

Under-ice acoustic navigation using real-time model-aided range estimation

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(Dated: 20 February 2022)

1 The long baseline (LBL) underwater navigation paradigm relies on the conversion
2 of recorded travel times into pseudoranges to trilaterate position. For real-time op-
3 erations, this conversion has assumed an isovelocity sound speed. For re-navigation
4 in post-processing, computationally and/or labor intensive acoustic modeling may
5 be employed to reduce uncertainty driven by multipath arrivals. This work demon-
6 strates a real-time ray-based prediction method of the effective sound speed along
7 a path from source to receiver to minimize vehicle position error. This method was
8 implemented for a small scale AUV-LBL system in March 2020, in the Beaufort Sea,
9 in total ice-covered conditions and a double ducted acoustic propagation environ-
10 ment. The vehicle was successfully deployed and recovered. Given the lack of GPS
11 data throughout the vehicle’s mission, however, the pseudorange performance is first
12 evaluated on connections between GPS-linked beacons. The real-time ranging error
13 between beacons is roughly 11 meters at distances up to 3 km. But a consistent over-
14 estimation in the real-time method provides insights for improved eigenray filtering
15 by the number of surface bounces. An operationally equivalent pipeline is used to
16 re-position the LBL beacons and re-navigate the AUV, using a modeled, historical,
17 and a locally observed sound speed profile. The median re-navigation error is 1.84 ± 2.19 RMS m. The improved trilateration performance for suggests that this ap-
18 proach effectively extends the single meter accuracy of the deployed GNSS units into
19 the water column.

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²¹ **I. INTRODUCTION**

²² Autonomous underwater vehicles (AUVs) are increasingly capable platforms to explore
²³ and sample the ocean, particularly for remote and/or dangerous regions. However, navi-
²⁴ gation uncertainty is a major challenge in considering AUVs as standard tools for oceano-
²⁵ graphic research. While land and air-based robots utilize information from Global Naviga-
²⁶ tion Satellite Systems (GNSS) to achieve stunning location accuracy and precision through-
²⁷ out the duration of their missions, AUVs cannot access GNSS fixes while underwater. There-
²⁸ fore, underwater vehicles have relied on any combination of dead reckoning, hydrodynamic
²⁹ models, inertial navigation systems, doppler velocity logs, and acoustic baseline positioning
³⁰ systems for navigation (Paull *et al.*, 2014). Limiting navigation error and drift requires an
³¹ AUV to periodically stall on the surface and obtain a GNSS fix to reset its position error.
³² This foolproof method of self-positioning is undesirable for stealth, adverse weather condi-
³³ tions, and mission efficiency, and inaccessible in a GNSS-denied situation like an ice-covered
³⁴ environment.

³⁵ Of underwater acoustic navigation systems, long baseline (LBL) is the most GPS-like
³⁶ in style and scale, and most appropriate for mitigating drift without overburdening com-
³⁷ putation or payload size on the vehicle (Van Uffelen, 2021). The state-of-the-art for LBL
³⁸ outsources depth to a pressure sensor and solves the two-dimensional localization problem
³⁹ with an isovelocity, linear scaling between one way travel time (OWTT) and range (Eustice
⁴⁰ *et al.*, 2006, 2007; Webster *et al.*, 2009, 2012). This assumption is valid for short scale
⁴¹ operations but oversimplifies propagation for larger and/or complex acoustic environments.

42 To achieve single meter, GNSS-like performance in a GNSS-denied environment, we demon-
 43 strate an embedded ray-based data processing algorithm to convert recorded OWTTs into
 44 pseudorange estimates. This methodology was integrated onto the AUV *Macrura*, deployed
 45 and recovered in the Beaufort Sea, in March 2020, during the Ice Exercise 2020 (ICEX20).
 46 A physics-driven methodology that received an *in situ* sound speed profile (SSP) was nec-
 47 essary despite the small operational domain because of the relatively high-risk mission en-
 48 vironment—total under-ice conditions and a variable double ducted acoustic environment.
 49 For consistency, we delineate specific definitions for timing, positioning, and navigation
 50 from [Howe et al. \(2019\)](#).

- 51 1. Timing is the ability to acquire and maintain accurate and precise time anywhere in
 52 the domain of interest within user-defined timeliness parameters
- 53 2. Positioning is the ability to accurately and precisely determine one's location refer-
 54 enced to a standard geodetic system
- 55 3. Navigation is the ability to determine current and desired position (relative or absolute)
 56 and apply corrections to course, orientation, and speed to attain a desired position
 57 anywhere in the domain of concern

58 Thus, navigation is inherently in real-time and depends on positioning; positioning depends
 59 on timing. We also suggest re-navigation and re-positioning as post-processed corollaries,
 60 which may include knowledge or processing capabilities not available *in situ*.

61 While RAFOS floats championed one way ranging for re-positioning ([Duda et al., 2006](#);
 62 [Rossby et al., 1986](#)), the ability to do so for navigation was facilitated by the advent of

the WHOI Micro-Modem (Singh *et al.*, 2006) and synchronized clocks (Rypkema *et al.*, 2017). AUV navigation efforts have achieved root mean square (RMS) localization error on the order of tens of meters relative to GNSS surface position over less than ten kilometers in shallow (Claus *et al.*, 2018; Eustice *et al.*, 2007; Kepper *et al.*, 2017) and deep water (Jakuba *et al.*, 2008; Kunz *et al.*, 2008; Webster *et al.*, 2009). However, these efforts all used a nominal sound speed for travel time conversion and the vehicles were limited to shallower isovelocity regimes.

Localization algorithms that do consider environmental or acoustic uncertainty tend to focus on longer and larger experiments, where spatio-temporal variability cannot be ignored. These methods have also been reserved for post-processing as they can be labor intensive, computationally heavy, and/or require additional information like contemporaneous data. For example, gliders navigating with kinematic flight models and equipped with acoustic modems were later unambiguously associated with predicted ray arrivals, resulting in roughly a kilometer error and hundred meters uncertainty over basin scale propagation (Van Uffelen *et al.*, 2013). A follow up study investigated how a single temporally and spatially averaged SSP could mitigate position error for a four month glider mission (Uffelen *et al.*, 2016). Wu *et al.* (2019) cross correlate three days of real acoustic records with synthetic ones generated through ocean model snapshots from HYCOM (Chassagnet *et al.*, 2007). While potentially applicable for various ocean states, this is reliant on model realism and impractical for real-time operations. Mikhalevsky *et al.* (2020) introduces a “cold start” algorithm that does not require prior knowledge of track, position, or sound speed information. The algorithm inputs a four-dimensional ocean model, constrained by tomography data, into a range dependent

85 ray code to isolate the last path detected in a full multipath pattern. Then, a representative
86 group speed is solved for alongside position in a least squares fashion. This approach is able
87 to re-position a floating hydrophone array with an error of 58 m and a standard deviation
88 of 32 m based on six sources 129–450 km away but remains to seen for real-time navigation.

89 The ICEX20 AUV deployment necessitated an environmentally- and physically-driven
90 relationship between recorded travel times and estimated pseudoranges due to the multipath
91 uncertainty brought upon by an increasingly observed double ducted environment in the
92 Beaufort Sea, which some refer to as the “Beaufort Lens” (Chen *et al.*, 2019; Chen and
93 Schmidt, 2020; Litvak, 2015).

94 Given that a lens introduces significant ray refraction, the Beaufort Lens is a shorthand for
95 the spatio-temporal variability of the local temperature and sound speed maxima generally
96 around 50 to 60 m in depth. A neutrally buoyant layer of warm Pacific Summer Water
97 creates a unique double ducted environment —the upper duct degrades signal coherence
98 due to intensified ice interaction and the lower duct effectively traps sound for long range
99 propagation (Poulsen and Schmidt, 2017). Modeling output (Duda *et al.*, 2021, 2019) and
100 experimental observations (Ballard *et al.*, 2020; Bhatt, 2021) suggest that, in the Beaufort
101 Sea, the duct is persistent and widespread but not necessarily continuous; it and its acoustic
102 effects can be non-existent, minimal, or drastic. Transmissions in the upper duct, between
103 the surface ice and the lens, experience minimal attenuation but degrade in signal coherence
104 with repeated reflections under the ice. In lower duct, between the lens and its conjugate
105 depth in the Atlantic water, roughly 200 m, sound above 350 Hz is trapped near losslessly
106 for long range propagation (Poulsen and Schmidt, 2017).

107 Thorough reviews of uncrewed vehicle operations in polar environments can be found in
108 (Norgren *et al.*, 2014) and (Barker *et al.*, 2020); there is no comparable work in the Arctic
109 for a short range AUV deployment in the Beaufort Lens. Seminal (Bellingham *et al.*, 1995;
110 Brooke, 1981; Hayes and Morison, 2002; Jackson, 1983; Light and Morison, 1989) and more
111 recent AUV deployments (Fossum *et al.*, 2021; Jakuba *et al.*, 2008; Kukulya *et al.*, 2010;
112 Kunz *et al.*, 2008; Plueddemann *et al.*, 2012; Timmermans and Winsor, 2013) witnessed the
113 classical upward refracting sound speed profile that is amenable to an isovelocity assumption.

114 Of note, despite different platforms and scales, are recent glider deployments in the
115 Canada Basin. In 2014, in partially ice-covered conditions, a long range LBL system with
116 WHOI Micro-Modems at 100 m exploited the lower duct for long range communication with
117 two gliders (Freitag *et al.*, 2016; Webster *et al.*, 2015). The sound speed value measured at
118 the time of reception was used to estimate pseudorange in post-processing. The beacon-to-
119 beacon performance was excellent, achieving contact at ranges greater than 200 km with
120 a position uncertainty of 40 m. The beacon-to-glider performance, however, deteriorated
121 due to missed contacts outside the duct, and was not described quantitatively. In 2017,
122 gliders were deployed in a region with no ice coverage (Graupe *et al.*, 2019). Ranges were
123 linearly scaled by a statistical description of sound speed observations taken during the
124 experiment, 1450 ± 6.5 m/s. This resulted in an error of 550 m, which was reduced by
125 a factor between 4 and 5, depending on the dive, using a post-processing acoustic arrival
126 matching method. Both cases exploit the lower duct for high fidelity communication at
127 long ranges. Unintuitively, the smaller nature of our deployment during ICEX20 is not a

¹²⁸ simplifying factor. For source depths typical to vehicle operations, 30 to 200 m, a shadow
¹²⁹ zone spans from 2 to 6 kilometers in range ([Schmidt and Schneider, 2016](#)).

¹³⁰ Compared to the previous small scale navigation efforts, the approach in this paper
¹³¹ integrates real-time model-aided data processing to estimate a representative sound speed
¹³² along a path from source to receiver, leveraging climatology, *in situ* data, and fast acoustic
¹³³ modeling. The paper is organized as follows. Section [II](#) details the experimental approach
¹³⁴ and conditions during ICEX20. Given that there is no GNSS ground truth for the vehicle
¹³⁵ position while underway, we first evaluate the real-time ranging performance of GPS-linked
¹³⁶ beacon-to-beacon communication events in section [III](#). Section [IV](#) uses insights from field
¹³⁷ data to introduce a new ray filtering algorithm to improve range estimation. Section [V](#)
¹³⁸ emulates the real-time processing pipeline to re-position beacon-to-beacon events and re-
¹³⁹ navigate AUV *Macrura*.

¹⁴⁰ **II. OVERVIEW OF THE ICEX20 EXPERIMENT**

¹⁴¹ The results from this paper derive from data taken while deploying the AUV *Macrura*, a
¹⁴² custom Bluefin-21, during the Ice Exercise 2020 (ICEX20), in the Beaufort Sea, from March
¹⁴³ 8th to 11th. The AUV deployment was supported by the Integrated Communication and
¹⁴⁴ Navigation Network (ICNN) ([Randeni et al., 2020, 2021; Schneider et al., 2020](#)), a special-
¹⁴⁵ ized implementation of the LBL solution. The ICNN was initially developed via numerous
¹⁴⁶ virtual experiments to ensure robust algorithms and interfaces between different hardware
¹⁴⁷ components. The simulation capabilities are largely physics-driven with a modular system of
¹⁴⁸ systems approach—an environmental simulator with sub-components for the ocean, includ-
¹⁴⁹ ing Arctic ice drift and ocean acoustic propagation; a vehicle simulator with sub-components
¹⁵⁰ for vehicle dynamics and navigation; a topside hardware simulator and acoustic communica-
¹⁵¹ tions simulator, both with a software-only configuration and a hardware-in-the-loop version
¹⁵² ([Schneider and Schmidt, 2018](#)). The virtual environment similarly emulates the interfaces
¹⁵³ between the real components to test the entire software pipeline.

