

# Contents

<i>Foreword</i>	<i>page</i> xiii
<i>Preface</i>	xv
<i>Notation</i>	xix

## PART I INTRODUCTION AND PREREQUISITES

<b>1</b>	<b>Sound Propagation through the Stochastic Ocean</b>	<b>3</b>
1.1	Introduction and Historical Background	3
1.1.1	Ocean Dynamics and Fluctuations	6
1.1.2	Theory of Sound Propagation	12
1.2	Three Decades of Development: Observations	14
1.2.1	Deep Water	14
1.2.2	Emergence of Shallow-Water Acoustics	21
1.2.3	Ocean Sound-Speed Spectrum	24
1.3	Three Decades of Development: Theory	25
1.3.1	Ray Theory	28
1.3.2	Weak Fluctuation Theory	30
1.3.3	Path Integral Theory	32
1.3.4	Coupled-Mode Transport Theory	34
1.4	Where We Stand	36
1.5	Utilizing this Book	37
<b>2</b>	<b>Acoustical Prerequisites</b>	<b>40</b>
2.1	Introduction	40
2.2	Fundamental Equations of Hydrodynamics	40
2.2.1	Parabolic Wave Equation	42
2.3	Character of the Oceanic Acoustic Waveguide	43
2.3.1	Canonical Sound-Speed Profiles	45
2.3.2	Sound-Speed Fluctuations due to Internal Waves	47

2.3.3	Example Profiles	47
2.3.4	Attenuation of Sound	49
2.4	Rays	52
2.4.1	Physical Picture and Basic Equations	52
2.4.2	Ray Theory: Asymptotic Analysis	55
2.4.3	Ray Amplitude and Stability	56
2.4.4	Ray Chaos: Introduction	59
2.4.5	Ehrenfest Theorem	61
2.4.6	Rays in a Range-Independent Ocean	63
2.5	Fresnel Zones and Ray Tubes	70
2.6	Born and Rytov Approximations	75
2.6.1	Relation to Amplitude and Phase	77
2.6.2	Relationship between Born and Rytov Solutions	77
2.7	Path Integrals	78
2.7.1	Variational Approach to Ray Propagation	78
2.7.2	Path Integrals: A Qualitative Discussion	80
2.7.3	Formulation of the Path Integral	80
2.7.4	Solution of the Parabolic Equation as a Path Integral	81
2.8	Normal Modes	82
2.8.1	Coupled Mode Equations	83
2.8.2	Adiabatic Theory	86
2.8.3	Vertical Modes, Horizontal Rays	87
2.8.4	Modes in a Range-Independent Ocean	88
	Appendix A Green's Functions and the Fresnel Zone	100
	Appendix B WKB Modes	102
<b>3</b>	<b>Stochastic Ocean Internal Waves</b>	<b>104</b>
3.1	Introduction	104
3.2	Fundamental Equations of Hydrodynamics	108
3.2.1	Ray Theory: Local Plane Waves	109
3.2.2	Modal Solutions	113
3.2.3	WKB Analysis	115
3.3	Garrett-Munk Internal-Wave Model	118
3.3.1	Other Useful Forms of the GM Spectrum	121
3.3.2	Maximum Internal-Wave Mode Number	123
3.3.3	Internal-Wave Correlation Scales	124
3.3.4	Modifications to GM	126
3.4	Observations	127
3.4.1	Deep Ocean: Mid-Latitude	128
3.4.2	Seamounts, Slopes, and Canyons	131
3.4.3	Continental Shelves	131

3.4.4	Arctic and High Latitude	133
3.4.5	Equator	134
3.5	Other Sources of Stochastic Sound-Speed Structure	135
3.5.1	Spice	136
3.5.2	Vortical Motions	139
	Appendix A An Internal-Wave Model with an Exponential Correlation Function	139
	Appendix B Monte Carlo Simulation	141
<b>4</b>	<b>Introduction to Acoustic Fluctuations</b>	<b>144</b>
4.1	Origin of Phase and Amplitude Fluctuations	144
4.1.1	Weak Fluctuations	145
4.1.2	Strong Fluctuations: Wave Front Folding and Interference	145
4.1.3	Ray Micro-multipath	147
4.1.4	A Simple Model of Microray Interference	153
4.1.5	Modal-multipath: Mode Coupling	156
4.2	Acoustic Sensitivity to Internal Waves	157
4.2.1	Ray/Internal Wave Resonance and Diffraction	159
4.2.2	Ray Chaos	165
4.2.3	Mode Coupling	167
4.3	Propagation Regimes and Signal Behavior	169
4.3.1	Definition of $\Phi$ and $\Lambda$ : Ray Propagation	170
4.3.2	Acoustic Behavior in Different $\Lambda - \Phi$ Regimes	175
4.3.3	Coherence Functions	181
	<b>PART II WAVE PROPAGATION THEORIES</b>	
<b>5</b>	<b>Ray Theory</b>	<b>187</b>
5.1	Introduction	187
5.2	Fundamental Equations: Displacement and Current	189
5.3	Travel-Time Variance	192
5.3.1	Ray-Tangent Approximation	195
5.3.2	Accuracy of Ray-Tangent Approximation	198
5.3.3	Observations	200
5.4	Other Ray-like Observables	202
5.4.1	Ray-Angle Variance	202
5.4.2	Ray-Intensity Variance	204
5.5	Scattering Along and Across the Wave Front	206
5.6	Ray Chaos	208
5.6.1	Nonlinear Response to Forcing	209
5.6.2	Features of Chaos and KAM Theory	212
5.6.3	Levels of Randomness: Ray Statistics	215

