实验题目3 四阶龙格-库塔(Runge-Kutta)方法

参考资料

julia ordinary differential equations tutorial https://diffeq.sciml.ai/stable/tutorials/ode_example/

intro to solving differential equations in julia https://www.youtube.com/watch?v=KPEqYtEd-zY

julia ode solver type: Runge-Kutta https://diffeq.sciml.ai/stable/solvers/ode_solve/#Explicit-Runge-Kutta-Methods

julia ode problem type https://diffeq.sciml.ai/stable/types/ode_types/#ode_prob

julia ode speed up perf https://diffeq.sciml.ai/stable/features/performance_overloads/#performance_overloads

julia ode common solver option https://diffeq.sciml.ai/stable/basics/common_solver_opts/#solver_options

代码实现

In [1]:
```
using DifferentialEquations
using Plots
using LaTeXStrings
using Statistics
using ImplicitEquations
```

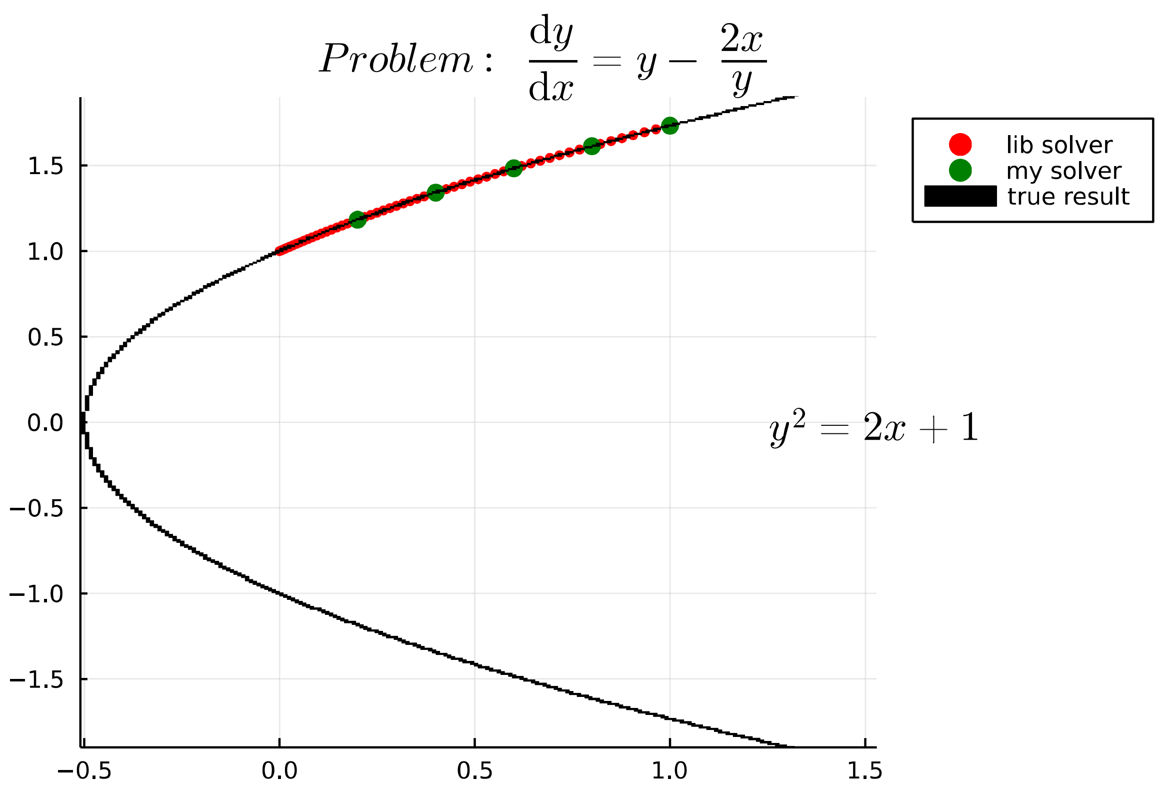
In [2]:
```
function rungekutta(f::Function, xspan, y0, num)
    a, b = xspan
    x0 = a
    h = (b - a) / num
    xs, ys = zeros(num), zeros(num)
    for n = 1:num
        K1 = h * f(x0, y0)
        K2 = h * f(x0 + h / 2, y0 + K1 / 2)
        K3 = h * f(x0 + h / 2, y0 + K2 / 2)
        K4 = h * f(x0 + h, y0 + K3)
        x1 = x0 + h
        y1 = y0 + 1 / 6 * (K1 + 2K2 + 2K3 + K4)
        xs[n], ys[n] = x0, y0 = x1, y1
    end
    println("Runge-Kutta:")
    println("x: $xs")
    println("y: $ys")
    xs, ys
end
```

Out[2]:
```
rungekutta (generic function with 1 method)
```

测试代码

Test 1 - Simple

In [3]:
```
f(y, p, x) = y - 2x / y
xspan = (0.0, 1.0)
y0 = 1.0
prob = ODEProblem(f, y0, xspan)
alg = RK4()
sol = solve(prob, alg, reltol=1e-8, abstol=1e-8)
plot(title=L"^^^^^^^^^^^^^ Problem:\ \frac{\mathrm{d} y}{\mathrm{d} x}=y-\frac{2x}{y}
plot!(sol.t, sol.u, seriestype=:scatter, markersize=3, msw=0, color=:red, label="li

f(x, y) = y - 2x / y
# xspan = (0.0, 1.0)
# y0 = 1.0
num = convert(Integer, 1.0 / 0.2)
xs, ys = rungekutta(f, xspan, y0, 5)

p = plot!(xs, ys, seriestype=:scatter, markersize=5, msw=0, color=:green, label="my
# display(p)
f(x, y) = y^2 - 2x - 1
p = plot!(f == 0.0, color=:green, linewidth=0.1, label="true result")  # \Equal[Tab
p = plot!(legend=:outertopright, xlim=(-0.51, 1.53), ylim=(-1.9, 1.9))
x = xlims(p)[2]
y = mean(ylims(p))
ymax = ylims(p)[2]
annotate!(x, y, L"y^2=2x+1", :black)
display(p)
```




```
Runge-Kutta:
x: [0.2, 0.4, 0.6000000000000001, 0.8, 1.0]
y: [1.183229287445307, 1.3416669298526065, 1.4832814583502616, 1.6125140416775265,
1.7321418826911932]
```

实验题目

In [4]:
```
function show_plot(p, f::Function, tspan, u0::Float64, reltol, abstol, dense::Bo
    prob = ODEProblem(f, u0, tspan)
    alg = RK4()
```

```julia
    sol = solve(prob, alg, reltol=1e-8, abstol=1e-8)
    if dense
        plot!(sol, seriestype=:scatter, markersize=1, msw=0, color=:red, label="lib
    else
        plot!(sol.t, sol.u, seriestype=:scatter, markersize=2, msw=0, color=:red, la
    end
end
function show_plot(p, f::Function, xspan, y0::Float64, iternum::Integer)
    xs, ys = rungekutta(f, xspan, y0, iternum)
    plot!(xs, ys, seriestype=:scatter, markersize=4, msw=0, color=:green, label="my
end
function show_plot(p, f::Function, show::Bool, text)
    x = xlims(p)[2]
    y = mean(ylims(p))
    annotate!(x, y, text, :black)
    if show
        p = plot!(f, color=:blue, label="true result")
    else
        p = plot!(f, color=:blue, label="true result")
    end
    p
end
function show_result(f1::Function, f2::Function, f3::Function, xspan, y0, itern
    println("\n\n" * title)
    for iternum in iternums
        print("\nIternum: $iternum\t")
        p = plot(legend=:outertopright, title=L"^^^^^^^^^^^^^^^^^^^^^" * title)
        p = show_plot(p, f1, xspan, y0, 1e-8, 1e-8, dense)
        p = show_plot(p, f2, xspan, y0, iternum)
        p = show_plot(p, f3, show, text)
        display(p)
    end
end
```

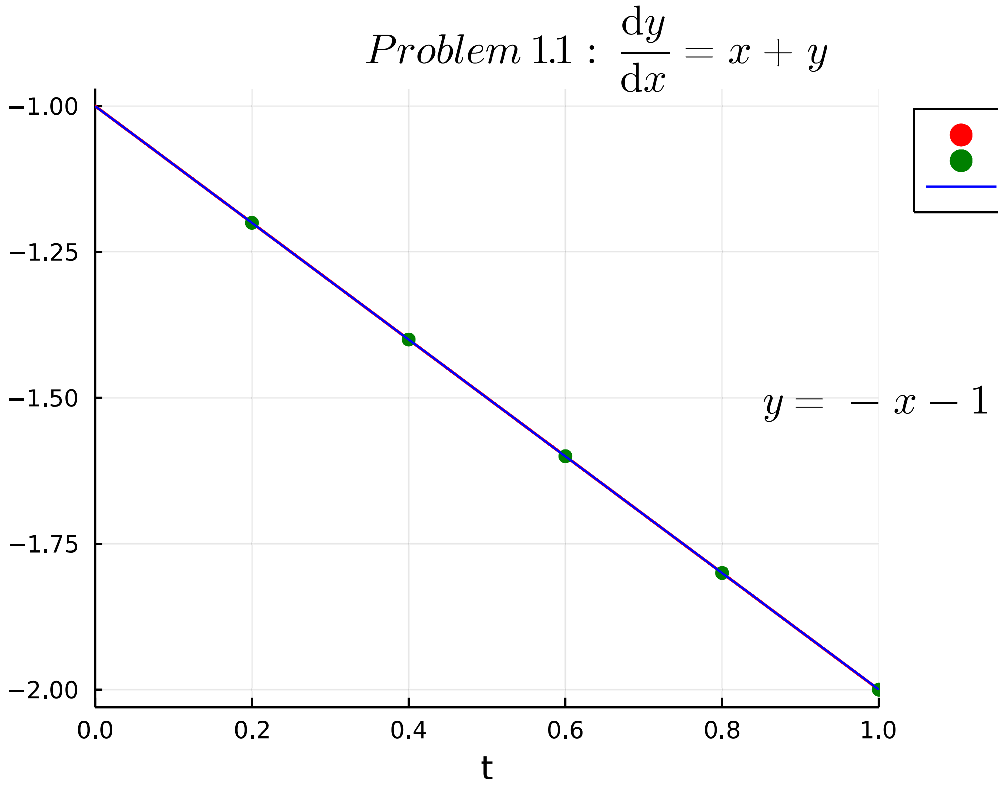
Out[4]:

