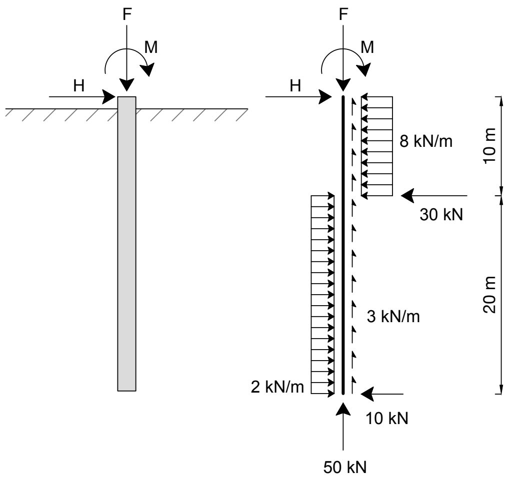

Figure 41: Pile and corresponding soil reactions.