Computational Modeling, Monitoring and Analysis of Economic Networks

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Abstract

This thesis proposes a novel formal framework for analyzing economic phenomena as dynamic networks of interconnected agents. This network-based perspective can capture contagion effects and structural changes often missed by traditional aggregate models.

The framework is built on four pillars: an algebraic construction, a dynamical overview, a behavior inference technique and an interactive real-time visualization. This integrated approach allows for a comprehensive analysis of money circulation from both a structural and a dynamic perspective.

The proposed approach provides a general and flexible theoretical foundation applicable to several economic contexts. It offers a robust basis for empirical integration with real-world data, the incorporation of advanced techniques such as machine learning and the development of exploratory tools for economic analysis and monitoring.

Keywords: economic network, graph-based modeling, discrete dynamical system, behavior inference, interactive real-time visualization.

Introduction

According to the first law of thermodynamics, *energy cannot be created or destroyed, only transformed.* This physical principle offers a useful analogy for economics.
Although this analogy is not exact, since central banks can create money and individuals can burn bills, this perspective frames the economy as a dynamic system of interconnected agents, where one agent's income corresponds to another's expenditure, illustrating how money merely flows.

Traditional macroeconomic models often overlook these explicit interactions. They tend to represent the economy in aggregate terms, without detailing how individual decisions propagate through the underlying network of monetary transfers.
This limits their ability to analyze contagion effects or structural changes.

To overcome these limitations, the novel framework is built upon four interconnected pillars, moving from a theoretical foundation to a practical application:

i. Graph-based modeling.

ii. Discrete-time dynamical systems.

iii. Behavioral inference.

iv. Interactive real-time visualization design.

Together, these components provide a comprehensive and operational framework to monitor real systems, extrapolate agent behavior, and evaluate economic scenarios in real-time.

Literature review

The thesis is situated at the intersection of six key research areas in computational economics and macroeconomic modeling. This review synthesizes classical and recent developments within these domains, identifying structural limitations that motivate the framework proposed.

Conceptual framework

Economic network theory
Foundational works by Jackson [Jackson, 2008] and subsequent contributions have shown how the structure of connections between agents influences phenomena such as information diffusion, market efficiency and financial shock propagation. However, most models in this strand are static, qualitative, or focused on specific topologies, lacking explicit monetary quantification. More recent studies like [Bargigli et al., 2015] and [Battiston et al., 2016] incorporate empirical multilayer network data but still lack explicit microeconomic formalization.

Thus, a framework that dynamically integrates endogenous decisions within network structures is still lacking.

Input-output modeling
Leontief's input-output framework [Leontief, 1936] has been central to structural analysis of sectoral interdependencies. Extensions such as [Lenzen et al., 2019] incorporate uncertainty and simulation but retain rigid technical coefficients and absence of autonomous agents. These constraints limit their ability to capture endogenous responses.

Therefore, a flexible input-output framework combining accounting structure with microeconomic heterogeneity is still required.

Discrete dynamical systems
Discrete dynamical systems provide a fundamental framework for analyzing time-based processes, especially through the Jacobian matrix [Elaydi, 2005]. The Perron–Frobenius theory [Perron, 1907, Frobenius, 1908] is crucial for extracting and interpreting eigenvalues and eigenvectors of non-negative matrices, ensuring the existence of a unique, positive dominant eigenvalue that governs the long-term behavior of such systems.

Despite these analytical strengths, a comprehensive framework that integrates microeconomic foundations and explicit network structures remains an open research frontier.

Observed behavior analysis
Economic behavior has traditionally been studied through revealed preference theory, including methods such as Inverse Reinforcement Learning (IRL) [Ng and Russell, 2000]. While macroeconomic models typically rely on Euler equations [Ramsey, 1928] involving several unobserved and indeterminate parameters, empirical approaches based on observed behavior are needed to infer a mathematical representation of the relationships between discretionary flows.

Agent-based simulation
Agent-based modeling (ABM) has allowed exploration of out-of-equilibrium dynamics, adaptive learning and emergent effects. Nonetheless, many ABMs rely on heuristic rules, empirical replication or ad hoc procedures. These methods often lack a formal algebraic framework that can ensure consistency between individual decisions and aggregate outcomes. [Assenza et al., 2020] highlight the need to link ABMs with micro-foundations principles [Gabaix, 2011], but most current approaches remain fragmented and not integrated.

Real-time visualization and monitoring
Interactive visualizations for economic networks have advanced considerably in fields such as data science and graphic design [Kirk, 2016], yet their adoption in formal macroeconomics remains limited. Although institutions like the ECB or Federal Reserve publish macroeconomic dashboards, these tend to be aggregate, static and loosely connected to endogenous structures or decision models.

A visual platform integrating real-time monitoring and dynamic simulation is still absent.

