Design of Public Transport Networks

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This paper describes the major features of an optimization model which can be used to design public transport networks. Design problems that can be solved with the model involve the redesign of either a part of a network or a complete network and the assignment of frequencies. The model consists of an additive procedure in which the decision to incorporate a route in the network or to increase the frequency of a route is based on an economic criterion which can also be regarded as an estimate of the Lagrange Multiplier of the optimization problem. A major advantage of the model is that the different design problems are solved with one single optimization process. Furthermore, the optimization process is kept understandable and the model is suited for use on a personal computer. Some results of the model are presented.

Due to the changing economic situation the financial constraints of public transport have become more and more important. The government is no longer willing to account for all deficits of the public transport companies. The policy has changed into granting a single subsidy with which the public transport companies have to offer service which can compete with other modes and transport facilities for those who cannot travel otherwise. Since the subsidy will be limited, the public transport companies will have to reconsider the service they are offering. Of course, it is also necessary to cut costs by improving the scheduling of personnel and vehicles, as well as the regularity of the service.

The design of the network deserves extra attention, as it is the network that determines the service offered. Moreover, the network is used as input for studies concerning other aspects such as timetables, scheduling, and regularity. Another reason for extra attention to the design of public transport networks is the fact that networks have often been adjusted by using simple design methods to meet changes in the city, e.g., new residential areas. Very little use was made of sophisticated tools such as traffic forecasting and assignment models. See for instance Chua and Sitcock (*1*) for a survey of planning techniques used in Great Britain.

The problem of the network design can be formulated as follows: which routes and which frequencies should be offered to fulfil the demand for public transport as well as possible, given a certain available budget.

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EXISTING DESIGN METHODS

The type of model mostly used for network design is an evaluation model in which an origin-destination (OD) matrix is assigned to a network and with which all kinds of evaluation characteristics are calculated. Such an evaluation model enables a systematic comparison between alternative network designs. Although the use of evaluation models must be considered as a major improvement to the quality of the planning process, the disadvantage remains that only a few alternatives can be compared because of the effort involved. Also, these alternatives will often be biased towards the existing network and the implicit ideas of the planner, although in some cases this might be considered as an advantage.

The disadvantage of a limited number of alternatives, however, does not apply to models which design a network as well as evaluate it. These so-called optimization models use operations research techniques to find a feasible network. The name optimization model is misleading, however, because most models do not find an optimum solution and even then it is questionable whether an optimum of a model, which is a mathematical description of reality, will be an optimum in reality. Therefore, the importance of these models does not derive from the fact that they find a (near) optimum solution, but rather that they help to find new and feasible alternatives.

Despite the advantage of generating new alternatives, the use of these optimization models is very limited. This limited use can be explained by several reasons, one of them being the overall lack of experience in using models in public transport studies, but when we take a close look at the optimization models which have been developed, some other reasons can be found.

EXISTING OPTIMIZATION MODELS

In the last two decades all kinds of optimization models for the design of public transport networks have been developed. These models can roughly be divided into six categories:

1. Analytical models (e.g., Holroyd [*2*], Kocur and Hendrickson [*3*]). These models use simplified networks to derive optimum relations for parameters of the public transport system, for instance headway and route-spacing.
2. Models determining which links should be used to construct routes for a public transport network (e.g., Billheimer and Gray [*4*], Rea [*5*]).
3. Models determining routes without considering the frequencies of the routes (e.g., Pierick and Wiegand [*6*], Simonis [*7*]).
4. Models assigning frequencies to a given set of routes

(e.g., Scheele [8], Furth and Wilson [9], Hagberg and Hasselström [10]).

5. Models determining routes in a first and assigning frequencies in a second stage (e.g., Lampkin and Saalmans [11], Dubois et al. [12]).

6. Models determining routes and frequencies simultaneously (Hasselström [13]).

The first two categories determine neither routes nor frequencies and are therefore unsuited for the problem we have formulated. The third and fourth categories solve only part of our problem, either routes or frequencies. Actually, there are only two categories of models suited to our design problem, categories 5 and 6.

Determining Routes and Assigning Frequencies Separately

The models of category 5 solve the network design problem in two stages. In the first stage the routes of the network are determined. The objective is to transport a maximum number of passengers given a fixed OD-matrix. In this stage Lampkin and Saalmans (*11*) consider trips without transfers, while Dubois et al. (*12*) consider all trips. In the second stage frequencies are assigned to the generated set of routes. The objective is to minimize the total travel time given the OD-matrix and the available number of vehicles. In the calculation of the travel time Dubois et al. (*12*) introduced the possibility of walking instead of using public transport. All the methods used are clearly heuristic, but those of Dubois et al. (*12*) are more sophisticated. The major disadvantage of these models, however, is the fact that they solve the problem of routes and frequencies separately, while there is a distinct relation between these two components of the public transport system. Moreover, a fixed OD-matrix is used, so the relation between supply and demand for public transport services is not taken into account.

Determining Routes and Assigning Frequencies Simultaneously

The model developed by Hasselström (*13*) does not have these disadvantages. It solves the problem in three stages. First, the model considers a link network and eliminates links seldom or never used by passengers (compare the models of category 2). The result is a concentrated network which is used in the second stage to generate a large set of possible routes. Finally, the route of the network are selected by assigning frequencies using linear programming. The objective is to maximize the number of transfers saved by changing from a link network (transfers at every node) to a public transport network (transfers only at intersections). Instead of a known OD-matrix, Hasselström (*13*) suggests the use of a desire matrix (i.e., an OD-matrix for the situation in which an ideal public transport system exists) in order to lessen the bias towards the network with which the OD-matrix is determined. The disadvantage of the model is that although routes and frequencies are determined simultaneously, two different optimization problems are formulated.