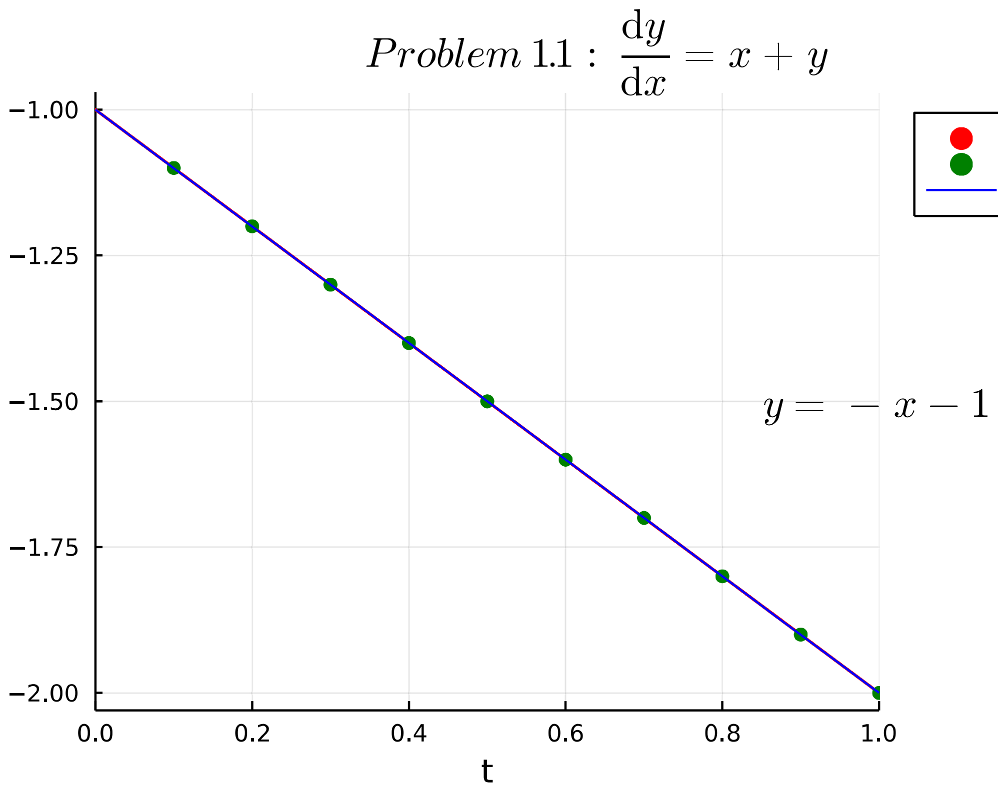
```
show_result (generic function with 1 method)
```

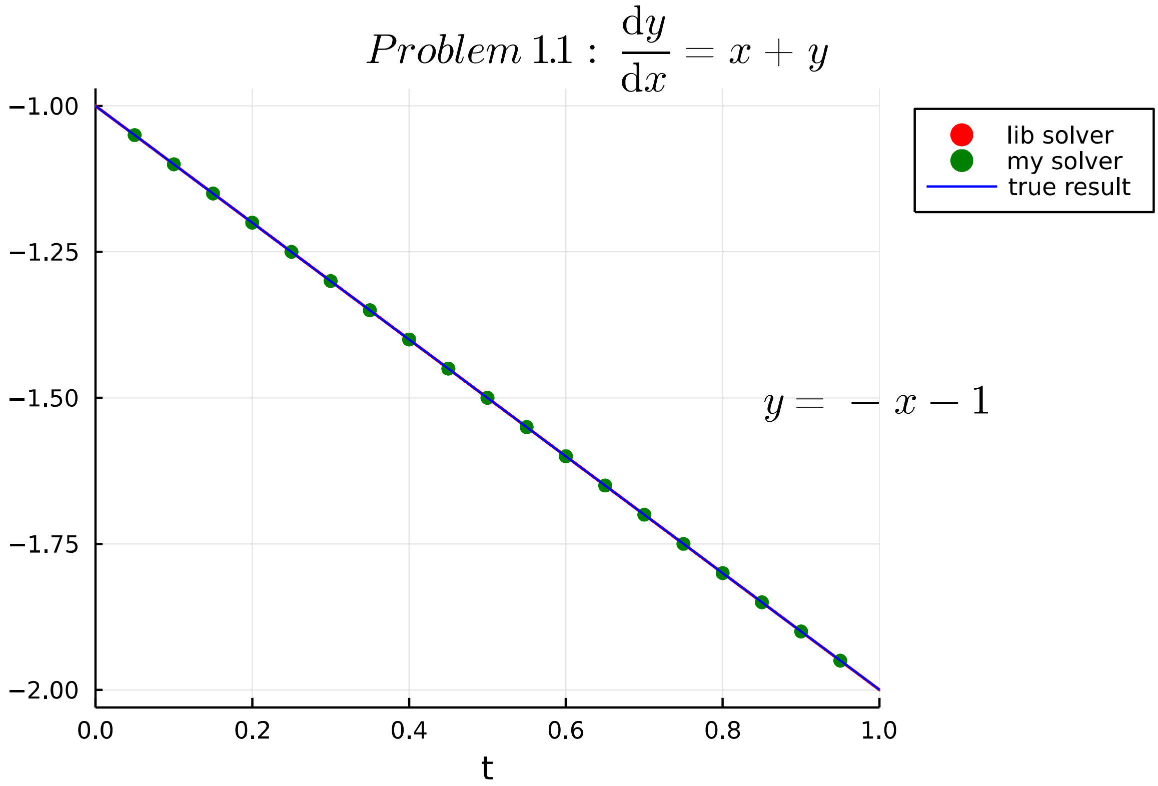
问题 1

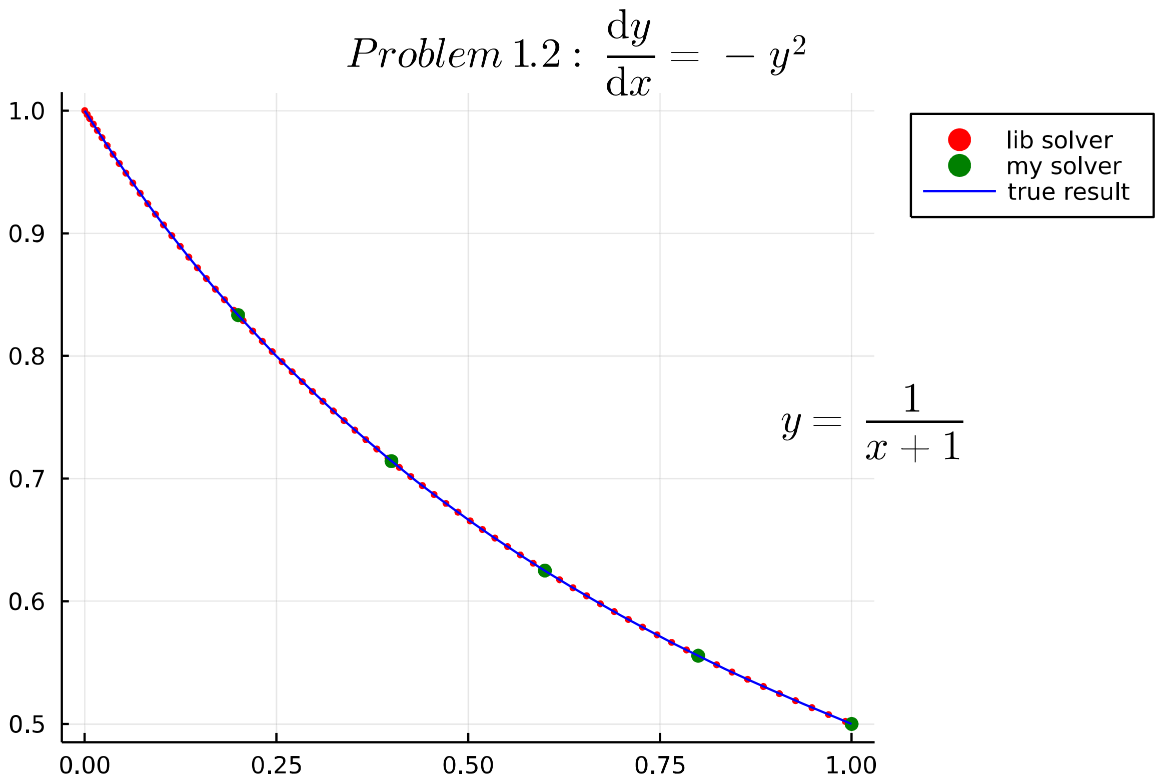
In [5]:

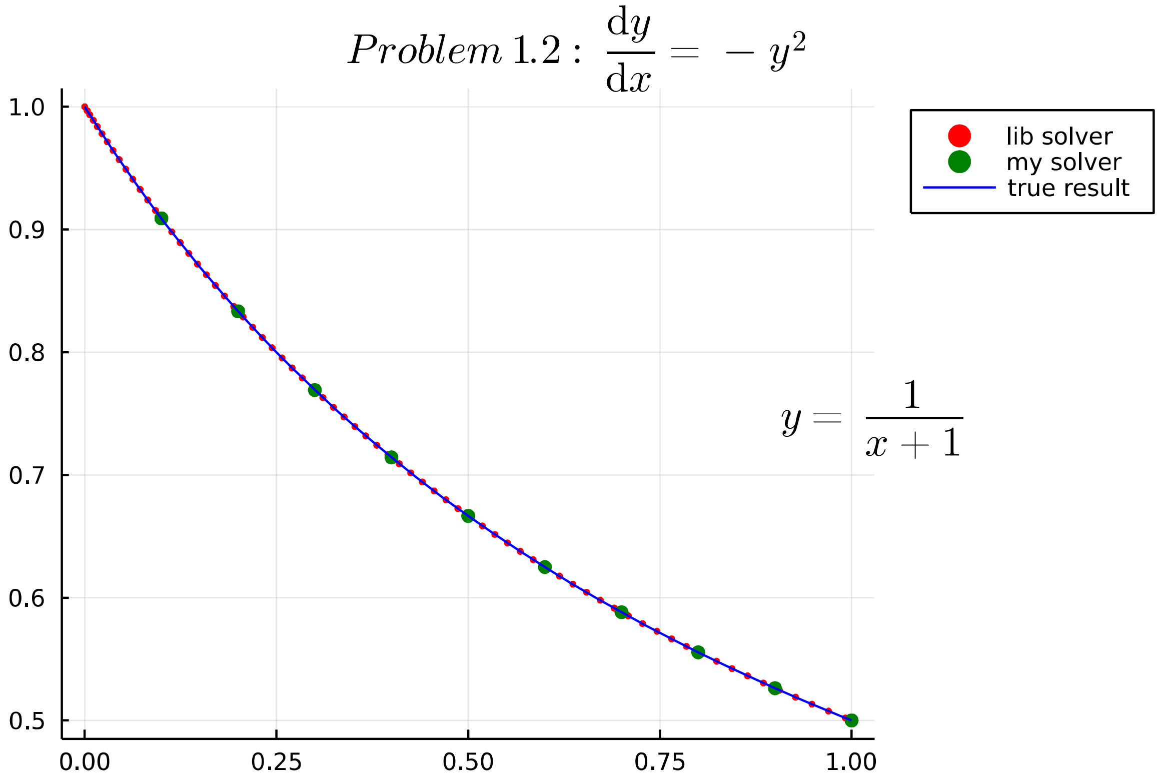
```julia
iternums = [5, 10, 20]

f1(y, p, x) = x + y    # lib RK4() solver
xspan = (0.0, 1.0)
y0 = -1.0
f2(x, y) = x + y       # my rungekutta() solver
f3(x) = -x - 1         # true result
show_result(f1, f2, f3, xspan, y0, iternums, true, true, L"Problem\ 1.1: \frac{\math

f1(y, p, x) = -y^2
xspan = (0.0, 1.0)
y0 = 1.0
f2(x, y) = -y^2
f3(x) = 1 / (x + 1)
show_result(f1, f2, f3, xspan, y0, iternums, true, false, L"Problem\ 1.2: \frac{\mat
```

