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Home work - 2

1) we have

$X_t = Z_t + \theta_1 Z_{t-1} + \dots + \theta_q Z_{t-q}$, $\{Z_t\} \sim WN(0,\sigma^2)$

a) prove it's stationary for all q

Soln.

mean of $\{X_t\}$

$E(X_t) = E(Z_t + \theta_1 Z_{t-1} + \dots + \theta_q Z_{t-q})$

$= \underline{\underline{0}}$

Autocovariance function of $\{X_t\}$

$\gamma(h) = Cov(X_{t+h}, t)$

$\Rightarrow$ ~~Cov(Zt+h + θ1 Zt+h-1 ... + Zt)~~

$\gamma(0) = Var(X_t) = (1 + \theta_1 + \theta_2)\sigma^2$

$\gamma(q) = Cov(X_t, X_{t+q})$

$\Rightarrow Cov(Z_t + \theta_1 Z_{t-1} \dots \theta_q Z_{t-q}, Z_{t+q} + \theta_1 Z_{t+q-1} + \dots + \theta_q Z_t)$

$\Rightarrow Cov(Z_t, Z_{t+q} + \theta_1 Z_{t+q-1} + \dots \theta_q Z_t) + \theta_1 Cov(Z_{t-1}, Z_{t+q} + \theta_1 Z_{t+q-1} + \dots \theta_q Z_t) + \dots + \theta_q Cov(Z_{t-q}, Z_{t+q} + \theta_1 Z_{t+q-1} + \dots + \theta_q Z_t)$

$\Rightarrow \theta_q \sigma^2 + \theta_{q-1}\theta_1\sigma^2 + \theta_2\theta_{q-2}\sigma^2 + \theta_3\theta_{q-3}\sigma^2 + \dots$

$\Rightarrow \sum_{i=1}^{q} (\theta_{q-i} \times \theta_i)\sigma^2$.

$$\gamma(h) = Cov(X, X_{t+h}) = \begin{cases} (1+\theta_1^2+\theta_2^2)\sigma^2, & h=0 \\ (\theta_1+\theta_1\theta_2)\sigma^2 & |h| = 1 \\ (\theta_2\sigma^2) & |h| = 2 \\ \sum_{i=1}^{q} (\theta_{q-i} \cdot \theta_i)\sigma^2 & h = q \\ 0 & h > q \end{cases}$$

∴ Auto covariance doesn't depend on time "t".

11 b) Auto correlation function of $\{X_t\}$.

we have

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)}$$

at $h=0$ $\therefore \rho(h) = 1$.

at $h=q$ $\rho(h) = \frac{\sum_{i=1}^{q}(\theta_i \theta_{q-i})\sigma^2}{1+\theta^2+\theta_2^2}$.

~~at h>q~~ at $|h|>q$.

There will not be same ~~Zt any~~ Z_t anymore b/n the two expressions so it is going to be zero since $Z(t)$ is $WN(0,\sigma^2)$.

$$\rho(h) = \begin{cases} 1 & \text{at } h=0 \\ \frac{\sum_{i}^{q}(\theta_i\theta_{q-i})\sigma^2}{(1+\theta_1^2+\theta_2^2)\sigma^2} & \text{at } h=q \\ 0 & \text{otherwise } |h|>q \end{cases}$$

2). we have.

$$X_t = 0.8X_{t-1} - 0.5X_{t-2} + 0.25X_{t-4} + 0.4X_{t-6}$$

where $Z_t \sim WN(0, \sigma^2)$

Generating function $\phi^6(x)$?

$$X_t - 0.8X_{t-1} + 0.5X_{t-2} - 0.25X_{t-4} - 0.4X_{t-6} = Z_t$$

apply backward shift operator on X_t

$$X_t \xrightarrow{x} X_{t-1} \xrightarrow{x} X_{t-2} \text{------.}$$

then

$$X_t - 0.8xX_t + 0.5x^2X_t - 0.25x^4X_t - 0.4x^6X_t = Z_t$$

$$\underbrace{(1 - 0.8x + 0.5x^2 - 0.25x^4 - 0.4x^6)}_{\Phi^6(x) \text{ Generating function.}}X_t = Z_t.$$

$$\Phi^6(x) = (1 - 0.8x + 0.25x^2 - 0.25x^4 - 0.4x^6)$$

③. $X_t = 0.9Z_{t-1} + Z_{t-3} + 1.2Z_{t-4}$, $\{Z_t\} \sim WN(0, \sigma^2)$

generating function of $\theta^4(x)$?

