Part 1.

$$\langle q_f | e^{-\frac{i}{\hbar}HT} | q_I \rangle = \int Dq\, e^{\frac{i}{\hbar}\int_0^T dt\, L(q,\dot q)} \rightarrow Z := \langle 0 | e^{-iHT} | 0 \rangle$$

$$\xrightarrow{\hbar\to 0} \frac{d}{dt}\left(\frac{\delta L}{\delta \dot q}\right) = \frac{\delta L}{\delta q} \quad (\text{Boundary})$$

Toy: $\langle x^{2n} \rangle := \left(\int_{-\infty}^{+\infty} dx\, e^{-\frac{1}{2}ax^2} x^{2n}\right) / \left(\int_{-\infty}^{+\infty} dx\, e^{-\frac{1}{2}ax^2}\right) = \frac{1}{a^n}(2n-1)!!$ ∴ Wick contraction

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{1}{2}ax^2 + Jx} = \left(\frac{2\pi}{a}\right)^{\frac{1}{2}} e^{J^2/2a}$$

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{1}{2}ax^2 + iJx} = \left(\frac{2\pi}{a}\right)^{\frac{1}{2}} e^{-J^2/2a}$$

$$\int_{-\infty}^{+\infty} dx\, e^{\frac{1}{2}iax^2 + iJx} = \left(\frac{2\pi i}{a}\right)^{1/2} e^{-iJ^2/2a}$$

$$\boxed{\begin{gathered}\int dx_1 \cdots dx_N\, e^{\frac{i}{2}x\cdot A x + iJx} = \left(\frac{(2\pi i)^N}{\det A}\right)^{1/2} e^{-\frac{i}{2}J\cdot A^{-1} J} \\ \langle x_i x_j \cdots x_k x_l \rangle = \sum_{\text{Wick}} (A^{-1})_{ab} \cdots (A^{-1})_{cd}\end{gathered}}$$

$$Z = \int D\varphi\, e^{\frac{i}{\hbar}S(\varphi)}. \quad S(\varphi) = \int d^dx \left[\tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}m^2\varphi^2 - \tfrac{g}{3!}\varphi^3 - \tfrac{\lambda}{4!}\varphi^4 + \cdots\right]$$

disturb the vacuum $+J\cdot\varphi$

$$\xrightarrow{\hbar\to 0} \underbrace{(\partial^2 + m^2)}\varphi(x) + \frac{g}{2}\varphi(x)^2 + \frac{\lambda}{3!}\varphi(x)^3 + \cdots = 0$$

Klein-Gordon eq. $\Rightarrow \varphi(\vec x, t) = \exp(i(\omega t - \vec k\cdot\vec x))$. $\omega^2 = \vec k^2 + m^2$

Energy Momentum Relation

Free Field $\mathcal{L}(\varphi) = \frac{1}{2}[(\partial\varphi)^2 - m^2\varphi^2] \sim$ Gaussian

Disturb $Z = \int D\varphi \exp\left(i\int d^4x \left(\tfrac{1}{2}[(\partial\varphi)^2 - m^2\varphi^2] + J\varphi\right)\right)$

(分部积分) $= \int D\varphi \exp\left(i\int d^4x\left[-\tfrac{1}{2}\varphi\underbrace{(\partial^2+m^2)}\varphi + J\varphi\right]\right) \rightarrow -(\partial^2+m^2)D(x-y) = \delta^{(4)}(x-y)$

(Gaussian) $= C\exp\left(-\frac{i}{2}\iint d^4x\, d^4y\, J(x)\underbrace{D(x-y)}J(y)\right) = Ce^{iW[J]} \rightarrow C = Z[J=0]$

$= C\sum \frac{i^s}{s!}W(J)^s \sim$ No interaction

$$D(x-y) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{k^2 - m^2 + i\varepsilon} \quad \text{propagator}$$

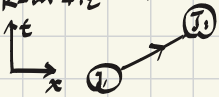
$$= -i\int \frac{d^3k}{(2\pi)^3 2\omega_k}\left[e^{-i(\omega_k t - \vec k\cdot\vec x)}\theta(x^0) + e^{i(\omega_k t - \vec k\cdot\vec x)}\theta(-x^0)\right]$$

