20171104 - Pset 6 - Q3

$\omega_1 \rightarrow (0,0), (0,1), (2,0), (3,2), (3,3), (2,2), (2,0)$

$\omega_2 \rightarrow (7,7), (8,6), (9,7), (8,10), (7,10), (8,9), (7,11)$

a) Prior probabilities

$P(\omega_1) = \frac{7}{14} = \frac{1}{2}$ $\qquad$ $P(\omega_2) = \frac{7}{14} = \frac{1}{2}$

b) Let μ_1, σ_1 be mean and covariance of ω_1

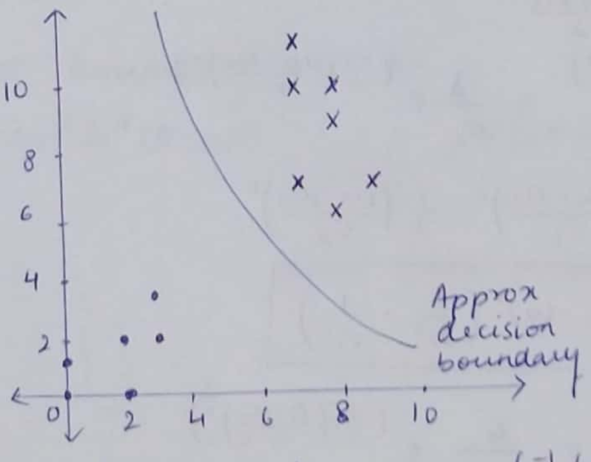
Let μ_2, σ_2 be mean and covariance of ω_2

$\mu_1 = \left(\frac{12}{7}, \frac{8}{7}\right)$ $\qquad$ $\mu_2 = \left(\frac{54}{7}, \frac{60}{7}\right)$

$$\sigma = \begin{bmatrix} (\bar{x}-\mu_x)^2 & (\bar{x}-\mu_x)(\bar{y}-\mu_y) \\ (\bar{x}-\mu_x)(\bar{y}-\mu_y) & (\bar{y}-\mu_y)^2 \end{bmatrix}$$

$$\sigma_1 = \begin{bmatrix} \frac{66}{49} & \frac{37}{49} \\ \frac{37}{49} & \frac{62}{49} \end{bmatrix} \qquad \sigma_2 = \begin{bmatrix} \frac{24}{49} & \frac{-27}{49} \\ \frac{-27}{49} & \frac{152}{49} \end{bmatrix}$$

c)



