

Supplementary information for

Global redistribution of income and household energy footprints: A computational thought experiment

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Contents of supplementary information

Content type	Number	Name	Page
Notes	1	Log-normal distribution	2
	2	The nature of prediction and our model	3
	3	Normalizing expenditure	4
	4	Sensitivity analysis elasticities	5-6
	5	Alternative distributions, alternative worlds?	7
Tables	1	Real-world vs. model	8
	2	Default parameters	9
	3	Constant elasticities weighted regression results	10
	4	Non-constant elasticities weighted regression results	11
Figures	1	GDP per capita and household consumption expenditure	12
	2	Energy intensity per quantile	13
	3	Budget share allocation	14
	4	Cross-country vs. within-country elasticities	15
	5	Total energy per consumption category	16
	6	Method of computing mean income for income groups	17
	7	Age dependency ratio vs. income	18
	8	Carbon emissions from transport vs. income inequality in 2011	19
	9	Share transport energy vs. share residential energy	20
	10	Relationship between sigma_X of the log-normal distribution and its Gini coefficient	21
	11	Exponential relationship between elasticities and scaling coefficients	22
	12	Log-normal model fit validation	23
	13	Income elasticities of food – the details	24
	14	Alternative distributions and total energy	25
References	1	Supplementary References	26-27

Supplementary Note 1: Log-normal distribution

We calculate income for 1000 income groups. These represent the entire world population as of 2011. In order to do so, we solve equation (1) the cumulative distribution function of the log-normal distribution for x . The parameters here are, p = cumulative population, x = income, μ is the mean and σ is the standard deviation of the logged income values. Erf denotes the error function.

$$p = \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left[\frac{\ln(x) - \mu}{\sqrt{2}\sigma} \right] \quad (1)$$

In order to translate between the mean and standard deviation of the “logged” incomes and the “non-logged” incomes we use the following equations (2) and (3) (Wicklin, 2014). The parameters are as follows: μ is the mean and σ is the standard deviation of the logged income values, while μ_x is the mean of the non-logged incomes and σ_x the standard variation of the non-logged incomes.

$$\mu = \ln \left(\frac{\mu_x^2}{\sqrt{\sigma_x^2 + \mu_x^2}} \right) \quad (2)$$

$$\sigma = \sqrt{\ln \left(1 + \frac{\sigma_x^2}{\mu_x^2} \right)} \quad (3)$$

From the relationship between the normal distribution and the error function follows directly the following form (Crow & Shimizu, 1988).

$$Gini\ coefficient = \operatorname{erf} \left(\frac{\sigma}{2} \right) \quad (4)$$

Supplementary Note 2: The nature of prediction and our model

The ultimate goal of any model is to make some sort of prediction. This means being able to say something about the behaviour of a system over time or over the range of another variable. The variable on whose basis we want to make a prediction is the degree of inequality. How suited is our model to make such a prediction? We have seen in section 3.3 that the uncertainty in some results is substantial but major trends are robust. Prediction however requires testing a model against data. An issue with our model is that it introduces circumstances that have no empirical equivalent, not on global scale nor on a national one. We isolate income redistribution as the only “control” variable and the overall size of the economy is preserved. There is no economy in history that ever experienced “pure” redistribution while holding total output constant. This is why we have little appropriate data to test against. We still attempt two more evaluations of this. Can we observe, for instance, a relationship between income inequality and the composition of energy consumption across countries, as is postulated by our model on a global level? Our model is one of global scale but country level data might provide an additional benchmark. Therefore we tested the relationship between income inequality and the share of transport energy out of the total national energy consumption. We find no significant relationship. Yet countries all around the world have vastly different income levels and energy demand is determined by that to a large degree. The relationship between income inequality and energy demand might be blurred by this. Thus, it would be too fast to conclude that the model makes a false prediction. Moreover, we do find a weak to moderate relationship between income inequality and carbon emissions from transport (R-Squared ~ 0.185) based on World Bank data, demonstrating that more unequal countries have a larger share of emissions from transport. Supplementary Figure 8 depicts that relationship.

The model itself relies on relationships inferred empirically and thus, provided that these relationships remain stable, the model should have predictive power. Another fact that we can clearly observe is that high-income countries do have a much greater share of their energy footprint in transport than low-income countries. Therefore, if the income gap between countries were to aggravate, let us say in the event that high-income nations like the U.S. and Germany further experience economic growth while nations like India or countries in Sub-Saharan Africa stagnate (e.g. because of climate hazards), it could be that rising income inequality is accompanied by a further increase in transport energy demand. Supplementary Figure 9 portrays the current trade-off between residential energy use and energy use in transport (including vehicle fuel and maintenance, vehicle purchases and package holiday) considering income a third dimension.

Supplementary Note 3: Normalizing expenditure

A caveat of the modelling principle applied is that with an elasticity larger than one, predicted expenditure per consumption category, at some point, outstrips the total expenditure that is used to predict it in the first place. This happens because then the expenditure per category rises faster than the total expenditure. Moreover, using power laws to predict expenditure per category does not suffice to precisely allocate total expenditure across consumption categories. They would not perfectly add up to 100% of total expenditure, even if no single category outstrips total expenditure. It is important to normalize the predicted values in such a way that 100% of total expenditure is composed of corresponding sub-shares. Therefore, the initial predictions, which do not take any predictive bounds into account yet, are only taken to generate the budget share that is spend per population segment on different consumption categories. These budget shares are subsequently used for splitting the predicted total expenditure. This is a simple normalization procedure that rescales the values to the proper interval. It does not change anything about the predicted proportions of consumption categories within the overall consumption.

At first, we use a given income per capita of group j to predict an expenditure per capita of group j through equation (5).

$$\text{total consumption expenditure}_j = 2.6 \text{ income}_j^{0.83} \quad (5)$$

From now on total consumption expenditure is denoted C_j . From this total expenditure value, we then can derive expenditure per consumption category i , denoted c_{ij} by similar power laws (all parameter values are found in Supplementary Table #2).

$$c_{ij} = a_i C_j^{b_i} \quad (6)$$

Then, we calculate the share s_{ij} of c_{ij} among the sum of all n predicted consumption categories.

$$s_{ij} = \frac{c_{ij}}{\sum_{i=1}^n c_{ij}} \quad (7)$$

Subsequently we can just multiply s_{ij} and C_j to arrive at the properly rescaled expenditure per category, here denoted z_{ij} .

$$z_{ij} = s_{ij} * C_j \quad (8)$$

Supplementary Note 4: Sensitivity analysis elasticities

We conducted a Monte-Carlo simulation in section 3.3 to investigate the robustness of results. This simulation however was only based on uncertainty results from Oswald, Owen, & Steinberger, 2020 and our own statistical models.

The literature is inconclusive about income elasticities of demand for products and services. In particular, the important energy related elasticities for transport fuel and residential fuels are disputed. The elasticities also vary over space and time. The range in the literature is large, for instance, for gasoline anything from 0.1 to 2, with most around 1 and shortly below, have been reported (Espey, 1998; Finke, Rosalsky, & Theil, 1983). Havranek and Kokes argue that high elasticities (>1) are due to publication bias and conclude that much lower elasticities for transport fuel of around 0.2 are close to the real average (Havranek & Kokes, 2015). Studies also usually distinguish between short-run and long-run elasticities. It has been shown that over the long-run, in countries as the U.K. for instance, income elasticities of demand decrease, which is possibly a consequence of rising average income overtime (Fouquet, 2014). At higher incomes and consumption levels, the marginal utility of consumption for many products and services diminishes, hence the decreasing income elasticities.