→ future, → past

$$W(J) = -\frac{1}{2}\iint d^4x\, d^4y\, J(x)D(x-y)J(y)$$

$J_1 + J_2$ (Fourier) $= -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4} J(-k)\frac{1}{k^2 - m^2 + i\varepsilon}J(k)$

$$J_2(-k)\frac{1}{k^2 - m^2 + i\varepsilon}J_1(k)$$



$x^0 = 0 \rightarrow$ decay $D(x) \sim e^{-m|\vec x|}$ → leak

$$\delta \quad W(J) = \left(\int dx^0\right)\int \frac{d^3k}{(2\pi)^3}\frac{e^{i\vec k\cdot(\vec x_1 - \vec x_2)}}{\vec k^2 + m^2} \xrightarrow[\langle 0|e^{-iHT}|0\rangle = e^{-iET}]{} E = -\int \frac{d^3k}{(2\pi)^3}\frac{e^{i\vec k\cdot(\vec x_1 - \vec x_2)}}{\vec k^2 + m^2} = -\frac{1}{4\pi r}e^{-mr} \sim \frac{1}{r} \Rightarrow \frac{1}{r^2}$$

attract+ potential force

Spin 1. $S(A) = \int d^4x \left(\tfrac{1}{2}A_\mu\underbrace{[(\partial^2+m^2)g^{\mu\nu} - \partial^\mu\partial^\nu]}A_\nu + A_\mu J^\mu\right)$

massive

$$\Rightarrow D_{\nu\lambda}(x) = \int \frac{d^4k}{(2\pi)^4}\frac{-g_{\nu\lambda} + k_\nu k_\lambda/m^2}{k^2 - m^2}e^{ikx}.$$

$k_\mu J^\mu(k) = 0 \Leftarrow \partial_\mu J^\mu(x) = 0$ repel-

$$\Rightarrow W(J) = -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}J^\mu(-k)\frac{-g_{\mu\nu} + k_\mu k_\nu/m^2}{k^2 - m^2 + i\varepsilon}J^\nu(k) = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}J^\mu(-k)\frac{1}{k^2 - m^2 + i\varepsilon}J_\mu(k).\quad E \sim \frac{1}{r}$$

Spin 2. $\rightarrow$
$$D_{\mu\nu,\lambda\sigma}(k) = \frac{(G_{\mu\lambda}G_{\nu\sigma} + G_{\mu\sigma}G_{\nu\lambda}) - \frac{2}{3}G_{\mu\nu}G_{\lambda\sigma}}{k^2 - m^2 (+i\varepsilon)},\quad G_{\nu\lambda}(k) = g_{\nu\lambda} - \frac{k_\nu k_\lambda}{m^2}$$

$$\Rightarrow W(T) = -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}T^{\mu\nu}(-k)D_{\mu\nu,\lambda\sigma}(k)T^{\lambda\sigma}(k) \approx -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}T^{00}(-k)\frac{1+1-2/3}{k^2 - m^2 + i\varepsilon}T^{00}(k)$$

$k_\mu T^{\mu\nu}(k) = 0 \Leftarrow \partial_\mu T^{\mu\nu}(x) = 0$ attract+

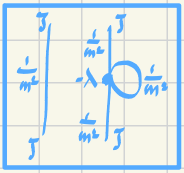
$$Z[J]=\int D\varphi\, \exp\left(i\int d^4x\left[\tfrac{1}{2}\left((\partial\varphi)^2-m^2\varphi^2\right)-\tfrac{\lambda}{4!}\varphi^4+J\varphi\right]\right)$$

anharmonious

Toy: $Z[J]=\int_{-\infty}^{+\infty}dq\, e^{-\frac{1}{2}m^2q^2-\frac{\lambda}{4!}q^4+Jq}=\int_{-\infty}^{+\infty}dq\, e^{-\frac{1}{2}m^2q^2+Jq}\left[1-\frac{\lambda}{4!}q^4+\frac{1}{2}\left(\frac{\lambda}{4!}\right)^2q^8+\cdots\right]$

$$=\left(1-\frac{\lambda}{4!}\left(\frac{d}{dJ}\right)^4+\frac{1}{2}\left(\frac{\lambda}{4!}\right)^2\left(\frac{d}{dJ}\right)^8+\cdots\right)\int_{-\infty}^{+\infty}dq\, e^{-\frac{1}{2}m^2q^2+Jq}$$

$$=e^{-\frac{\lambda}{4!}\left(\frac{d}{dJ}\right)^4}\int_{-\infty}^{+\infty}dq\, e^{-\frac{1}{2}m^2q^2+Jq}=\left(\frac{2\pi}{m}\right)^{\frac{1}{2}}\underbrace{e^{-\frac{\lambda}{4!}\left(\frac{d}{dJ}\right)^4}e^{\frac{1}{2m^2}J^2}}_{\approx \tilde{Z}[J]}$$