5.7	The $\alpha$ Parameter	216
5.7.1	Action Angle Variables Revisited	217
5.7.2	Ray Stability	219
5.7.3	Travel-Time Stability	221
5.8	Travel-Time Statistics: Random Walk Model	222
5.8.1	Eigenray Constrained Model	224
5.8.2	Unconstrained Model	225
5.9	Ray Chaos in Observations	226
5.9.1	Nature of Chaotic Rays and Time Fronts	226
5.9.2	Acoustic Field Statistics	231
5.10	Wave Chaos	235
5.11	Summary	237
	Appendix A Calculation of $dz/d\theta$ in a Waveguide	239
<b>6</b>	<b>Weak Fluctuation Theory</b>	<b>241</b>
6.1	Introduction	241
6.2	Spectra of Phase and Log-Amplitude: No Waveguide	242
6.2.1	Incident Plane Wave	242
6.2.2	Point Source	246
6.3	Spectra of Phase and Log-Amplitude: Ocean Waveguide	247
6.3.1	Local Straight Ray Approximation	247
6.3.2	A More Rigorous Approach	248
6.3.3	Depth/Time Observation Plane	252
6.3.4	Variances of Phase and Log-Amplitude	253
6.3.5	An Example Calculation	254
6.4	$\Lambda - \Phi$ Revisited	256
6.4.1	Spectral Average, $\{m^2\}$	257
6.4.2	Fresnel Zone	257
6.4.3	Border of the Unsaturated Regime	259
6.5	Observations	260
6.5.1	Cobb Seamount and MATE	261
6.5.2	AFAR	265
6.5.3	AATE	267
6.5.4	AET-87	268
6.6	Summary	270
<b>7</b>	<b>Path Integral Theory</b>	<b>271</b>
7.1	Introduction	271
7.2	Path Integral Theory	273
7.2.1	A Simple Example	275
7.2.2	A Note on the PE Approximation	276
7.3	Mean Pressure	277

7.4	Mutual Coherence Functions: Space and Time Separations	278
7.4.1	A Broadband Microray Coherence Theory	281
7.4.2	Evaluation of Phase Structure Function	282
7.4.3	Depth Separations	284
7.4.4	Time Separations	285
7.4.5	Horizontal Separations	286
7.4.6	Relation to Signals with Multiple Deterministic Paths	287
7.4.7	Observations: Single Path	288
7.4.8	Observations: Multiple Paths, Long Range, and Shallow Water	292
7.4.9	Internal-Wave Tomography	300
7.5	Mutual Coherence Functions: Frequency Separations	303
7.5.1	Evaluation of $Q(\Delta q)$	307
7.5.2	Observations	310
7.6	Time-Lagged Intensity Covariance	312
7.6.1	Observations	313
7.7	Intensity Coherence and Spectra	314
7.7.1	Separations in Time	318
7.7.2	Separations in Depth	320
7.7.3	Separations in Frequency	321
7.7.4	Observations	321
7.8	Intensity Moments: Microray Focusing Parameter	323
7.8.1	Observations	328
7.9	Summary	329
	Appendix A Path Integrals as Products of Delta Functions	329
	Appendix B Solution of the Gaussian Path Integral for Frequency	
	Correlations	330
	Appendix C GM Spectral Averages for Path Integral Equations	331
<b>8</b>	<b>Mode Transport Theory</b>	<b>333</b>
8.1	Introduction	333
8.2	Solutions to the Coupled Mode Equation	335
8.2.1	Adiabatic Solution	338
8.3	Coupling Matrix Correlation Function	338
8.3.1	Scattering Matrix	340
8.4	Mean Pressure	341
8.4.1	Adiabatic Theory	342
8.4.2	Transport Theory	342
8.4.3	Solution for Mean Pressure	344
8.4.4	Connection between Transport and Adiabatic Theory	345
8.5	Mean Intensity	346
8.5.1	Adiabatic Theory	346

8.5.2	Transport Theory	347
8.5.3	Mode Energy	349
8.5.4	Two-Mode Example	352
8.5.5	Hybrid Theory	354
8.5.6	Monte Carlo Validation	355
8.5.7	Observations	360
8.6	Mutual Coherence Functions: Space and Time Separations	365
8.6.1	Depth Separations	368
8.6.2	Temporal Separations	371
8.6.3	Transverse Separations	373
8.6.4	Observations	376
8.7	Mutual Coherence Function: Frequency Separations	382
8.7.1	Adiabatic Approach	384
8.7.2	Effects of Coupling	385
8.8	Intensity Variance	387
8.8.1	Adiabatic Approach and Hybrid Theory	390
8.8.2	Transport Theory	391
8.8.3	Asymptotic Behavior	391
8.8.4	Two-Mode Example	394
8.8.5	Observations	397
8.9	Summary	399
	Appendix A Integrals Related to Standard Scattering Matrices	399
	Appendix B Integrals Related to MCFs with Temporal and Transverse Lags	400
	Appendix C Random Surface Gravity Waves	401
	<i>References</i>	403
	<i>Index</i>	417