¹⁵⁴ **A. The Integrated Communication and Navigation Network**

¹⁵⁵ The ICNN is comprised of four ice buoys in a loose rectangle, roughly 2 km away from
¹⁵⁶ a central ice camp with a topside computer, as shown in Fig. 1. The AUV and each ice
¹⁵⁷ buoy are outfitted with a WHOI Micro-Modem ([Singh et al., 2006](#)), with a four-element
¹⁵⁸ receiver array, a single transmitter, and one-tenth of a millisecond resolution. Acoustic
¹⁵⁹ messages were sent with a 10 kHz carrier frequency, 5 kHz bandwidth, and phase-shift

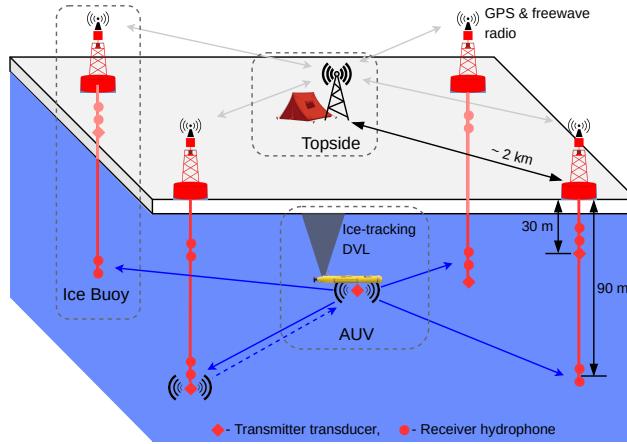


FIG. 1. A schematic overview of the Integrated Communication and Navigation Network (ICNN), which provides joint data-transfer and tracking between AUV and a human decision maker at Topside.

160 keying (PSK)modulation on a time-division multiple access schedule with a thirty-second
 161 cycle, giving room for two-way communication throughout the mission volume. The receive
 162 and transmit elements were split between shallow and deeper depths—30 and 90 m—to
 163 provide better coverage across the shadow zone. While each buoy only has one transmit
 164 depth, all buoys have both receive depths but the active receive layer is consistent across all
 165 buoys. The design of the ICNN enables a self-adapting network to transmit and receive at
 166 the optimal depth to maintain contact with the AUV ([Schneider *et al.*, 2020](#)). The buoys
 167 do not encompass the full horizontal range of the vehicle but are positioned to minimize
 168 overlap in trilateration for spherical positioning ([Deffenbaugh *et al.*, 1996a](#)).

169 To balance competing uses of the acoustic channel, the network uses a single synchronized
 170 digital communication packet to provide both tracking and data to the operator.

171 1. The AUV, running an ice-tracking DVL and an onboard hydrodynamic model, broad-
172 casts its perceived location on a scheduled, time-synchronized message via WHOI
173 Micro-Modem

174 2. Four ice buoys, each outfitted with a WHOI Micro-Modem, receive messages from the
175 AUV and send that information over freewave radio to a Topside computer

176 3. The topside computer converts travel times into pseudorange estimates using a stochas-
177 tic embedded prediction of the horizontal group velocity via BELLHOP ray tracing
178 code ([Porter, 2011](#)) using a sound speed profile provided by an updatable Virtual
179 Ocean ([Bhatt *et al.*, 2022; Schneider and Schmidt, 2018](#))

180 4. The topside computer calculates a new position by trilaterating the range estimates

181 5. The position differential, not the absolute position, is broadcast to the vehicle to
182 update its navigation solution and be robust to latency and intermittency

183 In the face of GNSS-denied navigation, this approach was anecdotally successful, as shown

184 in Fig. 2. The AUV *Macrura* was deployed through a hydrohole from an ice camp but re-

185 covered through an emergency hydrohole. A random disk error stalled *Macrura* underneath

186 the ice but did not prevent it from transmitting its location. Due to an incoming storm, a

187 team placed a physical marker on the ice at the location. Three days later, *Macrura* was

188 found within a meter of the marker. We view the emergency recovery as qualitative proof

189 of the robustness of this navigation approach. Nonetheless, this paper specifically addresses

190 the third and fourth steps—the conversion of travel times into pseudoranges and its effect

191 on trilateration. By focusing on pseudorange estimates between GPS-tracked beacons, and

re-running the trilateration pipeline, the results are decoupled from all other mechanisms in the ICNN.

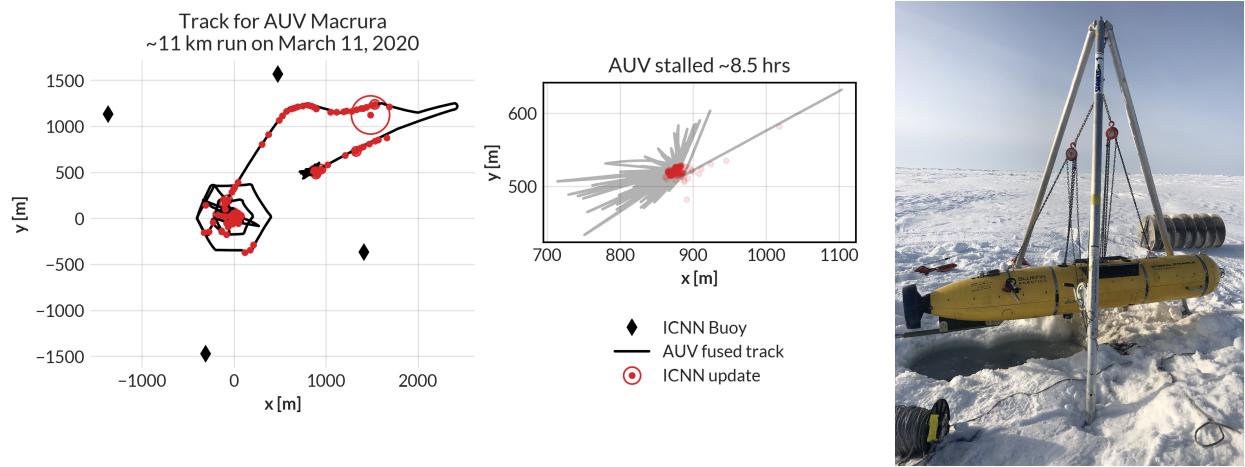


FIG. 2. The under-ice mission track for AUV *Macrura*, including the position updates as it stalled underneath the ice overnight. A marker was placed on the ice at the vehicle’s estimated self-location. It was recovered after a three day storm within a meter of the marker.

194 B. ICEX20 sound speed conditions

An important component to our navigation solution is an accurate estimation of a representative SSP for the ocean volume. Previous field experience, during the Ice Exercise 2016 (ICEX16), demonstrated the negative effects of the Beaufort Lens on tracking and communication (Schmidt and Schneider, 2016). Fig. 3 shows historical, modeled, and *in situ* sound speed data for both ICEX16 and ICEX20. These three input streams were selected to mirror the information available on a submarine (personal conversation with LT B. Howard and LT CDR D. Goodwin). In the field, the SSP information was shared with the vehicle via basis

representation compression on a lightweight digital acoustic message (Bhatt *et al.*, 2022). All modeled data comes from HYCOM (Chassignet *et al.*, 2007), which does not seem to capture the forcing mechanisms that cause the Beaufort Lens. For ICEX16, the data-driven profile was sourced from nearby Ice Tethered Profilers (ITP) after the field experiment (Krishfield *et al.*, 2008; Toole *et al.*, 2011) and exhibits a fairly low lens; the historical profile is from the World Ocean Atlas. For ICEX20, the chosen weights (data-driven) profile derives from an estimate of initial CTD casts taken on site, showing an intense warm water intrusion; the baseline (historical) profile, showing moderate ducted conditions, comes from the average of March 2013 ITP data. This month best matched sea ice and sound speed conditions at the beginning of ICEX20 (Bhatt *et al.*, 2022). It is important to note that all profiles that do show the Beaufort Lens do so with different local sound speed maxima at different depths, reflective of the wide range of lens properties observed for all ITP data in the region. The variability of the lens height and prominence is the main reason an updatable SSP was integrated into the ICNN solution.

During ICEX20, the HYCOM profile was available but never deployed. For post-processing comparison, we introduce both the HYCOM profile and an isovelocity case, 1441.8 ± 3.7 m/s, as the mean and standard deviation of the observed sound speed profile over the first 200 m.

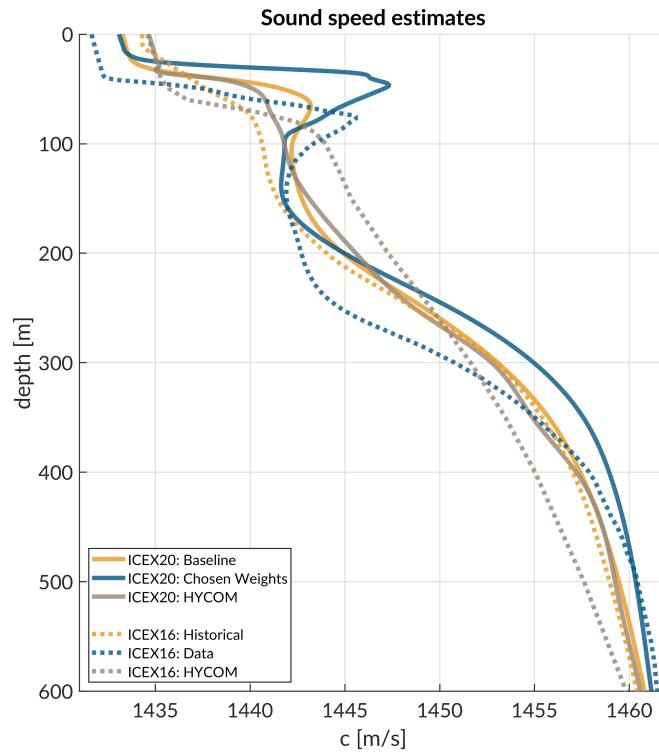


FIG. 3. Sound speed conditions for historical (baseline), data (chosen weights), and HYCOM model for both ICEX16 and ICEX20.

220 **III. REAL-TIME PSEUDORANGE ANALYSIS**

221 Because the vehicle's navigation solution during a mission can only be evaluated on the
222 basis of the error estimates sent, a sister experiment for validating the real-time ranging
223 approach was implemented. Ice buoy modems were run as "virtual vehicles" at a fixed
224 depth, receiving position updates from the other beacons as well as a camp site modem
225 lowered to 20 m. Figure 4 shows successful events sorted by source depth. In this analysis,
226 we assume there is insignificant displacement between the GNSS puck surface expression
227 and subsurface modem; this is supported by unusually low observed ice drift rates, just 0.7
228 cm/s on average throughout the mission.

229 **A. Minimal bounce criteria (MBC)**

230 The fundamental challenge to implement GNSS-like navigation, especially in an acousti-
231 cally complex propagation environment, is characterizing a single sound speed to compensate
232 for the effects of ray refraction and reflection. The use of the acoustic modem network for
233 tracking relies on the accurate estimates of travel times between the submerged platform
234 and LBL beacons, supported by clock synchronization and a pre-determined scheduling of
235 acoustic events. For the Beaufort Lens in particular, the strong multipath effects make it
236 virtually impossible to deterministically predict the modem's detected arrival time.

