20171104 : Pset 04, Mathematical Foundations - 01

We are given: $P(B1) = 0.6 \quad P(B2) = 0.4$

Conditional probabilities: $P(M|B1) = 2/3 \quad P(A|B1) = 1/3$

$P(M|B2) = 1/3 \quad P(A|B2) = 2/3$

We want to find $P(B2|M)$

Using Bayes Theorem,

$$P(B2|M) = \frac{P(M|B2) \times P(B2)}{P(M|B2) \times P(B2) + P(M|B1) \times P(B1)}$$

$$= \frac{\frac{1}{3} \times 0.4}{\frac{1}{3} \times 0.4 + \frac{2}{3} \times 0.6}$$

$$= \frac{0.4}{0.4 + 1.2}$$

$$= \frac{1}{4}$$

Probability that mango was picked from bag 2 $= \frac{1}{4} = 0.25$

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Pset 04, Mathematical Foundations-02

$B = P^{-1}AP \Rightarrow A = PB^{-1}P$

If $Ax = \lambda x$ where λ is eigenvalue & x is eigenvector of A,

$P^{\bullet}BP^{-1}x = \lambda x$

$BP^{-1}x = P^{-1}(\lambda x) \Rightarrow B(P^{-1}x) = \lambda(P^{-1}x)$

Let $\underline{P^{-1}x = v}$

$\therefore Bv = \lambda v$

$\Rightarrow$ v is eigenvector of B & λ is eigenvalue.

$\therefore$ Every eigenvalue of A is an eigenvalue of B and vice-versa

$\Rightarrow$ Proved that A and B have same eigenvalues

If x is eigenvector of A, then eigenvector of $B = P^{-1}x$

$\Rightarrow$ Similar matrices <u>have same eigenvalues</u> but <u>do not have same eigenvectors.</u>

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Pset 4 - Mathematical Foundations - 3

1) $\mu_1 = [0.5, 0.6, 0.2]^T$ $\quad \mu_2 = [0.3, 0.2, 0.5]^T$

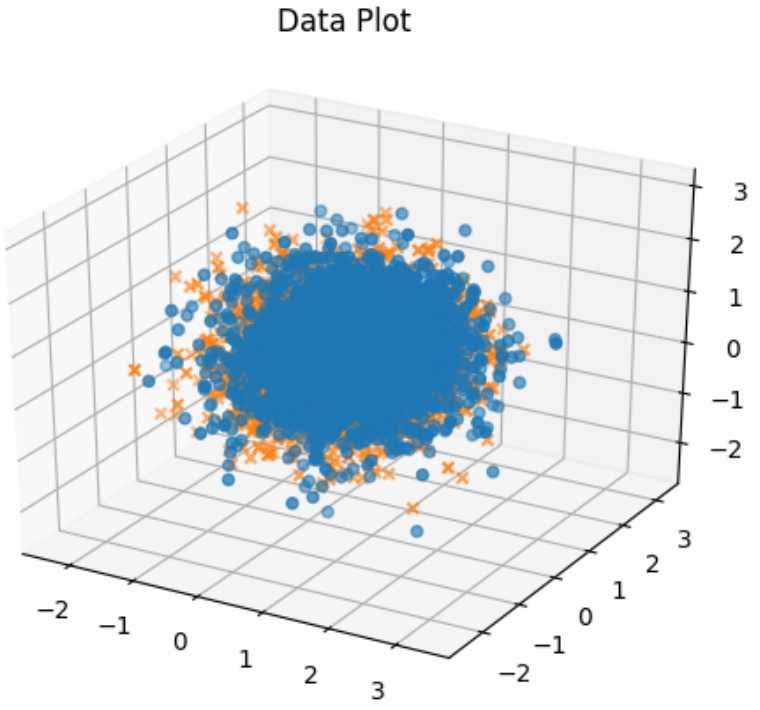
$$\Sigma = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

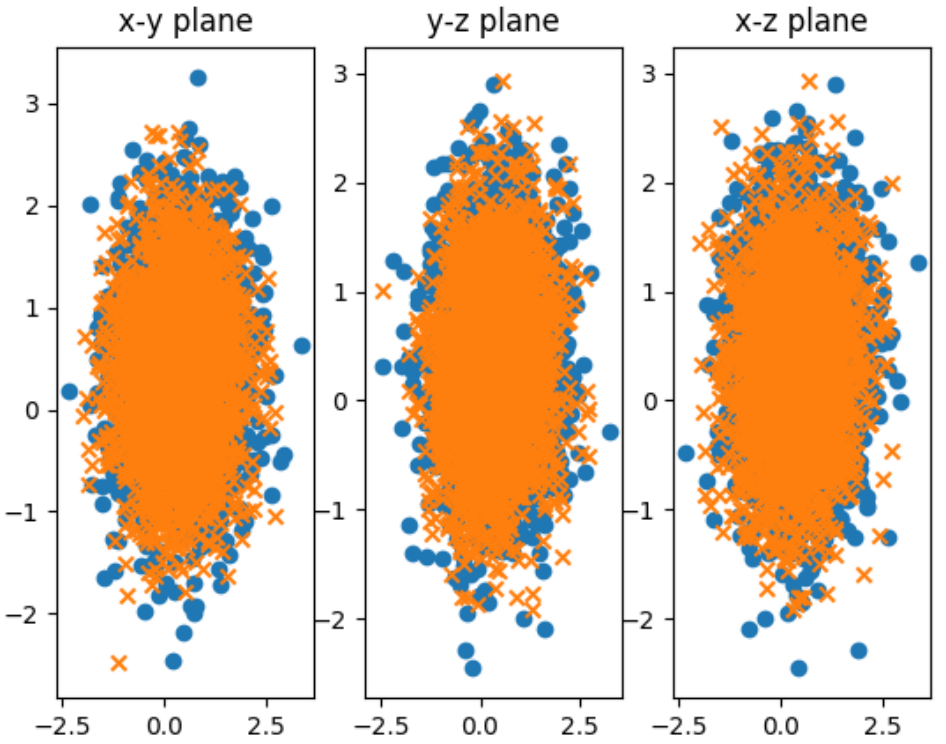
2) $\mu_1 = [0.2, 0.9, 0.4]^T$ $\quad \mu_2 = [0.5, 0.1, 0.7]^T$

