EXTRA NOTES - DISCUSSION

CLARIFICATION: To verify independence of random variables, we need to two things:

1. Verify that the support set is a "rectangle," i.e., it can be written as a Cartesian cross product.
2. write the joint density (OR joint cdf) as the product of the marginal densities (OR marginal CDFs).

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$$\begin{tikzcd}
f_X(x) \arrow[d, bend right, "\int dx"'] \\
F_X(x) \arrow[u, bend right, "\frac{d}{dx}"']
\end{tikzcd}$$

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$$\begin{tikzcd}
 & F(x,y) \arrow[dl, bend right, "\lim_{y\to+\infty} F(x,y)"'] \arrow[dr, bend left, "\lim_{x\to+\infty} F(x,y)"] \arrow[d, bend left, "\frac{\partial}{\partial x}\frac{\partial}{\partial y}"] & \\
F_X(x) \arrow[d, bend left, "\frac{d}{dx}"] & & F_Y(y) \arrow[d, bend right, "\frac{d}{dy}"'] \\
f_X(x) \arrow[u, bend left, "\int dx"] & f(x,y) \arrow[u, bend left, "\iint dx\,dy"] \arrow[l, "\int dy"] \arrow[r, "\int dx"'] & f_Y(y) \arrow[u, bend right, "\int dy"']
\end{tikzcd}$$

FORAY INTO PATHOLOGY: Random variables may or may not have a finite mean or finite variance.

Suppose X is a r.v. with pmf

$$p_X(x) = \begin{cases} \frac{6}{\pi^2}\frac{1}{x^2} & \text{if } x = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \mathbb{E}[X] = \sum_{i=1}^{\infty} x\,p(x) = \sum_{i=1}^{\infty} \frac{6}{\pi^2}\cdot x \cdot \frac{1}{x^2}$$

$$= \frac{6}{\pi^2} \sum_{i=1}^{\infty} \frac{1}{x} = +\infty$$

harmonic series

NOTE: Lyapunov's Inequality means that if $\mathbb{E}[|X|^p] < \infty$, then $\mathbb{E}[|X|^s] < \infty$ for $s < p$. Similarly, if $\mathbb{E}[|X|^p] = +\infty \Rightarrow \mathbb{E}[|X|^s] = +\infty$ for $s \geq p$.

EX: Consider the pdf $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$, $-\infty < x < \infty$. It has $\mathbb{E}[|X|] = +\infty$. ☹