Use of Optimization Models

All optimization models discussed have rarely been used in practice. Most models have been employed only in the projects they were designed for, or in the tests described in the presentations. The model of Hasselström (*13*) forms part of the VOLVO-package, which contains a variety of models for planning public transport networks (e.g., Andréasson [14]), and has been used more often (Arnström [15], HTM [16] and Harris and Haywood [17]). The major disadvantages of all optimization models, however, are the complex structure (e.g., several optimization problems within one model) and the limited accessibility for planners as the models can be used only on a mainframe.

A NEW MODEL

The disparities between the capabilities of optimization models in the design process and the practical situation combined with the need to improve the design process are the reasons for developing a new model. If a model is to be used as a tool in the design process, it should fulfil the following requirements:

1. It should be suited for several design problems ranging from short-term analyses to long-term decisions, e.g., assigning frequencies, designing part of a network and designing a complete network,
2. It should be easily accessible and understandable for the user (i.e., the planner).

The model presented in this paper is an attempt to serve as such a model. It is suited for use on a personal computer and special attention is given to the interactive design process.

Moreover, the optimization model will be included in a software package for the design of public transport networks. This package will also contain a model for the determination of an OD-matrix, an evaluation, model and interactive programs to arrange the necessary input. Activities for which the package can be used are as follows:

1. Evaluating a network,
2. Assigning frequencies,
3. Designing or redesigning part of a network,
4. Designing or redesigning a complete network.

For activities 2, 3 and 4, the optimization model can be used. The optimization process is structured to be simple and understandable.

OPTIMIZATION PROBLEM IN WORDS

The main objective is to design a network which can fulfil the demand for public transport as well as possible. It is obvious that this objective cannot be used in an optimization model, as it is unclear what is meant by "as well as possible." Does a network qualify as "good" if it offers services which can compete with other modes, or if it is especially suited to the needs of people who cannot travel otherwise? The decision on what is meant by "as well as possible" is a political one, however, and should not be made within an optimization model.

An objective suited to an optimization model and for both interpretations of "as well as possible" is maximizing the number of passengers, given a certain budget. It is a well-known fact that transfers negatively affect the number of passengers. Recent research in the Netherlands shows a penalty of 6 minutes, not including the waiting time at the transfer point (Van der Waard et al. [*18*]). Therefore, maximizing the number of passengers is more or less equivalent to minimizing transfers, especially in middle-sized cities such as those in the Netherlands. Although minimizing transfers is a commonly used objective (see e.g., Hasselström [*13*]), it is preferable to maximize the number of direct trips. The objective of maximizing direct trips makes it possible to use a description in which the demand for public services depends on the quality of the services offered, while the objective of minimizing transfers requires a fixed OD-matrix. Therefore, we choose to maximize the number of direct trips.

The major constraint of the problem is the available budget. Since there is a strong relation between the available budget and the number of vehicles that can be put into operation, the optimization problem can be formulated as follows:

Maximize the number of direct trips given a certain fleet size.

A special aspect of the public transport system is the use of different vehicle types (e.g., bus, tram). As the vehicle type influences both generalized costs and total costs, this aspect will also be included in the optimization model. Of course it is possible that, by maximizing the number of direct trips, networks may be developed which offer very poor transfer facilities, resulting in far fewer passengers than the highest number desirable. Therefore, additional constraints, such as a maximum number of routes or a minimum frequency, may be necessary. The decision as to which constraints must be imposed depends on the characteristics of the demand pattern and the specific network.

RELATION BETWEEN SUPPLY AND DEMAND FOR PUBLIC TRANSPORT SERVICES

The formulated objective makes it necessary to describe the relation between supply and demand for public transport services. It is not possible to use elasticities which are based on empirical research of the behavior of passengers as a result of changes in the public transport system. Usually these elasticities have constant values and are time and place dependent. Therefore, a direct demand model is formulated, which is based on the simultaneous distribution–modal split model (see, e.g., Wilson [*19*]). The relation between supply and demand for public transport is described by the deterrence function.

The simultaneous distribution–modal split model can be formulated as (see, e.g., Wilson [*19*]):

$$T_{ij} = r \cdot o_i \cdot d_j \cdot F_{ij} \qquad \forall i,j \tag{1}$$

where

T_{ij} = number of trips between nodes i and j,
r = constant term
o_i = factor for the generation of node i,
d_j = factor for the attraction of node j,
F_{ij} = value of the deterrence function for all modes for OD-pair i-j.

Constrained by:

$$\sum_i T_{ij} = D_j \qquad \forall j \quad \text{and} \quad \sum_j T_{ij} = o_i \qquad \forall i \tag{2}$$

where

D_j = arrivals at zone j,
O_i = departures from zone i.

F_{ij} can be written as:

$$F_{ij} = \sum_v [F_v(C_{ijv})] \qquad \forall i,j \tag{3}$$

where

F_v = the deterrence function for mode v,
C_{ijv} = the generalized costs for OD-pair i-j with mode v.