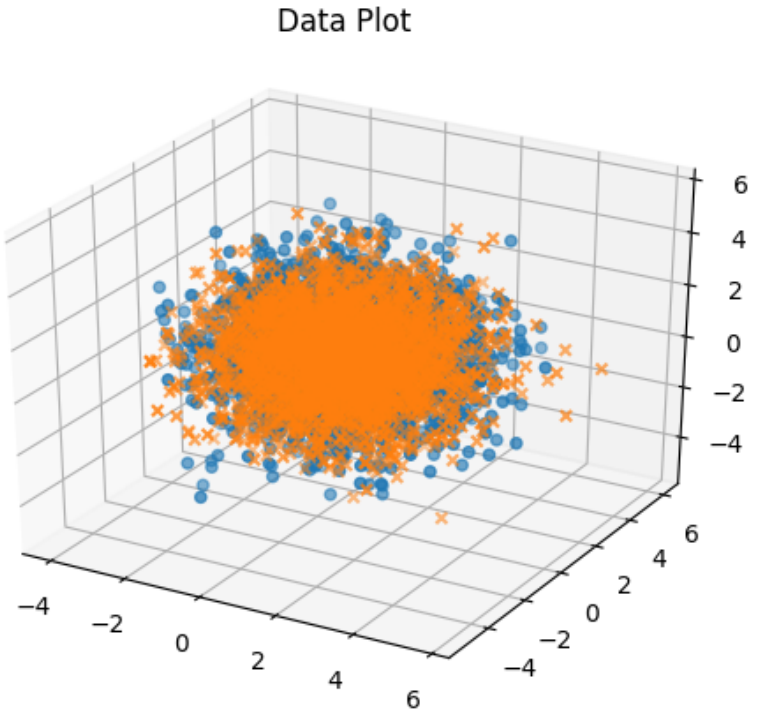
$$\Sigma = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

