Up to speed in Statics?

Exercises for the revision of Statique I throughout the Civil Engineering curriculum

SOLUTIONS

Katrin Beyer
Baptiste Vincens
Igor Tomić

Version 1.0

ECOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Architecture, Civil and Environmental Engineering (ENAC)

Contents

Imprint

Please cite as: Beyer K, Vincens B, Tomić I (2021). *Up to speed in Statics? Exercises for the revision of Statique I throughout the Civil Engineering curriculum*, Exercise booklet and solutions, Version 1.0, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland. https://github.com/eesd-epfl/Statique_I.git.

Foreword

The first course of structural mechanics in any civil engineering curriculum introduces important principles, such as the free body diagram, static equilibrium and the internal forces in structural members, and then applies these principles to statically determinate systems. At EPFL, these concepts are taught in the course Statique I in the second semester of the Bachelor curriculum (BA2). This introductory course lays the foundation to many other civil engineering courses, and engineers in practice or research who design or analyse any type of structure need to have these techniques and concepts at their fingertips.

The idea behind this booklet is twofold: First, we want to reinforce these first principles taught in Statique I throughout the curriculum by applying them to systems treated in the BA3-6 courses that build on Statique I. The presented exercises can serve as a basis for self-study when preparing for a new course or as a homework assignment in week 1 of this course. Second, this booklet is also intended to give Statique I students an idea of how the principles they are learning will be used in later courses and in engineering practice. For these purposes, we have put together a small set of exercises for each course that repeat important concepts and introduce others. In the future, we plan to expand this booklet with exercises on statically indeterminate systems, covered in Statique II.

We would like to thank Prof. Dr Alain Nussbaumer for the exercises on steel structures, Dr Olivier Burdet for his input in general and for the exercises on concrete structures and bridges in particular, and Dr Giovanni De Cesare for his help with the hydraulics exercises.

This booklet is accompanied by a solution manual containing example solutions for all problems. To facilitate re-usage of the material, all source files (Latex files and images) are shared. All material including the pdf-file of this document can be downloaded from the following GitHub repository: https://github.com/eesd-epfl/Statique_I.git.

Katrin Beyer
Baptiste Vincens
Igor Tomić

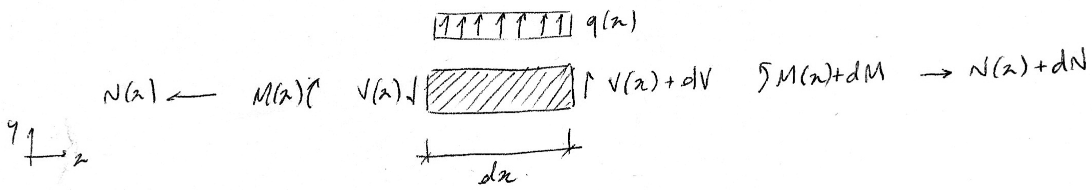
Earthquake Engineering and Structural Dynamics Laboratory (EESD)
School of Architecture, Civil and Environmental Engineering (ENAC)
École Polytechnique Fédérale de Lausanne (EPFL)

Lausanne, June 9, 2021

1 Structural mechanics (BA3)

1.1 Differential equations of equilibrium for beams

First, we draw an infinitesimal element of a beam of length dx with the load $q(x)$ and the internal forces.



Using the equilibrium equation $\sum F_y = 0$, we obtain:

$$q(x) \cdot dx - V(x) + V(x) + dV = 0$$

$$-q(x) \cdot dx = dV$$

$$\frac{dV}{dx} = -q(x)$$

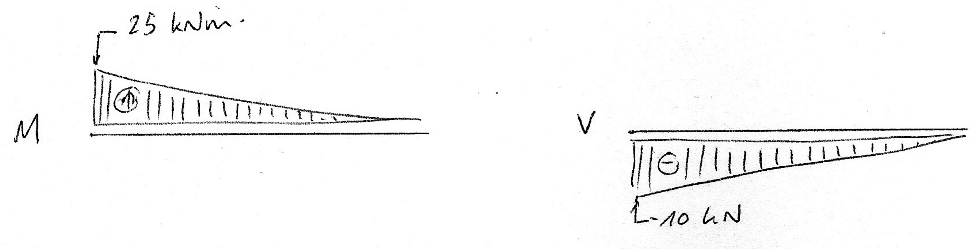
To obtain other differential equations of equilibrium, we have to use the other equilibrium equations on the same infinitesimal element of beam.

1.2 Internal force diagrams for beams and cantilevers

1

For a cantilever beam, we know the following:

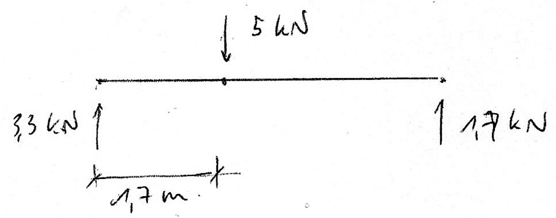
- at the fixed end, the moment is maximum
- at the free end, the moment is 0
- if the load is uniformly distributed, the moment diagram follows a second order parabola
- instinctively, we know that the beam will bend downwards so the diagram should be drawn upwards of the beam
- if we replace the distributed load by an equivalent concentrated load $q \cdot L$, we can calculate the maximum moment: $M_{max} = q \cdot L \cdot \frac{L}{2} = 25\ kNm$
- if the load is uniformly distributed, the shear diagram will be linear
- knowing that $\frac{dM}{dx} = -V(x)$, it implies that if the moment diagram has a negative slope, the shear force is negative
- the maximum value of the shear diagram is $V_{max} = q \cdot L = 10\ kN$



2

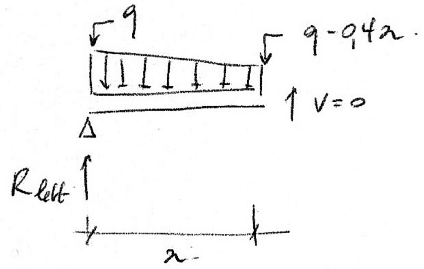
First, we compute the reaction forces:

- the equivalent concentrated force is $\frac{q \cdot L}{2} = 5\ kN$
- it is located at a distance $\frac{1}{3} \cdot L = 1.7\ m$ from the left support
- using equilibrium, we find the following reactions:



When a beam is subjected to distributed loads, it is good practice to first draw the shear diagram as it tells us where the maximum moment will be.

- as the distributed load is triangular, the shear diagram is a second order parabola
- as $\frac{dV}{dx} = -q$, the slope of the shear diagram is maximum at the left support and 0 at the right support.
- we need to solve the following equation to determine where the shear force is 0:



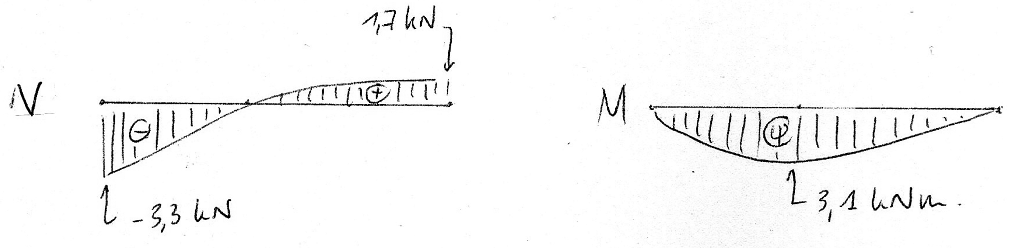
$$R_{left} - \frac{q + (q - 0.4 \cdot x)}{2} \cdot x = 0$$

we obtain $x = 2.1\ m$

Then, we can draw the bending moment diagram.

- it will follow a third order parabola
- the moment is 0 at extremities
- the maximum is obtained when $V = 0$, which is at $x = 2.1\ m$
- we can calculate:

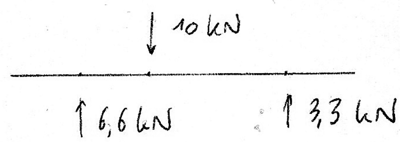
$$M_{max} = R_{left} \cdot 2.1 - \frac{(q - 0.4 \cdot 2.1) \cdot 2.1^2}{2} - \frac{0.4 \cdot 2.1^2}{2} \cdot \frac{2 \cdot 2.1}{3} = 3.1\ kNm$$



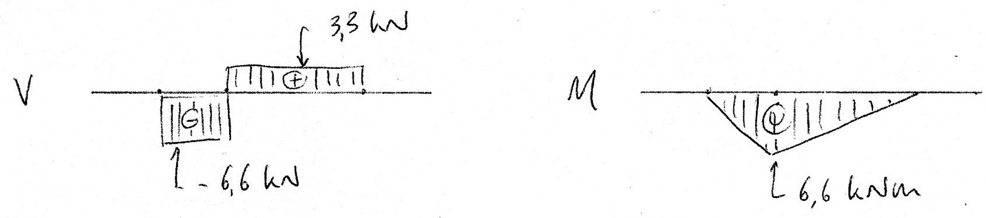
3

For this kind of situations, the easiest is to use the principle of superposition. We begin by treating the case where the beam is only subjected to the concentrated load.

- First, we calculate the reactions using equilibrium:

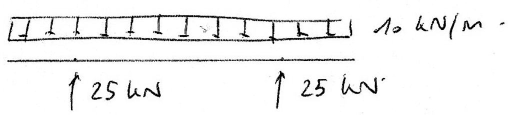


- then the shear and moment diagrams are easily computed:

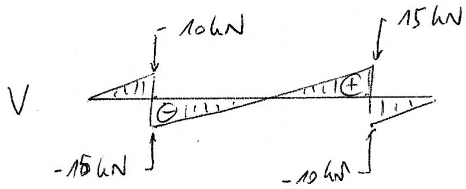


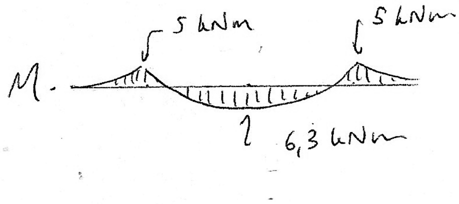
Then we can treat the case where the beam is only subjected to the distributed load.

- the reactions are easily computed



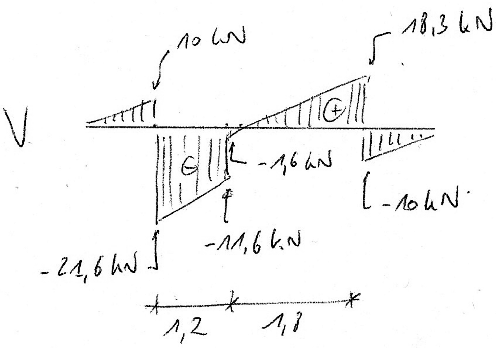
- the shear and bending moment diagrams are also easily computed

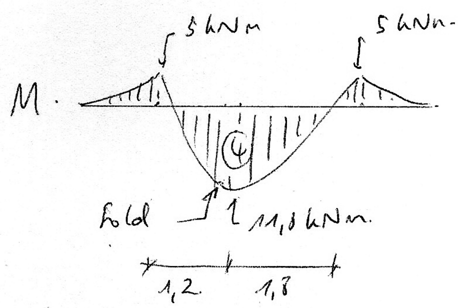




Finally, we can sum up the diagrams.

- to find where the maximum of the moment diagram will be, we have to find where the shear force is 0. Using the shear diagram, we easily find it is 1.8 m away from the right support.

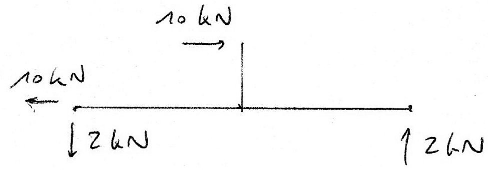




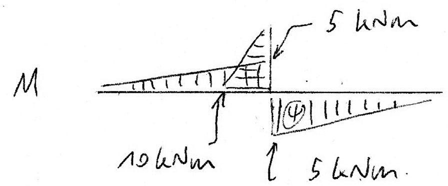
4

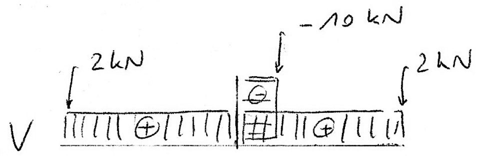
First, we use a simple equilibrium of moment to calculate the reaction forces.

One way of looking at it is simply to see that the moment created by F is $M = F \cdot 1 = 1\ kNm$ and it has to be balanced by two vertical forces at the supports which are 5 m apart. These forces are of opposite direction and have a value $R = \frac{M}{5} = 2\ kN$. Their direction at each support is found instinctively by feeling how the system wants to move. In this case, the right support wants to go downwards and the left one upwards. The horizontal forces can only be supported by the left support as the other one is sliding.



When there are only concentrated loads on the system, it is usually a good idea to start by drawing the moment diagram and then to use it to draw the shear diagram. If the moment diagram slopes downwards, the shear force is negative, else it is positive.





1.3 Portal frames and support systems

1

We name the reaction forces for the left support $R_{l,x}$, $R_{l,y}$ and M_l and for the right support $R_{r,x}$ and $R_{r,y}$. The connection forces at the hinge for frame 3 are $R_{h,x}$ and $R_{h,y}$. In the calculations, we assume that all the forces are in the positive direction of the x and y axis. However, they are drawn in their actual direction in the figures.

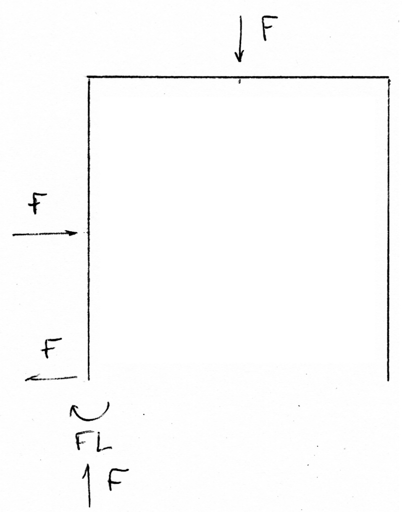
Frame 1

First, we calculate the moment M_l using equilibrium of moments around the fixed end.

$$M_l - F \cdot \frac{L}{2} - F \cdot \frac{L}{2} = 0$$
$$M_l = -F \cdot L$$

Then, using equilibrium of forces along both axes, we can calculate the other reactions.

$$R_{l,x} = -F$$
$$R_{l,y} = F$$



Frame 2

Using equilibrium of moments around the left support, we can first calculate the reaction at the right support.

$$R_{r,y} \cdot L - F \cdot \frac{L}{2} - F \cdot \frac{L}{2} = 0$$

$$R_{r,y} = F$$

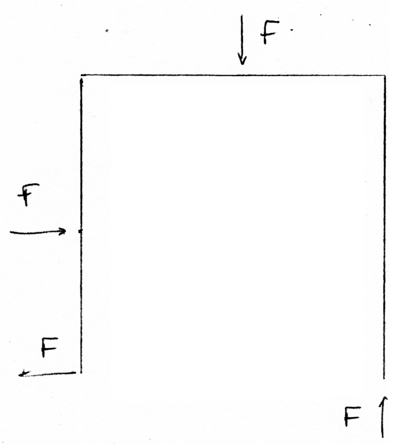
Then, using equilibrium of vertical forces, we obtain:

$$R_{l,y} + R_{r,y} - F = 0$$

$$R_{l,y} = 0$$

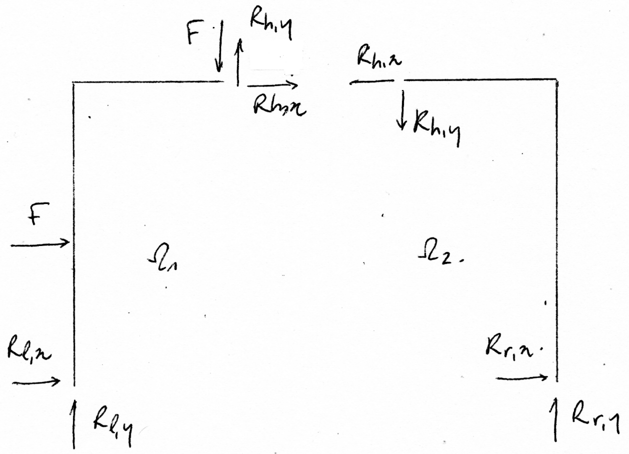
Finally, using equilibrium of horizontal forces:

$$R_{l_x} = -F$$



Frame 3

In this case, we need to explicitly solve a system of equations. The first step is to dislocate the structure at the hinge so that the connection forces appear. We obtain 2 systems: Ω_1 and Ω_2.



First, we express $R_{h,x}$ in function of $R_{h,y}$ using equilibrium of moments at the right support on system Ω_2.

$$R_{h,x} \cdot \frac{L}{2} + R_{h,y} \cdot \frac{L}{2}$$

$$R_{h,x} = -\frac{R_{h,y}}{2}$$

Then using equilibrium of forces along both axes on system Ω_2, we also express the reaction forces at the right support in function of $R_{h,y}$.

$$R_{r,x} = Rh, x = -\frac{R_{h,y}}{2}$$

$$R_{r,y} = R_{h,y}$$

Then, using equilibrium of moments around the left support, we can calculate the value of $R_{h,y}$ and deduce $R_{h,x}$.

$$-F \cdot \frac{L}{2} - F \cdot \frac{L}{2} - R_{h,x} \cdot L + R_{h,y} \cdot \frac{L}{2} = 0$$

$$R_{h,y} = F$$

$$R_{h,x} = -\frac{F}{2}$$

Then, using equilibrium of forces along both axes on system Ω_1, we calculate the reaction forces at the left support.

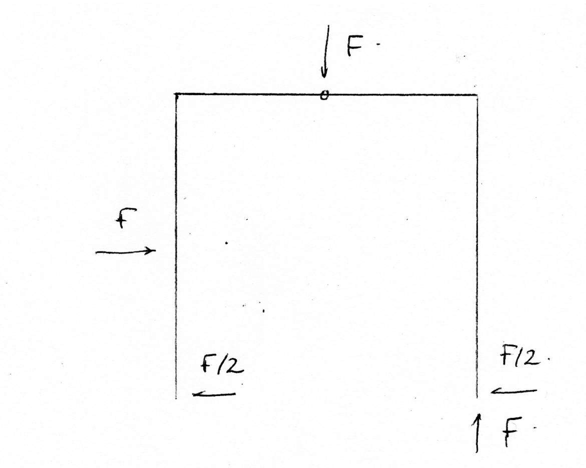
$$R_{l,x} = -F - R_{h,x} = -\frac{F}{2}$$

$$R_{l,y} = F - Rh, y = 0$$

Finally, the reactions at the right support are:

$$R_{r,x} = -\frac{F}{2}$$

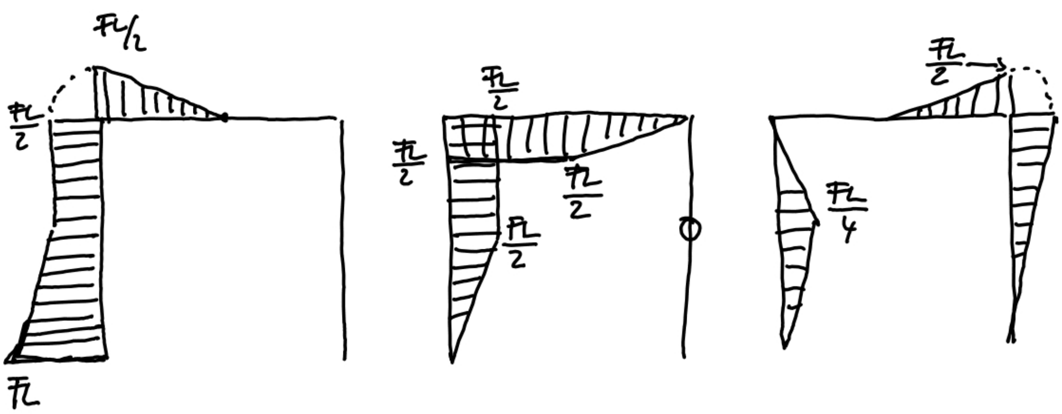
$$R_{r,y} = F$$



2

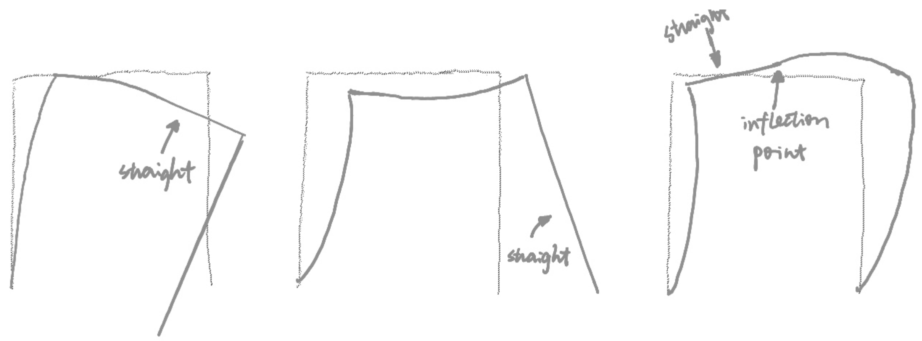
As there are only concentrated forces acting on the frame, the moment diagram is linear. We also know that at the supports, the moment is 0 and that the maximums and minimums will be either at the angles of the frame or at the position of concentrated loads.