Finally, the number of trips by public transport can be calculated with the following equation:

$$T_{ijp} = T_{ij} \cdot \frac{F_p(C_{ijp})}{F_{ij}} = r \cdot o_i \cdot d_j \cdot F_p(C_{ijp}) \qquad \forall i,j \tag{4}$$

where

T_{ijp} = number of trips by public transport between nodes i and j,
F_p = the deterrence function for public transport,
C_{ijp} = the generalized costs for OD-pair i-j with public transport.

We will assume that a small change in the public transport system will only affect the number of trips by public transport, and will neither affect the total number of trips for an OD-pair (T_{ij}) nor the value of the deterrence function for all modes (F_{ij}). This assumption is acceptable for situations where 10–20 percent of all trips are made by public transport (e.g., in the Netherlands). The values of o_i and d_j are known, so by using equation (4) it is possible to calculate the number of trips. The values of o_i and d_j have to be determined for a situation which is comparable with the new situation. In case of an existing public transport system, these values can be calculated with an observed OD-matrix and for instance a weighted Poisson model (e.g., Hamerslag et al. [*20*]). When large changes are expected to occur, such as new residential areas, a traffic forecasting model should be used to calculate the values of o_i and d_j.

OBJECTIVE FUNCTION

Given is a set of possible routes Y with characteristics such as:

1. f_y = frequency of route y,
2. s_y = vehicle type used on route y (e.g. bus, tram),
3. N_y = set of nodes connected by route y,
4. T_y = in-vehicle times between the nodes of set N_y.

Only f_y will be used as the decision variable in the optimization process; all other characteristics are assumed to be fixed for each route. For instance, if a route can be used by two vehicle types, two identical routes have to be included each with a different vehicle type.

The set of possible routes Y can be generated in several ways, for instance with the method described by Ceder and Wilson (*21*), or such a set can be developed manually, using the interactive programs that will be included in the package. The final package will contain a model for the generation of routes.

The objective is to maximize the number of public transport passengers who can travel without transfers:

$$\max_{f_y} \sum_{I} \left\{ \sum_{J} [r \cdot o_i \cdot d_j \cdot F_P(C_{ijp})] \right\} \tag{5}$$

The generalized costs for an OD-pair are determined by the set of routes S_{ij} which offer a direct trip for the OD-pair *i-j*. Therefore:

$$F_p(C_{ijp}) = G(S_{ij}) \tag{6}$$

with S_{ij} = a set of routes with $i \in N_y$, $j \in N_y$ and $f_y \geq 0$ for $\forall \ y \in S_{ij}$, and $S_{ij} \in Y$.

When equation (6) is substituted in (5) the objective can be written as:

$$\max_{f_y} \sum_{I} \left\{ \sum_{J} [r \cdot o_i \cdot d_j \cdot G(S_{ij})] \right\} \tag{7}$$

The description of the public transport system results in a complicated analytical formulation of the objective. In order to derive a formulation which is more suitable for analytical analyses, a somewhat simplified description is used. For instance, let us assume an exponential function for F_p:

$$F_p = a \cdot \exp\left[-b \cdot (C_{ijp} + c)\right] \tag{8}$$

with a, b, and c as the coefficients.

The generalized costs can be written as:

$$C_{ijp} = g_{ij} + (60 \cdot h) / \left[\sum_{y \in S_{ij}} (f_y) \right] \tag{9}$$

with

- g_{ij} = a constant for OD-pair *i-j*, determined by the time to access and to egress the system and the time spent in the vehicle,
- h = a parameter for the calculation of the waiting time (including the weight of the waiting time).

Of course this description of the generalized costs is too simple in case there are several routes available for the OD-pair *i-j*, but it is sufficient to illustrate the problem. Equation (7) can then be written as:

$$\max_{f_y} \sum_{I} \left(\sum_{j} \left(r \cdot o_i \cdot d_j \cdot a \cdot \exp\left\{ -b \cdot \left[g_{ij} + \left[60 \cdot h / \left(\sum_{y \in S_{ij}} (f_y) \right) + c \right] \right] \right\} \right) \right) \tag{10}$$

CONSTRAINTS

The constraints of the problem are the available budget (*S1*) and the number of vehicles per vehicle type (*S2*). Furthermore, the possible frequencies are restricted to a limited set of integer values in order to make it easy for the passenger to memorize headways (*S3*), and of course only an integer number of vehicles can be assigned to a route (*S4*). These constraints can be written as:

$$S1: \sum_{s} \left\{ k_s \cdot \left[\sum_{y} (nv_y \cdot b_y) \right] \right\} \leq K \tag{11}$$

with

- K = the available budget,
- nv_y = the number of vehicles that is necessary for route y,
- b_y = a binary variable that indicates whether route y will be included in the summation ($b_y = 1$ if $s_y = s$; otherwise, $b_y = 0$),
- k_s = a factor for the costs of using a vehicle of type s.

$$S2: \sum_{y} (nv_y \cdot b_y) \leq mnv_s \qquad \forall \ s \tag{12}$$

with mnv_s = available number of vehicles of type s.

$$S3: \ f_y \in f \qquad \forall \ y \tag{13}$$

with f = set of possible (integer) frequencies.

$$S4: \ nv_y - 1 < (f_y \cdot nvf_y) \leq ny_y \qquad \forall \ y \tag{14}$$

with nvf_y = the number of vehicles that is necessary for the frequency of one vehicle per hour on route y.