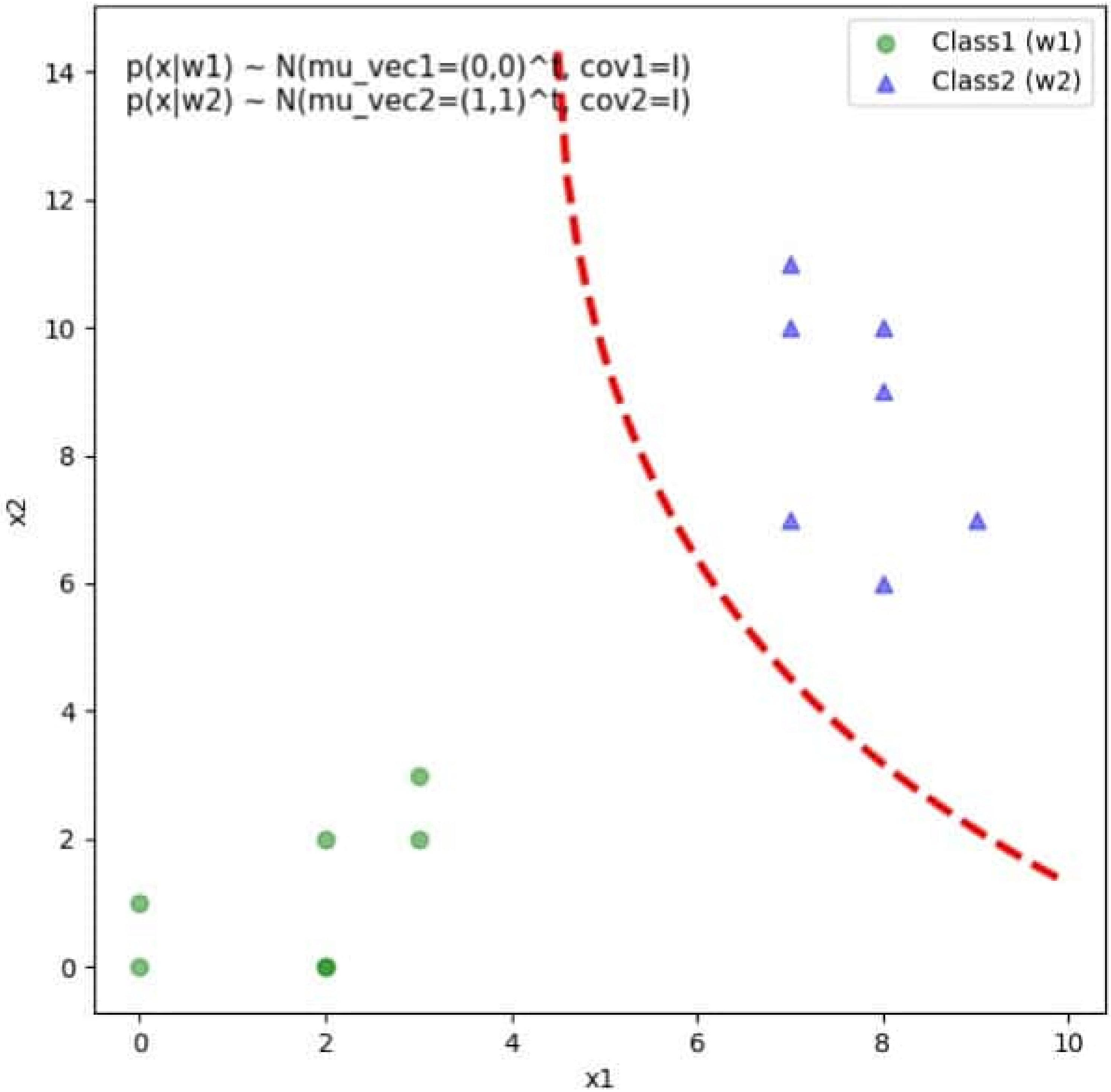
$|\sigma_1| = \frac{389}{343}$

$|\sigma_2| = \frac{417}{343}$

$$\frac{1}{|\sigma_1|^{1/2}} e\left(\frac{-1}{2}(\theta-\mu_1)^T \sigma_1^{-1}(\theta-\mu_1)\right) = \frac{1}{|\sigma_2|^{1/2}} e\left(\frac{-1}{2}(\theta-\mu_2)^T \sigma_2^{-1}(\theta-\mu_2)\right)$$

$$\left[x-\frac{12}{7}, y-\frac{8}{7}\right] \begin{bmatrix} \frac{434}{389} & \frac{-259}{389} \\ \frac{-259}{389} & \frac{462}{389} \end{bmatrix} \begin{bmatrix} x-\frac{12}{7} \\ y-\frac{8}{7} \end{bmatrix} - \left[x-\frac{54}{7}, y-\frac{60}{7}\right] \begin{bmatrix} \frac{1064}{417} & \frac{189}{417} \\ \frac{189}{417} & \frac{168}{417} \end{bmatrix} \begin{bmatrix} x-\frac{54}{7} \\ y-\frac{60}{7} \end{bmatrix} = \ln\left(\frac{417}{389}\right)$$

$$1.23x^2 + 1.92xy - 0.67y^2 + 38.43x + 11.54y - 205 = 0$$



p(x|w1) ~ N(mu_vec1=(0,0)^t, cov1=I)
p(x|w2) ~ N(mu_vec2=(1,1)^t, cov2=I)
Class1 (w1)
Class2 (w2)
x1
x2
0
2
4
6
8
10
12
14

e) Given reclassification of ω_1 into ω_2 is twice as costly, the decision boundary will move towards ω_2 expanding region of ω_1.

The decision boundary will be -

$$\frac{1}{|\Sigma_1|} e^{-\frac{1}{2}((\theta-\mu_1)^T \Sigma_1^{-1} (\theta-\mu_1))} = \underbrace{2}_{\text{gives extra penalty}} \times \frac{1}{|\Sigma_2|} e^{-\frac{1}{2}((\theta-\mu_2)^T \Sigma_2^{-1} (\theta-\mu_2))}$$