237 Instead, for each individual modem i , an embedded stochastic tracking framework is used
238 to provide a running estimate of the effective sound speed $c_{i,j}$ for the conversion from travel
239 time to range from modem j , with the ultimate goal of matching the implied horizontal

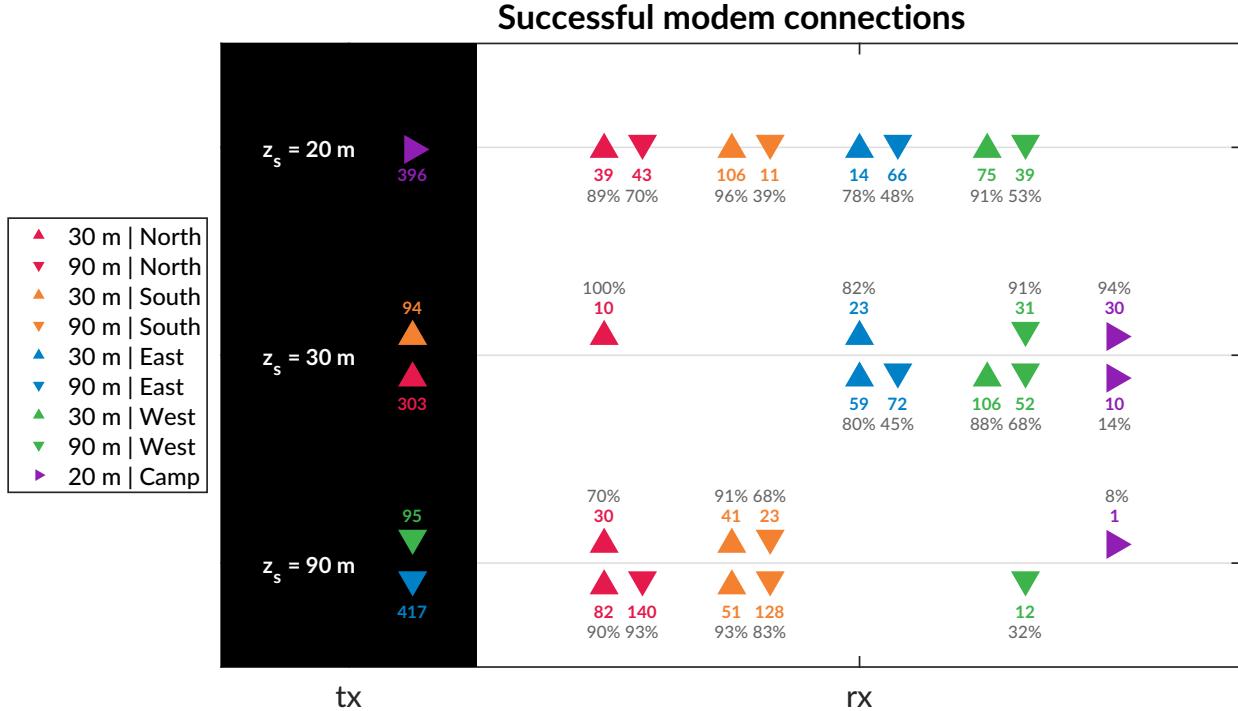


FIG. 4. An overview of the modem experiment by source and receiver depth and position. The black column on the left, *tx*, shows the source depth, z_s , with the total number of transmissions. The column on the right, *rx*, shows the receivers with the number of successful decodings and the percent success rate, defined as the ratio of decoded to detected. The orientation of the triangles—sideways, upwards, and downwards—corresponds to depths of 20, 30, and 90 m.

²⁴⁰ effective sound speed, i.e., the GPS-recorded distance between two nodes divided by the
²⁴¹ modem-recorded one way travel time between them.

²⁴² In the ICEX20 configuration, the acoustic tracking is running on the topside computer,
²⁴³ which controls the ICNN. Here we assume that the effective sound speeds $c_{i,j}$ are smoothly

²⁴⁴ varying over the course of a vehicle mission, i.e., with respect to range, mission time, and
²⁴⁵ the thirty-second frequency.

²⁴⁶ When the topside tracking framework receives a message, with a time delay, Δt , it will
²⁴⁷ request a new estimate for $c_{i,j}$ along with its standard deviation. The effective sound speed
²⁴⁸ is predicted using the vehicle's reported depth and the extrapolated navigation solution for
²⁴⁹ range, \hat{r} , as inputs to the ray tracing program, which returns an impulse response estimate
²⁵⁰ in the form of ray travel times dt_j and amplitudes a_j .

²⁵¹ The initial call to BELLHOP is over a local grid centered at the range and depth posited
²⁵² by the onboard tracking solution. The grid, compared to a point solver, adds redundancy
²⁵³ in resolving the actual multipath structure for a reliable acoustic path without overtaxing
²⁵⁴ onboard computational time and memory. It is initialized as 11×11 points spanning 10
²⁵⁵ m horizontally and 20 m vertically. The horizontal dimension reflects the accumulated
²⁵⁶ vehicle position error given a thirty-second communication cycle; the vertical dimension
²⁵⁷ reflects how, computationally, eigenrays of the same timefront seem to stack vertically in
²⁵⁸ the water column. For each grid point, BELLHOP produces a number of arrivals resulting
²⁵⁹ from multiple propagation paths. Using only the N_0 rays with neither surface nor bottom
²⁶⁰ bounces, it will then estimate the current effective sound speed c from a power weighted
²⁶¹ average of the ray travel times,

$$c = \frac{\hat{r} \sum_{n=1}^{N_0} a_n^2}{\sum_{n=1}^{N_0} dt_n a_n^2}, \quad (1)$$

²⁶² and the associated weighted standard deviation,

$$\sigma_c \simeq \sqrt{\frac{\sum_{n=1}^{N_0} (dt_n - \hat{r}/u)^2 a_n^2}{\sum_{n=1}^{N_0} a_n^2}} \frac{u^2}{\hat{r}} \quad (2)$$

²⁶³ If no direct paths exist, i.e. $N_0 = 0$, then the effective speed is computed using the same
²⁶⁴ algorithm for the ray arrivals with one bounce, and so on.

²⁶⁵ Finally, the pseudorange is calculated simply as

$$r_{i,j} = c_{i,j} \Delta t_{i,j} \quad (3)$$

²⁶⁶ Thus the MBC method assumes the signal detected by the modem will be dominated by
²⁶⁷ a set of paths with the least number of boundary interactions. Importantly, this stochastic,
²⁶⁸ ensemble method for group velocity calculation can run in real-time, appearing to be orders
²⁶⁹ of magnitude faster than other post-processing methods which seek to determine the specific
²⁷⁰ ray itself that best matches a prominent indicator from the arrival structure. The BELLHOP
²⁷¹ simulation that runs this calculation uses 3600 rays with launch angle fan of -60 to 60 degrees,
²⁷² a representative depth dependent sound speed profile, and a range dependent bathymetry.

²⁷³ B. Pseudorange error metrics

²⁷⁴ The sister modem experiment generated 811 beacon to beacon communication events with
²⁷⁵ their own real-time MBC group velocity predictions. Given the complexity of the ICNN
²⁷⁶ system, this experiment did not collect an exhaustive set of data across all buoy, source
²⁷⁷ depth, receive depth, and sound speed combinations. The algorithm generally overestimates
²⁷⁸ pseudoranges because it resolves the effective sound speed for the most direct path.

²⁷⁹ Fig. 5 shows the range error boundary for both SSP inputs used in ICEX20. A promising
²⁸⁰ sign that the MBC method adapts sound speed somewhat intelligently is the lack of error
²⁸¹ growth as travel time increases. The baseline SSP ($n=243$ events) has an absolute pseudor-

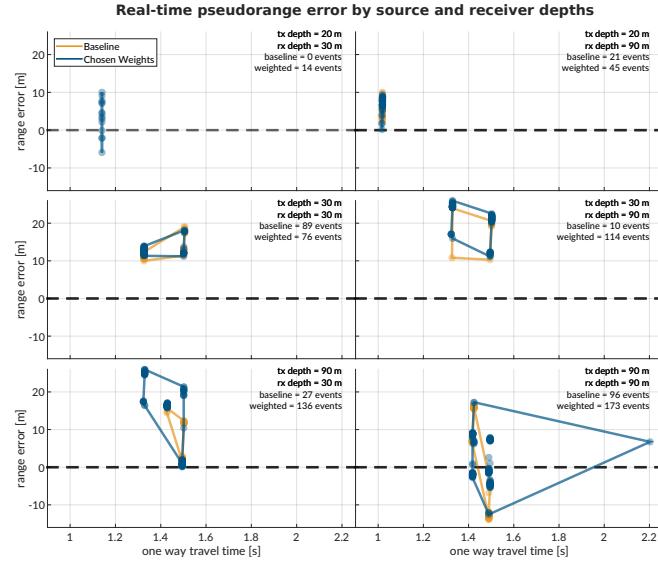


FIG. 5. The real-time range error by source (rows: 20, 30, and 90 m) and receiver (columns: 30 and 90 m) pairings for both sound speed estimates used during ICEX20. The amount of communication events is notated in the top right of each panel. The dashed line indicates no range error, and the boundary drawn indicates scope of the range error as a function of one way travel time.

ange error of 11.38 ± 4.23 m; the weighted SSP ($n=568$), 11.36 ± 8.12 m. The discrepancy between these two is largely due to outlier events only contained in the weighted SSP set. Where there is overlap between sound speed conditions used for the real-time MBC, the pseudorange error difference is no more than a few meters. The overarching results show that sound speed estimates derived from eigenrays for a local grid, as opposed to a singular point, are accurate enough to support vehicle navigation. While the NBC looks for just the least complex multipath, the high density of launch angles almost always guarantees a direct path. Nonetheless, the consistent overestimation of pseudorange invites further analysis into acoustic arrival matching.

291 **C. Eigenray identification for beacon-to-beacon events**

292 Accounting for ice movement between beacons creates nominal ranges with small vari-
293 ability. Figs. 6, 7, and 8 show eigenrays for three sound speed environments for source
294 depths of 20, 30, and 90 m, respectively. Eigenrays were initially found using the built-in
295 BELLHOP protocol with a launch angle fan of 2400 rays between -60 and 60 degrees. Sepa-
296 rately, recorded travel times between beacons were clustered with 1 millisecond boundaries
297 such that some source-receiver pairs had multiple, distinct travel times to approximate. The
298 BELLHOP eigenray returns were then filtered such that one was selected per travel time
299 cluster, in the hopes that the eigenray will converge to the receiver locations for the most
300 realistic sound speed input. It should be noted that bottom bounces were recovered but
301 filtered out. The three source depths create distinct ray geometries with respect to the three
302 sound speed inputs.

303 **1. Source depth of 20 m**

304 For a source at 20 m in depth, shown in Fig. 6, reliable eigenrays are found for all sound
305 speed inputs. Rays refract upwards and neatly intersect with the shallow and deep receiver
306 locations between 1.5 and 1.8 km in range. However, the ray paths for the shallow receivers
307 change both in the number of surface interactions and where the surface interactions occur
308 with respect to range across the SSPs. As the Beaufort Lens strengthens, the chosen paths to
309 the second farthest shallow buoy (North, in red) interact with the surface more and become
310 distinct. The weighted SSP shows the most interesting effects for the deeper receivers. The

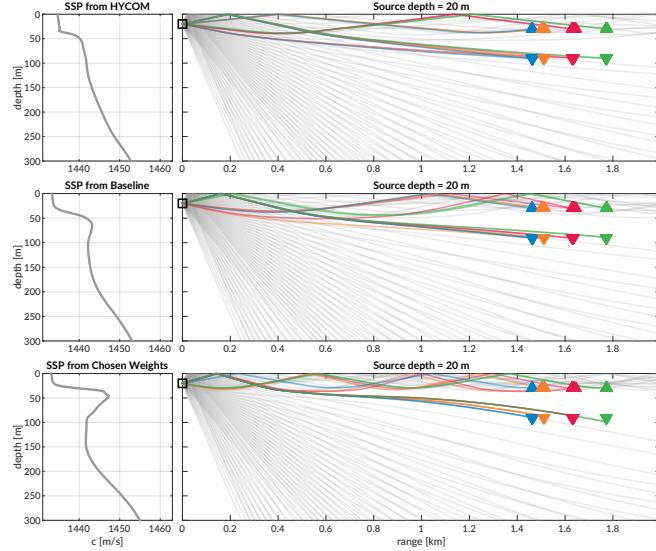


FIG. 6. Eigenrays for beacon-to-beacon events for each sound speed with a nominal source depth of 20 m over a total ray fan in gray. The beacons are highlighted in color/marker coding in Fig. 4.

311 ray paths all interact with the surface and the eigenrays for the Northern (red) and Western
312 (green) buoys are in fact the same ray.

313 **2. Source depth of 30 m**

314 The ray geometries from the 30 m source, shown in Fig. 6, show a similar degradation
315 of eigenray identification with increased ducting. Receptions span 1.5 to 3.2 km. Once
316 again, eigenrays for HYCOM and the baseline SSP are visually appropriate. Rays for the
317 weighted SSP show how the surface channel intensifies ice interactions and how the shadow
318 zone denies reliable acoustic paths. Pointedly, the increasing number of surface reflections to
319 the farthest shallow buoy (North, in red) crystallize the MBC's tendency for overestimation.
320 For the HYCOM, baseline, and weighted SSP inputs, the most appropriate eigenrays show
321 2, 3, and 4 surface interactions.

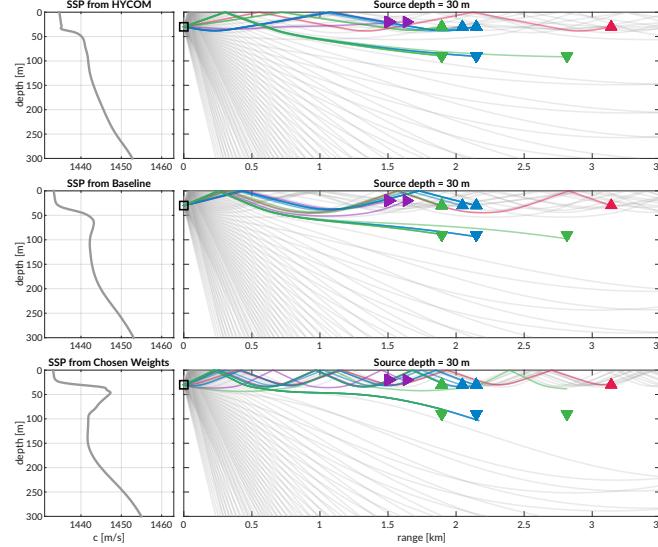


FIG. 7. Eigenrays for beacon-to-beacon events for each sound speed with a nominal source depth of 30 m over a total ray fan in gray. The beacons are highlighted in color/marker coding in Fig. 4.

