

**DEPARTMENT OF ELECTRICAL AND
ELECTRONIC ENGINEERING**

Passive Tracking of Targets

Third (Four if applicable) year project report is submitted in part fulfilment of the requirements of the degree of Bachelor (Master if applicable) of Engineering.

Abstract

This thesis discusses the feasibility including method and performance analyse of passive tracking. Compared with other locating technologies such as radar, passive tracking receive target's signal rather than broadcast any signal that may reveal the position of receiver. In addition to invisibility, passive tracking method needs no transmitter, so it is more simple and economical.

However, without transmitter, passive tracking asks for much stronger requirements on the errors processing to improve the bearing accuracy, so it is very important to find appropriate locating method and algorithm according to the case.

The research hence will explain how to locate target by passive technologies, present simulations and results for target locating and tracking both in ideal and realistic situation, discuss how to use feasible algorithm to process the noise, and analyse the performance such as the relationship between bearing and accuracy to see is there any important rule. There are also some performance improvements ideas will be given based on the simulation.

Contents

Abstract.....	2
Chapter 1: Introduction	5
1.1 Aims.....	5
1.2 Objectives.....	5
1.3 Deliverables.....	6
1.4 Structure of the whole project.....	6
Chapter 2: Background	7
2.1 Radar	7
2.2 Passive tracking.....	9
2.3 GPS	11
2.4 RFID	11
Chapter 3: Ideal passive tracking method & simulation.....	12
3.1 Bearing Calculation	13
3.2 Locating model.....	14
3.3 Tracking Algorithm.....	15
3.4. Ideal 2D Simulation	18
3.41. 2D Static Locating ideal simulation	18
3.42. 2D Linear motion ideal simulation.....	19
3.43. 2D Curve motion ideal simulation	19
3.5. Ideal 3D Simulation	20
3.51. 3D Static Locating ideal simulation	20
3.52. 3D linear motion ideal simulation.....	20
3.53. 3D Curve motion ideal simulation	21
Chapter 4: Realistic passive tracking method & simulation	22
4.1 Noise & Error.....	22
4.2 Realistic 2D Simulation with noise	23
4.21. 2D Static Simulation with noises.....	23
4.22. 2D linear motion simulation with noises	24
4.23. 2D Curve motion simulation with noises.....	25
4.3 Realistic 3D Simulation with noise	26
4.31. 3D Static Locating Simulation with noises	26
4.32. 3D linear motion simulation with noises	27
4.33. 3D Curve motion simulation with noises.....	28

Chapter 5: Discussion, Analysis & Improvements	29
5.1 Accuracy & Monte Carlo simulation	29
5.2 Receivers arrangements analysis	32
5.3 Multiple targets tracking.....	37
5.4 Improvements.....	39
5.41 Least square improvement in MATLAB.....	39
5.42 Moving windows	40
Chapter 6: Future Work	43
6.1 Other error in realistic passive tracking	43
6.2 passive tracking applications in the future	43
Chapter 7: Conclusion & Reflections on Time plan.....	44
References	46

Chapter 1: Introduction

1.1 Aims

The project is aimed at analysing the feasibility and performance of using passive tracking techniques instead of radar to locate and track a target, and it will focus on locating simulation of different motion models and on comprehensive locating analysis base on realistic situation, such as how to improve the accuracy when dealing with the noise, how to locate multiple targets at the same time, etc.

1.2 Objectives

The Objectives can be mainly divided into four sections according to the aims: research and background, ideal locating, realistic locating and performance analysis.

Research & Background

- Understand the development, application and basic operation principle of radar and passive locating techniques.
- Understand the characteristics and difference between different locating techniques
- Learn how to use simulation tool-‘MATLAB 2015a’.
- Research on basic related mathematics, signal and noise processing background.

Ideal locating

- Identify the locating method
- Build the locating model
 - 2D model
 - 3D model
- Design simulation of target tracking both in 2D and 3D by using MATLAB
 - Static
 - Linear motion model
 - Curve motion model

Realistic locating

- Explain how the noise in realistic situation will affect the locating
- Simulate target tracking with noise both in 2D and 3D by MATLAB
- Design a way processing the noise to improve the tracking by using MATLAB
 - Static
 - Linear motion model
 - Curve motion model

Performance analyse & Developments

- Evaluate the tracking accuracy based on the simulation results
- Evaluate the relationship between accuracy and parameters.
- Multiple targets & more receivers cases

1.3 Deliverables

A comprehensive and detailed report including:

- Provide the method how to locate and track target by passive tracking techniques.
- Tracking simulation including static, linear and curve both in 2D and 3D surface.
- Provide design and simulation details with respect to the noise processing.
- Provide accuracy and feasible analysis of passive tracking
- Provide receivers arrangement strategies
- Multiple targets tracking simulation
- Ideas of what could be improved in the passive tracking

1.4 Structure of the whole project

The thesis can be mainly divided into seven chapters:

Chapter 1 is the introduction section of whole project. Aims, objectives and deliverables of this project are stated.

Chapter 2 started with the introduction to radar; then the back ground knowledge, development and theories of operation of passive tracking techniques will be illustrated. Besides, several related location techniques will be described such as GPS and RFID, and comparison between these techniques will be discussed.

Chapter 3 elaborates the complete process of passive tracking, beginning with the target signal received, and then come to how to locate based on the received bearing information, and how to track the whole path of target. The main part of this project—tracking simulation will be presented.

Chapter 4 will introduce the noise that tracking system will face when trying to locate target in realistic situation, and then present the processing simulation when the random noise be added into the bearing.

Chapter 5 will begin with a detailed analyse and discussion based on the simulation results in chapter 3 and 4. Figure out the factors that will have an impact on the locating accuracy, Following this, the ideas of improvement would be discussed. Furthermore, multiple targets tracking cases will be also analysed in this chapter.

Chapter 6 will explain ideas and thinking of future work. Several questions should be thought. For example, is there any potential possibility of improving the accuracy? If there can be a possibility. What are the methods? Are there any errors had been neglected?

Chapter 7 is the conclusion and provide reflections on time plan to check the whole project management.

Chapter 2: Background

2.1 Radar

Radar (comes from 'Radio Detection and Ranging') is one of the most commonly locating technologies that can track target position by the principle of electromagnetic waves reflection. The main developments of radar can be traced in a series of related events from 1922 to 1941. In 1922, Guglielmo Marconi, one of the radio inventors, introduced a new concept about how to use echo of radio wave to detect target. After few years in 1935, Robert Watson-Watt presented a device which can not only send out radio wave but receive the echo, and at that time it is the first true sense of radar in the world. Nowadays, radar can be classified as two main types according to different positioning methods: active radar and passive radar^[1].

Active radar, be equipped with transmitter and receptor can detect target's position by broadcasting electromagnetic wave and receiving echo by reception antennas. Active radar is widely used in many areas. For example, AESA (Active Electronically Scanned Array) technology take advantage of the feature that active radar can make every single irradiator has the combination of emission and reception. AESA is widely used in many military airborne radar systems such as F15, F16 Fighting Falcon.^[2]

When the echo is received, the distance between target and active radar can be calculated by a simple mechanism (see fig. 1): The RF wave speed is equal to light speed which is known; the time spent during the whole trip from emission to target then come back to receptor is determined; the distance to the object can be found by the following equation^[3]:

*Distance= the time delay between the emission of the signal and its reception*light speed /2*

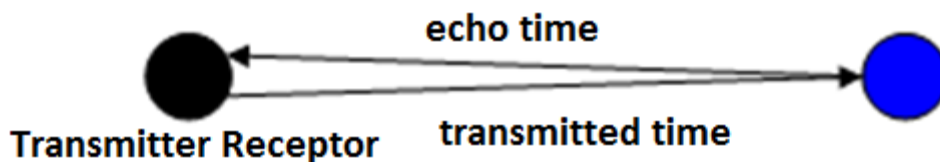


Figure 2.11 Active radar mechanisms^[3]

Passive radar

Unlike active radar, passive radar receive the signal from transmitter in different location rather than emit signal itself(also called as bistatic radar because emit antenna and receive antenna are serparate). Since passive radar has no active transmitter,it is relatively low cost and convenient, furthermore, it will not reveal itself or be detected when working, these advantages make it very popular in investigative applications . The following figure shows the schematic of simple passive radar system, if the detection range is too large to cover, bistatic(one transmitter+one receptor) can be changed to multistatic(more receptors are used) to improve the locating accuracy.

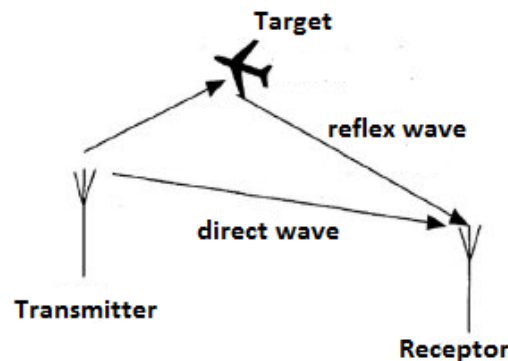


Figure 2.12 passive radar mechanisms ^[3]

The history of passive radar can be dated back to 1935 when British scientist Robert Watson-Watt tried to use passive system (monostatic receptor only), with the shortwave from BBC transmitter, to detect a target 'Handley Page Heyford bomber' 12km away. During World War II-era, Germany locating system 'Kleine Heidelberg' tried to use passive radar to detect but failed to get accurate location because the processing ability was deficient at that time^[4].

Currently, there are many countries that very keen to investigate and implement the technique of passive radar. For example, the 'silent sentry' passive detecting system of Lockheed Martin company in American^[5]. This kind of passive radar uses continuous wave signal between 50 MHz and 80 MHz, which are launching by commercial FM radio station and Television, to detect, track and monitor the moving target in the region. Its tracking range can reach 180km for target with 10.2m reflection area, the tracking range can by extended to 220km after improvement. Moreover, it can track at least 200 targets at the same time and has about 15m resolution range separation.



Figure 2.13 Silent sentry passive detecting system ^[5]

2.2 Passive tracking

What passive tracking and passive radar share in common is that they only receive signals rather than sending any radio waves, so it ensures a good invisibility. Besides, passive tracking can intercept radar emission; therefore it is frequently used for acquiring intelligence for the military as one method of anti-investigation: develop corresponding coping strategy by locating the space position of signal source.

Currently, there are three main passive tracking methods: ^[6]:

Angle of arrival (AOA) measurement:

This is primary locating method used in the project, if one knows the distance between two receivers and received the target signal bearing information, the relative location between target and receivers can be determined by triangulation calculation, more details will be provided in Chapter 3.

Known the distance L between two receivers, once two bearing angle information are received, the position of target can be estimated by triangulation calculation:

$$R1 = \frac{L * \sin\theta_2}{\sin(\theta_2 - \theta_1)}, \quad R2 = \frac{L * \sin\theta_1}{\sin(\theta_2 - \theta_1)}$$

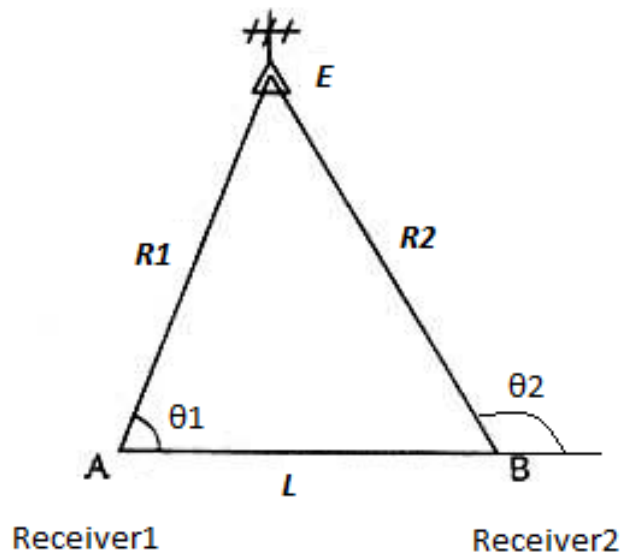


Figure 2.21 the schematic of AOA

Time Difference of Arrival (TDOA) measurement:

It is generally consists of a main station and several secondary stations. The observatory will measure the time of arrival of the emitter signal and then transmits it to the main station. Following this, the time difference of the arrival of the signal to observatory station will be calculated by the main station. The spatial position of the target can be defined by gathering all the results of time differences.

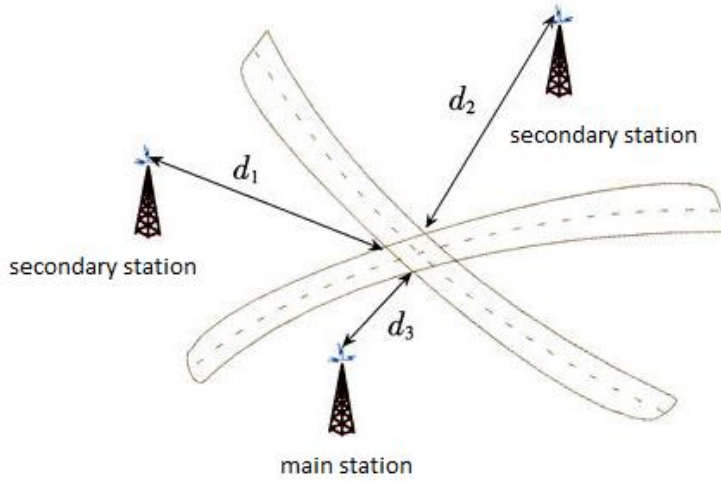


Figure 2.22 the schematic of TDOA ^[6]

AOA and TDOA measurement:

The main station measures the time difference between signal arrivals to the main station and transmission from secondary station to main station, meanwhile azimuth of the target can be measured. Then the distance between targets and receivers can be determined. For example, t_d is the **time difference** between signal arrivals to the main station and signal transmitted from B to A, and bearing angle θ is measured, the location can be worked out as:

$$ct_d = r + L - R \quad R = \frac{ct_d(1 - \frac{ct_d}{2})}{ct_d - L(1 - \cos\theta)}$$

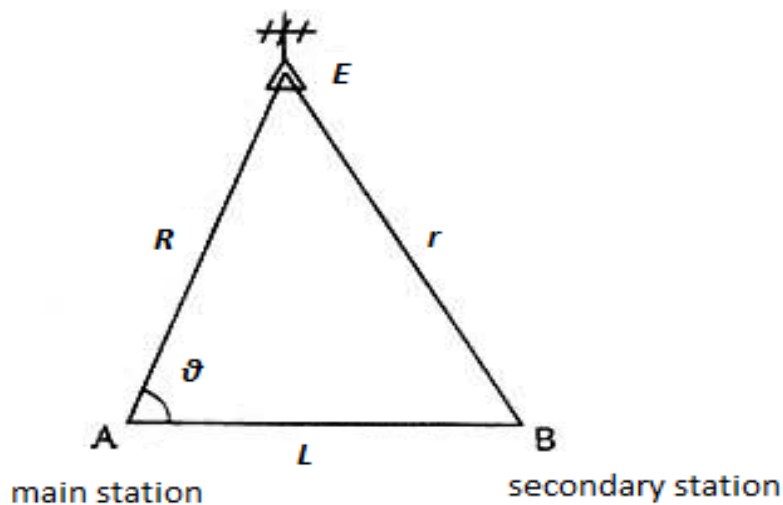


Figure 2.23 the schematic of combination of AOA and TDOA combination

2.3 GPS

Nowadays, GPS (Global Positioning System) is a very popular locating technique with space-based navigation system. Comparing with radar techniques, GPS uses **satellite** in the space rather than antennas in the ground to transmit signal; the operation system is a **passive tracking** system.

General principles of GPS including firstly it consists of 24 working satellites, which ensures at least four satellites can be observed wherever and whenever around the world. The distance between satellites that the location is already known and receiver is measured, then the location of receiver can be found by integrating data from satellites. It is highly inefficient to search satellites in the context of the whole sky, so it will take several minutes for the initial start-up when using GPS for locating. That is the reason why a large circle will appear first when opening a digital map application, a precise point is then positioned. However, if a rough range of location is known before using the GPS, the time takes for searching satellites can be reduced to a great extent ^[7].

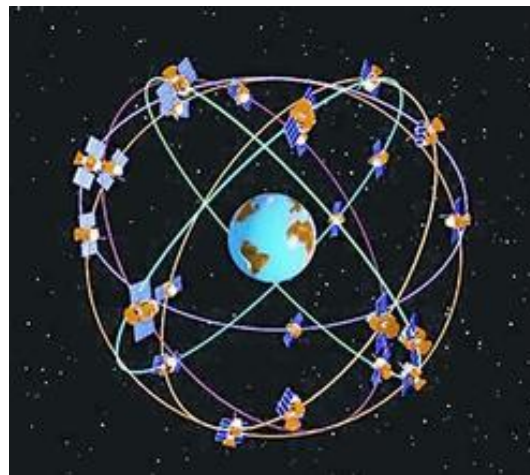


Figure 2.3 the sketch of GPS

2.4 RFID

RFID (Radio Frequency Identification) is a communication technology using radio to identify and track target. Attach the tag with electronic information on target, receiver can read all those electronic information by radio waves. It is widely used in logistical management such as production tracking, package tracking, etc. Unlike radar and GPS, RFID has a higher level of accuracy but the range of coverage is limited, so it is suitable for indoor locating ^[8].

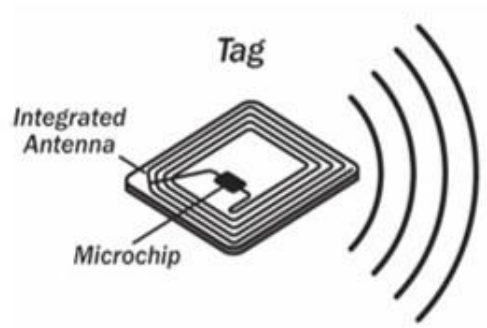


Figure 2.4 RFID tag schematic ^[8]

Chapter 3: Ideal passive tracking method & simulation

The overall process of passive tracking can be summarized as the following flowchart: For example, in a classic locating model (two receivers and one target), once the target signal is received, bearing information is determined, hence the relative location of target can be worked out by locating method which is triangulation. To track the target, appropriate tracking algorithm should be used for processing the real time location (evaluated the location before each iteration of the loop).

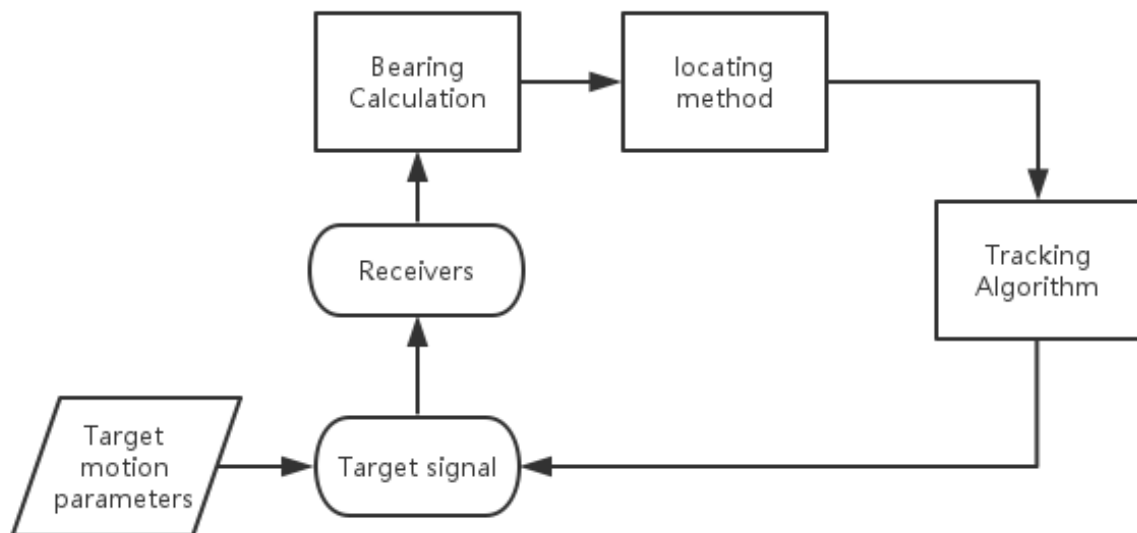


Figure 3.0 Flowchart of passive tracking

Since the target signal and parameters have been already decided, the processes behind are very important for passive tracking. In other words, the key of the passive tracking consist of three main parts as the flowchart above shows: **Bearing calculation**, **locating method**, and **tracking algorithm**.

Note that the Receivers arrangement strategies can also have significant impacts on the tracking performance, and would be discussed in the Chapter 5.