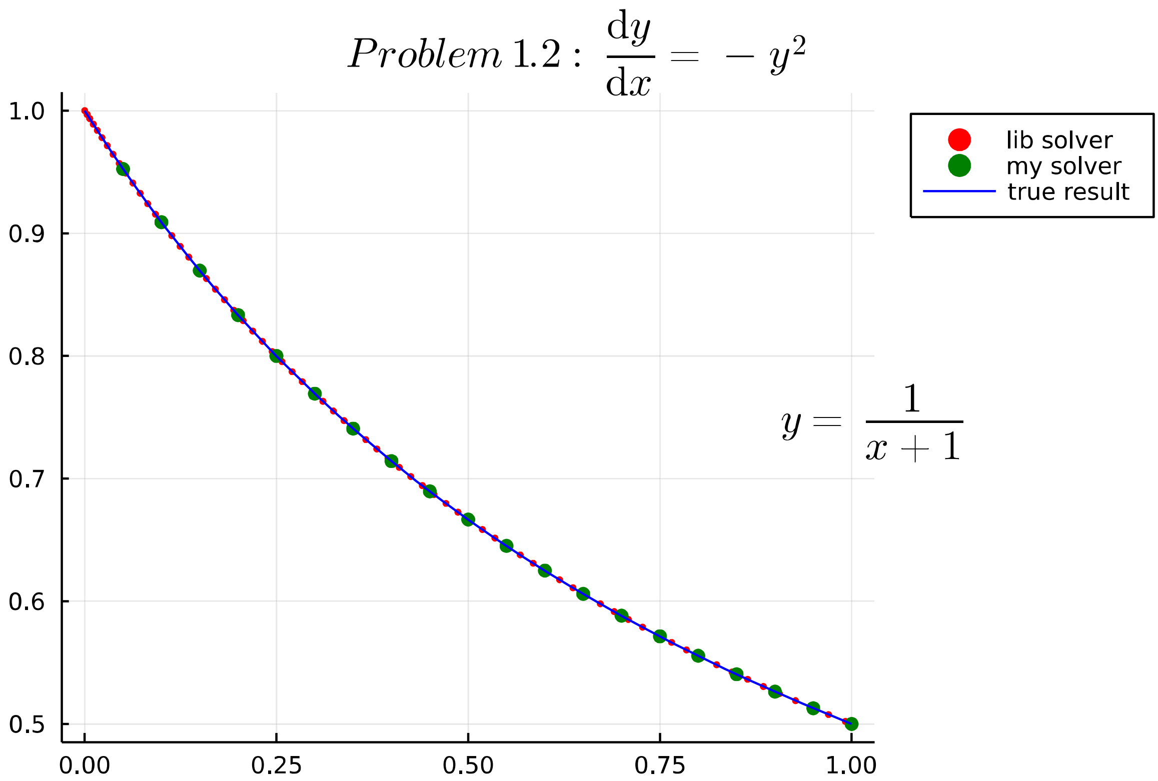

$Problem\,1.1:\ \frac{\mathrm{d}y}{\mathrm{d}x} = x + y$
lib solver
my solver
true result
$y = -x - 1$
−1.00
−1.25
−1.50
−1.75
−2.00
0.0
0.2
0.4
0.6
0.8
1.0
t


$Problem\,1.1:\ \frac{\mathrm{d}y}{\mathrm{d}x} = x + y$
lib solver
my solver
true result
$y = -x - 1$
−1.00
−1.25
−1.50
−1.75
−2.00
0.0
0.2
0.4
0.6
0.8
1.0
t


$Problem\,1.1: \frac{\mathrm{d}y}{\mathrm{d}x} = x + y$
−1.00
−1.25
−1.50
−1.75
−2.00
0.0
0.2
0.4
0.6
0.8
1.0
t
lib solver
my solver
true result
$y = -x - 1$


$Problem\,1.2: \frac{\mathrm{d}y}{\mathrm{d}x} = -y^2$
1.0
0.9
0.8
0.7
0.6
0.5
0.00
0.25
0.50
0.75
1.00
lib solver
my solver
true result
$y = \frac{1}{x+1}$


$Problem\ 1.2: \frac{\mathrm{d}y}{\mathrm{d}x} = -y^2$
lib solver
my solver
true result
$y = \frac{1}{x+1}$
1.0
0.9
0.8
0.7
0.6
0.5
0.00
0.25
0.50
0.75
1.00

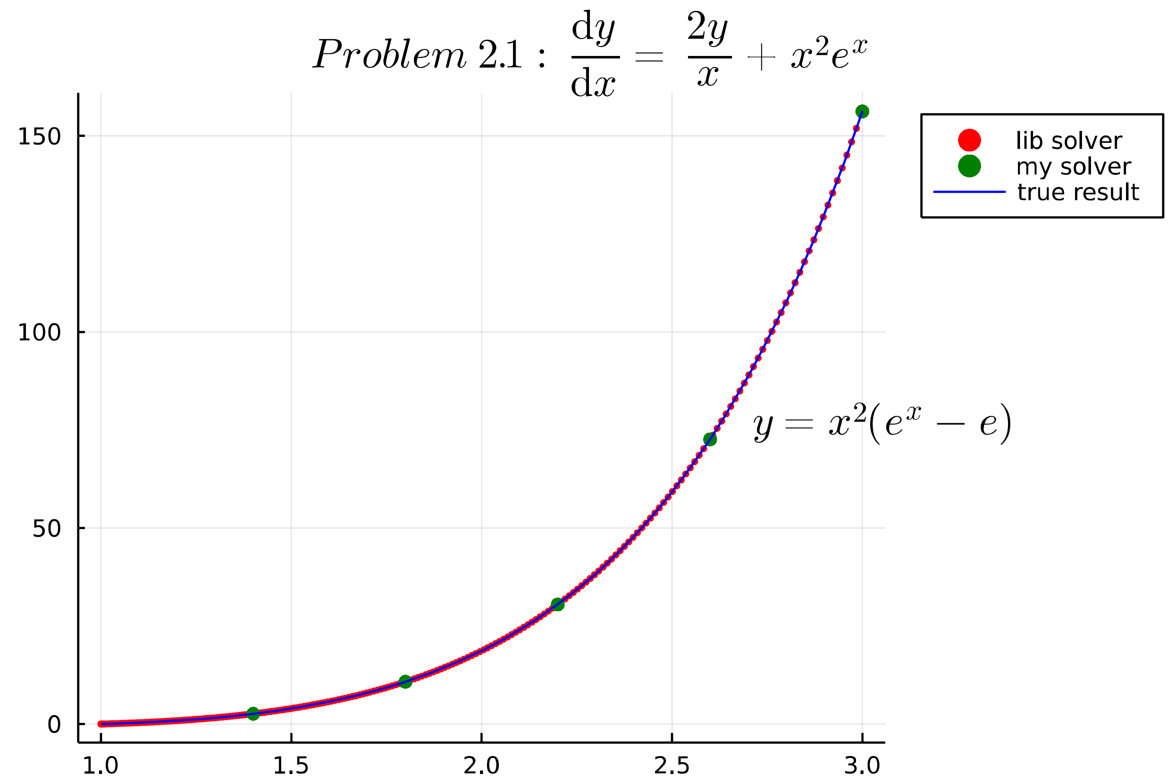

$Problem\ 1.2: \frac{\mathrm{d}y}{\mathrm{d}x} = -y^2$
lib solver
my solver
true result
$y = \frac{1}{x+1}$
1.0
0.9
0.8
0.7
0.6
0.5
0.00
0.25
0.50
0.75
1.00