The maximum and minimum values are simply calculated by cutting the frame at the key areas and calculating the moment generated by the reaction forces and external loads at this point.



3

The curvature of the deformed shape is derived from the moment diagram. When the moment is 0, the frame keeps its original shape. The right angles at the corners of the frame are preserved.

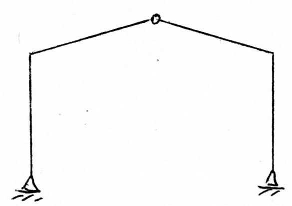


4

Observing the moment diagram and the displaced shape, we instantly see that the third frame is more efficient as the internal forces are lower than for the 2 remaining frames.

The displaced shape shows that displacements are kept low as opposed to the first frame which bends down significantly and the second one which undergoes significant lateral displacements.

This type of frames is commonly used today in timber construction for large spaces like gymnasiums. It is quite an efficient shape and the frame can be transported in 2 pieces to the construction site which is convenient. The behaviour of the frame is rendered more efficient if the cross beam has a double slope.



1.4 Inverse problem

1

As $\frac{dM}{dx} = -V(x)$, we can compute the shear diagram by calculating the slope of the moment diagram on each continuous section. As a rule, if the moment diagram slopes downwards, V is negative and if it slopes upwards, V is positive.

For the sections where the moment diagram is linear, the slope is quickly computed using the values at the extremes. However, on the section where the moment diagram follows a second order parabola, we determine the expression of the moment and differentiate it to obtain the expression of the shear force. Other methods could also be used (property of the point of intersection of the tangents of a stretch of parabola).

We fix $x = 0$ at the level of the third support. The expression of the moment diagram is:

$$M(x) = a + b \cdot x + c \cdot x^2$$

As we know the values of $M(x)$ at the third support, at the local maximum and at the fourth support, we can solve the following system:

$$\begin{cases} M(0) = a = -\frac{q \cdot L^2}{8} \\ M(\frac{5 \cdot L}{8}) = a + b \cdot \frac{5 \cdot L}{8} + c \cdot \frac{25 \cdot L^2}{64} = \frac{9 \cdot q \cdot L^2}{128} \\ M(L) = a + b \cdot L + c \cdot L^2 = 0 \end{cases}$$

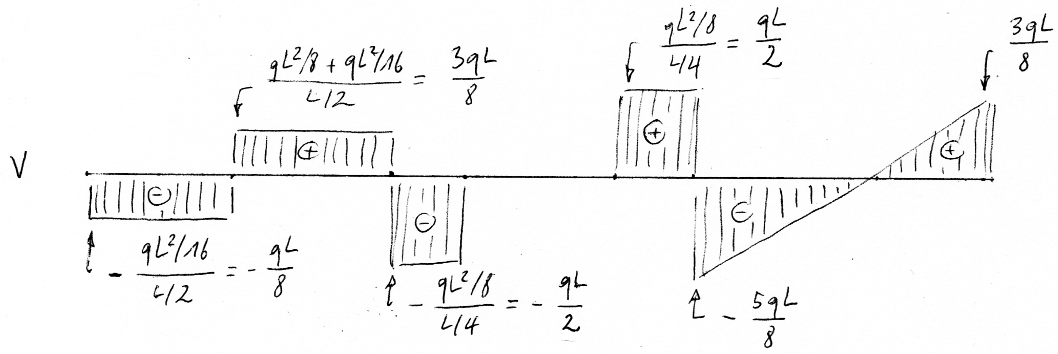
We find: $M(x) = -\frac{q \cdot L^2}{8} + \frac{5 \cdot q \cdot L}{8} \cdot x - \frac{q}{2} \cdot x^2$

Therefore, $V(x) = -\frac{dM(x)}{dx} = -\frac{5 \cdot q \cdot L}{8} + q \cdot x$

We obtain the values at the extremities:

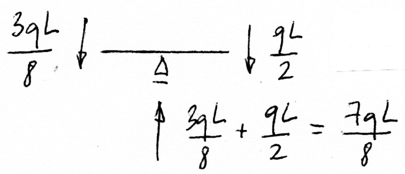
$$V(0) = -\frac{5}{8} \cdot q \cdot L$$

$$V(L) = \frac{3}{8} \cdot q \cdot L$$

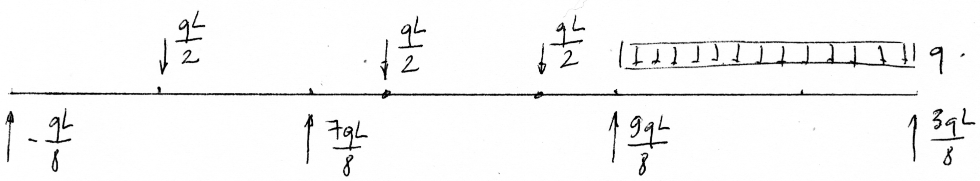


2

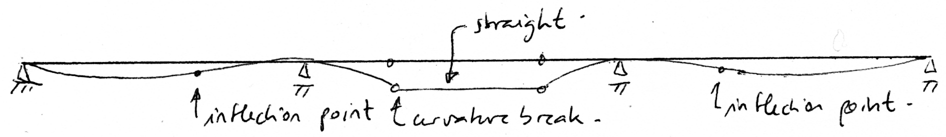
A concentrated load applied on a beam causes a leap in the shear diagram with amplitude equal to the value of the force. To determine the direction of the concentrated load, the easiest way is to draw a small section of the beam and to draw the shear forces on either side. Using equilibrium of vertical forces, we instantly know the direction of the external load. An example of this procedure is given for the reaction at the second support.



When the shear diagram is linear, its slope is the value of the distributed load as $\frac{dV}{dx} = -q(x)$.



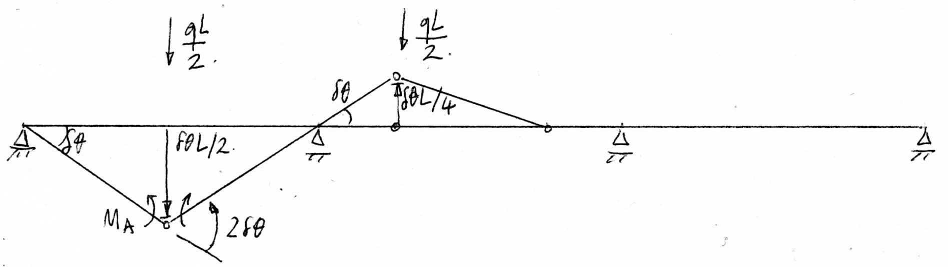
3



4

Moment at point A

We begin by placing a hinge at point A and externalising the moment, obtaining the mechanism sketched below. The virtual rotation $\delta\theta$ at the first support is used to express the rotation of the hinge at point A and the displacement of the external forces.



Then we can calculate the internal and external virtual work:

$$\delta W_{int} = -M_A \cdot 2 \cdot \delta\theta$$

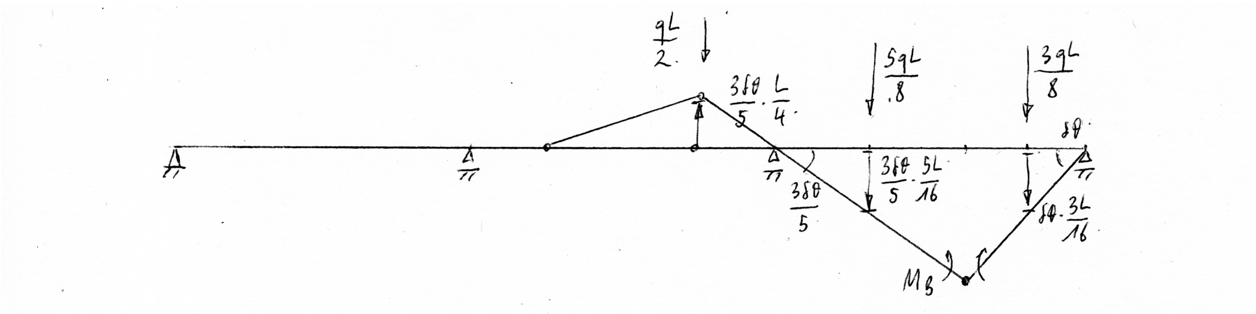
$$\delta W_{ext} = \frac{q \cdot L}{2} \cdot \frac{\delta\theta \cdot L}{2} - \frac{q \cdot L}{2} \cdot \frac{\delta\theta \cdot L}{4}$$

As $\delta W_{int} + \delta W_{ext} = 0$, we obtain:

$$M_A = \frac{1}{2} \cdot \left(\frac{q \cdot L}{2} \cdot \frac{L}{2} - \frac{q \cdot L}{2} \cdot \frac{L}{4} \right) = \frac{q \cdot L^2}{16}$$

Moment at point B

We place a hinge at point B and externalise the moment M_B, obtaining the mechanism sketched below. The virtual rotation $\delta\theta$ at the fourth support is used to express the rotation of the hinge at point B and the displacement of the external forces. The distributed load is reduced to 2 concentrated loads on either side of the hinge.



$$\delta W_{int} = -M_B \cdot \left(\delta\theta + \frac{\delta\theta \cdot 3}{5}\right) = -M_B \cdot \frac{8 \cdot \delta\theta}{5}$$

$$\delta W_{ext} = -\frac{q \cdot L}{2} \cdot \frac{\delta\theta \cdot 3}{5} \cdot \frac{L}{4} + \frac{5 \cdot q \cdot L}{8} \cdot \frac{\delta\theta \cdot 3}{5} \cdot \frac{5 \cdot L}{16} + \cdot \frac{3 \cdot q \cdot L}{8} \cdot \delta\theta \cdot \frac{3 \cdot q \cdot L}{8} = \delta\theta \cdot \frac{9 \cdot q \cdot L^2}{80}$$

We obtain:

$$M_B = \frac{5}{8} \cdot \frac{9 \cdot q \cdot L^2}{80} = \frac{9 \cdot q \cdot L^2}{128}$$

2 Statique II (BA4)

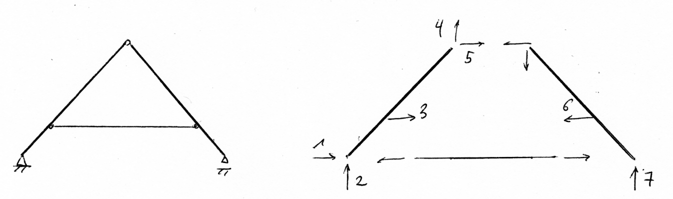
2.1 Isostatic, hyperstatic systems, and mechanisms

The first step to determine if a structure is isostatic or hyperstatic is always to verify that the system is not a mechanism. If it is not the case, the structure should then be dislocated and the connection and reaction forces externalised.

Then, the number of unknowns can be counted (making sure that we count one unknown for both an action and a reaction in the case of connection forces). The number of equation can then be determined by counting 3 for beam elements and 1 for bars.

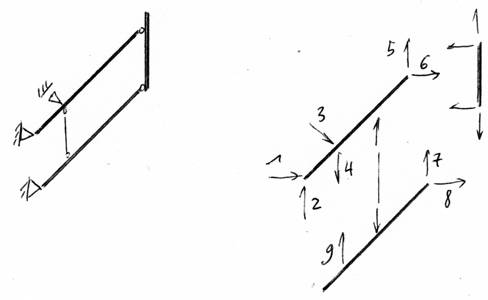
Bars are drawn with a lighter line to distinguish them from the beams. When a bar is connected to a hinged support, it can be reduced to a rolling support with its reaction in the direction of the bar.

System 1



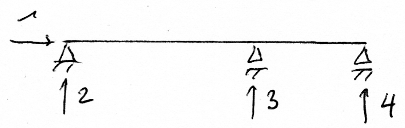
- 7 unknowns
- $3 + 3 + 1 = 7$ equations
- The system is isostatic

System 2



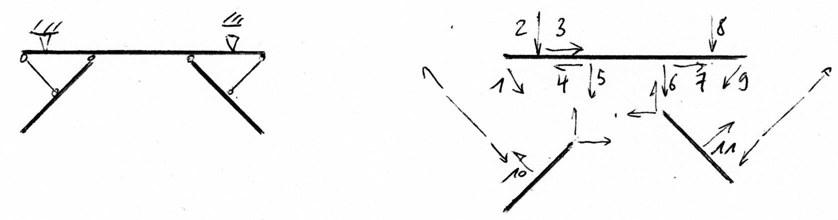
- 8 unknowns
- $3 + 3 + 3 + 1 = 10$ equations
- The system is hyperstatic with degree $10 - 8 = 2$

System 3



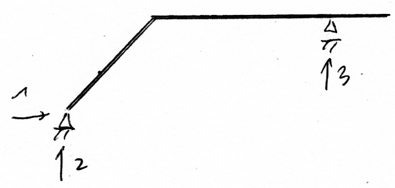
- 4 unknowns
- 3 equations
- The system is hyperstatic with degree $4 - 3 = 1$

System 4



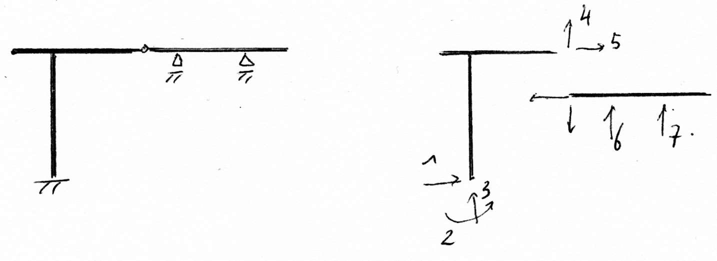
- 11 unknowns
- $3+3+3+1+1 = 11$ equations
- The system is isostatic

System 5



- 3 unknowns
- 3 equations
- The system is isostatic

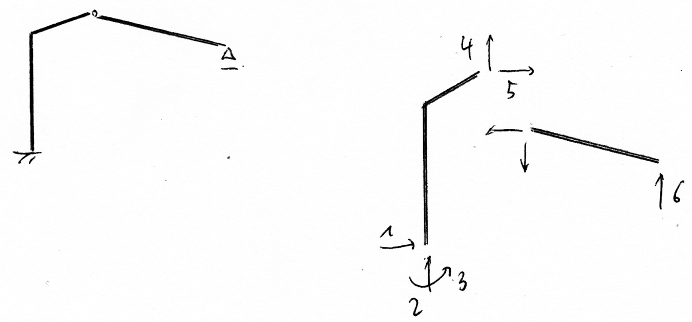
System 6



- 7 unknowns

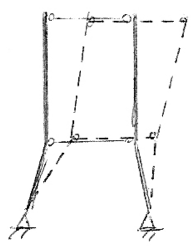
- 6 equations
- The system is hyperstatic with degree $7 - 6 = 1$

System 7



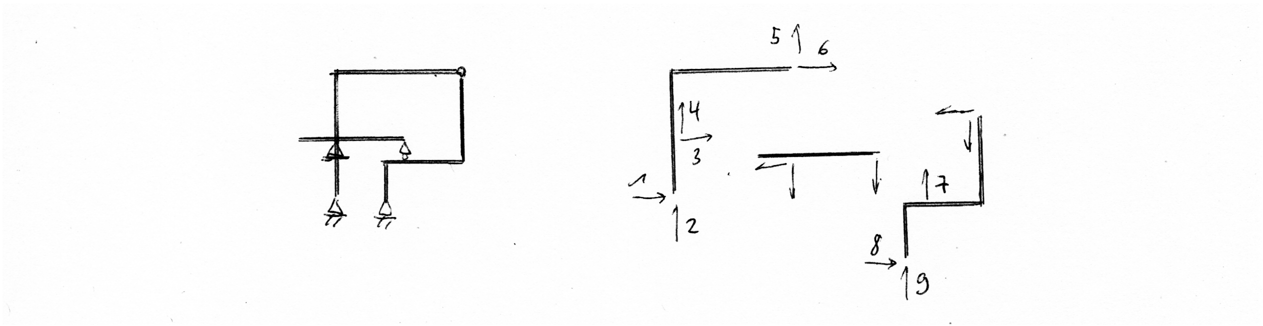
- 6 unknowns
- $3 + 3 = 6$ equations
- The system is isostatic

System 8



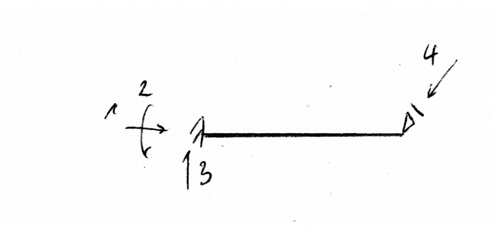
It is a mechanism. The placement of the bars do not prevent lateral displacement of the structure.

System 9



- 9 unknowns
- $3 + 3 + 3 = 9$ equations
- The system is isostatic

System 10



- 4 unknowns
- 3 equations
- The system is hyperstatic with degree $4 - 3 = 1$

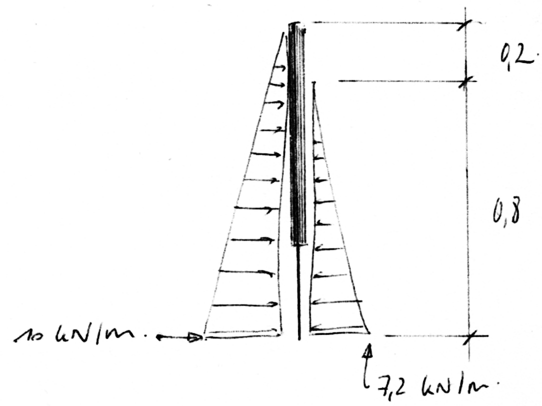
3 Hydraulics (BA6)

3.1 Trappe séparant deux réservoirs

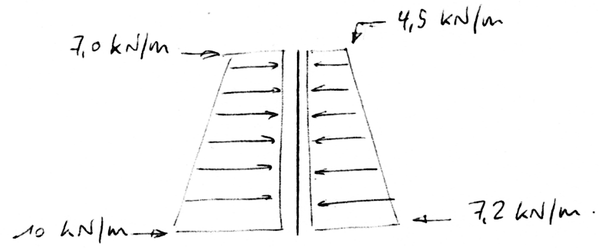
1

First, the lateral distributed load exerted by the liquids should be computed. These loads are triangular with slope $\gamma_{.} \cdot g$. We perform the calculation on an assumed 1 m long representative segment. The maximum values (at the bottom of the tank) are:

- left: $q_{1,max} = \gamma_1 \cdot g \cdot h_1 = 1.0 \cdot 10^3 \cdot 10 \cdot 1.0 = 10.0\ kN/m$
- right: $q_{2,max} = \gamma_2 \cdot g \cdot h_2 = 0.9 \cdot 10^3 \cdot 10 \cdot 0.8 = 7.2\ kN/m$

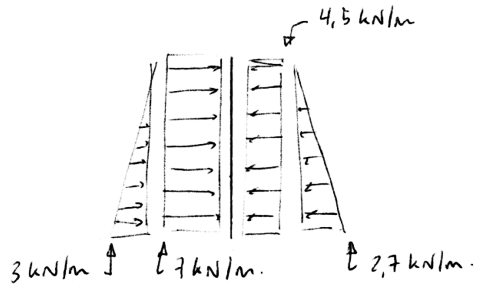


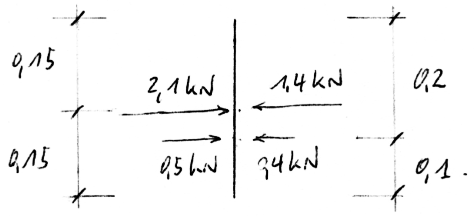
As we only focus on the trap door, we can cut around it and calculate the extremity values.



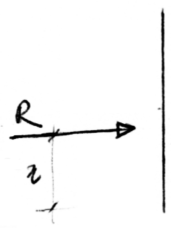
2

The first step is to replace the distributed loads by equivalent concentrated loads. It is easier to separate the distributed load into a uniform load and a triangular load using the principle of superposition as the position of the concentrated loads can be found more quick this way than calculating based on a trapesoid.