322 **3. Source depth of 90 m**

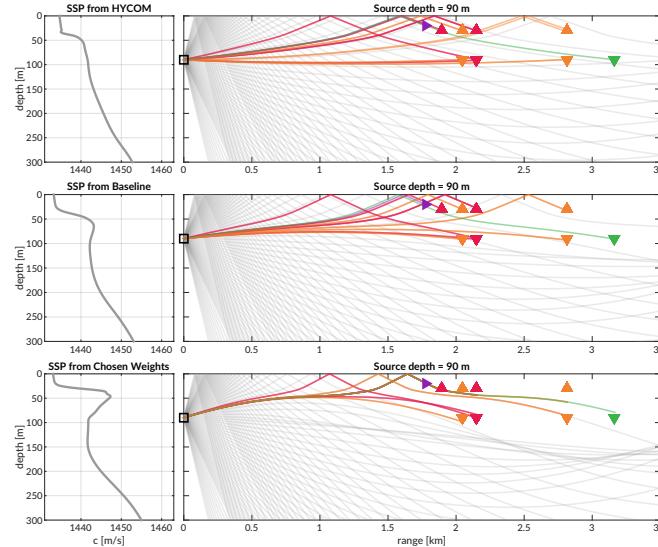


FIG. 8. Eigenrays for beacon-to-beacon events for each sound speed with a nominal source depth of 90 m over a total ray fan in gray. The beacons are highlighted in color/marker coding in Fig. 4.

323 Lastly, Fig. 8 shows ray geometries from the 90 m source, elucidating a different extent
324 of the shadow zone. While the receiver locations are similar to that of the 30 m source
325 depth, the deeper source depth effectively negates the upper duct and places the upper (and
326 some of the lower) receivers in unreliable acoustic paths. The HYCOM eigenrays still show
327 the most reliable acoustic paths that deteriorate with increasing ducted conditions. The
328 lack of direct paths from the observed SSP further points out the shortcomings of the MBC
329 approach.

330 The goal of the MBC algorithm was to provide a reliable, physically intuitive interpre-
331 tation of the acoustic propagation without taking on the additional burden of regularly
332 identifying specific paths that may connect any given source-receiver pair in the network.
333 Its performance was adequate for vehicle navigation and would have likely sufficed if it were
334 not for the prominence of the duct observed relative that of other model and data products.

³³⁵ **IV. POST-PROCESSED PSEUDORANGE ANALYSIS**

³³⁶ From all events recorded during the modem test experiment, there are 1242 successfully
³³⁷ decoded beacon-to-beacon events. Only these events are used to evaluate ranging accuracy,
³³⁸ as the ICNN was not configured to use receptions with failure flags. Thus, a post-processing
³³⁹ analysis that emulates the real-time navigation engine was run to overcome the unequal
³⁴⁰ distribution of communication events with respect to depth, range, and sound speed status.

³⁴¹ It is important to note that the value for the extrapolated range, \hat{r} , is only tracked when
³⁴² the modem runs the vehicle behavior; thus we replace \hat{r} with the GPS-tracked range for all
³⁴³ modem events. Because \hat{r} converges to the correct solution, a comparison of \hat{r} with the GPS-
³⁴⁴ tracked range shows a normal, zero-centered distribution within the bounds of GPS drift.

³⁴⁵ The present analysis therefore seeds realistic but “omniscient” knowledge of the extrapolated
³⁴⁶ range and leverages the post-processing pipeline to more thoroughly evaluate the acoustic
³⁴⁷ range estimate for all modem events, with three relevant sound speed sources, and both
³⁴⁸ group velocity criterion. Accordingly, the results in this section evaluate the utility of the
³⁴⁹ algorithms and sound speed sources, divorced from their role in the ICNN while maintaining
³⁵⁰ real-time relevance.

³⁵¹ **A. Nearest bounce criteria (NBC)**

³⁵² As shown in the eigenray traces of Fig. 7, the extent of ray bending and repeated
³⁵³ reflections is extremely dependent on the sound speed profile. An isovelocity approach
³⁵⁴ would completely miss this nuance; our field-tested approach that only resolved the simplest

355 path is unlikely to resolve the one that triggers modem detection. Based on this insight, a
 356 new algorithm, the nearest bounce criteria (NBC), is a slight modification from the MBC
 357 and includes multipath as a new dimension of information to exploit. This metric, while
 358 run in post-processing, adds a negligible amount of computation for real-time efficacy.

359 Given a running estimate for the horizontal group velocity $u_{i,j}$ between nodes i and j ,
 360 the navigation system has an extrapolated value for range, \hat{r} , and a recorded travel time,
 361 $\Delta t_{i,j}$. Instead of using only the N_0 rays with neither surface nor bottom bounces to estimate
 362 group velocity and subsequently moving to incremental number of bounces only when no
 363 valid direct path solutions exist, we solve for the power weighted average of the ray travel
 364 time for the N_k rays with k bounces,

$$t_k = \frac{\sum_{n=1}^{N_k} dt_n a_n^2}{\sum_{n=1}^{N_k} a_n^2}, \quad (4)$$

365 find the nearest matching power weighted average to recorded travel time,

$$t_{i,j,k} = \min_{k=0,1,2,\dots} |t_k - \Delta t_{i,j}| \quad (5)$$

366 predict a group velocity,

$$u_{i,j} = \frac{\hat{r}}{t_{i,j,k}} \quad (6)$$

367 and estimate the range as was done previously.

$$r_{i,j} = u_{i,j} \Delta t_{i,j} \quad (7)$$

368 This method selects a different group velocity based on the multipath arrival structure,
 369 as the detected arrival is not always the first arrival or the direct path and could even be
 370 masked by noise or blocked temporarily (Deffenbaugh *et al.*, 1996b). We manually cap the

³⁷¹ number of bounces to 4 because of the smaller operational scale and the attenuation accrued
³⁷² with many surface interactions. Bottom bounces are not encoded separately because of ray's
³⁷³ tendency to refract upward, not due to information limitations.

³⁷⁴ **B. Effective sound speed predictions**

³⁷⁵ The minimal and nearest bounce algorithms are applied with the three sound speed inputs
³⁷⁶ shown in Fig. 7. The resulting predicted group velocities for all source depths are shown in
³⁷⁷ Fig 9.

³⁷⁸ The goal of the group velocity estimation is to converge towards the implied sound speed,
³⁷⁹ i.e. the GNSS-derived range divided by the recorded travel time. For a 30 m receiver depth,
³⁸⁰ the NBC shows more overlap with data-derived values as it classifies multipath more cor-
³⁸¹ rectly. For a 90 m receiver depth, the overlap is less accurate due to computational con-
³⁸² straints of a limited fan of rays entering the shadow zone rendering a less reliable simulated
³⁸³ times of arrival packet.

³⁸⁴ As the environmental and ray filtering method become better representations of the real
³⁸⁵ ocean, the lower the expected mismatch is between the implied and estimated effective
³⁸⁶ sound speeds. Analysis shows that the higher multipath classification produces more ac-
³⁸⁷ curate sound speed predictions, likely driven by a tighter and/or sparser bundle of rays.
³⁸⁸ However, that data are too small to draw significant conclusions. Discontinuities in mul-
³⁸⁹ tipath classification verify our hypothesis for its importance to a smoothly varying group
³⁹⁰ velocity, as shown in the cluster for a receiver depth of 30 m, where HYCOM jumps from

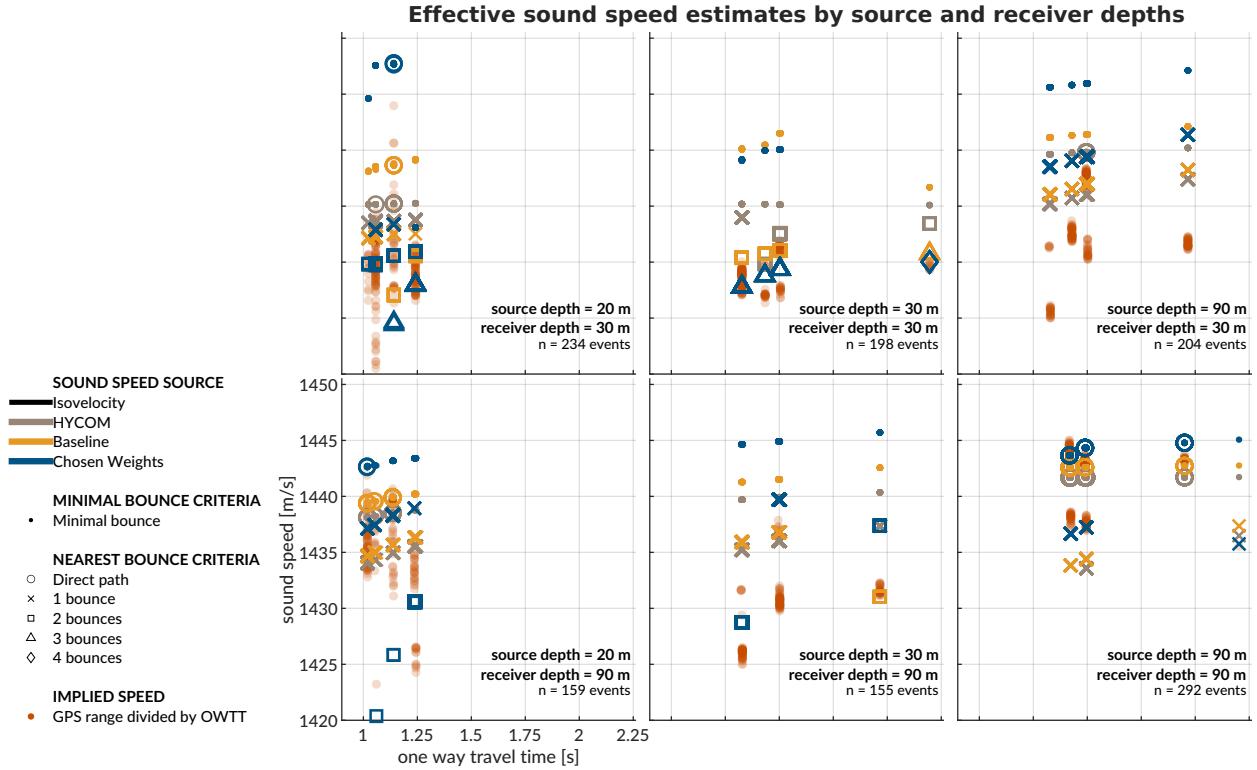


FIG. 9. A comparison of group velocity predictions for all beacon to beacon events in post-processing with a source depth of 30 m, with group velocity on the y-axis and recorded travel time on the x-axis. The left panel is for a receiver depth of 30 m; the right panel for 90 m. The sound speed source is indicated by color. The minimal and nearest bounce criterion are distinguished by different marker shapes, compared to the separately colored red dots showing the naive, data-driven group velocity calculation.

391 one to two bounces amidst the baseline SSP and weighted SSP smoothly increasing while
 392 consistently seeing two and three bounces, respectively.