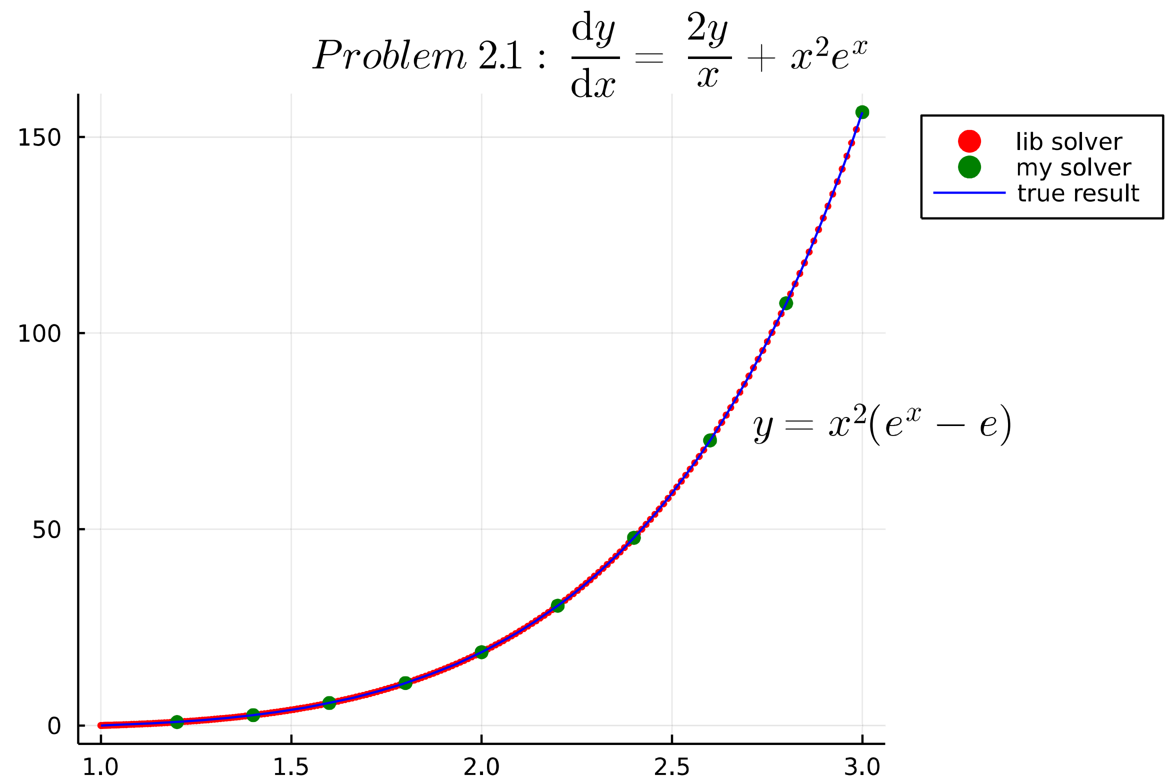
```
$Problem\ 1.1: \frac{\mathrm{d} y}{\mathrm{d} x} = x + y$

Iternum: 5      Runge-Kutta:
x: [0.2, 0.4, 0.6000000000000001, 0.8, 1.0]
y: [-1.2, -1.4, -1.5999999999999999, -1.7999999999999998, -1.9999999999999998]

Iternum: 10      Runge-Kutta:
x: [0.1, 0.2, 0.30000000000000004, 0.4, 0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999
999999999, 0.9999999999999999]
y: [-1.1, -1.2000000000000002, -1.3000000000000003, -1.4000000000000004, -1.50000000
00000004, -1.6000000000000005, -1.7000000000000006, -1.8000000000000007, -1.90000000
00000008, -2.000000000000001]

Iternum: 20      Runge-Kutta:
x: [0.05, 0.1, 0.15000000000000002, 0.2, 0.25, 0.3, 0.35, 0.39999999999999997, 0.449
99999999999996, 0.49999999999999994, 0.5499999999999999, 0.6, 0.65, 0.70000000000000
01, 0.7500000000000001, 0.8000000000000002, 0.8500000000000002, 0.9000000000000002,
0.9500000000000003, 1.0000000000000002]
y: [-1.05, -1.1, -1.1500000000000001, -1.2000000000000002, -1.2500000000000002, -1.3
000000000000003, -1.3500000000000003, -1.4000000000000004, -1.4500000000000004, -1.5
000000000000004, -1.5500000000000005, -1.6000000000000005, -1.6500000000000006, -1.7
000000000000006, -1.7500000000000007, -1.8000000000000007, -1.8500000000000008, -1.9
000000000000008, -1.9500000000000008, -2.000000000000001]


$Problem\ 1.2: \frac{\mathrm{d} y}{\mathrm{d} x} = -y^2$

Iternum: 5      Runge-Kutta:
x: [0.2, 0.4, 0.6000000000000001, 0.8, 1.0]
y: [0.8333390356230387, 0.7142921304635431, 0.6250058936085341, 0.5555606879341864,
0.5000044061582258]

Iternum: 10      Runge-Kutta:
x: [0.1, 0.2, 0.30000000000000004, 0.4, 0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999
999999999, 0.9999999999999999]
y: [0.9090911863322196, 0.8333337288430721, 0.7692312057532864, 0.7142861538927614,
0.6666670910658625, 0.6250004009491739, 0.5882356685808391, 0.5555559031832142, 0.52
63161112642581, 0.500000297580231]

Iternum: 20      Runge-Kutta:
x: [0.05, 0.1, 0.15000000000000002, 0.2, 0.25, 0.3, 0.35, 0.39999999999999997, 0.449
99999999999996, 0.49999999999999994, 0.5499999999999999, 0.6, 0.65, 0.70000000000000
01, 0.7500000000000001, 0.8000000000000002, 0.8500000000000002, 0.9000000000000002,
0.9500000000000003, 1.0000000000000002]
y: [0.9523809630269818, 0.9090909268125394, 0.8695652397267474, 0.8333333585722669,
0.8000000269481106, 0.7692307970520927, 0.7407407688506038, 0.7142857422771115, 0.68
96552000061802, 0.6666666936699812, 0.6451613166117316, 0.6250000254966355, 0.606060
6307199913, 0.5882353179191637, 0.571428594368805, 0.5555555776432456, 0.54054056179
28818, 0.5263158099136128, 0.5128205324747149, 0.5000000188974524]
```

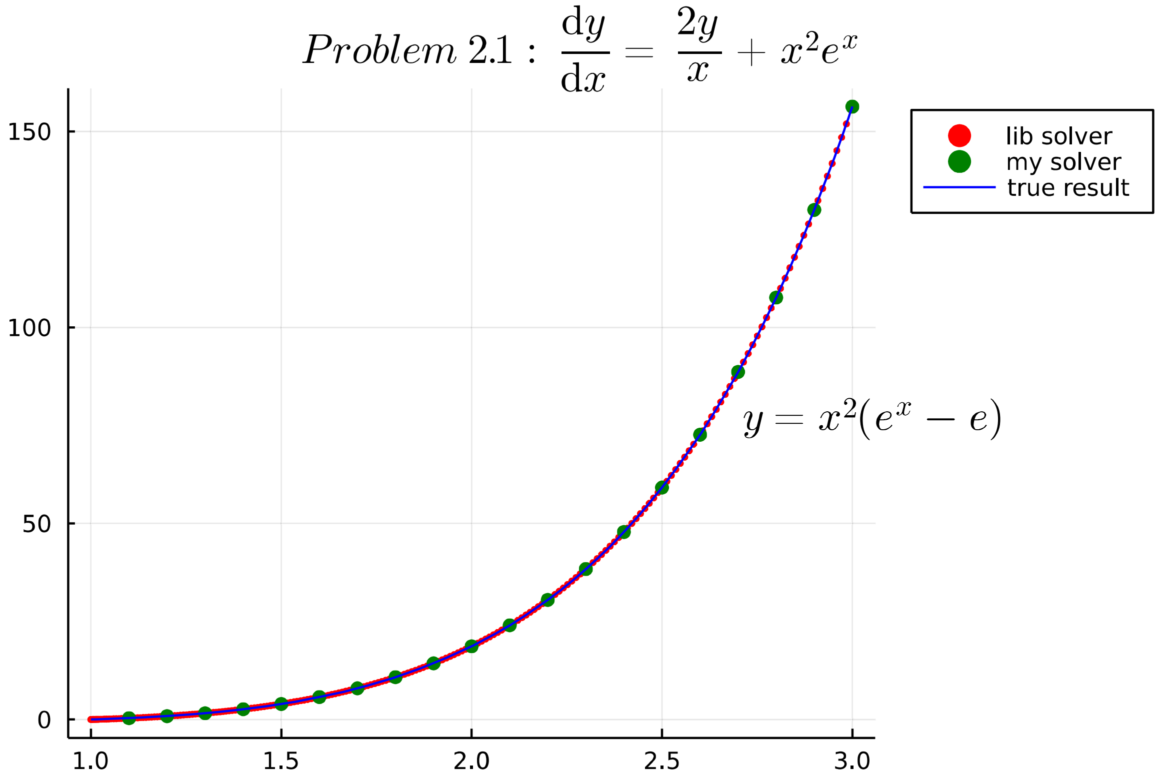
问题 2

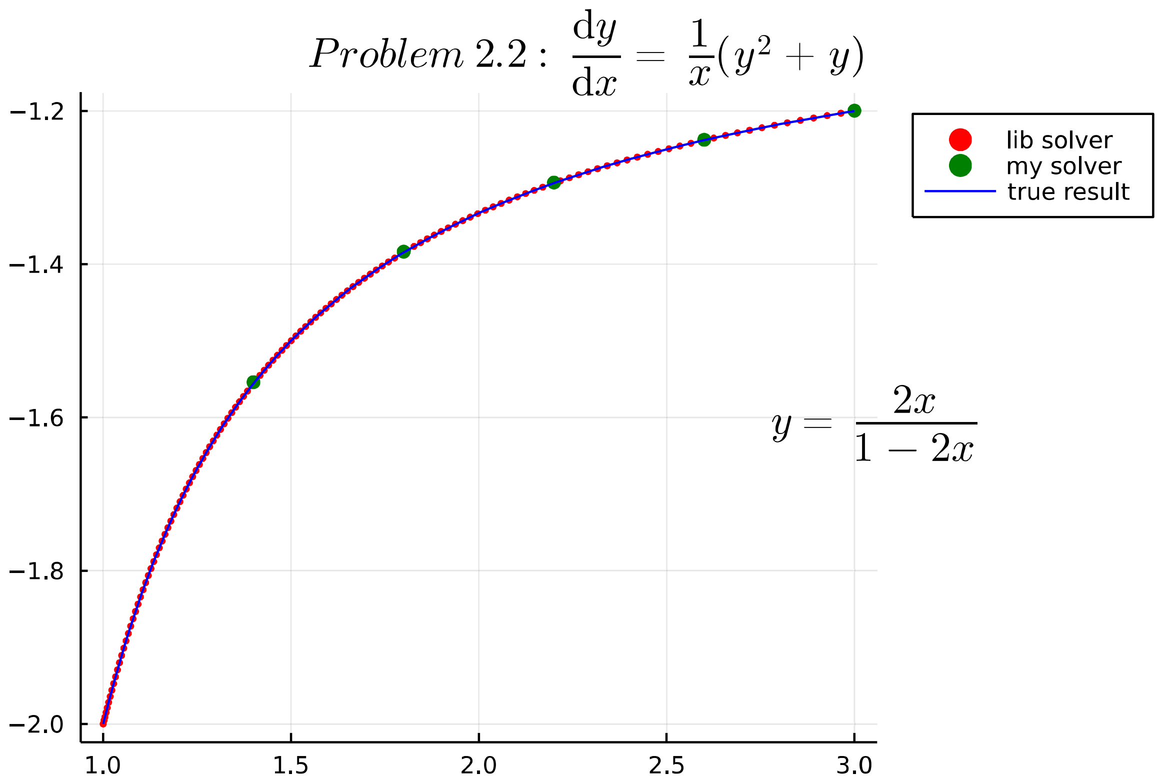
In [6]:
```
iternums = [5, 10, 20]

f1(y, p, x) = 2 * y / x + x^2 * exp(x)
xspan = (1.0, 3.0)
# !不需要换元，也不需要改xspan，当x最左边的值确定的时候，就是初值的位置
y0 = 0.0
f2(x, y) = 2 * y / x + x^2 * exp(x)
f3(x) = x^2 * (exp(x) - exp(1))
show_result(f1, f2, f3, xspan, y0, iternums, true, false, L"Problem\ 2.1:\frac{\math
```