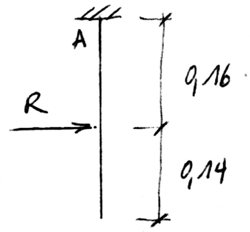
Using equilibrium around the x axis, we see that the resultant will have values: $2.1 + 0.5 - 1.4 - 0.4 = 0.8\ kN$. Now, to determine the position z of the resultant, we use the fact that the moment created by the group of force at the foot of the trap door should be the same as the one created by the resultant.



$$0.5 \cdot 0.1 - 0.4 \cdot 0.1 + 2.1 \cdot 0.15 - 1.4 \cdot 0.15 = R \cdot z$$
$$z = 0.14\ m$$

3

The static system of the trap door with the resulting external load is as follow.

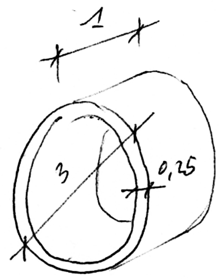


The moment at point A is: $M_A = R \cdot (0.3 - 0.14) = 0.13\ kNm$

3.2 Tunnel immergé

1

We begin by calculating the volume of concrete there is in a 1 m deep slice of tunnel.



$$v_c = \left(\pi \cdot 1.5^2 - \pi \cdot (1.5 - 0.25)^2\right) \cdot 1.0 = 2.2\ m^3/m$$

Then the weight of concrete is:

$$q_c = v_c \cdot g \cdot \gamma_b = 2.2 \cdot 10 \cdot 2.5 \cdot 10^3 = 55\ kN/m$$

2

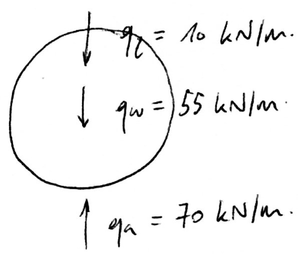
Archimedes' buyant force for a 1 m deep slice of tunnel is $q_a = v_a \cdot g \cdot \gamma_e$ where v_a is the volume of displaced fluid.

Here, $v_a = \pi \cdot 1.5^2 = 7\ m^3/m$

Therefore, $q_a = 7.0 \cdot 10 \cdot 1.0 \cdot 10^3 = 70\ kN/m$

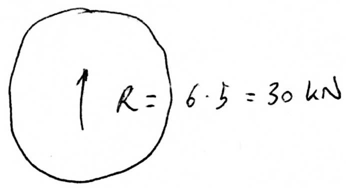
3

The loads applied on the tunnel are as follows.

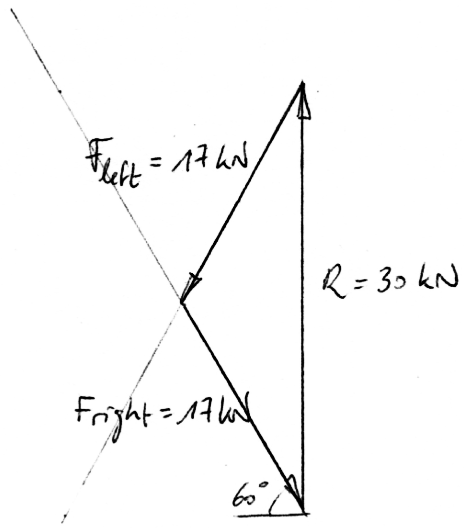


The resultant is $r = q_a - q_l - q_w = 5\ kN/m$

As the tunnel is maintained every 6 m, the force that needs to be balanced by the cables will be:



Using graphic statics, we can quickly compute the axial load in the cables: 17 kN



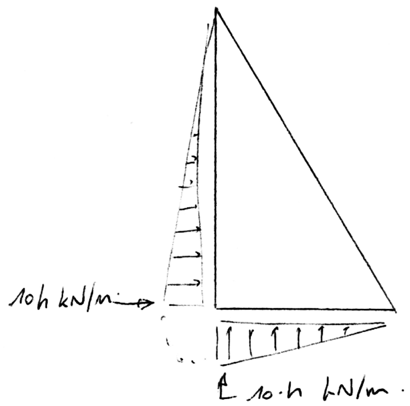
3.3 Barrage de la Grande-Dixence

1

The lateral thrust exerted by water on the side of the dam is a triangular load with maximum value $h \cdot \gamma_e \cdot g = h \cdot 1000 \cdot 10 = 10 \cdot h\ kN/m^2$.

Additionally, we admit that there is also an upward thrust created by water that infiltrated in the soil underneath the dam. This load is also triangular and its maximum value is also $10 \cdot h\ kN/m^2$

We assume performing the calculation on 1 m long representative segment of the dam.



These loads can be reduced to equivalent concentrated loads that can be used to calculate the moment around B as depicted below.

2

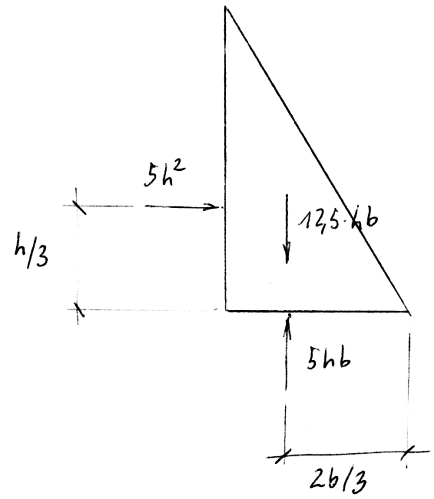
The volume of concrete for a 1 m deep slice of dam is $v_c = \frac{b \cdot h}{2}$

Then, the weight of the dam is $v_c \cdot \gamma_c \cdot g = \frac{b \cdot h}{2} \cdot 2500 \cdot 10 = 12.5 \cdot b \cdot h\ kN/m$

The centre of mass of the dam is that of a triangle, that is $\frac{2}{3} \cdot b$ away from B.

3

Once all the external loads are simplified into concentrated loads and drawn on the cross section of the dam, we can simply calculate the moment around B.



$$M_B = -5 \cdot h^2 \cdot \frac{h}{3} - 5 \cdot h \cdot b \cdot \frac{2 \cdot b}{3} + 12.5 \cdot h \cdot b \cdot \frac{2 \cdot b}{3}$$

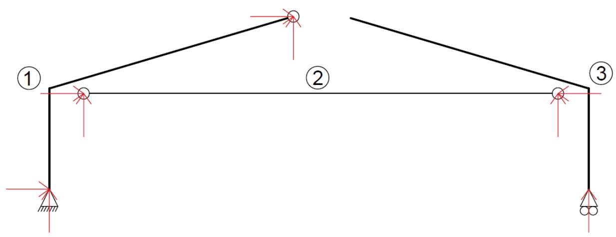
The dam does not uplift if and only if $M_B \geq 0$. We find $h \leq \sqrt{3} \cdot b$

4 Structures en métal (BA4)

4.1 Cadre avec ferme sous-tendue

1

- Nombre d'éléments: 3
- Nombre d'équations d'équilibre par élément (dans un plan): 3
- Nombre d'équations d'équilibre: $N = 3 \cdot 3 = 9$
- Nombre de réaction d'appuis: 3
- Nombre de réactions aux nœuds: 6
- Nombre d'inconnues statiques: $U = 3 + 6 = 9$
- Système à N équations pour U inconnues, $U - N = 9 - 9 = 0$, le système est isostatique



Si l'on supprime le tirant nous avons:

- Nombre d'éléments: 2
- Nombre d'équations d'équilibre par élément (dans un plan): 3
- Nombre d'équations d'équilibre: $N = 2 \cdot 3 = 6$
- Nombre de réaction d'appuis: 3
- Nombre de réactions aux nœuds: 2
- Nombre d'inconnues statiques: $U = 3 + 2 = 5$
- Système à N équations pour U inconnues, $U - N = 5 - 6 = 0$, la structure est instable

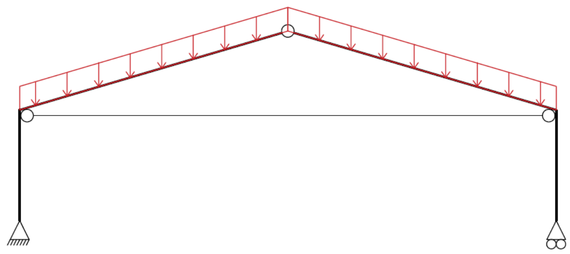
2

La répartition de la charge étant symétrique et parallèle à l'axe vertical nous pouvons dire:

$$R_{Ay} = R_{By} = q_{neige} \cdot \frac{L}{2} = 2 \cdot \frac{17}{2} = 17\ kN$$
$$R_{Ax} = R_{Bx} = 0$$

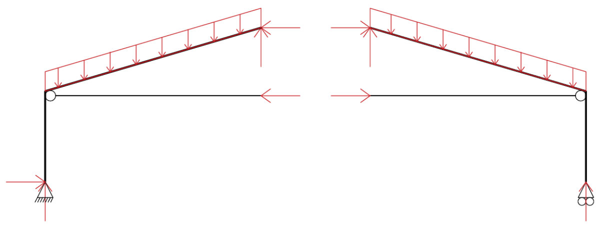
3

Il est dans un premier temps nécessaire de rabattre la charge sur les pans de la ferme. L'angle entre la poutre principale de la ferme et le tirant $\alpha = arctan\left(\frac{2.5}{17/2}\right)$.



La charge de neige rabattue est alors $q_{neight,1} = 2 \cdot cos(\alpha) = 1.92\ kN/m$.

On découpe ensuite la structure en deux parties et on pose nos équations d'équilibre. De plus nous savons que le tirant ne peut reprendre qu'un effort de traction ce qui réduit le nombre d'inconnues.



Forces de liaison

Sur la partie de droite:

$$\sum F_x = 0 = F_{x,sup} = F_{x,inf}$$

$$\sum F_y = 0 = F_{y,sup} + R_{B,y} - q_{neige} \cdot \frac{17}{2}$$

$$F_{y,sup} = -R_{B,y} + q_{neige} \cdot \frac{17}{2} = -17 + 2 \cdot \frac{17}{2} = 0 \; kN$$

$$\sum M_{sommet} = 0 = R_{By} \cdot \frac{17}{2} - q_{neige} \cdot \frac{(17/2)^2}{2} + F_{x,inf} \cdot 2.5$$

$$F_{x,inf} = -28.9 \; kN$$

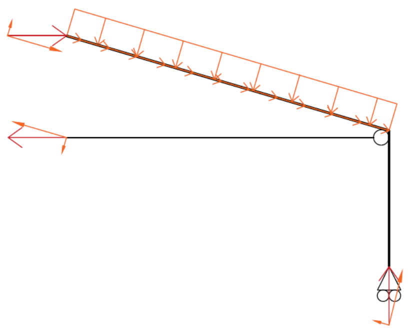
$$F_{x,sup} = -F_{x,inf} = 28.9 \; kN$$

Efforts dans le tirant

Nous pouvons déjà conclure que, aucune charge n'étant appliquée sur le câble, l'effort de traction sera constant sur toute la longueur et donc $N = 28.9 \; kN$.

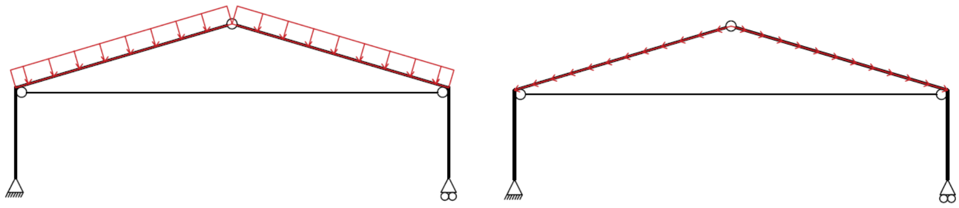
Efforts dans la poutre principale

Nous avons alors système suivant.



Afin de déterminer les efforts dans la poutre principale, nous devons décomposer les forces perpendiculairement et parallèlement au système d'axes local de la poutre:

- $q_{neige,1,perp} = 1.92 \cdot cos(\alpha) = 1.84 \; kN/m$
- $q_{neige,1,para} = 1.92 \cdot sin(\alpha) = 0.541 \; kN/m$



L'effort normal au sommet est alors égal à la composante parallèle à l'élément de $F_{x,sup}$ soit

$$N = -F_{x,sup} \cdot cos(\alpha) = -27.7\ kN$$

L'effort tranchant au sommet est alors égal à la composante perpendiculaire à l'élément de $F_{x,sup}$ soit

$$V = F_{x,sup} \cdot sin(\alpha) = 8.15\ kN$$

Le moment au sommet est nul car il y a une rotule.

A mi travée, l'effort normal est

$$N = -F_{x,sup} \cdot cos(\alpha) - q_{neige,1,para} \cdot \frac{17/2}{cos(\alpha)} \cdot \frac{1}{2} = -30.1\ kN$$

A mi travée, l'effort tranchant est

$$V = F_{x,sup} \cdot sin(\alpha) - q_{neige,1,perp} \cdot \frac{17/2}{cos(\alpha)} \cdot \frac{1}{2} = 0\ kN$$

A mi travée, le moment est

$$M = -F_{x,sup} \cdot sin(\alpha) \cdot \frac{17/2}{cos(\alpha)} \cdot \frac{1}{2} + q_{neige,1,perp} \frac{17/2}{cos(\alpha)} \cdot \frac{1}{2} \cdot \frac{1}{2} = -18.1\ kNm$$

A l'angle, l'effort normal est

$$N = -F_{x,sup} \cdot cos(\alpha) - q_{nege,1,para} \cdot \frac{17/2}{cos(\alpha)} = -32.5\ kN$$

A l'angle, l'effort tranchant est

$$V = F_{x,sup} \cdot sin(\alpha) - q_{neige,1,perp} \cdot \frac{17/2}{cos(\alpha)} = -8.15\ kN$$

A l'angle, le moment est

$$M = -F_{x,sup} \cdot sin(\alpha)\frac{17/2}{cos(\alpha)} + q_{neige,1,perp} \cdot \frac{17/2}{cos(\alpha)} \cdot \frac{1}{2} = 0\ kNm$$

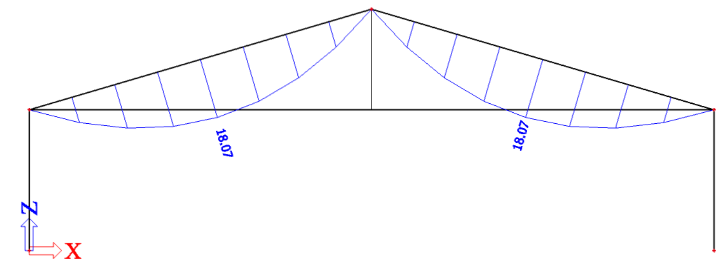


Diagramme des moments

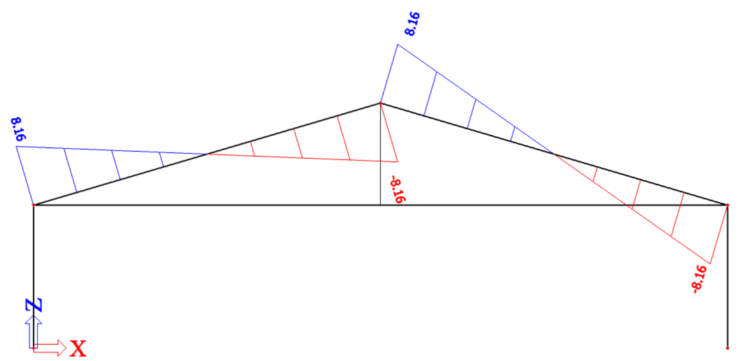


Diagramme de l'effort tranchant

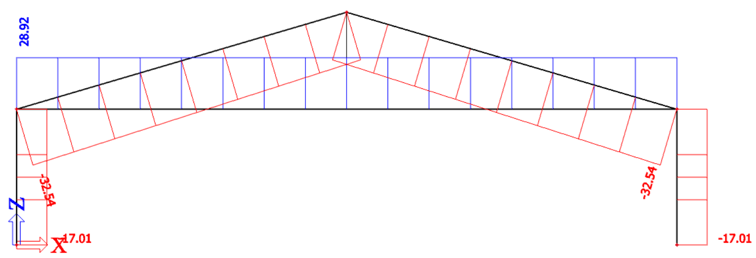


Diagramme de l'effort normal

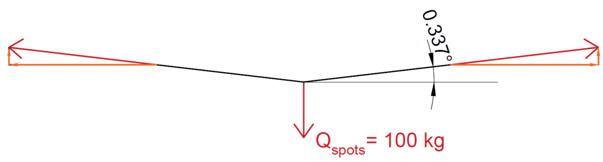
4

La force dans le tirant est relativement grande en comparaison des autres efforts dans la structure. Ce type de système devient avantageux lorsque l'on veut soulager les appuis de la structure en évitant ainsi de leur introduire une charge horizontale importante (le système isostatique est alors un cadre sur deux appuis fixes sans tirant). Pour donner un exemple, les murs en maçonnerie sont généralement très peu résistants dans leur axe transversal et ils ne peuvent généralement pas reprendre les efforts horizontaux engendrés par un système similaire sans tirant, c'est pourquoi dans le cas d'un bâtiment en briques avec un toit à deux pans il peut être préférable d'introduire un tirant tel que dans le système présenté.

5

- Nombre d'éléments: 3
- Nombre d'équations d'équilibre par élément (dans un plan): 3
- Nombre d'équations d'équilibre: $N = 3 \cdot 3 = 9$
- Nombre de réaction d'appuis: 4
- Nombre de réactions aux nœuds: 6
- Nombre d'inconnues statiques: $U = 4 + 6 = 10$
- Système à N équations pour U inconnues, $U - N = 10 - 9 = 1$, le système est hyperstatique de degré 1.

6



Pour calculer l'effort dans le tirant on isole le nœud ou s'applique la charge.

- $Q_{spots} = 100\ kg = 1\ kN$

- Angle entre le câble et l'horizontale: $\beta = arctan\left(\frac{0.05}{17/2}\right) = 0.337°$

La charge Q_{spots} est reprise par les composantes verticales des forces de traction dans le câble. Le système étant symétrique nous pouvons en déduire que la force de traction à droite est égale à la force de traction à gauche dans le câble. Ainsi si T est la force de traction dans le câble, T_x sa composante horizontale et T_y sa composante verticale, nous avons:

$$\sum F_y = 0 = -2 \cdot T_y + Q_{spots}$$

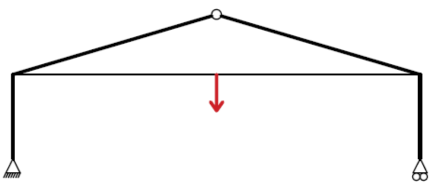
$$T_y = \frac{Q_{spots}}{2} = 0.5\ kN$$

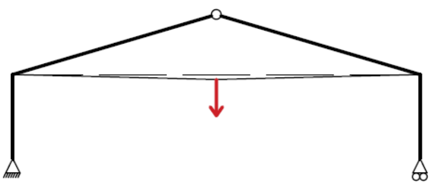
$$T = \frac{T_y}{sin(\beta)} = 85\ kN$$

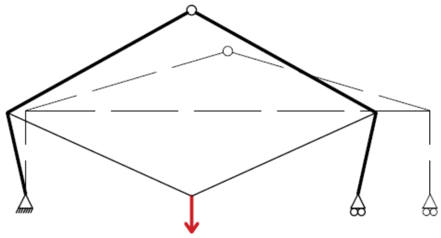
$$T_x = T \cdot cos(\beta) = 85\ kN$$

7

Le cas présenté dans cet exemple est un cas particulier de la statique. En effet les barres considérées dans la plupart des cas ont un comportement dit « linéaire ». Celles-ci peuvent en effet être soumises à des efforts en traction identiques aux efforts de compression ainsi qu'à des moments. Cependant les câbles sont des cas particuliers puisqu'ils ne peuvent pas reprendre d'efforts de compression ni de moments (ou alors négligeables). Il est dit qu'ils ont un comportement « non-linéaire ». Ainsi tant que les charges qui sont appliquées sur le cadre créent un effort de traction dans le câble, le calcul est « linéaire ». Si les charges appliquées sur le cadre engendrent une « compression » du câble, celui-ci ne pouvant pas reprendre cet effort on observe une déformation de la structure. Celle-ci peut se déformer jusqu'à effondrement ou rester en état d'équilibre intermédiaire mais pour vérifier cela il convient d'effectuer ce que l'on appelle un calcul « non-linéaire » qui est itératif. Ainsi, l'exemple présenté est similaire à une nième étape du calcul itératif non-linéaire, la première étant un calcul avec un câble non déformé.







Cela revient au même que de comprimer le câble depuis l'extérieur. La « non-linéarité » du câble va permettre des déformations très importantes. Nous pouvons d'ailleurs vérifier ce phénomène de la même manière que pour la question 1 ; le système devient en effet un mécanisme lorsque le câble est « comprimé » ($U - N = 5 - 6 = -1$) et il est alors nécessaire que la structure soit appuyée sur deux appuis fixes pour être isostatique.