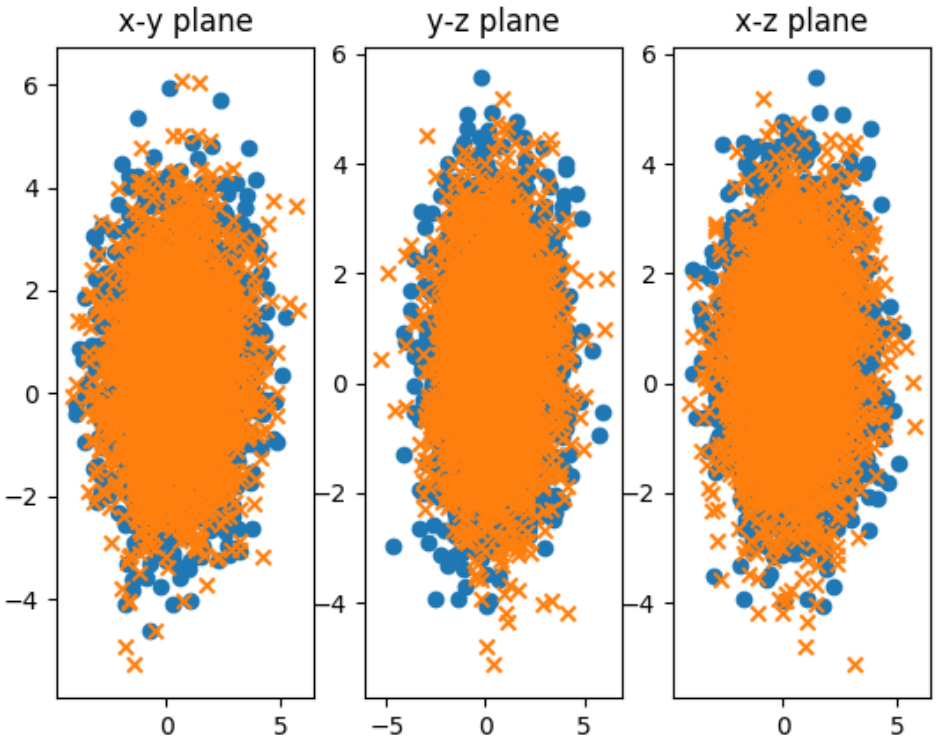
3) $\mu_1 = \mu_2 = [0.7, 0.6, 0.3]^T$

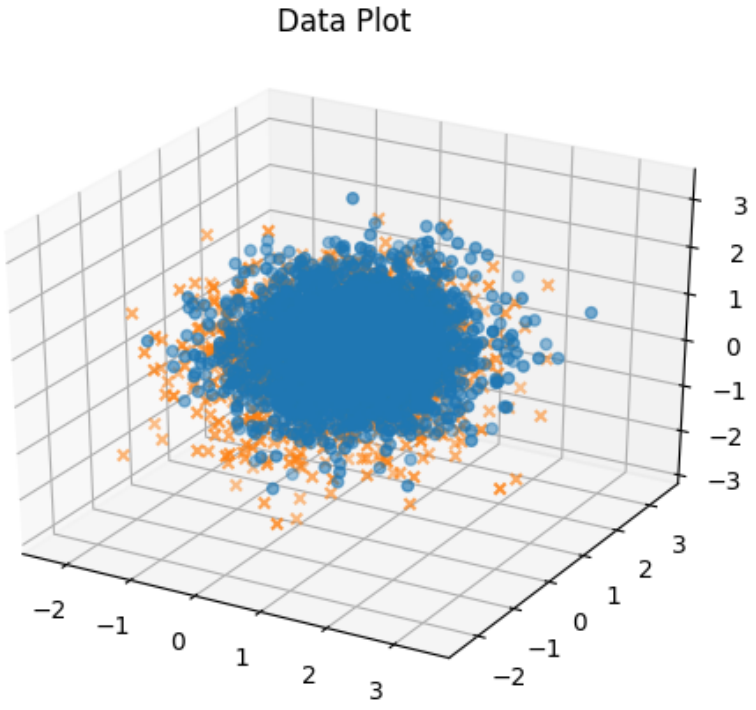
$$\Sigma_1 = \begin{bmatrix} 0.6 & 0. & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}$$

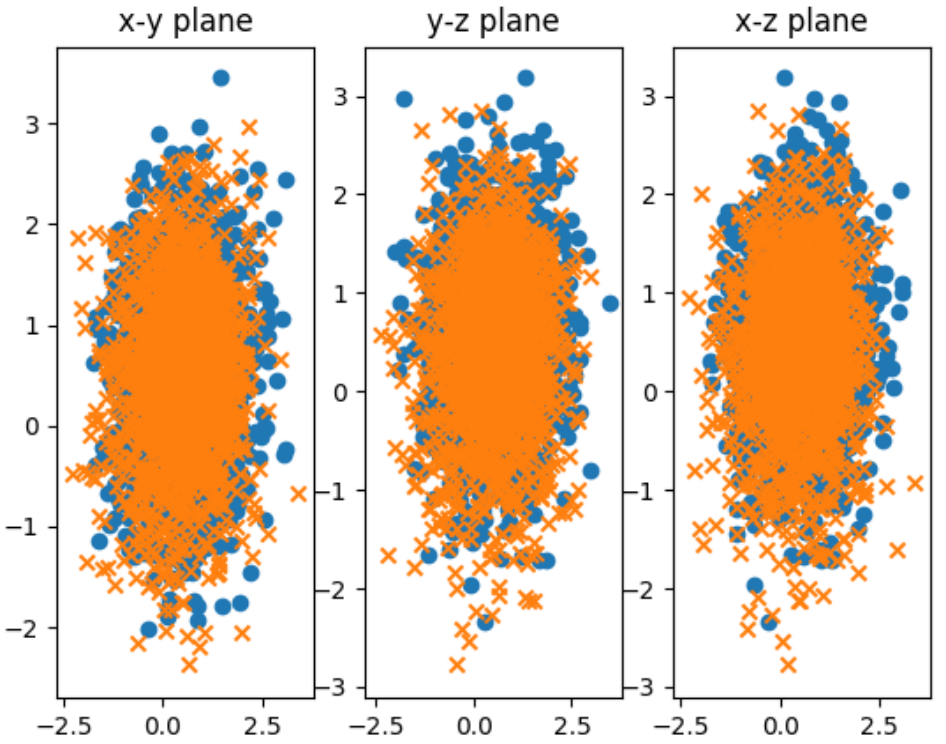

Data Plot
3
2
1
0
−1
−2
−2
−1
0
1
2
3
3
2
1
0
−1
−2


x-y plane
y-z plane
x-z plane
3
2
1
0
−1
−2
−2.5
0.0
2.5


Data Plot
6
4
2
0
−2
−4
−4
−2
0
2
4
6
6
4
2
0
−2
−4


x-y plane
y-z plane
x-z plane
6
4
2
0
−2
−4
0
5
−5
0
5
0
5


Data Plot
3
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
3
2
1
0
−1
−2
−3


x-y plane
y-z plane
x-z plane
3
2
1
0
−1
−2
−3
−2.5
0.0
2.5