PRACTICAL - 1

Topic: Limits and Continuity

1. $$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$= \lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$= \lim_{x\to a} \frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$= \frac{1}{3}\lim_{x\to a} \frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$= \frac{1}{3} \frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}} \qquad = \frac{2}{3\sqrt{3}}$$

2. $\lim\limits_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$= \lim\limits_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \dfrac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$= \lim\limits_{y \to 0} \dfrac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$= \lim\limits_{y \to 0} \dfrac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$= \dfrac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})}$

$= \dfrac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$

$= \dfrac{1}{\sqrt{a}(2\sqrt{a})} = \dfrac{1}{2a}$

3. $\lim\limits_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \dfrac{\pi}{6} = h$

$x = h + \dfrac{\pi}{6}$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cosh\cdot\frac{\sqrt{3}}{2} - \sin h\,\frac{1}{2} - \sqrt{3}\left(\sin h\frac{\sqrt{3}}{2} + \cos h\cdot\frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$\lim_{h\to 0} \frac{\left(\cos h\frac{\sqrt{3}}{2} + \frac{\sin h}{2}\right) - \sqrt{3}\left(\frac{1}{2}\cos h - \frac{\sqrt{3}}{2}\sin h\right)}{6h}$$

$$\lim_{h\to 0} \frac{\frac{\sqrt{3}}{2}\cos h + \frac{\sin h}{2} - \frac{\sqrt{3}\cos h}{2} + \frac{3\sin h}{2}}{6h}$$

$$\lim_{h\to 0} \frac{4\sin h}{2(6h)}$$

$$\lim_{h\to 0} \frac{4\sin h}{12h}$$

$$= \frac{1}{3}$$

4. $\lim\limits_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing numerator & denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5})+\sqrt{x^2-3})}$$

$$\lim_{x\to\infty}\frac{8}{2}\frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit we get,

$=4$

5. (i) $f(x)=\dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0<x\le\dfrac{\pi}{2}$

$=\dfrac{\cos x}{\pi-2x}$ for $\dfrac{\pi}{2}<x<\pi$

at $x=\dfrac{\pi}{2}$

$$f(\pi/2)=\frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2\left(\frac{\pi}{2}\right)}}\qquad\therefore f\left(\frac{\pi}{2}\right)=0$$

f at $x=\dfrac{\pi}{2}$ defined.

(ii) $\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}^+} \frac{\cos x}{\pi - 2x}$

By substituting method

$$x - \frac{\pi}{2} = h$$

$$x = h + \frac{\pi}{2}$$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(\frac{2h + \pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h \to 0} \frac{-\sin h}{-2h}$$

$$= \frac{1}{2}$$

b) $\lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^-} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x\to\frac{\pi}{2}^-} \dfrac{2\sin x\cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x\to\frac{\pi}{2}^-} \dfrac{2\sin x\cos x}{\sqrt{2}\,\sin x}$

$\lim_{x\to\frac{\pi}{2}^-} \dfrac{2\cos x}{\sqrt{2}}$

$\dfrac{2}{\sqrt{2}} \lim_{x\to\frac{\pi}{2}^-} \cos x$

$\therefore$ LHL $\neq$ RHL

$\therefore$ f is not continuous at $x = \pi/2$

(ii) $f(x) = \left.\begin{cases} \dfrac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \leq x \leq 6 \\ = \dfrac{x^2-9}{x+3} & 6 \leq x < 9 \end{cases}\right\}$ at $x = 3$ and $x = 6$.

at $x = 3$

(i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ defined

(ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is defined at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3}$

$\therefore$ LHL = RHL

f is continuous at $x=3$

for $x=6$.

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

2] $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3.$

$\lim_{x\to 6^-} x+3 = 3+6 = 9.$

$\therefore$ LHL $\neq$ RHL

Function is not continuous.

6.(i) $f(x) = \dfrac{1-\cos 4x}{x^2}$ $x<0$ $\Big\}$ at $x=0$.

$= k$ $x=0$

Soln: f is continuous at $x=0$.

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$$

$$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k.$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$k = 8.$$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$ $\Big\}$ at $x=0$.

$= k$ $x=0$

Soln:

$$f(x) = (\sec^2 x)^{\cot^2 x}.$$

$$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}.$$

$$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$$

We know that,

$$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$$

$\therefore e$

$\therefore k = e$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \dfrac{\pi}{3}$ $\Big\}$ at $x = \dfrac{\pi}{3}$

$= k$ $x = \dfrac{\pi}{3}$

$x - \dfrac{\pi}{3} = h$

$x = h + \dfrac{\pi}{3}$

where $h \to 0$

$$f\left(\frac{\pi}{3} + h\right) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} + h\right)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \tan h}}{-3h}.$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3}\tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$\lim_{h \to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \lim_{h \to 0} \frac{1}{(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \frac{1}{(1 - \sqrt{3}(0))}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

(i)
$$f(x) = \left.\begin{cases} \dfrac{1 - \cos 3x}{x\tan x} & x \neq 0 \\ 9 & x = 0 \end{cases}\right\} \text{ at } x = 0.$$

$$f(x) = \frac{1 - \cos 3x}{x\tan x}$$

$$\lim_{x \to 0} \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$\lim_{x\to 0} \frac{\dfrac{2\sin^2\frac{3x}{2}}{x^2} \times x^2}{x \cdot \dfrac{\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$= 2 \times \frac{9}{4}$$

$$= \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \quad g = f(0)$$

$\therefore$ f is not continuous at $x=0$.

Redefine function.

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x+\tan x} & x \neq 0. \\ \dfrac{9}{2} & x=0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$.

(ii) $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\quad x \neq 0$ } at $x=0$.
$= \pi/6$ $\quad x=0$ }

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0} \frac{3\cdot e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

f is continuous at $x=0$.

8. $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $\quad x=0$.

is continuous at $x=0$.

$\therefore$ Given

f is continuous at $x=0$.

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

9.