Contributions

The proposed model offers an integrated framework linking mathematical formalization, steady-state analysis, observed behavior analysis and real-time visualization. This theoretical foundation opens new pathways for developing institutional tools aimed at macroeconomic supervision and policy design.

Economic network theory:
Formalizes the economy as a complete directed graph where each node represents an economic agent and each edge a quantified monetary flow. This representation captures structured direct and indirect interactions among agents.

Input-output modeling:
Extends traditional input-output approaches with a dynamic matrix structure incorporating dynamic coefficients and microeconomic heterogeneity. This enables simulation of responses to exogenous shocks.

Discrete dynamical systems:
Integrates established discrete-time modeling techniques and control theory within a network-based economic framework, enabling the analysis of system-wide dynamics, the steady-states convergence and propagation through agent-level interactions and structural interdependencies, providing a foundation for policy-oriented diagnostics.

Observed behavior analysis:
Building on Euler equation, the optimal ratio of discretionary flows are reformulated in terms of the propensity factor, which can be inferred from observed data.

Agent-based simulation:
The framework provides a consistent algebraic basis enabling simulation of heterogeneous rational-agent networks, ensuring traceability between micro-foundations and aggregate behavior.

Real-time visualization and monitoring:
Develops an interactive interface based on normalized matrix representation that dynamically visualizes monetary flows. This tool facilitates diagnosis of structural imbalances and early detection of anomalies in agents' preferences.

Benchmarking against existing models

Compared to standard modeling approaches:

Micro-level heterogeneity: Offers greater detail than DSGE and IO models, while remaining on par with ABM frameworks.

Explicit network interactions: Provides a fully formalized treatment, unlike DSGE and IO models, and captures network structure more systematically than typical ABMs.

Dynamic and qualitative analysis: Extends dynamic analysis beyond IO and ABM models by formalizing networks dynamics, while complementing DSGE frameworks through explicit heterogeneity.

Marginal rate of substitution inference: Introduces the adoption of observational data to deduce Euler's conditions, a feature not found in DSGE, ABM, or IO frameworks.

Shock adaptability: Exhibits greater flexibility than DSGE and IO models, comparable to ABMs but within a coherent and formal setup.

Real-time visualization: Integrates native real-time visualization capabilities, setting it apart from other models.

Computational scalability: Requires similar ABMs computational resources and more than IO models, available for dimensionality reduction.

Institutional applicability: At an exploratory stage, but with promising potential for real-time monitoring and policy diagnostics.

Graph-based modeling

Algebraic formulation

An economic network is modeled as a complete directed graph $G = (A, I)$, where:

- $A = \{a_1, a_2, \ldots, a_n\}$ is the set of agents, with cardinality n.
- $I = A \times A$ is the set of all directed interactions, of size n^2.

Each ordered pair $(a_i, a_j) \in I$ represents a monetary flow from agent a_i to agent a_j, with diagonal pairs (a_k, a_k) representing intertemporal savings of a_k.

The system is represented algebraically by the flow matrix

$$S_t = \left[s_t^{i,j}\right]_{i,j \in A} \in \mathbb{R}_{\geq 0}^{n \times n} \quad \forall t \in \mathbb{N},$$

where $s_t^{i,j}$ denotes the monetary flow from a_i to a_j at time t and diagonal entries $s_t^{k,k}$ capture the intertemporal savings of a_k.

Roles, constraints and net monetary flows

Agents are classified by an exogenous role function $k : A \to \{-1, 0, 1\}$, reflecting their capacity to create or destroy money.
The accounting constraint for each agent at time t is defined as

$$R_t^a := \sum_{\substack{j \in A \\ j \neq a}} \left[s_t^{a,j}\right] + s_t^{a,a} - \sum_{\substack{i \in A \\ i \neq a}} \left[s_t^{i,a}\right] - s_{t-1}^{a,a} \quad \forall t \in \mathbb{N}.$$

This satisfies

$$\begin{cases} R_t^a \leq 0 & \text{if } k(a) = -1 \quad \text{(destroyer)}, \\ R_t^a = 0 & \text{if } k(a) = 0 \quad \text{(neutral)}, \\ R_t^a \geq 0 & \text{if } k(a) = 1 \quad \text{(creator)}, \end{cases}$$

where system net monetary flows are computed as

$$\sum_{a \in A} R_t^a \quad \forall t \in \mathbb{N}.$$

Dynamic network properties

Modeling the economy as a dynamic network provides a fundamental framework for analyzing both the existence and convergence to steady-states and the propagation of exogenous shocks on propensity factors (Ω_t).

Let the network be represented as a discrete-time linear system:

$$x_t\left(\Omega_t\right) = V\left(\Omega_t\right) J\left(\Omega_t\right) V^{-1}\left(\Omega_t\right) \quad x_{t-1}\left(\Omega_{t-1}\right)$$

where x_t denotes the state vector comprising all discretionary flows, V is the matrix of eigenvectors, and J is the Jordan canonical form.