One issue is that all these results are at the country level and studies did not investigate elasticities over the range of various countries taken together. This makes comparison difficult because it is a fundamentally different question to ask how the consumption of a good varies within one country or over the entire globe. We also work with a specific product aggregation. For example, vehicle fuel and maintenance are one consumption category in our model and this might make the elasticity behave differently from other reported results.

Despite challenges in comparison, the here applied elasticities of demand agree with a lot of the literature. Food (as a product bundle) for example is a basic good with an elasticity smaller than 1 and purchases of private vehicles a luxury good with elasticities far greater than 1. The general tendency for package holiday and all sorts of financial services to exhibit elasticities greater than 1 is also in agreement with the literature on luxury consumption and international travel. We also conducted a comparison to average within-country elasticities from Oswald et al., 2020 and the differences are rather small (see Supplementary Figure 4). We do differ with a significant share of the literature on income elasticities of residential energy and transport fuels. Both are often reported to be significantly lower than 1 (Havranek & Kokes, 2015; Schulte & Heindl, 2016). A fuel elasticity for our model lower than 1 makes no sense given that globally private transport vehicles are definitely not a basic good and we need to account for the vast number of people in the Global South not owning cars etc. Still we conduct a sensitivity analysis of our results with both energy-related elasticities put notably lower. Once we set only the elasticity of heat and electricity down to 0.7 from 0.88 (sensitivity run 1) and once we set down both, the elasticity of heat and electricity as well as the elasticity of vehicle fuel (sensitivity run 2 – vehicle fuel elasticity is put down to 1.2 from 1.77). All other parameters are kept the same way as in the default settings. All results refer to a variant of section 3.1 of the main text.

We find that in sensitivity run 1, the overall energy demand increases and energy inequality decreases slightly. Moreover, the total energy-expenditure elasticity drops from slightly above 1 to slightly below one. This is an important qualitative change in the behaviour of the model. However, the difference is not yet large enough to cause a dramatic shift in re-distributional outcomes. The energy costs of equity is with 10% still quite moderate. The trade-off between transport and residential energy still exists, though is of lower magnitude. In the second sensitivity run, the re-distributional differences are larger. The energy costs of equity increases considerably to 17%. Now, lower-income people tend to spend more on energy in general but richer people not drastically more. The trade-off between residential

and transport energy further diminishes. This world could be loosely interpreted as one in which income inequalities are not treated at all but high-income people around the world grow more environmentally conscious avoiding more of energy-intensive transport — an unlikely scenario. Sensitivity run 1 illustrates that even under a qualitative change major results remain stable. Sensitivity run 2 illustrates that, on the contrary, re-distributional consequences could be substantially different if spending patterns of people were to change radically and across various sectors.

Result	Default parameters	Sensitivity run 1	Sensitivity run 2
Gini coefficient total energy 2011	0.57	0.53	0.49
Total energy demand 2011 in Exajoules	209	241	233
Energy costs of equity (going from $\sigma_X = 26800$ down to $\sigma_X = 2680$)	6.7%	10%	17%
Over all energy-expenditure elasticity	1.06	0.95	0.92
Over all energy-income elasticity	0.88	0.78	0.76
Share transport $\sigma_X = 53600$	32%	28%	20%
Share transport $\sigma_X = 2680$	18%	14%	16%
Share heating and electricity $\sigma_X = 53600$	37%	45%	50%
Share heating and electricity $\sigma_X = 2680$	48%	58%	57%

Supplementary Note 5: Alternative distributions, alternative worlds?

Another source of uncertainty is whether the observed effects of changes in inequality (effect on total energy demand and so forth) are somehow unique features of the log-normal model. The parameterization of the lognormal model allows us to control the standard deviation but no other feature of the distribution, as for example the overall shape. We approached reduced inequality in an alternative way already in section 3.2 by setting specific minima and maxima, and the overall influence of income redistribution on energy remains the same – there are modest increases in aggregate demand, sectoral shifts from transport to residential and less energy poverty. Assuming a consistent shape as in section 3.1 is probably a realistic assumption. The global economy is a single interconnected system and absolute growth rates across income groups vary but are proportional to the level of income (Alvaredo, Chancel, Piketty, Saez, & Zucman, 2018a)—much like in theoretical Brownian motion models that can generate lognormal distributions (Hajargasht & Griffiths, 2013). Yet, so far, this is not an exhaustive investigation of the relationship between shape and inequality and its influence on energy demand. Other parameterizations of the global income distribution allow for other ways to control the shape of the distribution and history demonstrated that the nature of the global income distribution can change (Lakner & Milanovic, 2016). Moreover, the “rules” of the global economy are not “cast in stone” forever but can be disrupted by social or technological change. These are ever more important considerations now that ecological and social crises are omnipresent and could potentially give rise to entirely different income distributions.

How would energy demand change as a function of the inequality if the distribution had an entirely different shape? We used three further models to investigate that question: 1) A Weibull distribution – because it represents the evolution from a skewed and unequal distribution to a symmetric and equal one 2) A normal distribution — this way we can test whether the observed relationship between income inequality and household energy demand is constant, even if the income distribution were symmetric instead of asymmetric and 3) A standard Pareto distribution — hereby, we can test how much of a difference it makes to the observed patterns, if income inequality under constant mean evolves according to the Pareto Index instead of the lognormal standard deviation. These tests are to be understood as additional thought experiments, not as an accurate representation of reality.

We find that the results based on the Weibull distribution and its shape (controlled by the shape parameter k which is inversely related to the Gini coefficient—if k goes up, the Gini goes down) do not differ much from the main results based on the lognormal model. All major trends remain the same. Even if income were to follow a normal distribution, changes in inequality, under constant mean income, would follow the same trends. Although the inequality range that can be meaningfully investigated under a normal distribution is narrow and just goes from a Gini coefficient of roughly 0.05 to 0.2 (otherwise the normal distribution would reach negative income values in the left tail). In this range, the consequences are extremely minor (under all distributions). If income were to follow a Pareto distribution, there is unsurprisingly extreme polarization between rich and poor, particularly if the Pareto Index is close to 1. Interestingly, none of the aggregate trends change significantly as compared to the lognormal model. There is the same trade off in total energy and the same trade-offs in sectoral composition between transport and residential energy use when going from high to low inequality. The only clearly outstanding feature of the Pareto distribution is that the extremely rich (e.g. the top 0.1%) receive extreme amounts of incomes and consume extreme amounts of energy and everyone else very little.

Supplementary Figure 14 depicts the inequality of all tested distributions vs. the total global household energy demand.

Supplementary Table 1: Real-world vs. model. The references for the real-world values in this table are Oswald et al., 2020, ourworldindata.org, the World Bank open data repository or our own calculation. A value 90% that of the actual real-world value is considered high in accuracy, 70-90% medium and everything below that is considered of low accuracy.