SOLUTION METHOD

The formulated problem has a non-linear objective, linear constraints and a great number of integer variables. There are no efficient algorithms available to solve the problem without simplifying the formulation. For that reason Lampkin and Saalmans (*11*) and Dubois et al. (*12*) solve the problem in two stages: first, determine the routes and second, assign the frequencies. But, as there is a distinct relation between routes and frequencies, it would be better to determine them simultaneously. Therefore a new method has been developed. This method can be described as follows:

0. Set all frequencies equal to 0 and determine the elements of the sets S_{ij}.

1. Determine for each route y the efficiency r_y of an increase of the frequency by calculating the ratio of the number of extra passengers as a result of this increase and the necessary costs:

$$r_y = \frac{\sum_{m} \left\{ \sum_{n} [r \cdot o_m \cdot d_n \cdot G(S_{mn2})] \right\} - \sum_{m} \left\{ \sum_{n} [r \cdot o_m \cdot d_n \cdot G(S_{mn1})] \right\}}{k_{s_y} \cdot nvf_y \cdot (f_{y2} - f_{y1})} \tag{15}$$

with

$m,n \in N_y$,
$f_{y1}, f_{y2} \in f$, and $f_{y1} < f_{y2}$,

r_y = the efficiency of route y,
S_{mn1} = set of routes available for OD-pair m-n; $f_y = f_{y1}$,
S_{mn2} = set of routes available for OD-pair m-n; $f_y = f_{y2}$,
k_s = factor for the costs of using a vehicle of type s ($s = s_y$).

2. Select the route with the highest efficiency ratio and increase the frequency of that route.
3. Check the constraints $S1$ and $S2$ (eq. [11] and [12]); if they are no longer met the process stops; otherwise, continue with step 1.

A special feature of the method is the possibility of assigning some routes a fixed frequency, e.g., routes of other public transport companies, or routes of a vehicle type of which vehicles are no longer available. Because the optimization problem is limited to passengers who are being offered a direct trip, the values of the r_ys and the value of the objective function can quickly and easily be obtained. By restraining the set of possible routes Y, the method can also be applied to other design problems; for example, if we are only interested in the assignment of frequencies, the set Y consists of the existing routes.

ANALYSIS OF THE METHOD

Lagrange Multiplier

Although the method is heuristic, the efficiency of a route (r_y) can be regarded as an estimate of the Lagrange Multiplier, which is introduced when the first constraint is included in the objective and the integer constraints are dropped. The Lagrange formulation can then be written as:

$$\max_{f_y} \sum_{1}\left\{\sum_{j}[r \cdot o_i \cdot d_j \cdot G(S_{ij})]\right\} - \mu \cdot \left\{\sum_{s}\left[k_s \cdot \left(\sum_{y}(f_y \cdot nvf_y \cdot b_y) - K\right)\right]\right\} \quad (16)$$

with μ = the Lagrange Multiplier.

An optimum will be found when the Kuhn-Tucker conditions are fulfilled. Therefore it is required that:

$$\frac{\delta\left\{\sum_{m}\left[\sum_{n}(r \cdot o_m \cdot d_n \cdot G(S_{mn}))\right]\right\}}{\delta f_y} - \mu \cdot \frac{\delta\left\{\sum_{s}\left[k_s \cdot \left(\sum_{y}(f_y \cdot nvf_y \cdot b_y) - K\right)\right]\right\}}{\delta f_y} = 0 \quad \forall y \quad (17)$$

with $m,n \in N_y$.

$$\mu \cdot \left\{\sum_{s}\left[k_s \cdot \left(\sum_{y}(f_y \cdot nvf_y \cdot b_y) - K\right)\right]\right\} = 0 \quad (18)$$

