A Communication Efficient Stochastic Multi-Block Alternating Direction Method of Multipliers

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1. Linearly Constrained Sto-Opt

- Llinearly constrained stochastic convex programs

$$\min_{\mathbf{x}_i\in\mathcal{X}_i,\forall i} \quad f(\mathbf{x})=\sum_{i=1}^N f_i(\mathbf{x}_i) \ \text{ s.t. } \sum_{i=1}^N \mathbf{A}_i\mathbf{x}_i=\mathbf{b}$$

- N arbitrary; Each $f_i(\mathbf{x}_i)\triangleq \mathbb{E}_\xi[f_i(\mathbf{x}_i;\xi)]$ with expensive true gradient but cheap unbiased stochastic gradient.
- **Applications**
  - **Large scale** linearly constrained **optimization**, e.g,. linear programs: Too large to store or solve on a single node such that each node i stores $\mathbf{A}_i$ and iteratively solve smaller sub-problem with inter-node computation.
  - Distributed machine learning: N nodes with distributed and possibly non-identical training data jointly train a common ML model.

2. Communication Efficient ADMM

- **ADMM** is effective and popular for distributed optimization, yet **suffers significant communication overhead** for passing Lagrange multipliers. Typically, a communication step follows immediately after a computation step.
- Communication usually costs much more time than SGD type computation.
- This paper develops **communication efficient** ADMM for multi-block stochastic ADMM such that **communication rounds are reduced without sacrificing convergence**.

3. Our Algorithm

Alg1 : Two-Layer Communication Efficient ADMM

1: **Input:** Algorithm parameters T, $\{\rho^{(t)}\}_{t\geq 1}$, $\{\nu^{(t)}\}_{t\geq 1}$ and $\{K^{(t)}\}_{t\geq 1}$.
2: Initialize arbitrary $\mathbf{y}_i^{(0)}\in\mathcal{X}_i,\forall i$, $\mathbf{r}^{(0)}=\sum_{i=1}^N \mathbf{A}_i\mathbf{y}_i^{(0)}-\mathbf{b}$, $\boldsymbol{\lambda}^{(0)}=\mathbf{0}$, and $t=1$.
3: **while** $t\leq T$ **do**
4: Each node i defines $\phi_i^{(t)}(\mathbf{x}_i)\triangleq$

$$f_i(\mathbf{x}_i)+\rho^{(t)}\langle \mathbf{r}^{(t-1)}+\frac{1}{\rho^{(t)}}\boldsymbol{\lambda}^{(t-1)}, \mathbf{A}_i\mathbf{x}_i-\frac{\mathbf{b}}{N}\rangle+\frac{\nu^{(t)}}{2}\|\mathbf{x}_i-\mathbf{y}_i^{(t-1)}\|^2$$

and **in parallel** updates $\mathbf{x}_i^{(t)},\mathbf{y}_i^{(t)}$ using local sub-procedure **Alg 2** via

$$(\mathbf{x}_i^{(t)},\mathbf{y}_i^{(t)})=\text{STO-LOCAL}(\phi_i^{(t)}(\cdot),\mathcal{X}_i,\mathbf{y}_i^{(t-1)},K^{(t)})$$

5: Each node i passes $\mathbf{x}_i^{(t)}$ and $\mathbf{y}_i^{(t)}$ between nodes or to a parameter server. Update $\boldsymbol{\lambda}^{(t)}$ and $\mathbf{r}^{(t)}$

$$\boldsymbol{\lambda}^{(t)}=\boldsymbol{\lambda}^{(t-1)}+\rho^{(t)}\Big(\sum_{i=1}^N \mathbf{A}_i\mathbf{x}_i^{(t)}-\mathbf{b}\Big)$$

$$\mathbf{r}^{(t)}=\sum_{i=1}^N \mathbf{A}_i\mathbf{y}_i^{(t)}-\mathbf{b}.$$

6: Update $t\leftarrow t+1$.
7: **end while**
8: **Output:** $\overline{\mathbf{x}}^{(T)}=\frac{1}{\sum_{t=1}^T\rho^{(t)}}\sum_{t=1}^T\rho^{(t)}\mathbf{x}^{(t)}$

- Each outer-loop iteration of our communication efficient ADMM (Alg 1) involves a SGD type sub-procedure Alg 2:

Alg2: STO-LOCAL($\phi(\mathbf{z}),\mathcal{Z},\mathbf{z}^{\text{init}},K$)