8

Le système expliqué à la question précédente montre que celui-ci est isostatique car l'on remplace le câble par des forces appliquées connues sur le cadre.

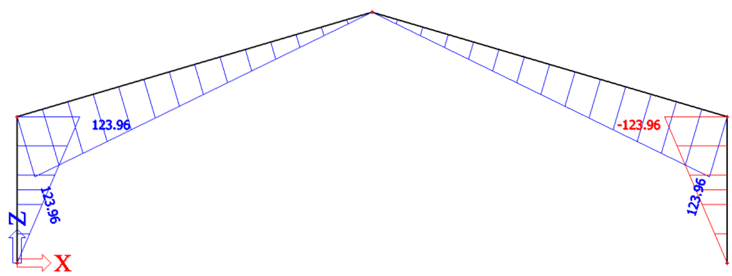


Diagramme des moments

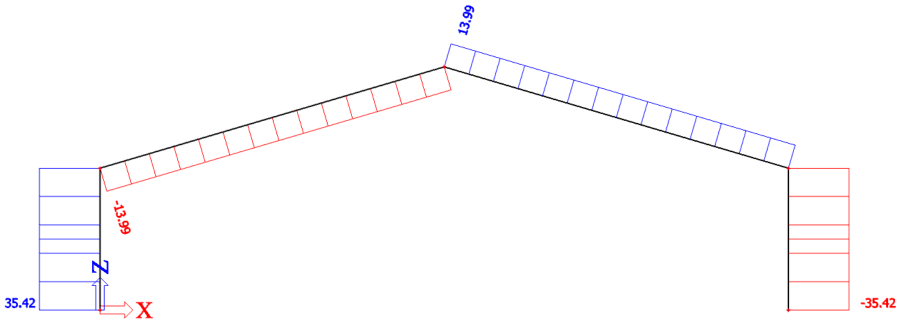


Diagramme de l'effort tranchant

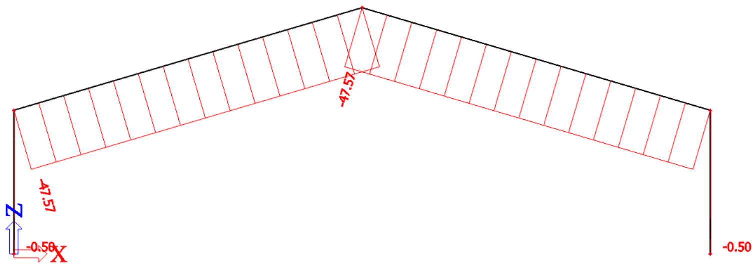


Diagramme de l'effort normal

9

Si la déformation (flèche) tend à être nulle, ou si l'angle du câble tend à être nul, alors la force de traction dans le câble tend vers l'infini. La composante verticale de cette force tend alors vers zéro et la composante horizontale tend vers l'infini, ce qui est physiquement impossible.

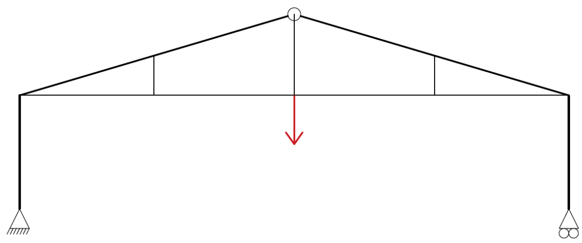
10

Même avec une petite charge verticale appliquée au milieu d'un câble tendu entre deux points fixe, la tension engendrée dans le câble peut-être très grande. Il convient donc de faire très attention à ce genre de situation ou les efforts engendrés dans la structure peuvent être déterminants par la suite lors du dimensionnement.

Note :

- Dans cette situation, en réalité le système va s'adapter (et peut être se rompre) par l'augmentation de la flèche du câble. Il faut effectuer un calcul itératif non-linéaire

- Cela explique pourquoi, souvent, ce genre de système contient des « suspentes » pour soutenir le câble ou tirant et retransmettre les charges verticalement et non horizontalement



4.2 Cadre avec pont-roulant

1

- Nombre d'éléments: 2
- Nombre d'équations d'équilibre par élément (dans un plan): 3
- Nombre d'équations d'équilibre: $N = 2 \cdot 3 = 6$
- Nombre de réaction d'appuis: 3
- Nombre de réactions aux nœuds: 3
- Nombre d'inconnues statiques: $U = 3 + 3 = 6$
- Système à N équations pour U inconnues, $U - N = 6 - 6 = 0$, le système est isostatique

Il est possible de tirer rapidement la même conclusion en observant le fait que le pont roulant est une poutre simple dont les réactions d'appuis sont des forces appliquées à un cadre appuyé comme une poutre simple. Ce sont donc deux systèmes isostatiques.

2

$$4.5\ to = 4500\ kg = 45\ kN$$
$$\sum F_x = 0 = R_{A,x}$$
$$\sum M_{z,A} = 0 = -(9 + 0.5) \cdot Q_y + 27 \cdot R_{By}$$
$$R_{By} = 15.8\ kN$$
$$\sum F_y = 0 = R_{A,y} + R_{B,y} - Q_y$$
$$R_{Ay} = 29.2\ kN$$

3

Pour cette question nous allons procéder au calcul en deux étapes : Dans une première étape nous allons calculer les efforts dans le pont roulant qui agit comme une poutre simple. Les réactions d'appuis du pont roulant seront ensuite introduites sur la structure comme des forces extérieures.

Réactions d'appuis dues au pont roulant

$\sum F_x = 0 = R_{C,x}$

$\sum M_{z,C} = 0 = -9 * Q_y + 26 * R_{\mathrm{Dy}} \rightarrow R_{\mathrm{Dy}} = \frac{9*Q_y}{26} = 15.6\ kN$

$\sum F_y = 0 = R_{C,y} + R_{D,y} - Q_y \rightarrow R_C = Q_y - R_{D,y} = 45 - 15.6 = 29.4\ kN$

Efforts au point C

$\sum F_{x,C} = 0 = N \rightarrow N = 0\ kN$

$\sum M_{z,C} = 0 = -R_{C,y} * 0 + M \rightarrow M = 0\ kNm$

$\sum F_{y,C} = 0 = R_{C,y} + V \rightarrow V = -R_{C,y} = -29.4\ kN$

Efforts à gauche du point d'application de la charge

$\sum F_{x,Q} = 0 = N \rightarrow N = 0\ kN$

$\sum M_{z,Q} = 0 = -R_{C,y} * 9 + Q_y * 0 + M \rightarrow M = 29.4 * 9 = 265\ kNm$

$\sum F_{y,Q} = 0 = R_{C,y} + V \rightarrow V = -R_{C,y} = -29.4\ kN$

Efforts à droite du point d'application de la charge

$\sum F_{x,Q} = 0 = N \rightarrow N = 0\ kN$

$\sum M_{z,Q} = 0 = -R_{C,y} * 9 + Q_y * 0 + M \rightarrow M = 29.4 * 9 = 265\ kNm$

$\sum F_{y,Q} = 0 = R_{C,y} - Q_y + V \rightarrow V = -R_{C,y} + Q_y = -29.4 + 45 = 15.6\ kN$

Efforts au point D

$\sum F_{x,D} = 0 = N \rightarrow N = 0\ kN$

$\sum M_{z,D} = 0 = -R_{C,y} * 26 + Q_y * 17 + M \rightarrow M = 29.4 * 26 - 45 * 17 = 0\ kNm$

$\sum F_{y,D} = 0 = R_{C,y} + V \rightarrow V = -R_{C,y} = -29.4\ kN$

Efforts à droite du point E

$\sum F_{x,E} = 0 = N \rightarrow N = 0\ kN$

$\sum M_{z,E} = 0 = R_{C,y} * 0.5 - M \rightarrow M = 29.4 * 0.5 = 14.7\ kNm$

$\sum F_{y,E} = 0 = R_{C,y} + V \rightarrow V = -R_{C,y} = -29.4\ kN$

Efforts en bas du point E

$\sum F_{x,E} = 0 = V \rightarrow V = 0\ kN$

$\sum M_{z,E} = 0 = M \rightarrow M = 0\ kNm$

$\sum F_{y,E} = 0 = R_{A,y} + N \rightarrow N = -R_{A,y} = -29.2\ kN$

Efforts en haut du point E

$\sum F_{x,E} = 0 = V \rightarrow V = 0\ kN$

$\sum M_{z,E} = 0 = M - R_{C,y} * 0.5 \rightarrow M = 29.4 * 0.5 = 14.7\ kNm$

$\sum F_{y,E} = 0 = R_{A,y} - R_{C,y} + N \rightarrow N = -R_{A,y} + R_{C,y} = -29.2 + 29.4 = 0.2\ kN$

Efforts en bas du point G

$\sum F_{x,G} = 0 = V \rightarrow V = 0\ kN$

$\sum M_{z,G} = 0 = M - R_{C,y} * 0.5 \rightarrow M = 29.4 * 0.5 = 14.7\ kNm$

$\sum F_{y,G} = 0 = R_{A,y} - R_{C,y} + N \rightarrow N = -R_{A,y} + R_{C,y} = -29.2 + 29.4 = 0.2\ kN$

Efforts à droite du point G

$\sum F_{x,G} = 0 = N \rightarrow N = 0\ kN$

$\sum M_{z,G} = 0 = M - R_{C,y} * 0.5 \rightarrow M = 29.4 * 0.5 = 14.7\ kNm$

$\sum F_{y,G} = 0 = R_{A,y} - R_{C,y} - V \rightarrow V = R_{A,y} - R_{C,y} = 29.2 - 29.4 = -0.2\ kN$

Efforts à gauche du point H

$\sum F_{x,H} = 0 = N \rightarrow N = 0\ kN$

$\sum M_{z,H} = 0 = M - R_{D,y} * 0.5 \rightarrow M = 15.6 * 0.5 = 7.8\ kNm$

$\sum F_{y,H} = 0 = R_{B,y} - R_{D,y} + V \rightarrow V = -R_{B,y} + R_{D,y} = -15.8 + 15.6 = -0.2\ kN$

Efforts en bas du point H

$\sum F_{x,H} = 0 = V \rightarrow V = 0\ kN$

$\sum M_{z,H} = 0 = M + R_{C,y} * 0.5 \rightarrow M = -15.6 * 0.5 = -7.8\ kNm$

$\sum F_{y,H} = 0 = R_{A,y} - R_{C,y} + N \rightarrow N = -R_{A,y} + R_{C,y} = -29.2 + 29.4 = 0.2\ kN$

Efforts à gauche du point F

$\sum F_{x,F} = 0 = N \rightarrow N = 0\ kN$

$\sum M_{z,F} = 0 = R_{D,y} * 0.5 + M \rightarrow M = -15.6 * 0.5 = -7.8\ kNm$

$\sum F_{y,F} = 0 = R_{D,y} - V \rightarrow V = R_{D,y} = 15.6\ kN$

Efforts en bas du point F

$\sum F_{x,F} = 0 = V \rightarrow V = 0\ kN$

$\sum M_{z,F} = 0 = M \rightarrow M = 0\ kNm$

$\sum F_{y,F} = 0 = R_{B,y} + N \rightarrow N = -R_{A,y} = -29.2\ kN$

Efforts en haut du point F

$\sum F_{x,F} = 0 = V \rightarrow V = 0\ kN$

$\sum M_{z,F} = 0 = M + R_{C,y} * 0.5 \rightarrow M = -15.6 * 0.5 = -7.8\ kNm$

$\sum F_{y,F} = 0 = -R_{B,y} + R_{D,y} + N \rightarrow N = R_{B,y} - R_{D,y} = 29.2 - 29.4 = -0.2\ kN$

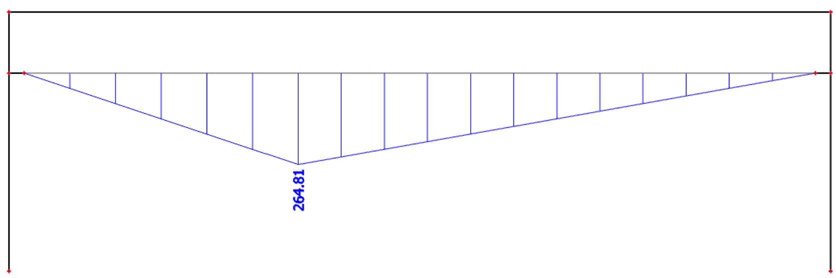


Diagramme des moments (pont roulant)

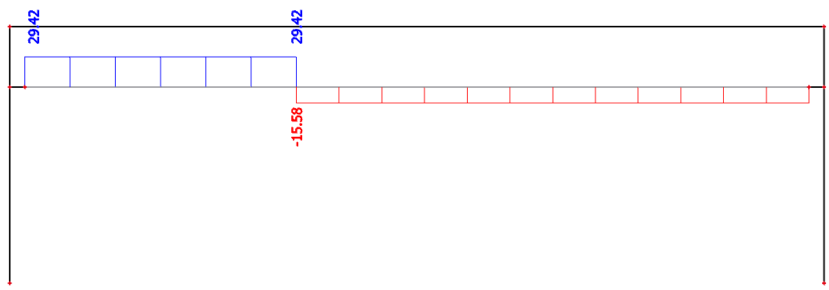


Diagramme de l'effort tranchant (pont roulant)

Diagramme de l'effort normal (pont roulant)

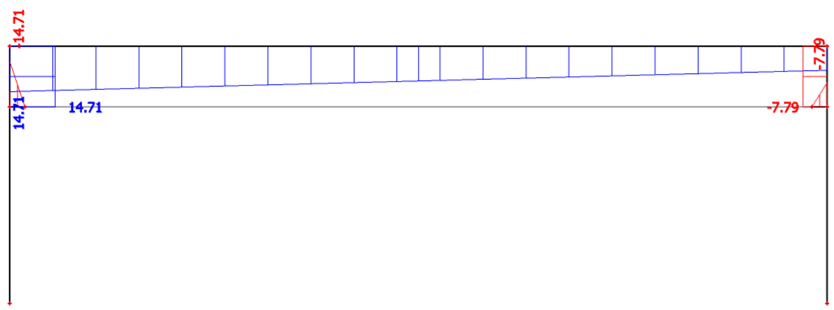


Diagramme des moments (cadre)

Diagramme de l'effort tranchant (cadre)

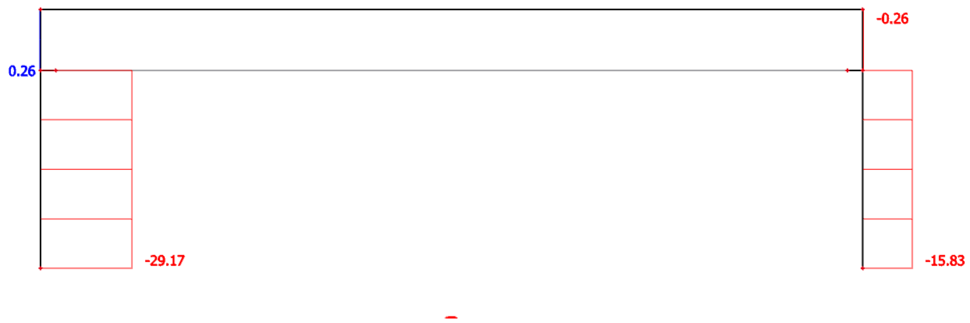


Diagramme de l'effort normal (cadre)

4

5

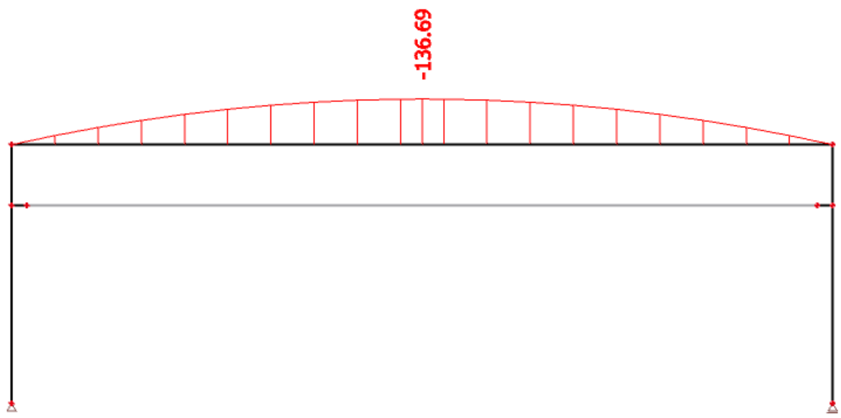


Diagramme des moments

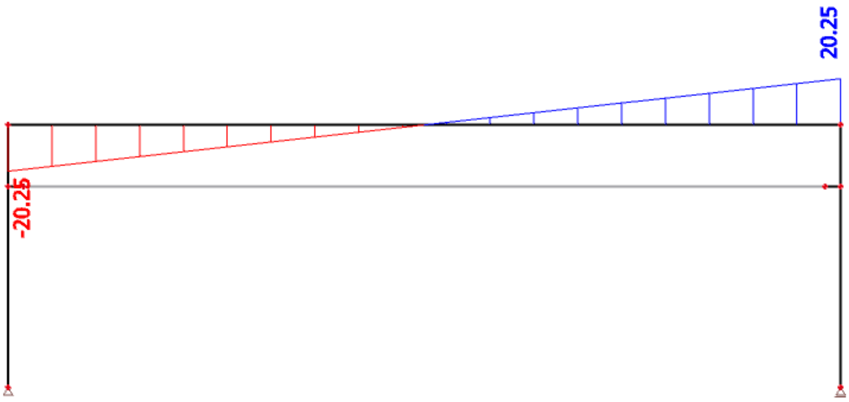


Diagramme de l'effort tranchant

Diagramme de l'effort normal

6

Les charges de vent peuvent créer des dépressions importantes. Les structures en acier étant généralement légères (en comparaison des structures en béton armé par exemple), celles-ci peuvent être sujettes à des efforts de traction importants qui risqueraient de générer des instabilités globales. Par exemple, dans la réalité, les appuis sont principalement dimensionnés pour reprendre des efforts de compression et non de traction et de telles situations peuvent être critiques et il est important de considérer les efforts de traction engendrés par le vent.

7

8

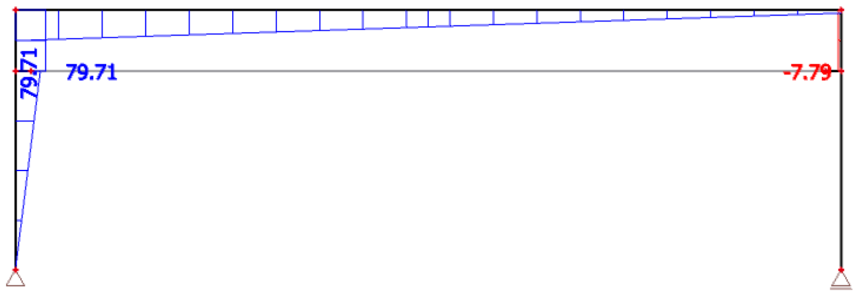


Diagramme des moments (cadre)

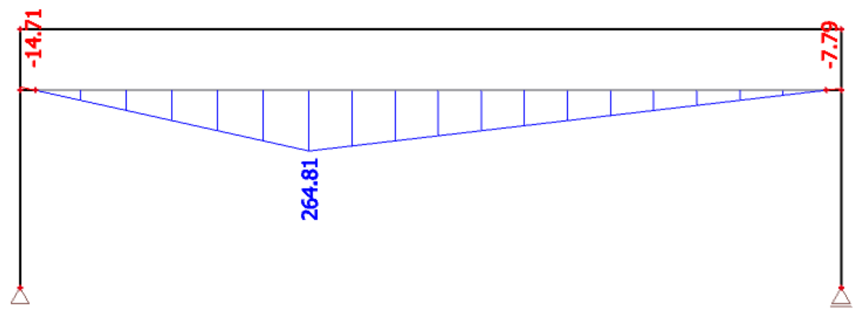


Diagramme des moments (pont roulant et corbeaux)

Diagramme de l’effort tranchant (cadre)

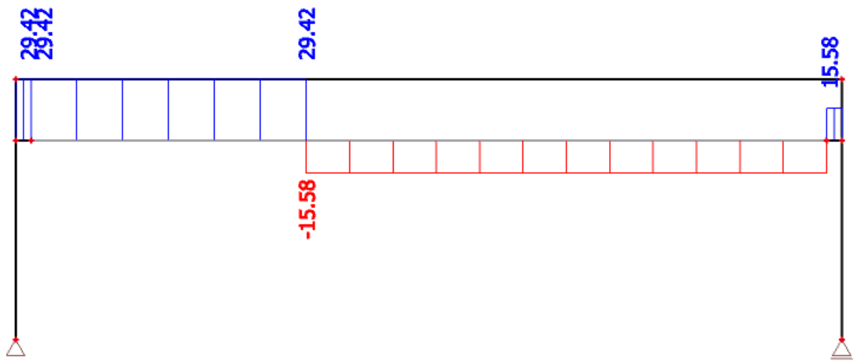


Diagramme de l’effort tranchant (pont roulant et corbeaux)

Diagramme de l’effort normal (cadre)

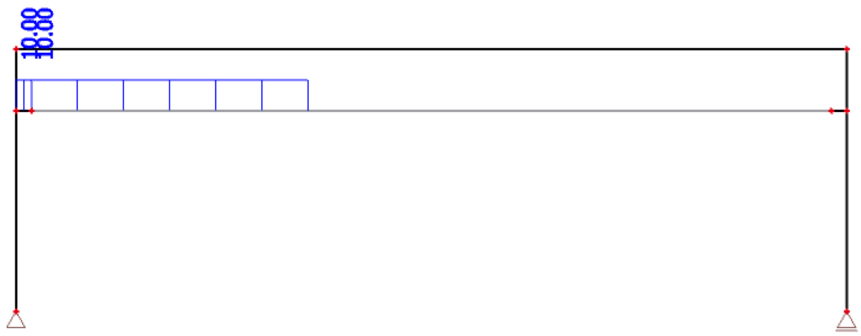


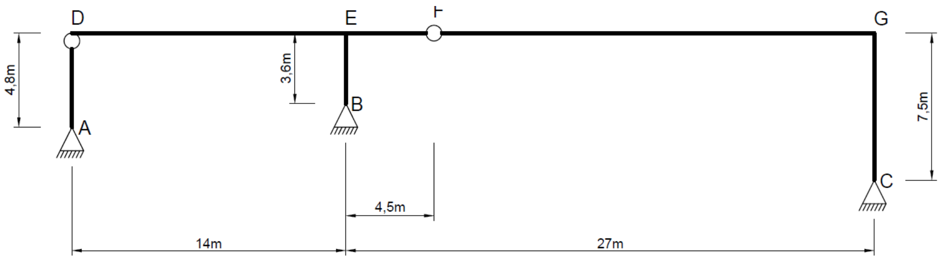
Diagramme de l'effort normal (pont roulant et corbeaux)

9

Les efforts engendrés par des charges dynamiques peuvent être très importantes. Ici par exemple, la force de freinage du pont roulant engendre des moments beaucoup plus importants dans les angles du cadre et sont donc déterminants pour le dimensionnement de la structure.