3.1 Bearing Calculation

Location of an object is not only consisting of one parameter, but consisting of more dimensions. There is Horizontal axis and vertical axis in 2D dimension and for 3D dimension there is one more coordinate for height. For tracking a moving target, time is also needed to be a considered as a variable, which make it 4D dimension (tracking algorithm will be discussed in the following section). Bearing includes that coordinate information. So once the bearing information is received, the coordinate of target can be determined by resolution of bearing (vector) and the known distance between receivers. For example in 2D surface, decompose the position vector 'v' as:

$$x = v \cdot \sin(30^\circ) \quad y = v \cdot \cos(30^\circ)$$

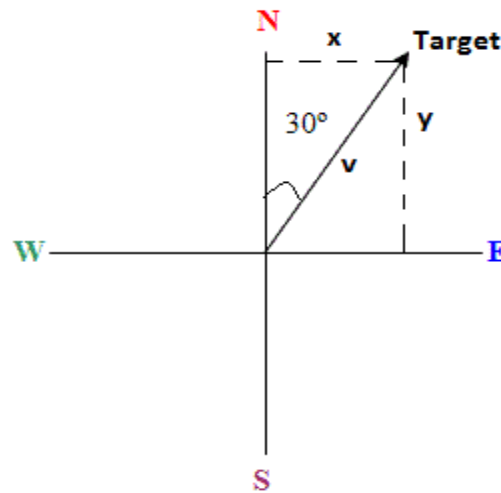


Figure 3.11 2D vector resolutions

In 3D surface, the coordinates of target can be determined by using two bearing angle as follows:

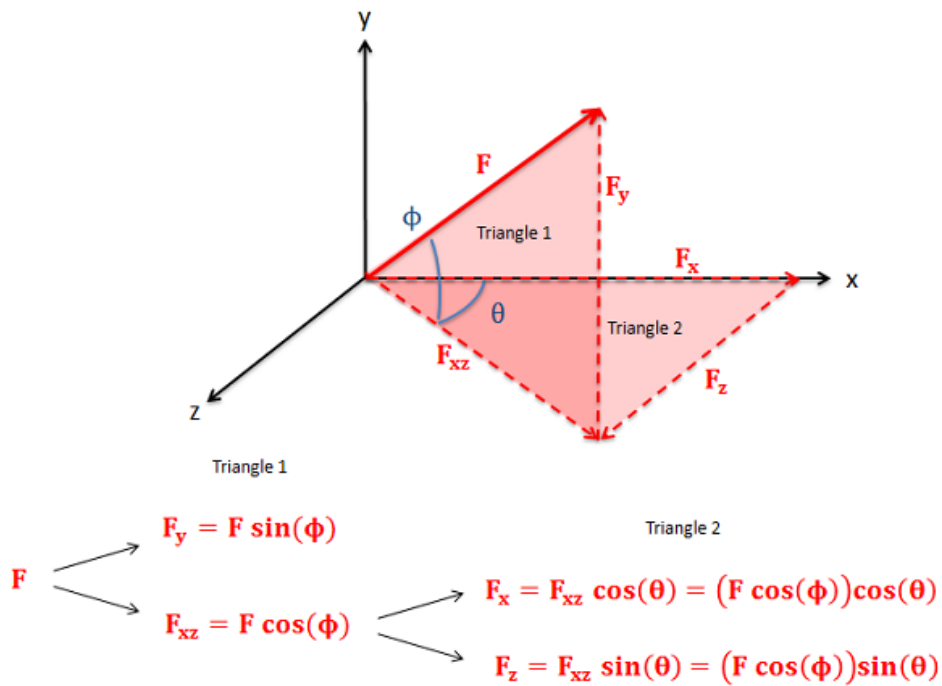


Figure 3.12 3D vector resolutions^[9]

3.2 Locating model

According to the discussion in last section, the coordinate of target can be determined by vector resolution, and the calculation depend on the vector 'v' is known. However, in passive tracking practice, the information of vector 'v' is unknown and the only known information is: direction relationship between target and receivers, and exact position of receivers. Now the locate model can be changed to the figure on the right:

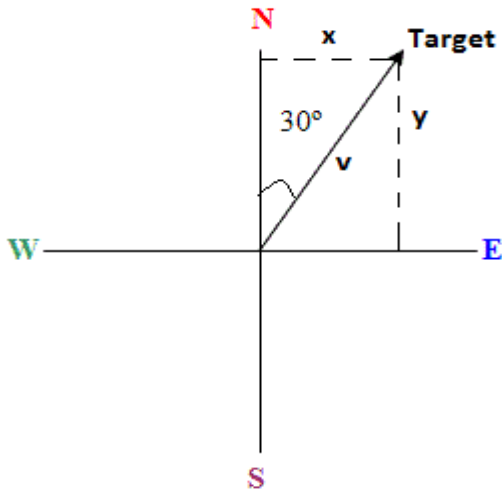


Figure 3.21 2D vector resolutions

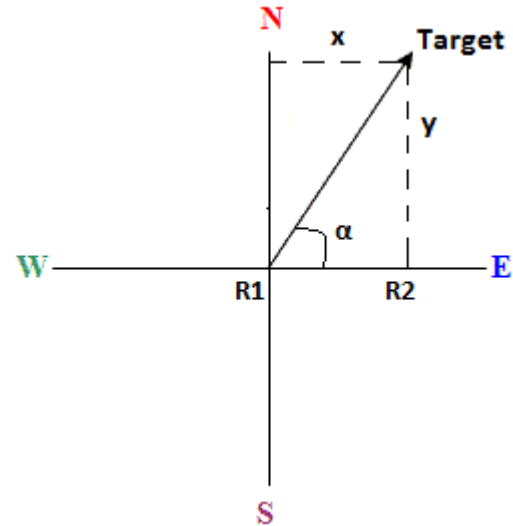


Figure 3.22 rewrite 2D vector resolutions

Assume that the coordinate: Receiver (X_i, Y_i) and Target (X_t, Y_t).

According to the plane-coordinate system^[10]: **slope of the line** = $\tan(\alpha) = \frac{Y_t - Y_i}{X_t - X_i}$

So the coordinate of target can be easily calculated based on the bearing angle α .

Similar in 3D space rectangular coordinates^[9]: only need to introduce another bearing angle β .

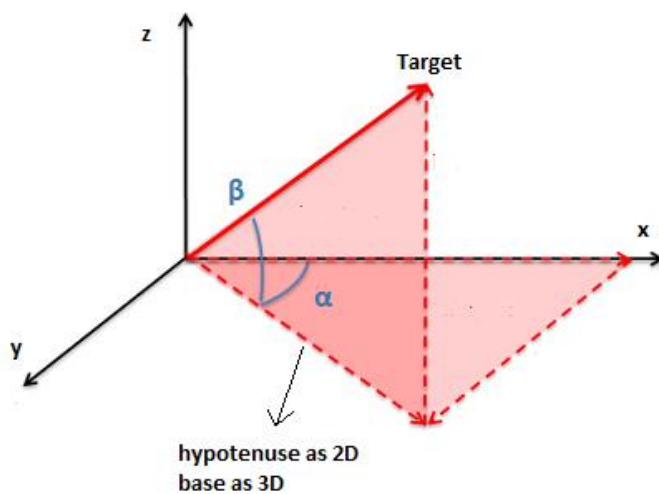


Figure 3.23 rewrite 3D vector resolutions

Receiver (X_i, Y_i, Z_i) and Target (X_t, Y_t, Z_t).

$$\tan(\alpha) = \frac{Y_t - Y_i}{X_t - X_i}$$

$$\tan(\beta) = \frac{Z_t - Z_i}{\sqrt{(X_t - X_i)^2 + (Y_t - Y_i)^2}}$$

3.3 Tracking Algorithm

In the previous section, target can be located by determining the coordinate using triangulation.

But for the moving target, locating system need to process continuous signals by applying tracking algorithm. Furthermore, sometime the measured signal will mix with noise which has impact on bearing accuracy, so appropriate statistical method need to be applied to locating calculation. It can be assumed as different mathematic model based on the relationship between locating system parameter. Select the combination of data series from signal source and receiver position information as the input of system mathematic model, appropriate data processing technique like least square can be applied to find the optimal estimated location of target. ^[11].

Estimating the variable by making the sum of error squares minimum, **least square** is a very effective way to determine variables, and it is well suited for tracking such a plot of signals measurements with noises, so it is the main algorithm used in this project.

According to the matrix principle of least square ^[12]:

Matrix form $\mathbf{AX} = \mathbf{F}$

$$\rightarrow A^T \mathbf{AX} = A^T \mathbf{F} \quad (\text{Where } \mathbf{X} \text{ is the optimal solution need to be found})$$

$$\rightarrow \mathbf{X} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{F}$$

In this project, based on the model in previous section, the corresponding least square parameters will be:

$$\begin{aligned} \tan(\alpha) &= \frac{Y_t - Y_i}{X_t - X_i} \rightarrow \tan(\alpha) X_t = Y_t - Y_i + \tan(\alpha) X_i \\ &\rightarrow \tan(\alpha) X_t - Y_t = -Y_i + \tan(\alpha) X_i \\ &\quad \text{Replace unknown data } x_t, y_t \\ (\tan(\alpha), -1) * \begin{pmatrix} X_t \\ Y_t \end{pmatrix} &= \tan(\alpha) X_t - Y_t = \tan(\alpha) X_i - Y_i \\ &\quad \text{matrix } \mathbf{A} \quad \text{matrix } \mathbf{X} \quad \mathbf{F} \\ &\quad \mathbf{AX} = \mathbf{F} \\ &\quad \text{'Linearization' to process locating signal} \end{aligned}$$

In order to make matrix calculation valid, the bearing $\tan(\alpha) = \frac{Y_t - Y_i}{X_t - X_i}$ will be calculated twice to get two equations for least square (since two receivers bearing information are needed), for example, known coordinates of two receivers (x_i, y_i) are (-100,-100) (100,100), target (x_t, y_t) and the bearing angle alpha, so MATLAB simulation program design will be:

`xi = [-100 100]; yi = [-100 100];`

For `ii=1: length (xi)`; `{alpha(ii) = atan((yt-yi(ii))/(xt-xi(ii)))}` then output the bearing information alpha to matrix A. (length command returns the length of the largest array dimension in xi)

Similarly, for example in 3D locating model, assume that the known coordinate of two receivers: $S_1(x_1, y_1, z_1)$, $S_2(x_2, y_2, z_2)$; received the bearing information: azimuth angle ϕ_1, ϕ_2 and pitching angle $\varepsilon_1, \varepsilon_2$. For the **target**(x, y, z) Locating model will be built easily as follow:

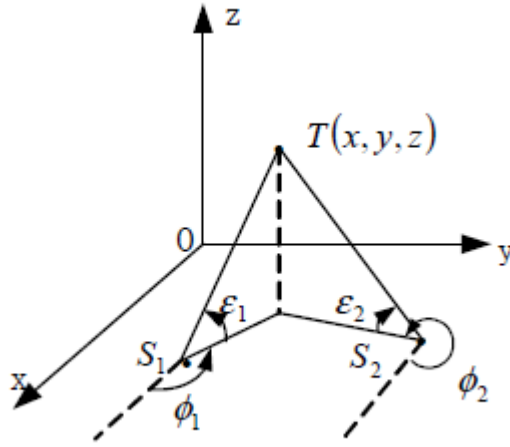


Figure 3.31 the schematic of 3D model passive tracking

Based on the triangulation relationship derived from previous chapter, using $\phi_1, \varepsilon_1, \phi_2$ (can also use other combinations) equations can be easily found:

$$\operatorname{tg} \phi_1 = \frac{y - y_1}{x - x_1} \quad \operatorname{tg} \varepsilon_1 = \frac{z - z_1}{y - y_1} \cdot \sin \phi_1 \quad \operatorname{tg} \phi_2 = \frac{y - y_2}{x - x_2}$$

Reformulate equations:

Rewrite in matrix form

$$\begin{cases} -x \cdot \operatorname{tg} \phi_1 + y = y_1 - x_1 \cdot \operatorname{tg} \phi_1 \\ y \cdot \operatorname{tg} \varepsilon_1 - z \cdot \sin \phi_1 = y_1 \cdot \operatorname{tg} \varepsilon_1 - z_1 \cdot \sin \phi_1 \\ -x \cdot \operatorname{tg} \phi_2 + y = y_2 - x_2 \cdot \operatorname{tg} \phi_2 \end{cases} \longrightarrow \begin{bmatrix} -\operatorname{tg} \phi_1 & 1 & 0 \\ 0 & \operatorname{tg} \varepsilon_1 & -\sin \phi_1 \\ -\operatorname{tg} \phi_2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 - x_1 \cdot \operatorname{tg} \phi_1 \\ y_1 \cdot \operatorname{tg} \varepsilon_1 - z_1 \cdot \sin \phi_1 \\ y_2 - x_2 \cdot \operatorname{tg} \phi_2 \end{bmatrix}$$

Back to the least square matrix form $\mathbf{AX} = \mathbf{F}$ where

$$\mathbf{A} = \begin{bmatrix} -\operatorname{tg} \phi_1 & 1 & 0 \\ 0 & \operatorname{tg} \varepsilon_1 & -\sin \phi_1 \\ -\operatorname{tg} \phi_2 & 1 & 0 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} y_1 - x_1 \cdot \operatorname{tg} \phi_1 \\ y_1 \cdot \operatorname{tg} \varepsilon_1 - z_1 \cdot \sin \phi_1 \\ y_2 - x_2 \cdot \operatorname{tg} \phi_2 \end{bmatrix}$$

Now, the 3D coordinate of target (X matrix) can be found by the way as mentioned before, the locating performance will depend on the bearing information (in this case $\phi_1, \varepsilon_1, \phi_2$ are used).

According to the derivation above, the target coordinate X_t Y_t (matrix X) can be easily found by matrix relationship through MATLAB. The whole tracking method and algorithm above can be implemented by MATLAB, the main programming flowchart as follows:

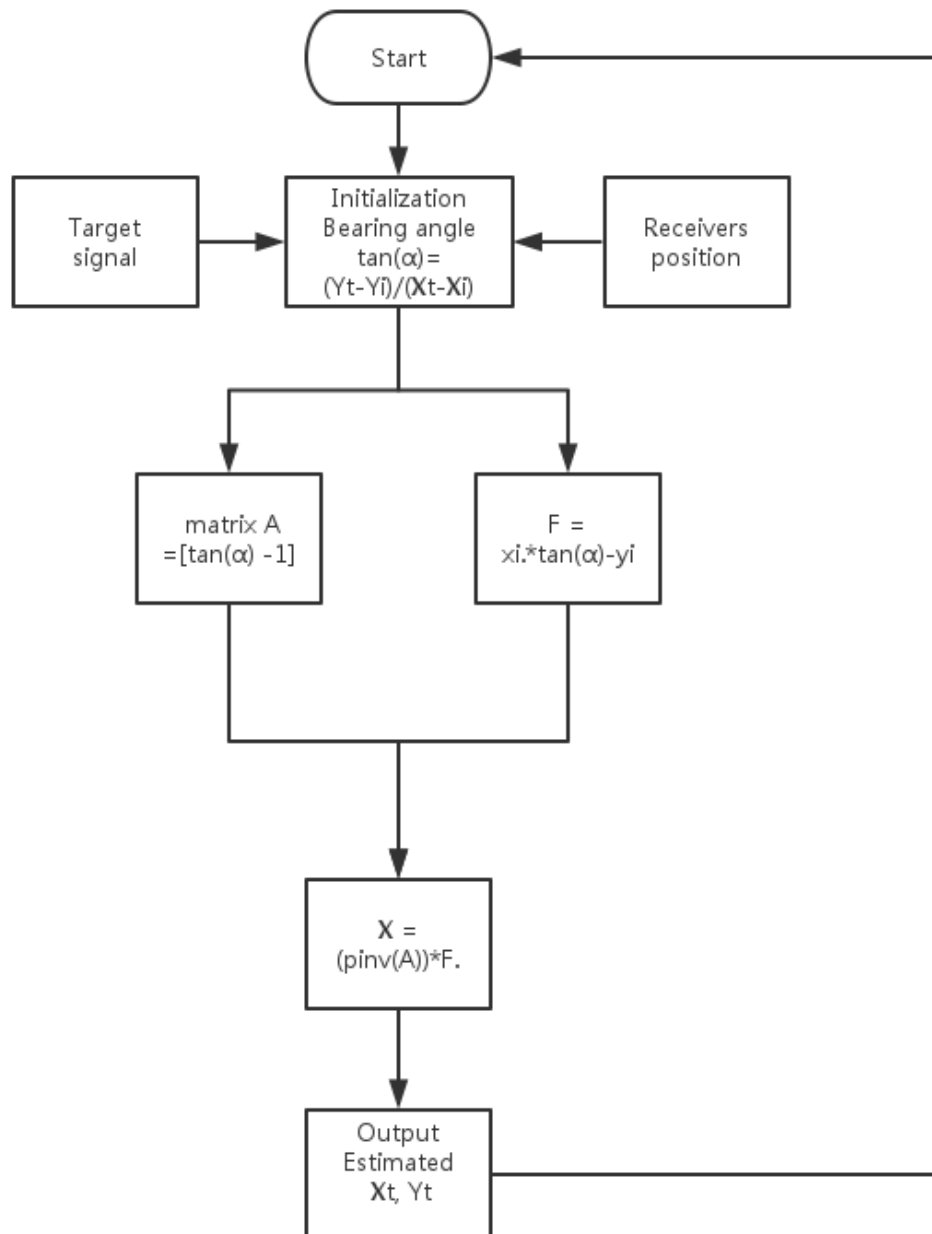


Figure 3.32 flowchart of tracking algorithm least square

3.4. Ideal 2D Simulation

3.41. 2D Static Locating ideal simulation

In order to investigate the feasibility of method and algorithm above for passive tracking, now run the target locating simulation by MATLAB without any noise and interference, if the method is exact, estimated location by least square should be precisely the same as the original target.

Static target parameter: Target coordinate: $x_t = 30$; $y_t = 50$;

Two receiver coordinates: $(-100, 100)$ and $(100, 100)$, sample by 100s:

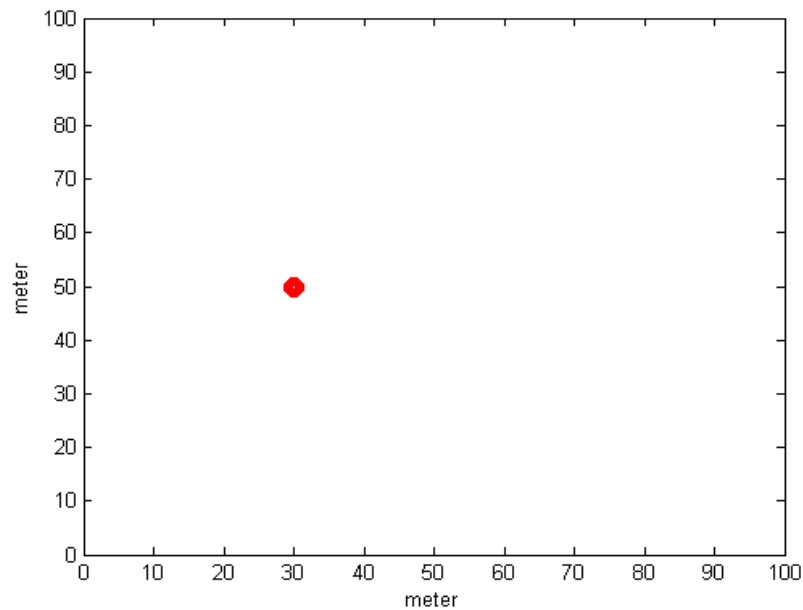


Figure 3.41 static target

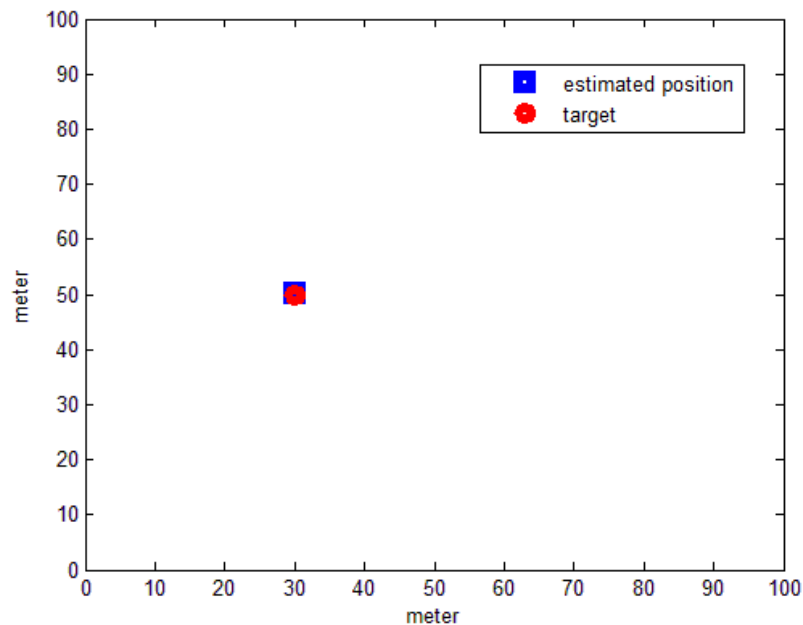


Figure 3.42 static target locating simulation

As the figures above show: without any noise and interference, estimated location is exactly the same as original target location.