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$$\lim_{x\to 0} \frac{(e^{x^2}-1)+(1-\cos x)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0} \left(\frac{\sin^2 x/2}{x}\right)^2$$

Multiply with 2 in numerator & denominator

$$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

9. $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \dfrac{\pi}{2}$

$f(0)$ is continuous at $x = \dfrac{\pi}{2}$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$= \lim_{x\to\pi/2} \frac{2-1+\sin x}{\cos^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \lim_{x\to\pi/2} \frac{1+\sin x}{1-\sin^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \lim_{x\to\pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1}+\sin x)}$$

$$\lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(22\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$\therefore f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}$$

AB
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PRACTICAL - 02

TOPIC : Derivative

Q1. Show that the fou function defined from R to R are differentiable

1) $\cot x$ 2) $\operatorname{cosec} x$ 3) $\sec x$

Q2. If $f(x) = 4x+1$ $x \le 2$

$= x^2 + 5$ $x > 0$ at $x = 2$ then find f is differentiable or not

Q3. If $f(x) = 4x + 7$ $x < 3$

$= x^2 + 3x +$ $x \geq 3$ at $x = 3$ then find f is differentiable or not.

Q4. If $f(x) = 8x - 5$ $x \le 2$

$= 3x^2 - 4x + 7$ $x < 2$ at $x = 2$ then find f is differentiable or not.

Solution:

Q1. 1) $\cot x$

$$f(x) = \cot x$$

$$= \lim_{x \to a} \frac{f(x) - f(x)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

Put $x-a=h$ $\quad x=a+h$ as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

formula : $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan a + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{-\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$= -\operatorname{cosec}^2 a$

$Df(a) = -\cos^2 a$

$\therefore$ f is differentiable $\forall a \in R$

2) cosec x

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$\lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Put $x - a = h$ $\quad x = a + h$ $\quad$ as $x \to a$ $\; h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times 1/2 \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2\cos\left(\frac{2a}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

3) $\sec x$

$$f(x) = \sec x$$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\sec(x) - \sec(a)}{x-a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x-a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x-a=h$ $\quad x=a+h$ as $x \to a$ $\quad h \to 0$

$$\therefore Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \cos a \cos(a+h)}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2 \sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{\cos a \cos(a+h) \cdot x - h/2}$$

$$= 1/2 \times -2 \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a$$

Q2 Soln :

LHD

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1 - (4 \times 2 + 1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$$Df(2^-) = 4$$

RHD $\quad Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$$= 2+2 = 4$$

$Df(2^+) = 4$ $\quad \because$ RHD = LHD

f is differentiable at $x=2$

Q3 Soln : RHD :

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 - 3 + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x(x + 6) - 3(x + 6)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x + 6)(x - 3)}{(x - 3)} = 3 + 6 = 9$$

$$Df(3^+) = 9$$

$$LHD = Df(3^-)$$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x - 12}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4(x - 3)}{(x - 3)}$$

$$Df(3^+) = 4$$

$$RHD \neq LHD$$

f is not differentiable at $x = 3$.

Q4 soln:

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD: $Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$

$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3 \times 2 + 2 = 8$

$Df(2^+) = 8$

LHD:

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$

$Df(2^-) = 8$

LHD = RHS

f is diff at $x = 3$.

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PRACTICAL - 03

TOPIC : Application of Derivative

1) Find the intervals in which function is increasing or decreasing

(i) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x - 5$

$\because$ f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5}/3)(x + \sqrt{5}/3) > 0$

$x = \pm\sqrt{\frac{5}{3}}$

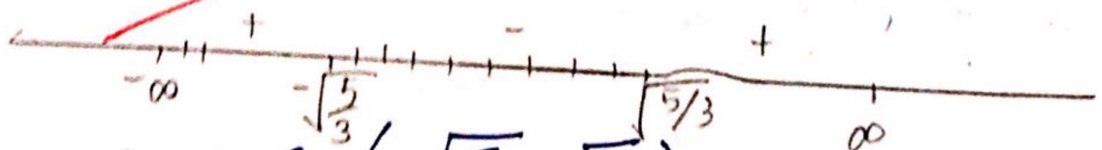
$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now f is decreasing if and only if $f'(x) < 0$

$f'(x) < 0$

$3x^2 - 5 < 0$

$x = \pm\sqrt{\frac{5}{3}}$



$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b] $f(x) = x^2 - 4x$

Soln : f is increasing if and only if $f'(n) > 0$

$\because f(x) = x^2 - 4x$

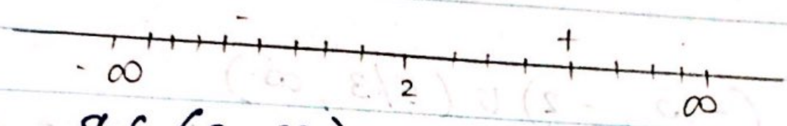
$f'(x) = 2x - 4$

$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x = 2$



$\therefore x \in (2, \infty)$

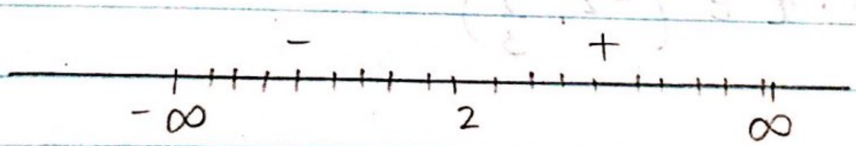
Now if f is decreasing if and only if $f'(x) < 0$

$\therefore 2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x = 2.$



$x \in (-\infty, 2)$

c] $f(x) = 2x^3 + x^2 - 20x + 4$

Soln :

f is increasing iff $f'(x) > 0$

$\because f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(x+2)(6x-10) > 0$

$x = -2, \frac{-5}{3}$

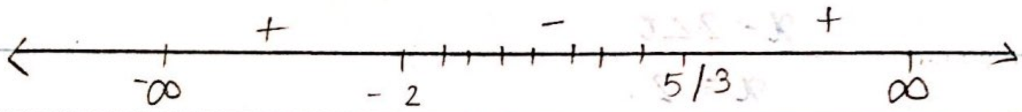
$\therefore x \in (-\infty, -2) \cup (5/3, \infty)$

Now f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$(x+2)(6x-10) < 0$

$x = -2, 5/3$



$\therefore x \in \left(-2, \frac{5}{3}\right)$

d] $f(x) = x^3 - 27x + 5$

If f is increasing iff $f'(x) > 0$

$\therefore f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$\therefore x = 3, -3$