Measure	Unit	Real world	Model	Accuracy in %	References real-world data
Distribution related measures					
Global income distribution shape	<i>None</i>	World Inequality Lab data on national income per adult equivalent	Log normal model cdf	99.6% (high)	(Alvaredo, Chancel, Piketty, Saez, & Zucman, 2018)
Global income Gini coefficient	<i>None</i>	0.6-0.7, 0.64	0.63	>90% (high)	(Anand & Segal, 2008; Milanovic, 2013), own calculation from (Alvaredo et al., 2018)
Global household expenditure Gini coefficient	<i>None</i>	0.59	0.54	>90% (high)	(Oswald et al., 2020) and extended version for this paper
Global final energy Gini coefficient	<i>None</i>	0.57	0.57	100% (high)	(Oswald et al., 2020) and extended version for this paper
People living below 1.9 \$ PPP in 2011	<i>Per cent</i>	13.5%	6%	40% (low)	(Beltekian & Ortiz-Ospina, 2018)
People living below 10 \$ PPP in 2011	<i>Per cent</i>	68%	50%	73% (medium)	(Beltekian & Ortiz-Ospina, 2018)
Scale related measures					
Global GDP (scale of the economy)	\$ PPP 2011	95.2 trillion	95.2 trillion	100% (high)	(World Bank, 2020c)
Global household expenditure (scale of expenditure)	\$ PPP 2011	46.72 trillion (deflated from 2017 dollars)	44 trillion	95% (high)	(World Bank, 2019)
Total household final energy consumption	Exajoule	231	209	90% (medium)	(Oswald et al., 2020) and extended version for this paper

Supplementary Table 2: Default parameters. Package holiday is the only category where standard parameters are derived from an unweighted cross country regression because it performs much better than the population-weighted version.

	elasticity (b)	coefficient (a)	MJ/\$
Income to expenditure	0.83	2.6	/
Food	0.62	7.745739	1.35
Alcohol and Tobacco	0.91	0.040184	1.21
Wearables	0.92	0.110155	2.88
Other housing	1.24	0.012748	1.30
Heating and Electricity	0.88	0.149723	35.77
Household Appliances and Services	1.03	0.030121	3.23
Health	1.04	0.028578	2.41
Vehicle Purchase	1.60	0.000073	2.05
Vehicle Fuel and Maintenance	1.77	0.000029	19.37
Other transport	0.84	0.076674	6.52
Communication	1.26	0.004269	1.96
Recreational items	1.56	0.000110	2.50
Package Holiday	2.05	0.00000023	6.86
Education & Finance & Other Luxury	1.25	0.014520	1.65

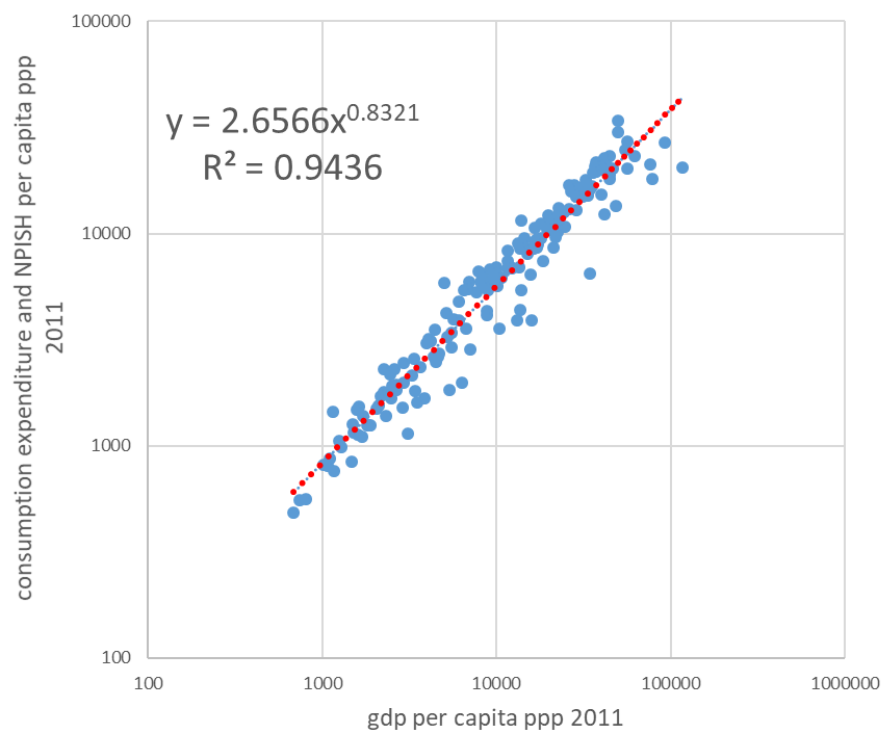
Supplementary Table 3: Constant elasticities weighted regression results

Consumption category	r-squared	b	a0	exp(a0) = a	se_b	CI 95% low	CI 95% high	b/se_b	N
Food	0.85	0.62	2.05	7.75E+00	0.01	0.60	0.65	46.77	379
Alcohol and Tobacco	0.69	0.91	-3.21	4.02E-02	0.03	0.85	0.97	28.64	375
Wearables	0.88	0.92	-2.21	1.10E-01	0.02	0.89	0.96	52.18	379
Other housing	0.83	1.24	-4.36	1.27E-02	0.03	1.18	1.29	42.78	379
Heating and Electricity	0.81	0.88	-1.90	1.50E-01	0.02	0.84	0.93	40.24	375
Household Appliances and Services	0.90	1.03	-3.50	3.01E-02	0.02	1.00	1.07	57.70	379
Health	0.72	1.04	-3.56	2.86E-02	0.03	0.98	1.11	31.46	378
Vehicle Purchase	0.75	1.60	-9.52	7.31E-05	0.05	1.50	1.69	31.74	342
Vehicle Fuel and Maintenance	0.85	1.77	-10.45	2.89E-05	0.04	1.70	1.85	46.70	375
Other transport	0.69	0.84	-2.57	7.67E-02	0.03	0.78	0.90	29.00	377
Communication	0.83	1.26	-5.46	4.27E-03	0.03	1.20	1.32	43.47	378
Recreational items	0.75	1.56	-9.12	1.10E-04	0.05	1.47	1.65	33.49	376
Package Holiday	0.41	1.48	-10.06	4.26E-05	0.11	1.27	1.69	13.67	274
Education & Finance & Other Luxury	0.89	1.25	-4.23	1.45E-02	0.02	1.21	1.30	54.27	379

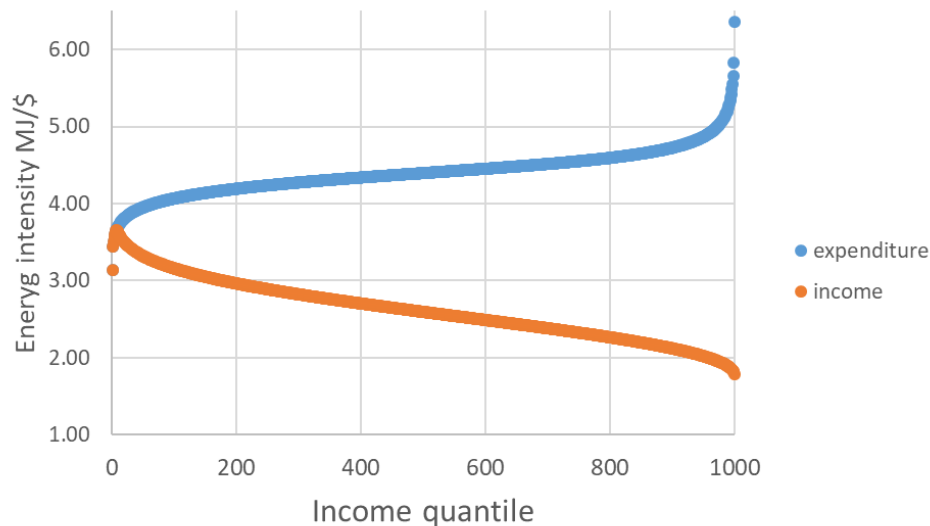
Supplementary Table 4: Non-constant elasticities weighted regression results. Non-constant elasticities over the global income spectrum exhibit high variation and are found not to be generally significant. An example for this is illustrated in Supplementary Figure 13 for the category food: Whereas the first two elasticities, in the lower half of the data, are significantly different from each other, this is not the case for the upper half. This might be a data resolution bias or a bias due to the vast heterogeneity in the underlying consumption surveys from the Global consumption database (World Bank, 2018) and the household budget surveys by Eurostat (Eurostat, 2015) which were used to compute elasticities.