3.42. 2D Linear motion ideal simulation

Parameter: Target coordinate: $x_t = 30 + 0.5 \cdot t$; $y_t = 50 + 0.5 \cdot t$;

Two receiver coordinates: (-100, 100) and (100, 100);

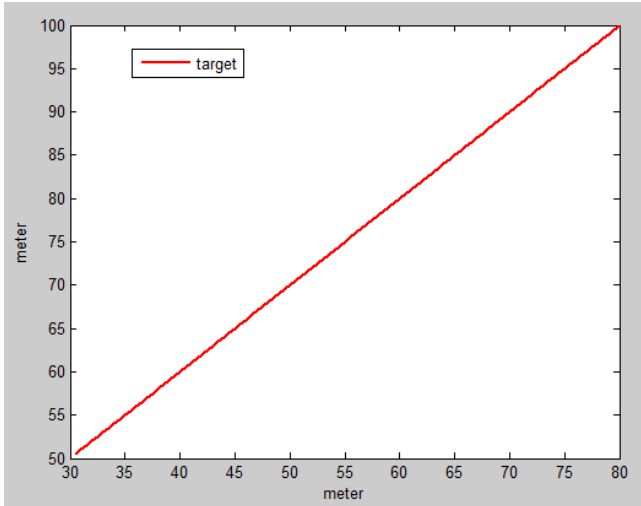


Figure 3.43 linear motion target

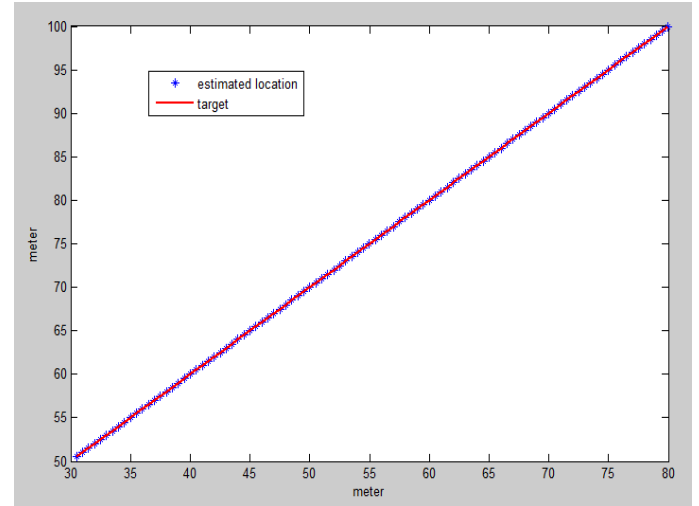


Figure 3.44 linear motion target locating simulation

3.43. 2D Curve motion ideal simulation

Parameter: Target coordinate: $x_t = 30 + 30 \cdot \cos(\phi(t))$; $y_t = 30 + 30 \cdot \sin(\phi(t))$

(cosd and sind returns the cosine and sine (expressed in degrees) of the $\phi(t)$ elements)

$\phi = \text{linspace}(30, 60, 100)$; Two receiver coordinates: (-100, 100) and (100, 100);

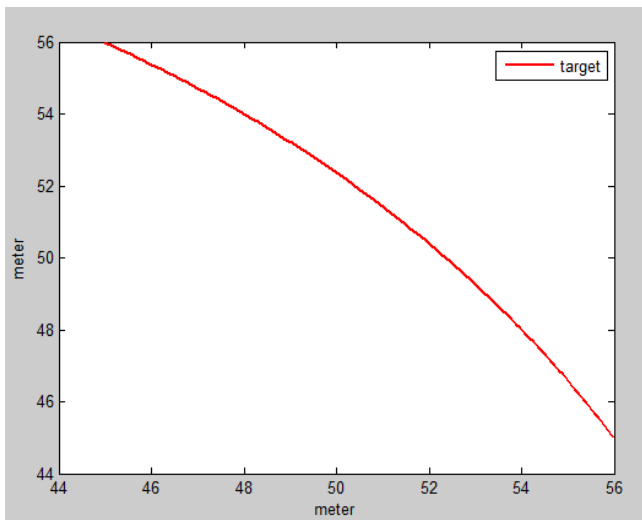


Figure 3.45 curve motion target

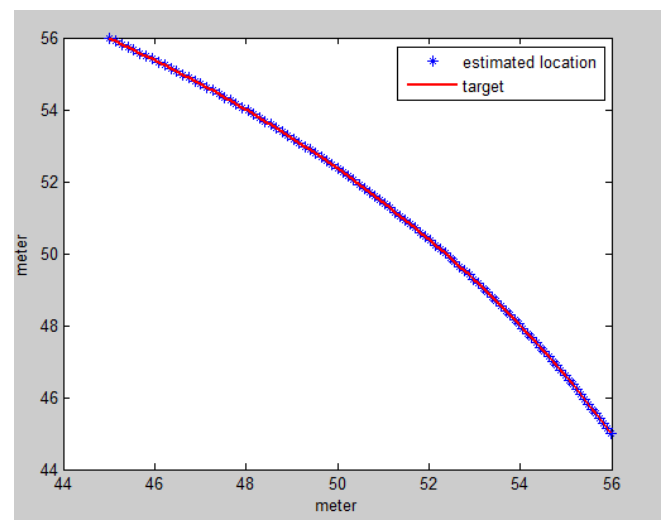


Figure 3.46 curve motion target locating simulation

As the figures above show: estimated track is exactly the same as original target track.

3.5. Ideal 3D Simulation

3.51. 3D Static Locating ideal simulation

Target: $x_t = 30$; $y_t = 50$; $z_t = 50$;

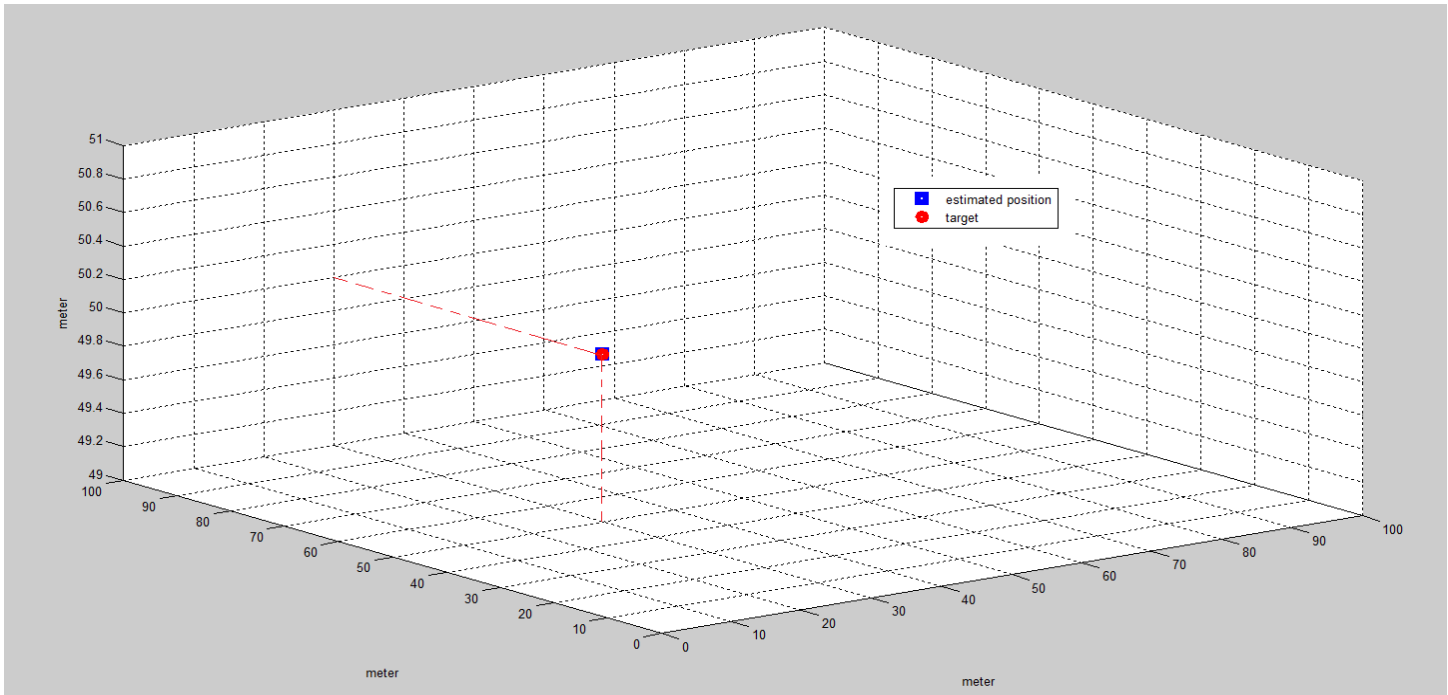


Figure 3.51 3D Static Locating ideal simulation

As the figures above show: without any noise estimated location is exactly the same as original target location in 3D simulation as well.

3.52. 3D linear motion ideal simulation

Target: $x_t = 30 + 0.5 \cdot t$; $y_t = 50 + 0.5 \cdot t$; $z_t = 50 + 0.5 \cdot t$;

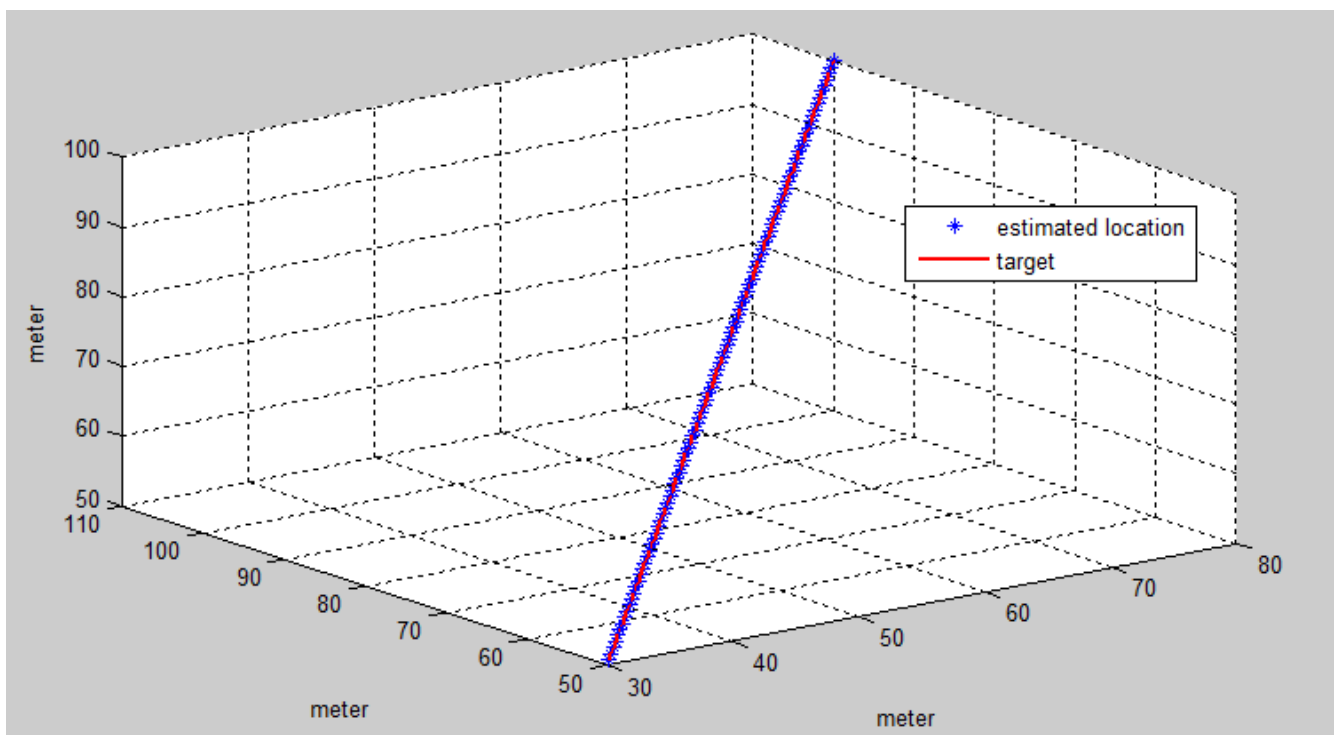


Figure 3.52 3D linear Locating ideal simulation

3.53. 3D Curve motion ideal simulation

Target: $x_t = 30 \cdot \cos(\phi(t))$; $y_t = 30 \cdot \sin(\phi(t))$; $z_t = 50 \cdot \cos(\phi(t))$;

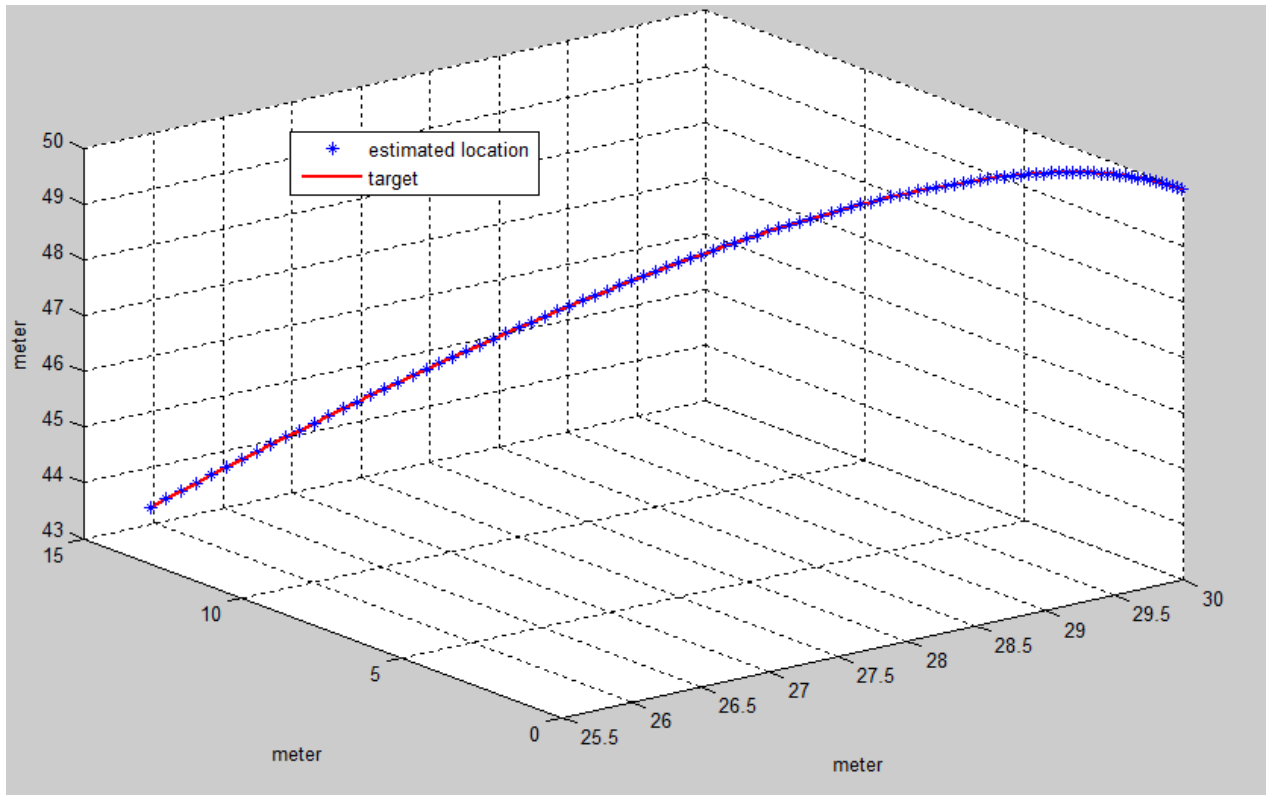


Figure 3.53 3D Curve motion ideal simulation

As the figures above show: all the estimated positions (blue trace) are same as the target (red trace). In ideal locating, if the motion model is exact without any noises and errors, least squares should make no difference to the data, because **the differences between measured data and estimated data have already been minimized.**

Therefore, this simulation demonstrated that in an ideal situation (Assume that signal source is accurate and stable, systematic error, noise or other errors are negligible), the passive tracking method and least square estimations algorithm can locate and track the target without any error. To evaluate the feasibility of applying passive tracking in real life, the most common interference—noise, will be introduced to the locating system in next chapter.

Chapter 4: Realistic passive tracking method & simulation

4.1 Noise & Error

In realistic locating, measurement errors are usually caused by the presence of noises, these noises can be considered as Gaussian in the project. Gaussian noise is statistical noise with same probability density function (PDF) as the Gaussian distribution (normal distribution)^[13].

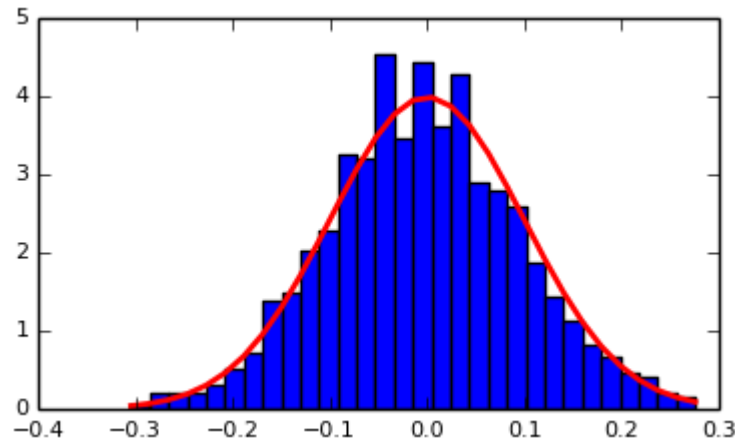


Figure 4.11 Gauss noise distribution

As the example figure above shows: noise satisfy Gaussian distribution, $P(n>0)=P(n<0)$, and the mean value of noise equal to 0.

As the discussion in chapter 3, adding noise into target location need to make sure the targets' entire coordinate will be affected by noise, a possible way to simulate this is adding gauss noise into bearing angle, which is the triangulation locating angle: $\tan(\alpha) = \frac{Y_t - Y_i}{X_t - X_i}$ now need to be changed to $\tan(\alpha) + X * \text{randn}(1, N)$ in the MATLAB^[14], where X is the noise intensity (generally set to 1%).

