

Recovering Probability Moments from Inflation Option Prices

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Quantitative Methods in Finance

- **Report scope**

- dataset: 1y inflation caps, floors, swaps; areas: EU, US
- objective: option price time series \rightarrow implied moments (mean, variance, skewness, kurtosis)
- focus: identification, stability; no forecasting

- **Deliverables**

- code: `code in github`
- outputs: `moments_main.csv`, `moments_direct_bkm.csv`, `moments_parametric_lognormal.csv`, `moments_parametric_normal.csv`

- **LLM disclosure**

- ChatGPT (GPT-5.2 Thinking)
- usage: diagnostic narratives and LaTeX bullet structuring
- no usage: data generation, result fabrication, code execution on hidden data

- **Literature review: method families & mappings**

1. **Prices \rightarrow pdf (risk-neutral density / state prices)**

- Breeden–Litzenberger (1978) (nonparametric; differentiation of call price curve; ~ 3315 cites)
- Jackwerth–Rubinstein (1996) (nonparametric; constrained recovery / smoothing; ~ 1614 cites)
- Aït-Sahalia–Lo (1998/1995) (nonparametric; kernel / smoothing; ~ 1591 cites)
- spline/shape-constrained smoothing (convexity/monotonicity enforcement)
- parametric densities (lognormal / mixtures) (pdf implied by calibrated parameters)

2. **Pdf \rightarrow moments (mean/var/skew/kurt)**

- numerical quadrature on grid (trapz) on truncated support
- analytic moments (parametric families; e.g., lognormal moments)
- parametric fit to recovered pdf (diagnostic; e.g., Normal fit on grid)

3. Prices \rightarrow moments (direct)

- Bakshi–Kapadia–Madan style moment integrals (2003) (~ 1858 cites)
- static replication using OTM option payoffs; numerical integration over strikes

• Citation-based ranking (high \rightarrow low; by approximate cites)

1. Prices \rightarrow pdf

- Breeden–Litzenberger (1978) (~ 3315)
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2. Prices \rightarrow moments

- Bakshi–Kapadia–Madan (2003) (~ 1858)
- parametric calibration families (model-dependent; citations dispersed)

3. Pdf \rightarrow moments

- numerical integration (standard; method-agnostic; citations dispersed)
- analytic moments (family-specific; citations dispersed)

• Implemented methods (cross-sectional per date/area)

1. Most popular replication: Breeden–Litzenberger (two-sided)

- file: `method_main.py` \rightarrow `moments_main.csv`
- inputs: caps + floors + swaps
- construction: call-equivalent curve (puts converted via parity)
- smoothing: spline across strikes
- density proxy: second derivative; clipping negatives; renormalization
- moments: numerical integration on strike grid
- gates: monotone decreasing call curve; convexity in strike; parity residual filter

2. Other replicated method: BKM direct moments (tail-stabilized)

- file: `method_moments_direct_bkm.py` \rightarrow `moments_direct_bkm.csv`
- inputs: caps + floors + swaps
- preprocessing: deduplicate strikes; PV \rightarrow undiscounted payoffs (divide by B)
- OTM selection: puts ($K \leq K_*$), calls ($K \geq K_*$)
- gates: parity residual filter; monotonicity of OTM payoffs; min strikes both wings
- stabilization: synthetic tail points (zero payoff); optional winsorization
- moments: raw moments (2–4) via strike integrals; central moments (var/skew/kurt)

3. Parametric benchmark: Lognormal calibration

- file: `method_parametric_lognormal.py` \rightarrow `moments_parametric_lognormal.csv`
- assumption: $X = I_T/I_0$ lognormal
- calibration: fit (μ, σ) to OTM caps+floors; bounded σ ; mean anchoring to K_*
- outputs: analytic mean/variance/skew/kurt; RMSE diagnostic
- gates: parity residual filter; monotonicity of call/put PV curves; min strikes both wings

4. Diagnostic benchmark: Normal fit to BL-proxy pdf

- file: `method_parametric_normal.py` → `moments_parametric_normal.csv`
 - steps: single-instrument price curve → spline → BL-proxy pdf → fit Normal on grid
 - filters: convexity/monotonicity; boundary-mass threshold; variance sanity caps
 - role: diagnostic only (positivity mismatch; truncation sensitivity)
- **Comparison logic (identical inputs)**
 - common keys: (date, area)
 - intersection sample: retain rows where all compared methods return valid moments
 - plots produced: time series of mean; time series of variance (log scale)
 - diagnostics stored: strike span proxies; counts of OTM puts/calls; tail parameters; parity residual pass/fail
- **Empirical interpretation: identification & stability**
 1. **Mean (first moment)**
 - strong cross-method agreement: $BL \approx \text{Lognormal} \approx \text{BKM}$
 - driver: near-ATM information; swap-anchored center (K_*); parity enforcement
 - Normal benchmark: flat/spiky behavior; diagnostic only
 2. **Variance (second moment)**
 - systematic gaps: BKM variance lower than BL/Lognormal in sparse-strike periods
 - driver: tail dependence; missing far-OTM strikes; truncation sensitivity
 - convergence episodes: improved strike coverage / liquidity (stress periods)
 3. **Skewness (third central moment)**
 - dependence: left-right tail balance (puts vs calls)
 - instability sources: wing asymmetry; sparse tail quotes; regularization sensitivity
 - interpretation: caution; conditional on balanced OTM coverage
 4. **Kurtosis (fourth central moment)**
 - dependence: extreme tails; highest sensitivity to missing strikes
 - instability sources: truncation; tail-point assumptions; outlier quotes
 - interpretation: very cautious; mainly qualitative tail-risk indicator
- **Disagreement diagnosis**
 - trigger: material divergence in mean/variance across methods
 - checks: OTM strike counts; observed strike span; tail-point usage; parity residual filter; monotonicity failures
 - primary cause: variance divergence aligned with limited far-OTM coverage; BKM downward bias
- **Main conclusion: best way to work on moments**
 1. **Mean**
 - preferred: any method using both wings + parity + swap anchoring
 - robustness: high; low tail dependence

2. Variance

- preferred: tail-regularized methods (parametric lognormal; smoothed two-sided BL)
- secondary: BKM with explicit tail stabilization; interpret as lower-bound under sparse tails

3. Skewness

- preferred: parametric/regularized methods; require balanced wing coverage
- caution: high sensitivity to asymmetric strike availability

4. Kurtosis

- preferred: parametric with strong tail structure; report with sensitivity flags
- caution: weakest identification; treat as qualitative