	weighted				non-weighted			
	b (elasticity)							
	1st quartile	2nd quartile	3rd quartile	4th quartile	1st quartile	2nd quartile	3rd quartile	4th quartile
Food	0.84	0.49	0.59	0.60	0.81	0.30	0.49	0.47
Alcohol and Tobacco	1.03	1.74	-0.31	0.84	1.17	0.83	0.55	-0.03
Wearables	1.06	1.56	1.31	0.76	0.91	1.03	1.11	0.19
Other housing	1.08	0.26	0.27	0.61	1.17	1.52	0.98	0.36
Heating and Electricity	1.01	0.57	0.57	0.42	1.08	0.86	0.36	0.21
Household Appliances and Services	0.92	1.04	1.29	0.82	1.06	1.24	1.17	0.75
Health	1.29	1.69	0.92	1.12	1.23	1.35	1.23	0.61
Vehicle Purchase	1.25	2.86	1.63	1.74	1.24	1.70	2.09	1.44
Vehicle Fuel and Maintenance	1.71	0.96	1.90	1.21	1.68	1.92	1.72	0.26
Other transport	1.29	1.24	0.53	0.69	1.28	0.62	0.53	1.20
Communication	1.29	1.56	1.00	0.54	1.50	1.19	0.82	0.10
Recreational items	1.40	2.19	1.88	1.03	1.73	2.16	1.85	0.46
Package Holiday	0.24	2.91	2.87	1.00	1.24	2.88	2.11	1.06
Education & Finance & Other Luxury	1.11	1.20	1.92	1.61	1.17	1.55	1.59	0.23
b confidence interval 95% upper value								
Food	0.94	0.74	0.96	0.76	0.89	0.62	0.94	0.78
Alcohol and Tobacco	1.28	2.39	0.58	1.19	1.43	1.46	1.34	0.51
Wearables	1.19	1.96	1.58	0.97	1.07	1.41	1.55	0.70
Other housing	1.29	0.88	1.03	0.93	1.48	2.24	1.77	1.10
Heating and Electricity	1.22	0.92	1.01	0.54	1.27	1.45	1.18	0.60
Household Appliances and Services	1.06	1.40	1.70	1.10	1.22	1.59	1.54	1.26
Health	1.59	2.26	1.59	1.47	1.58	2.04	2.02	1.45
Vehicle Purchase	1.72	3.73	2.95	2.12	1.88	3.01	3.68	2.38
Vehicle Fuel and Maintenance	2.07	1.61	2.38	1.45	2.06	2.52	2.32	0.83
Other transport	1.48	1.58	1.38	1.25	1.54	1.22	1.38	1.84
Communication	1.57	2.01	1.22	0.69	1.75	1.59	1.20	0.66
Recreational items	1.84	2.96	2.66	1.29	2.05	2.98	2.79	1.29
Package Holiday	1.41	4.85	4.21	1.56	2.30	4.16	3.76	2.33
Education & Finance & Other Luxury	1.32	1.58	2.32	1.78	1.35	1.94	2.04	0.97
b confidence interval 95% lower value								
Food	0.74	0.23	0.22	0.45	0.72	-0.03	0.03	0.15
Alcohol and Tobacco	0.77	1.10	-1.20	0.48	0.92	0.21	-0.25	-0.58
Wearables	0.92	1.16	1.04	0.55	0.74	0.65	0.68	-0.32
Other housing	0.87	-0.36	-0.50	0.29	0.86	0.80	0.18	-0.38
Heating and Electricity	0.81	0.21	0.13	0.30	0.90	0.27	-0.46	-0.18
Household Appliances and Services	0.78	0.68	0.87	0.54	0.89	0.88	0.79	0.24
Health	1.00	1.12	0.25	0.78	0.89	0.66	0.45	-0.23
Vehicle Purchase	0.79	1.99	0.30	1.37	0.59	0.40	0.50	0.49
Vehicle Fuel and Maintenance	1.35	0.32	1.41	0.97	1.31	1.32	1.11	-0.31
Other transport	1.09	0.89	-0.33	0.13	1.02	0.01	-0.32	0.56
Communication	1.01	1.10	0.78	0.39	1.24	0.80	0.45	-0.45
Recreational items	0.96	1.43	1.10	0.76	1.41	1.34	0.91	-0.36
Package Holiday	-0.93	0.97	1.52	0.44	0.18	1.60	0.46	-0.21
Education & Finance & Other Luxury	0.90	0.81	1.53	1.44	0.98	1.16	1.14	-0.52