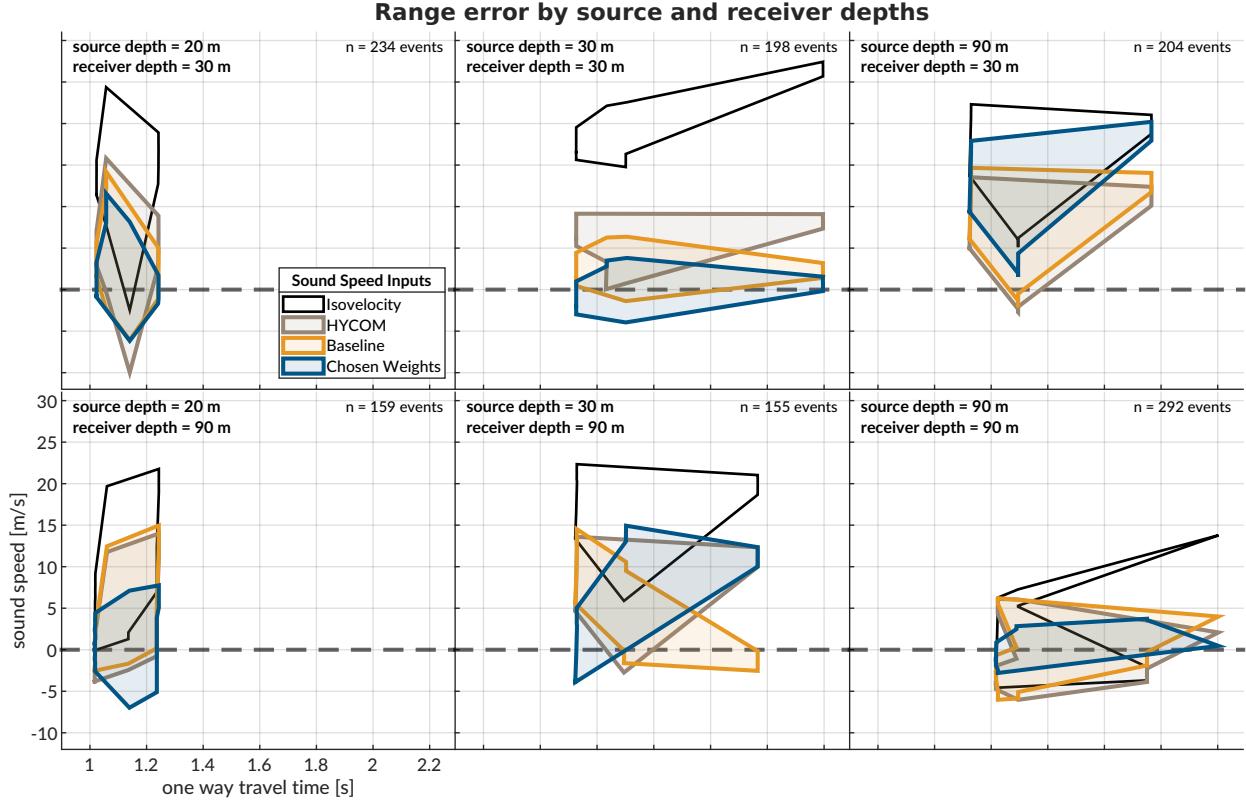


FIG. 10. The post-processed range error for source depths of 20, 30, and 90 m, and receiver depths of 30 and 90 m. The dashed gray line shows no error. The shaded region connects the range performance across all events.

393 **C. Pseudorange error metrics**

394 Fig. 10 shows the directional range error footprints for all three sound speed inputs with
 395 respect to OWTT, separated by source and receiver depth configurations. The weighted
 396 SSP range error generally has the smallest and most zero-centered footprint. The one case
 397 it does not is for the source-receiver pairings between 30 and 90 m in depth. The increased
 398 error for these reciprocal transmission paths is most likely driven by the computational
 399 artifacts encountered when propagating through the steep sound speed gradients of the lens

⁴⁰⁰ and through the shadow zone. All other source depth pairings are significantly improved
⁴⁰¹ using the chosen weights compared to HYCOM or the baseline.

⁴⁰² When using a linear scaling to convert travel time into range, any offset between the
⁴⁰³ assumed sound speed and the horizontal group velocity produces unconstrained error with
⁴⁰⁴ increasing receiver distance. Most importantly, we see the consequences of the adaptive
⁴⁰⁵ group velocity in that range error does not strictly increase with OWTT.

⁴⁰⁶ The improvement from MBC to NBC is most evident for the realistic sound speed; while
⁴⁰⁷ the HYCOM SSP improves from a median absolute range error of 6.41 to 4.61 m, the
⁴⁰⁸ baseline SSP improves from 10.30 to 2.27 m, and the weighted SSP improves from 13.28
⁴⁰⁹ to 2.12 m. Table I shows further statistics on the absolute range error by SSP and group
⁴¹⁰ velocity algorithm. The order of magnitude improvement in the ducted SSPs demonstrate
⁴¹¹ the effectiveness of the algorithm exploiting the multipath conditions.

⁴¹² As shown in table I, there is a striking maximum range error of 1491 m for the weighted
⁴¹³ SSP in the minimal bounce criteria. There are 10 events from South transmitting at 30 m
⁴¹⁴ depth to North receiving at 30 m depth. The OWTT spread is from 2.1958 to 2.1963 s; the
⁴¹⁵ naive group velocity is 1429.3 to 1430.1 m/s; and the GPS-tracked range is from 3138.54 m
⁴¹⁶ to 3140.87 m. This example ends up being an excellent case study for how sound speed and
⁴¹⁷ multipath fidelity work in concert to minimize range error. The large error in this instance
⁴¹⁸ is driven by the MBC unexpectedly defaulting to a bottom bounce with a much greater
⁴¹⁹ OWTT. The NBC classifies the multipath as 4 bounces, reducing the range error from

⁴²⁰ greater than a kilometer to less than a meter. While there is no actual way of knowing if
⁴²¹ this is the correct multipath structure, the range error is remarkably small, at 0.025%. This

	Baseline		Chosen Weights		HYCOM	
	MBC	NBC	MBC	NBC	MBC	NBC
minimum [m]	0.01	0.00	0.00	0.00	0.11	0.01
25th % [m]	4.96	0.99	6.26	0.95	3.30	2.25
median [m]	10.30	2.27	13.28	2.12	6.41	4.61
75th % [m]	15.81	5.51	19.75	4.11	10.92	7.46
maximum [m]	22.52	14.96	1491	20.21	19.55	15.81

TABLE I. A comparison of range estimation metrics for each sound speed source and group velocity estimation algorithm for all 1283 beacon to beacon events via post-processing. The 0th (minimum), 25th, 50th (median), 75th, and 100th (maximum) percentiles are shown to the range resolution afforded by the WHOI Micro-Modem. There are a few outliers that drive the mean to be higher than the median.

422 pattern of not choosing the minimal observed bounce structure is consistent across all SSPs;
 423 the baseline goes from 1 to 3 bounces and HYCOM goes from 0 to 2 bounces. Notably,
 424 the baseline and HYCOM range errors are never egregiously large, but are nonetheless
 425 improved with the NBC algorithm. Thus, for acoustically complex environments, the NBC
 426 has a disproportionately positive impact as the estimated SSP approaches the desired SSP.

⁴²⁷ **V. TRILATERATION FOR ICEX20 FIELD DATA**

⁴²⁸ To overcome potentially intermittent acoustic communication, the operational paradigm
⁴²⁹ of the ICNN computes corrections relative to the trilaterated position estimates transmitted
⁴³⁰ by the vehicle, rather than transmitting the updated positions themselves. The reliability
⁴³¹ of the correction is directly linked to how accurately the travel time measurements are
⁴³² converted to pseudoranges. This section aims to resolve that tension by reevaluating the
⁴³³ trilateration results with respect to the MBC and NBC algorithms. The MBC/NBC sound
⁴³⁴ speed estimates were tracked independently for each transmitter-receiver pair; although the
⁴³⁵ sound speed was expected to be locally smooth near a given receiver, no such assumption
⁴³⁶ was enforced between distinct acoustic links.

⁴³⁷ **A. Re-positioning beacon to beacon events**

⁴³⁸ When the beacons ran as virtual vehicles, the ICNN did not have access to that buoy's
⁴³⁹ GPS data stream except for what was sent via digital acoustic message. The static nature of
⁴⁴⁰ the experiment means that the initial estimate transmitted to the ICNN was in fact a ground
⁴⁴¹ truth position. Therefore, a distribution of corrections from the ICNN, as shown in Fig. 11,
⁴⁴² reflects positioning accuracy. The NBC clearly outperforms the MBC, with almost 80% of
⁴⁴³ the corrections below 6 meters and the median within the deployed GNSS puck precision
⁴⁴⁴ of 3 meters. By contrast, the MBC shows roughly 20% within the GNSS puck precision,
⁴⁴⁵ and separate peaks from 9–12 meters and 21–27 meters. This trimodal nature reflects the
⁴⁴⁶ distribution of reflections on the ice surface.

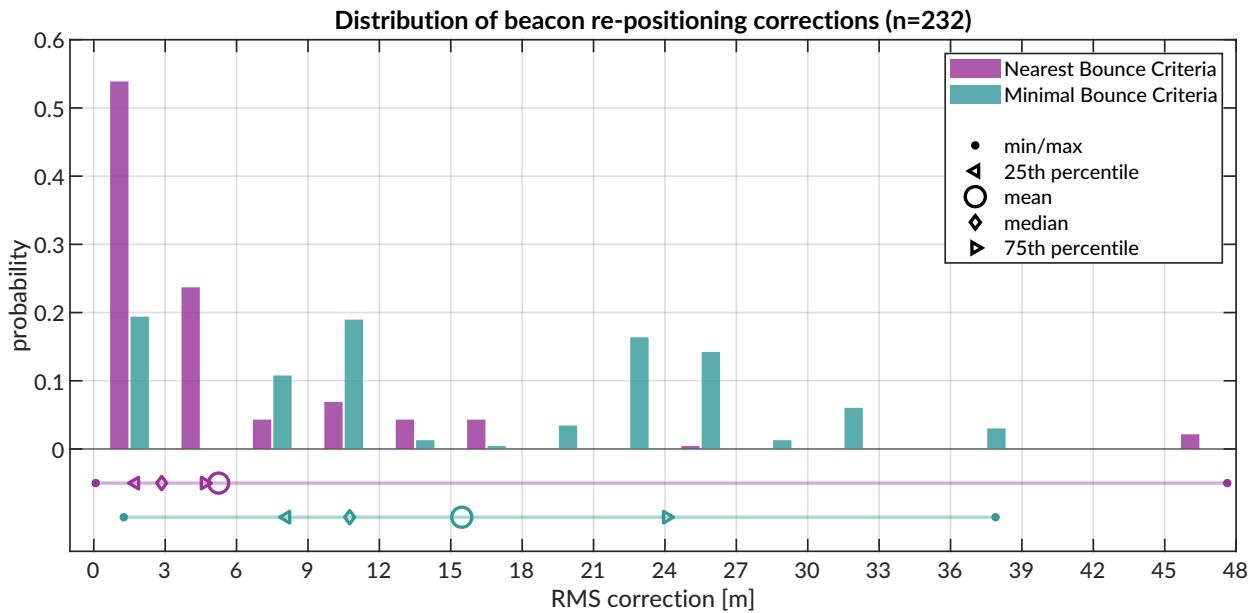


FIG. 11. Trilateration events; however, 264 entries are 2-beacon solutions, 22 are 3-beacon, and 2 are 4-beacon. When limited to only two ranging circles, the trilateration chooses the closer of the two intersection points. The x-axis is cut off at the upper limit for the NBC algorithm.

447 In several events, the MBC is unable to accurately estimate the effective sound speed for
 448 one of the acoustic links, leading to a large positioning error. The NBC, however, better
 449 resolves an approximation of the acoustic path. For example, in some trilateration solutions
 450 for the Eastern buoy, the MBC shows a correction of more than a kilometer; the NBC is
 451 two order of magnitudes less.

452 **B. Re-navigating AUV *Macrura***

453 Up to this point, pseudorange estimation and localization have been evaluated on GPS-
 454 linked beacon-to-beacon connections to validate the NBC algorithm. This analysis ports the
 455 MBC and NBC algorithms to re-navigate the AUV *Macrura*.

456 The AUV dataset clearly exhibits instances where a receiver detects the same transmission
457 more than once. This is not surprising considering the complex multipath provided by
458 the Beaufort Lens. The 11 hour vehicle mission contains 3260 transmissions, 12938 total
459 detections, and 4704 successful receptions. Allowing receptions with PSK errors would
460 almost double the number of recorded multipath arrivals exploited for positioning, if a real-
461 time solution could correctly parse paths from different arrivals in the same thirty-second
462 cycle. Thus it remains a future endeavor to explore how failure mode information from
463 acoustic modems could be used to identify unsuccessful but otherwise trustworthy arrivals
464 to augment trilateration samples.

465 The following performance analysis is constrained to what the vehicle acted on in real-
466 time. AUV *Macrura* and the ICNN ran an adaptive depth behavior to maintain acoustic
467 communication on the insight that cross-layer links were more likely to fail than same-layer
468 ones. Accordingly, the vehicle dove deeper than 50 meters about 20% of the time it was
469 underway.

470 In contrast to the modem tests, where position correction illustrated re-positioning ac-
471 curacy, the re-navigation corrections are less valuable in the absence of GNSS ground truth.
472 The correction magnitude necessarily depends on the vehicle's internal navigation estimate,
473 which is prone to larger errors from sensor drifts, ocean currents, and other error not cap-
474 tured in the hydrodynamic model. Thus, larger corrections are not necessarily indicative of
475 worse performance. Navigation accuracy may be better described by trilateration error, the
476 RMS of the remaining pseudorange errors from each acoustic link.

