Efrem - Ghebreab.

Home work # 3.

① for ARMA(1,1) $X_t - 0.8X_{t-1} = Z_t + 0.6Z_{t-1}$
$Z_t \sim WN(0, \sigma^2)$

a) prove this process is causal & invertible.
$X_t - 0.8X_{t-1} = Z_t + 0.6Z_{t-1}$ ARMA(1,1).
then. the AR model.
Generating function.
$(1 - 0.8x)X_t = (1 + 0.6x)Z_t$.
then. $(1 - 0.8x) = \Phi'(x)$
$1 - 0.8x = 0$
$\boxed{AR(1) \rightarrow x = 1.25}$

$|x| = 1.25 > 1$
so ARMA(1,1) is Causal

MR(1)
$\theta'(x) = 1 + 0.6x$.
$1 + 0.6x = 0$.
$\boxed{x = \frac{1}{0.6} = 1.66 > 1}$
so ARMA(1,1) is invertible

$\therefore$ $\boxed{\text{ARMA(1,1) is both invertible and Causal.}}$

[illegible].

4 b)

i) compute coef. ψ_1, ψ_2, ψ_3 in the equiv $MA(\infty)$.

$$X_t = \sum_{j=0}^{\infty} \psi_i Z_{t-1}$$

we have. $ARMA(p,q)$: $\Phi(B) X_t = \Theta(B) Z_t$.

$$\Phi(B) \sum_{j=0}^{\infty} \psi_i Z_{t-1} = \Theta(B) Z_t .$$

$$\Downarrow$$

$$\Phi(B) \; \psi(B) \cancel{Z_t} = \Theta(B) \cancel{Z_t} .$$

$$\Phi(B) \psi(B) = \Theta(B)$$

→ we have. $X_t - 0.8 X_{t-1} = Z_t + 0.6 Z_{t-1}$

then.

$$(1 - 0.8x)(1 + \psi_1 x + \psi_2 x^2 + \dots) = 1 + 0.6x$$

$$1 + (\psi_1 - 0.8)x + (\psi_2 - 0.8\psi_1)x^2 + (\psi_3 - 0.8\psi_2)x^3 \dots$$
$$= 1 + 0.6X$$

$\psi_0 = 1$.

$\psi_1 = 0.8 + 0.6 = 1.4$.

$\psi_2 = 0.8 \psi_1 = 0.8 \times 1.4 = 1.12$

$\psi_3 = 0.8 \psi_2 = 0.8 \times 1.2 = 0.89$

The ~~DA~~ ARMA(1,1) is equivalent MA(∞)

$$\boxed{X_t = 1 + 1.4 Z_t + 1.12 Z_{t-1} + 0.89 Z_{t-2} \dots}$$

1) b)

ii) compute coeff. $\lambda_1, \lambda_2, \lambda_3$ in eq AR(∞)

$$\cancel{X}_t = \sum_{j=0}^{\infty} \lambda_j X_{t-1}$$

we have:

ARMA (P,q)

$$\Phi(B) X_t = \Theta(B) Z_t \quad , \quad Z_t = \lambda(B) X_t$$

$\Downarrow$ from equivalent AR(∞)

then

$$\Phi(B)\cancel{X_t} = \Theta(B)\lambda(B)\cancel{X_t}$$

$$\Phi(B) = \Theta(B)\lambda(B)$$

given $X_t - 0.8X_{t-1} = Z_t + 0.6Z_{t-1}$

then.

$$1 - 0.8x = (1+0.6x)(1+\lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + \dots)$$

$$1 - 0.8x = 1 + (\lambda_1 + 0.6)x + (\lambda_2 + 0.6\lambda_1)x^2 + (\lambda_3 + 0.6\lambda_2)x^3 + \dots)$$

then.

$\lambda_0 = 1$

$\lambda_1 + 0.6 = -0.8 \Rightarrow \lambda_1 = -1.4$

$\lambda_2 + 0.6\lambda_1 = 0 \Rightarrow \lambda_2 = -\lambda_1 \cdot 0.6 = -(-1.4 \times 0.6)$

$= 0.84$

$\lambda_3 + 0.6\lambda_2 = 0 \Rightarrow \lambda_3 = -(0.6 \times 0.84) = -0.50$

The ARMA(1,1) equivalent AR(∞)

$$\boxed{Z_t = (1 + -1.4X_t + 0.84X_{t-1} - 0.5X_{t-2} + \dots}$$

Time series Homework Answers to Coding questions
<u>The codes and results are in notebook file</u>

2. Number 2 is coding in a notebook

3.

(a) Perform order selection based on AIC and BIC and provided the choice of orders fromboth ICs (max p=4, max q=4). Do they agree in this case?

ANS--> Both AIC and BIC provided order selection of (1, 0). Yes their agree both at a min order (1,0).

(b) Perform order selection based on RMSE and MAE and cross validation(use the func-tions you have defined from question 2, and use 67% for split) provided the choice oforders. Do they agree?

ANS --> Best ARMA(1, 0) RMSE=0.976 , Best ARMA(0, 4) MAE=0.782 . Both MAE and MSE provided best orders at (0,4). So Yes They agree.

(c) Use a set of orders of your choice from part(a) as your model 1, estimate this ARMAprocess and write out the estimated ARMA equation using the summary table.

ANS --> My mean is 0.6001 and Then the ARMA(1,0) model is Xt = Xt = 0.6001+0.8673(X_{t-1} - 0.6001) +Z_t

(d) Use a set of orders of your choice from part(b) as your model 2, estimate this ARMAprocess and write out the estimated ARMA equation using the summary table.

ANS--> ARMA (0,4) Model: X_t = 0.6818 + 0.9019Z_{t-1} + 0.8486Z_{t-2} + 0.5347Z_{t-3} + 0.2391Z_{t-4} + Zt

(e) Plot forecast of 20 steps out of the given data using model 1 and model . Just by observing how the forecasting following the originaldata, which model do you think performed better? or they performed similarly?

ANS --> Model 1 predicted from AIC and BIC seems to perform better since the observed data and the forecast are going close tat each step(the estimation is pretty close to the observed data). In the second model which was based on the RMSE and MAE there is some difference in the forecast and the observed data.

(f) We have defined the function to use one-step rolling cross validation. Read it verycarefully and define function(s) in python to evaluate ARMA performance by usingh=2, 3 and 4 steps rolling cross validation(you don't have to follow how I did it), andreport the RSME of 1-step, 2-step, 3-step and 4-step forecasting for model 1 and model2:RMSE1-step2-step3-step4-stepModel 1Model 2

Model 1

ARMA(1, 0), Step=1, RMSE=0.988
ARMA(1, 0), Step=2, RMSE=0.989
ARMA(1, 0), Step=3, RMSE=1.020
ARMA(1, 0), Step=4, RMSE=1.069

Model 2

ARMA(0, 4), Step=1, RMSE=0.976
ARMA(0, 4), Step=2, RMSE=1.859
ARMA(0, 4), Step=3, RMSE=1.859
ARMA(0, 4), Step=4, RMSE=1.859