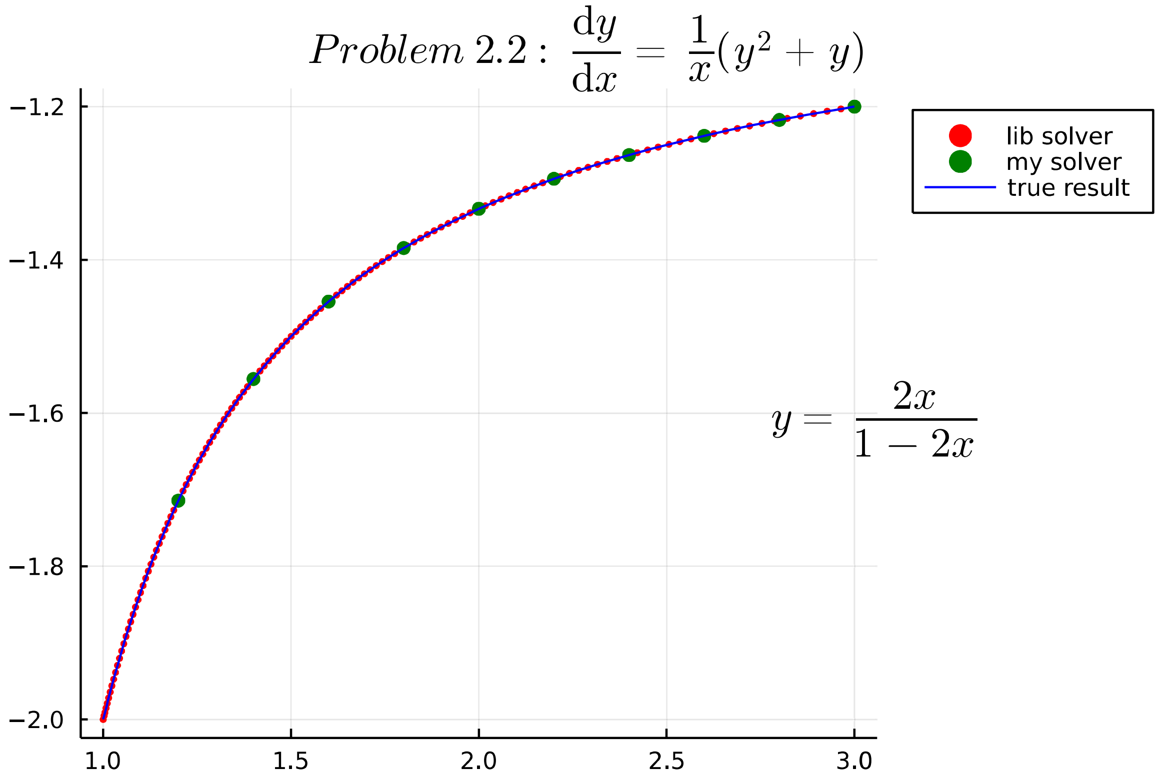
```
f1(y, p, x) = (y^2 + y) / x
xspan = (1.0, 3.0)
y0 = -2.0
f2(x, y) = (y^2 + y) / x
f3(x) = 2x / (1 - 2x)
show_result(f1, f2, f3, xspan, y0, iternums, true, false, L"Problem\ 2.2:\frac{\math
```

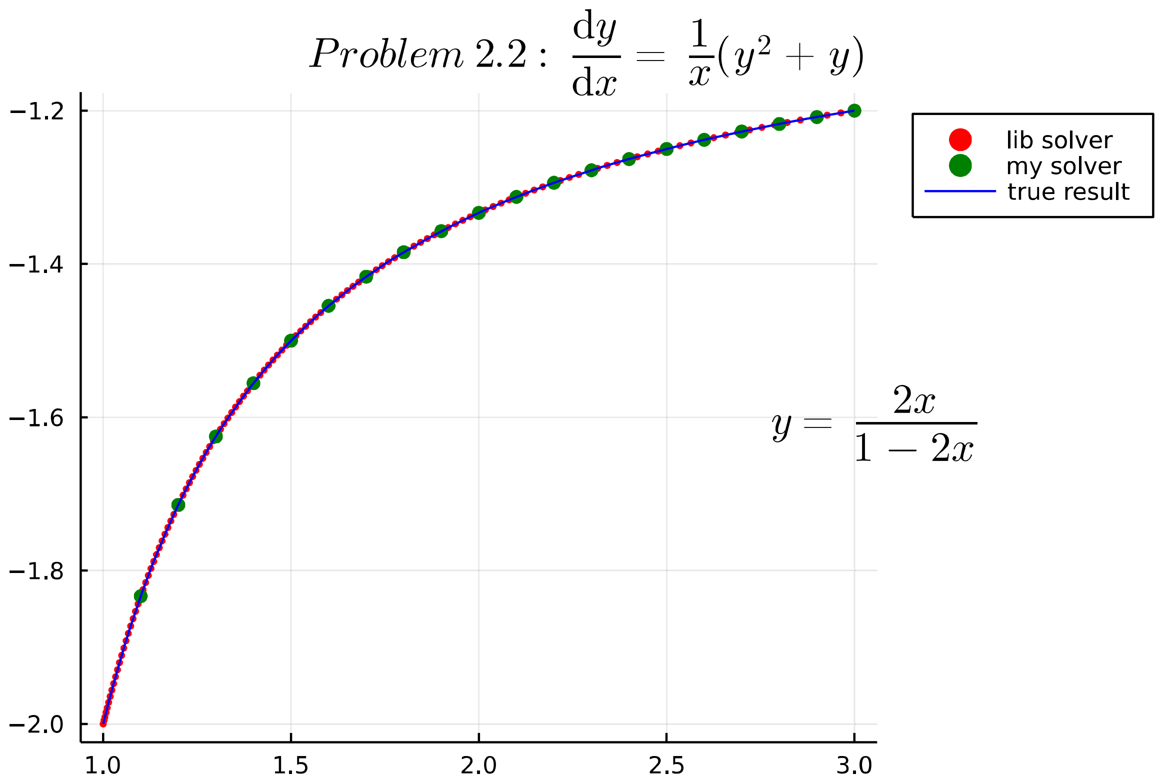





$Problem\ 2.1 : \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y}{x} + x^2e^x$
lib solver
my solver
true result
$y = x^2(e^x - e)$
150
100
50
0
1.0
1.5
2.0
2.5
3.0


$Problem\ 2.2 : \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}(y^2 + y)$
lib solver
my solver
true result
$y = \frac{2x}{1 - 2x}$
−1.2
−1.4
−1.6
−1.8
−2.0
1.0
1.5
2.0
2.5
3.0


$Problem\ 2.2:\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}(y^2 + y)$
lib solver
my solver
true result
$y = \frac{2x}{1-2x}$
−1.2
−1.4
−1.6
−1.8
−2.0
1.0
1.5
2.0
2.5
3.0

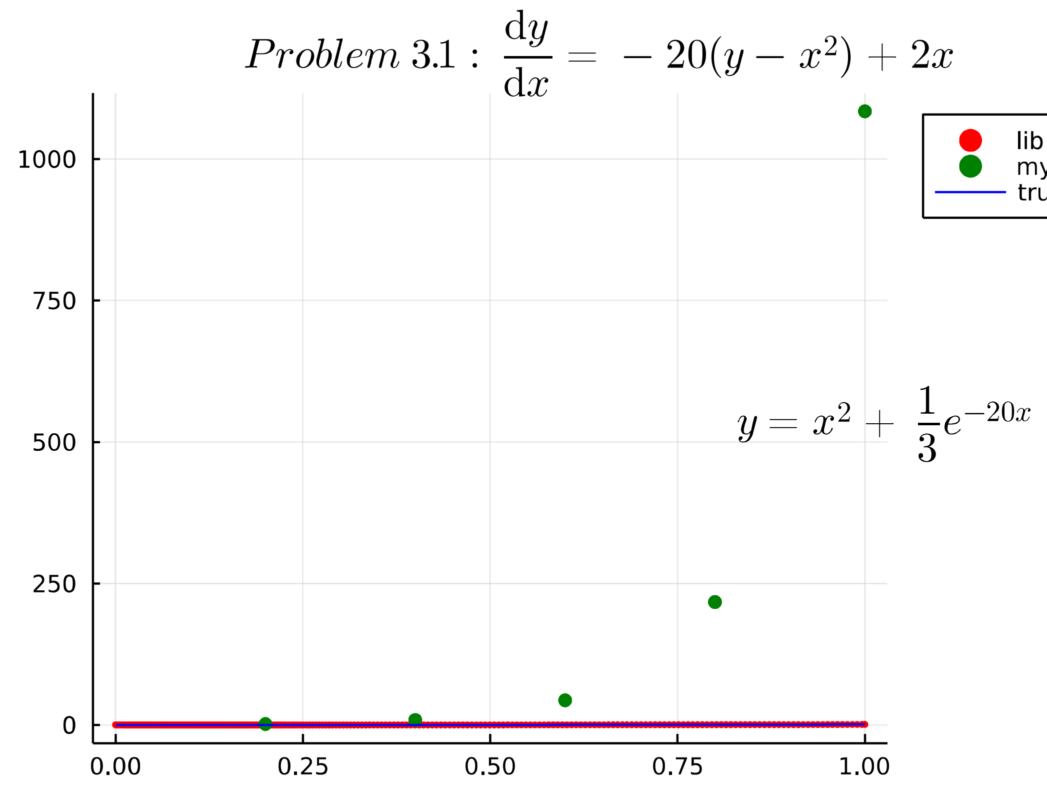

$Problem\ 2.2:\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}(y^2 + y)$
lib solver
my solver
true result
$y = \frac{2x}{1-2x}$
−1.2
−1.4
−1.6
−1.8
−2.0
1.0
1.5
2.0
2.5
3.0