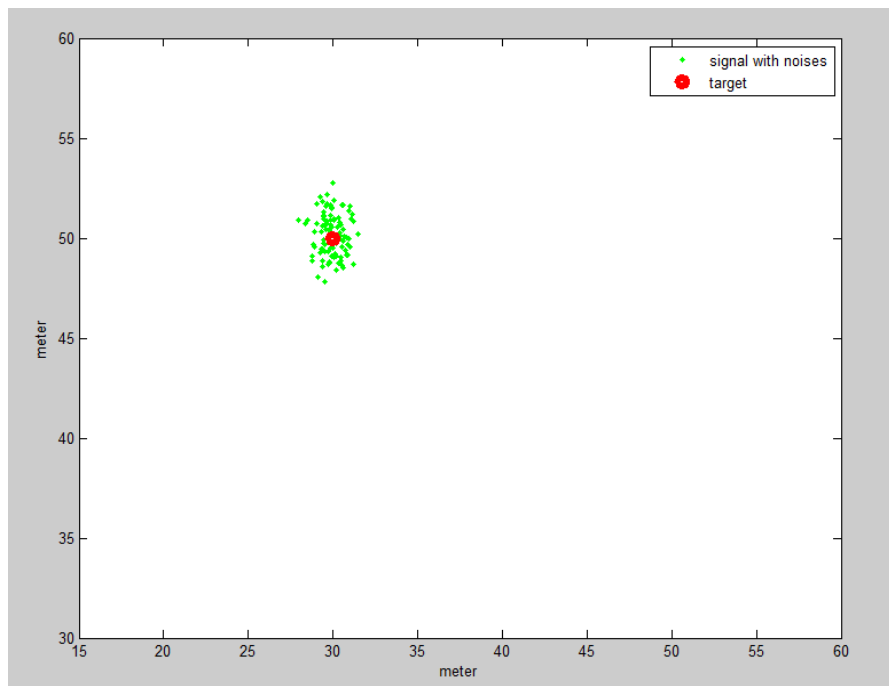


Figure 4.12 static target with noises

4.2 Realistic 2D Simulation with noise

4.21. 2D Static Simulation with noises

The target parameters and receivers arrangement are same as the ideal simulation in the previous chapter:

Target coordinate: $x_t = 30$; $y_t = 50$; sample by 100s, bearing angle $+0.01\text{randn}(1,N)$

Two receiver coordinates: $(-100, 100)$ and $(100, 100)$;

The locating method and algorithm, the only difference is random noises have added to bearing angle, now check the measured signal and apply least square for the data:

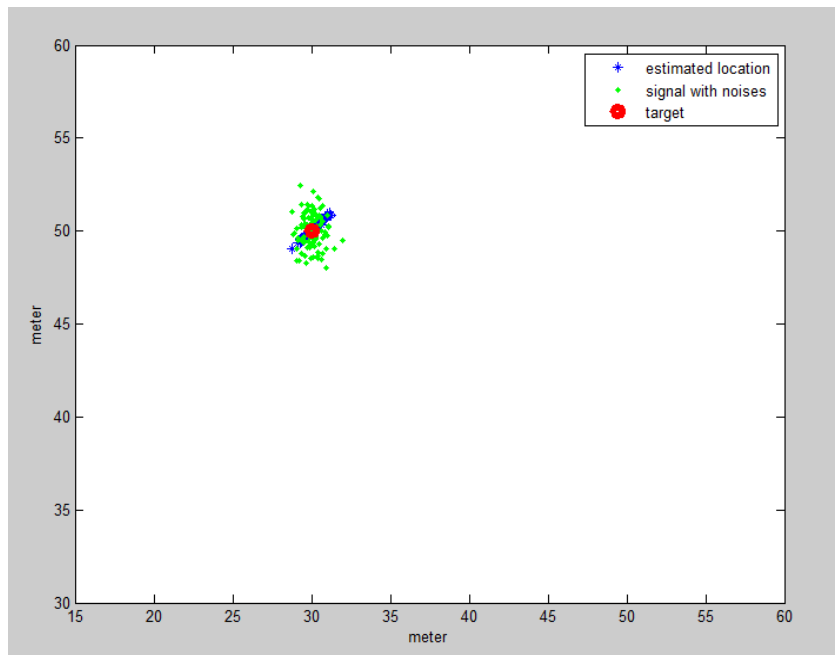


Figure 4.21 2D Static locating simulation with noises

As the figure above shows: after the noises added into bearing angle, measured locations of target distribute randomly (green points) with centre at original location (red point), the vertical error is about $\pm 2.5/50 = \pm 5\%$ and the horizontal error is about $\pm 1.5/30 = \pm 5\%$ (RMS error will be discussed in detail later).

After fitting data by least square, since data with smallest sum of square error are selected, the locating accuracy is increased visibly, and locating errors are reduced correspondingly (as the blue points in the figure). It can be also observed from the figure that the distribution of estimated location is linear, this is because the model used for least square is a linear model (See Chapter 3.3, matrix $AX=F$).

Introduce noise into linear motion bearing angle ($\alpha + 0.01 \text{randn}(1, N)$) and track the motion.

Parameter: Target coordinate: $x_t = 30 + 0.5t$; $y_t = 50 + 0.5t$;

Two receiver coordinates: $(-100, 100)$ and $(100, 100)$;

4.22. 2D linear motion simulation with noises

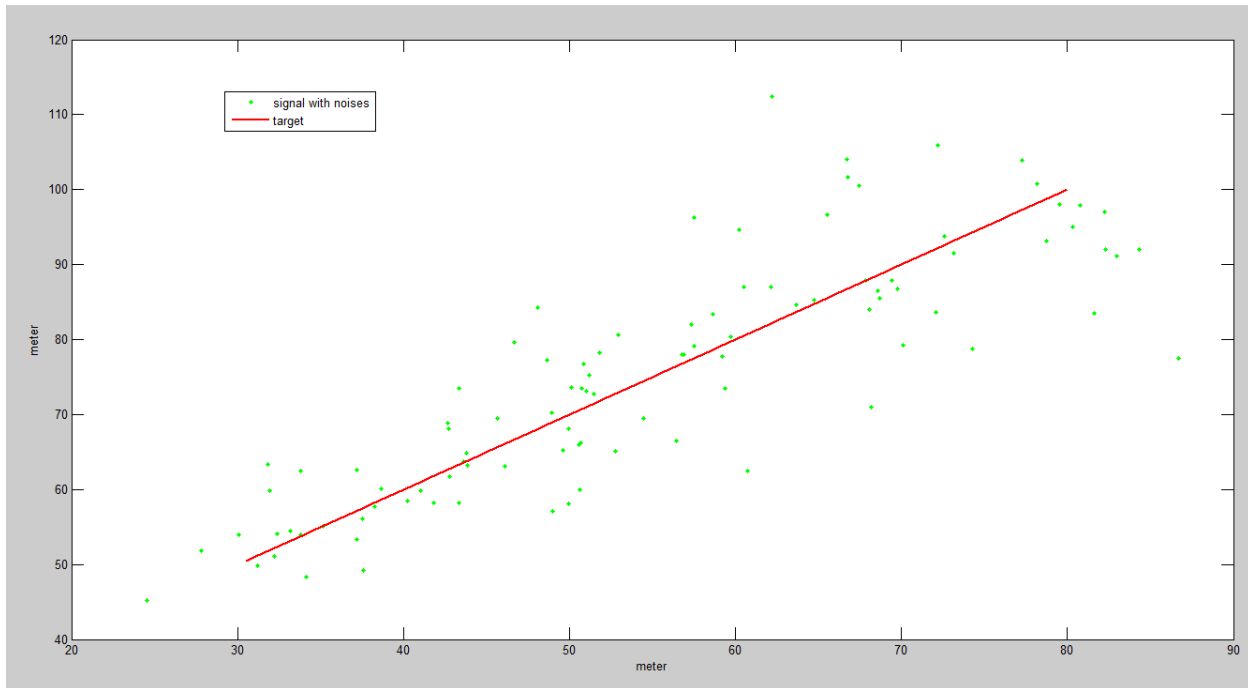


Figure 4.221 2D linear motion with noises

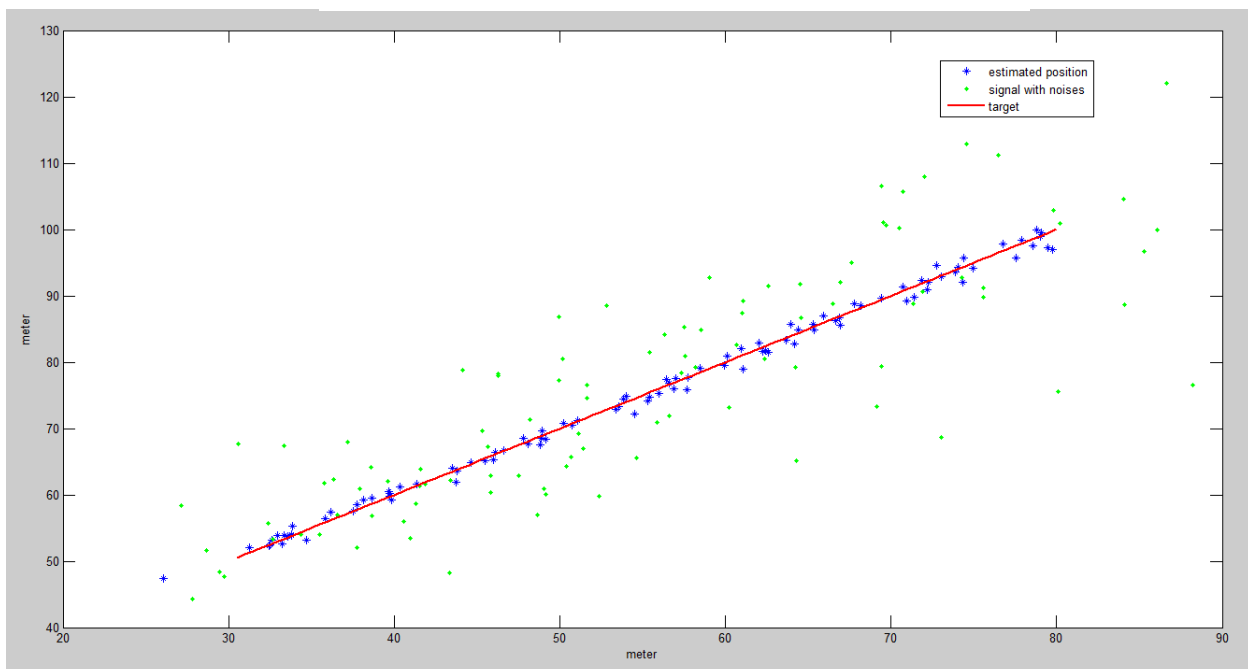


Figure 4.222 2D linear motion tracking simulation with noises

As the figure 4.222 shows: estimated position by least square reduces the error range, improve the locating accuracy (blue trace is essentially coincident with target original trace)

Parameter: Target coordinate: $x_t = 30 + 30 \cdot \cos(\phi(t))$; $y_t = 30 + 30 \cdot \sin(\phi(t))$

(cosd and sind returns the cosine and sine (expressed in degrees) of the $\phi(t)$ elements)

$\phi = \text{linspace}(30, 60, 100)$; Two receiver coordinates: (-100, 100) and (100, 100);

4.23. 2D Curve motion simulation with noises

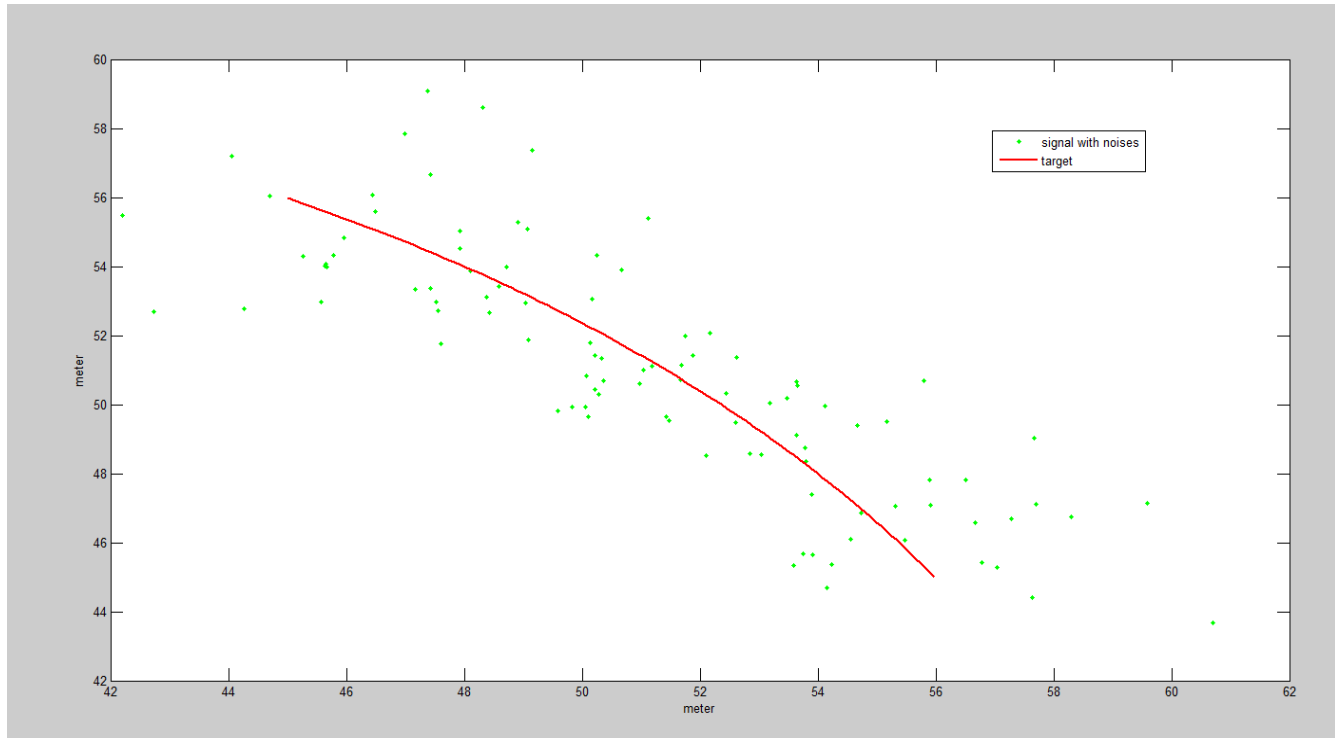


Figure 4.231 2D curve motion with noises

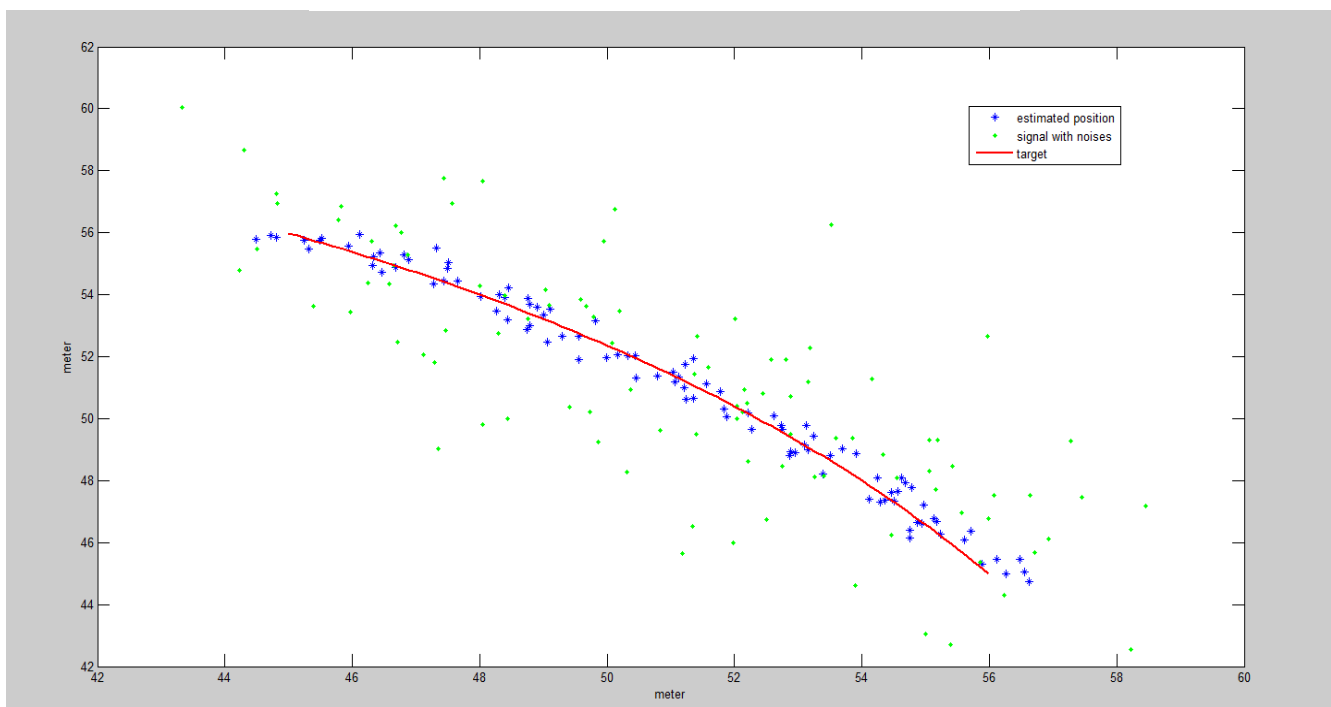


Figure 4.232 2D curve motion tracking simulation with noises

After least square, locating accuracy is highly improved as the blue trace shows.

4.3 Realistic 3D Simulation with noise

4.31. 3D Static Locating Simulation with noises

Target: $x_t = 30$; $y_t = 50$; $z_t = 50$; sample by 100s, both two bearing angles $+0.01\text{randn}(1,N)$

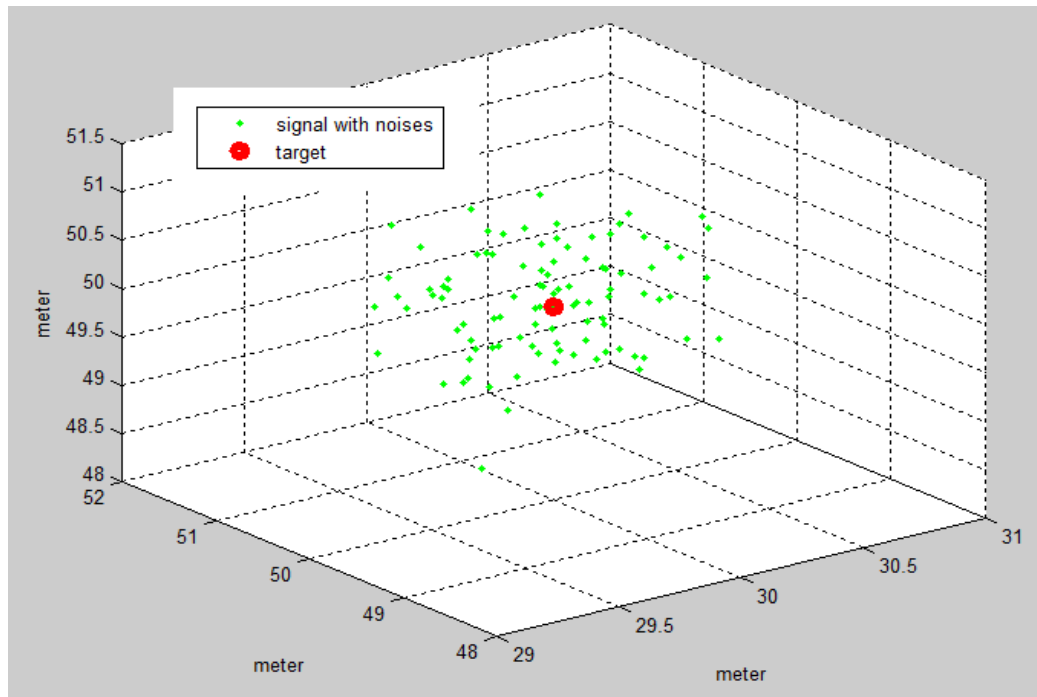


Figure 4.311 3D Static target with noises

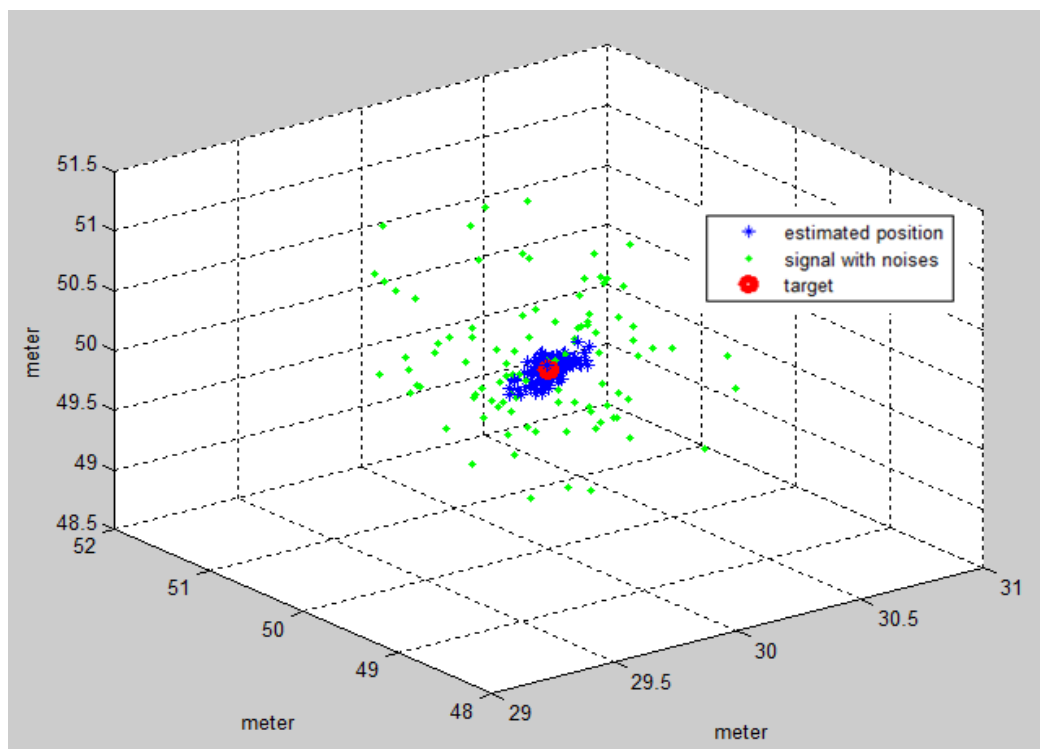


Figure 4.312 3D Static locating simulation with noises

Similar to 2D simulation, as the blue trace, least square reduce the noise range significantly.