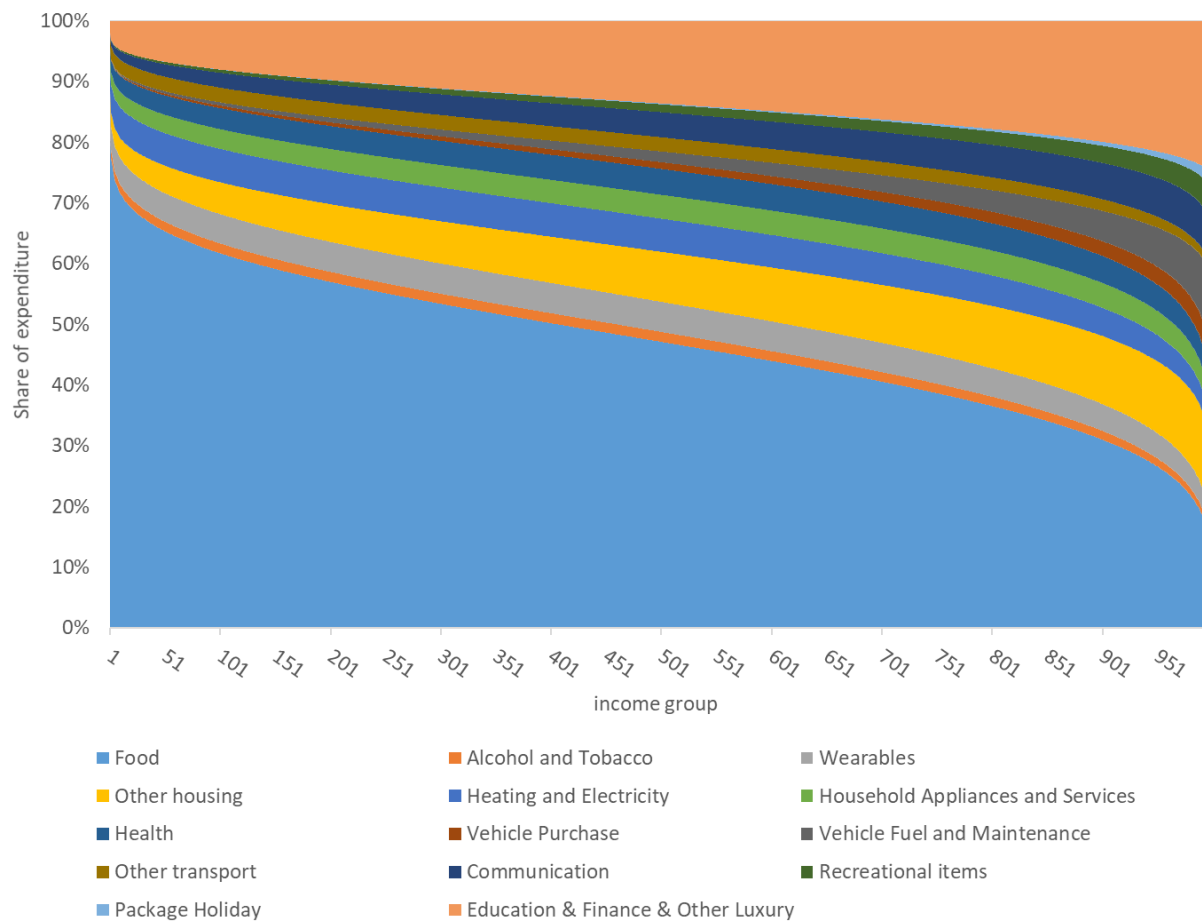
Supplementary Figure 1: GDP per capita and household consumption expenditure. This empirical relationship is used for modelling income to expenditure and is based on data by the World Bank (World Bank, 2019, 2020c).



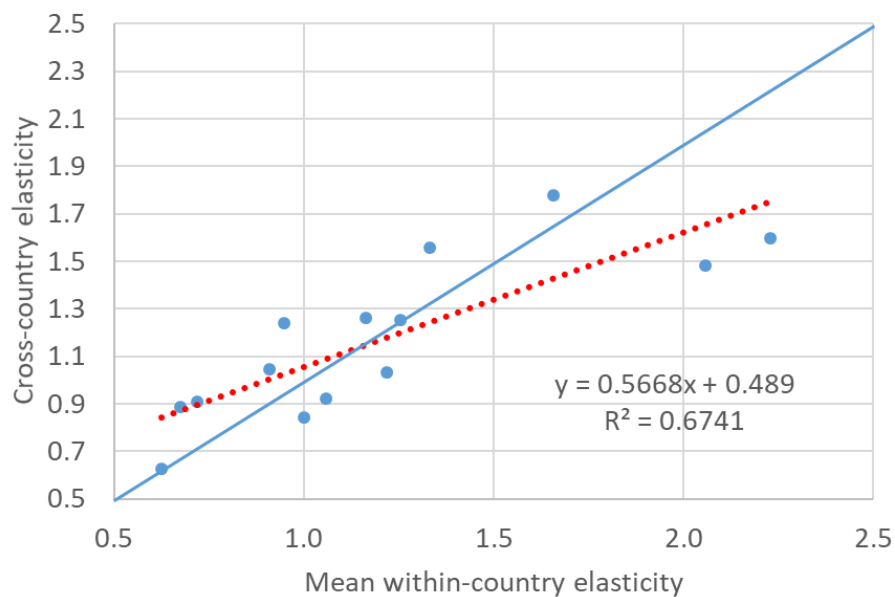
Supplementary Figure 2: Energy intensity per income group. Our average energy intensity of expenditure increases slightly with quantile and sharply at the beginning and at the end of the distribution. This is a result of using homogenous energy intensities across the entire global distribution. This also means that our average energy-expenditure elasticity is slightly large than one (1.06). It is important to note that our energy intensity is of household consumption only. Moreover, our average income-energy elasticity is significantly less than one and therewith clearly in line with the literature (0.88). Average energy intensity of income thus decreases. This difference between income and expenditure energy elasticity is because households spend a decreasing share of their entire income when incomes rise. Usually the energy-GDP elasticity is measured to be less than one varying between 0.7-1, yet results remain uncertain and sometimes insignificant (Liddle & Huntington, 2019). On a country level, it has been measured that consumption-based energy elasticities of household expenditure can be larger than one (Oswald et al., 2020). This also has been measured and affirmed for carbon elasticities of household income (Hubacek, Baiocchi, Feng, Sun, & Xue, 2017). Results are mixed however and there are also a variety of countries with elasticities <1 . Since in our model the overall expenditure elasticity being larger than unity is an “emergent” feature due to (empirical) energy intensities and elasticities on a product level we keep it this way and argue that it is a reasonable assumption. We test however for sensitivity of major results in supplementary note 4.



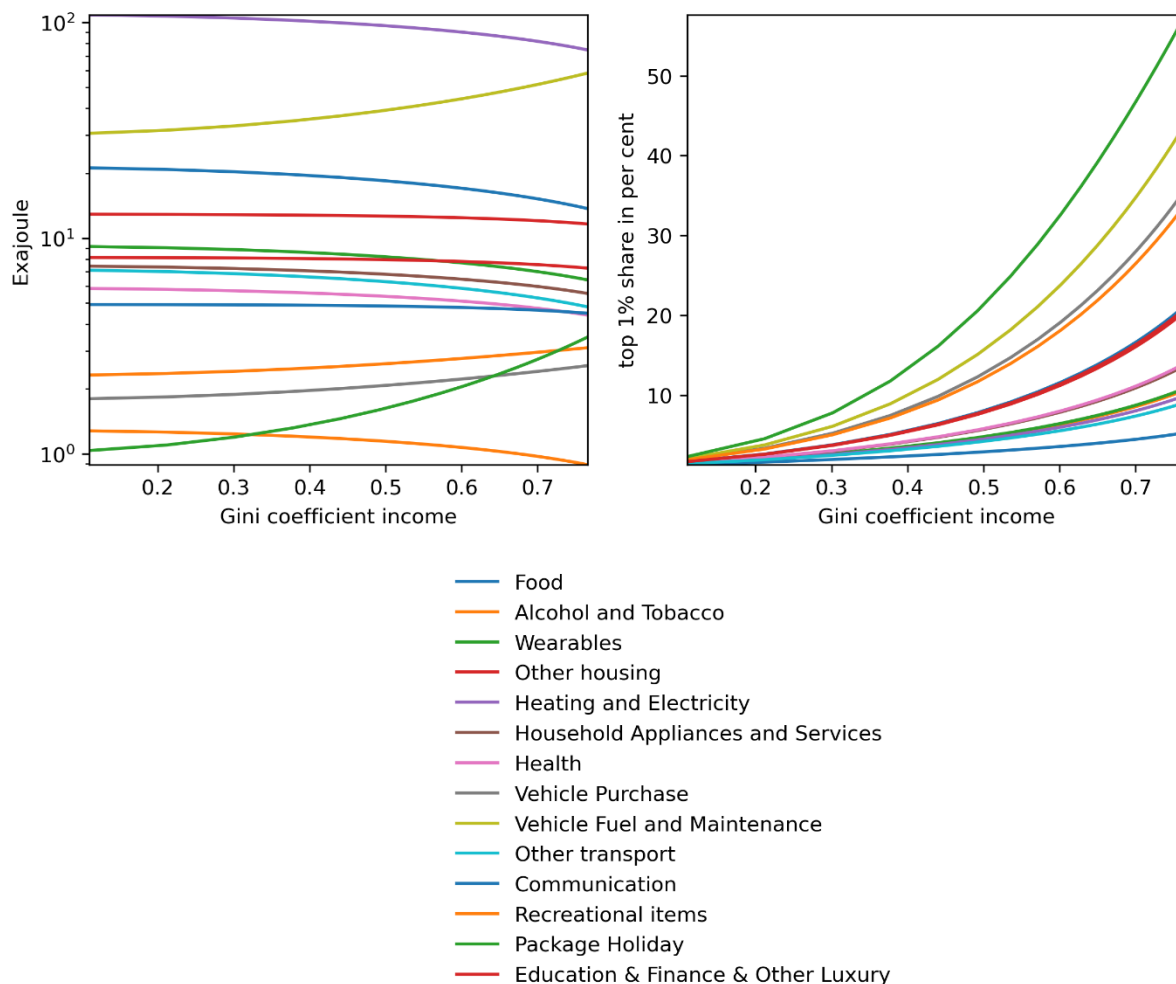
Supplementary Figure 3: Budget share allocation. Our model fulfils the important empirical law “Engel’s law”. With increasing income the share spent on food decreases. This figure depicts the budget allocation of expenditure over all 1000 income groups (1000 is the max. of the x-axis). The budget share is sensitive to the estimated parameters.