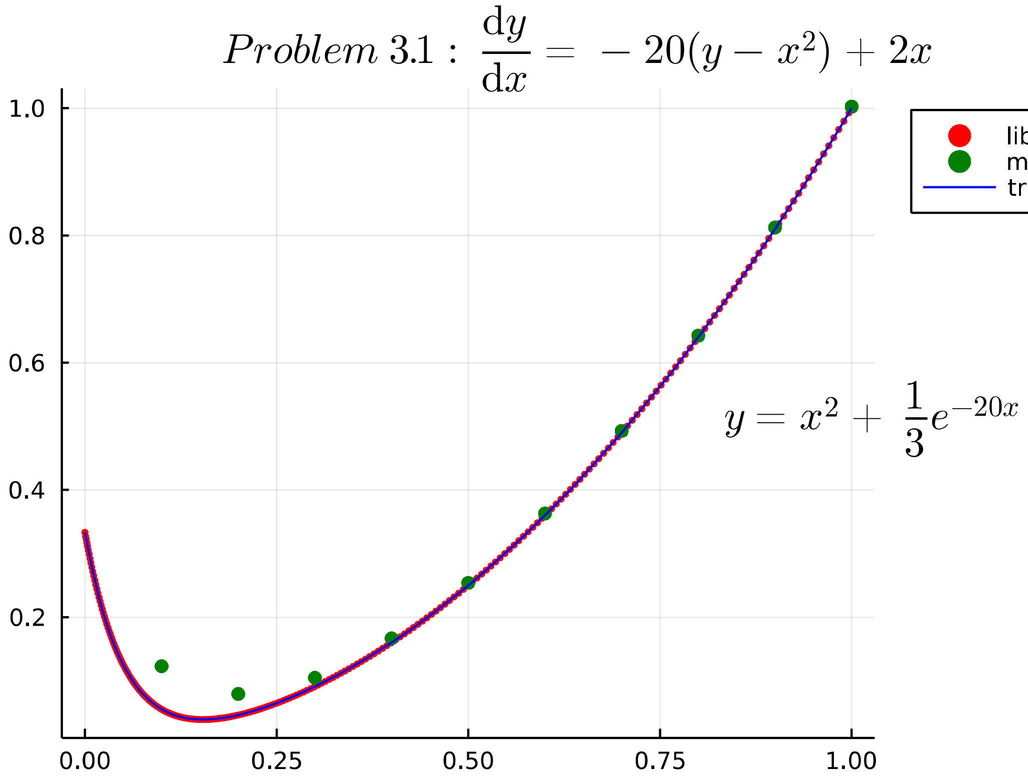
```
$Problem\ 2.1:\frac{\mathrm{d} y}{\mathrm{d} x}=\frac{2y}{x}+x^2 e^x$

Iternum: 5      Runge-Kutta:
x: [1.4, 1.7999999999999998, 2.1999999999999997, 2.5999999999999996, 2.9999999999999996]
y: [2.613942792503426, 10.776313166418575, 30.491654203794223, 72.58559860601221, 156.22519827584796]

Iternum: 10      Runge-Kutta:
x: [1.2, 1.4, 1.5999999999999999, 1.7999999999999998, 1.9999999999999998, 2.1999999999999997, 2.4, 2.6, 2.8000000000000003, 3.0000000000000004]
y: [0.8663791119740196, 2.619740520468712, 5.719895279538559, 10.79201759748925, 18.6808523645173, 30.521598135366503, 47.83236583269365, 72.634503537672, 107.60885199118545, 156.2982574428725]

Iternum: 20      Runge-Kutta:
x: [1.1, 1.2000000000000002, 1.3000000000000003, 1.4000000000000004, 1.5000000000000004, 1.6000000000000005, 1.7000000000000006, 1.8000000000000007, 1.9000000000000008, 2.000000000000001, 2.100000000000001, 2.200000000000001, 2.300000000000001, 2.4000000000000012, 2.5000000000000013, 2.6000000000000014, 2.7000000000000015, 2.8000000000000016, 2.9000000000000017, 3.0000000000000018]
y: [0.3459102873064402, 0.8666216927288839, 1.6071813476640324, 2.620311305871806, 3.9676018979880405, 5.720879324244457, 7.963771792604615, 10.793501783648523, 14.32293572758867, 18.6829265676522, 24.02498941966458, 30.524355889829184, 38.38345866002602, 47.83590478093721, 59.15100382752139, 72.6389257808317, 88.65657333091889, 107.61426438930823, 129.9833331156541, 156.30477188083754]


$Problem\ 2.2:\frac{\mathrm{d} y}{\mathrm{d} x}=\frac{1}{x}(y^2+y)$

Iternum: 5      Runge-Kutta:
x: [1.4, 1.7999999999999998, 2.1999999999999997, 2.5999999999999996, 2.9999999999999996]
y: [-1.5539889980952382, -1.3836172899114931, -1.2934015269193302, -1.237540157935232, -1.1995479584579267]

Iternum: 10      Runge-Kutta:
x: [1.2, 1.4, 1.5999999999999999, 1.7999999999999998, 1.9999999999999998, 2.1999999999999997, 2.4, 2.6, 2.8000000000000003, 3.0000000000000004]
y: [-1.714245180451154, -1.5555228848496194, -1.4545197492007562, -1.3845945062866782, -1.3333158560752736, -1.2941026605729438, -1.263144798904635, -1.2380836211681467, -1.2173808733204385, -1.1999905397087856]

Iternum: 20      Runge-Kutta:
x: [1.1, 1.2000000000000002, 1.3000000000000003, 1.4000000000000004, 1.5000000000000004, 1.6000000000000005, 1.7000000000000006, 1.8000000000000007, 1.9000000000000008, 2.000000000000001, 2.100000000000001, 2.200000000000001, 2.300000000000001, 2.4000000000000012, 2.5000000000000013, 2.6000000000000014, 2.7000000000000015, 2.8000000000000016, 2.9000000000000017, 3.0000000000000018]
y: [-1.8333328294259301, -1.7142851698413297, -1.6249995001712725, -1.5555551110526054, -1.4999996057103289, -1.454545102841952, -1.416666350536796, -1.3846150982409504, -1.3571425958336794, -1.333333093327103, -1.3124997782659393, -1.2941174411362102, -1.2777775856503875, -1.2631577147371074, -1.24999983073609, -1.2380950783953766, -1.22727257614238, -1.2173911609365085, -1.2083331969086475, -1.1999998699271444]
```

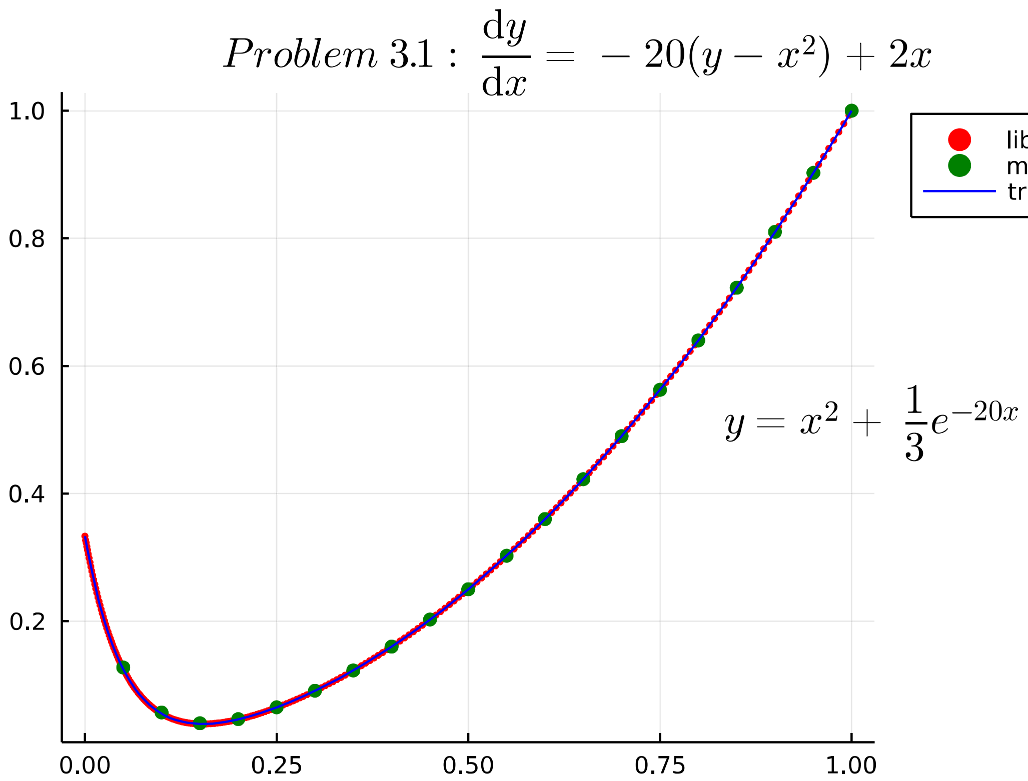
问题 3

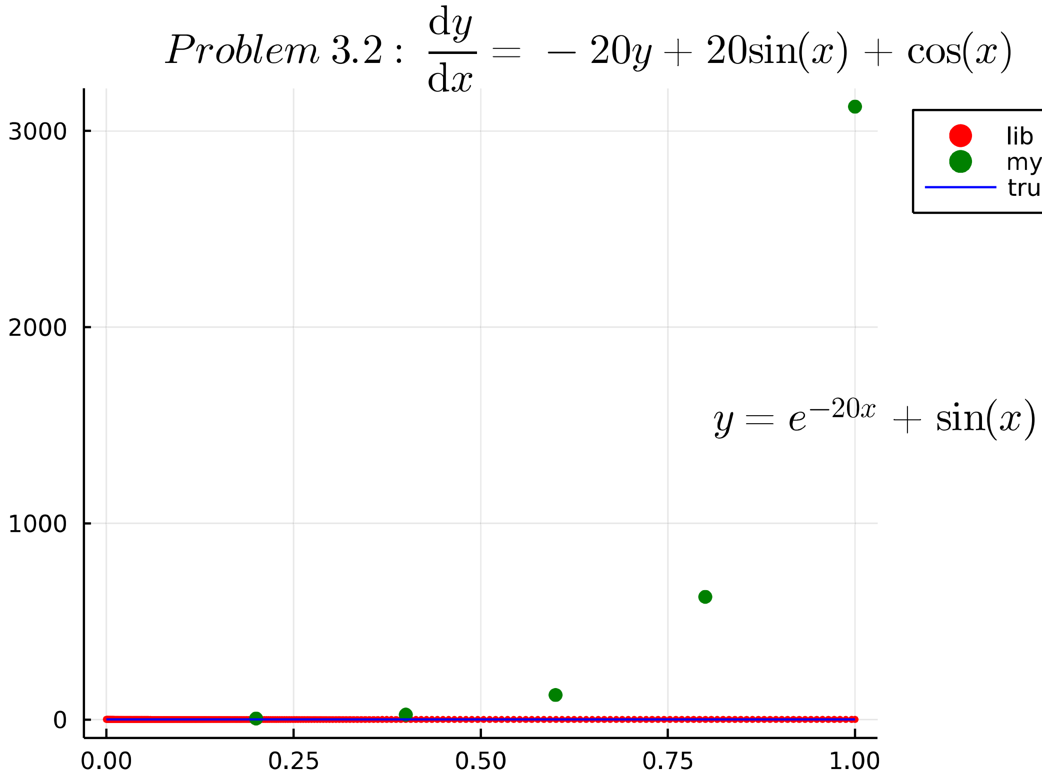
In [7]:
```julia
f1(y, p, x) = -20(y - x^2) + 2x
xspan = (0.0, 1.0)
y0 = 1 / 3
f2(x, y) = -20(y - x^2) + 2x
f3(x) = x^2 + 1 / 3 * exp(-20x)
```

```
show_result(f1, f2, f3, xspan, y0, iternums, true, false, L"Problem\ 3.1: \frac{\mat


f1(y, p, x) = -20y + 20sin(x) + cos(x)
xspan = (0.0, 1.0)
y0 = 1.0
f2(x, y) = -20y + 20sin(x) + cos(x)
f3(x) = exp(-20x) + sin(x)
show_result(f1, f2, f3, xspan, y0, iternums, true, false, L"Problem\ 3.2: \frac{\mat


f1(y, p, x) = -20(y - exp(x)sin(x)) + exp(x) * (sin(x) + cos(x))
xspan = (0.0, 1.0)
y0 = 0.0
f2(x, y) = -20(y - exp(x)sin(x)) + exp(x) * (sin(x) + cos(x))
f3(x) = exp(x) * sin(x)
show_result(f1, f2, f3, xspan, y0, iternums, true, false, L"Problem\ 3.3: \frac{\mat
```