4.32. 3D linear motion simulation with noises

Target: $x_t = 30 + 0.5 \cdot t$; $y_t = 50 + 0.5 \cdot t$; $z_t = 50 + 0.5 \cdot t$

Sample by 100s, both two bearing angles $+0.01 \cdot \text{randn}(1, N)$

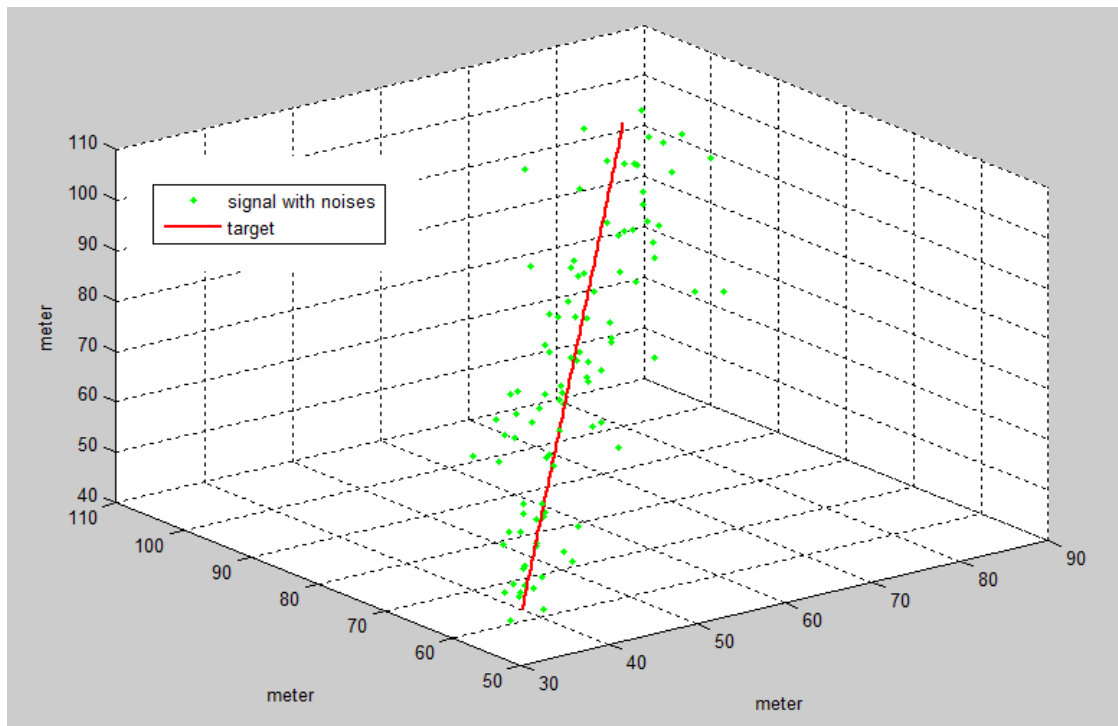


Figure 4.321 3D linear motion with noises

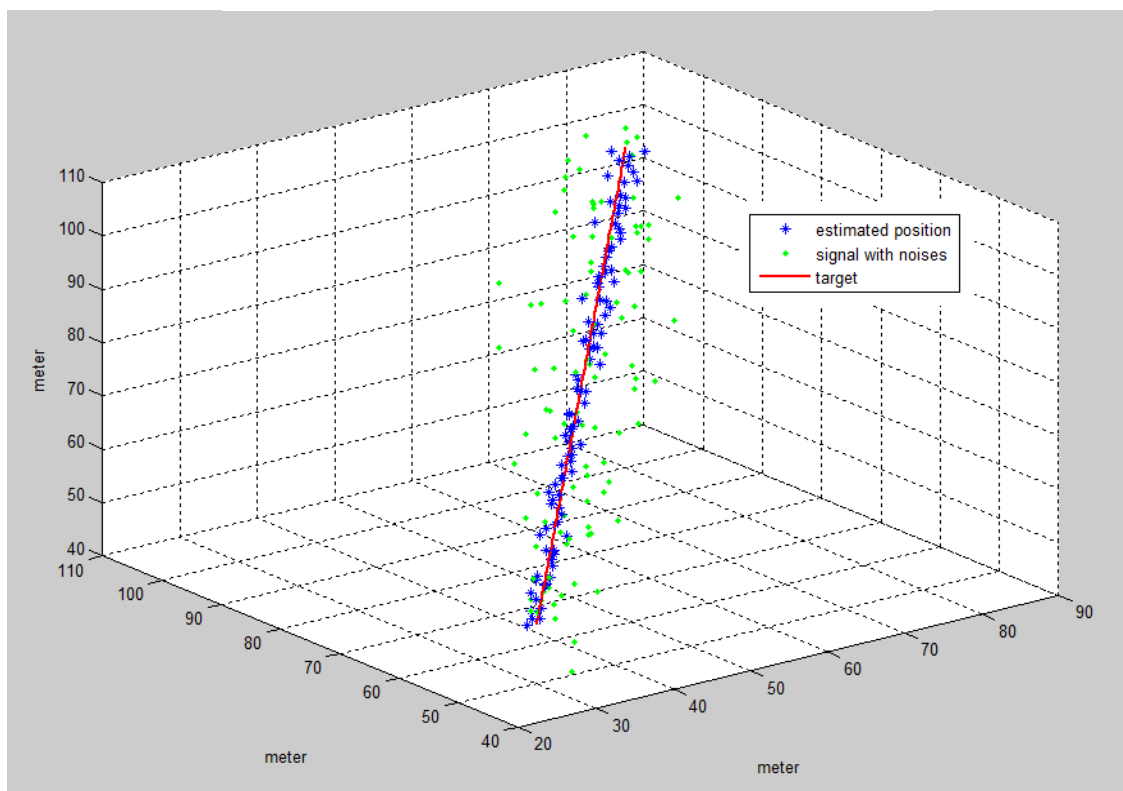


Figure 4.322 3D linear motion tracking simulation with noises

As the figure above: estimated track using least square (blue) is close to original target track (red).

4.33. 3D Curve motion simulation with noises

Target: $x_t = 30 \cdot \cos(\phi(t))$; $y_t = 30 \cdot \sin(\phi(t))$; $z_t = 50 \cdot \cos(\phi(t))$;

Sample by 100s, both two bearing angle $+0.01 \cdot \text{randn}(1, N)$

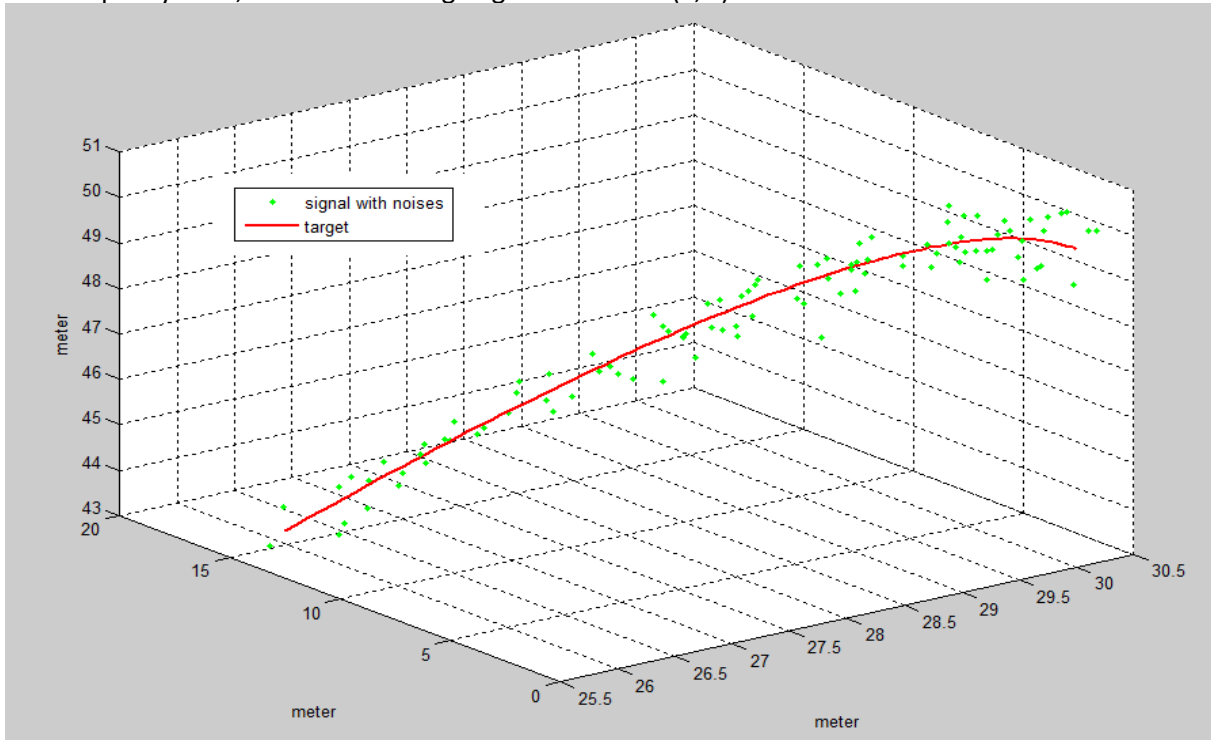


Figure 4.331 3D curve motion with noises

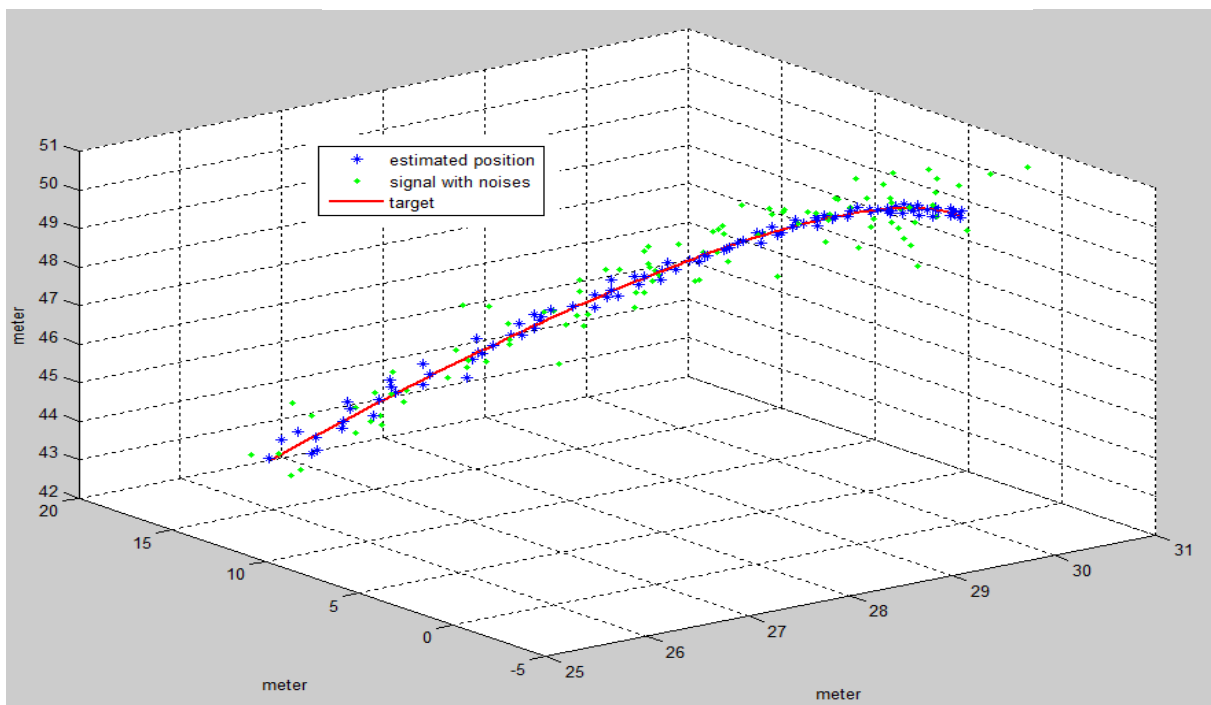


Figure 4.332 3D curve motion tracking simulation with noises

On the whole, as the figures above show: after adding Gauss noise into passive tracking system, sample signal was disrupted apparently, but least square will help with dealing with the noise, reduce the error position range and improve locating accuracy. It shows the feasibility of using passive location in the realistic environment.

Chapter 5: Discussion, Analysis & Improvements

5.1 Accuracy & Monte Carlo simulation

To evaluate the accuracy of realistic passive tracking simulation, ^[15] RMS (Root-mean-square) error need to be introduced, RMS error calculation in this project is:

$$RMSE = \sqrt{(Xt - Xout)^2 + (Yt - Yout)^2};$$

Where Xt, Yt is the actual coordinate of target, Xout, Yout is the coordinate of estimated location
In tracking process, the equation can be rewritten as:

$$RMSE(t) = \sqrt{(Xt(t) - Xout(t))^2 + (Yt(t) - Yout(t))^2};$$

'randn' function in MATLAB is used for the random noise simulation, it means that even if in the same simulation every time results are different, because error depends on the noise distribution which varies at every second in each simulation.

In this case, to investigate how error associate with target motion and to check the feasibility and stability. **Monte Carlo method** can be used to find the law of random distribution by repeating simulation about 100 times ^[16], the flowchart of Monte Carlo simulation by MATLAB as follows:

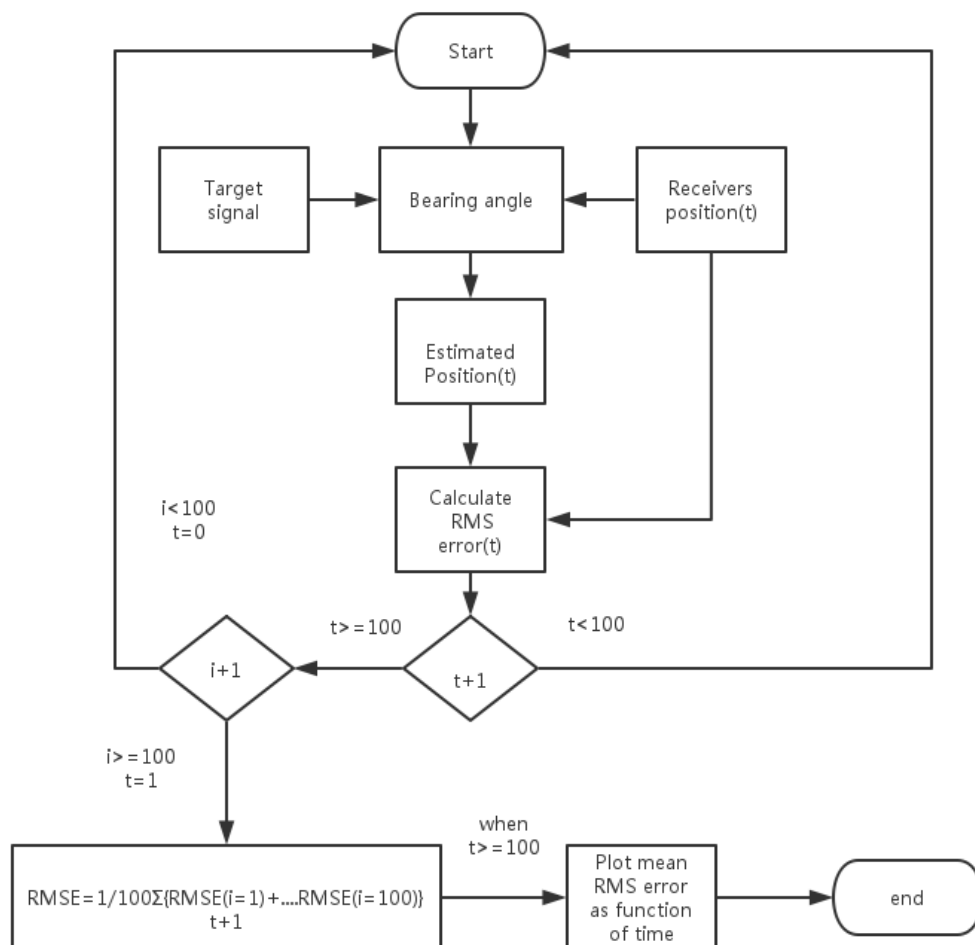


Figure 5.11 flowchart of Monte Carlo simulation

Now check the simulation results:

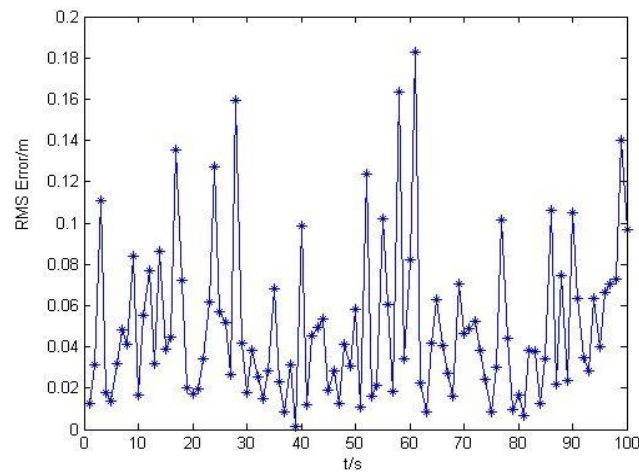


Figure 5.12 Monte Carlo simulation of 2D static target tracking

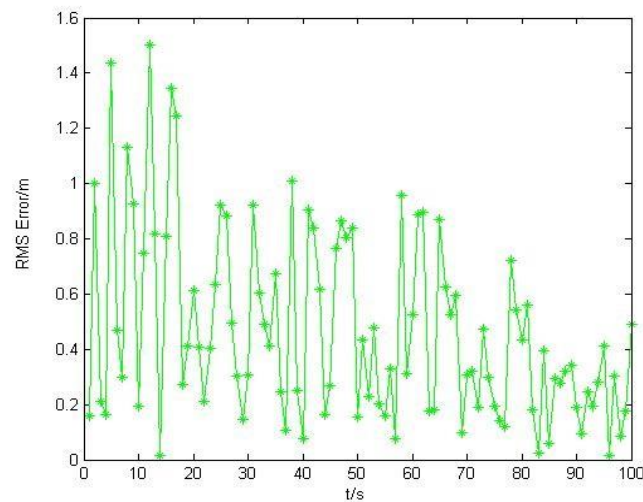


Figure 5.13 Monte Carlo simulation of 2D linear motion target tracking

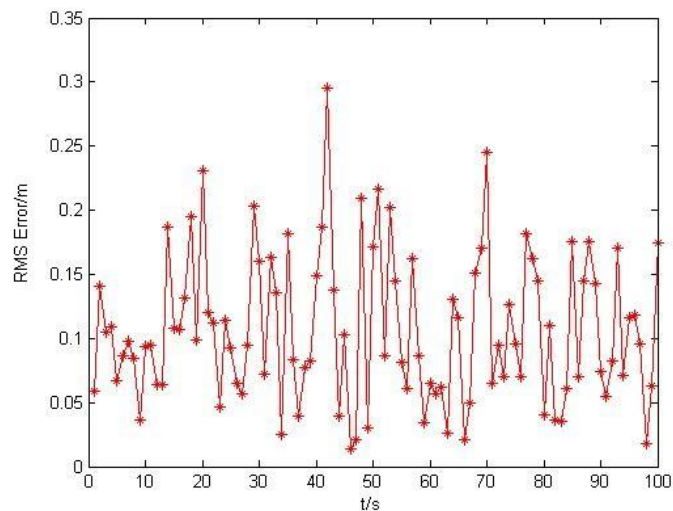


Figure 5.14 Monte Carlo simulation of 2D curve motion target tracking

As the figures above show: compared with the corresponding original target parameter, estimated locations are very accurate, especially for static model, most of the RMSE is 0.1m, the error is about $0.1/40=0.25\%$ of target actual position.