4.3 Sporthalle Buchholz

1



2

Nombre d'éléments: 3

Nombre d'équations d'équilibre par élément (dans un plan): 3

Nombre d'équations d'équilibre N = 3 x 3 = 9

Nombre de réaction d'appuis: 6

Nombre de réactions aux nœuds: 4

Nombre d'inconnues statiques U = 6 + 4 = 10

Système à N équations pour U inconnues U − N = 10 − 9 = 1 système hyperstatique de degré 1

3

Cette donnée supplémentaire élimine une inconnue dans le système d'équations d'équilibre. Le nombre d'inconnues de réaction d'appuis passe alors de 6 à 5. Ainsi le nombre d'équations d'équilibre est égal au nombre d'inconnues et le système peut alors être résolu de la même manière qu'un système isostatique.

4

Nous avons donc:

$$V_B = 41.35\ kN$$

$$H_A = 0\ kN$$

Pour résoudre le problème, Il faut séparer le système en 3 sous-systèmes, en effectuant une coupe en D et en F et en extériorisant les forces au droit de ces rotules. Nous obtenons alors 9 équations d'équilibre pour 8 inconnues (2 réactions verticales, 2 horizontales, 2 forces en D et 2 en F).

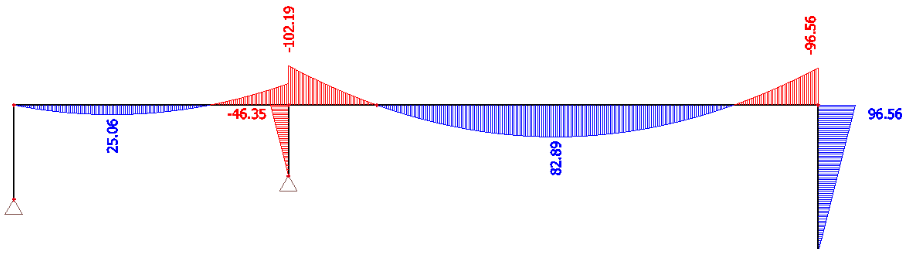


Diagramme des moments

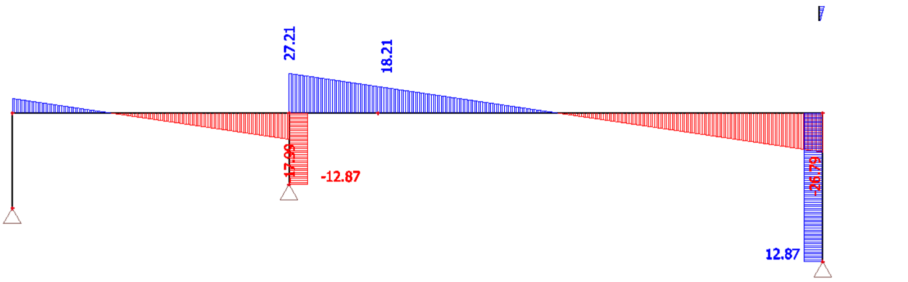


Diagramme de l'effort tranchant

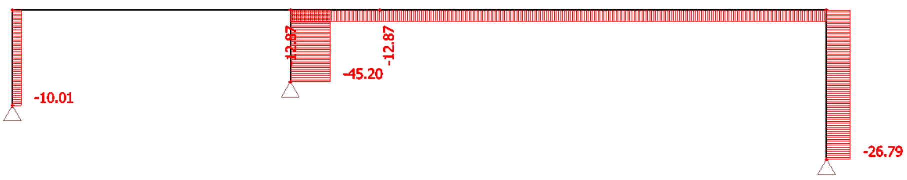
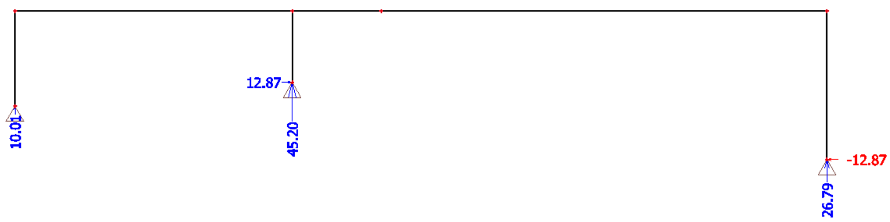


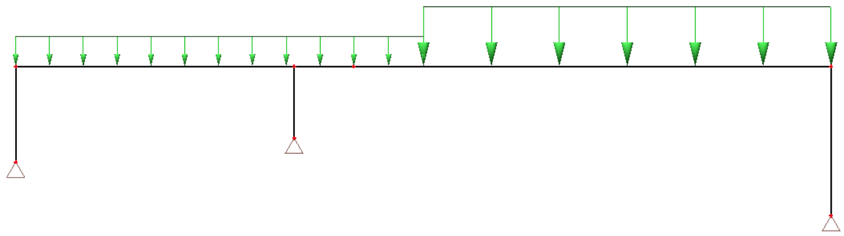
Diagramme de l'effort normal



Réactions d'appuis

5

Nous avons donc la répartition de charge de neige suivante,



Nous savons que,

$$V_A = 4.37\ kN$$

$$V_B = 30.41\ kN$$

$$V_C = 26.72\ kN$$

et de l'exercice précédent,

$$H_A = 0\ kN$$

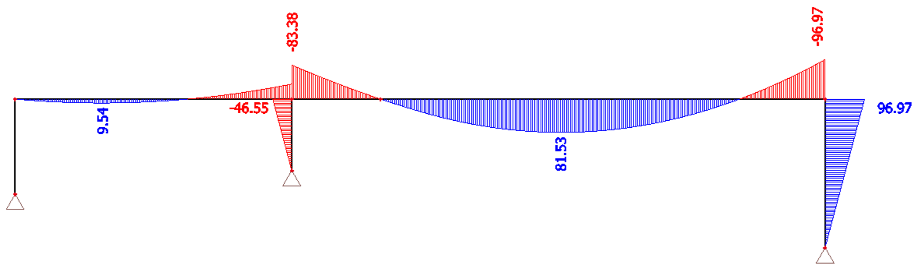


Diagramme des moments

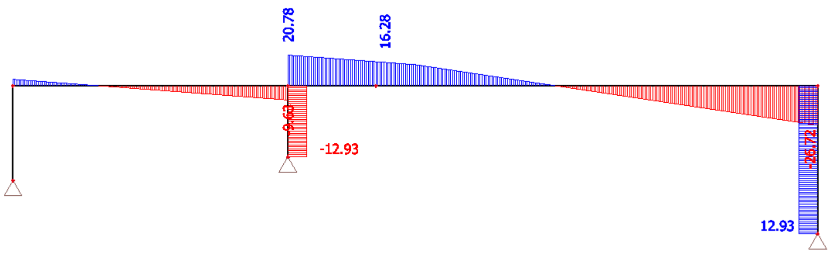


Diagramme de l'effort tranchant

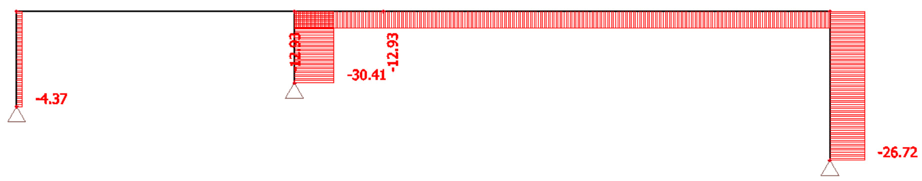
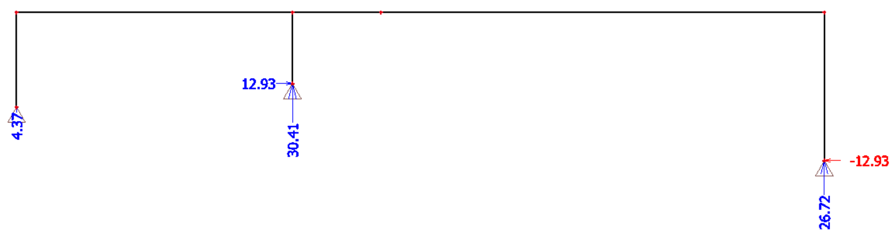


Diagramme de l'effort normal



Réactions d'appuis

6

Nous voyons que la partie droite de la structure (à droite du point F) est soumise aux mêmes efforts que dans la question précédente. La rotule au point F rend en effet la partie droite de la structure quasiment indépendante vis-à-vis de la partie de gauche. La charge verticale étant restée quasiment identique, les efforts dans cette partie n'ont en effet quasiment pas varié.

La partie de gauche est, quant à elle, dépendante des charges qui lui sont appliquées directement mais aussi des charges qui sont appliquées sur la partie de droite car celle-ci est appuyée par le biais de la rotule en F sur la partie gauche de la structure.

7

Nous voyons tout de suite au point que la colonne de gauche est «soulagée» par l'application d'une charge plus importante sur la travée principale. Nous pouvons donc en déduire que cet effet sera encore plus important dans le cas où la charge de neige est totalement supprimée de la travée de gauche. Si l'on fait l'hypothèse que la variation est linéaire, ce qui n'est pas le cas mais donne une estimation, on peut même supposer que l'effort dans la colonne n'est plus de la compression mais de la traction.

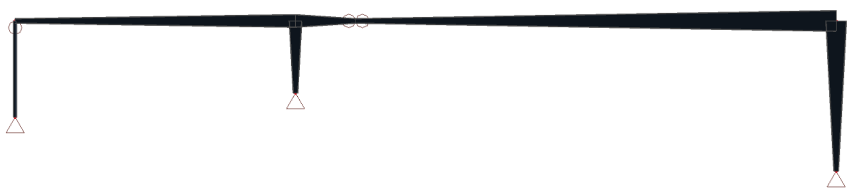
Nous pouvons vérifier cela dans la note donnée au point b., dans laquelle nous donnons la solution où la colonne est soumise à un effort de traction sous charge de neige. En réalité le poids propre de la structure viendra certainement compenser l'effort de traction dans le cas présent mais il est important de vérifier tous les cas possibles lors d'un dimensionnement.

8

Une première approche simple avec des inerties constantes est importante pour comprendre le fonctionnement global de cette structure. Cette première approche est rapide et permet de tirer certaines conclusions quant à la répartition des efforts. Il est cependant important de comprendre que plus les variations d'inertie sont grandes, et plus la structure est hyperstatique, plus le modèle et les résultats obtenus sont erronés. C'est pourquoi il est alors important dans un deuxième temps d'effectuer une vérification en considérant les sections variables.

Le fait d'augmenter les inerties aux angles de cadre a pour conséquence d'attirer encore plus les moments à ces endroits (les efforts se concentrent aux endroits les plus rigides). Par conséquent, il est probable que le moment en travée de la portée de 27 m va diminuer et se reporter sur les nœuds E et G, i.e. moments négatifs plus élevés. Il n'est pas possible de mieux connaître la répartition des efforts sans modèle de calcul avec inerties variables.

Note: Ci-après les calculs de la question 4 en tenant compte des inerties variables.



Modèle à inerties variables

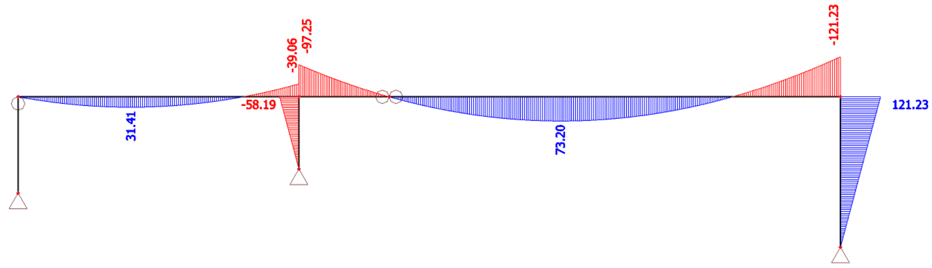


Diagramme des moments

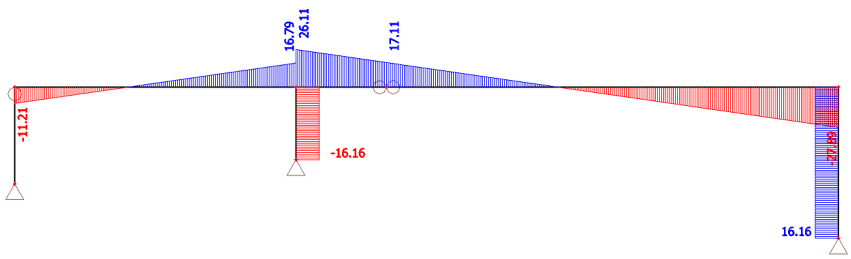


Diagramme de l'effort tranchant

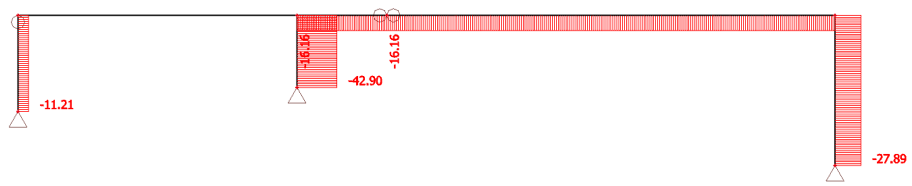


Diagramme de l'effort normal

Nous voyons dans la comparaison entre les résultats obtenus à la question 4 et ceux obtenus ici que, comme nous le pensions, plus l'inertie d'un élément ou d'une partie d'élément est grande (et par la même la rigidité) plus celui-ci attirera les efforts. Dans le cas présent, les angles de cadre ont une inertie plus grande et reprennent donc plus d'efforts que dans un modèle avec inerties constantes. On passe de 82.89 kNm en travée à 73.20 kNm. C'est l'angle de cadre de droite, nœud G, qui voit son moment augmenter le plus, de 96.56 kNm à 121.23 kNm.

C'est la raison pour laquelle il ne faut pas négliger cette deuxième étape, car si les angles de cadres ont étés dimensionnés selon les calculs du modèle à inertie constante dans une première étape, il se peut que les efforts supplémentaires obtenus après une vérification à l'aide d'un modèle à inertie variable invalident le dimensionnement réalisé à la première étape. Le dimensionnement/vérification est donc un processus itératif.

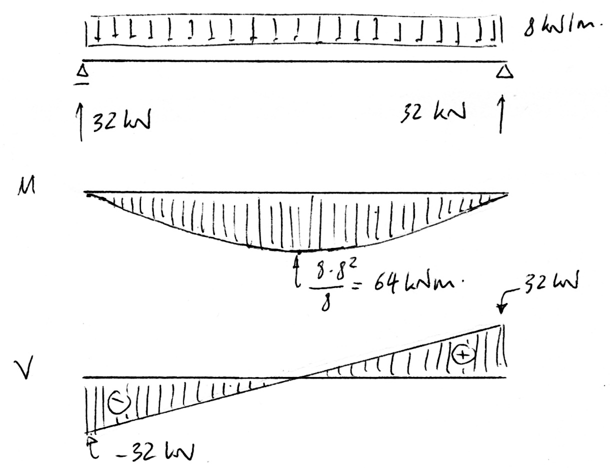
5 Structures en béton (BA5)

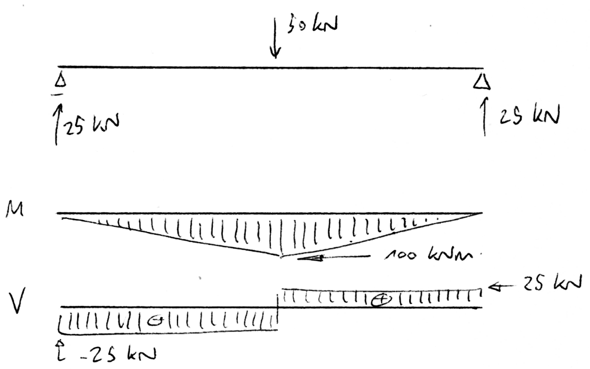
5.1 Static analysis of a frame

1

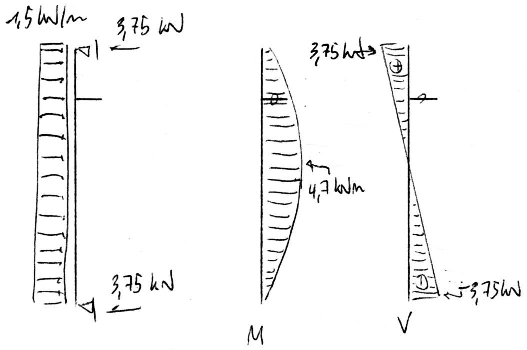
If we look closely at this structure, we can see that the beam and the two columns can be analysed separately with regards to the moment and the shear force. This means that we can solve the three parts independently. Additionally, we propose to use the principle of superposition and calculate separately the internal forces diagrams for the distributed loads and the concentrated loads, summing up the diagrams in the end. The normal force diagram is left for later as it can be obtained quickly without using the principle of superposition.