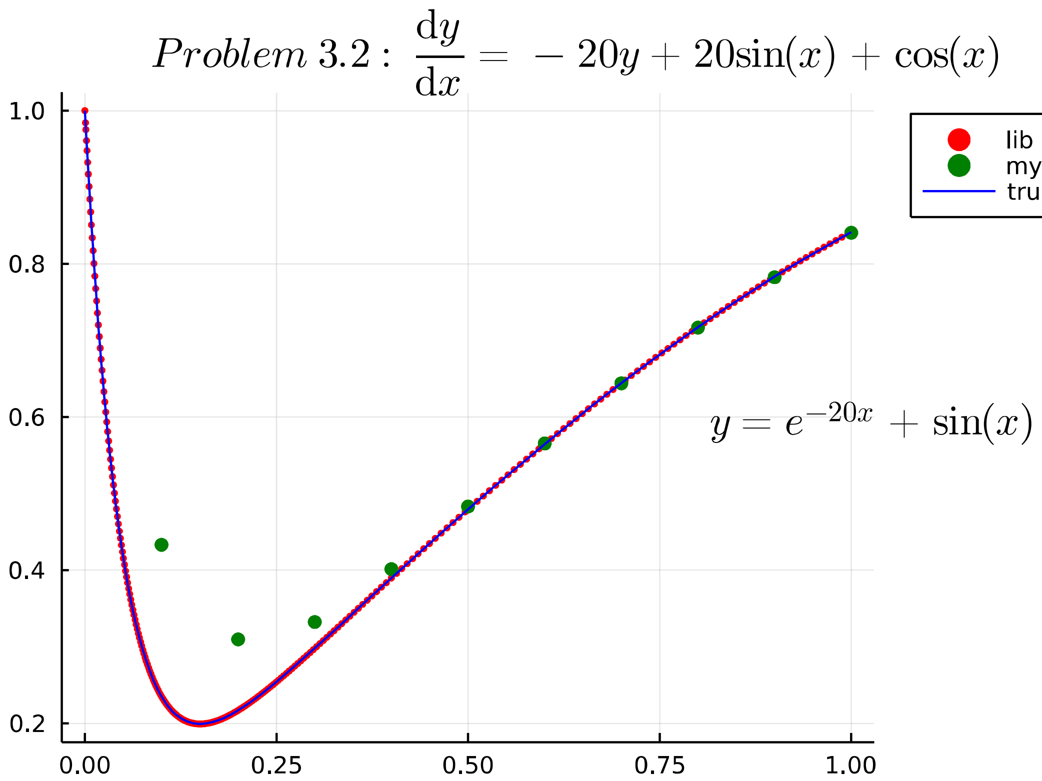



$Problem\ 3.1:\ \frac{\mathrm{d}y}{\mathrm{d}x} = -20(y - x^2) + 2x$
lib solver
my solver
true result
$y = x^2 + \frac{1}{3}e^{-20x}$
1.0
0.8
0.6
0.4
0.2
0.00
0.25
0.50
0.75
1.00

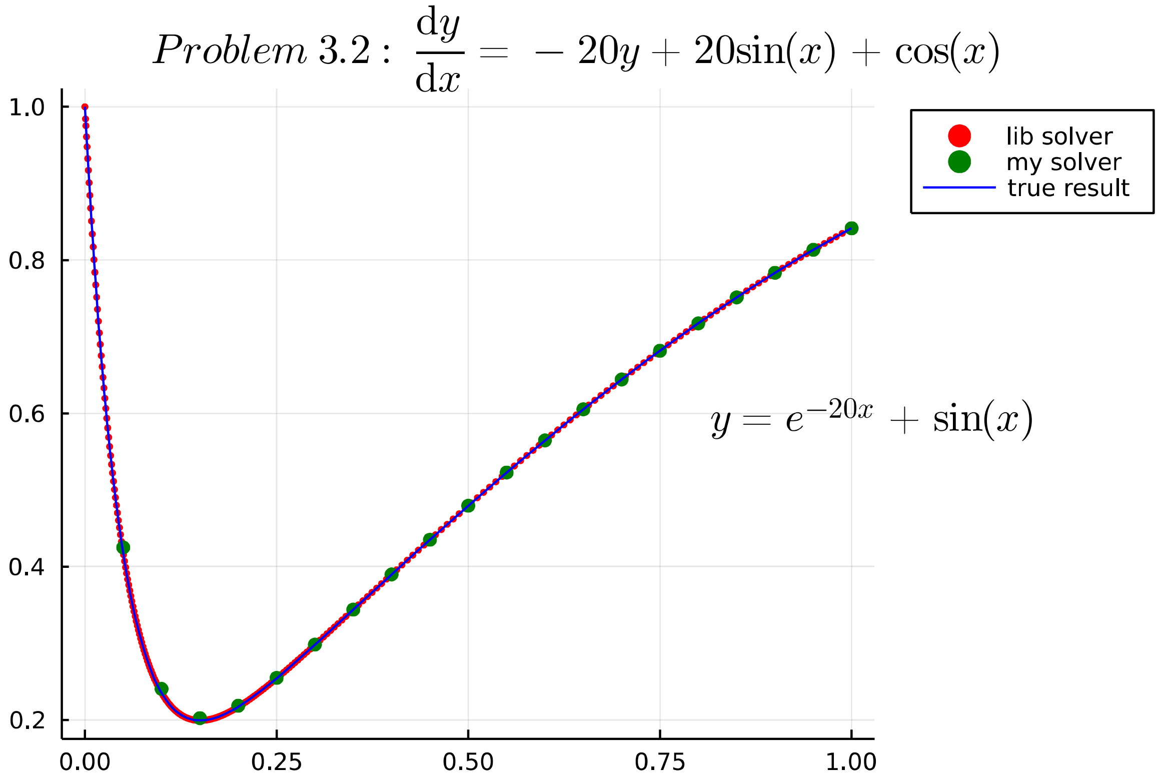

$Problem\ 3.1:\ \frac{\mathrm{d}y}{\mathrm{d}x} = -20(y - x^2) + 2x$
lib solver
my solver
true result
$y = x^2 + \frac{1}{3}e^{-20x}$
1.0
0.8
0.6
0.4
0.2
0.00
0.25
0.50
0.75
1.00

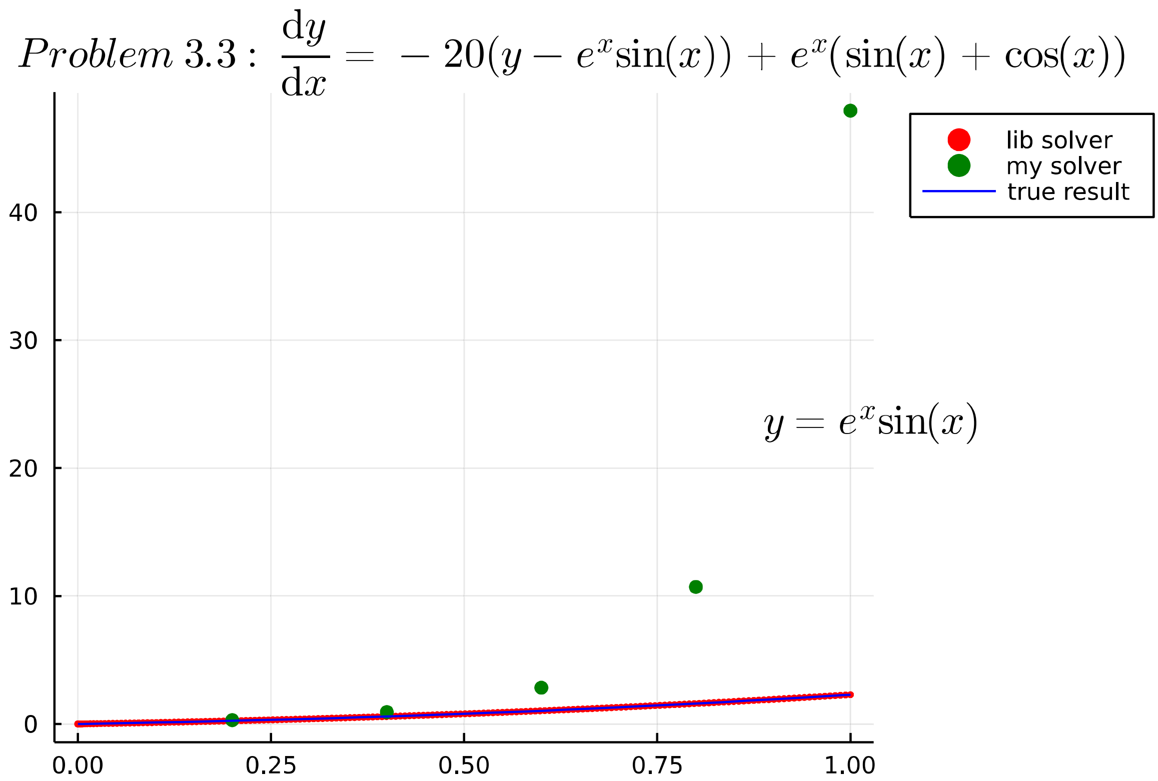


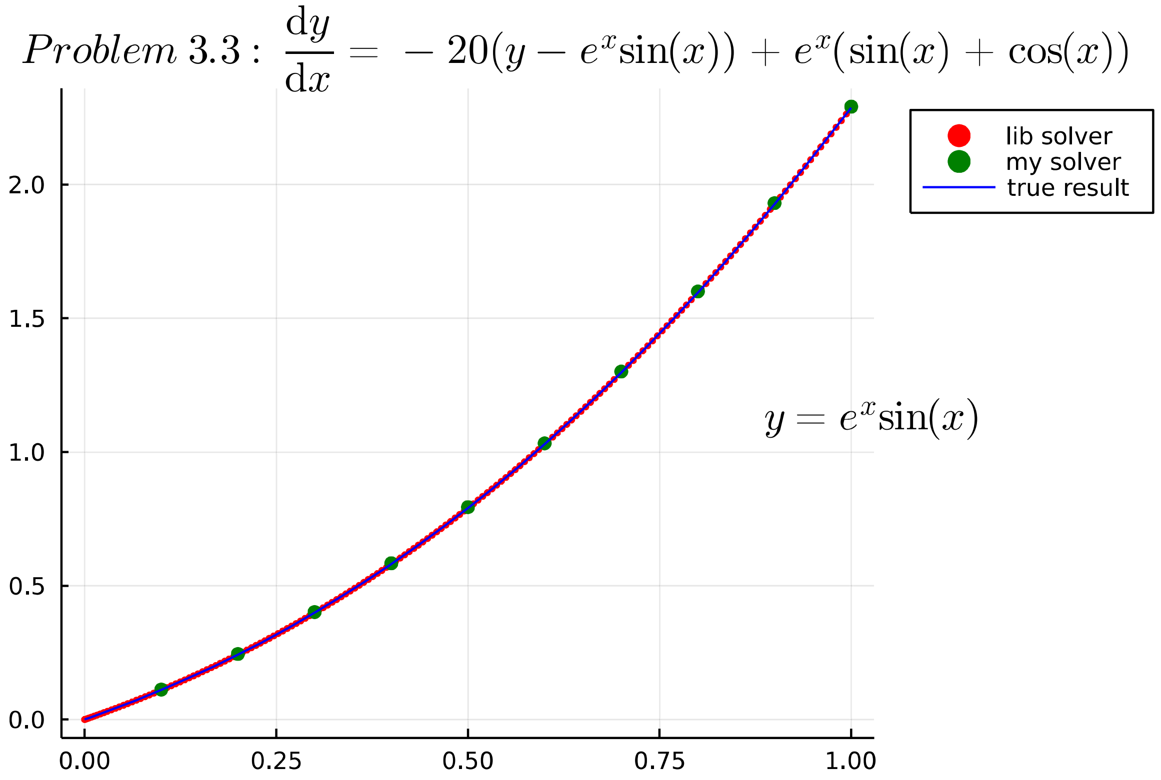
$Problem\ 3.2: \frac{\mathrm{d}y}{\mathrm{d}x} = -20y + 20\sin(x) + \cos(x)$
3000
2000
1000
0
0.00
0.25
0.50
0.75
1.00
lib solver
my solver
true result
$y = e^{-20x} + \sin(x)$