$\therefore\ x \in (-\infty, -3) \cup (3, \infty)$

Now f is decreasing iff $f'(x) < 0$

$3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$x = 3\ ,\ -3$

$x \in (-3, 3)$

e] $f(x) = 69 - 24x - 9x^2 + 2x^3$

f is increasing iff $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

$-24 - 18x + 6x^2 > 0$

$6(-4 - 3x + x^2) > 0$

$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) 1 (x-4) > 0$

$(x-4)\ (x+1) > 0$

$x = 4, -1$

$x \in (-\infty, -1) \cup (4, \infty$

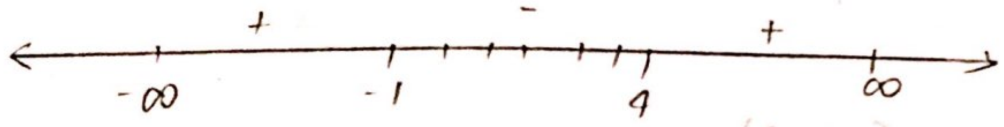
Now f is decreasing if and only if $f'(x) < 0$

$\therefore -24 - 18x + 6x^2 < 0$

$6(-4 - 3x + x^2) < 0$

$(x-4)(x+1) < 0$

$x = 4, -1$



$x \in (-1, 4)$

Q2] Find the intervals in which function is concave upwards and concave downwards

a] $y = 3x^2 - 2x^3$

Sol^n:

$\therefore f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

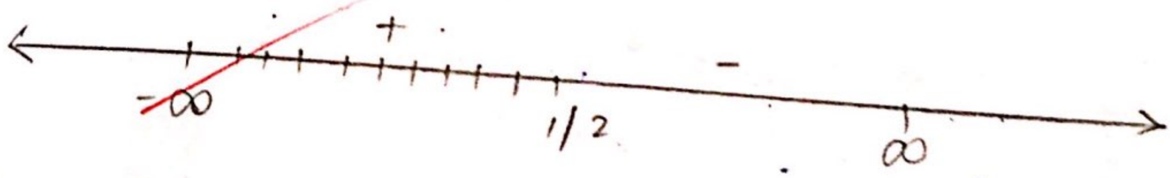
$\because f$ is concave upward iff $f''(x) > 0$

$\therefore 6 - 12x > 0$

$\therefore 6(1 - 2x) > 0$

$\therefore 1 - 2x > 0$

$\therefore -(2x - 1) > 0$

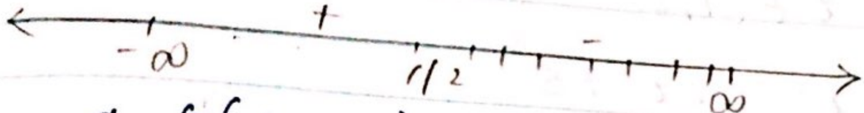


$x \in \left(-\infty, \frac{1}{2}\right)$

f is concave downward iff $f''(x) < 0$

$\therefore 6(1-2x) < 0$

$\therefore -(2x-1) < 0$



$x \in \left(\frac{1}{2}, \infty\right)$

b] $f = x^4 - 6x^3 + 12x^2 + 5x + 7$

Sol^n $y = f(x)$

$\therefore f(x) = x^4 - 6x + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

$\therefore$ f is concave upward iff $f''(x) > 0$

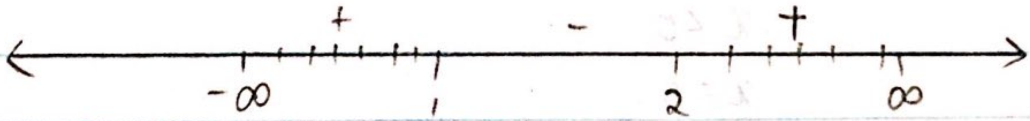
$\therefore 12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$(x-2)(x-1) > 0$

$x = 2, 1$



$x \in (-\infty, 1) \cup (2, \infty)$

f is concave downward iff $f''(x) < 0$

$12x^2 - 36x + 24 < 0$

$12(x^2 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$(x-2)(x-1) < 0$

$x = 2, 1$

$x \in (1,2)$

c] $y = x^3 - 27x + 5$

$\therefore y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

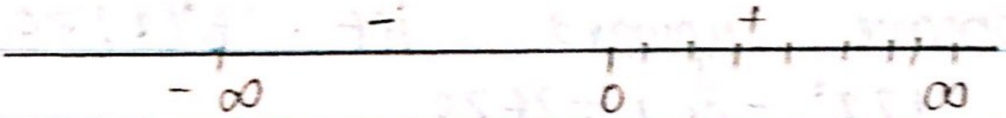
$f''(x) = 6x$

$\therefore$ f is concave upward iff $f''(x) > 0$

$6x > 0$

$x > 0$

$x = 0$



$x \in (0, \infty)$

$\therefore$ f is concave downward iff

$f''(x) < 0$

$6x < 0$

$x < 0$

$x = 0.$

$\therefore x \in (-\infty, 0)$

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d] $y = 69 - 24x - 9x^2 + 2x^3$

Soln $\therefore y = f(x)$

$$f(x) = 69 - 24x - 9x^2 + 2x^3$$

$$f'(x) = -24 - 18x + 6x^2$$

$$f''(x) = -18 + 12x$$

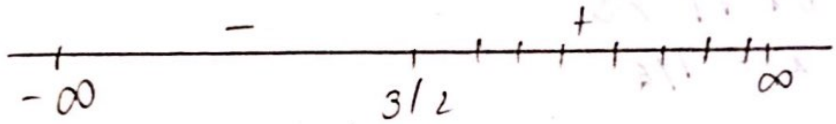
$\therefore f$ is concave upward iff $f''(x) > 0$

$$-18 + 12x > 0$$

$$6(2x - 3) > 0$$

$$2x - 3 > 0$$

$$x = 3/2$$



$$x \in \left(\frac{3}{2}, \infty\right)$$

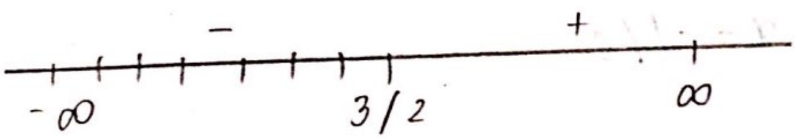
$\therefore f$ is concave downwards iff $f''(x) < 0$.

$$\therefore -18 + 12x < 0$$

$$6(2x - 3) < 0$$

$$2x - 3 < 0$$

$$x = 3/2$$



$$\therefore x \in \left(-\infty, \frac{3}{2}\right)$$

e] $y = 2x^3 + x^2 - 20x + 4$

Sol^n $\therefore y = f(x)$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

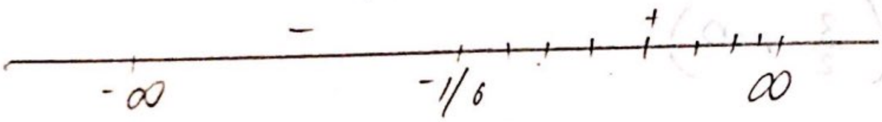
$\therefore$ f is concave upwards iff $f''(x) > 0$