- all of the noises are on the same side

$$X_t = 0.9Z_{t-1} + Z_{t-3} + 1.2Z_{t-4}$$

apply backward shift operator on Z_t, then

$$Z_t \xrightarrow{x} Z_{t-1} \xrightarrow{x} Z_{t-2} \text{ ----.}$$

then,

$$X_t = 0.9xZ_t + x^3Z_t + 1.2x^4Z_t.$$

$$X_t = \underbrace{(0.9x + x^3 + 1.2x^4)}Z_t.$$

$$X_t = \theta^4(x)Z_t.$$

$$\therefore \theta^4(x) = 0.9x + x^3 + 1.2x^4.$$

(5) Determine whether or not stationary AR process.

a) $X_t = -0.2X_{t-1} + 0.48X_{t-2} + Z_t$.

$X_t + 0.2X_{t-1} - 0.48X_{t-2} = Z_t$.

$X_t + 0.2xX_t - 0.48x^2X_t = Z_t$.

- Generating function.

$(1 + 0.2x - 0.48x^2) = \Phi^{-2}(x)$

- check the roots.

$1 + 0.2x - 0.48x^2 = 0$.

$-0.48x^2 + 0.2x + 1 = 0$.

$\cancel{\text{roots}}\ x = \frac{-0.2 \pm \sqrt{0.2^2 - 4\times(-0.48)\times 1}}{2\times -0.48}$.

$x = \frac{-0.2 \pm \sqrt{1.96}}{-0.96}$.

$x = \underline{1.67}$ or $\underline{-1.25}$.

$|x| > 1$. $\Rightarrow$ $1.67 > 1$ & $1.25 > 1$.

So $\{x_t\}$ is stationary process.

5(b). $X_t = 0.3X_{t-1} - 0.8X_{t-2} + \varepsilon_t$.

$X_t - 0.3X_{t-1} + 0.8X_{t-2} = \varepsilon_t$.

$\Rightarrow$ apply backward shift operator on X_t, then

$X_t - 0.3x X_t + 0.8x^2 X_t = \varepsilon_t$.

$\Rightarrow$ Generating function

$1 - 0.3x + 0.8x^2 = \phi^2(x)$.

$\Rightarrow$ check the roots.

$1 - 0.3x + 0.8x^2 = 0$.

$0.8x^2 - 0.3x + 1 = 0$.

$$x = \frac{0.3 \pm \sqrt{(-0.3)^2 - 4\times(.8)(1)}}{2\times 0.8}.$$

$$= \frac{0.3 \pm \sqrt{-3.11}}{1.6}$$

$$= 0.1875 \pm 1.1022i$$

$\underline{\underline{|x| = 1.118}}$. > 1.

$\therefore$ $\{X_t\}$ is a stationary AR process

In [ ]:

4. Simulate the above MA process in Python and run the ACF plot. What did you observe from the ACF plot? Does it give a suggestion of stationary or the order of the MA process?

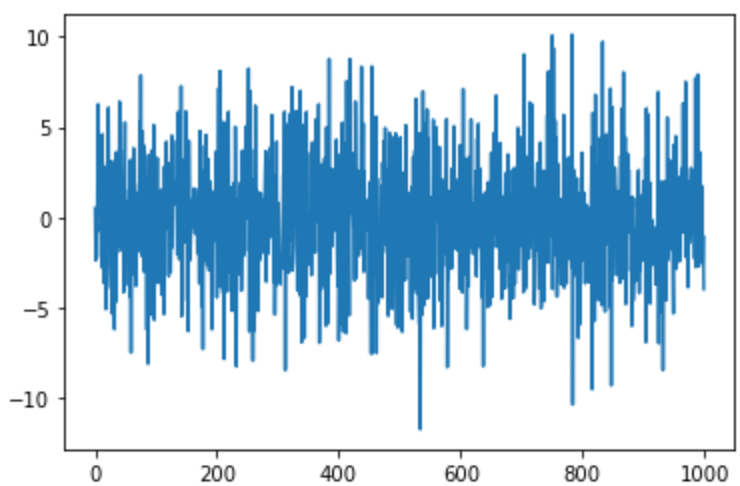
ANS --> From the ACF plot on MA(4), we can observe that there is 'shut off' around order h =3. This gives a good suggestion/estimation about the order of the MA process.