477 Fig. 12 shows the correction magnitudes and trilateration errors for events with three or
 478 more receptions during AUV operations. Whereas the MBC has a fairly bimodal nature,
 479 with peaks centered around 10–15 and 35–40 m, the NBC favors smaller corrections, from
 480 5–20 m, and has a long tail. The distribution of corrections are much larger than the
 481 distribution of RMS error. It is apparent that, while both methods are quite successful,
 482 there is strong evidence that the NBC achieves single meter accuracy.

483 **C. Investigating potential GNSS noise**

484 The fact that the bulk of the best performing re-navigation error exists within the pre-
 485 cision of the GNSS unit deployed invites a further look into GNSS noise. In the Arctic,
 486 GNSS performance worsens due to poor constellation coverage, larger ionospheric effects,
 487 and multipath interference. The National Security Implications of Climate Change for U.S.
 488 Naval Forces (Council, 2011) details some of the limitations of the Global Positioning Sys-
 489 tem (GPS) at polar latitudes. Radio infrastructure that provides position corrections and
 490 references does not regularly extend to polar regions. The effect is minor for surface platform
 491 navigation —roughly 15 m of horizontal precision has been displayed at the North Pole—but
 492 is significant enough to register against the modem’s detected travel times. Figure 13 zooms
 493 in on the GNSS and OWTT noise relative to the ice movement for two pairs of modem buoy
 494 connections. The two panels indicate the GPS drift as $\delta R = \sqrt{\delta x^2 + \delta y^2}$ and temporal drift,
 495 δt , relative to the median OWTT recorded between the two modems. The dashed line is
 496 scaled by a group velocity of 1440 m/s, such that if there were ideal sensor measurements
 497 with no drift, all events should exist on or near the line.

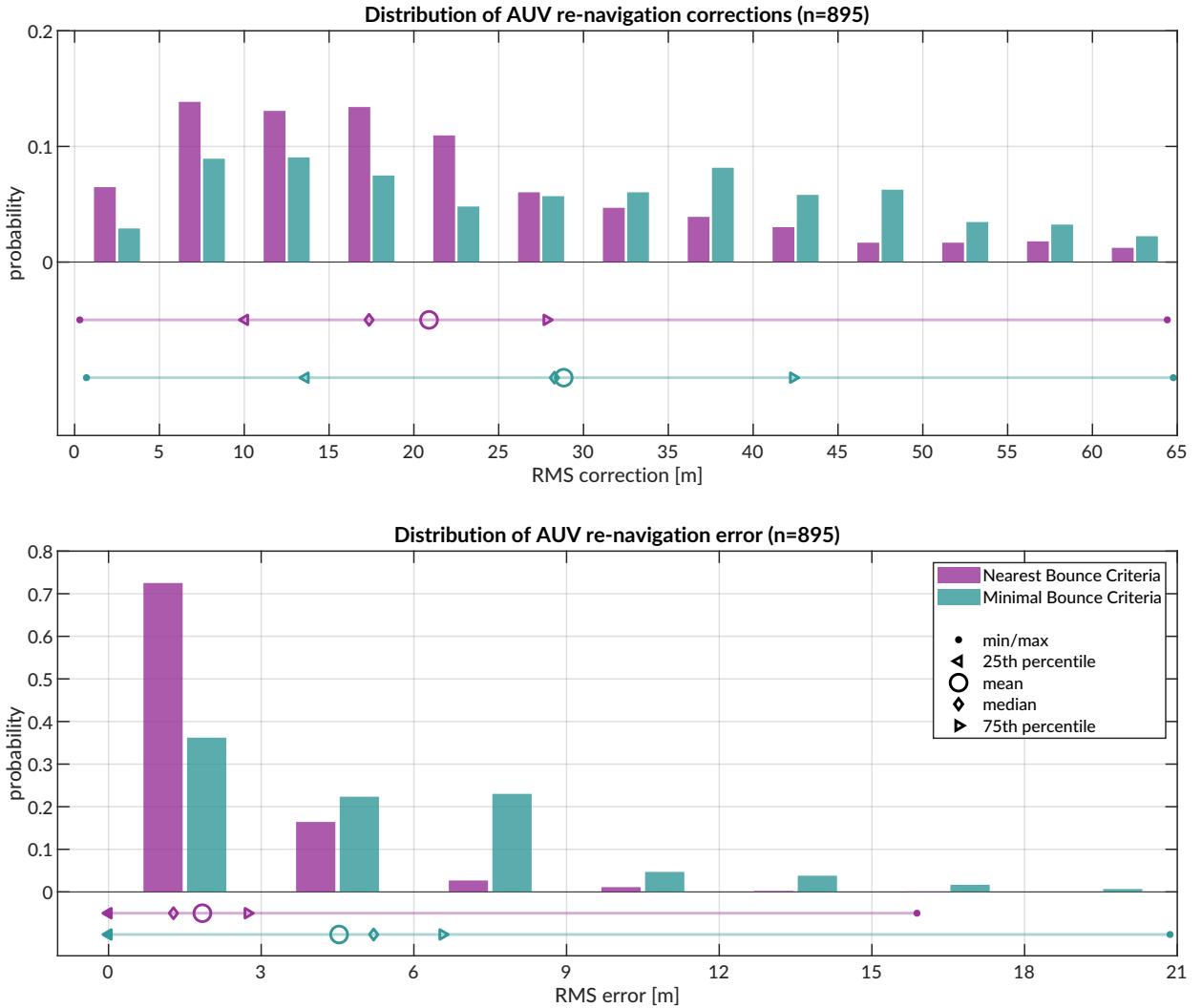


FIG. 12. Distribution of vehicle re-navigation corrections (top) and error (bottom) computed in post-processing for the Minimum and Nearest Bounce Criterion.

498 The top panel shows the connections between the North and East buoys. There is relative, i.e. non-rigid, ice movement between the North and East buoys, evidenced by events spanning the dashed line. But the height of the scatter plot is indicative of the precision of the GPS signal, as it remains consistent across many arrival time bands. Naturally, some minor offsets between these vertical bands relate to different operational configurations of

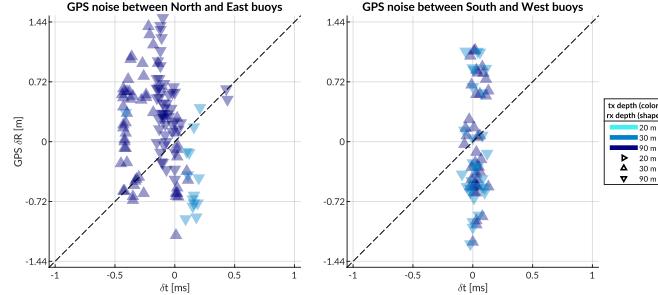


FIG. 13. A comparison of GPS drift (y-axis) versus OWTT drift (x-axis), colored by source and receiver depth. The physical link between North and East are shown on the top; South and West is on the bottom.

503 source and receiver depth. However, the large majority of events show vertical banding for
 504 the same nominal δt , indicating the amount of GPS drift.

505 This idea of GPS drift relative to the same OWTT measurements is further indicated by
 506 events between the other two buoys, South and West, in the bottom panel. These buoys are
 507 moving in a more rigid ice floe and there is minimal impact by source and receiver depth
 508 on the spread of OWTT. The GPS drift is much larger relative to the OWTT drift, which
 509 is sensitive to acoustic scattering, multipath, and/or environmental microstructure.

510 These are just two subsets of the physical links that cover all four GPS modem buoys. The
 511 GPS at camp is the least accurate due to the human activity and infrastructure occluding
 512 the physical puck.

513 VI. DISCUSSION

514 Given the computational constraints of real-time modeling, the gridded approach facil-
 515 itates enough multipath classification to build in a “ray ensemble” of characteristic group

516 velocities. This result is not always possible when aiming to find eigenrays to just an indi-
517 vidual point, even with a higher density of launch angles. An important takeaway for those
518 interested in ray-based localization is leveraging a local grid to give BELLHOP tolerance
519 for finding solutions that otherwise may not be found in a center or single point solution.
520 The limitations of numerical computation, particularly for a complex environment, are more
521 adeptly addressed by accepting some uncertainty in position than by prescribing an exact
522 solution. Even though BELLHOP is a greedy estimator for eigenrays, the additional data
523 created is a negligible burden.

524 Underwater navigation research is broadly motivated by acquiring GPS-like navigation
525 accuracy in GPS-denied conditions. The Arctic, while remote, is the perfect place to test
526 mature navigation technologies in real GPS-denied conditions.

527 Range estimation is an essential step of acoustic localization and navigation. Current
528 approaches in real-time underwater acoustic navigation simplify the non-linear relationship
529 between a sound speed profile and acoustic propagation with a deterministic sound speed.
530 Some post-processing approaches attempt to leverage the acoustic arrival structure via var-
531 ious ray methods, but often use a singular SSP for simplicity, even over long term missions
532 or dynamic conditions. Thus, the conversion from travel time to range, particularly for
533 real-time vehicle deployments, can be pre-conditioned for error growth as the OWTT/range
534 increases.

535 For GPS-denied navigation, especially in hostile environments like the Arctic, tolerance
536 for error is close to none. This work addresses a critical need in acoustic navigation by

537 retooling acoustic arrival methods generally deemed too complex or labor intensive for real-
 538 time, ray-based range estimation to achieve GPS-like positioning.

539 We hypothesize and validate that the embedded stochastic prediction of a single group
 540 velocity is a smoothly varying function of range, source and receiver depth pairings, as
 541 well as multipath structure. We employ a GPS-tracked experiment to have a ground truth
 542 comparison for real-time localization algorithms. The real-time system achieves GPS-like
 543 navigation for an AUV without taking into account multipath structure; the ranging error
 544 improves by an order of magnitude with the suggested multipath adaptability, minimizing
 545 range error to single meters. Post-processing analysis shows that this method of ranging is
 546 sensitive to GPS drift itself; thus embedding a stochastic prediction of the horizontal group
 547 velocity has an outsized benefit to minimizing trilateration error.

548 There are many avenues through which this approach can be further refined and tested for
 549 field operations. Amongst them is defining the uncertainty grid for BELLHOP via stochastic
 550 or data-driven measures such as the distance traveled by the AUV between ICNN updates
 551 or the magnitude of the position corrections by the ICNN. Another is to entirely remove the
 552 seeded range and instead rely on the submerged asset's depth and recorded OWTT to find
 553 high probability fields in range.

554 The literature in underwater acoustic navigation and positioning is either real-time or
 555 physics-based. In this paper we demonstrate a field-tested approach that is both real-time
 556 and physics-based; this is achieved by coupling data streams with fast acoustic modeling.
 557 The methods exploit the upward refracting nature and the total ice cover of the Arctic
 558 environment to achieve remarkable ranging accuracy and precision. It transforms multipath,

559 widely considered as an obstacle for acoustic ranging, into a new information content to
 560 refine ranging accuracy. We believe that this work enables more accurate range estimation,
 561 localization, and/or navigation for any field experiment given known source and receiver
 562 depths.

563 Performance in other acoustic environments may require introducing a different thresh-
 564 olded metric to sort time of arrivals. Given the NBC algorithm's improvement with increased
 565 multipath, its effectiveness is likely only challenged by the valid operational scales of a range
 566 independent propagation environment. For mesoscale operations, like that of many glid-
 567 ers, the group velocity criteria may need to be modified to better account for variability
 568 driven by range dependent propagation through internal waves, eddies, or even bathymetric
 569 changes like underwater volcanoes. BELLHOP simulations provide other relevant eigenray
 570 information, like time and angle of arrival, that is ripe for statistical and machine learning
 571 methods to classify a representative group velocity. A bespoke and fast ray tracing method,
 572 which was used during ICEX20 to evaluate the acoustic fit of sound speed profile parame-
 573 terization ([Bhatt *et al.*, 2022](#)), can easily report back the number of turning points instead
 574 of the number of bounces for multipath classification.

575 This approach will start to break down in extremely dynamic environments, like fast
 576 moving fronts. Realistic *in situ* considerations of the acoustic environment may not be pos-
 577 sible without complete through-the-sensor integration of acoustic dat and/or hyper realistic
 578 ocean models.

579 Many approaches to underwater navigation combine it with acoustic tomography, i.e.,
 580 a joint estimation of both source and receiver locations and the ocean volume between

them. There has been considerable success at this effort in post-processing methods, which utilize intensive—and due to the non-linearity of sound propagation, often brute force—computational methods. For vehicle operations, fast tomography is the ideal implementation, in that one can fully consider how sound speed structure, horizontally and vertically, influences sound propagation. AUVs can serve as moving sources to better image the ocean volume (Deffenbaugh, 1997; Elisseeff *et al.*, 2002), where mobile tomography and navigation converge on the same set of component technologies: position estimation, sound speed parameterization estimation, ray path identification, and vehicle path optimization.