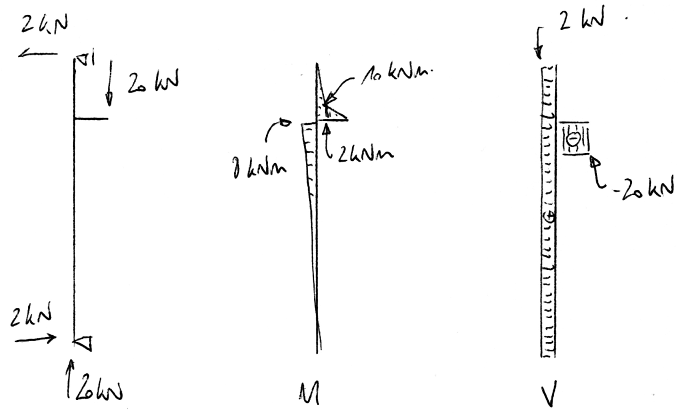
First for the beam we obtain the following diagrams:



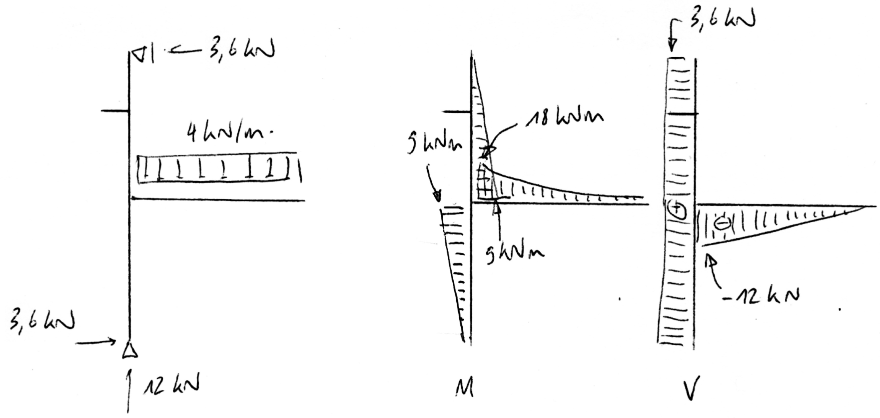


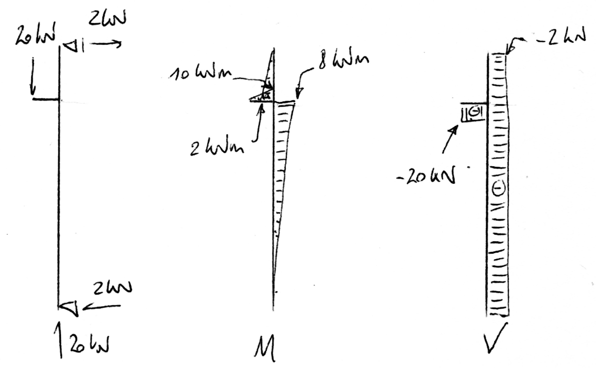
Then for the left-hand column:



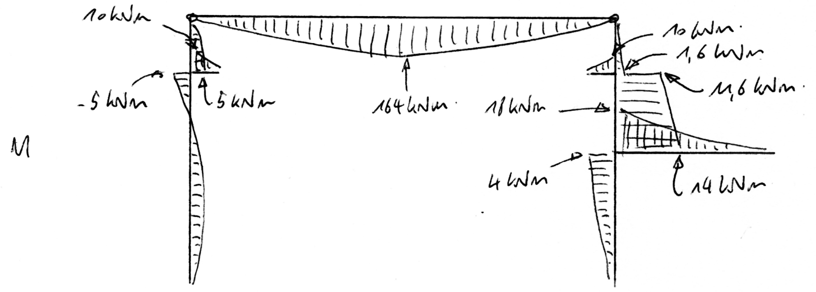


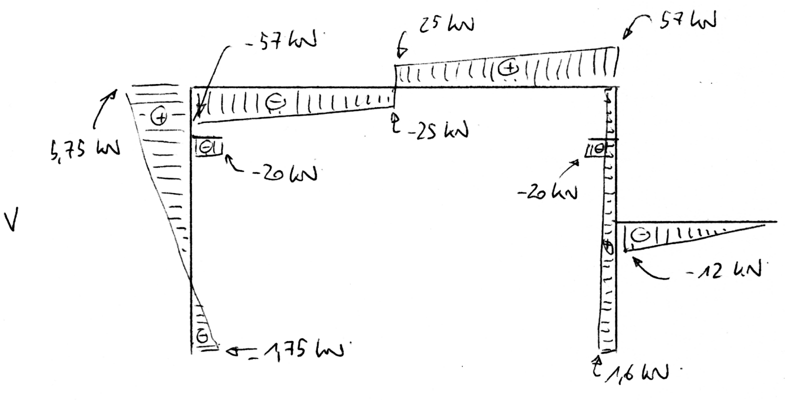
For the right-hand beam:

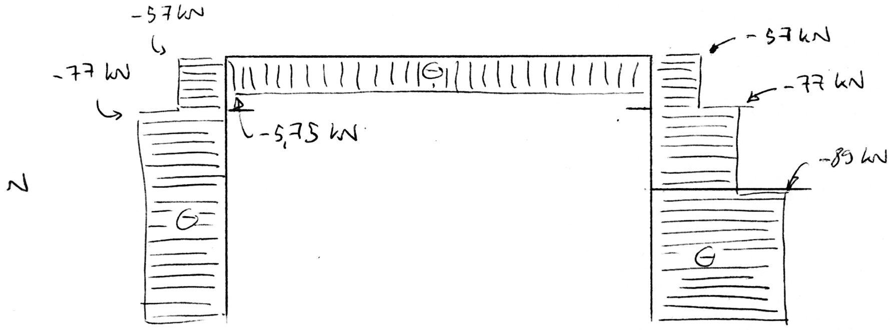




Finally, we can sum up the bending moment and shear force diagrams. The normal force diagram is obtained by using the shear diagram. For each node — starting from the hinges and going down — we can use equilibrium of vertical forces. This gives us the normal force diagram in the columns. Using equilibrium of horizontal forces around the left-hand corner of the frame will give us the normal force inside the beam which is transmitted to the support at the right-hand corner of the frame.

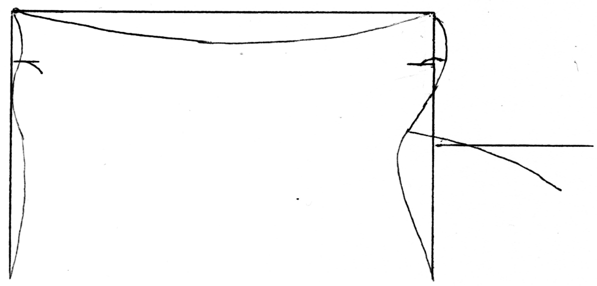






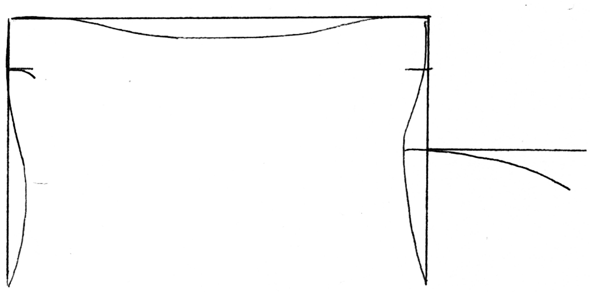
2

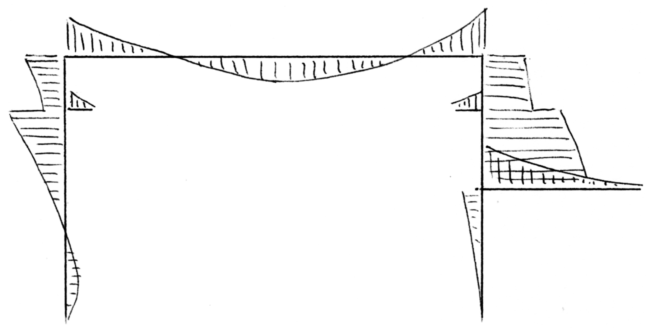
The deformed shape is obtained from the moment diagram. At the hinges, the right angle should not be conserved.



3

If we remove the two hinges, we know that the frame will deform differently. We begin by drawing the deformed shape instinctively. The main difference is that the right angle will be conserved at the corners of the frame. This has an effect on the curvature and therefore on the moment diagram. We can consider that the shape of the moment diagram does not change but the values at the corners of the frame are not forced to 0. Using that, we can modify the moment diagram obtained previously and adapt it so it produces the deformed shape of the hyperstatic frame.





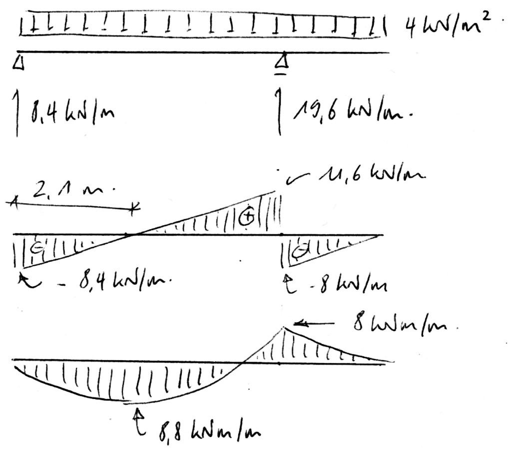
4

As we see in the deformed shape, the displacements both in the beam and in the column are smaller, the structure is working together rather than apart to transmit the bending moment which means that it becomes more rigid. Additionally, the effect of developing a negative moment at the corner of the frame is that the mid-span moment of the beam is reduced. This means that smaller cross-section dimensions or/and reinforcement for this beam can be chosen. In the columns however, the behaviour is not really improved. Overall, monolithic structures are preferred when using reinforced concrete as producing rigid corners is easier with this material than making hinges and the overall behaviour is much improved.

5.2 Simplified analysis of a concrete slab

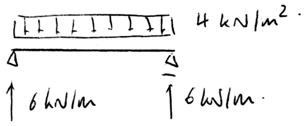
1

First, we calculate the reactions of the beam using equilibrium of moments. Second, we calculate the shear diagram which gives us the point where the moment is maximum in the 5 m span. Third, we can draw the moment diagram. All the exterior and internal forces are usually expressed per meter of slab. This means that distributed loads are in kN/m^2, shear force in kN/m and moments in kNm/m. In this case, the strip is already 1 m wide so there is no difference between expressing them per meter or not.



2

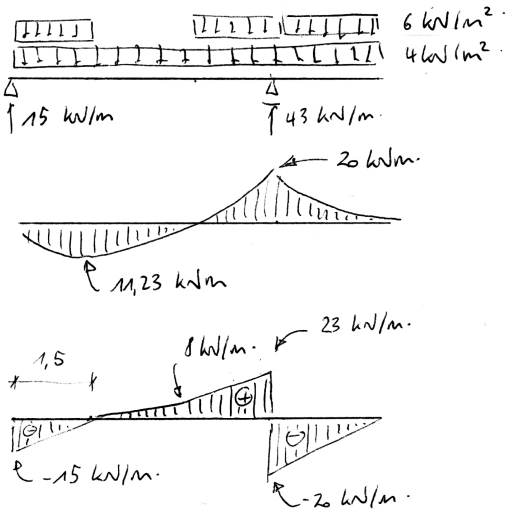
Strips 1 have different width but we express the exterior and internal forces per meter of slab.



3

The reactions of Strips 1 which are applied to Strips 2 need to be distributed on the leaning surface. As they were expressed in kN/m, we only need to divide them by the width of Strips 2 so in turn, the reactions and internal forces of Strips 2 are expressed per meter of slab. As in this case, the width is 1 m, this is instantaneous.

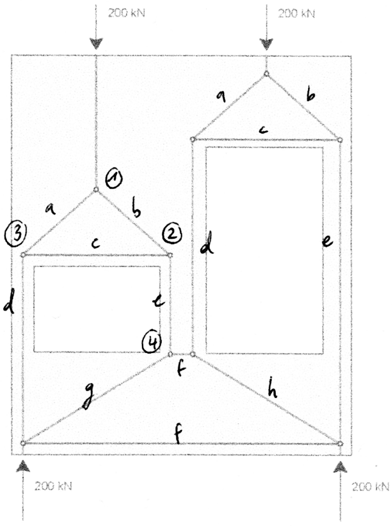
Similarly to question 1, we calculate the internal forces and reactions.

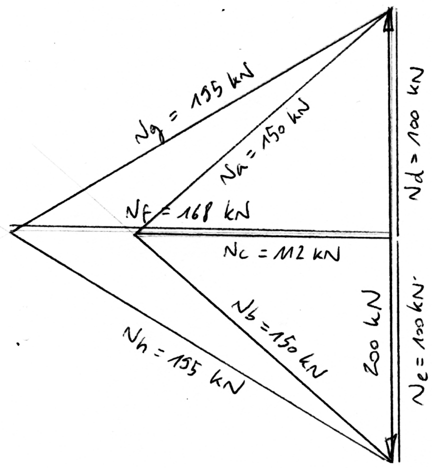


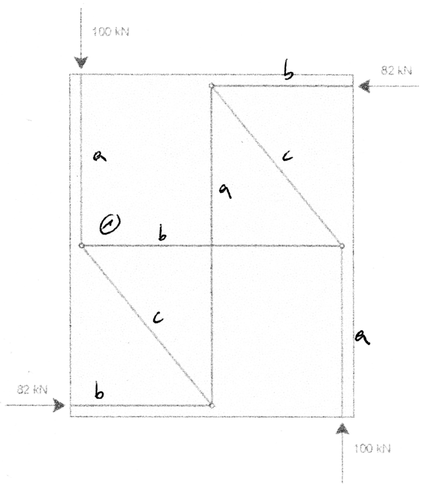
We can see how this model produces both the larger moments in the slab and larger reactions on the wall. It means this effect of force redistribution around openings needs to be taken into account. However, this model is conservative as the real behaviour is more complex. For instance, we could have chosen to widen Strips 1 which would reduce both moment and reactions. We also could make Strips 1 hyperstatic by supporting it on more strips which also would be favourable. To model a more realistic behaviour of the slab, we need a finite element analysis.

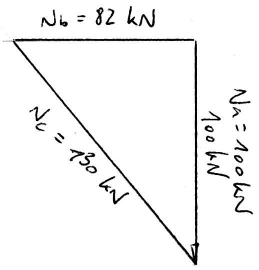
5.3 Strut-and-tie models

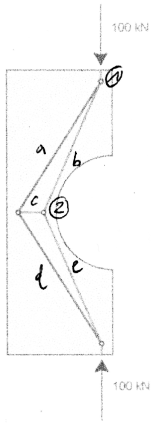
For the three strut-and tie models, we number the nodes in the order in which we solve them. When there is no number, we did not need to solve this node to compute all the forces because of a symmetry axis or a similar reason. Each bar is given a letter to link it with the polygon of forces. When two bars have the same letter, it means they have the same normal force.

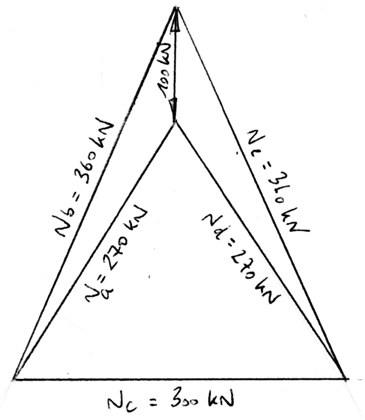









100 kN
1
a
b
c
2
d
e
100 kN

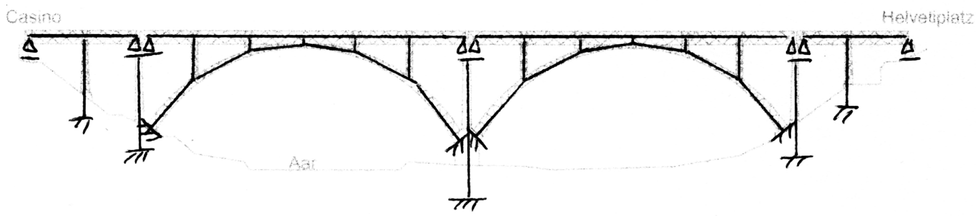

100 kN
Nb = 360 kN
Ne = 360 kN
Na = 270 kN
Nd = 270 kN
Nc = 300 kN

6 Conception des ponts (BA6)

6.1 Defining a static system

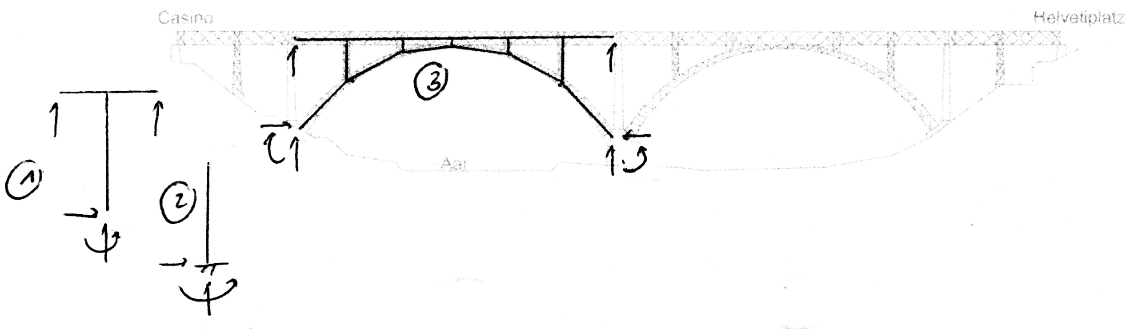
Kirchenfeldbrücke

The static system we propose for this bridge is the following. The expansion joints on each concrete pier and at the abutments are modelled by sliding supports. If the arches where hinged at the base, there would be a narrowing of their width near it. As this is not the case, we can assume that the arches are fixed at their supports. Similarly, the props which connect the arches with the deck do not seem hinged either at the deck or at the arch.



This bridge is actually composed by several structures:

- The lateral spans which connect the abutments to the beginning of the arches (1). The number of unknowns is 5 and the number of equations 3, they are 2 times hyperstatic.

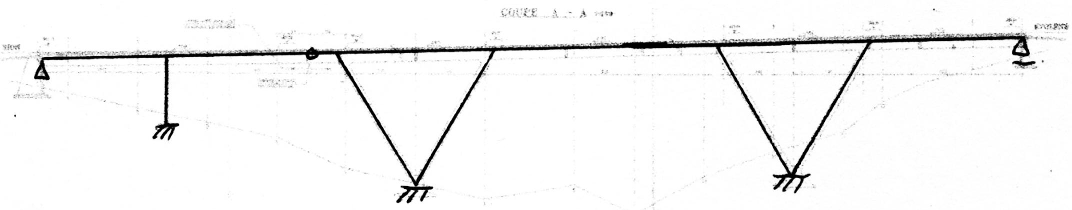


- The concrete piers which are isostatic cantilevers (2)
- The arches themselves (3). There are 8 unknowns and 3 equations so these are 5 times hyperstatic. Additionally, they are 12 times

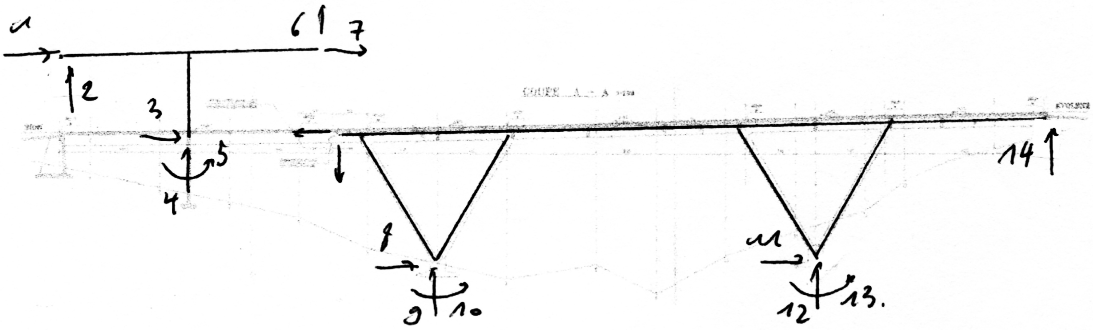
internally hyperstatic. Indeed, if the reactions forces are given to us, we would need to remove four props in order to be able to compute all the internal forces in the structure.

Pont de la Luette

If we look closely at the right-hand abutment, we can see that there is an expansion joint there. This bridge is called a “fixed bridge" as it is held at one abutment and free to move at the other. Additionally, there is a Gerber joint which is usually modelled by a hinge. In a concrete bridge, the piers are usually fixed at the top and at the bottom unless we can see a bearing.

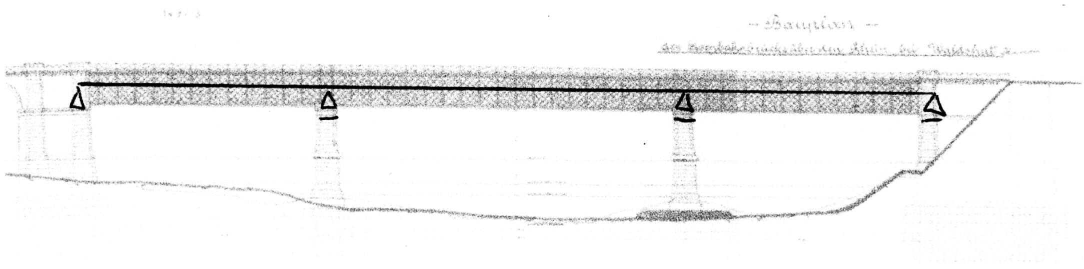


To determine the degree of hyperstaticity of this bridge, we dislocate the structure and count the number of unknows. Here we have 14 unknowns and 6 equations. The system is 8 times hyperstatic. Additionnally, it is 6 times hyperstatic internally. Indeed, if all the reactions are known, the internal forces inside of the struts cannot be computed using only the equilibrium equations.