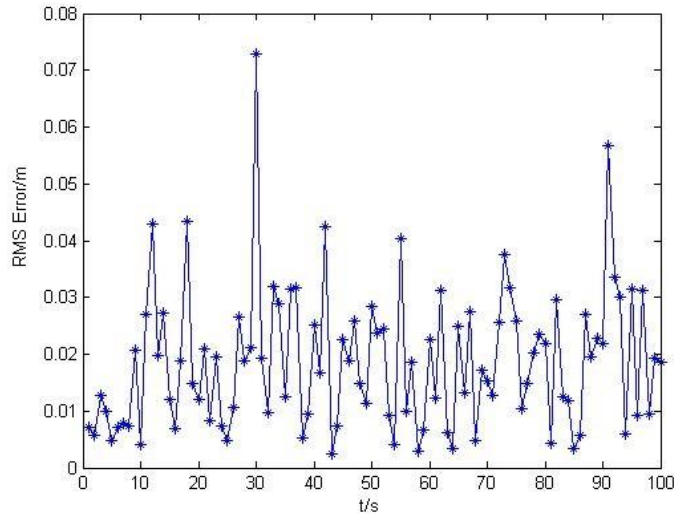


Figure 5.15 Monte Carlo simulation of 3D static target tracking

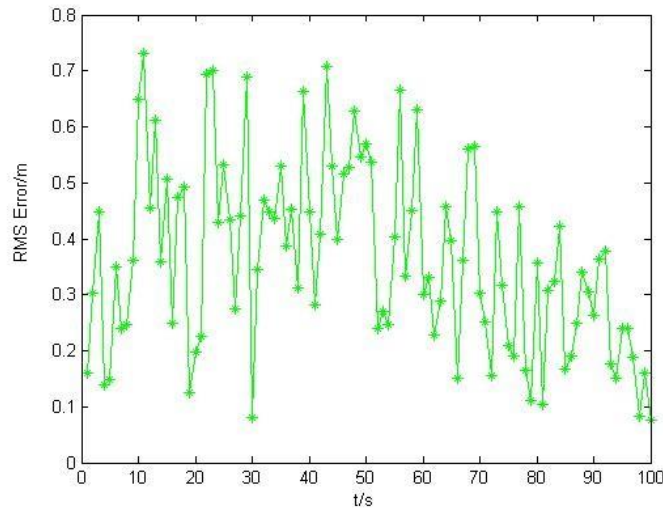


Figure 5.16 Monte Carlo simulation of 3D linear motion target tracking

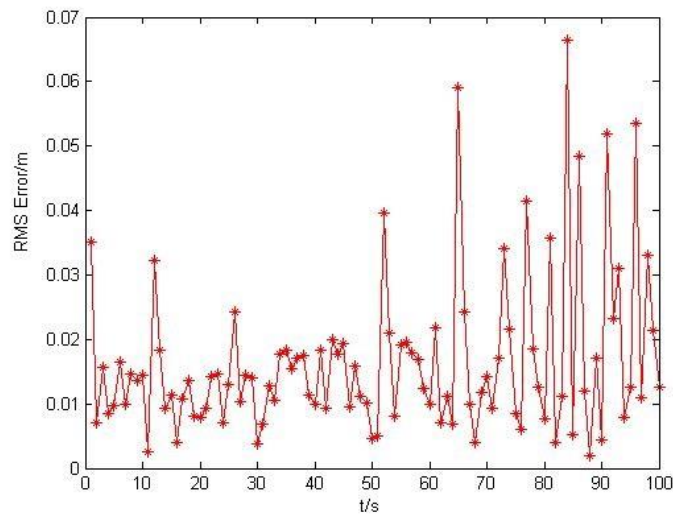


Figure 5.17 Monte Carlo simulation of 3D curve motion target tracking

For all three motion models, the RMS errors are smaller than the errors in 2D models. Compared with the 2D model, 3D model are more accurate, it is because receivers can received more measured information of target (with one more dimensional coordinate).

5.2 Receivers arrangements analysis

It is worth to mention that in the simulation chapter, when doing linear motion simulation with speed higher than before, for example the ship speed 8.3m/s, tracking simulation without noise results are still perfect as the figure shows:

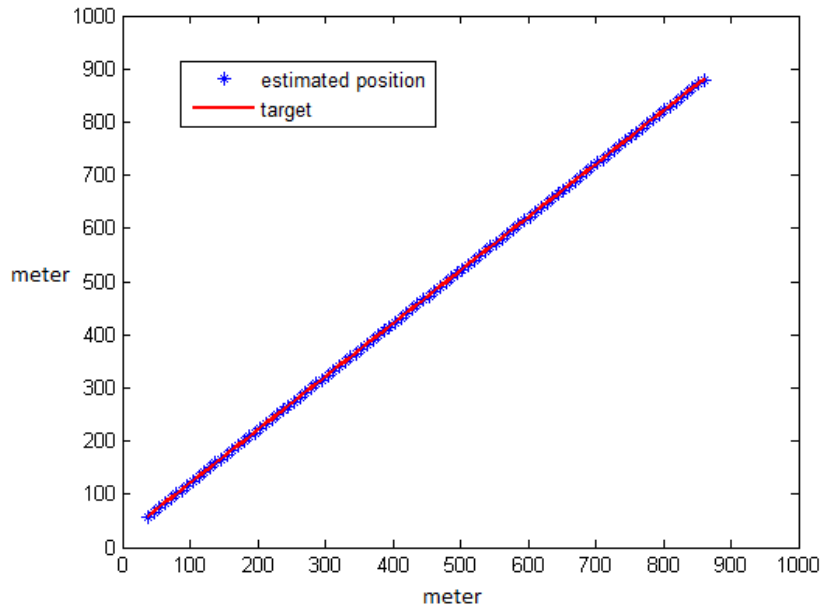


Figure 5.21 2D ideal simulation of linear motion with higher speed

However, there is something interesting in realistic tracking simulation with noises:

With the target moving farther and farther, noise intensity is increasing significantly (As the larger ellipse area shows). This is mainly because error depends on the relative location relationship between target and receivers; it will be investigated by following simulations.

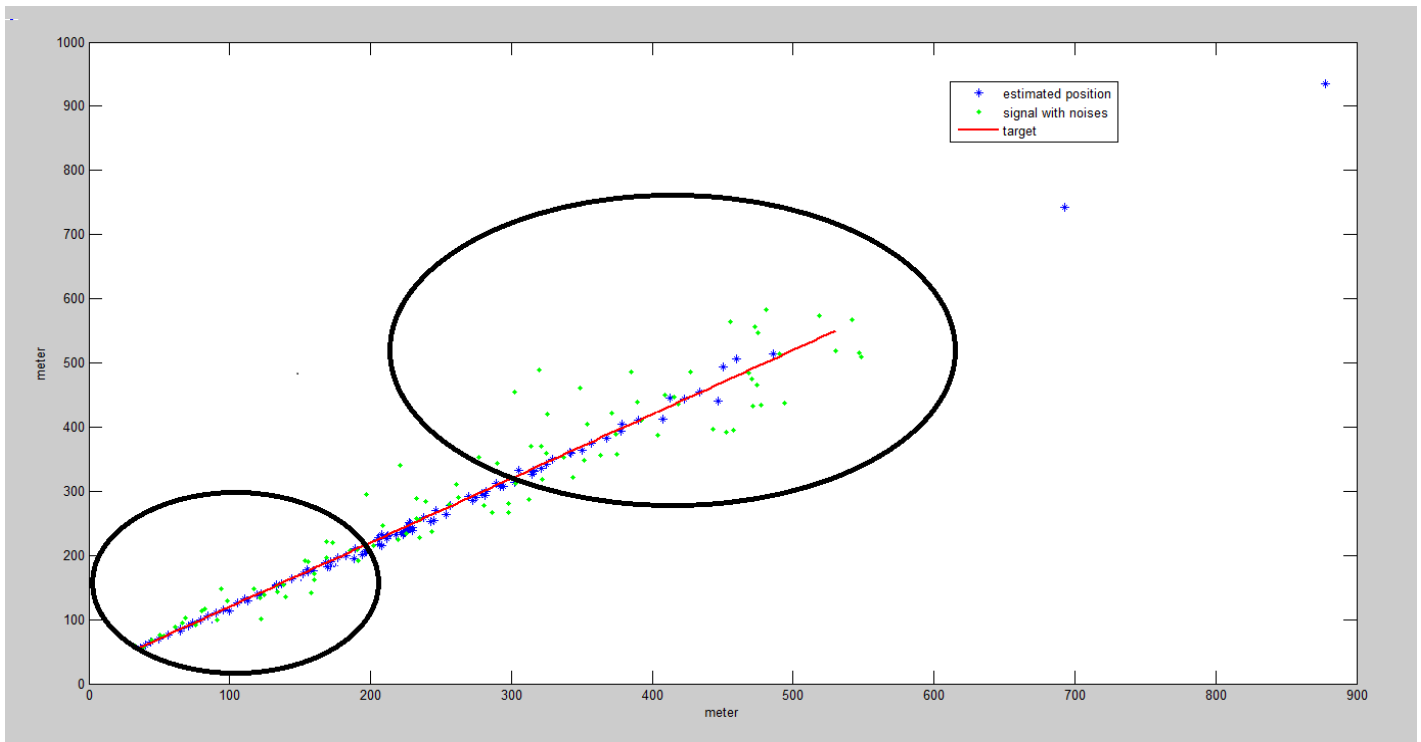


Figure 5.22 2D realistic simulation of linear motion with higher speed

The reason why noise increase happened in figure 5.22 is that the target is getting farther and farther to receivers. Check the simulation parameter: two receivers (-100 -100) and (100 -100), target motion: $X_t=50+8.3t$ $Y_t=50+8.3t$, with the target moving farther away, the receiver cannot cover the locating area anymore. Furthermore, apparently the target trace is parallel to two receivers, and the motion is coincide to the connection between two receivers, it will affect the bearing angle calculation.

To identify how the relative location relationship between target and receivers impact on accuracy, following factors need to be taken into consideration:

Factor1: Distance between two receivers.

Factor2: Relationship between target direction and route between two receivers.

Based on the previous locating model, reduce the distance between two receivers to 50m, as the figure 5.24 shows: RMS error grows exponentially, which is about 100 times greater than before.

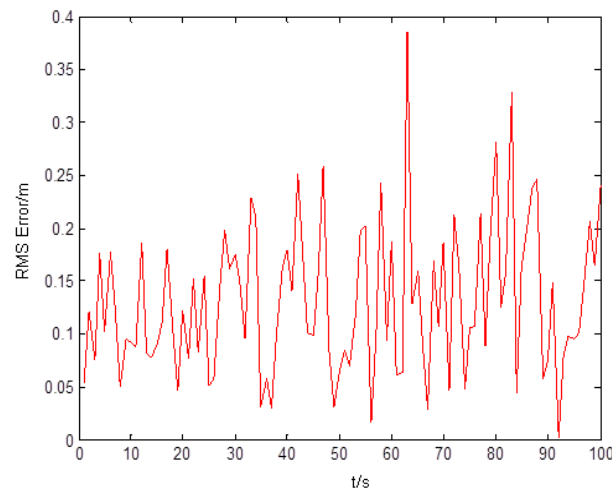


Figure 5.23 RMSE with 200m Receivers distance

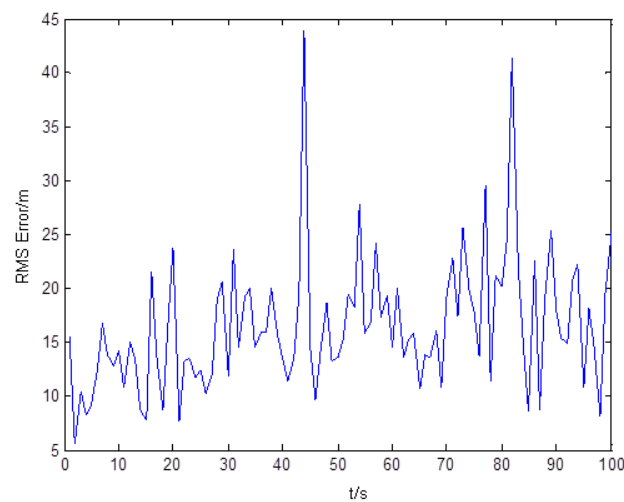


Figure 5.24 RMSE with 50m Receivers distance

Now vary the **RMS error as the function of distance between receivers:**

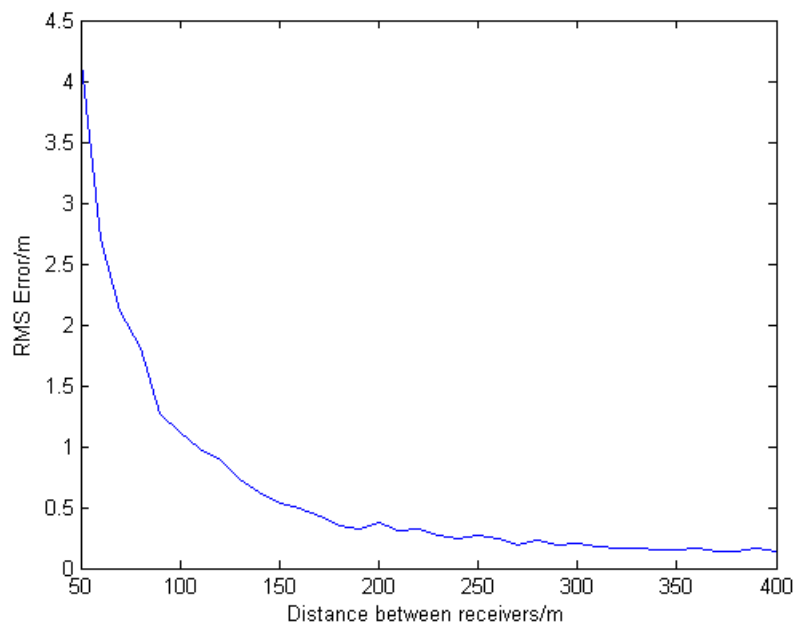


Figure 5.25 RMSE 2D as function of distance

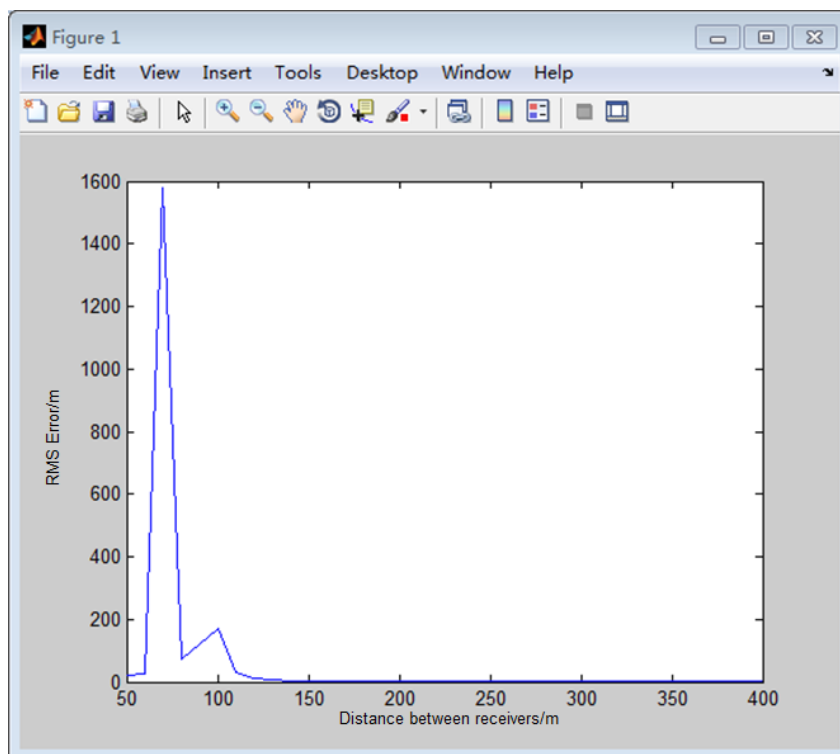


Figure 5.26 RMSE 3D as function of distance

It can be easily observed that: with the increasing distance between receivers, the RMS error is smaller and locating performance is more accurate. In addition, if the receivers get too close to each other, the passive tracking estimated location calculation would be invalid (as the steep peak in the figure) which is because of the least square data are based on the triangulation calculation of bearing angle.

The case needs more receivers:

Two receivers: $[-10,0]$ $[10,0]$, target: $[0,200]$. RMSE now are very large as the following figure:

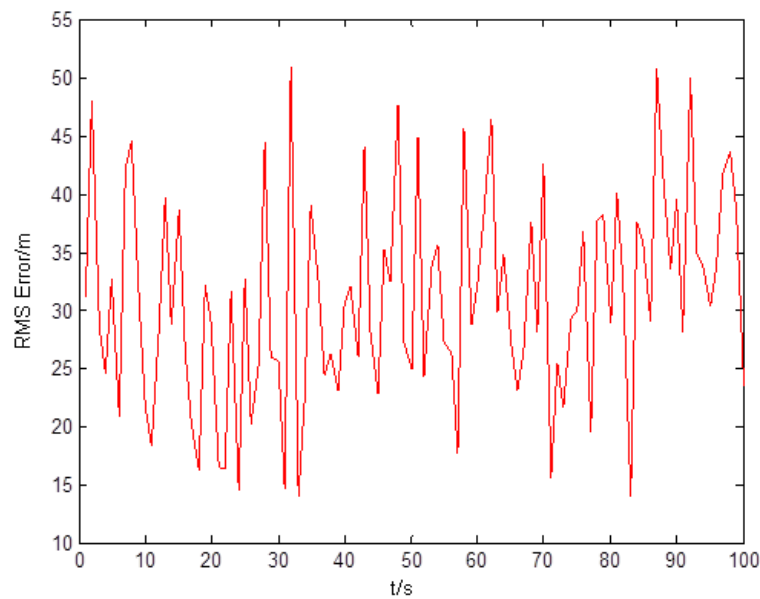


Figure 5.27 RMSE with two receivers: $[-10,0]$ $[10,0]$

Add a new receiver: $[10,100]$, RMSE decrease significantly, accuracy increase as the figure below:

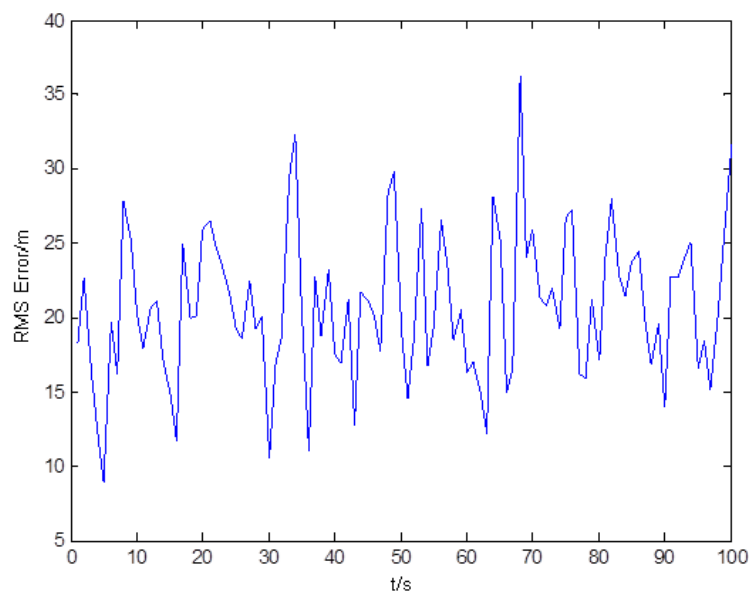


Figure 5.28 RMSE with three receivers: $[-10,0]$ $[10,0]$ $[10,100]$

The figure 5.27 and 5.28 show: RMSE decrease due to the change of receiver distribution. This is a typical case that passive tracking system needs more receivers: when the two receiver locating coverage area is not enough to the target motion range, another receiver should be used for extending the locating range to reduce RMSE and improve locating accuracy.

Now vary the RMS error as the function of target moving direction:

Case 1: Two receivers: $[-100\ 0]$, $[100\ 0]$, when bearing is 90° target is parallel to the line between receivers, where the RMSE is largest.

