

Recovering Probability Moments from Inflation Option Prices

Ethan Benchetrit
Evelyn Fagbola
Hanane Larbi
Lorenzo Marasco
Eduardo Schutt

Quantitative Methods in Finance

• Report scope

- dataset: 1y inflation caps, floors, swaps; areas: EU, US
- objective: option price time series → implied moments (mean, variance, skewness, kurtosis)
- focus: identification, stability; no forecasting

• Deliverables

- code: [code in github](#)
- outputs: `moments_main.csv`, `moments_direct_bkm.csv`, `moments_parametric_lognormal.csv`, `moments_parametric_normal.csv`

• LLM disclosure

- ChatGPT (GPT-5.2 Thinking)
- usage: diagnostic narratives and LaTeX bullet structuring
- no usage: data generation, result fabrication, code execution on hidden data

• Literature review: method families & mappings

1. Prices → pdf (risk-neutral density / state prices)

- Breeden–Litzenberger (1978) (nonparametric; differentiation of call price curve; ~3315 cites)
- Jackwerth–Rubinstein (1996) (nonparametric; constrained recovery / smoothing; ~1614 cites)
- Aït-Sahalia–Lo (1998/1995) (nonparametric; kernel / smoothing; ~1591 cites)
- spline/shape-constrained smoothing (convexity/monotonicity enforcement)
- parametric densities (lognormal / mixtures) (pdf implied by calibrated parameters)

2. Pdf → moments (mean/var/skew/kurt)

- numerical quadrature on grid (trapz) on truncated support
- analytic moments (parametric families; e.g., lognormal moments)
- parametric fit to recovered pdf (diagnostic; e.g., Normal fit on grid)

3. Prices → moments (direct)

- Bakshi–Kapadia–Madan style moment integrals (2003) (~ 1858 cites)
- static replication using OTM option payoffs; numerical integration over strikes

• Citation-based ranking (high → low; by approximate cites)

1. Prices → pdf

- Breeden–Litzenberger (1978) (~ 3315)
- Jackwerth–Rubinstein (1996) (~ 1614)
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2. Prices → moments

- Bakshi–Kapadia–Madan (2003) (~ 1858)
- parametric calibration families (model-dependent; citations dispersed)

3. Pdf → moments

- numerical integration (standard; method-agnostic; citations dispersed)
- analytic moments (family-specific; citations dispersed)

• Implemented methods (cross-sectional per date/area)

1. Most popular replication: Breeden–Litzenberger (two-sided)

- file: `method_main.py` → `moments_main.csv`
- inputs: caps + floors + swaps
- construction: call-equivalent curve (puts converted via parity)
- smoothing: spline across strikes
- density proxy: second derivative; clipping negatives; renormalization
- moments: numerical integration on strike grid
- gates: monotone decreasing call curve; convexity in strike; parity residual filter

2. Other replicated method: BKM direct moments (tail-stabilized)

- file: `method_moments_direct_bkm.py` → `moments_direct_bkm.csv`
- inputs: caps + floors + swaps
- preprocessing: deduplicate strikes; PV → undiscounted payoffs (divide by B)
- OTM selection: puts ($K \leq K_*$), calls ($K \geq K_*$)
- gates: parity residual filter; monotonicity of OTM payoffs; min strikes both wings
- stabilization: synthetic tail points (zero payoff); optional winsorization
- moments: raw moments (2–4) via strike integrals; central moments (var/skew/kurt)

3. Parametric benchmark: Lognormal calibration

- file: `method_parametric_lognormal.py` → `moments_parametric_lognormal.csv`
- assumption: $X = I_T/I_0$ lognormal
- calibration: fit (μ, σ) to OTM caps+floors; bounded σ ; mean anchoring to K_*
- outputs: analytic mean/variance/skew/kurt; RMSE diagnostic
- gates: parity residual filter; monotonicity of call/put PV curves; min strikes both wings

4. Diagnostic benchmark: Normal fit to BL-proxy pdf

- file: `method_parametric_normal.py` → `moments_parametric_normal.csv`
- steps: single-instrument price curve → spline → BL-proxy pdf → fit Normal on grid
- filters: convexity/monotonicity; boundary-mass threshold; variance sanity caps
- role: diagnostic only (positivity mismatch; truncation sensitivity)

- **Comparison logic (identical inputs)**

- common keys: (date, area)
- intersection sample: retain rows where all compared methods return valid moments
- plots produced: time series of mean; time series of variance (log scale)
- diagnostics stored: strike span proxies; counts of OTM puts/calls; tail parameters; parity residual pass/fail

- **Empirical interpretation: identification & stability**

1. **Mean (first moment)**

- strong cross-method agreement: $BL \approx Lognormal \approx BKM$
- driver: near-ATM information; swap-anchored center (K_*); parity enforcement
- Normal benchmark: flat/spiky behavior; diagnostic only

2. **Variance (second moment)**

- systematic gaps: BKM variance lower than $BL/Lognormal$ in sparse-strike periods
- driver: tail dependence; missing far-OTM strikes; truncation sensitivity
- convergence episodes: improved strike coverage / liquidity (stress periods)

3. **Skewness (third central moment)**

- dependence: left-right tail balance (puts vs calls)
- instability sources: wing asymmetry; sparse tail quotes; regularization sensitivity
- interpretation: caution; conditional on balanced OTM coverage

4. **Kurtosis (fourth central moment)**

- dependence: extreme tails; highest sensitivity to missing strikes
- instability sources: truncation; tail-point assumptions; outlier quotes
- interpretation: very cautious; mainly qualitative tail-risk indicator

- **Disagreement diagnosis**

- trigger: material divergence in mean/variance across methods
- checks: OTM strike counts; observed strike span; tail-point usage; parity residual filter; monotonicity failures
- primary cause: variance divergence aligned with limited far-OTM coverage; BKM downward bias

- **Main conclusion: best way to work on moments**

1. **Mean**

- preferred: any method using both wings + parity + swap anchoring
- robustness: high; low tail dependence

2. Variance

- preferred: tail-regularized methods (parametric lognormal; smoothed two-sided BL)
- secondary: BKM with explicit tail stabilization; interpret as lower-bound under sparse tails

3. Skewness

- preferred: parametric/regularized methods; require balanced wing coverage
- caution: high sensitivity to asymmetric strike availability

4. Kurtosis

- preferred: parametric with strong tail structure; report with sensitivity flags
- caution: weakest identification; treat as qualitative