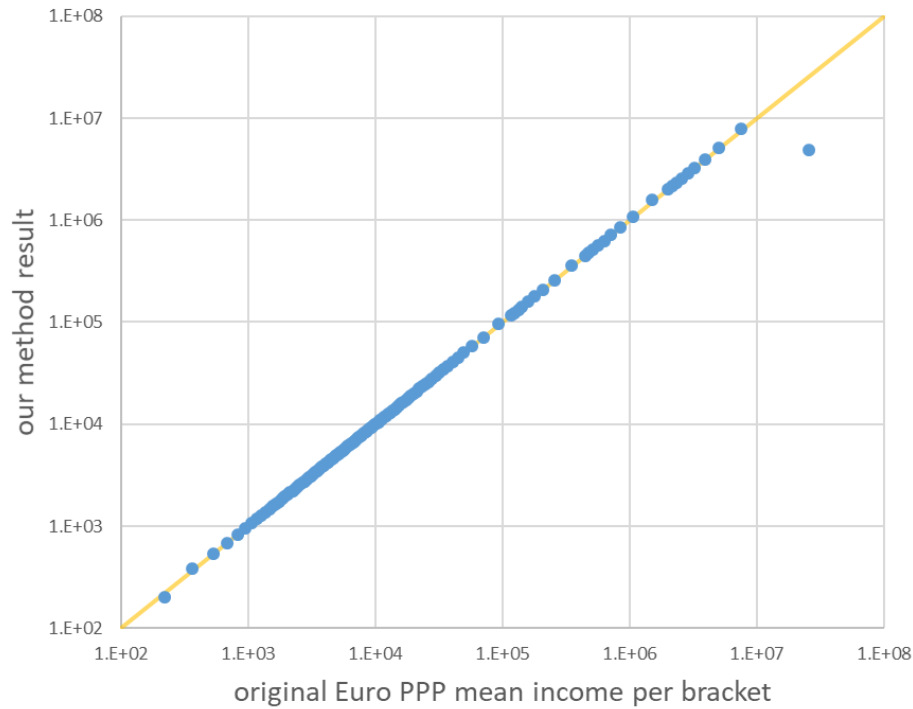
Supplementary Figure 4: Cross-country vs. within-country elasticities. Here we correlated the population-weighted cross-country international elasticities of consumption and average within-country elasticities from Oswald et al., 2020. The linear correlation is good (with unweighted elasticities even better at an R-Squared of 0.87) and illustrates that consumption with increasing income within countries (national-scale) behaves similarly to consumption across the entire world (global-scale). The most notable difference between within-country and cross-country elasticity is in vehicle purchases and package Holiday which both are larger than 2 in within-country results and rather around 1.5 in a cross-country perspective (when weighted). In the unweighted case, package holidays is still above 2 and vehicle purchase is ~1.9. The red dotted line is the linear fit. The blue continuous line is the one-to-one line.



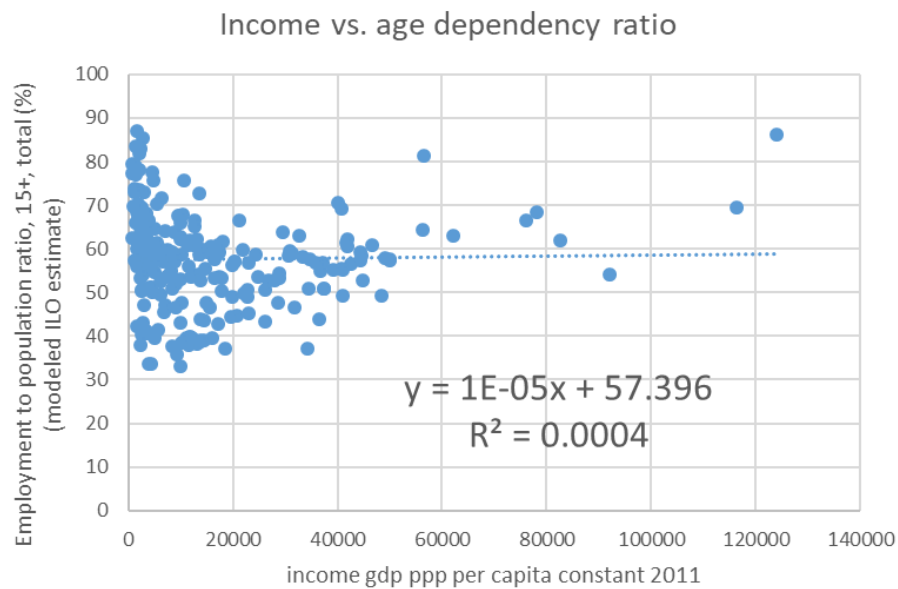
Supplementary Figure 5: Total energy per consumption category. This figure is complementary to Figure 4 and Figure 5 of the main paper. It illustrates the absolute energy consumption per consumption category as a function of the income Gini coefficient as well as the energy share of the top 1% global income earners.



