Recovering probability moments from inflation option prices

Ethan Benchetrit, Evelyn Fagbola, Hanane Larbi,
Lorenzo Marasco, Eduardo Schutt

Quantitative Methods in Finance - Prof. Eric Vansteenberghe

January 12, 2026

Project scope and objectives

- **Dataset:** 1-year zero-coupon inflation caps, floors, and swaps (EU and US areas).
- **Main goal:** Recovering implied moments (mean, variance, skewness, kurtosis) from risk-neutral distributions.
- **Methodology:** Focus on identification and stability across different dates.
- **Scientific boundary:** This is a recovery project, not a forecasting exercise.

Literature review: classification per task

1. **Task 1.3.1: Prices → PDF (Nonparametric)**
   - Breeden-Litzenberger (1978): second derivative method (∼3,315 cites).
   - Jackwerth-Rubinstein (1996): constrained recovery and smoothing (∼1,614 cites).
2. **Task 1.3.2: PDF → moments (Analytical/Numeric)**
   - Numerical quadrature: standard integration (trapezoidal rule).
   - Analytic formulas: moments for parametric families like Lognormal.
3. **Task 1.3.3: Prices → moments (Direct)**
   - Bakshi-Kapadia-Madan (2003): direct extraction via replication (∼1,858 cites).

Data processing and quality gates

- **Scaling:** Prices divided by 100 to get price-per-1 units.
- **Reference strikes:** ATM gross strike K^* defined by 1y inflation swap rates.
- **No-arbitrage filters:**
  - Put-call parity residual check ($PARITY\_EPS$ gate).
  - Shape constraints: enforcing monotonicity and convexity.
  - Strike span diagnostics: checking K_{min} and K_{max} coverage.

Method 1: Breeden-Litzenberger (1978) (1)

1. Data cleaning and quality control

- **OTM selection**: Using Out-of-the-Money options to capture the most reliable information.
- **Parity check**: Verifying the Put-Call parity to ensure market consistency.
- **Data alignment**: Converting Floor prices into Call equivalents:

$$C = P + B(F - K)$$

2. Price curve construction

- **Cubic splines**: Creating a smooth curve from discrete market points.
- **Financial constraints**:
  - Downward slope: Prices must decrease as the strike increases.
  - Convexity: The curve must be convex to avoid arbitrage opportunities.

Method 1: Breeden-Litzenberger (1978) (2)

3. The Breeden-Litzenberger method

The probability density $f(S_T)$ is calculated using the second derivative of the Call price curve:

$$f(S_T) = e^{rT} \frac{\partial^2 C}{\partial K^2}$$

Concept: The curvature of the price curve reveals the market's inflation expectations.

4. Market insights

- **Skewness**: Measures if the market fears high inflation (right side) or low inflation (left side).
- **Kurtosis**: Indicates the risk of extreme economic shocks (fat tails).
- **Validation**: The total sum of probabilities must be equal to 1.

Method 2 : Bakshi-Kapadia-Madan (2003)

1. From Discrete Prices to Moments

The BKM method uses **Numerical Integration** (Trapezoidal rule) to recover moments directly:

$$m_n = K_0^n + \int_0^{K_0} g''(K)P(K)dK + \int_{K_0}^{\infty} g''(K)C(K)dK$$

- $g''(K)$: Quadrature weights (2, $6K$, or $12K^2$).
- $P(K), C(K)$: Undiscounted Put and Call prices.

2. Numerical Illustration

Example for a Call at $K = 1.05$ (Price $= 0.001$):

- **Variance contribution**: $0.001 \times 2 = 0.002$.
- **Kurtosis contribution**: $0.001 \times (12 \times 1.05^2) \approx 0.013$.