$12x + 2 > 0$

$2(6x + 1) > 0$

$6x + 1 > 0$

$x = -1/6$



$\therefore x \in \left(\frac{-1}{6}, \infty\right)$

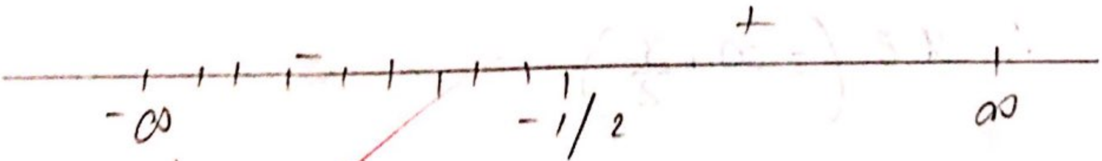
$\therefore$ f is concave downward iff $f''(x) < 0$

$12x + 2 < 0$

$2(6x + 1) < 0$

$6x + 1 < 0$

$x = -1/6$



$x \in \left(-\infty, \frac{-1}{6}\right)$

PRACTICAL - 4

TOPIC : Application of derivative & Newton's Method

Q1 Find maximum & minimum value of following

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

Q2 Find the root of the following equation by Newton's (take 4 iteration only) correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2,3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1,2]$

8) i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ Function reached minimum value at $x = 2$, and $x = -2$.

ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore 15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

ii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x - 2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$.

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$.

$6(x^2 - x - 2) = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$.

iii

$\therefore$ f has minimum value at $x=2$

$$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$$
$$= 2(8) - 3(4) - 24 + 1$$
$$= 16 - 12 - 24 + 1$$
$$= -19$$

Q2 (i)

$$\therefore f''(-1) = 12(-1) - 6$$
$$= -12 - 6$$
$$= -18 < 0$$

$\therefore$ f has maximum value at $x = -1$

$$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$$
$$= -2 - 3 + 12 + 1$$
$$= 8$$

$\therefore$ f has maximum value 8 at $x = -1$ and f has minimum value -19 at $x = 2$.

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ii) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\qquad \underline{x_0 = 0} \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method

$x_{n+1} = x_n + f(x_n)/f'(x_n)$

$\therefore x1 = x_0 - f(x_0)/f'(x_0)$

$x1 = 0 + 9.5/55$

$x1 = 0.1727$

$\therefore f(x1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 6.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

$\therefore$ The root of the equation is $\underline{0.1712}$

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(ii) $f(x) = x^3 - 4x - 9$ $[2,3]$

$f'(x) = 3x^2 - 4$

$f(2) = 23 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 33 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial appmoximation

$\therefore$ By Newton's Method,

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 6.596 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = \underline{2.7015}$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9 = -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0050$

$= 2.7065$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1,2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(7)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$

$= 2 - 2.2/5.2$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= \underline{6.6755}$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$\therefore x_2 = x_1 + f(x_1) / f'(x_1)$
$= 1.577 + 0.6755 / 8.2164$
$= 1.577 + 0.0822$
$= \underline{1.6592}$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$

$f(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2) / f(x_2)$
$= 1.6592 + 0.0204 / 7.7143$
$= 1.6592 + 0.0026$
$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$
$= 1.8618 + \dfrac{0.0004}{7.6977}$

$= \underline{1.6618}$

15/9/2020

PRACTICAL - 5

TOPIC : Integration

Q1) Solve the following integration.

i) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

$\int \dfrac{1}{x^3+2x-3}\, dx$

$= \int \dfrac{1}{\sqrt{x^2+2x-3}}\, dx$

$= \int \dfrac{1}{\sqrt{x^2+2x+1-4}}$

$a^2 + 2ab + b^2 = (a+b)^2$

$= \int \dfrac{1}{\sqrt{(x+1)^2-4}}\, dx$

Substitute put $x+1 = t$

$dx = \dfrac{1}{t} \times dt$ where $t = 1$ $t = x+1$

$\int \dfrac{1}{\sqrt{t^2-4}}\, dt$

using,

$\int \dfrac{1}{\sqrt{x^2-a^2}}\, dx = \ln\left(|x + \sqrt{x^2-a^2}|\right)$

$= \ln\left(|t + \sqrt{t^2-4}|\right)$

$t = x+1$

$= \ln\left(|x+1| + \sqrt{(x+1)^2 - 4} | \right)$

$= \ln\left(|x+1| + \sqrt{x^2 + 2x - 3} | \right)$

$= \ln\left(|x+1| + \sqrt{x^2 + 2x - 3} | \right) + c$

2) $\int (4e^{3x} + 1)\, dx$

$= \int 4e^{3x}\, dx + \int 1\, dx$

$= 4\int e^{3x}\, dx + \int 1\, dx$ # $\int e^{dx} dx = \frac{1}{2} \times e^{dx}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\, dx - \int 3\sin(x)\, dx + \int 5x^{1/2}\, dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4. $\int \frac{x^3 + 3x + 4}{\sqrt{x}} \, dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}} \, dx$

Split the denominator.

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} \, dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}} \, dx.$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} dx.$

$= \frac{x^{\cancel{5/2}+1}}{\cancel{5}/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5. $\int t^7 \times \sin(2t^4) \, dt$

Put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int \cancel{t^7} \times \sin(2t^4) \times \frac{1}{2 \times 4t^3} \, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4} \, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8} \, du =$

$= \frac{t^4 \times \sin(2t^4)}{8} \, du$

Substitute t^4 with $\frac{u}{2}$

$= \int \frac{u/2 \times \sin(\ )}{8} du$

$= \int \frac{u \times \sin(4)}{2} / 8 \quad du$

$= \int \frac{4 \times \sin(4)}{16} du$

$= \frac{1}{16} \int 4 \times \sin(4) \; du$

$\int u dv = uv - \int v du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1 du \qquad V = -\cos(u)$

$= \frac{1}{16} (u \times (-\cos(u)) - \int -\cos(u) du$

$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u) du$

\# $\int \cos x \; dx = \sin(x)$

$= \frac{1}{16} \times (4 \times (-\cos(4)) + \sin(4))$

Return the substitution $u = 2t4$

$= 1/16 \times (2t^4 \times (-\cos(2t4)) + \sin(2t^4))$

$= \frac{-t^4 \times \cos(2t4)}{8} + \frac{\sin(2t4)}{16} + C$

vi) $\int \sqrt[3]{x}\,(x^2-1)\,dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\; dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$

