

# 作业四

1. (1) 令  $f(x) = 1, x, x^2$  时有

$$\begin{cases} H_1 + H_0 + H_1 = 2h \\ H_1(-h) + H_0(0) + H_1(h) = 0 \\ H_1(h^2) + H_0(0) + H_1(h^2) = \frac{2}{3}h^3 \end{cases} \Rightarrow \begin{cases} H_1 = \frac{1}{3}h, H_0 = \frac{4}{3}h, H_1 = \frac{1}{3}h \\ \int_{-h}^h f(x) dx \approx \frac{1}{3}hf(-h) + \frac{4}{3}hf(0) + \frac{1}{3}hf(h) \end{cases}$$

令  $f(x) = x^3$ , 左边 = 0 = 右边 = 0

令  $f(x) = x^4$ , 左边 =  $\frac{2}{5}h^5 \neq$  右边 =  $\frac{2}{3}h^5$ , 故代数精度为 3 次

(2) 令  $f(x) = 1, x^2, x^4$  时有

$$\begin{cases} H_1 + H_0 + H_1 = 4h \\ -hH_1 + 0 + hH_1 = 0 \\ h^2H_1 + 0 + h^2H_1 = \frac{16}{3}h^3 \end{cases} \Rightarrow \begin{cases} H_1 = \frac{8}{3}h, H_0 = -\frac{4}{3}h, H_1 = \frac{8}{3}h \\ \int_{-2h}^{2h} f(x) dx \approx \frac{8}{3}hf(-h) - \frac{4}{3}hf(0) + \frac{8}{3}hf(h) \end{cases}$$

令  $f(x) = x^3$ , 左边 = 0 = 右边 = 0, 故代数精度为 2 次

令  $f(x) = x^4$ , 左边 =  $\frac{64}{5}h^5 \neq$  右边 =  $\frac{16}{3}h^5$

(3) 由于有两个待定参数,  $f(x) = 1$  时成立. 令  $f(x) = x, x^2$ .

$$\begin{cases} \frac{1}{3}(1 + 2x_1 + 3x_2) = 0 \\ \frac{1}{3}(1 + 2x_1^2 + 3x_2^2) = \frac{2}{3} \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1+\sqrt{6}}{5}, x_2 = \frac{3+\sqrt{6}}{5} \\ x_1 = \frac{1-\sqrt{6}}{5}, x_2 = \frac{3-\sqrt{6}}{5} \end{cases}$$

$$\Rightarrow \begin{cases} \int_{-1}^1 f(x) dx \approx \frac{1}{3}f(-1) + 2f\left(\frac{1+\sqrt{6}}{5}\right) + 3f\left(\frac{3+\sqrt{6}}{5}\right) \\ \int_{-1}^1 f(x) dx \approx \frac{1}{3}f(-1) + 2f\left(\frac{1-\sqrt{6}}{5}\right) + 3f\left(\frac{3-\sqrt{6}}{5}\right) \end{cases}$$

令  $f(x) = x^3$ , 左边 = 0 = 右边 = 0, 故代数精度为 2 次

(4) 令  $f(x) = 1, x$  时恒成立, 令  $f(x) = x^2$ .

$$\frac{h}{2}(0+h^2) + ah^2(0-2h) = \frac{1}{3}h^3 \Rightarrow a = \frac{1}{12}$$

$$\Rightarrow \int_0^h f(x) dx \approx \frac{1}{2}h[f(0) + f(h)] + \frac{1}{12}h^2[f'(0) - f'(h)]$$

令  $f(x) = x^3$ , 左边 =  $\frac{1}{4}h^4$  = 右边 =  $\frac{1}{4}h^4$

令  $f(x) = x^4$ , 左边 =  $\frac{1}{5}h^5 \neq$  右边 =  $\frac{1}{6}h^5$ , 故代数精度为 3 次

9.

```
1 using Printf
2 function romberg(f::Function, xlim, n, ε)
3     a, b = xlim
4     h = b - a
5     T = zeros(n, n)
6     T[1, 1] = 1 / 2 * h * (f(a) + f(b))
7     for i = 1:n
8         tmpsum = 0
9         jmax = 2^(i - 1)
10        for j = 1:jmax
11            tmpsum += f(a + (j - 1 / 2) * h)
12        end
13        T[i+1, 1] = 1 / 2 * T[i, 1] + 1 / 2 * h * tmpsum
14        for m = 1:i
15            T[i+1, m+1] = (4^m * T[i+1, m] - T[i, m]) / (4^m - 1)
16        end
17        for m = 1:i
18            @printf("%12.9f\t", T[i, m])
19        end
20        @printf("\n")
21        if i > 1 && abs(T[i+1, i+1] - T[i, i]) < ε
22            @printf("Accuracy requirement satisfied.\n\n")
23            break
24        end
25        h /= 2
26    end
27 end
```

```
1 f(x) = 2 / sqrt(pi) * exp(-x)
2 ε = 1e-6
3 xlim = 0, 1
4 romberg(f, xlim, 20, ε)
5 f(x) = exp(-x^2)
6 ε = 1e-6
7 xlim = 0, 0.8
8 romberg(f, xlim, 20, ε)
```

```
1 Problem 9.1
2 0.771743332
3 0.728069946      0.713512151
4 0.716982762      0.713287034      0.713272026
5 Accuracy requirement satisfied.
6 Problem 9.2
7 0.610916970
8 0.646316000      0.658115677
9 0.654851153      0.657696204      0.657668239
10 0.656966396      0.657671477      0.657669829      0.657669854
11 Accuracy requirement satisfied.
```

- 9.1, 取  $I = 0.713272026$
- 9.2, 取  $I = 0.657669854$