$$\mu \geq 0 \quad (19)$$

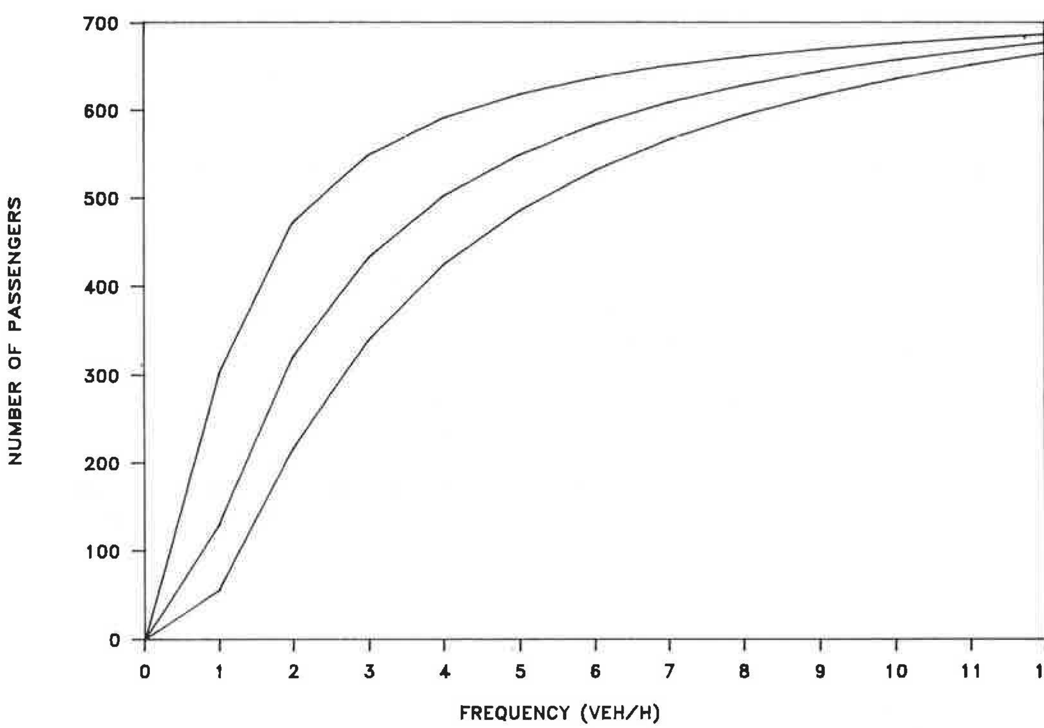

FIGURE 1 Possible relations between the number of passengers and the frequency offered.

Using equation (17) the Lagrange Multiplier μ can be written as:

$$\mu = \frac{\delta\left\{\sum_{m}\left[\sum_{n}(r \cdot o_m \cdot d_n \cdot G(S_{mn}))\right]\right\}}{k_s \cdot nvf_y \cdot \delta f_y} \quad \forall y \qquad (20)$$

If we take the limit of equation (15) as $(f_{y2} - f_{y1})$ approaches zero, the resemblance between the equations (15) and (20) is obvious. From this point of view the method is based on minimizing the variance between the values of r_y by increasing the frequency of the route with the largest r_y. The values of r_y will decrease gradually and finally converge to a solution in which they are more or less equal to each other and consequently equal μ.

Concavity

If the objective function is concave over the decision variables (f_y), the Kuhn-Tucker conditions are sufficient to determine the optimum. For an exponential, as well as for a lognormal deterrence function it can be shown that the objective is concave for frequencies greater than a certain value, depending on the coefficient being used (see Figure 1). The concavity of the objective function also guarantees that an increase of f_y will result in a decrease of r_y, and consequently that the method converges to a solution.

Quality of the Solution

As the method can be used for several design problems, there are two aspects that have to be analyzed:

1. The assignment of frequencies,
2. The selection of routes.

Both aspects have been analyzed with the use of the simplified objective described with equation (10). When we restrict the problem to assigning frequencies only, we can derive an alternative solution technique. By introducing the first constraint (11) in the objective (10) the Lagrange equation is derived. In the optimum situation the Kuhn-Tucker conditions should be fulfilled. These conditions result in a set of non-linear equations, which because of the concavity of the objective can be solved using Newton-Raphson (see Simmons [22]). For these analyses a simple network (Figure 2 and Table 1) has been used.

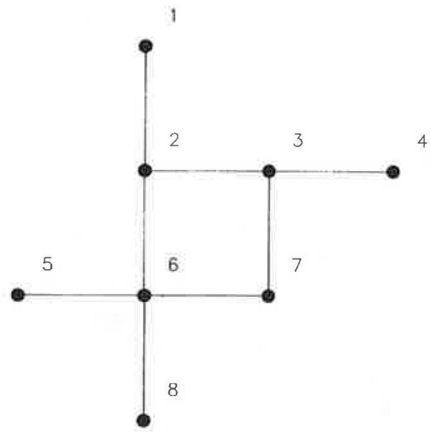

FIGURE 2 Network for analysis of the method.

For several sets of routes the frequencies were determined using Newton-Raphson as well as the new method with $f_{y2} - f_{y1} = 0.1$ and with $f_{y2} - f_{y1} = 1$. The results show that with a small stepsize the new method gives the same results as Newton-Raphson. If we use the integer stepsize, the results are quite satisfactory. The results are shown in Table 2.

The selection of routes is more difficult to analyze as the method used to analyze the assignment of frequencies cannot be used for the selection of routes. Therefore this has been done by comparing the first four selected routes with the results of every possible combination of four routes from the set Y. For each combination the Newton-Raphson method was used to determine the frequencies and the value of the objective. This comparison showed that the selected four routes were the best combination. Moreover, this analysis showed

TABLE 1 VALUES OF o_i AND d_j

Zone	o_i	d_j
1	2	4
2	10	10
3	2	2
4	2	2
5	4	2
6	10	10
7	4	4
8	2	4

TABLE 2 COMPARISON OF CALCULATED FREQUENCIES FOR A TEST NETWORK

	Frequency		
Route	Newton-Raphson	$(f_{y2} - f_{y1}) = 0.1$	$(f_{y2} - f_{y1}) = 1.0$
1-2-6-5	1.9	1.9	2.0
1-2-6-7	2.3	2.3	2.0
4-3-7-6-5	1.4	1.4	1.0
4-3-2-6-8	2.4	2.4	3.0
No. of direct trips	696.5	696.2	692.9

that 3 percent of the combinations were near-optimal, i.e., within 2 percent difference from the optimum solution. Analyses with a more realistic network gave similar results.