$= \int x^{5/2} - x^{1/2}\, dx$

$= \int x^{5/2} dx - x^{1/2}\, dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$= I_1 \cdot \dfrac{x^{5/2+1}}{5/2+1} = \dfrac{x^{7/2}}{7/2} = \dfrac{2x^{7/2}}{7} = \dfrac{2\sqrt{x^7}}{7} = \dfrac{2x^3\sqrt{x}}{7}$

$= I_2 = \dfrac{x^{1/2+1}}{1/2+1} = \dfrac{x^{3/2}}{3/2} = \dfrac{2x^{3/2}}{3/2} = \dfrac{2\sqrt{x^3}}{3}$

$= \dfrac{2x^3\sqrt{x}}{7} + \dfrac{2\sqrt{x^3}}{3} + c$

viii) $\displaystyle\int \frac{\cos x}{3\sqrt{\sin(x)^2}}\quad dx$

$= \displaystyle\int \frac{\cos x}{\sin x^{2/3}}\quad dx$

Put $t = \sin(x)$

$t = \cos x$

$= \displaystyle\int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)}\quad dt$

$= \dfrac{1}{\sin x^{3/2}}\quad dt$

$= \dfrac{1}{t^{2/3}}\quad dt$

$$I = \int \frac{1}{t^{2/3}} \, dt = \frac{-1}{(2/3-1)\, t^{2/3-1}} = \cancel{\frac{-1}{(2/3-1)\, t^{2/3-1}}}$$

$$= \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$$= 3\sqrt[3]{t}$$

Return substitution $t = \sin(x)$

$$= 3\sqrt[3]{\sin(x)} + c$$

(x) $\displaystyle\int \frac{x^2-2x}{x^3-3x^2+1} \, dx$

Put $x^3-3x^2+1 = dt$

$$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x} \, dt$$

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x} \, dt$$

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)} \, dt$$

$$= \int \frac{1}{x^3-3x^2+1} \times 1/3 \, dt$$

$$= \cancel{\int \frac{1}{3(x^3-3x^2+1)} \, dt}$$

$$= 1/3 \int 1/t \, dt \qquad \int 1/x \, dx = \ln|x|$$

$$= 1/3 \times \ln|t| + c$$

$$= 1/3 \times \ln(|x^3-3x^2+1|) + c$$

PRACTICAL - 6

$y = \sqrt{4 - x^2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$

$= \frac{-x}{\sqrt{4-x^2}}$

$I = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$

$= \int_{-2}^{2} \sqrt{\frac{1+x^2}{4-x^2}}\, dx.$

$= \int_{-2}^{2} \frac{\sqrt{4-x^2+x^2}}{4-x^2}$

$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$

$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\, dx.$

$= 2\left[\sin^{-1}(x/2)\right]^{2}$

$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$

$= 2\left[\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right] \quad = 2\pi$

3] $y = x^{3/2}$ $\quad x \in [0,4]$

$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

$L = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^4 \sqrt{1+\frac{9}{4}x}\, dx$

$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$

$= \frac{1}{2}\left[\frac{[4+9x]^{3/2}}{3/2} \times \frac{1}{9} \right]_0^4$

$= \frac{1}{27}\left[(4+9x)^{3/2} \right]$

$= \frac{-1}{27}\left[(4+0)^{7/2} - (4+31)^{7/5} \right]$

$= \frac{1}{27}\left(-40^{3/2} - 8 \right)$ units

4] $x = 3\sin t$ $\quad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t$ $\quad \frac{dy}{dt} = -3\sin t$

$I = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} 3\, dt$

$= 3\int_0^{2\pi} dt$

$= 3[x]_0^{2\pi}$

$= 3[2\pi - 0]$

$= 6\pi$ units

5] $x = \frac{1}{6} y^3 + \frac{1}{2}$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$I = \int_0^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_0^2 \sqrt{1 + \frac{(y^4-1)^2}{4y^4}}\, dy$

$= \int_0^2 \sqrt{\frac{(y^4-1) + 4 \times y^4 \times 1}{4y^4}}\, dy$

$\int_0^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$

$\int_0^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2} \int_1^2 y^2 dy + \frac{1}{2} \int_1^2 y^{-2} dy$

$= \frac{1}{2} \left[\frac{y^3}{3} - \frac{y^{-1}}{1} \right]_1^2$

$= \frac{1}{2} \left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1 \right]$

$= \frac{1}{2} \left[\frac{7}{3} + \frac{1}{2} \right]$

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$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$

$= \frac{1}{2}\left[\frac{17}{6}\right]$

$= \frac{17}{12}$ units

Q2
1) $\int_0^2 e^{x^2}\,dx$ with $n = 4$

$l = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.4877	54.5982
	y_0	y_1	y_2	y_3	y_4

$\int_0^2 e^{x^2}\,dx = \frac{2}{3}\left[(y_0 + y_4) + 4(y_1 + y_3 + 2(y_2))\right]$

$= \frac{0.5}{3}\left[(1 + 54.5982) + 5(1.284 + 9.487 + 2.23885\right]$

$= \frac{0.5}{3}\left[55.5982 + 43.0866 + 5.436\right]$

$= \int_0^2 e^{x^2}\,dx = 17.3535.$

ii) $\int_0^4 x^2 dx \qquad n = 4$

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2 dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$$

$$= \frac{1}{3}\left[0+16+4(1+9)+2\times 4\right]$$

$$= \frac{1}{3}\left[16+4(10)+8\right]$$

$$= \frac{64}{3}$$

$$= \int_0^4 x^2 dx = 21.3333$$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \qquad n=6$

$h = \frac{\frac{\pi}{3} - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	0.4167	0.4585	0.7071	0.8017	0.8752	0.93[illegible]
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi/18}{3}\left[0.4167 + 0.9306 + 4(0.4167 + 0.7071 + 0.9752) + 2(0.5848 + 0.8017)\right]$$

$$= \frac{\pi}{54}\left[1.3473 + 4(1.999) + 2(1.3865)\right]$$

$$= \frac{\pi}{54}\left[1.3473 + 7.996 + 2.773\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$$

PRACTICAL - 7

Topic : Differential Equation.

