

Homework 3. Answer.

1. ARMA(1,1) $X_t - 0.8 X_{t-1} = Z_t + 0.6 Z_{t-1}$

(a) Generating function for AR:

$$\Phi(x) = 1 - 0.8x$$

$$\text{roots } x = 1.25 \quad |x| > 1$$

so the process is causal (stationary)

Generating function for MA

$$\Theta(x) = 1 + 0.6x$$

$$\text{roots } x = -1.67 \quad |x| > 1$$

so the process is invertible

(b) (i) MA(∞)

$$X_t = \Psi(B) Z_t$$

$$\Phi(B) \Psi(B) Z_t = \Theta(B) Z_t$$

$$\text{so } \Phi(x) \Psi(x) = \Theta(x)$$

$$(1 - 0.8x)(\psi_0 + \psi_1 x + \psi_2 x^2 + \dots) = 1 + 0.6x$$

$$\psi_0 = 1$$

$$\psi_1 - 0.8 = 0.6 \Rightarrow \psi_1 = 1.4$$

$$\psi_2 - 0.8 \psi_1 = 0 \Rightarrow \psi_2 = 1.12$$

$$\psi_3 - 0.8 \psi_2 = 0 \Rightarrow \psi_3 = 0.896$$

4ii) $Z_t = \Lambda(B) X_t$

$$\Phi(B) X_t = \Theta(B) \Lambda(B) X_t$$

$$1 - 0.8x = (1 + 0.6x)(\lambda_0 + \lambda_1 x + \lambda_2 x^2 + \dots)$$

$$\lambda_0 = 1$$

$$\lambda_1 + 0.6 = -0.8 \Rightarrow \lambda_1 = -1.4$$

$$\lambda_2 + 0.6\lambda_1 = 0 \Rightarrow \lambda_2 = 0.84$$

$$\lambda_3 + 0.6\lambda_2 = 0 \Rightarrow \lambda_3 = -0.504$$

Q2: see python

Q3: see python

(a) AIC and BIC chose AR(1) or ARMA(1,0)

(b) RMSE and MAE chose MA(4) or ARMA(0,4)

Note: when choose different max p and max q, this model might be different.

$$(c) \quad X_t - 0.6001 = 0.8673 (X_{t-1} - 0.6001) + z_t$$

$$(d) \quad X_t - 0.6818 = 0.9019 z_{t-1} + 0.8486 z_{t-2} \\ + 0.5347 z_{t-3} + 0.2391 z_{t-4} + z_t$$

(e) see plots.

Both converge to the estimated mean, where

AR converges a little slower.

(f) RMSE	1 step	2 step	3 step	4 step
ARMA(0,1,0)	0.988	1.373	1.647	1.765
ARMA(0,1,4)	0.976	1.375	1.679	1.825