ADDITIONAL FEATURES

Although the test showed good results for the method, there are indications that the purely additive nature of the method might have a negative effect on the optimal quality of the selected network. Therefore, an exchange routine to check the solution has been introduced. This routine is based on the interpretation of the efficiency r_y as an estimate of the Lagrange Multiplier μ and checks whether the solution can be improved by replacing a selected route with another. If an optimum solution has been found it will not be possible to improve the solution in this way because the efficiency of the routes will be more or less equal. Moreover, an interactive routine is developed which can be used to analyze a solution by fixing frequencies, dropping routes or introducing extra routes, and to restart the optimization process, for example, to assign frequencies or to select alternative routes given the adapted solution. Therefore, the model does not present the solution, but allows the planner to play around with an opti mized solution. Besides, the possibility of using different starting sets Y, developed manually or with the use of a route generation model, also offers different solutions from which the planner may choose.

INCORPORATION OF THE PUBLIC TRANSPORT SYSTEM IN THE MODEL

Another aspect which determines the quality of the model is the description of the public transport system. Some special features of this description will be discussed in this paragraph.

The area that is the subject of the study is divided into zones, which are located around the stops. For each zone an access- and egress-time is determined. Trips from or to the study area are supposed to enter or leave at fictional zones located at the major transfer points between the local and regional public transport system.

The generalized costs consist of the weighted sum of the time-elements of a trip by public transport, namely the access- and egress-time, the in-vehicle time and the waiting time. The in-vehicle time is weighted with a coefficient which depends on the vehicle type. The waiting time can be calculated with several formulas, so it is possible to account for the expected regularity of the route, for example.

A special situation occurs when several routes offer a direct trip for an OD-pair. In some models the frequencies of the routes are added, but this is clearly a very optimistic approach. We will use an approach similar to that of Lampkin and Saalmans (*11*), but instead of calculating an average waiting time we also take amount of the possibility of bunched arrivals of vehicles. In this approach it is assumed that a passenger uses the first vehicle that arrives at the stop. This assumption has often been criticized (e.g., Marguier and Ceder [*23*]), but this criticism is not supported by empirical evidence.

All kinds of routes can be used in the set of possible routes: one-way and two-way routes, routes with loops, express routes and so on. Moreover, it is possible to use different deterrence functions to account for the different behavior of separate groups of travelers.

EXAMPLES

The model which has been described is suitable for a personal computer (Olivetti M24, MS-DOS, 640KB) and can be used for a network consisting of 250 nodes with a maximum of 150 zones, and a maximum of 750 possible routes.

Fictional Network

As an example of the design process using the optimization model the network of Figure 3 is used. A set of possible routes Y was generated manually, and consists of 75 routes. Two alternative demand patterns are considered: a midday period and an evening peak hour. For the midday period a network is designed which offers 1205 passengers a direct trip, given a fleet size of 10 vehicles. This solution cannot be improved using the exchange routine. For the evening peak hour two networks were developed: a complete new network and a network which uses the midday network as a base network. A comparison of these two networks shows that adding new constraints, such as the use of a base network, results in less

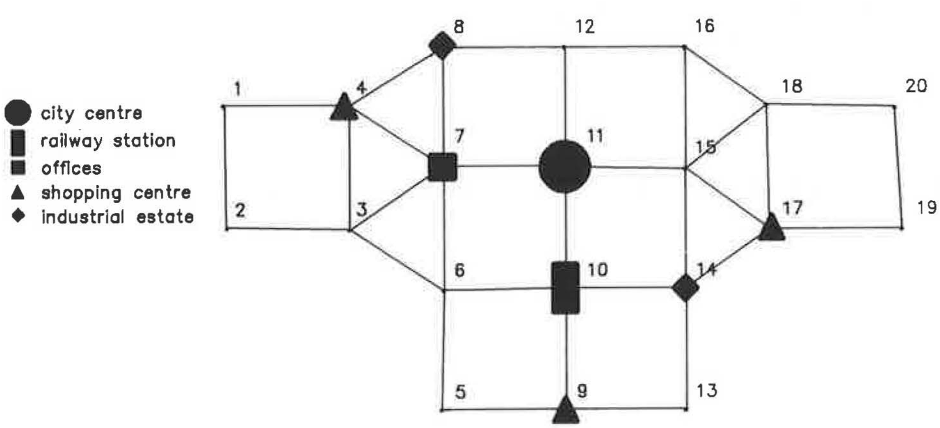

FIGURE 3 Test network.

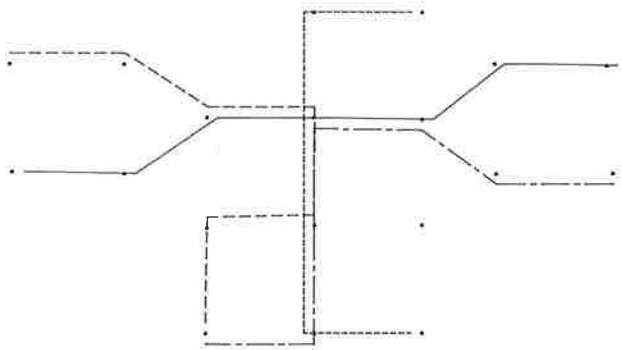

FIGURE 4 Selected network with four routes (midday).