In [21]:

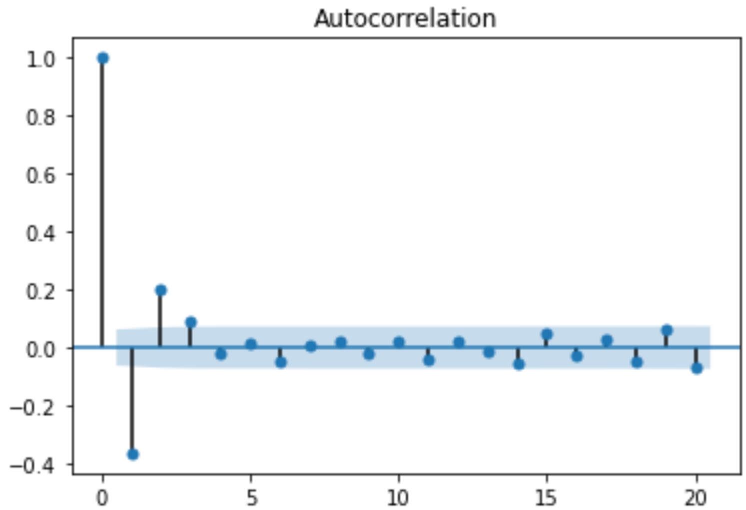
```
#simulate a MA process
ar1 = np.array([1])
ma1 = np.array([0.9, 1,1.2])
mMA_object1 = ArmaProcess(ar1, ma1)
ma_4 = MA_object1.generate_sample(nsample=1000)
plt.plot(ma_4)
```

Out[21]:

```
[<matplotlib.lines.Line2D at 0x138967880>]
```



In [22]:
```
plot_acf(ma_4, lags=20)
plt.show()
```



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6. Simulate the above AR processes in Python then run the ADF test for stationary, report pvalues and test conclusion.

ANS --> p-values for both a and b are 0.00, in conclusion both of the AR processes are stationary as the p-values are < 0.05.

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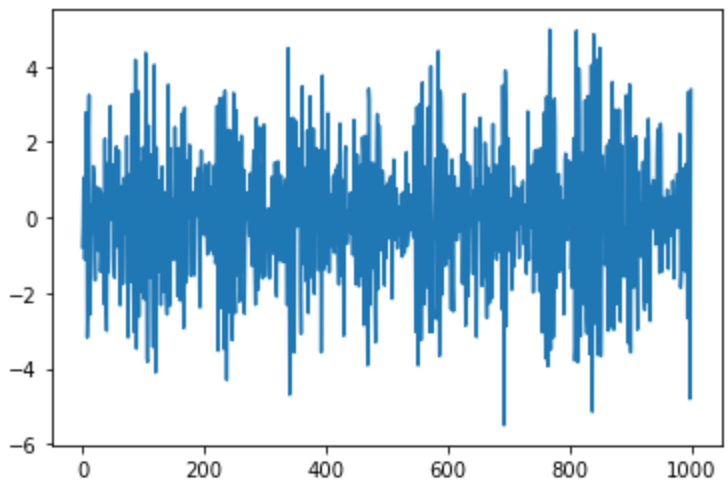
In [52]:
```
# 6.b)

ar1 = np.array([1, -0.3, 0.8])
ma1 = np.array([1])
AR_object1 = ArmaProcess(ar1, ma1)
ar_2b = AR_object1.generate_sample(nsample=1000)
plt.plot(ar_3)
```

Out[52]: [<matplotlib.lines.Line2D at 0x13b3a9a90>]



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7. The dataset profit.csv recorded the profits (in $k) of an investment product in 200 days(positive number shows increased price compared to original price, negative number showsdropped price from original price).

(a) Provide the Time Series line plot, ACF plot for lagsh=0, 1, ..., 20, and PACF plotfor lagsh=0, 1, ..., 20. (Hint: when h gets larger, ACF and PACF plots don't pro-vide a close-to-unbiased estimate anymore, therefore numbers become unstable andunreliable.It's reasonable to just examine the beginning part of the plots)•In python, to choose the lags up to 20:plot_ac f(data,lags=20)plot_pac f(data,lags=20)