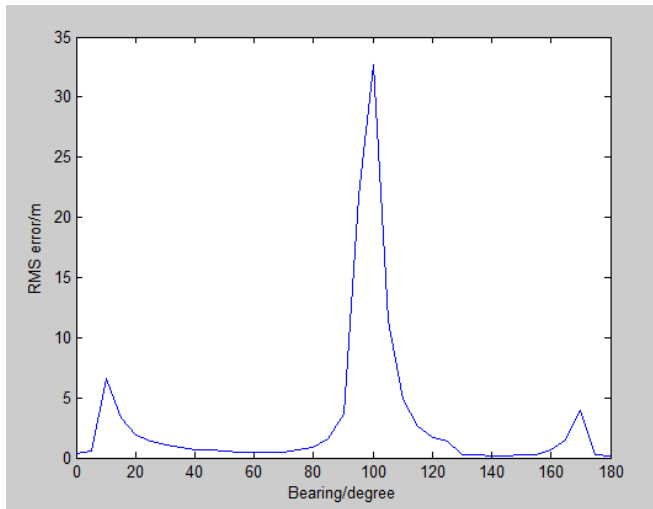


Figure 5.291 Case1: RMSE with receivers: $[-100\ 0]$ $[100\ 0]$

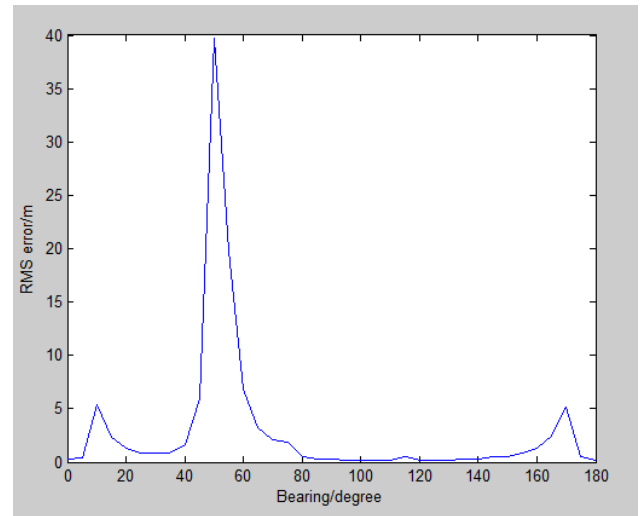


Figure 5.292 Case2: RMSE with receivers: $[-100\ 100]$ $[100\ 100]$

Case 2: Two receivers $[-100\ -100]$, $[100\ 100]$, when bearing is 45° target is parallel to the line between receivers, where the RMSE is largest.

The noise is added into bearing angle ($\alpha(t) = \text{atan}((y_t - y_i(t)) / (x_t - x_i(t))) + 0.001 * \text{randn}(1,1)$ where x_t, y_t is the coordinate of actual target, x_i, y_i is the coordinate of receiver), when the target is parallel to the line between receivers, tangent close to minimum, at this time noise has the greatest effect on the tracking algorithm.

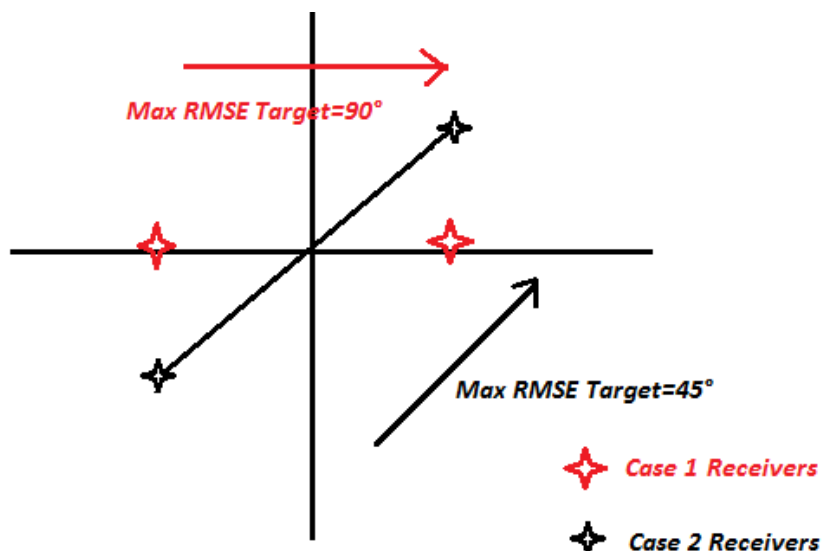


Figure 5.293 the sketch of accuracy test

Overall, the receiver arrangement rule should obey: **distance** between receivers should be larger; it means the locating coverage should be **as large as possible**. Moreover, the locating accuracy will be better if the target is located **vertically against the line between receivers**.

5.3 Multiple targets tracking

When multiple targets are on non-intersect trajectories, and none of them beyond the area that receivers' arrangement still valid, these targets can be tracked normally as previous simulation (See figure 5.31 and 5.32).

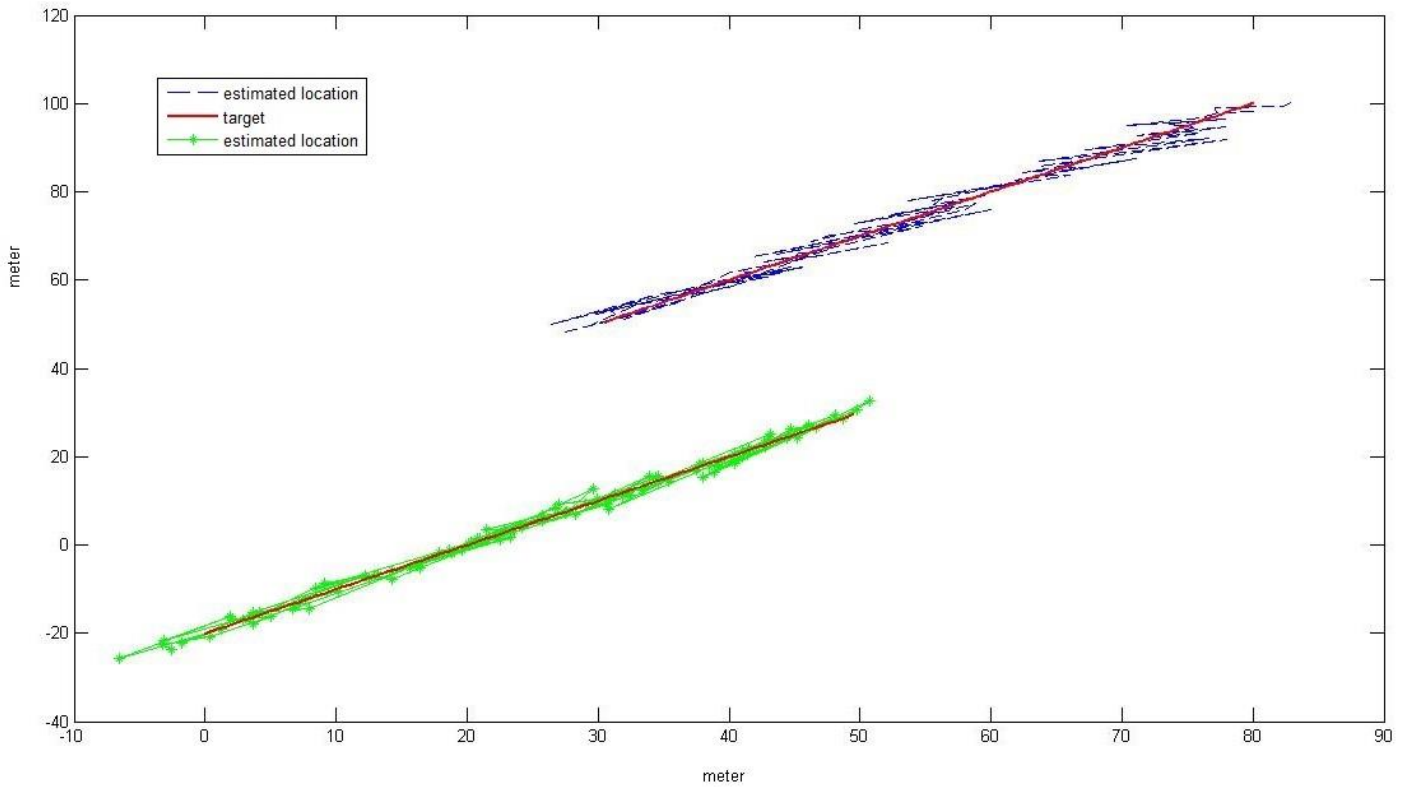


Figure 5.31 multiple targets tracking

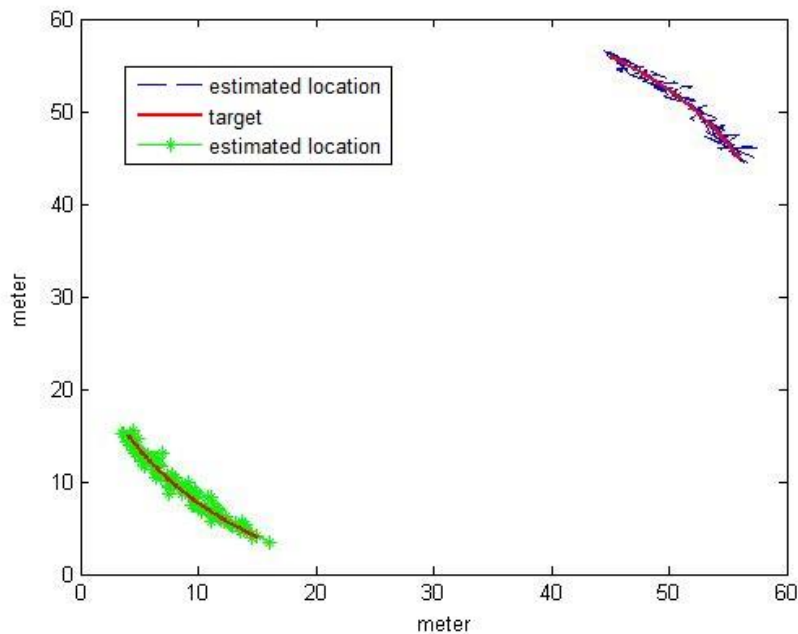


Figure 5.32 multiple targets tracking

As the figure 5.31 and 5.32 show: two targets can be located correctly as previous simulation, because the receivers' distribution is appropriate and there is no mutual interference between multiple targets.

However, when targets crossing each other, there is remarkable interference:

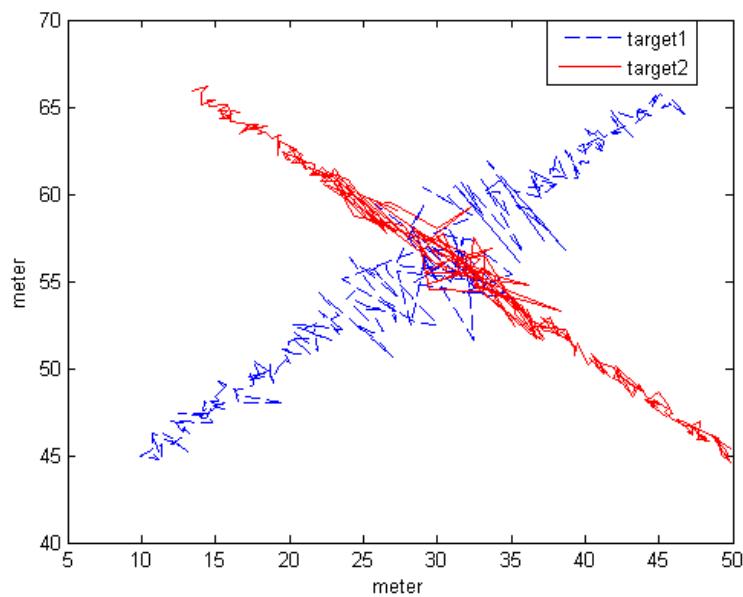


Figure 5.33 multiple targets crossing

As the figure 5.33 shows: there is a significant interference around the crossing point.

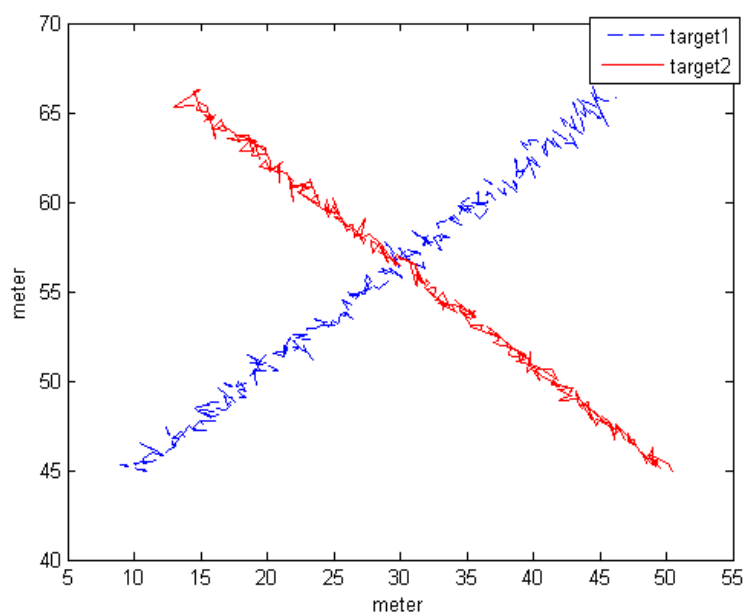


Figure 5.34 multiple targets crossing smoothing

The crossing situation is very complicated in practice, for example superposition of multiple signals might occur. In simulation, researcher use ^{[17][19]} **forecasting** and **smoothing** to process: when targets are crossing each other, the bearing accuracy decreases significantly, at the same time, least square need to keep operating and uninterrupted. On the other hand, outliers ^[18] may appear during the crossing, so it needs to be forecasted and smoothed.

5.4 Improvements

5.41 Least square improvement in MATLAB

Through simulation and debugging continually by using MATLAB, author discovered a new way to improve the least square performance: increase the receiver measurement equations, the measured bearing information will be twice in every single loop (for example, in MATLAB change $\text{xi} = [-100\ 100]$ to $\text{xi} = [-100\ 100\ -100\ 100]$); it can improve the optimal solution searching.

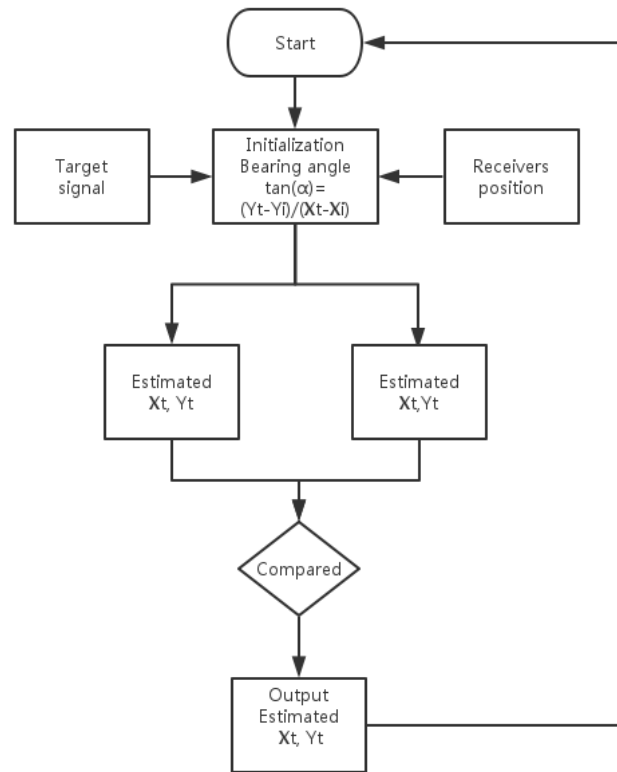


Figure 5.411 flowchart of least square improvement

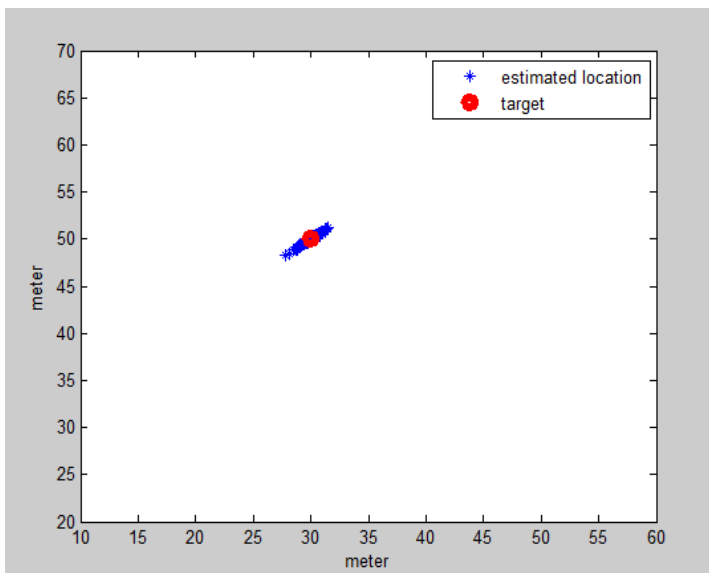


Figure 5.412 static simulation results before improvement

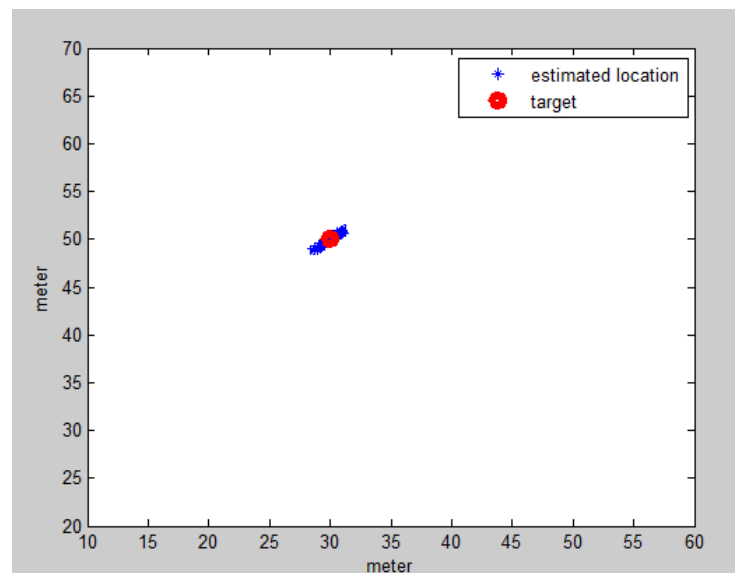


Figure 5.413 static simulation results after improvement

As the figure on the right show: incorrect estimated area is reduced, accuracy increases, because more measured equations are used for least square.

5.42 Moving windows

Moving windows just like low pass filter, which is widely used in smoothing data series.

The principle of moving windows can be summarised as follow:

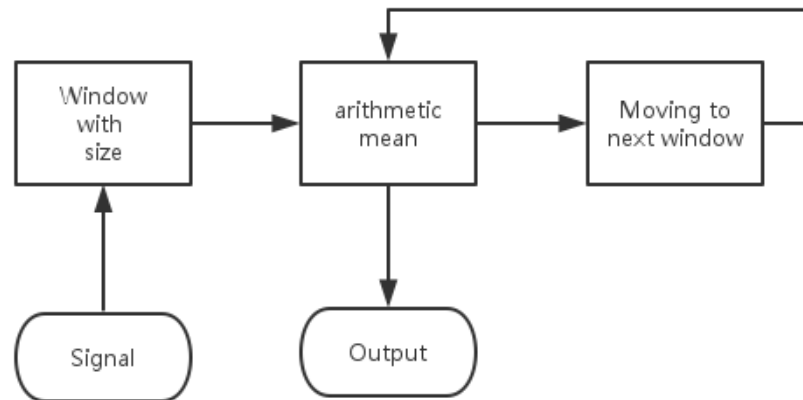


Figure 5.421 flowchart of moving windows

Now apply moving windows to least square to see the simulation results:

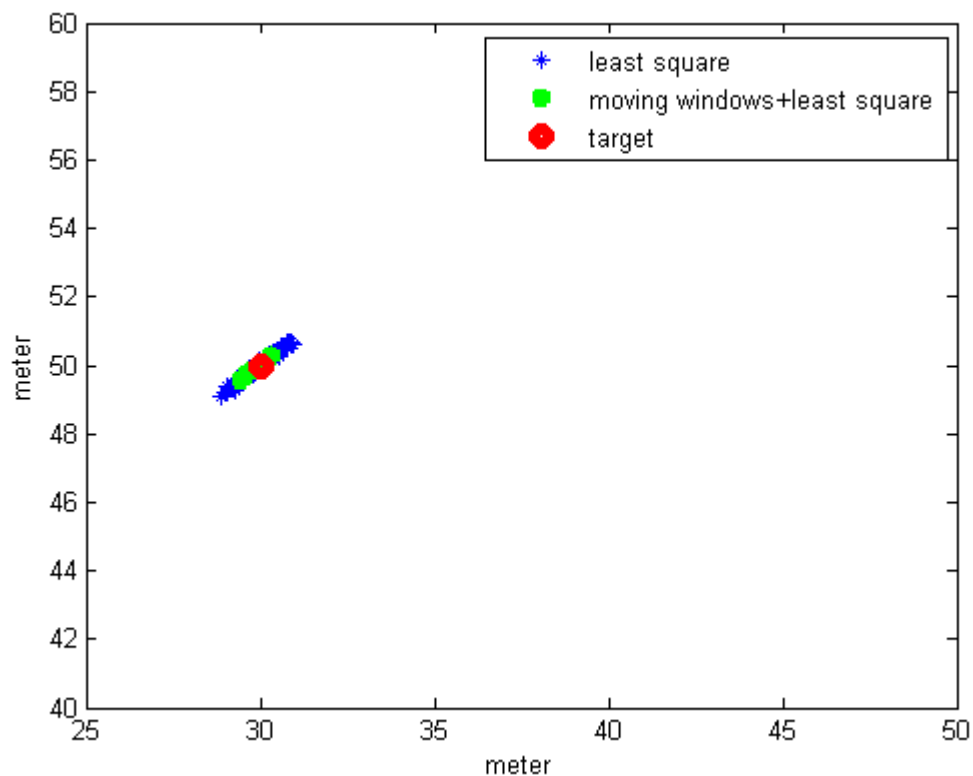


Figure 5.422 Static simulation by using moving windows

As the green trace shows: due to the use of moving average windows, estimated area is closer to original target, and locating accuracy is increase.