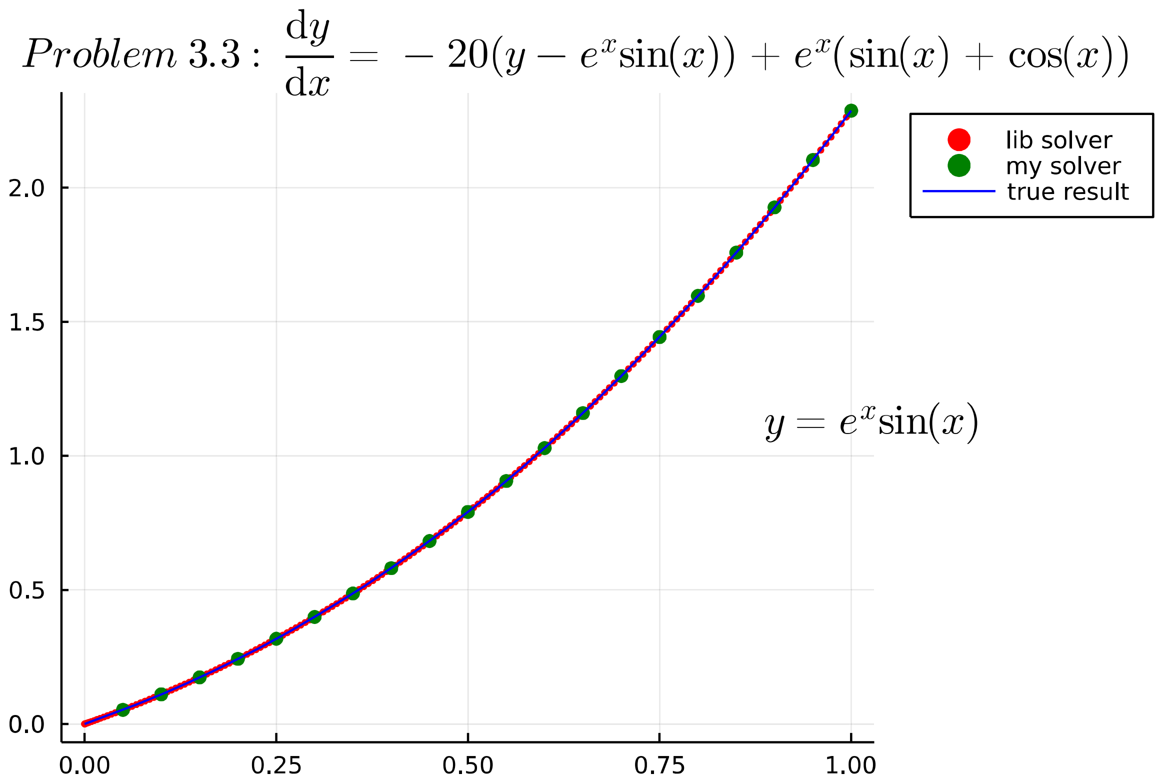


$Problem\ 3.2: \frac{\mathrm{d}y}{\mathrm{d}x} = -20y + 20\sin(x) + \cos(x)$
1.0
0.8
0.6
0.4
0.2
0.00
0.25
0.50
0.75
1.00
lib solver
my solver
true result
$y = e^{-20x} + \sin(x)$


$Problem\ 3.2: \frac{\mathrm{d}y}{\mathrm{d}x} = -20y + 20\sin(x) + \cos(x)$
1.0
0.8
0.6
0.4
0.2
0.00
0.25
0.50
0.75
1.00
lib solver
my solver
true result
$y = e^{-20x} + \sin(x)$


$Problem\ 3.3: \frac{\mathrm{d}y}{\mathrm{d}x} = -20(y - e^x\sin(x)) + e^x(\sin(x) + \cos(x))$
40
30
20
10
0
0.00
0.25
0.50
0.75
1.00
lib solver
my solver
true result
$y = e^x\sin(x)$


$Problem\ 3.3:\ \frac{\mathrm{d}y}{\mathrm{d}x} = -20(y - e^x\sin(x)) + e^x(\sin(x) + \cos(x))$
lib solver
my solver
true result
$y = e^x\sin(x)$
2.0
1.5
1.0
0.5
0.0
0.00
0.25
0.50
0.75
1.00


$Problem\ 3.3:\ \frac{\mathrm{d}y}{\mathrm{d}x} = -20(y - e^x\sin(x)) + e^x(\sin(x) + \cos(x))$
lib solver
my solver
true result
$y = e^x\sin(x)$
2.0
1.5
1.0
0.5
0.0
0.00
0.25
0.50
0.75
1.00

```
$Problem\ 3.1: \frac{\mathrm{d} y}{\mathrm{d} x}=-20(y-x^2)+2x$

Iternum: 5      Runge-Kutta:
x: [0.2, 0.4, 0.6000000000000001, 0.8, 1.0]
y: [1.7600000000000002, 8.813333333333336, 43.68000000000001, 217.29333333333338, 1084.3200000000002]

Iternum: 10     Runge-Kutta:
x: [0.1, 0.2, 0.30000000000000004, 0.4, 0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999, 0.9999999999999999]
y: [0.12277777777777779, 0.07925925925925928, 0.10475308641975309, 0.16658436213991773, 0.25386145404663923, 0.36295381801554644, 0.4926512726718488, 0.6425504242239496, 0.8125168080746498, 1.00250560269155]

Iternum: 20     Runge-Kutta:
x: [0.05, 0.1, 0.15000000000000002, 0.2, 0.25, 0.3, 0.35, 0.39999999999999997, 0.44999999999999996, 0.49999999999999994, 0.5499999999999999, 0.6, 0.65, 0.7000000000000001, 0.7500000000000001, 0.8000000000000002, 0.8500000000000002, 0.9000000000000002, 0.9500000000000003, 1.0000000000000002]
y: [0.12755208333333334, 0.056946614583333346, 0.04015706380208334, 0.04667348225911459, 0.0650546391805013, 0.09101007302602132, 0.12293086071809133, 0.16021365610261756, 0.2026322043718149, 0.2501016599727639, 0.30259020582311974, 0.3600859105170032, 0.42258429977720957, 0.49008369574978694, 0.5625834692395035, 0.6400833842981473, 0.7225833524451387, 0.8100833405002607, 0.9025833360209314, 1.000083334341183]


$Problem\ 3.2: \frac{\mathrm{d} y}{\mathrm{d} x}=-20y+20\sin(x)+\cos(x)$

Iternum: 5      Runge-Kutta:
x: [0.2, 0.4, 0.6000000000000001, 0.8, 1.0]
y: [5.197338106220029, 25.376170704380762, 125.48681526112966, 625.3120955171343, 3123.7951509471586]

Iternum: 10     Runge-Kutta:
x: [0.1, 0.2, 0.30000000000000004, 0.4, 0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999, 0.9999999999999999]
y: [0.43313899649719434, 0.309660468004797, 0.33232466670513466, 0.4014139712639834, 0.4830743414705462, 0.565435279659871, 0.6439890044827506, 0.7167223470605888, 0.7824991512012693, 0.840525720595564]

Iternum: 20     Runge-Kutta:
x: [0.05, 0.1, 0.15000000000000002, 0.2, 0.25, 0.3, 0.35, 0.39999999999999997, 0.44999999999999996, 0.49999999999999994, 0.5499999999999999, 0.6, 0.65, 0.7000000000000001, 0.7500000000000001, 0.8000000000000002, 0.8500000000000002, 0.9000000000000002, 0.9500000000000003, 1.0000000000000002]
y: [0.424978518601945, 0.24045622213059903, 0.20216843904311116, 0.21843866341230506, 0.25481165110875065, 0.2982910222151891, 0.3439285510509592, 0.38979533635034436, 0.4350961733328589, 0.4794626228591924, 0.5226880879214766, 0.5646286383723231, 0.6051659863042762, 0.6441937625688693, 0.6816125254669342, 0.7173280378596233, 0.7512507634216402, 0.783295813201471, 0.8133830538368221, 0.8414372688602679]


$Problem\ 3.3: \frac{\mathrm{d} y}{\mathrm{d} x}=-20(y-e^x \sin(x))+e^x (\sin(x) + \cos(x))$

Iternum: 5      Runge-Kutta:
x: [0.2, 0.4, 0.6000000000000001, 0.8, 1.0]
y: [0.29864621275013403, 0.9272198700273476, 2.835477338896381, 10.710885330937327, 47.941446381632616]

Iternum: 10     Runge-Kutta:
x: [0.1, 0.2, 0.30000000000000004, 0.4, 0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999, 0.9999999999999999]
```

```
y: [0.11205510913037421, 0.2451165144244346, 0.4017780966782987, 0.5840969565792278, 0.793822052967138, 1.0324183053426443, 1.3010149883505369, 1.6003210120174918, 1.9305210337840668, 2.291156923060078]

Iternum: 20     Runge-Kutta:
x: [0.05, 0.1, 0.15000000000000002, 0.2, 0.25, 0.3, 0.35, 0.39999999999999997, 0.44999999999999996, 0.49999999999999994, 0.5499999999999999, 0.6, 0.65, 0.7000000000000001, 0.7500000000000001, 0.8000000000000002, 0.8500000000000002, 0.9000000000000002, 0.9500000000000003, 1.0000000000000002]
y: [0.05259503995574239, 0.11040898628183947, 0.17370939051652695, 0.24274900092603802, 0.31777169155820517, 0.3990135524673282, 0.48670206962488405, 0.5810544890982305, 0.6822757724973431, 0.7905562930220894, 0.9060693250287787, 1.0289683441295572, 1.1593841416649548, 1.2974217527850616, 1.4431571960299754, 1.5966340222275905, 1.7578596709840264, 1.9268016337536287, 2.1033834233412323, 2.2874803506747066]
```

思考题

1. 对实验 1，数值解和解析解相同吗？为什么？试加以说明。

   是相同的，因为解是线性函数，能够通过所得数值解的两个点确定直线的方程，即等价于得到了解析解

2. 对实验 2，N 越大越精确吗？试加以说明。

   从实验的结果来看，并不是，因为当n=5的时候已经获得足够精确的数值解了，再增大n的值只是增加了计算量，却不能再明显提高结果的精度，得不偿失

3. 对实验 3，N 较小会出现什么现象？试加以说明

   当n较小的时候所得数值解和正确结果相差较大，结果失真，说明在一定条件下确实需要更大的n来更好的获得数值解。具体的条件取决于待求解微分方程性质。