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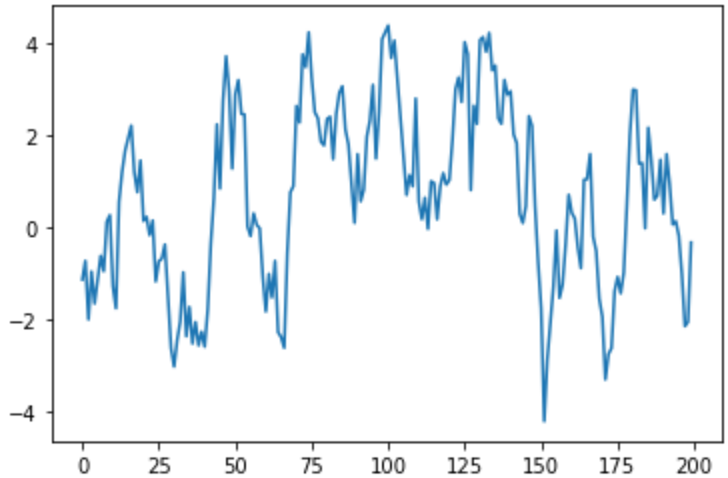
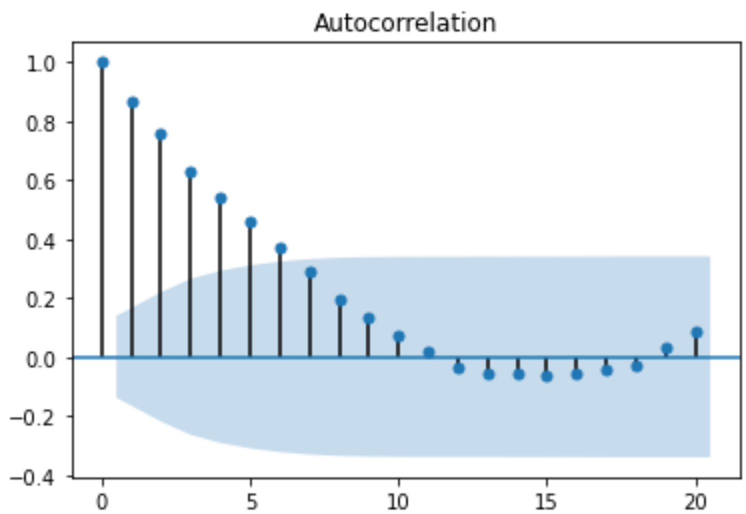
In [47]:
```
data = pd.read_csv('profit.csv')
# data
```

In [50]:
```
plt.plot(data.Profit)
plt.show()
```

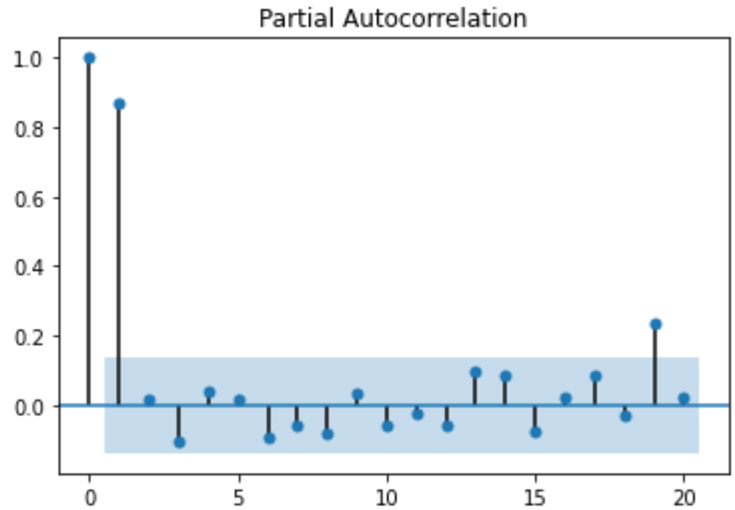


In [49]:
```
plot_acf(data.Profit, lags=20)
plt.show()
```



In [ ]:

In [46]:
```
plot_pacf(data.Profit, lags=20)
plt.show()
```



(b) Based on the plots from (a),i. Do you think this is a stationary process? Briefly justify your answer.Perform anADF test to verify your observation from the plots. ii. Do you think this is an AR process? If so, what would be your choice of orderp? iii. Do you think this is a MA process? If so, what would be your choice of order q?

i. Do you think this is a stationary process?

ANS --> This is stationary process because there is faster decay of the ACF plot towards zero. According to the ADF test p-value = 0.003352 which is less than 0.05 so it supports the conclusion from the plots.

ii. Do you think this is an AR process?

ANS --> This is an AR process becuase the ACF for an AR(p) process tails off toward zero very slowly as is shown in the graph of ACF, but in the PACF plot we can observe that there is a clear 'shut off' around order h =1 and the order for the AR process is 1(AR(1)).

iii. Do you think this is a MA process? If so, what would be your choice of order q?

ANS --> This is not MR process because in the ACF plot there is no clear shut off i.e there is no sudden change to zero so we can conclude that it is not MA process but there is in the PACF plot which is highly suggestive of

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