Waldshut–Koblenz Rhine Bridge

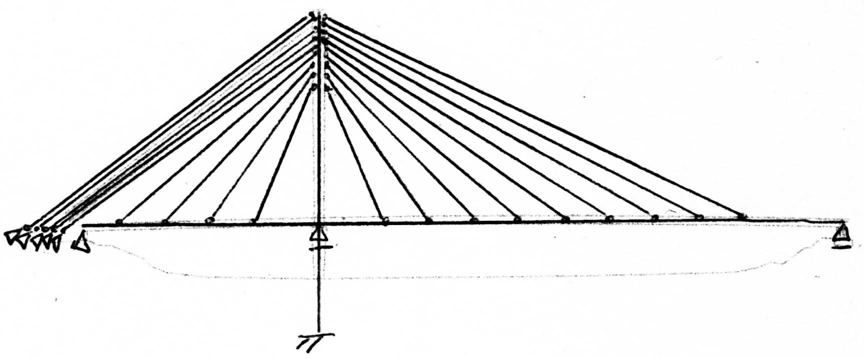
This bridge is a continuous beam. We can assume that the bridge is held laterally only at one abutment.



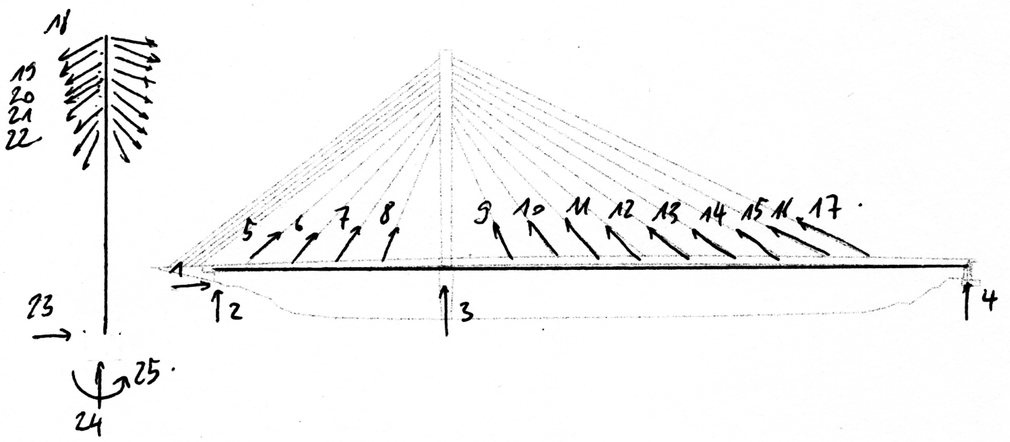
Instantly, we see that the bridge is two times hyperstatic.

Pont Eric-Tabarly

We can see that the bridge is held laterally at the left-hand abutment and left free to expand at the right-hand one. The support of the deck at the level of the mast is usually a sliding support. The cables cannot withstand any other forces than traction so we can model them as bars (hinged at both extremities). The five cables at the left-end extremity of the bridge are called "retaining stays" as their function is to hold the tip of the mast. They are anchored directly in the abutment and do not support the deck.

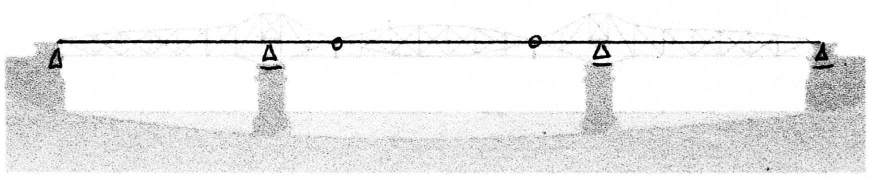


The structure is dislocated and the number of reactions counted. Here we simplified the system and considered the deck was directly connected to the mast without taking the cables into account. We obtain 25 unknowns and 6 equations, meaning that the bridge is 19 times hyperstatic.



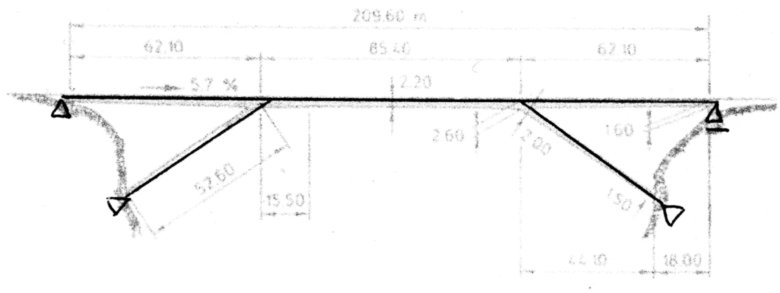
Mainbrüucke Haßfurt

First we can consider that this bridge is only held laterally at one abutment. Then we see that it is a cantilever beam with two hinges in the middle span. We can easily show that this structure is isostatic.



Pont de la Dala

First, we can see that the bridge is held laterally at the left support and that there is an expansion joint at the right abutment. As the struts narrow down near their base, we can conclude that the supports at the base of the struts are hinged. However, the struts widen as they meet the deck so they are fixed there. We can count the number of reactions and find that the structure is 4 times hyperstatic.



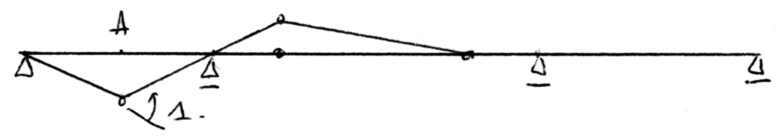
209.60 m
62.10
65.40
62.10
5.7 %
2.20
2.60
2.00
1.60
52.60
15.50
1.50
44.10
18.00

6.2 Longitudinal design

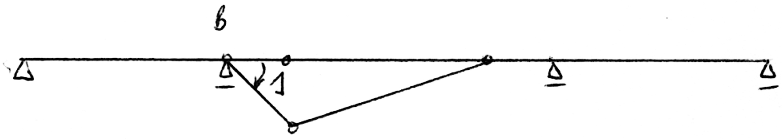
1

To draw the line of influence of the moment at point A, we introduce a hinge at point A and apply a unit rotation to it. In this case, the value we can read at a given point is the moment created at A by a unit load located at this point.

The direction in which we draw the line is not too important because we can always find the sign of the moment instinctively after reading the value if needed. In many cases, only the shape of the line of influence is necessary and not its values.



To draw the line of influence of the moment at point B, we do the same as previously.



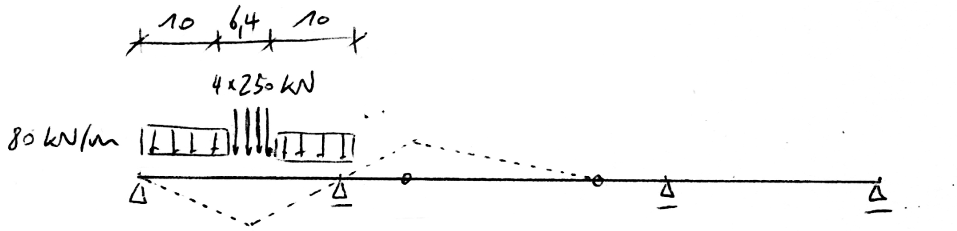
2

The defining load model is the one which creates the most important moment at A or B. Using the lines of influence, we can place both load models where they seem most unfavourable separately. We choose their placement by making sure the most important loads are where the moment they create (given by the line of influence) is the most important.

To compare the two models, we need to calculate their effect separately. This is done by multiplying the value of each load by its effect which can be read on the line of influence (it is the same procedure as when we compute the virtual work of a load when we use the Theorem of virtual displacements).

Moment at point A

At point A, we can introduce model 1 this way to obtain the maximum effect. The most important loads are the concentrated forces so they are placed around the maximum of the line of influence. To the right, the line of influence changes sign so no load should be applied there as they would have a favourable effect on the moment at point A.

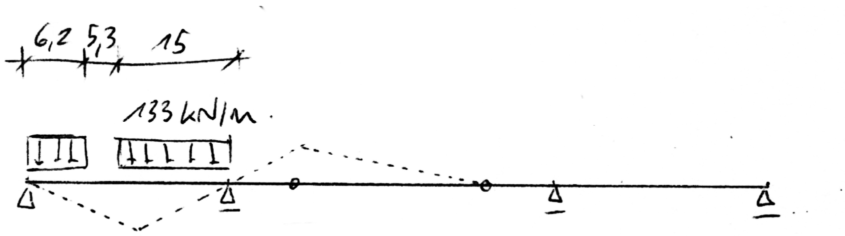


We know that the rotation at point A of the line of influence is 1 so the rotation at each support is $\theta = 0.5$.

Therefore:

$$M_{A,1} = 2 \cdot 80 \cdot 10 \cdot \theta \cdot \frac{10}{2} + 2 \cdot 2 \cdot 250 \cdot \theta \cdot (10 + 1.6) = 9800 \; kNm$$

Similarly, we introduce the loads of model 2 in the most unfavourable manner and compute the moment at point A they create.



$$M_{A,2} = 133 \cdot 6.2 \cdot \theta \cdot \frac{6.2}{2} + 133 \cdot 1.75 \cdot \theta \cdot \left(6.2 + 5.3 + \frac{1.75}{2}\right) + 133 \cdot 13.25 \cdot \theta \cdot \frac{13.25}{2}$$

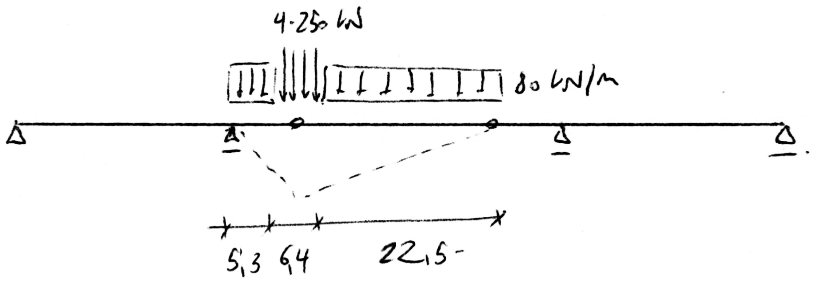
Finally,

$$M_{A,2} = 8556 \; kNm$$

The first model is determinant in this case and should be used to design the bars of the lateral spans.

Moment at point B

At point B, we can introduce model 1 this way to obtain the maximum effect. In this case, the rotation at point B is $\theta_1 = 1$ so the rotation at the second hinge is $\theta_2 = \frac{\theta_1 \cdot 8.5}{25.7} = 0.33$.

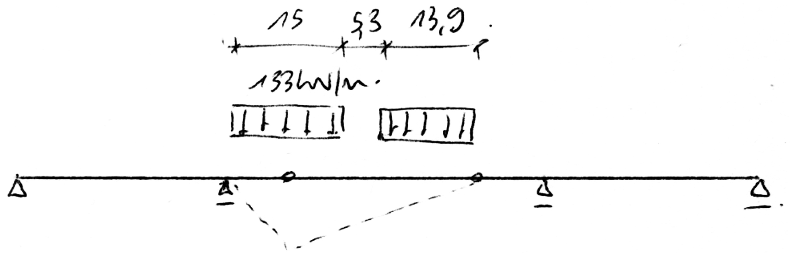


$$M_{B,1} = 80{\cdot}5.3{\cdot}\theta_1{\cdot}\frac{5.3}{2}+2{\cdot}250{\cdot}\theta_1{\cdot}(5.3+1.6)+2{\cdot}250{\cdot}\theta_2{\cdot}(22.5+1.6)+80{\cdot}22.5{\cdot}\theta_2{\cdot}\frac{22.5}{2}$$

Finally,

$$M_{B,1} = 15233\ kNm$$

The second model is introduced this way.



$$M_{B,2} = 133{\cdot}8.5{\cdot}\theta_1{\cdot}\frac{8.5}{2}+133{\cdot}6.5{\cdot}\theta_2{\cdot}\left(13.9+5.3+\frac{6.5}{2}\right)+13.9{\cdot}133{\cdot}\theta_2{\cdot}\frac{13.9}{2}$$

Finally,

$$M_{B,2} = 15449\ kNm$$

The second model is determinant in this case so it should be used to design the bars which are directly above the struts.

3

To calculate the moments in bars 1 and 2, there is no need to take in consideration the geometry of the truss.

For a uniformly loaded beam, we know that the moment is maximum at mid-span. Additionally, we know that a moment M is a pair of forces F with equal value $F = \frac{M}{z}$ but with inverse directions. z is the level arm or distance between the two forces.

In this case, the loads are equivalent to a uniformly distributed load $q = \frac{6 \cdot 400}{25.7} = 93.3\ kN/m$.

The mid-span moment is $M = \frac{q \cdot L^2}{8} = \frac{93.3 \cdot 25.7^2}{8} = 7703\ kNm$.

Then, the normal force in members 1 and 2 is the same and its value is $\frac{M}{z} = \frac{7703}{4.3} = 1791\ kN$.

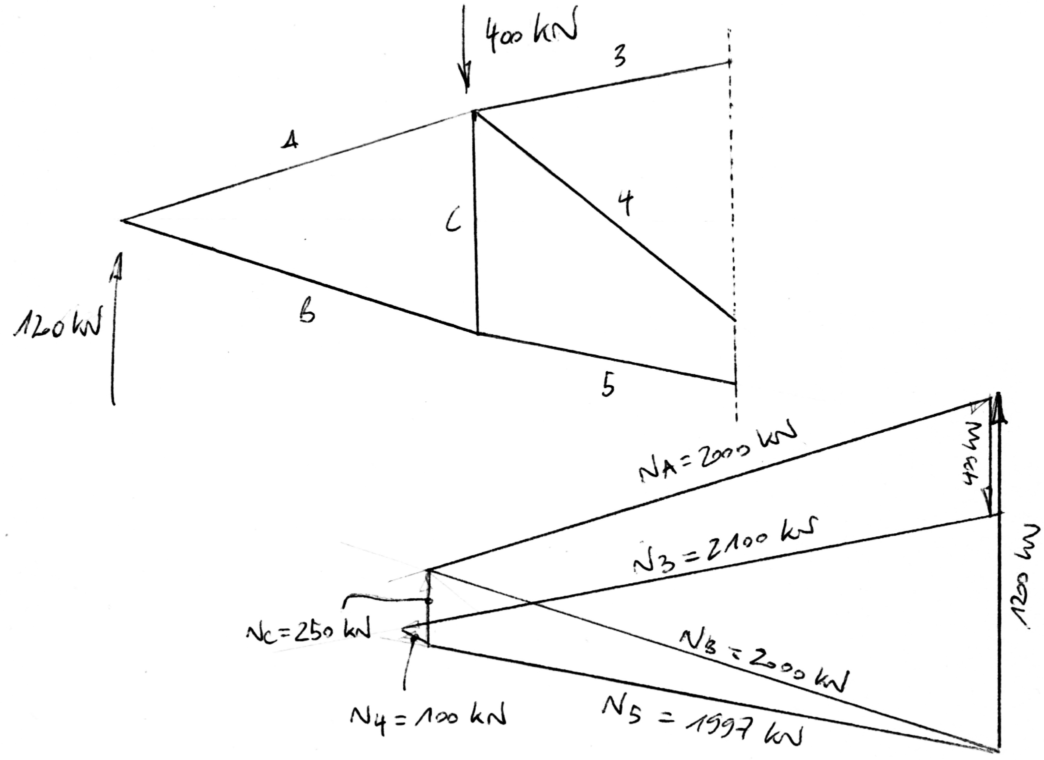
We can find instinctively the direction of the forces: as the beam will deflect downwards, bar 2 will be in traction and bar 1 in compression.

4

Graphic statics is a very powerful and quick way to calculate forces inside of a truss because it is based on drawing rather than on trigonometry and solving equations. It can be done on a piece paper with a satisfactory precision but also on a CAD software to obtain exact values.

Here we only need to calculate of few internal forces. We begin by calculating the reaction at the left support: $R = \frac{6 \cdot 400}{2} = 1200\ kN$. Then, we can make a section through bars 3, 4 and 5 and draw the resulting left-hand part of the truss on a piece of paper. Then, we first solve the node at the left support, secondly the bottom node to the right of the left support and finally the top node.

The normal forces are approximately: $N_3 = 2100\ kN$, $N_4 = 100\ kN$ and $N_5 = 1997\ kN$.

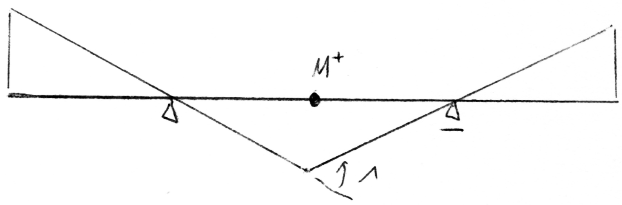


400 kN
3
1
C
4
120 kN
B
5
NA = 2000 kN
400 kN
N3 = 2100 kN
1200 kN
Nc = 250 kN
NB = 2000 kN
N4 = 100 kN
N5 = 1997 kN

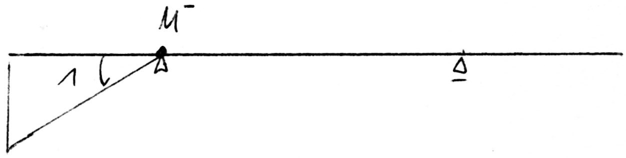
6.3 Design of a cross-section

1

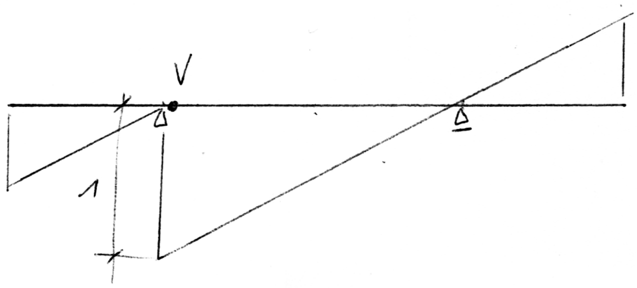
The line of influence of the mid-span moment is obtained by placing a hinge at mid-span and introducing a unit rotation. In this case, we do not need the numerical values of the line of influence but only their shape to understand the functioning of the slabs and where to place the loads.



For the line of influence of the moment at the support, we place a hinge and introduce a unit rotation over the support.

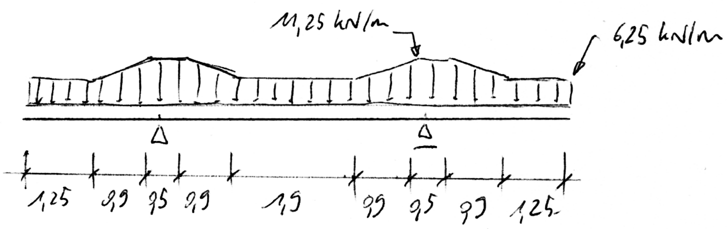


For the line of influence of the shear force at the left-hand support, we introduce a unit displacement at the right-hand side of the support but keeping the lines parallel as differential rotation is not permitted.



2

When the slab is 25 cm thick, its weight for a 1 meter deep slice is $0.25 \cdot 25 = 6.25\ kN/m$. Similarly, when it becomes 45 cm thick, it weights $0.45 \cdot 25 = 11.25\ kN/m$. Therefore, the self-weight of the slab can be modelled with the following load.



To simplify the calculations, we can average this load over the length of the slab. The total load is:

$$G = 6.25 \cdot (1.9 + 1.25 \cdot 2) + 11.25 \cdot (2 \cdot 0.5) + 4 \cdot 0.9 \cdot \frac{11.25 + 6.25}{2} = 70.25\ kN$$

Therefore, the average distributed load is:

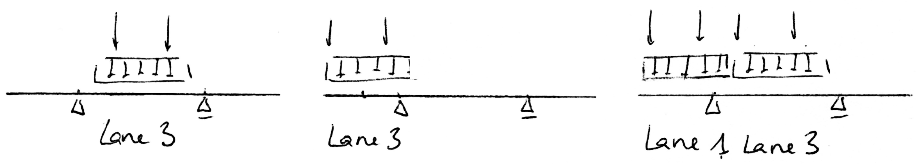
$$g = \frac{G}{L} = \frac{70.25}{9} = 7.8\ kN/m$$

3

To determine the most unfavourable placement of lanes, we follow these steps:

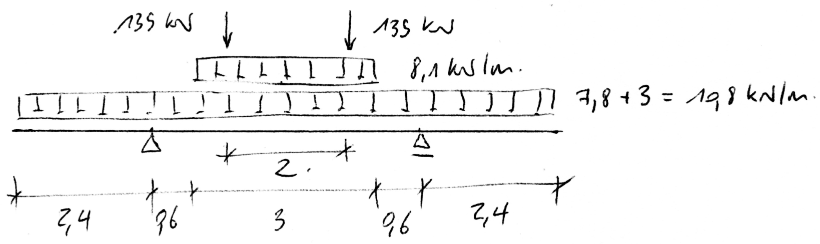
- We place only lanes where their load is unfavourable
- We always place the heavier lane (Lane 3) where it has the most important effect
- In general, the heaviest loads are the concentrated axle forces. When possible, one of the axle load should be placed at the maximum of the line of influence to have the most important effect. However, in the cases where the line of influence is symmetrical (e.g. for mid-span moment lines of influence), the same effect is achieved by placing the load symmetrically around the maximum.