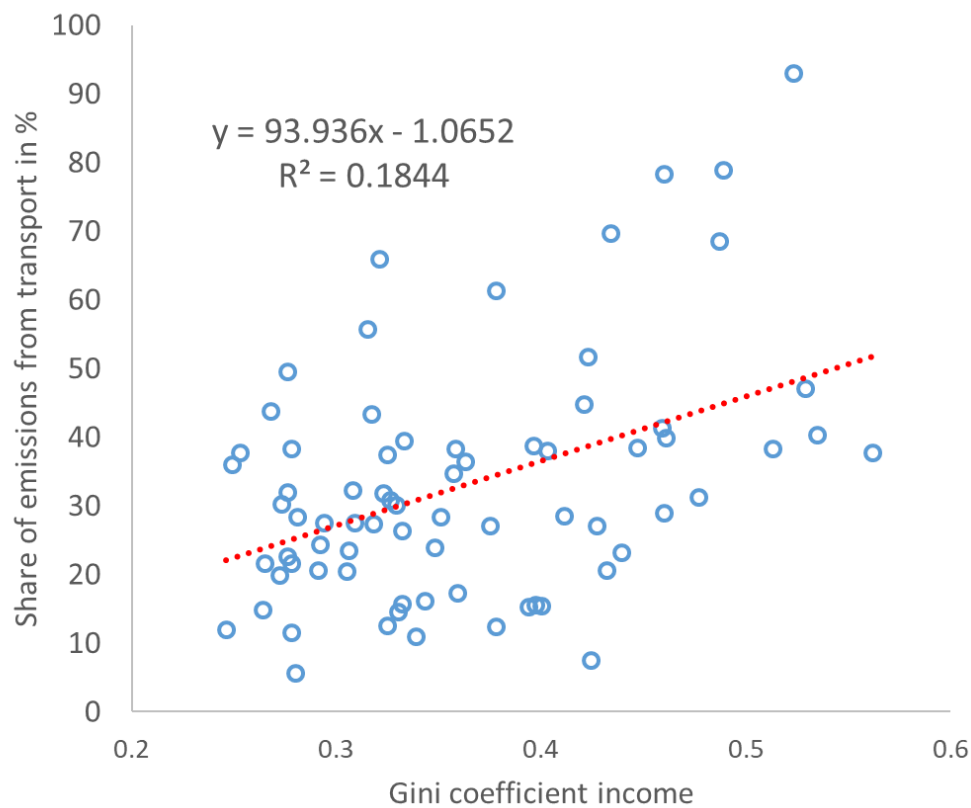
Supplementary Figure 6: Method of computing mean income for income groups. We just take the lower and upper bound of an income bracket and take the average to compute mean income per group. We compared our method, applied to World Inequality Database data (Alvaredo et al., 2018) using their lower and upper bounds, against their original results of mean incomes. The comparison results into a one-to-one relationship, except at the very long tails of the distribution (PPP refers to PPP Euro here). We also tested a version of our simulations applying high resolution numerical integration to the log-normal CDF. The results only change negligibly.



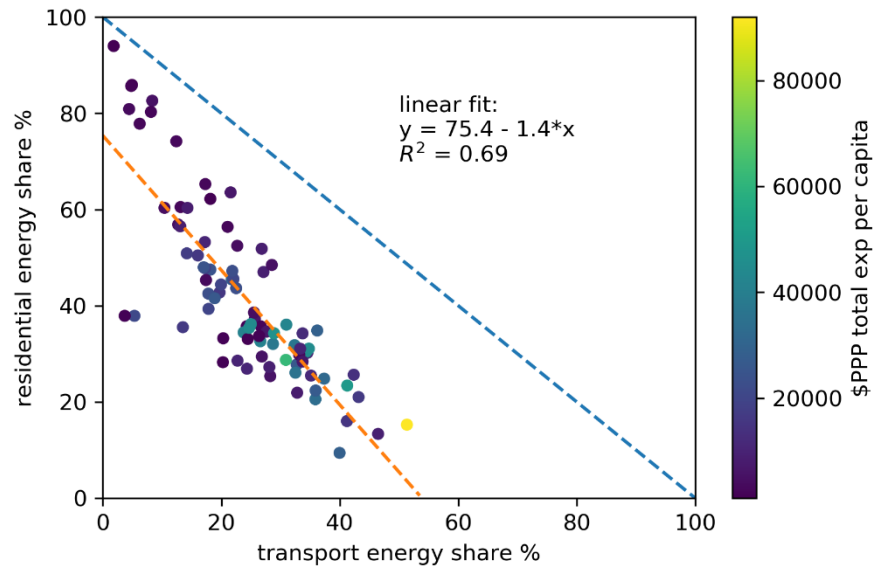
Supplementary Figure 7: Age dependency ratio vs. income. We tested whether there is a relationship between the income level of a country (World Bank, 2020c) and its age dependency ratio (World Bank, 2020a) in order to translate per adult equivalent data to per capita data (because it is assumed that the adult population is sufficiently close to the working age population). There is high variation and no significant relationship.



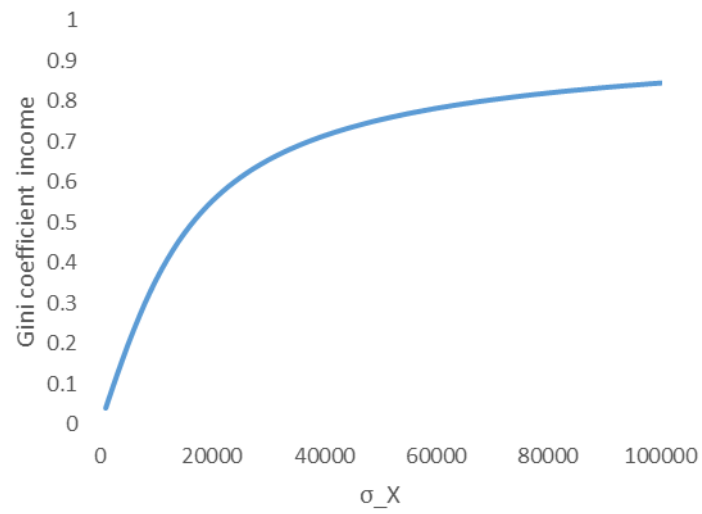
Supplementary Figure 8: Carbon emissions from transport vs. income inequality in 2011. The correlation between emissions from transport and income inequality was observed for instance here (Tomkiewicz, 2019) and can be reproduced through World Bank open-source data (World Bank, 2020d, 2020b). The fit is weak to moderate but, considering that many different income levels are included, a fit that explains nearly a fifth of transport emissions is quite remarkable.



Supplementary Figure 9: Share transport energy vs. share residential energy. The linear fit is the dashed orange line. The dashed blue line depicts the one-hundred-percent-frontier of consumption. The trade-off between residential and transport energy is clear. Yet there is a growing gap between the consumption frontier and the trade-off, implying that consumption overall diversifies. The source for the data is Oswald et al., 2020.

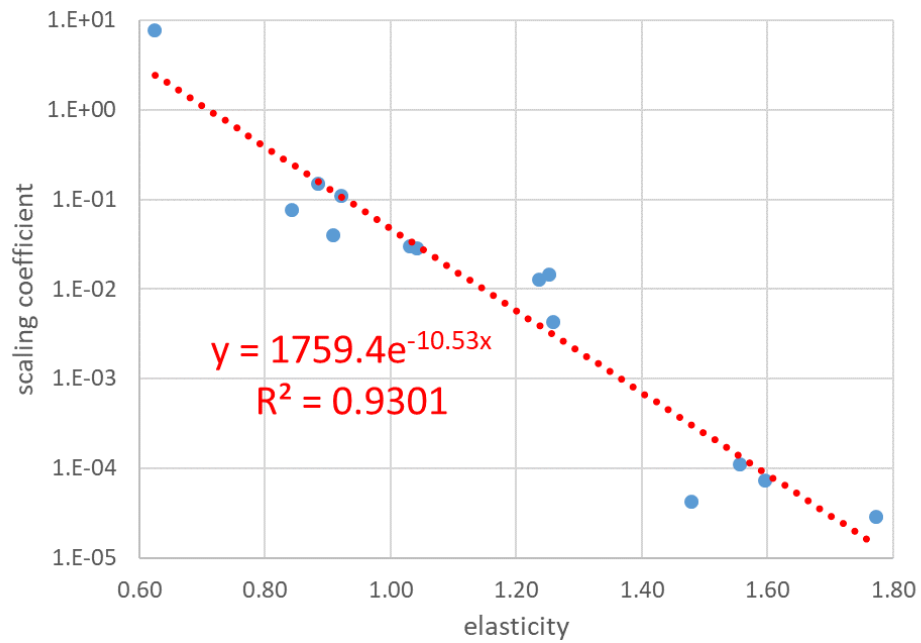


Supplementary Figure 10: Relationship between σ_X of the log-normal distribution and its Gini coefficient. This graph relates to supplementary equations number three and four.