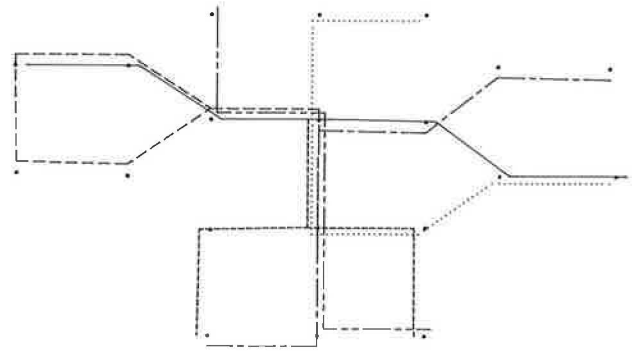

FIGURE 5 Selected network with six routes (peak).

optimal solutions. On the other hand, using a base network has the advantage that the network remains recognizable for the passenger. The major point, however, is that the optimization model can be used for both strategies. Results of the tests are shown in Figures 4, 5 and 6, and in Table 3.

Existing Network

The optimization model is also tested with data from the city of Groningen in the Netherlands (170,000 inhabitants). This network consists of 182 nodes and 115 zones (Figure 7). Three different starting sets Y were used. The first set consists of the current routes run by the local public transport company. The second set was constructed by splitting the existing routes at the city center and connecting them in all possible ways. The third set was derived with the use of basic design principles. The shortest routes from the city center and the railway station to 14 termini were determined and these route-segments were combined in such a way that each route passes the railway station and the city center. The optimization model was used to determine the best possible network based on the possibilities contained in sets 2 and 3, given the demand pattern for the morning peak hour. The first set, the existing routes, is used for comparison.

The results of these tests can be found in Table 4. They clearly indicate that the optimization method is suited for realistic situations. Sets 2 and 3 yield similar results for the number of direct trips, an increase of 300 trips. Set 3, however, is clearly the best solution when the total number of trips is included. This is due to the basic design principles used to construct set 3, because of which a network can be developed offering good transfer facilities.

These analyses show that the set Y is an additional constraint; set 2, which is determined by the network, yields a lower result compared to set 3, which is developed with fewer constraints. It is up to the planner to decide which constraints will have to be included in the set of possible routes. It should be noted that during these tests the number of routes was not used as a constraint. The method itself stopped selecting routes at 9, respectively 7 routes.

CONCLUSIONS

We have presented a new optimization model for the design of public transport networks. Special features of the optimization method are as follows:

1. The simultaneous selection of routes, assignment of frequencies and the determination of the number of passengers,
2. The single optimization process which can be used for several design problems, ranging from short-term analyses to long-term decisions,
3. The application on a personal computer, which together with the interactive approach and the inclusion in a software

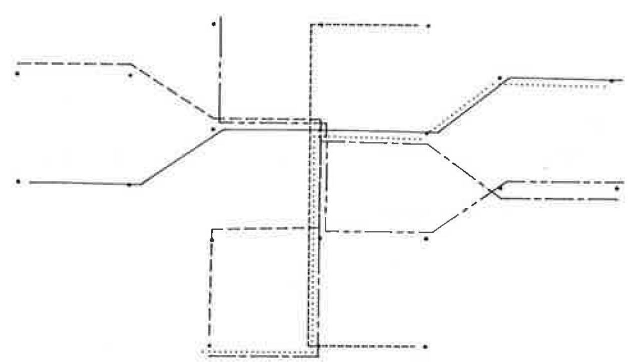

FIGURE 6 Selected network with six routes using a base network (peak).

TABLE 3 CHARACTERISTICS OF THE RESULTS WITH THE TEST NETWORK

Period of Day	Base Network	Direct Trips	No. of Vehicles	No. of Routes
Mid-day	No	1,205	10	4
Peak	Yes	1,529	15	6
Peak	No	1,543	15	6

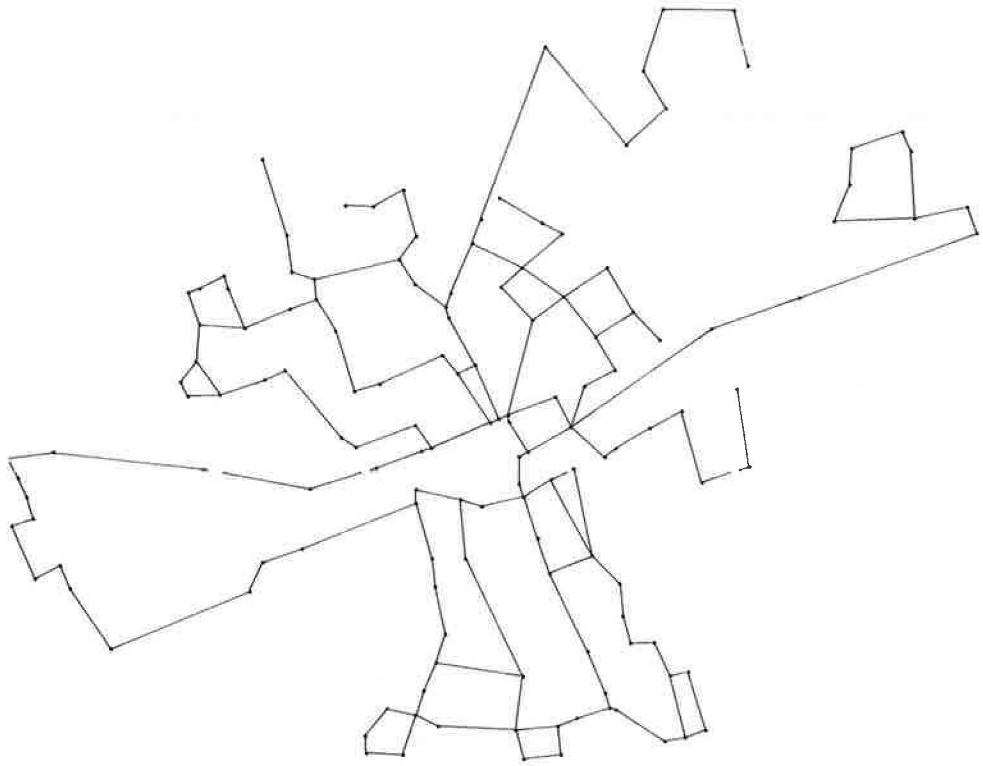

FIGURE 7 Network of Groningen.