Parametric Benchmarks & Model Validation

Lognormal Calibration

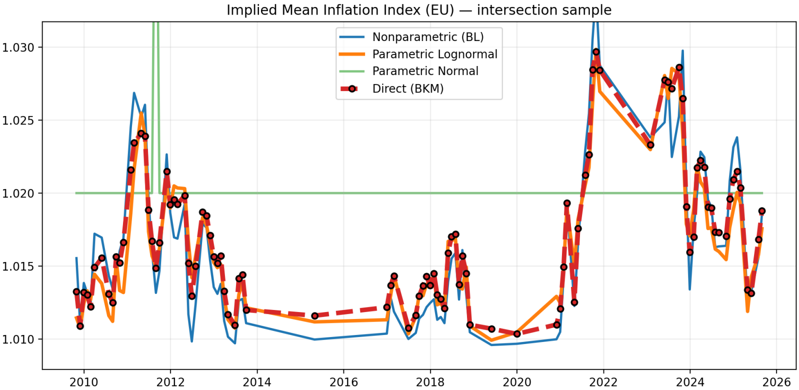
- **Methodology:** Fitting (μ, σ) to Out-of-the-Money (OTM) caps and floors by minimizing pricing RMSE.
- **Martingale constraint:** Use of a penalty anchor to ensure the mean is consistent with the **forward swap rate** (K^*).
- **Tail stability:** Preferred for variance recovery in sparse-strike environments as it provides structural regularization for the tails.

Normal Fit

- **Process:** Gaussian fit performed on the density proxy recovered via the Breeden-Litzenberger (BL) method.
- **Role:** Strictly diagnostic; helps identify **truncation bias** and sensitivity to boundary-mass thresholds.
- **Limitations:** Used to flag positivity mismatches and artifacts caused by limited strike ranges.

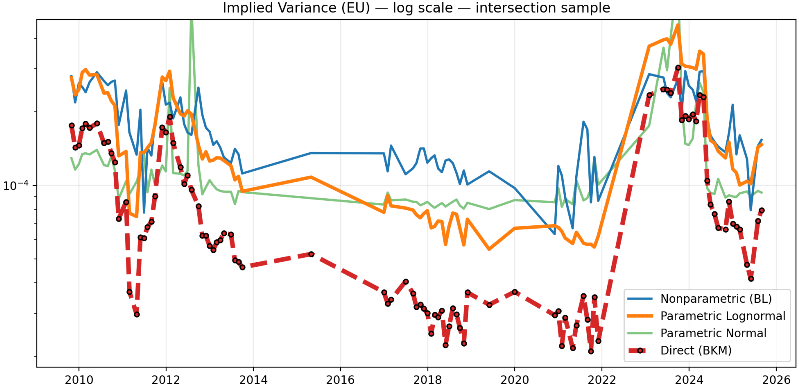
Empirical results: implied mean (1st moment)

- **Method convergence:** High agreement across BL, BKM, and Lognormal.
- **Identification:** Results are strongly anchored by liquid ATM info and swap rates.



Empirical results: implied variance (2nd moment)

- **Divergence diagnosis:** Gaps appear when strike coverage is limited.
- **BKM bias:** Downward bias due to missing far-OTM strikes in the tails.
- **Context:** Methods tend to converge during high-liquidity stress events.

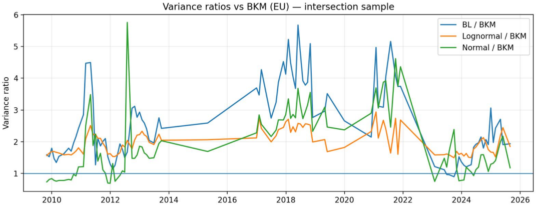


Skewness, kurtosis, and stability

- **Instability factors:** High sensitivity to tail balance and sparse quotes.
- **Kurtosis behavior:** Most volatile moment; depends heavily on extreme strikes.
- **Our take:** Higher moments serve as qualitative risk indicators rather than exact levels.

Diagnostic findings on variance gaps

- **Large variance gaps:** BL variance is often 2–5.7$\times$ higher than BKM on the EU sample.
- **Edge-mass analysis:** divergences are not explained by boundary issues; BL density is not boundary-dominated on worst dates.
- **Parity consistency:** overlapping-strike residuals remain very small (10^{-4} to 10^{-3}) during these divergence periods.
- **Strike centrality:** K^* remains well-positioned near the center (0.38–0.57), not at the edges of the strike range.
- **Model parameters:** lognormal σ bounds are not binding on the dates with the largest gaps.
- **Common-support impact:** gaps between BL and BKM remain high even when the comparison is restricted to overlapping strikes.



Final conclusion and takeaways

- **Mean:** Robust identification across methods using swap anchoring.
- **Variance:** Tail-regularized methods (Lognormal/Smoothed BL) are better for sparse data.
- **Direct BKM:** Best interpreted as a lower-bound for variance in illiquid periods.
- **Final word:** Careful strike-span monitoring is essential for reliable moment recovery.