Q1]

1. $x \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q = (x) = \frac{e^x}{x}$

$I \cdot F = e^{\int 1/(x)\, dx}$

$= e^{\int 1/x \, dx}$

$= e^{\ln x}$

$I \cdot F = x$

$y(IF) = \int Q(x)\,(I \cdot F)\,dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int\int e^x \, dx + 1$

$xy = e^x + c$

Q2 $e^{x}\frac{dy}{dx} + 2e^{x}y = 1$

$\frac{dy}{dx} + 2\frac{e^{x}}{e^{x}}y = \frac{1}{e^{x}}$

$\frac{dy}{dx} + 2y = \frac{1}{e^{x}}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \quad Q(x) = e^{-x}$

$\int P(x)\,dx$

$IF = e^{\int 2dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)\,(I.F)\,dx + c.$

$y \cdot e^{2x} \int e^{-x+2x}\,dx + c$

$= \int e^{x}\,dx + c$

$y \cdot e^{2x} = e^{x} + c$

$x \frac{dy}{dx} = \frac{\cos x - \cancel{\#} 2y}{x}$

iii) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \qquad Q(x) = \frac{\cos x}{x^2}$

$IF = e \int P(x)\, dx$

$= e \int 2/x\, dx$

$= e^{2/x}$ x

$= \ln x^2$

$IF = x^2$

$y(I.F) = \int Q(x)\ (I.F)\, dx + c$

$= \int \frac{\cos x}{x^2} - x^2\, dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

i) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both sides)

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e \int P(x)\, dx$

$= e \int 3/x\ dx$

$$= e^{3/x} \quad x$$

$$i.a \quad = e^{\ln x^3}$$

$$IF = x^3$$

$$y(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$$

$$= \int \sin x\,dx + c$$

$$x^3 y = -\cos x + c$$

v) $$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$P(x) = 2$ $\qquad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int 2\,dx}$$

$$= e^{2x}$$

$$y(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int 2xe^{-2x}\,e^{2x}\,dx + c$$

$$= \int 2x\,dx + c$$

$$ye^{2x} = x^2 + c$$

vi) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$

$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

vii) $\frac{dy}{dx} = \sin^2(x - y + 1)$

Put $x - y + 1 = v$

Differentiating on both sides

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

viii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

Put $2x + 3y = v$

$$2 + \frac{3dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{(v-1)}{(v+2)}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$\int \left(\frac{v+2}{v+1}\right) dv = 3\,dx$

$\int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3x$

$v + \log|x| = 3x + c$

$2x + 3y + \log|2x+3y+1| = 3x + c$

$3y = x - \log|2x+3y+1| + c.$

25/01/2020

PRACTICAL NO: 8

Using Euler's method find the following.

1) $\frac{dy}{dx} = y + e^{x} - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$

$f(x) = y + e^{x} - 2$, $x_0 = 0$

$y(0) = 2$, $h = 0.5$

$y(0.2) = ?$

n	x_n	y(0)	$F(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	3.14787	3.5743
2	1	3.5443	4.2925	5.7205
3	1.5	5.7205	8.2021	9.3215

$y(0) = 9.8215$

2) $\frac{dy}{dx} = 1 + y^2$, $(y(0) = 0$, $n = 0.2$ Find $y(1)$

$x_0 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	0	1
1	0.2	0.2	0.4472	1.0894
2	0.4	0.408	0.6059	1.2105
3	0.6	0.6412	0.7040	1.3513
4	0.8	0.9934	0.7694	1.5051
5	1	1.2939		

$y(1) = 1.5051$

4) $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$ find $y(2)$

for $h = 0.5$ & $h = 0.25$

$h = 0.5$ & $h = 0.25$, $y_0 = 2$, $x_0 = 1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	2	4
1	1.5	1.5	4	7.875
2	2	2	7.875	

$y(2) = 7.875$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.88 75	4.4218
2	1.5	4.4218	59.6969	19.3360
3	1.75	19.3360	122.3482	299.9966
4	2	299.966		

$y(1) = 299.9966$

5. $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$ Find $y(1.2)$ with $h = 6.2$

$y(0) = 1$ $x(0) = 1$ $h = 0.2$

23/01/2020

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6.
1	1.2	3.6		

$y(1) = 3.6.$

PRACTICAL - 9

Limits and Partial Order Derivatives

1) Evaluate the foll limits

i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

Applying limit

$$\frac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$$

$$\frac{-64+3+1-1}{4+5} = \frac{-61}{9}$$

ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

Applying limit

$$\frac{(0+1)\ (2)^2+(0)^2-4(2)}{2+3(0)}$$

$$\frac{1\,(4-8)}{2} = \frac{-4}{2} = -2$$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$\Rightarrow \lim_{(x,y,z)\,(1,1,1)} \dfrac{(x)^2-(yz)^2}{x^2(x-yz)}$

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{(x+yz)(x-yz)}{x^2(x-yz)}$ $\left[\because (a)^2-b^2=(a+b)(a-b)\right]$

ii.a

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

$$\frac{1+(1)(1)}{1^2} = 2$$

Q2 Find fx, fy, for each of the following

(i) $f(x,y) = xy\, e^{x^2+y^2}$

$$f(x) = \frac{\partial f}{\partial x}$$

$$= \frac{\partial (xy\, e^{x^2+y^2})}{\partial x}$$

$$= y \frac{\partial (x \cdot e^{x^2+y^2})}{\partial x}$$

$$= y\left[x \cdot \frac{d}{dy}\left(e^{x^2+y^2}\right) + e^{x^2+y^2} \frac{d}{dx}(x)\right]$$

$$\left[\because \frac{d}{dx}(uv) = uv' + v \cdot u'\right]$$

$$= y\left[x \cdot e^{x^2+y^2} \cdot 2x + e^{x^2+y^2}(1)\right]$$

$$= y \cdot e^{x^2+y^2}[2x+1]$$

Now, $f(y) = \frac{\partial f}{\partial y}$

$$= \frac{\partial\left(xy\, e^{x^2+y^2}\right)}{\partial y}$$

(ii)

(iii)

=

=

=

=

$= x \cdot \frac{\partial}{\partial y}(y \cdot e^{x^2+y^2})$

$= x\left[y \cdot \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{d}{dy}(y)\right]$ 7(i)

$\because \left[\frac{d}{d}(uv) = u \cdot v^1 + vu'\right]$

$= x \cdot [2y^2 \cdot e^{x^2+y^2} + e^{x^2+y^2}]$

$= x \cdot e^{x^2+y^2}[2y^2+1]$

ii) $f(x,y) = e^x \cos y$

$\therefore f(x) = e^x \cos y$

$\therefore f(y) = e^x \frac{d}{\partial y}(\cos y)$

$= e^x(-\sin y)$

$= -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$

$= x^3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q3) Using defination find values of $f_x \cdot f_y$ at (0,0) for $f(x,y) = \frac{2x}{1+y^3}$

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

where $(a,b) = (0,0)$

$$\therefore f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{2h - 0}{2} = 2$$

Similarly

$$f_y(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0$$

$\therefore f_x = 2$, $f_y = 0$

$f(f_y)(0,0)$.