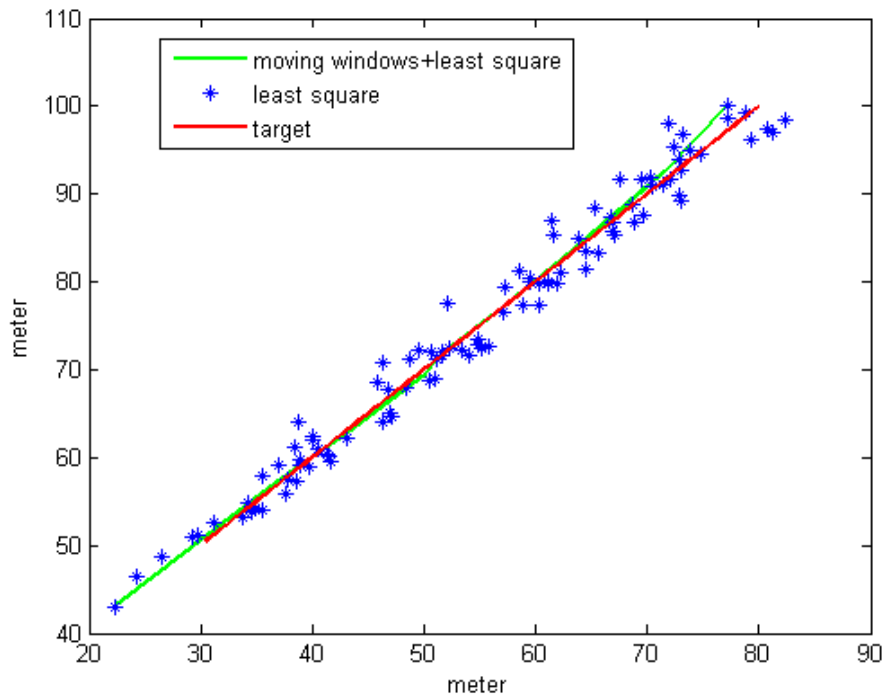


Figure 5.423 linear simulations by using moving windows

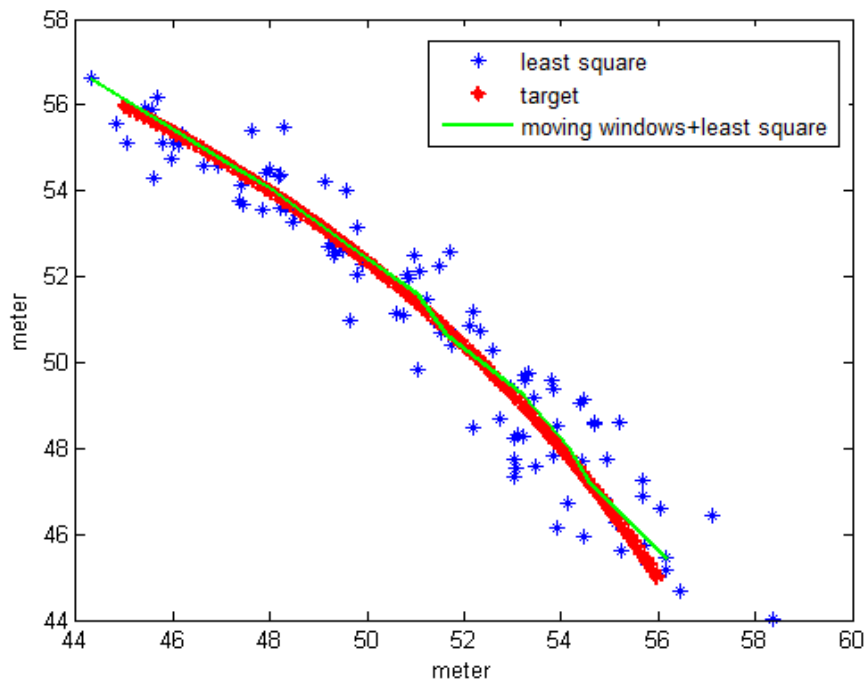


Figure 5.424 curve simulation by using moving windows

As the figure 5.423 and figure 5.424 show:

The least square performance is increasing significantly after data series was divided into several windows with specific window size, to find the mean then **process window by window**.

For example, in the MATLAB curve motion simulation, once estimated location matrix 'X_out' has been calculated, moving window can be applied as follow:

```
N = floor(100/10);
for ii=2:N-1

    X_out_window(ii,1) = mean(X_out(1+(ii-1)*10:10+(ii-1)*10,1)) ;
    X_out_window(ii,2) = mean(X_out(1+(ii-1)*10:10+(ii-1)*10,2)) ;

end
X_out_window(1,1) = X_out(1,1);
X_out_window(N,1) = X_out(end,1);
X_out_window(1,2) = X_out(1,2);
X_out_window(N,2) = X_out(end,2);
```

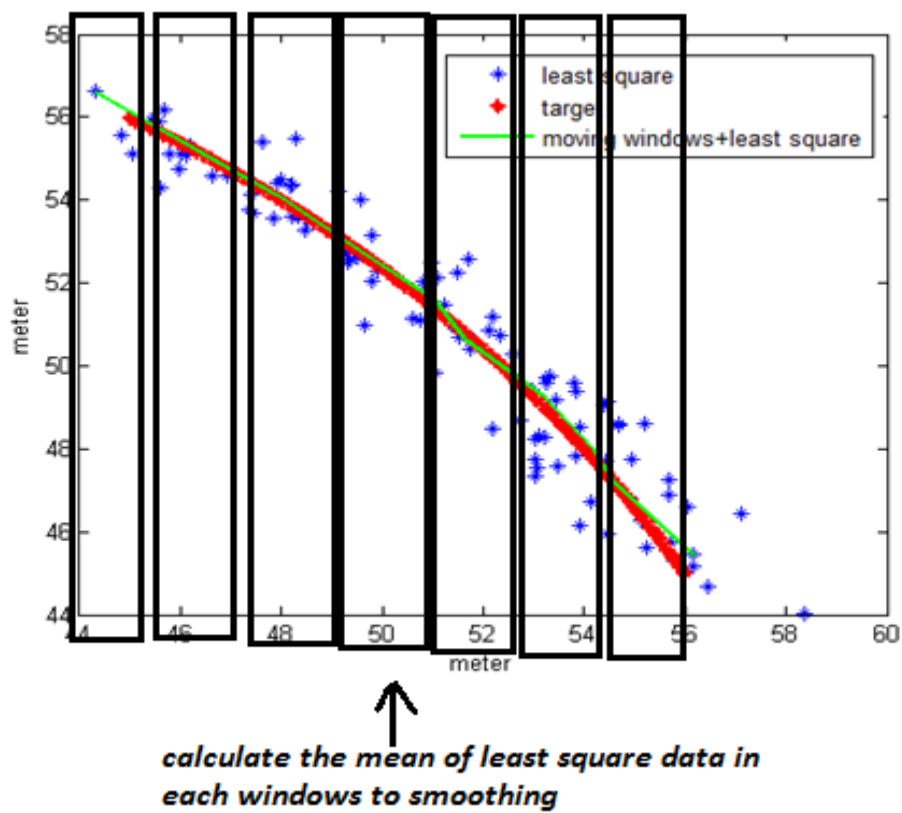


Figure 5.425 the schematic of moving window

Chapter 6: Future Work

6.1 Other error in realistic passive tracking

- In addition to error caused by noise, system error is also a factor affecting locating accuracy: such as the antenna is skewed and the receptor addressing error, etc. With the development of locating technologies, the equipment can have high precision and make these system errors negligible.
- In terms of the target motion simulation, all the motion models have already been initialized by MATLAB, but the motion model will be more complicated in realistic tracking, such as the acceleration model, singer model and current statistical model (CS)^[20], etc.
- In order to investigate the multiple targets crossing situation clearly, the simulation of multiple target tracking is simplified. In realistic passive locating, appropriate data association algorithm including static association such as reference line method^[21], and dynamic association such as Hough Transform algorithm^[22].

6.2 passive tracking applications in the future

Passive tracking technologies are increasingly being used in positioning systems. One of them is Galileo global navigation system, which is owned by Europe. The complete Galileo navigation system consists of 24 operational satellites and 6 in-orbit spares. The ground infrastructure consists of 2 Galileo Control Centres (GCC) and 29 Galileo Sensor Stations (GSS), such receiver's distribution ensure navigation service with high accuracy and coverage to both military and civil. The service includes using Synthetic Aperture Radar (SAR) to monitor passing through the ground, it can be well applied in environmental monitoring, surveying and other fields. By the end of 2016, the initial services will be available, and the whole project is scheduled for completion by 2018^[23].

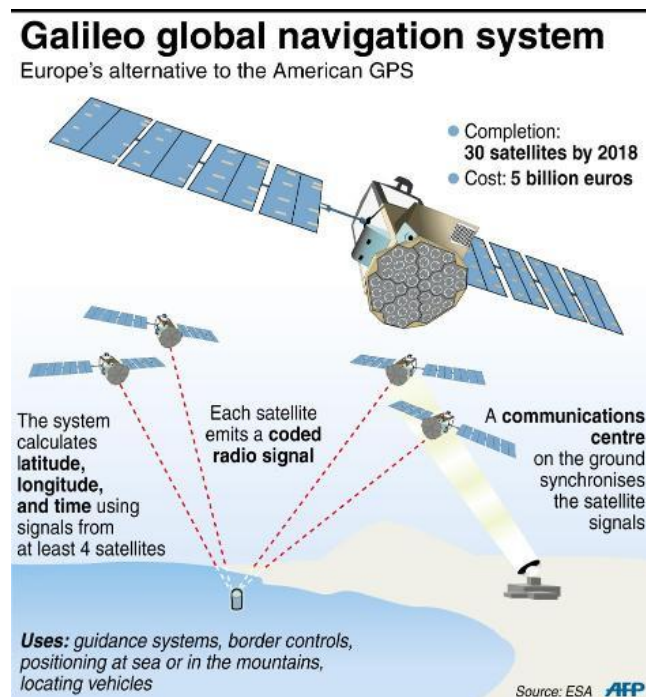


Figure 6 the schematic of Galileo global navigation system^[23]

Chapter 7: Conclusion & Reflections on Time plan

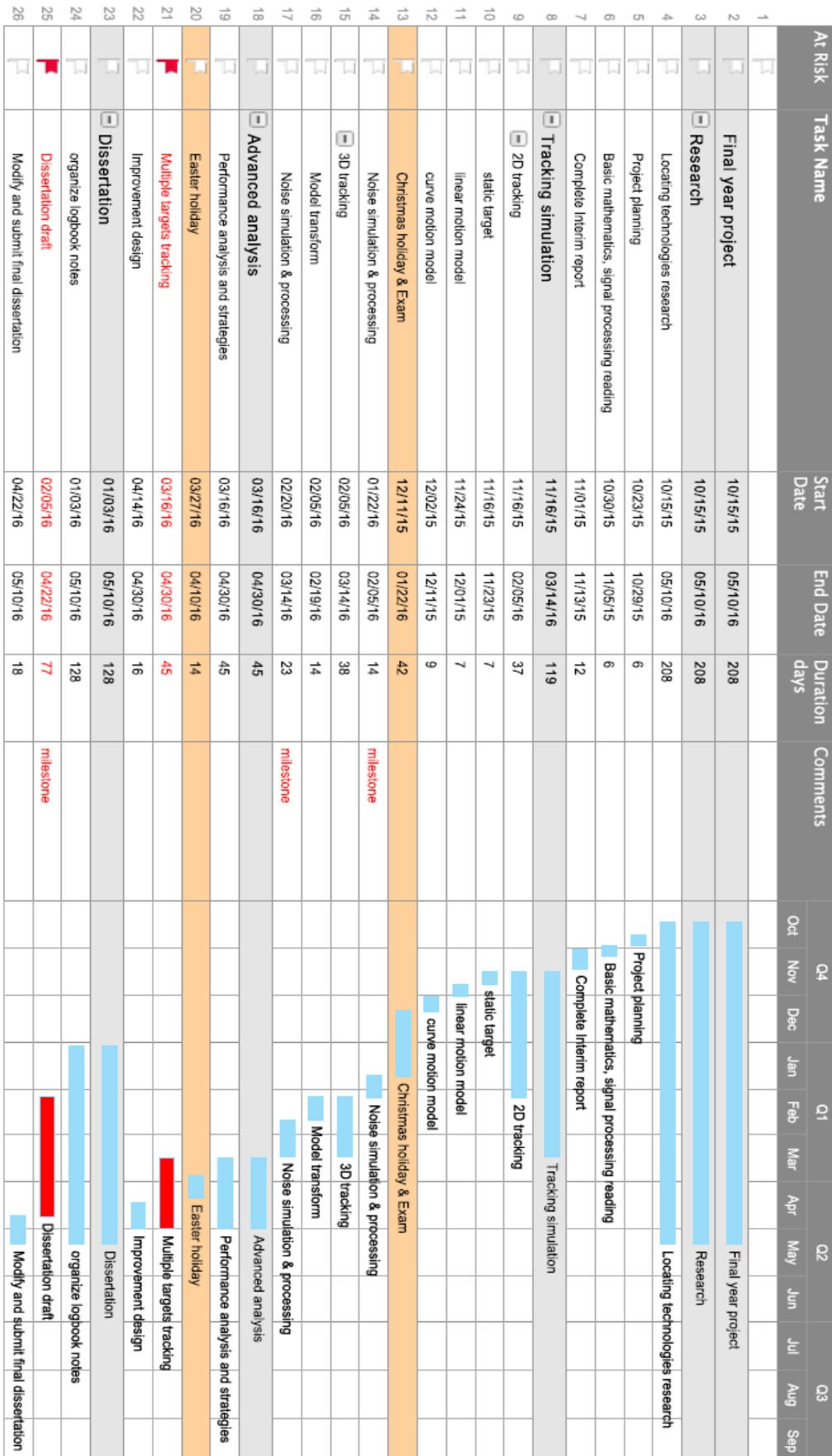


Figure 7 Project time plan

According to the time plan, the project management can be summarized by the following points:

1. Important milestone planned for both 2D and 3D locating in ideal and realistic simulation is achieved on time satisfyingly, which is very helpful for further analysis.
2. All tasks are completed and several new ways of performance improvement are found beyond expectation. For example, the combination of moving windows and least square to process the signal with noises can improve locating accuracy significantly.
3. Extra time was needed for investigating and simulating the idea of using the combination of moving windows and least square to process the signal with noises. To reduce the risk and ensure the following planned schedule can be finished on time, the researcher made use of 14 days during the Easter holiday.
4. The time used for study the case of more targets crossing was more than expected, therefore less time can be used for writing up the draft compared with the time plan. Research solved the writing risk by making use of the weekend.

In summary, all the objectives and tasks are achieved successfully. Besides, there are many extra outcomes beyond expected. Based on the research and analysis in Chapter 5, in order to improve the performance and accuracy of locating, firstly it is important to ensure the accuracy of measured bearing information, then distance between receivers should be big enough. Moreover, the locating accuracy will be better if the target is located vertically against the line between receivers. In light of the tracking algorithm, least square is selected in this project; however, the research also found that there is high potential for accuracy improvements such as combined with moving window. On the whole, as long as appropriate locating method and tracking algorithm are applied, passive tracking is a feasible, stable and accurate technique to determine target position, and since no active transmitter is needed in the passive tracking system, it is relatively low cost and convenient.

(Words account: 6809)

References

- [1]Page R M. The early history of radar[J]. Proceedings of the IRE, 1962, 50(5): 1232-1236.
- [2]Gruener W, Toernig J P, Fielding P J. Active-electronically-scanned-array based radar system features[C]//Radar 97 (Conf. Publ. No. 449). IET, 1997: 339-343.
- [3]Active vs Passive Radar, Nutaq, [Online]. Available: <http://www.nutaq.com/blog/active-vs-passive-radar>, last accessed Apr 2th 2016
- [4] Kuschel H, O'Hagan D. Passive radar from history to future[C]//Radar Symposium (IRS), 2010 11th International. IEEE, 2010: 1-4.
- [5]Joe Kurian, "Silent Sentry Innovative Technology for Passive, Persistent Surveillance," Lockheed Martin Corporation, United States of America, 06-05.
- [6] Yuesong Lin; Anke Xue; Jixin Qian, Passive tracking and data association algorithm for multiple targets with single sensor, Intelligent Control and Automation, 2004. WCICA 2004. Fifth World Congress on , Volume: 6 , June 15-19, 2004, Pages:5392 – 5395
- [7] Masumoto Y. Global positioning system: U.S. Patent 5,210,540[P]. 1993-5-11.
- [8] Choosing the Right RFID Technology, barcodesinc, [Online]. Available:<https://www.barcodesinc.com/info/buying-guides/rfid.htm> last accessed Apr 5th 2016
- [9] Jacob Moore, "Vectors Converting between Forms in 3D," adaptivemap. [Online]. Available: <http://adaptivemap.ma.psu.edu/websites/vectormath/vectors/vectors.html>. Last accessed Apr 22th 2016
- [10] Introduction to Vectors. [Online]. Available: <http://omega.albany.edu:8008/mat214dir/vectors-dir/cornell-lecture.html> , Last accessed 1 October 2015, at 10:26.
- [11] Björck, Å. (1996). Numerical Methods for Least Squares Problems. SIAM. ISBN 978-0-89871-360-2.
- [12] Allen G I, Grosenick L, Taylor J. A generalized least-square matrix decomposition[J]. Journal of the American Statistical Association, 2014, 109(505): 145-159.
- [13] Gaussian Noise, ZTH zurich [Online]. Available: <http://www.nari.ee.ethz.ch/teaching/wirelessIT/handouts/gaussian.pdf>.Last accessed Apr 22th 2016
- [14] Generate white Gaussian noise, Mathworks [Online]. Available:<http://uk.mathworks.com/help/comm/ref/wgn.html?requestedDomain=www.mathworks.com>. Last accessed Apr 22th 2016
- [15] Root-Mean-Squared Error, CIRP,[Online]. Available: <https://www.cirp.net/component/cirppubli/?task=form1&Itemid=341>. Last accessed Apr 22th 2016
- [16] Hastings W K. Monte Carlo sampling methods using Markov chains and their applications[J]. Biometrika, 1970, 57(1): 97-109.

- [17] Reid D B. An algorithm for tracking multiple targets[J]. Automatic Control, IEEE Transactions on, 1979, 24(6): 843-854.
- [18] Taek Song; Speyer, J.; A stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearings only measurements. Automatic Control, IEEE Transactions on , Volume: 30 Issue: 10 , Oct 1985. Page(s): 940 -949
- [19] Lu C T, Kou Y, Zhao J, et al. Detecting and tracking regional outliers in meteorological data[J]. Information Sciences, 2007, 177(7): 1609-1632.
- [20] Rong Li, X.; Jilkov, V.P.; Survey of maneuvering target tracking . part I: dynamic models, Aerospace and Electronic Systems, IEEE Transactions on , Volume: 39 , Issue: 4 , Oct. 2003, Pages:1333 – 1364
- [21] Jing Meng, a new method exclude outlier in multiple target crossing, Systems Engineering and Electronics, 1999,Vo.121,No.4,pages:20~23
- [22] Alexiev, K.M.; Bojilov, L.V.; A hough transform track initiation algorithm for multiple passive sensors [J], Proceedings of the Third International Conference on Information Fusion, Volume: 1 , 2000, Page(s): TuB2_11 -TuB2_16
- [23] What is Galileo, ESA,[Online]. Available:
http://www.esa.int/Our_Activities/Navigation/The_future_-_Galileo/What_is_Galileo .Last accessed Apr 22th 2016