$\frac{1}{(m^2)^4}(-\lambda)J^4$

$$Z[J,\lambda]=Z(J=0,\lambda)e^{W[J,\lambda]}=Z(J=0,\lambda)\sum\frac{1}{N!}(W[J,\lambda])^N$$

$$Z(J)=\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}dq_1\cdots dq_N\exp\left(-\tfrac{1}{2}qAq-\tfrac{\lambda}{4!}q^4+Jq\right)=Z(0,0)\sum\frac{1}{s!}J_{i_1}\cdots J_{i_s}G^{(s)}_{i_1\cdots i_s}$$

$$\Rightarrow G^{(2)}_{ij}(\lambda=0)=\frac{1}{Z(0,0)}\int_{-\infty}^{+\infty}\prod dq_l\, e^{-\frac{1}{2}qAq}q_iq_j\overset{\text{Wick}}{=}(A^{-1})_{ij}$$

$$G^{(4)}_{ijkl}=\frac{1}{Z(0,0)}\int_{-\infty}^{+\infty}\prod dq_l\, e^{-\frac{1}{2}qAq}q_iq_jq_kq_l\left(1-\tfrac{\lambda}{4!}q^4+O(\lambda^2)\right)$$

$$(\text{Wick})=(A^{-1})_{ij}(A^{-1})_{kl}+\cdots-\lambda\sum_n(A^{-1})_{in}(A^{-1})_{jn}(A^{-1})_{kn}(A^{-1})_{ln}+\cdots+O(\lambda^2)$$

$$Z[J]=Z[0,0]\,e^{-\frac{i}{4!}\lambda\int d^4w\,(\delta/i\delta J(w))^4}e^{-\frac{i}{2}\iint d^4x\,d^4y\,J(x)D(x-y)J(y)}$$

$$=Z[0,0]\sum_{s=0}^{\infty}\frac{i^s}{s!}\int dx_1\cdots dx_s\,J(x_1)\cdots J(x_s)\,G^{(s)}(x_1,\cdots,x_s)$$

$$=\sum_{s=0}^{\infty}\frac{i^s}{s!}\int dx_1\cdots dx_s\,J(x_1)\cdots J(x_s)\int D\varphi\, e^{i\int d^4x\left[\frac{1}{2}\left((\partial\varphi)^2-m^2\varphi^2\right)-\frac{\lambda}{4!}\varphi^4\right]}\varphi(x_1)\cdots\varphi(x_s)$$

$$\Rightarrow G(x_1,x_2)=\frac{1}{Z(0,0)}\int D\varphi\, e^{i\int\cdots}\varphi(x_1)\varphi(x_2)$$

$D(x-y)=D(y-x)$

$$=\underline{iD(x_1-x_2)}-i\lambda\int d^4w\,D(x_1-w)D(x_2-w)+\cdots+O(\lambda^2)$$

$$G(x_1,x_2,x_3,x_4)=\underline{iD(x_1-x_2)D(x_3-x_4)+\cdots}-i\lambda\int d^4w\,D(x_1-w)\cdots D(x_4-w)+\cdots+O(\lambda^2)$$

$$\int d^4w = \int\prod_{i=1}^{4}dk_i\,(2\pi)^4\delta^{(4)}(k_1+k_2-k_3-k_4)$$

$\Rightarrow G$ corrects D

Feynman Rules in Momentum Space

line k ~ internal line $i/(k^2-m^2+i\varepsilon)$ ~ integrate internal line $\frac{d^4k}{(2\pi)^4}$

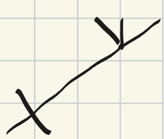
vertex $-i\lambda$ ~ $(2\pi)^4\delta^{(4)}\left(\sum_{in}k-\sum_{out}k\right)$

factor: count symmetry

can be ignored

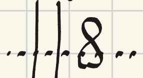
$\Rightarrow \mathcal{M}(\times)=-i\lambda$

$$\mathcal{M}(\,|\!\!\circ\,)=-i\lambda\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\varepsilon}$$



2 mesons → 4 mesons

divergence in the loop



vacuum fluctuation

Heisenberg's Approach to QM:

$L=\frac{1}{2}\dot{q}^2-V(q)$, $p=\frac{\delta L}{\delta \dot{q}}=\dot{q}$. $H=p\dot{q}-L=\frac{1}{2}p^2+V(q)$