Q4) Find all second order partial derivative of f. Also verify whether $f_{xy} = f_{yx}$

(i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial (y^2 - xy)}{x^2}$$

$$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy) \cdot \frac{d}{dx}(x^2)}{(x^2)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - u \cdot v'}{v^2} \right]$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4} = \frac{x(xy - 2y^2)}{x^4}$$

$$f(x) = \frac{xy-2y^2}{x^3}$$

$$f(y) = \frac{\partial f}{\partial y} = \partial\left(\frac{y^2 - xy}{x^2}\right)$$

$$= \frac{\partial\left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$$

$$= \partial\left(\frac{y^2}{x^2} - \frac{y}{x}\right) / \partial y$$

$$= \frac{1}{x^2} 2y - \frac{1}{x}$$

$$f(y) = \frac{2y-x}{x^2}$$

$$f(x,x) = \frac{\partial\left(\frac{xy - 2y^2}{x^3}\right)}{\partial x}$$

$$= \frac{x^3 \frac{d}{dx}(xy-2y^2) - (xy-2y^2)\frac{d}{dx}(x^3)}{(x^3)^2}$$

$$= \frac{x^3(y) - (xy-2y^2)(3x^2)}{6}$$

$$= \frac{\cancel{x^3y - 3x^3y} + 6x^2y^2}{6}$$

$$= \frac{6x^2y^2 - 2x^3y}{x^6} = \frac{x^2(6y^2-2xy)}{x^6}$$

$$= \frac{6y^2-2xy}{x^4}$$

15

$$f(yy) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial y}$$

$$= \frac{\frac{1}{x^2}\partial(2y-x)}{\partial y} = \frac{1}{x}(2) = \frac{2}{x^2}$$

$$f(xy) = \frac{\partial\left(\frac{xy-2y^2}{x^3}\right)}{\partial y} = \frac{\partial\left(\frac{xy}{x^3} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{\partial\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{1}{x^3} - \frac{1}{x^3}2(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3 - 4x^2}{x^5}$$

$$= \frac{x^2(x-4y)}{x^5}$$

$$= \frac{x-4y}{x4}$$

$$f(yx) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(\frac{2y}{x^2} - \frac{1}{x}\right)}{\partial x}$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= \frac{-4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{-4yx^2 + x^3}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$\therefore f(xy) = f(yx) = \frac{x-4y}{x4}$$

Hence verified

$$f(x,y) = x^3+3x^2y^2 - \log(x^2+1)$$

$$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial (x^3+3x^2y^2 - \log(x^2+1)}{\partial x}$$

$$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$$

$$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial (x^3+3x^2y^2 - \log(x^2+1)}{\partial y}$$

$$= 0 + 3(2y(x^2) + 0$$

$$f(y) = 6x^2y$$

$$f(xx) = \frac{\partial}{\partial x} fx = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{(x^2+1)^2}\right]$$

$$\left[\because \frac{d}{dx} u/v = \frac{v \cdot u' - u - v'}{v^2}\right]$$

$$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$$

$$= 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$$

$f(yy) = \frac{\partial}{\partial y} fy \quad = \frac{\partial (6x^2 y)}{\partial y}$

$= 6x^2(1) \quad = 6x^2$

$f(xy) = \frac{\partial \left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$

$= 0 + 6x(2y)$

$= 12xy$

$f(yx) = \frac{\partial}{\partial x} fy$

$= \frac{\partial (6x^2 y)}{\partial x}$

$= 12xy$

$\therefore f(xy) = f(yx) = 12xy$

Hence verified

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial (\sin(xy) + e^{x+y})}{\partial x}$

$= \cos(xy(y) + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial (\sin(xy) + e^{x+y})}{\partial y}$

$= \cos(xy)(x) + e^{x+y}(1)$

$= x\cos xy + e^{x+y}$

$f(xx) = \frac{\partial fx}{\partial x} = \frac{\partial(y\cos xy + e^{x+y})}{\partial x}$

$= y\cos xy\,(y) + e^{x+y}\,(1)$

$= y^2\cos(xy + e^{x+y}$

$f(yy) = \frac{\partial}{\partial y} fy = \frac{\partial(x\cos xy + e^{x+y})}{\partial y}$

$= x\cos xy(x) + e^{x+y}\,(1)$

$= x^2\cos xy + e^{x+y}$

$f(xy) = \frac{\partial fx}{\partial y} = \frac{\partial(y\cos xy + e^{x+y})}{\partial y}$

$= y[-\sin(xy)\,(x) + \cos xy(1)] + e^{x+y}(1)$

$[\because \frac{d}{dx}(uv) = u\cdot v' + v\cdot u']$

$= -xy\sin(xy) + \cos xy + e^{x+y}$

$f(yx) = \frac{\partial fy}{\partial x} = \frac{\partial(x\cos xy + e^{x+y})}{\partial x}$

$= \cos xy\,(1) + x(-\sin(xy))\,(y) + e^{x+y}$

$= -xy\sin(xy) + \cos(xy) + e^{x+y}$

$\therefore f(xy) = f(yx) = -xy\sin(xy) + \cos(xy) + e^{x+y}$

Q 5] Find the linearization of $f(x,y)$ at given point

(i) $f(x,y) = \sqrt{x^2 + y^2}$ at $(1,1)$

$$f(1,1) = \sqrt{(1)^2 + (1)^2}$$
$$= \sqrt{2}$$

$$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$$

$$f_x(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$f_y(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$
$$= \frac{2 + x - 1 + (y-1)}{\sqrt{2}}$$
$$= \frac{2 + x + y - 2}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0\left(\sin\left(\frac{\pi}{2}\right)\right)$