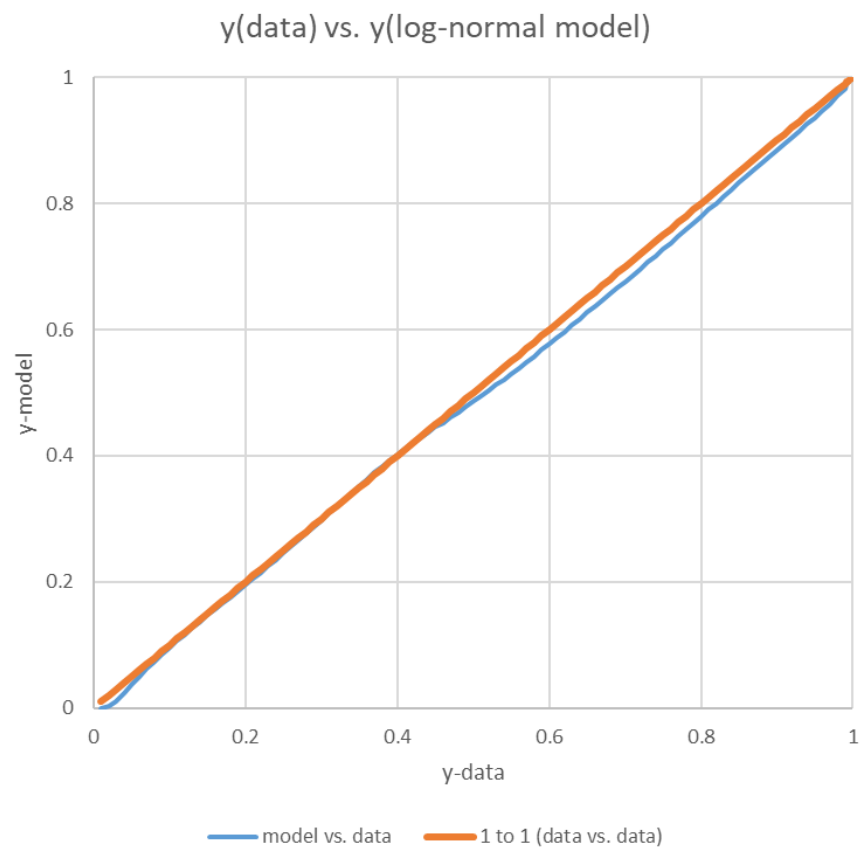


Supplementary Figure 11: Exponential relationship between elasticities and scaling coefficients.

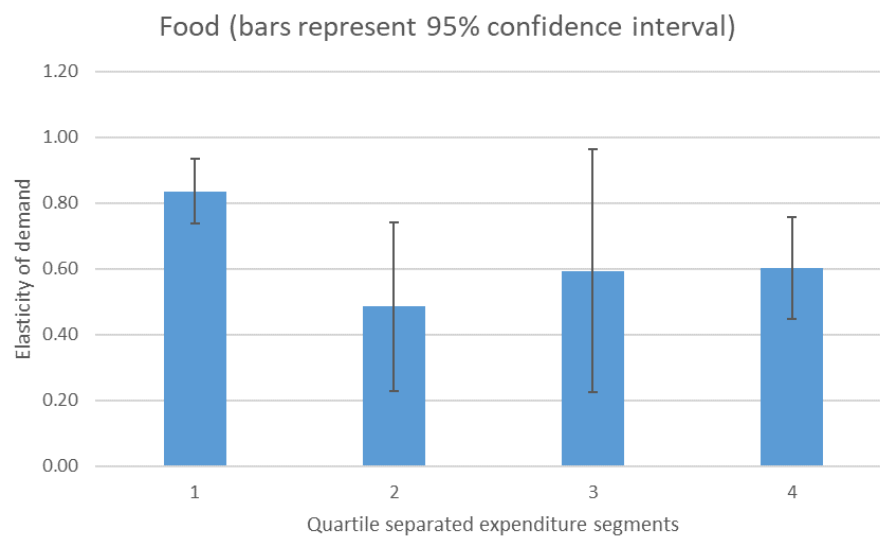
This relation is applied in the sensitivity analysis and the Monte-Carlo simulation. If the elasticity is altered we adjust the scaling coefficient accordingly.



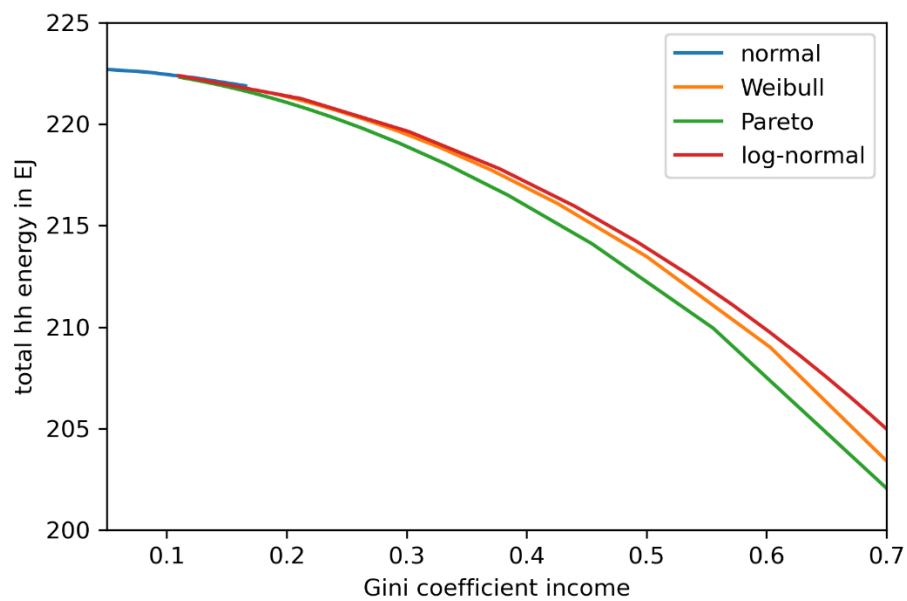
Supplementary Figure 12: Log-normal model fit validation. The y -dimension represents the cumulative population as in Figure 2 of the main paper. The orange line depicts the case if model and data would match perfectly. The blue line depicts how they actually relate. We minimized the residual sum of squares as a function of the log-normal standard deviation (the only free parameter in our log-normal model because the mean is fixed) in order to achieve the best fit.



Supplementary Figure 13: Income elasticities of food – the details



Supplementary Figure 14: Alternative distributions and total energy



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