$\rightsquigarrow [p,q]=-i$ (equal t). $\frac{dp}{dt}=i[H,p]=-V'(q)$, $\frac{dq}{dt}=i[H,q]=p \rightsquigarrow O(t)=e^{iHt}O(0)e^{-iHt}$

$\rightsquigarrow \ddot{q}=-V'(q)$

$a:=\frac{1}{\sqrt{2\omega}}(\omega q+ip)$, $a^\dagger:=\frac{1}{\sqrt{2\omega}}(\omega q-ip) \rightsquigarrow [a,a^\dagger]=1$

$\frac{da}{dt}=i[H,a]=-i\sqrt{\frac{\omega}{2}}(ip+\frac{1}{\omega}V'(q)) \rightsquigarrow |0\rangle$ ground state

Harmonic Oscillator

if $V'(q)=\omega^2 q$, $\frac{da}{dt}=-i\omega a$, and $L=\frac{1}{2}\dot{q}^2-\frac{1}{2}\omega^2q^2$, $H=\frac{1}{2}(p^2+\omega^2q^2)=\omega(a^\dagger a+\frac{1}{2})$

$L=\sum_a \frac{1}{2}\dot{q}_a^2-V(q_1,\cdots,q_N)$, $p_a=\frac{\delta L}{\delta \dot{q}_a}=\dot{q}_a$, $H=\sum p_a\dot{q}_a-L=\frac{1}{2}\sum p_a^2+V(q_1,\cdots,q_N)$

$[p_a(t),q_b(t)]=-i\delta_{ab}$

$L=\int d^Dx\,(\frac{1}{2}(\dot{\varphi}^2-(\nabla\varphi)^2-m^2\varphi^2)-u(\varphi))$, $\pi(\vec{x},t)=\frac{\delta L}{\delta\dot{\varphi}(\vec{x},t)}=\dot{\varphi}(\vec{x},t)$

$H=\int d^Dx\,\pi(\vec{x},t)\dot{\varphi}(\vec{x},t)-L=\int d^Dx(\frac{1}{2}(\pi^2+(\vec{\nabla}\varphi)^2+m^2\varphi^2)+u(\varphi))$

$[\pi(\vec{x},t),\varphi(\vec{x}',t)]=-i\delta^{(D)}(\vec{x}-\vec{x}')$, $[\pi(\vec{x},t),\pi(\vec{x}',t)]=0$, $[\varphi(\vec{x},t),\varphi(\vec{x}',t)]=0$

$\xrightarrow[\text{expand}]{\text{Fourier}} \varphi(\vec{x},t)=\int\frac{d^Dk}{\sqrt{(2\pi)^D2\omega_k}}\left(a(\vec{k})e^{-i(\omega_kt-\vec{k}\cdot\vec{x})}+a^\dagger(\vec{k})e^{i(\omega_kt-\vec{k}\cdot\vec{x})}\right)$ $k^0=\omega_k=+\sqrt{\vec{k}^2+m^2}$, $\rho(k)=\sqrt{(2\pi)^D2\omega_k}$

$\rightsquigarrow [a(\vec{k},t),a^\dagger(\vec{k}',t)]=\delta^{(D)}(\vec{k}-\vec{k}') \rightsquigarrow |0\rangle$ ground state. $|\vec{k}\rangle:=a^\dagger(\vec{k})|0\rangle$

eg. $\langle 0|\varphi(\vec{x},t)|\vec{k}\rangle=\frac{1}{\sqrt{(2\pi)^D2\omega_k}}e^{-i(\omega_kt-\vec{k}\cdot\vec{x})}$

time order $i\epsilon$

$\langle 0|T\varphi(\vec{x},t)\varphi(0,0)|0\rangle=\int\frac{d^Dk}{(2\pi)^D2\omega_k}\left(\theta(t)e^{-i(\omega_kt-\vec{k}\cdot\vec{x})}+\theta(-t)e^{+i(\omega_kt-\vec{k}\cdot\vec{x})}\right)=iD(\vec{x})$

$\frac{d^Dk}{\omega_k}$ is Lorentz-invariant. (Caution: Here $\langle 0|\cdots|0\rangle$ doesn't mean path integral)

Meson Scattering $\vec{k}_1+\vec{k}_2\longrightarrow\vec{k}_3+\vec{k}_4$ in order λ with $u(\varphi)=\frac{\lambda}{4!}\varphi^4$.