$= 1 - \frac{\pi}{2}$

$f(x) = -1 + y\cos x$

$f(y) = 1$

$fx(\frac{\pi}{2}, 0) = -1 + 0\cos\left(\frac{\pi}{2}\right)$

$= -1$

$fy(\frac{\pi}{2}, 0) = 1$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= y - x - 1$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$\rightarrow f(1,1) = \log(1) + \log(1)$

$= 0 + 0 = 0$

$f(x) = \frac{1}{x}$ $\qquad$ $f(y) = \frac{1}{y}$

$f_x(1,1) = 1$ $\qquad$ $f_y(1,1) = 1$

$(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a-b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

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Find the directional derivative of the foll funct^n at given points & in the direction of given vector.

i) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1) \quad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along u is $\dfrac{u}{|u|} = \dfrac{1}{\sqrt{10}}(3,-1)$

$= \left(\dfrac{3}{\sqrt{10}}, \dfrac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\dfrac{3}{\sqrt{10}}, \dfrac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\dfrac{3}{\sqrt{10}}, \dfrac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \dfrac{3}{\sqrt{10}}\right), \left(-1 - \dfrac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1 + \dfrac{3}{\sqrt{10}}\right) + 2\left(-1 - \dfrac{h}{\sqrt{10}}\right) - 3$

$= 1 + \dfrac{3}{\sqrt{10}} - 2 - \dfrac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4u + \dfrac{h}{\sqrt{10}}$

$$Duf(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-4} + \cancel{h}/\sqrt{10} + \cancel{4}}{\cancel{h}}$$

$$Duf(a) = \frac{1}{\sqrt{10}}$$

(ii) $f(x) = y^2 - 4x + 1$ $\quad a = (3,4)$ $u = i + 5j$

Hence, $u = i + 5j$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

Unit vector along u is $\frac{v}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{x,y}\, f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}}}{h}$$

$$= \frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

(iii) $2x + 3y$ $\quad a = (1,2)$, $u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

Unit vector along u- is $\frac{V}{|u|} = \frac{1}{5}(3,4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$$f(a) = f(1,2) = 2(1) + 3(2) = 8$$

$$f(a + hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$$

$$f(a + hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8.$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{12 \cancel{h}}{5} + \cancel{h} - \cancel{h}}{\cancel{h}}$$

$$= \frac{12}{5}$$

Q2- Find gradient vector for the following funct^n at given point

(i) $f(x,y) = x^y + y^x = a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^x \log y, x^y \log x + x y^{x-1})$

$f(1,1) = (1 + 0, 1 + 0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x \right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2) \right)$

$= \left(\frac{1}{2}, \frac{\pi}{\cancel{4}_2}(-\cancel{2}) \right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2} \right)$

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iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{(1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0})$

$= (0 - e^0, 0 - e^0, -1, -e^0)$

$= (-1, -1, -2)$

Q3 Find the equation of tangent & normal to each of the foll using curves at given points

$x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \cancel{2x} + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1,0) =$ $\therefore x_0 = 1, y_0 = 0$

eq^n of tangent

$$f_x(x - x_0) + f_y(y - y_0) = 0$$

$$f_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 0$$
$$= 1(2) + 0$$
$$= 2$$

$$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$$
$$= 0 + 1 \cdot 1$$
$$= 1$$

$$2(x-1) + 1(y-0) = 0$$
$$2x - 2 + y = 0$$
$$2x + y - 2 = 0$$ It is the required eq^n of tangent

eq^n of Normal

$$= ax + by + c = 0$$
$$= bx + ay + d = 0$$

$$1(1) + 2(y) + d = 0$$
$$\therefore 1 + 2y + d = 0$$ at $(1,0)$
$$= 1 + 2(0) + d = 0$$
$$d + 1 = 0$$
$$\therefore d = -1$$

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78

(ii) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f_x = 2x+0-2+0+0$

$= 2x-2$

$f_y = 0+2y-0+3+0$

$= 2y+3$

$(x_0, y_0) = (2,-2) \therefore x_0 = 2, y = -2$

$f_x(x_0, y_0) = 2(2)-2 = 2$

$f_y(x_0, y_0) = 2(-2)+3 = -1$

eq^n of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2)+(-1(y+2)) = 0$

$2x-2-y-2=0$

$2x-y-4=0$ → It is required eq^n

eq^n of Normal

$= ax+by+c=0$

$bx+ay+d=0$

$= -1(x)+2(y)+d=0$

$-x+2y+d=0$ at $(2,-2)$

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$\therefore d=6$

Q4. Find the eq^n of tangent & normal list to each of the foll surface.

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$fx = 2x - 0 + 0 + z$

$fx = 2x + z$

$fy = 0 - 2z + 3 + 0$

$= 2z + 3$

$fz = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0]$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$

$fx(x_0, y_0, z_0) = 2(2) + 0 = 4$

$fy(x_0, y_0, z_0) = 2(0) + 3 = 3$

$fz(x_0, y_0, z_0) = -2(1) + 2(= 0$

eq^n of tangent

$fx(x_0 - x_0) + fy(y_0 - y_0) + fz(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$ → This is required eq^n of tangent

Eq^n of normal at $(4, 3, -11)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}.$$

$3xyz - x - y + z = -4$

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$ at $(1,-1,2)$

$f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0 \rightarrow$ This is req eq^n of tangent

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x - 1}{-7} = \frac{y + 1}{5} = \frac{z - 2}{-2}$$

Q 5 ~~10~~. Find the local maxima, & minima for the fou functions

i. $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$fx = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$fy = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$fx = 0$
$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ —— 1

$fy = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4 \rightarrow$ 2

Multiplying eq^n 1 with 2
$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$

Substitute value of x in eq^n 1
$2(0) - y = -2$
$\cancel{-}y = \cancel{-}2$
$\therefore y = 2$//

$\therefore$ Critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

for

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore$ f has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

ii $f(x, y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \rightarrow (1)$

Xing eq^n (1) with 3

(2) with 4

$12\cancel{x^2} + 9y = 0$

$-\cancel{12}x^2 - 8y = -0$

$11y = 0$

$u = 0$

Substitute value of y in eq^n ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (5)^2$

$= 0 \cdot 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

iii $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$\therefore f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = \frac{-2}{2} \quad \therefore x = -1$

$f_y = 0$

$-2y + 8 = 0$

$y = \frac{\cancel{-8}\ 4}{\cancel{-2}}$

$\therefore y = 4$

$\therefore$ Critical point is $(-1,4)$

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$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70 = \underline{\underline{33}}$

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