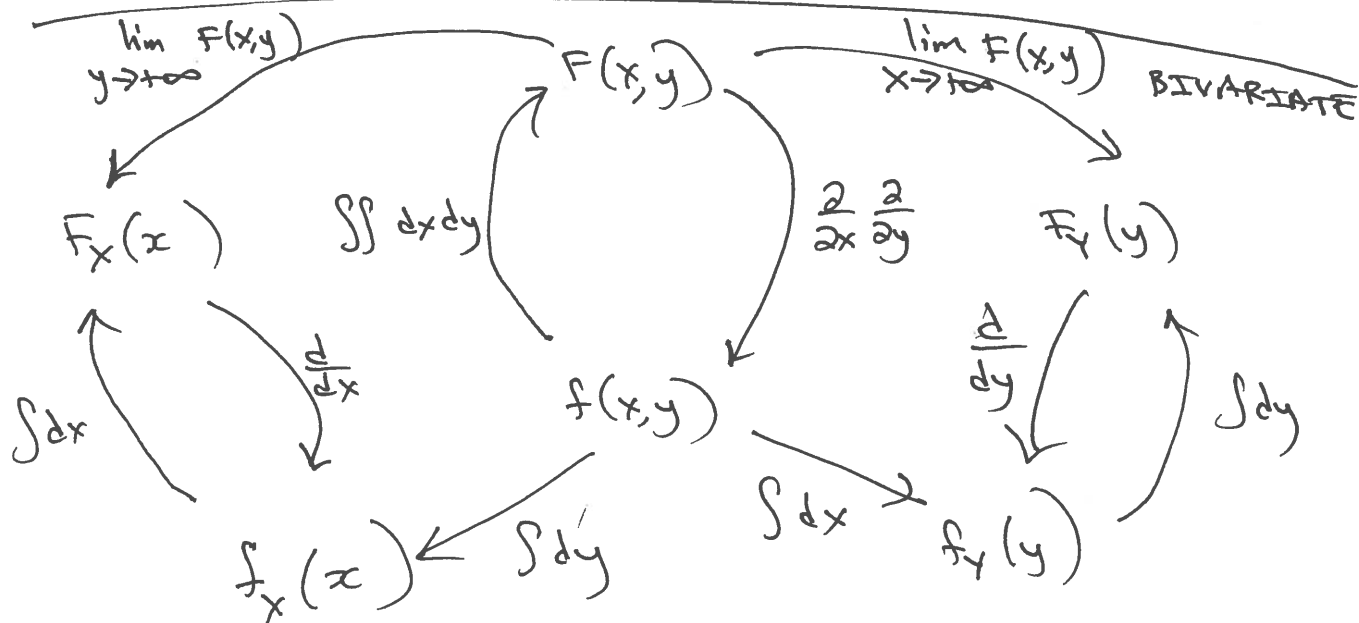
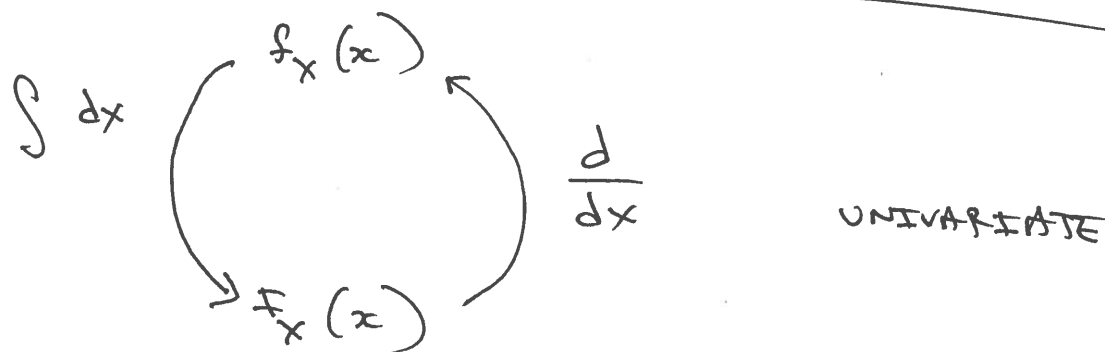


CLARIFICATION: To verify independence of random variables, we need to two things:

- ① Verify that the support set is a "rectangle," i.e., it can be written as a Cartesian cross product.
- ② write the joint density (or joint cdf) as the product of the marginal densities (or marginal cdfs).



FORAY INTO PATHOLOGY: Random variables

may or may not have a finite mean or finite variance.

Suppose X is a r.v. with pmf

$$P_X(x) = \begin{cases} \frac{6}{\pi^2} \frac{1}{x^2} & \text{if } x=1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \mathbb{E}[X] = \sum_{i=1}^{\infty} x p(x) = \sum_{i=1}^{\infty} \frac{6}{\pi^2} \cdot x \cdot \frac{1}{x^2}$$

$$= \frac{6}{\pi^2} \left(\sum_{i=1}^{\infty} \frac{1}{x} \right) = +\infty$$

→ harmonic series

NOTE: Lyapunov's Inequality means that if $\mathbb{E}[|X|^p] < \infty$, then $\mathbb{E}[|X|^s] < \infty$ for $s < p$.
Similarly, if $\mathbb{E}[|X|^p] = +\infty \Rightarrow \mathbb{E}[|X|^s] = +\infty$ for $s \geq p$.

EX: Consider the pdf $f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$, $-\infty < x < \infty$.
It has $\mathbb{E}[|X|] = +\infty$. (∞)