But there are overwhelming challenges, operationally and computationally, for fast, mobile tomography to become a realistic endeavor. Addressing the spatial and temporal scales of what can be solved deterministically and what must be solved stochastically imposes a resolution constraint on the utility of gridded models—resolving fine features inaccurately (or with a false sense of confidence) could be more harmful than assuming range independence. Given that AUV operations are often on small spatial and temporal scales, the added benefit of a gridded model is quite small, and in cases like the Arctic, may actually mischaracterize the ocean volume. For gliders, with longer and larger operational scales, an ocean model may provide more useful information. Currently gliders are low power and do not have the storage or computational power to run a full-scale, realistic ocean model. A lightweight representation of the key environmental and acoustic features, passed through the same manner of acoustic message from the modem experiment, may drastically improve glider navigation.

602 **ACKNOWLEDGMENTS**

603 We acknowledge the significant operational effort spearheaded by the LAMSS ICEX20
 604 team and all our collaborators. Bhatt was funded by a National Defense, Science, and
 605 Engineering Graduate Fellowship. This work was supported by the Office of Naval Research
 606 322-OA under ICEX20 (N00014-17-1-2474) and Task Force Ocean (N00014-19-1-2716).

607

608 Ballard, M. S., Badiey, M., Sagers, J. D., Colosi, J. A., Turgut, A., Pecknold, S., Lin, Y.-T.,
 609 Proshutinsky, A., Krishfield, R., Worcester, P. F., and Dzieciuch, M. A. (**2020**). “Tem-
 610 poral and spatial dependence of a yearlong record of sound propagation from the Canada
 611 Basin to the Chukchi Shelf,” The Journal of the Acoustical Society of America **148**(3),
 612 1663–1680, <http://asa.scitation.org/doi/10.1121/10.0001970http://files/814/>
 613 [Ballardetal.-2020-Temporalandspatialdependenceofayearlongreco.pdf](#), doi: [10.](https://doi.org/10.1121/10.0001970)
 614 [1121/10.0001970](https://doi.org/10.1121/10.0001970).

615 Barker, L. D. L., Jakuba, M. V., Bowen, A. D., German, C. R., Maksym, T., Mayer,
 616 L., Boetius, A., Dutrieux, P., and Whitcomb, L. L. (**2020**). “Scientific challenges and
 617 present capabilities in underwater robotic vehicle design and navigation for oceanographic
 618 exploration under-ice,” Remote Sensing **12**(16), 1–31, doi: [10.3390/RS12162588](https://doi.org/10.3390/RS12162588).

619 Bellingham, J., Leonard, J., Vaganay, J., Goudey, C., Atwood, D., Consi, T., Bales, J.,
 620 Schmidt, H., and Chryssostomidis, C. (**1995**). “Auv operations in the arctic,” in *Sea Ice
 621 Mechanics and Arctic Modeling Workshop*.

- 622 Bhatt, E. C. (2021). “A Virtual Ocean framework for environmentally adaptive, em-
 623 bedded acoustic navigation on autonomous underwater vehicles,” Ph.D. thesis, Mas-
 624 sachusetts Institute of Technology and Woods Hole Oceanographic Institution Joint Pro-
 625 gram, <https://hdl.handle.net/1912/27309>, doi: [10.1575/1912/27309](https://doi.org/10.1575/1912/27309).
- 626 Bhatt, E. C., Howard, B., and Schmidt, H. (2022). “An Embedded Tactical Decision Aid
 627 Framework for Environmentally Adaptive Autonomous Underwater Vehicle Communica-
 628 tion and Navigation,” IEEE Journal of Oceanic Engineering .
- 629 Brooke, J. (1981). “Arcs (autonomous remotely controlled submersible),” in *Proceedings of*
 630 *the 1981 2nd International Symposium on Unmanned Untethered Submersible Technology*,
 631 IEEE, Vol. 2, pp. 28–28.
- 632 Chassignet, E. P., Hurlburt, H. E., Smedstad, O. M., Halliwell, G. R., Hogan, P. J., Wall-
 633 craft, A. J., Baraille, R., and Bleck, R. (2007). “The HYCOM (HYbrid Coordinate
 634 Ocean Model) data assimilative system,” Journal of Marine Systems **65**(1-4), 60–83, doi:
 635 [10.1016/J.JMARSYS.2005.09.016](https://doi.org/10.1016/J.JMARSYS.2005.09.016).
- 636 Chen, R., Poulsen, A., and Schmidt, H. (2019). “Spectral, spatial, and tem-
 637 poral characteristics of underwater ambient noise in the Beaufort Sea in 1994
 638 and 2016,” The Journal of the Acoustical Society of America **145**(2), 605–
 639 614, <https://asa.scitation.org/doi/full/10.1121/1.5088601http://files/757/>
 640 [Chenetal.-2019-Spectral,spatial,andspacecharacteristicsof.pdf](https://files/757/Chenetal.-2019-Spectral,spatial,andspacecharacteristicsof.pdf), doi: 10.
 641 [1121/1.5088601](https://doi.org/10.1121/1.5088601).
- 642 Chen, R., and Schmidt, H. (2020). “Temporal and spatial charac-
 643 teristics of the Beaufort Sea ambient noise environment,” The Jour-

- 644 nal of the Acoustical Society of America **148**(6), 3928–3941, <https://doi.org/10.1121/10.0002955>
- 645 //asa.scitation.org/doi/full/10.1121/10.0002955http://files/755/
- 646 [ChenandSchmidt-2020-TemporalandspatialcharacteristicsoftheBeaufou.pdf](#), doi:
647 [10.1121/10.0002955](https://doi.org/10.1121/10.0002955).
- 648 Claus, B., Kepper, J. H., Suman, S., and Kinsey, J. C. (2018). “Closed-loop one-way-travel-
649 time navigation using low-grade odometry for autonomous underwater vehicles,” Journal
650 of Field Robotics **35**(4), 421–434, doi: [10.1002/rob.21746](https://doi.org/10.1002/rob.21746).
- 651 Council, N. R. (2011). *National Security Implications of Climate Change for U.S. Naval Forces* (The National Academies
652 Press, Washington, DC), <https://www.nap.edu/catalog/12914/national-security-implications-of-climate-change-for-us-naval-forces>.
- 653 Deffenbaugh, M. (1997). “Optimal Ocean Acoustic Tomography and Navigation with Mov-
654 ing Sources,” Ph.D. thesis, MIT-WHOI Joint Program in Oceanography/Applied Ocean
655 Science and Engineering.
- 656 Deffenbaugh, M., Bellingham, J. G., and Schmidt, H. (1996a). “Relationship between
657 spherical and hyperbolic positioning,” Oceans Conference Record (IEEE) **2**, 590–595, doi:
658 [10.1109/OCEANS.1996.568293](https://doi.org/10.1109/OCEANS.1996.568293).
- 659 Deffenbaugh, M., Schmidt, H., and Bellingham, J. G. (1996b). “Acoustic positioning in a
660 fading multipath environment,” in *OCEANS 96 MTS/IEEE Conference Proceedings. The
661 Coastal Ocean-Prospects for the 21st Century*, IEEE, Vol. 2, pp. 596–600.
- 662 Duda, T. F., Morozov, A. K., Howe, B. M., Brown, M. G., Speer, K., Lazarevich,
663 P., Worcester, P. F., and Cornuelle, B. D. (2006). “Evaluation of a long-range joint

- 666 acoustic navigation / thermometry system," in *Oceans 2006*, pp. 1–6, <http://files/939/Dudaetal.-2006-EvaluationofaLong-RangeJointAcousticNavigati.pdf> <http://files/940/4099137.html>, doi: [10.1109/OCEANS.2006.306999](https://doi.org/10.1109/OCEANS.2006.306999).
- 669 Duda, T. F., Zhang, W. G., and Lin, Y.-T. (2021). "Effects of Pacific Summer Water layer
670 variations and ice cover on Beaufort Sea underwater sound ducting," *The Journal of the
671 Acoustical Society of America* **149**(4), 2117–2136, doi: [10.1121/10.0003929](https://doi.org/10.1121/10.0003929).
- 672 Duda, T. F., Zhang, W. G., Lin, Y.-T., and Newhall, A. E. (2019). "Long-
673 range sound propagation in the Canada Basin," <http://files/565/Dudaetal.-Unknown-LONG-RANGESOUNDPROPAGATIONINTHECANADABASIN.pdf>.
- 675 Elisseeff, P., Schmidt, H., and Xu, W. (2002). "Ocean acoustic tomography as a data assimilation problem," *IEEE Journal of Oceanic Engineering* **27**(2), 275–282, <http://files/438/Elisseeff,Schmidt,Xu-2002-OceanAcousticTomographyasaDataAssimilationProblem.pdf>, doi:
676
677
678
679 [10.1109/JOE.2002.1002482](https://doi.org/10.1109/JOE.2002.1002482).
- 680 Eustice, R. M., Whitcomb, L. L., Singh, H., and Grand, M. (2006). "Recent advances in
681 synchronous-clock one-way-travel-time acoustic navigation," *Oceans 2006* doi: [10.1109/OCEANS.2006.306931](https://doi.org/10.1109/OCEANS.2006.306931).
- 683 Eustice, R. M., Whitcomb, L. L., Singh, H., and Grund, M. (2007). "Experimental results in synchronous-clock one-way-travel-time acoustic navigation for autonomous underwater vehicles," in *Proceedings - IEEE International Conference on Robotics and Automation*, pp. 4257–4264, <http://files/877/Eusticeetal.-2007-ExperimentalResultsinSynchronous-ClockOne-Way-.pdf> <http://files/877/Eusticeetal.-2007-ExperimentalResultsinSynchronous-ClockOne-Way-.pdf>

- 688 <http://files/878/4209752.html>, doi: [10.1109/ROBOT.2007.364134](https://doi.org/10.1109/ROBOT.2007.364134).
- 689 Fossum, T. O., Norgren, P., Fer, I., Nilsen, F., Koenig, Z. C., and Ludvigsen, M. (2021).
- 690 “Adaptive sampling of surface fronts in the arctic using an autonomous underwater ve-
- 691 hicle,” IEEE Journal of Oceanic Engineering **46**(4), 1155–1164, doi: [10.1109/JOE.2021.3070912](https://doi.org/10.1109/JOE.2021.3070912).
- 692
- 693 Freitag, L., Ball, K., Partan, J., Koski, P., and Singh, S. (2016). “Long range acoustic
- 694 communications and navigation in the Arctic,” OCEANS 2015 - MTS/IEEE Washington
- 695 2–6, doi: [10.23919/oceans.2015.7401956](https://doi.org/10.23919/oceans.2015.7401956).
- 696 Graupe, C. E., Van Uffelen, L. J., Webster, S. E., Worcester, P. F., and Dzieci-
- 697 uch, M. A. (2019). “Preliminary results for glider localization in the Beau-
- 698 fort Duct using broadband acoustic sources at long range,” in *OCEANS 2019*
- 699 *MTS/IEEE Seattle, OCEANS 2019*, pp. 1–6, <http://files/763/Graupeetal.-2019-Preliminaryresultsforgliderlocalizationinthe.pdf>
- 700 <http://files/763/Graupeetal.-2019-Preliminaryresultsforgliderlocalizationinthe.pdf>
- 701 <http://files/912/Graupeetal.-2019-Preliminaryresultsforgliderlocalizationinthe.pdf>
- 702 <http://files/764/8962637.html> <http://files/913/8962637.html>, doi: [10.23919/OCEANS40490.2019.8962637](https://doi.org/10.23919/OCEANS40490.2019.8962637).
- 703
- 704 Hayes, D. R., and Morison, J. H. (2002). “Determining turbulent vertical velocity, and
- 705 fluxes of heat and salt with an autonomous underwater vehicle,” Journal of Atmospheric
- 706 and Oceanic Technology **19**(5), 759–779.
- 707 Howe, B. M., Miksis-Olds, J., Rehm, E., Sagen, H., Worcester, P. F., and Haral-
- 708 abus, G. (2019). “Observing the Oceans Acoustically,” Frontiers in Marine Science **6**,
- 709 426, <https://www.frontiersin.org/article/10.3389/fmars.2019.00426/full>, doi:

710 10.3389/fmars.2019.00426.

711 Jackson, E. (1983). “Autonomous remotely controlled submersible “ARCS”,” in *Proceedings*
 712 *of the 1983 3rd International Symposium on Unmanned Untethered Submersible Technol-*

713 *ogy*, IEEE, Vol. 3, pp. 77–88.