1: **Input:** μ: strong convexity modulus of $\phi(\mathbf{z})$; Algorithm parameters: $k_0>0$; $\gamma^{(k)}=\frac{2}{\mu(k+k_0)},\forall k\in\{1,2,\ldots,K\}$.
2: Initialize $\mathbf{z}^{(0)}=\mathbf{z}^{\text{init}}$ and $k=1$.
3: **while** $k\leq K$ **do**
4: Observe an unbiased gradient $\boldsymbol{\zeta}^{(k)}$ such that $\mathbb{E}[\boldsymbol{\zeta}^{(k)}]=\partial\phi(\mathbf{z}^{(k-1)})$ and update $\mathbf{z}^{(k)}$ via

$$\mathbf{z}^{(k)}=\mathcal{P}_{\mathcal{Z}}\Big[\mathbf{z}^{(k-1)}-\gamma^{(k)}\boldsymbol{\zeta}^{(k)}\Big]$$

where $\mathcal{P}_{\mathcal{Z}}[\cdot]$ is the projection onto $\mathcal{Z}$.
5: **end while**
6: **Output:** $(\widehat{\mathbf{z}},\mathbf{z}^{(K)})$ where $\widehat{\mathbf{z}}$ is the time average of $\{\mathbf{z}^{(0)},\ldots,\mathbf{z}^{(K)}\}$ defined in Lemmas 1 or 2.

- Each iteration of Alg 2 is a cheap SGD update.

5. Performance Analysis

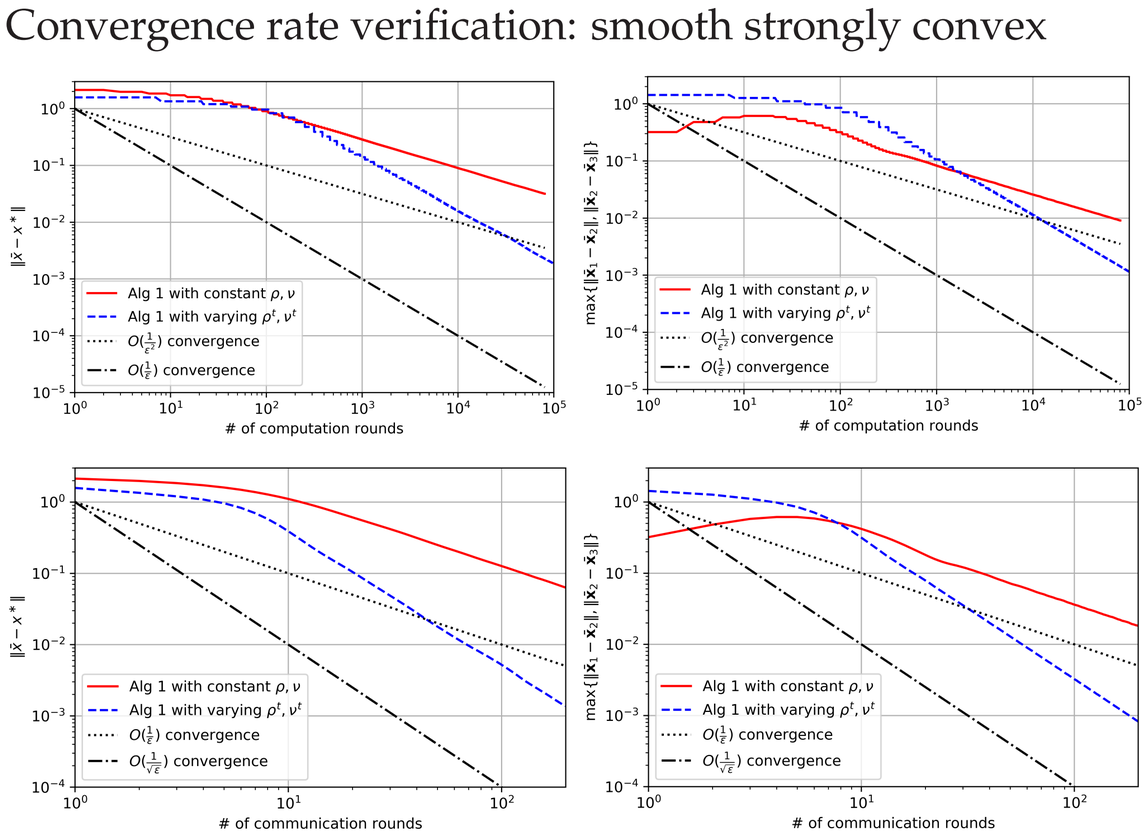
Under corresponding algorithm parameter rules, to achieve an $O(\epsilon)$ accuracy solution

- **General Convex**: Alg 1 uses $\tilde{O}(1/\epsilon^2)$ SGD update rounds and $\tilde{O}(1/\epsilon)$ inter-node communication rounds.
- **Strongly Convex**: Alg 1 uses $\tilde{O}(1/\epsilon)$ SGD update rounds and $\tilde{O}(1/\sqrt{\epsilon})$ inter-node communication rounds.

Using Alg 1, the # of communication rounds is only the square root of that of computation (SGD update) rounds. The achieved computation complexity is the lowest possible for stochastic convex opt but the communication complexity is lower than existing stochastic ADMM.

6. Experiments

- Convergence rate verification: smooth strongly convex



- Setting algorithm parameters in line with our theory yields the proven convergence rates