The following determinant placement of loads were found (from left to right: mid-span moment, moment at the support, shear force).



4

To calculate the design mid-span moment, the loads we place on the beam are: the averaged self-weight of 7.8 kN/m, the asphalt load of 3 kN/m, the distributed traffic loads 8.1 kN/m and the two axle loads $\frac{Q}{2} = 135\ kN$.



We first calculate the reaction forces:

$$R = \frac{10.8 \cdot 9 + 8.1 \cdot 3 + 135 \cdot 2}{2} = 196\ kN$$

We can then calculate the mid-span moment by making a section at mid-span and using equilibrium of one of the fragments:

$$M = -10.8 \cdot \frac{(2.4 + 0.6 + 3/2)^2}{2} + 196 \cdot (0.6 + 3/2) - 8.1 \cdot \frac{(3/2)^2}{2} - 135 \cdot 2/2 = 158\ kNm$$

5

This type of model is very appropriate for distributed forces (both dead weight and distributed traffic loads). However, it is not entirely realistic when we model concentrated traffic loads.

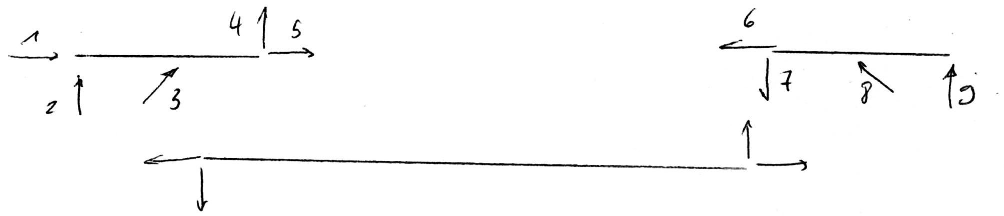
Here we make the assumption that the concentrated forces are distributed on the 1 m thick slice of slab. In reality, they act on a square of $40 \cdot 40\ cm$. Therefore the behaviour of the slab is very different than for distributed loads. The concentrated loads will be supported in several directions and a diffusion of the moment will take place over the depth of the slab.

Therefore, to take these loads into account, we need a 2D model of the slab which will give us the moments produced in the slab in all directions. However, the model we used can still be used for distributed loads and the effects can be summed afterwards for the design.

6.4 Optimisation of the geometry of a portal bridge

1

To check if the structure is isostatic, we first verify that it is not a mechanism. As this is not the case, we can dislocate the structure at the supports and hinges and externalise the reactions. We obtain the following:



The number of unknowns is 9 and the number of equations $3 \cdot 3 = 9$. The structure is isostatic.

2

To determine the value of a, we express the moments for a uniformly distributed load q at points A and B in function of a and solve the equation so they are equal.

The middle section can be seen as a simple beam. Therefore, the moment at mid-span is:

$$M_A = \frac{q \cdot (20 - 2 \cdot a)^2}{8}$$

The moment at B is created by the reaction of the simple beam and the uniformly distributed load between B and the hinge:

$$M_B = \frac{q \cdot (20 - 2 \cdot a)}{2} \cdot a + \frac{q \cdot a^2}{2}$$

We solve to find a when $M_A = M_B$ and find $a = 2.9\ m$.

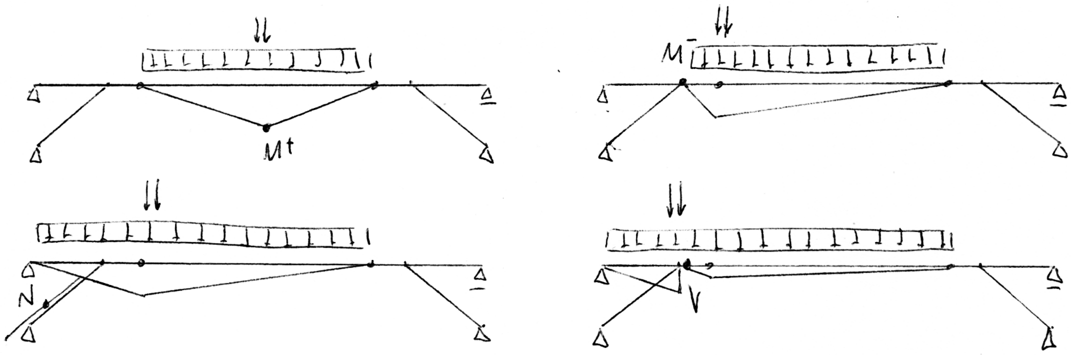
This optimisation is a good idea as it means we can keep the same dimensions for the longitudinal beams without it being oversized.

However, we should also verify that the reactions at the supports are never in traction as this poses construction and durability issues.

3

We draw the lines of influence by placing a simple cut which corresponds to the internal forces for which we want to draw the line at the point we want to check. We then introduce a displacement. Here, we do not really need to read values but only to understand where the loads should be placed so they are the most unfavourable.

We place the uniformly distributed loads where they are unfavourable and the concentrated loads near the maximum.

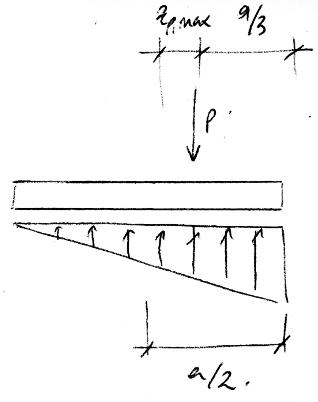


7 Ouvrages géotechniques (BA6)

7.1 Flat Foundation

One dimensional case

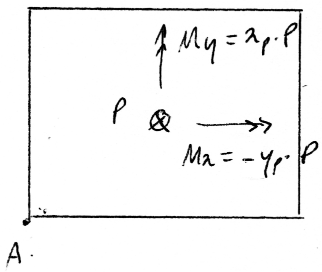
The maximum eccentricity of P without uplifting of the foundation is obtained for the following soil pressure distribution:



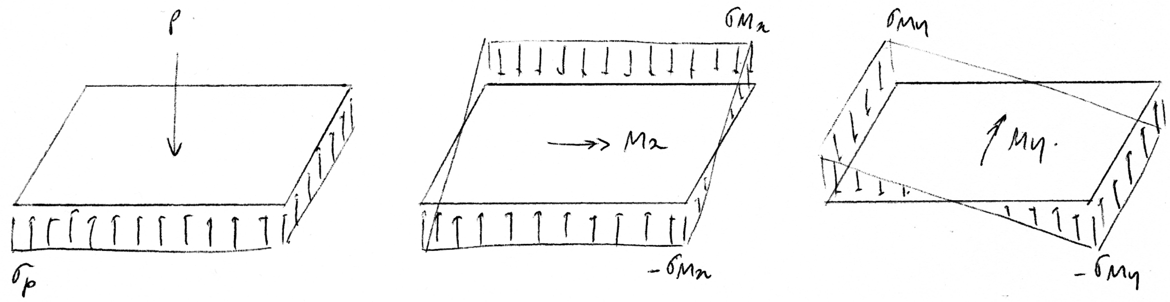
Thus, $x_{P,max} = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$ and the condition of no uplift is fulfilled when $x_P \in \left[-\frac{a}{6}, \frac{a}{6}\right]$.

Two dimensional case

The eccentric axial load is statically equivalent to a centred axial force and two bending moments.



The soil pressure resulting from each of the 3 forces (P, M_x and M_y) are as follows.



- $\sigma_P = -\frac{P}{a \cdot b}$
- $M_x = \frac{1}{2} \cdot \sigma_{M_x} \cdot \frac{b}{2} \cdot a \cdot \frac{2 \cdot b}{3} = \frac{1}{6} \cdot \sigma_{M_x} \cdot b^2 \cdot a$, therefore, $\sigma_{M_x} = \frac{6 \cdot M_x}{b^2 \cdot a}$
- $M_y = \frac{1}{2} \cdot \sigma_{M_y} \cdot \frac{a}{2} \cdot b \cdot \frac{2 \cdot a}{3} = \frac{1}{6} \cdot \sigma_{M_y} \cdot a^2 \cdot b$, therefore, $\sigma_{M_y} = \frac{6 \cdot M_y}{a^2 \cdot b}$

For $x_P > 0$ and $y_P > 0$, the first corner to uplift would be corner A. The contact stress at this point is:

$$\sigma_A = -\sigma_P + \sigma_{M_x} - \sigma_{M_y} = \frac{P}{a \cdot b} + \frac{6 \cdot M_x}{b^2 \cdot a} + \frac{6 \cdot M_y}{a^2 \cdot b} = -\frac{P}{a \cdot b} + \frac{6 \cdot P \cdot y_P}{b^2 \cdot a} + \frac{6 \cdot P \cdot x_P}{a^2 \cdot b}$$

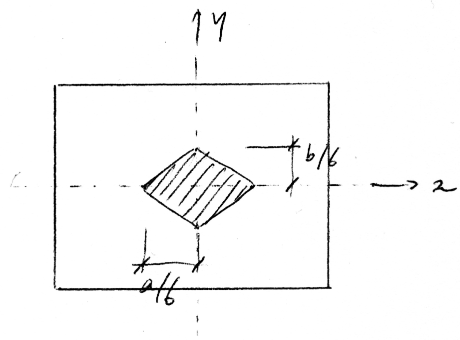
The maximum eccentricity for which the foundation just does not lift up is reached when $\sigma_A = 0$, that is to say when:

$$-\frac{P}{a \cdot b} + \frac{6 \cdot P \cdot y_P}{b^2 \cdot a} + \frac{6 \cdot P \cdot x_P}{a^2 \cdot b} = 0$$

or when:

$$-1 + \frac{6 \cdot y_P}{b} + \frac{6 \cdot x_P}{a} = 0$$

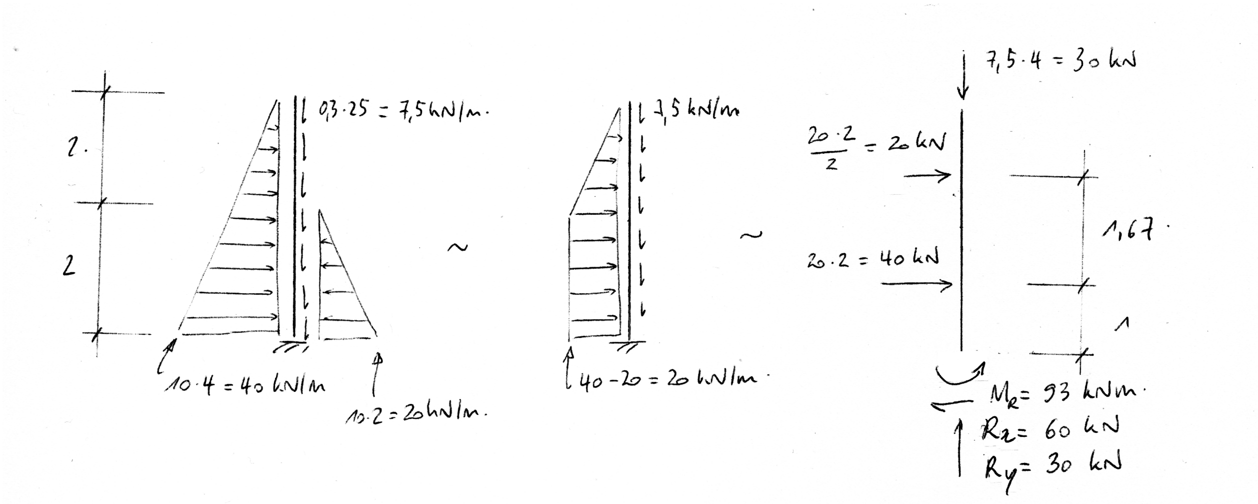
The function $\frac{y_P}{b} + \frac{x_P}{a} = \frac{1}{6}$ is represented below. Note that it was derived for only $x_P > 0$ and $y_P > 0$. The space is symmetrical about the x and y axes.



7.2 Dimensionnement d'une paroi rectangulaire et de sa foundation

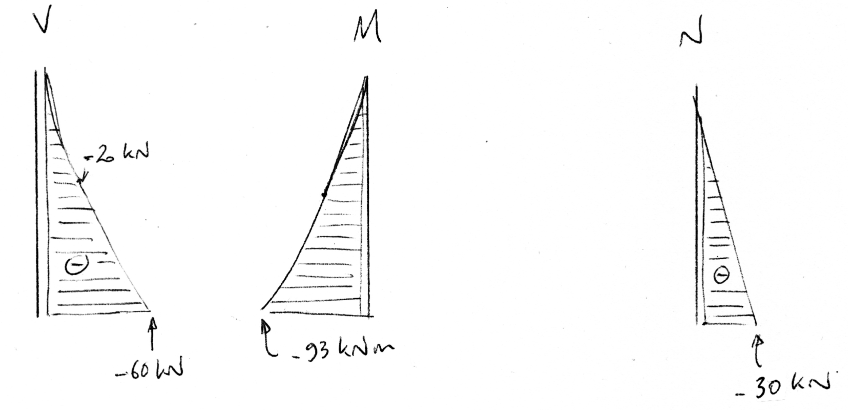
1

The forces applied as follows. We can sum up the hydraulic pressure on either side to simplify the calculations. Once this is done, the forces are reduced to concentrated loads and the reactions can be computed using equilibrium.



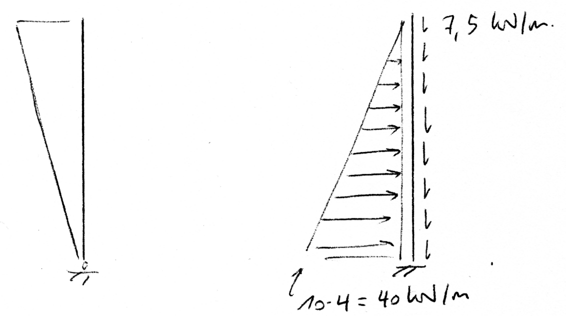
Now using the differential equations for equilibrium, we know that:

- The N diagram will be triangular as the self-weight of the wall is uniformly distributed
- The V diagram will be triangular when the hydraulic pressure is constant and will follow a parabola when the pressure is triangular
- The M diagram will be a second order parabola when the shear force is triangular and a third order parabola when the shear force follows a second order parabola



2

To determine the defining load case, it is always a good idea to trace the line of influence. To draw the line of influence of the maximum moment, we introduce a rotation at the base of the wall. We instantly see that the defining load case is when one of the reservoirs is full and the other empty. We choose to fill reservoir 1 so we have $h_1 = 4\ m$ and $h_2 = 0\ m$.



The moment at the base of the wall for this load case is $M = \frac{40 \cdot 4}{2} \cdot 4 \cdot \frac{1}{3} = 107\ kNm$.

3

Using equilibrium of vertical forces, we know that F_y is the total weight of the wall and the foundations. That is:

$$F_y = (0.3 \cdot 4 + 0.3 \cdot b) \cdot 25$$

Similarly, we understand that F_x is the shear force at the base of the wall. That is $F_x = \frac{40 \cdot 4}{2} = 80\ kN$

The moment created by F_x and F_y around the base of the wall should be equal to M in order to prevent the wall from rotating. That is to say:

$$F_x \cdot \frac{b}{2} - F_y \cdot 0.15 = M$$

or:

$$(0.3 \cdot 4 + 0.3 \cdot b) \cdot 25 - 80 \cdot 0.15 = 107$$

finally, we find:

$$b = 4.0\ m$$

7.3 Internal forces of a pile

We first calculate F, H and M by using the equilibrium equations:

- Equilibrium of moments around the top of the pile (we take forces from top to bottom):

  $$M - 8 \cdot 10 \cdot \frac{10}{2} - 30 \cdot 10 + 2 \cdot 20 \cdot \left(10 + \frac{20}{2}\right) - 1 \cdot (10 + 20) = 0$$

  we find:

  $$M = -200\ kNm$$

- Equilibrium of forces along x:

  $$H - 8 \cdot 10 - 30 + 2 \cdot 20 - 10 = 0$$

  we find:

  $$H = 80\ kN$$

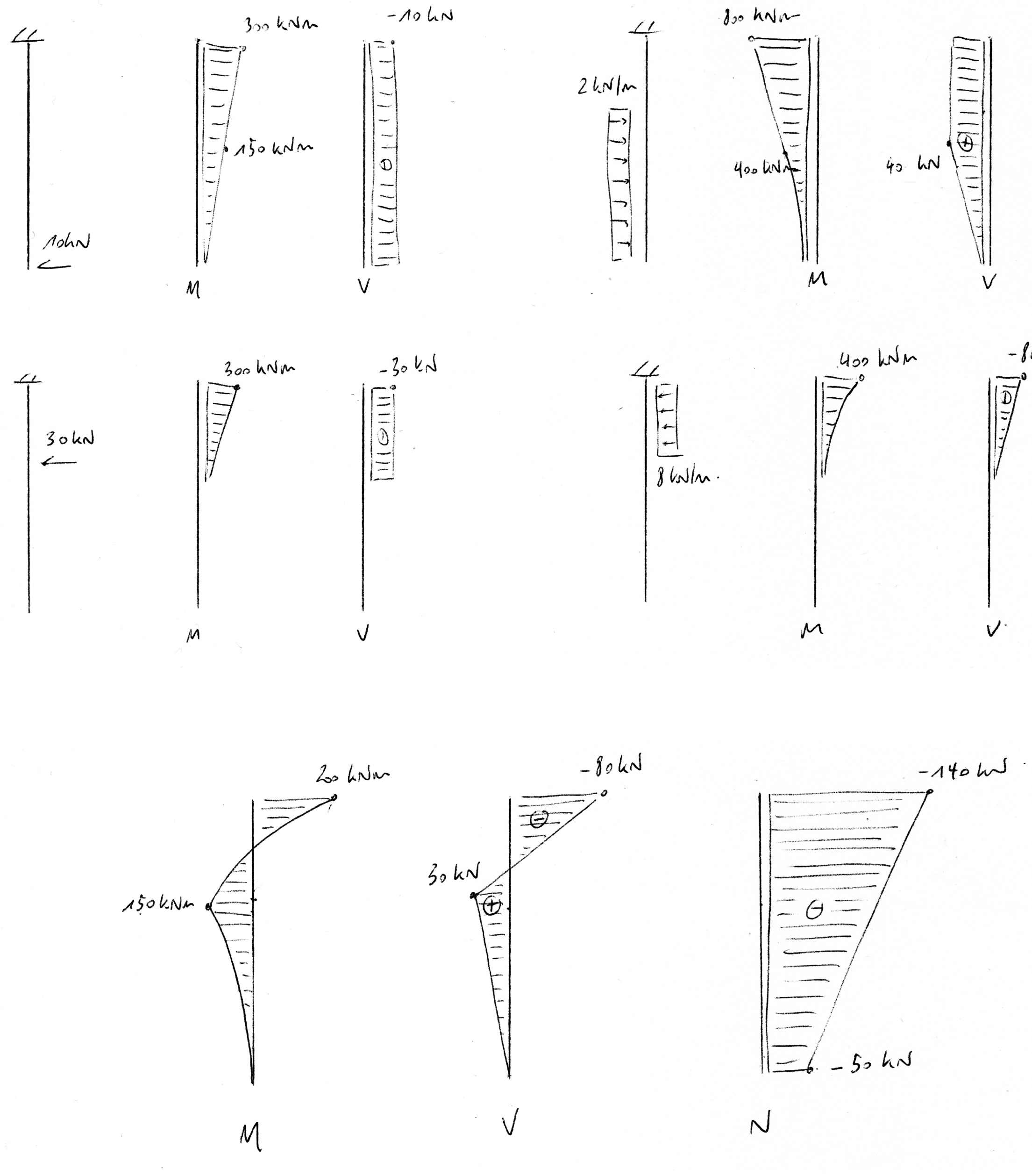
- Equilibrium of forces along y:

  $$F + 3 \cdot (10 + 20) + 50 = 0$$

  we find:

  $$F = 140\ kN$$

To calculate the moment and shear diagrams, the easiest is to use the principle of superposition. For the six loads applied to the pile, we calculate the moment and the shear diagram and then we sum all of them up. The normal force diagram is linear as the distributed load is uniformly distributed and we know the extremity values: 50 kN at the bottom, 140 kN at the top.


300 kNm
-10 kN
150 kNm
10 kN
M
V
800 kNm
2 kN/m
400 kNm
40 kN
M
V
300 kNm
-30 kN
30 kN
M
V
400 kNm
-80 kN
8 kN/m.
M
V
200 kNm
-80 kN
-140 kN
30 kN
150 kNm
-50 kN
M
V
N