$\langle\vec{k}_3\vec{k}_4|e^{-iHT}|\vec{k}_1\vec{k}_2\rangle\approx\langle\vec{k}_3\vec{k}_4|e^{i\int d^4x\,\mathcal{L}(x)}|\vec{k}_1\vec{k}_2\rangle$

$\rightsquigarrow -i\frac{\lambda}{4!}\int d^4x\langle\vec{k}_3\vec{k}_4|\varphi^4(x)|\vec{k}_1\vec{k}_2\rangle \longrightarrow \left(\prod_{a=1}^4\frac{1}{\rho(k_a)}\right)\int d^4x\,e^{i(k_3+k_4-k_1-k_2)\cdot x}$

$=\left(\prod_{a=1}^4\frac{1}{\rho(k_a)}\right)(2\pi)^4\delta^4(k_3+k_4-k_1-k_2)$

def: $S_{fi}=\langle f|e^{-iHT}|i\rangle$.

$S_{fi}=\delta_{fi}+iT_{fi} \rightsquigarrow iT_{fi}=(2\pi)^4\delta^4\left(\sum_f k-\sum_i k\right)\left(\prod_a\frac{1}{\rho(k_a)}\right)\mathcal{M}(f\leftarrow i)$

Complex Scalar Field.

$L=\partial\varphi^\dagger\partial\varphi-m^2\varphi^\dagger\varphi$. $\pi(\vec{x},t)=\frac{\delta L}{\delta\dot{\varphi}(\vec{x},t)}=\dot{\varphi}^\dagger(\vec{x},t)$. $[\pi(\vec{x},t),\varphi(\vec{x}',t)]=-i\delta^{(D)}(\vec{x}-\vec{x}')$

$\varphi(\vec{x},t)=\int\frac{d^Dk}{\sqrt{(2\pi)^D2\omega_k}}\left(a(\vec{k})e^{-i(\omega_kt-\vec{k}\cdot\vec{x})}+b^\dagger(\vec{k})e^{i(\omega_kt-\vec{k}\cdot\vec{x})}\right)$

$J_\mu:=i(\varphi^\dagger\partial_\mu\varphi-\partial_\mu\varphi^\dagger\varphi)\rightsquigarrow\partial_\mu J^\mu=0\rightsquigarrow Q=\int d^Dx\,J_0(x)=\int d^Dk\,(a^\dagger(\vec{k})a(\vec{k})-b^\dagger(\vec{k})b(\vec{k}))$. conserved charge

particle anti-particle

$\rightsquigarrow [Q,\varphi]=\varphi$, $e^{i\theta Q}\varphi e^{-i\theta Q}=e^{i\theta}\varphi$

Energy of the Vacuum $\langle 0|H|0\rangle$ (No Disturbed)

$$\langle 0|\varphi(\vec{x},t)\varphi(\vec{x},t)|0\rangle = \langle 0|\varphi(0,0)\varphi(0,0)|0\rangle = \lim_{x,t\to 0}\langle 0|\varphi(\vec{x},t)\varphi(0,0)|0\rangle = \int\frac{d^Dk}{(2\pi)^D 2\omega_k}$$

$$\to \langle 0|H|0\rangle = V\int\frac{d^Dk}{(2\pi)^D 2\omega_k}\frac{1}{2}(\omega_k^2+k^2+m^2) = V\int\frac{d^Dk}{(2\pi)^D}\frac{1}{2}\hbar\omega_k \rightsquigarrow H-\langle 0|H|0\rangle.$$

Time Order $\Leftarrow$ Path Integral vs. Operators

$$\int Dq(t)\,A[q(t_1)]\,e^{i\int_0^T dt\,L(\dot{q},q)}$$

$$= \langle q_F|e^{-iH(T-t_1)}A[\hat{q}]e^{-iHt_1}|q_I\rangle$$

Schrodinger

$$= \langle q_F|e^{-iHT}A[\hat{q}(t_1)]|q_I\rangle$$

Heisenberg

$$\int Dq(t)\,A[q(t_1)]B[q(t_2)]\,e^{i\int_0^T dt\,L(\dot{q},q)} = \langle q_F|e^{-iHT}T[A[\hat{q}(t_1)]B[\hat{q}(t_2)]]|q_I\rangle$$

$$\boxed{\begin{aligned}&\rightsquigarrow \langle q_F|e^{-iHT}T[\textstyle\prod \hat{O}_H(t_j)]|q_I\rangle = \int_{q(0)=q_I}^{q(T)=q_F} Dq(t)\prod O(q(t_j))\,e^{i\int_0^T dt\,L}\\ &\langle 0|T[\textstyle\prod \hat{O}_H(t_j)]|0\rangle = \frac{1}{Z(0)}\int D\phi\,\prod O[\phi(t_j)]\,e^{iS[\phi]}\end{aligned}}$$