TABLE 4 CHARACTERISTICS OF THE RESULTS WITH THE NETWORK OF GRONINGEN

Set	Direct Trips	Total Trips	No. of Vehicles	No. of Routes
1	3,494	5,242	55	10
2	3,818	5,393	56	9
3	3,805	5,746	55	7

package, enables the use of the model by the planner independently,

4. The possibility of taking into account all kinds of additional constraints, such as a base network, existing routes, a maximum number of routes, etc.

The method has proved to give good results with test networks and with actual data. Further research will be carried out to develop a model to generate a proper set of possible routes which can be used as input for the optimization model.

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REFERENCES

1. T. A. Chua and D. T. Silcock. The Practice of British Bus Operators in Planning Urban Bus Services. *Traffic Engineering and Control*, Vol. 22, No. 2, pp. 66–70, 1982.
2. E. M. Holroyd. The Optimum Bus Service: a Theoretical Model for a Large Uniform area, 3rd OECD Int. Symp. Theory of Road Traffic Flow, pp. 308–328, 1967.
3. G. Kocur and C. Hendrickson. Demand of Local Bus Service with Demand Equilibration, *Transportation Science*, Vol. 16, No. 2, pp. 149–170, 1982.
4. J. W. Billheimer and P. Gray. Network Design with Variable and Fixed Cost Elements. *Transportation Science*, Vol. 7, No. 1, pp. 49–74, 1973.
5. J. C. Rea. Designing Urban Transport Systems: An Approach to the Route Technology Selection Problem. *Highway Research Record* 417, pp. 48–59, 1972.
6. K. Pierick and K.D. Wiegand. Methodical Formulations for an Optimization of Railway Long-Distance Passenger Transport. *Rail International* 6, pp. 328–333, 1976.
7. C. Simonis. Optimierung von Omnibuslinien, Stadt Region Land B26, 1981.
8. S. Scheele. A Supply Model for Public Transport Services. *Transportation Research*, 14B, 3, pp. 133–148, 1982.
9. P. G. Furth and N. H. M. Wilson. Setting Frequencies on Bus Routes: Theory and Practice. *Transportation Research Record 818*, pp. 1–7, 1981.
10. B. Hagberg and D. Hasselström. *A Method to Assign Frequencies and Vehicle Types to Fixed Route Urban Public Transport Systems*. Volvo Transportation Systems, Gothenburg, 1980.
11. W. Lampkin and P.D. Saalmans. The Design of Routes, Service Frequencies and Schedules for a Municipal Bus Undertaking: A

Case Study. *Operational Research Quarterly*, Vol. 18, No. 4, pp. 376–397, 1967.
12. D. Dubois, G. Bel, and M. Llibre. A Set of Methods in Transportation Network Synthesis and Analysis. *Journal Operational Research Society*, Vol. 30, No. 9, pp. 797–808, 1979.
13. D. Hasselström. A Method for Optimization of Urban Bus Route Networks. Volvo Transportation Systems, Gothenburg, 1979.
14. I. Andréasson. New Interactive Computer Methods for Public Transport Analysis. *PTRC Proceedings, Sem. M*, University of Warwick, 1980.
15. M. Arnstrom. Interactive Planning—the Gothenburg Experience. *PTRC Proceedings, Sem. M*, University of Warwick, 1981.
16. Haagsche Tramweg Maatschappij, HTM lijnennet studie 1983: aanpak van de studie (in Dutch), The Hague, 1984.
17. I. D. H. Harris and P.J. Haywood. The Coventry Bus Study. *PTRC Proceedings, Sem. M*, University of Warwick, 1982.
18. J. Van der Waard, Th. J. H. Schoemaker and P.B.L. Wiggenraad. Quality Weighs! (in Dutch). *Proceedings Verkeerskundige Werkdagen*, Driebergen, 1987.
19. A. G. Wilson. The Use of Entropy Maximizing Models in the Theory of Trip Distribution, Modal Split and Route Split. *Journal of Transport Economics and Policy*, Vol. 2, No. 1, pp. 108–126, 1969.
20. R. Hamerslag, J.P. Roos and M. Kwakernaak. *Analysis of Accidents in Traffic Situations by Means of the Weighted Multiproportional Poisson Model*. Transportation Research Board, 1982.
21. A. Ceder and N. H. M. Wilson. Bus Network Design. *Transportation Research*, Vol. 20B, No. 4, pp. 331–344, 1988.
22. D. M. Simmons. *Nonlinear Programming for Operations Research*. Prentice Hall, New Jersey, 1975.
23. P. H. J. Marguier and A. Ceder. Passenger Waiting Strategies for Overlapping Bus Routes. *Transportation Science* Vol. 18, No. 3, pp. 207–230, 1984.

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