714 Jakuba, M. V., Roman, C. N., Singh, H., Murphy, C., Kunz, C., Willis, C., Sato,

715 T., and Sohn, R. A. (2008). “Long-baseline acoustic navigation for under-ice
 716 autonomous underwater vehicle operations,” *Journal of Field Robotics* **25**(11-12),

717 861–879, <https://onlinelibrary.wiley.com/doi/full/10.1002/rob.20250>
<https://onlinelibrary.wiley.com/doi/abs/10.1002/rob.20250>

718 doi: 10.1002/rob.20250, doi: 10.1002/ROB.20250.

719 Kepper, J. H., Claus, B. C., and Kinsey, J. C. (2017). “MEMS IMU and one-
 720 way-travel-time navigation for autonomous underwater vehicles,” in *OCEANS*

721 2017 - Aberdeen, Vol. 2017-Octob, pp. 1–9, <http://files/550/Kepper,Claus,Kinsey-2017-MEMSIMUandOne-Way-Travel-TimeNavigationforAutonomousUnderwaterVehicles.pdf>

722 doi: 10.1109/OCEANSE.2017.8084842.

723 Krishfield, R., Toole, J., Proshutinsky, A., and Timmermans, M. L. (2008). “Automated
 724 ice-tethered profilers for seawater observations under pack ice in all seasons,” *Journal of*
 725 *Atmospheric and Oceanic Technology* **25**(11), 2091–2105, doi: 10.1175/2008JTECH0587.

726 1.

727 Kukulya, A., Plueddemann, A., Austin, T., Stokey, R., Purcell, M., Allen, B., Littlefield, R.,
 728 Freitag, L., Koski, P., Gallimore, E. et al. (2010). “Under-ice operations with a remus-100
 729 auv in the arctic,” in *2010 IEEE/OES Autonomous Underwater Vehicles*, IEEE, pp. 1–8.

- 732 Kunz, C., Murphy, C., Camilli, R., Singh, H., Bailey, J., Eustice, R., Jakuba, M., Nakamura,
 733 K., Roman, C., Sato, T., Sohn, R., and Willis, C. (2008). “Deep sea underwater robotic
 734 exploration in the ice-covered Arctic ocean with AUVs,” in *2008 IEEE/RSJ International*
 735 *Conference on Intelligent Robots and Systems*, IEEE, pp. 3654–3660, <http://files/875/Kunzetal.-2008-Deepseaunderwaterroboticexplorationintheice.pdf>
 736 <http://files/968/Kunzetal.-2008-Deepseaunderwaterroboticexplorationintheice.pdf>
 737 <http://ieeexplore.ieee.org/document/4651097/>, doi: [10.1109/IROS.2008.4651097](https://doi.org/10.1109/IROS.2008.4651097).
- 739
- 740 Light, R., and Morison, J. (1989). “The autonomous conductivity-temperture vehicle: First
 741 in the seashuttle family of autonomous underwater vehicle’s for scientific payloads,” in
 742 *Proceedings OCEANS*, Vol. 3, pp. 793–798, doi: [10.1109/OCEANS.1989.586683](https://doi.org/10.1109/OCEANS.1989.586683).
- 743 Litvak, A. (2015). “Acoustics of the deepwater part of the arctic ocean and of russia’s
 744 arctic shelf,” *Herald of the Russian Academy of Sciences* **85**, 239–250, doi: [10.1134/S1019331615030144](https://doi.org/10.1134/S1019331615030144).
- 745
- 746 Mikhalevsky, P. N., Sperry, B. J., Woolfe, K. F., Dzieciuch, M. A., and Worces-
 747 ter, P. F. (2020). “Deep ocean long range underwater navigation,” *The Jour-*
 748 *nal of the Acoustical Society of America* **147**(4), 2365–2382, <http://asa.scitation.org/doi/10.1121/10.0001081>
 749 <http://files/631/Mikhalevskyetal.-2020-Deepeceanlongrangeunderwaternavigation.pdf>, doi: [10.1121/10.0001081](https://doi.org/10.1121/10.0001081).
- 750
- 751 Norgren, P., Lubbad, R., and Skjetne, R. (2014). “Unmanned underwater vehicles in Arctic
 752 operations,” in *22nd IAHR International Symposium on Ice*.

- 753 Paull, L., Saeedi, S., Seto, M., and Li, H. (2014). *AUV navigation and*
 754 *localization: A review*, **39**, pp. 131–149, <http://files/127/Paulletal.-2014-AUVnavigationandlocalizationAreview.pdf>.
- 755
- 756 Plueddemann, A. J., Kukulya, A. L., Stokey, R., and Freitag, L. (2012). “Autonomous
 757 Underwater Vehicle Operations Beneath Coastal Sea Ice,” IEEE/ASME Transactions
 758 on Mechatronics **17**(1), 54–64, doi: [10.1109/TMECH.2011.2174798](https://doi.org/10.1109/TMECH.2011.2174798) conference Name:
 759 IEEE/ASME Transactions on Mechatronics.
- 760 Porter, M. B. (2011). “The BELLHOP Manual and User’s Guide,” HLS Research, , 2010
 761 1–57, <http://oalib.hlsresearch.com/Rays/HLS-2010-1.pdf>.
- 762 Poulsen, A. J., and Schmidt, H. (2017). “Acoustic noise properties in the rapidly changing
 763 Arctic Ocean,” **070005**(2016), 070005, doi: [10.1121/2.0000552](https://doi.org/10.1121/2.0000552).
- 764 Randeni, S., Schneider, T., and Schmidt, H. (2020). “Construction of a
 765 high-resolution under-ice AUV navigation framework using a multidisci-
 766 plinary virtual environment,” in *2020 IEEE/OES Autonomous Underwater
 767 Vehicles Symposium, AUV 2020*, pp. 1–7, <http://files/689/Randenietal.-2020-Constructionofahigh-resolutionunder-iceAUVna.pdf>, doi: [10.1109/AUV50043.2020.9267950](https://doi.org/10.1109/AUV50043.2020.9267950).
- 768
- 769
- 770 Randeni, S., Schneider, T., Schmidt, H., Bhatt, E., and Viquez, O. (2021). “A high-
 771 resolution AUV navigation framework with integrated communication and tracking for
 772 under-ice deployments,” *Field Robotics* (in review).
- 773 Rossby, T., Dorson, D., and Fontaine, J. (1986). “The RAFOS System,” *Journal of Atmo-*
 774 *spheric and Oceanic Technology* **3**, 148–162.

- 775 Rypkema, N. R., Fischell, E. M., and Schmidt, H. (2017). “One-Way Travel-Time Inverted
 776 Ultra-Short Baseline Localization for Low-Cost Autonomous Underwater Vehicles,” in *2017*
 777 *IEEE International Conference on Robotics and Automation (ICRA)*, Singapore, pp. 4920–
 778 4926.
- 779 Schmidt, H., and Schneider, T. (2016). “Acoustic communication and navigation in
 780 the new Arctic-A model case for environmental adaptation,” 3rd Underwater Com-
 781 munications and Networking Conference, Ucomms 2016 <http://files/583/Schmidt,Schneider-2016-AcousticCommunicationandNavigationintheNewArctic-AModelCaseforEnvironment.pdf>, doi: [10.1109/UComms.2016.7583469](https://doi.org/10.1109/UComms.2016.7583469).
- 784 Schneider, T., and Schmidt, H. (2018). “NETSIM: A realtime virtual ocean hardware-
 785 in-the-loop acoustic modem network simulator,” in *2018 4th Underwater Communi-*
 786 *cations and Networking Conference, UComms 2018*, pp. 1–5, <http://files/1047/SchneiderandSchmidt-2018-NETSIMARealtimeVirtualOceanHardware-in-the-l.pdf>, doi: [10.1109/UComms.2018.8493188](https://doi.org/10.1109/UComms.2018.8493188).
- 789 Schneider, T., Schmidt, H., and Randeni, S. (2020). “Self-Adapting Under-Ice Inte-
 790 grated Communications and Navigation Network,” 2020 5th Underwater Communica-
 791 tions and Networking Conference, UComms 2020 5, <http://files/607/Schneideretal.-Self-AdaptingUnder-IceIntegratedCommunications.pdf>.
- 793 Singh, S., Grand, M., Bingham, B., Eustice, R., Singh, H., and Freitag, L. (2006).
 794 “Underwater acoustic navigation with the WHOI Micro-Modem,” in *Oceans 2006*,
 795 IEEE, pp. 1–4, <http://ieeexplore.ieee.org/document/4099008/> <http://files/774/Singhetal.-2006-UnderwaterAcousticNavigationwiththeWHOIMicro.pdf>, doi: [10.1109/OCEANS.2006.1700001](https://doi.org/10.1109/OCEANS.2006.1700001).

797 1109/OCEANS.2006.306853.

798 Timmermans, M.-L., and Winsor, P. (2013). “Scales of horizontal density structure in the
799 chukchi sea surface layer,” Continental Shelf Research **52**, 39–45.

800 Toole, J. M., Krishfield, R. A., Timmermans, M. L., and Proshutinsky, A. (2011). “The
801 Ice-Tethered profiler: Argo of the Arctic,” Oceanography **24**(3), 126–135, doi: [10.5670/oceanog.2011.64](https://doi.org/10.5670/oceanog.2011.64).

803 Uffelen, L. J. V., Howe, B. M., Nosal, E.-M., Carter, G. S., Worcester, P. F., and Dzieci-
804 uch, M. A. (2016). “Localization and subsurface position error estimation of gliders using
805 broadband acoustic signals at long range,” IEEE Journal of Oceanic Engineering **41**(3),
806 501–508.

807 Van Uffelen, L. J. (2021). “Global Positioning Systems: Over Land and Under Sea,” Acous-
808 tics Today **17**(1), 52, doi: [10.1121/at.2021.17.1.52](https://doi.org/10.1121/at.2021.17.1.52).

809 Van Uffelen, L. J., Nosal, E.-M., Howe, B. M., Carter, G. S., Worcester, P. F., Dzieciuch,
810 M. A., Heaney, K. D., Campbell, R. L., and Cross, P. S. (2013). “Estimating uncertainty
811 in subsurface glider position using transmissions from fixed acoustic tomography sources,”
812 The Journal of the Acoustical Society of America **134**(4), 3260–3271, doi: [10.1121/1.4818841](https://doi.org/10.1121/1.4818841).

814 Webster, S. E., Eustice, R. M., Singh, H., and Whitcomb, L. L. (2009). “Prelim-
815 inary deep water results in single-beacon one-way-travel-time acoustic navigation
816 for underwater vehicles,” 2009 IEEE/RSJ International Conference on Intelligent
817 Robots and Systems, IROS 2009 2053–2060, <http://files/416/Websteretal.-2009-Preliminarydeepwaterresultsinsingle-beaconone-way-travel-timeacousticnavigationf>

- 819 pdf, doi: [10.1109/IROS.2009.5354457](https://doi.org/10.1109/IROS.2009.5354457).
- 820 Webster, S. E., Eustice, R. M., Singh, H., and Whitcomb, L. L. (2012). “Advances in
821 single-beacon one-way-travel-time acoustic navigation for underwater vehicles,” Interna-
822 tional Journal of Robotics Research **31**(8), 935–950, doi: [10.1177/0278364912446166](https://doi.org/10.1177/0278364912446166).
- 823 Webster, S. E., Freitag, L. E., Lee, C. M., and Gobat, J. I. (2015). “Towards real-time
824 under-ice acoustic navigation at mesoscale ranges,” in *Proceedings - IEEE International
825 Conference on Robotics and Automation*, June, IEEE, pp. 537–544, [http://files/625/
826 Websteretal.-2015-Towardsreal-timeunder-iceacousticnavigationat.pdf](http://files/625/Websteretal.-2015-Towardsreal-timeunder-iceacousticnavigationat.pdf) <http://files/641/Websteretal.-2015-Towardsreal-timeunder-iceacousticnavigationat.pdf>
827 <http://files/835/Websteretal.-2015-Towardsreal-timeunde>, doi: [10.1109/ICRA.2015.7139231](https://doi.org/10.1109/ICRA.2015.7139231).
- 830 Wu, M., Barmin, M. P., Andrew, R. K., Weichman, P. B., White, A. W., Lavely, E. M.,
831 Dzieciuch, M. A., Mercer, J. A., Worcester, P. F., and Ritzwoller, M. H. (2019).
832 “Deep water acoustic range estimation based on an ocean general circulation model:
833 Application to PhilSea10 data,” The Journal of the Acoustical Society of America
834 **146**(6), 4754–4773, [https://asa.scitation.org/doi/10.1121/1.5138606](https://doi.org/10.1121/1.5138606) <http://files/947/Wuetal.-2019-Deepwateracousticrangeestimationbasedonano.pdf>
835 <http://files/948/1.html>, doi: [10.1121/1.5138606](https://doi.org/10.1121/1.5138606).