$Z[J] \neq \tilde{Z}[J]$

different $G^{(n)}(\ldots)$

the same S-matrix

Casimir Effect (Energy of the Vacuum with disturbance)

$|\xleftrightarrow{d}|\xleftrightarrow{\;L-d\;}|$

$E = f(d) + f(L-d)$. $f(d) = \frac{1}{2}\sum\omega_n = \frac{\pi}{2d}\sum_{n=1}^{\infty} n$ (Boundary Condition)

$$\rightsquigarrow f(d) = \frac{\pi}{2d}\sum n e^{-an/d} = -\frac{\pi}{2}\frac{\partial}{\partial a}\frac{1}{1-e^{-a/d}} = \frac{\pi}{2d}\frac{e^{a/d}}{(e^{a/d}-1)^2} = \frac{\pi d}{2a^2} - \frac{\pi}{24d} + \frac{\pi a^2}{480 d^3} + O\left(\frac{a^4}{d^5}\right)$$

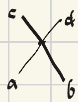
$$F = -\frac{\partial E}{\partial d} = -\left(\frac{1}{2\pi a^2} + \frac{\pi}{24d^2} - \frac{1}{2\pi a^2} - \frac{\pi}{24(L-d)^2} + \cdots\right) \underset{a\to 0}{=} -\frac{\pi}{24}\left(\frac{1}{d^2} - \frac{1}{(L-d)^2}\right) \underset{L\gg d}{=} -\frac{\pi\hbar c}{24 d^2}$$

$$L = \frac{1}{2}[(\partial\varphi_1)^2 + (\partial\varphi_2)^2] - \frac{1}{2}m^2(\varphi_1^2+\varphi_2^2) - \frac{\lambda}{4}(\varphi_1^2+\varphi_2^2)^2 \rightsquigarrow SO(2)\ \text{Symmetry}$$

internal

$$L = \frac{1}{2}[(\partial\vec{\varphi})^2 - m^2\vec{\varphi}^2] - \frac{\lambda}{4}(\vec{\varphi}^2)^2 \rightsquigarrow SO(N)\ \text{Symmetry}$$

$$\Rightarrow D_{ab}(x) = \delta_{ab}D(x) \Longrightarrow$$



$$\sim -2i\lambda(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$$

$$R = e^{\theta\cdot T}.\quad \theta\cdot T = \sum\theta^A T^A \rightsquigarrow \vec{\varphi}\to R\vec{\varphi} = \sum\theta^A T^A\vec{\varphi}$$

$$\varphi_a \to R_{ab}\varphi_b \simeq (1+\theta^A T^A)_{ab}\varphi_b$$

$$\delta\varphi_a = \theta^A T^A_{ab}\varphi_b \underset{\text{Vary } L}{\Longrightarrow} J^A_\mu = \frac{\delta L}{\delta\partial_\mu\varphi_a}\delta\varphi_a = \partial_\mu\varphi_a(T^A)_{ab}\varphi_b$$

conserved currents

Metric. $g'_{\lambda\sigma}(x')\frac{\partial x'^\lambda}{\partial x^\mu}\frac{\partial x'^\sigma}{\partial x^\nu} = g_{\mu\nu}(x)$. $g^{\mu\nu}g_{\nu\rho} = \delta^\mu_\rho$. $g = \det g_{\mu\nu} = g'\left[\det\left(\frac{\partial x'}{\partial x}\right)\right]^2$, $d^4x\sqrt{-g} = d^4x'\sqrt{-g'}$

$\varphi(x) = \varphi'(x')$. $\partial_\mu\varphi(x) = \frac{\partial x'^\lambda}{\partial x^\mu}\partial'_\lambda\varphi'(x')$. $A_\mu(x) = \frac{\partial x'^\lambda}{\partial x^\mu}A'_\lambda(x')$

$T^{\mu\nu}(x) = \frac{-2}{\sqrt{-g}}\frac{\delta S_M}{\delta g_{\mu\nu}(x)}$. $E = P^0$, $P^i = \int d^3x\sqrt{-g}\,T^{0i}(x)$

photon $\rightarrow$ $L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2)$. $T_{00} = \frac{1}{2}(\vec{E}^2+\vec{B}^2)$, $T_{0i} = (\vec{E}\times\vec{B})_i$, $\operatorname{tr}T = 0$