Steady-state existence and convergence

A steady-state is a state that remains invariant over time in the absence of exogenous shocks

$$x_{t-1}\left(\Omega_{t-1}\right) = x_t\left(\Omega_t\right) = x^*\left(\Omega^*\right) \quad \forall t \in \mathbb{N}$$

and its properties are determined by the spectral radius of the system matrix J

$$\rho(J) := \max_{\lambda \in \sigma(J)} |\lambda|.$$

A linear dynamical system has at least one trivial steady-state ($x^* = 0$). But, if J has at least one diagonal term equal to one, then the system has an additional non-trivial steady-state ($x^* \neq 0$).

After a shock perturbation:
If $\rho(J) < 1$, the system is asymptotically convergent to $x^* = 0$.
If $\rho(J) = 1$ with Jordan blocks associated of size one, the system is non-convergent and do not return to the previous steady-state but reaches a new one.
Otherwise, the system diverges.

Shock Response

To assess the propagation effects of exogenous shocks on propensity factors, the derivative of the state vector is used to capture infinitesimal sensitivities.
Global effects are quantified by comparing trajectories with and without the shock.

The derivative provides a theoretical measure of how infinitesimal shocks propagate through the network:

$$\frac{\partial x_t}{\partial \Omega_t} = \frac{\partial}{\partial \Omega_t}\left[VJV^{-1}\right] x_{t-1} \quad \forall t \in \mathbb{N},$$

such that

$$x_t\left(\Omega_t + \Delta\Omega_t\right) \approx x_t\left(\Omega_t\right) + \frac{\partial x_t}{\partial \Omega_t}\Delta\Omega_t \quad \forall t \in \mathbb{N}.$$

The effect of an exogenous shock on propensity factors over time can be quantified by comparing the trajectory with the shock ($\tilde{\Omega}_t$) to the baseline scenario without the shock.

Shock response analysis can be divided into short and long term.

Short-term analysis, where the horizon is finite

$$\Delta x_t = x_t\left(\tilde{\Omega}_t\right) - x_t\left(\Omega_t\right) \quad \forall t \in \mathbb{N}.$$

Long-term analysis, where the horizon tends to infinity

$$\Delta x_\infty = \lim_{t\to\infty}\left[x_t\left(\tilde{\Omega}_t\right) - x_t\left(\Omega_t\right)\right].$$

Consequently, short-term dynamics are determined by eigenvectors associated with $\lambda > 0$ while, long-term dynamics are determined by eigenvectors associated with $\lambda \geq 1$.
This is because eigenvectors with $\lambda = 0$ vanish immediately, and those with $\lambda < 1$ decay over time.

Taken together, this dual framework provides a comprehensive tool-set to analyze both the local propagation of shocks and their global effects over time.

Behavior inference

In a monitored environment, where flows are continuously observed, the relationships among optimal flows can be inferred and used for deduce the dynamical properties of the system. Two steps must be considered in order to succeed.

Discretionary flows identification

Discretionary flows (s_t^a) are money transfers that directly affect the agent's utility function.
Their identification relies on economic reasoning and the institutional context.

Attention is restricted to the simplified case where each agent has two discretionary flows $s_t^a = \{p_t^a, q_t^a\}$. This assumption allows Euler's equations (EE) to be expressed as the optimal margin rate of substitution.

Relating via propensity factor

EE computes the optimal ratio between the discretionary flows for each agent

$$EE_t^a := \frac{p_t^a}{q_t^a}, \quad \forall a \in A, t \in \mathbb{N}.$$

By economic intuition, propensity factor, which reflects agents' preferences, can be expressed relative to net income (I_t^a):

$$\Omega_t^a = \frac{p_t^a}{I_t^a} \quad \forall a \in A,\ t \in \mathbb{N}$$

From the accounting constraint, it follows that

$$I_t^a := -R_t^a \quad \forall s_t^{i,j} \notin s_t^a, a \in A, t \in \mathbb{N},$$

or equivalently

$$I_t^a = p_t^a + q_t^a \quad \forall a \in A,\ t \in \mathbb{N}.$$

So, propensity factor can be expressed as

$$\Omega_t^a = \left(1 + \frac{q_t^a}{p_t^a}\right)^{-1} \quad \forall a \in A, t \in \mathbb{N}$$

and EEs becomes

$$q_t^a = \left(\frac{1}{\Omega_t^a} - 1\right) p_t^a \quad \forall a \in A, t \in \mathbb{N}.$$

Visualization framework

Although agent flows may depend on future expectations, their evolution can be approximated using a fixed propensity factor (Ω_t^a).

Discretionary flows are then simulated as

$$p_t^a = \Omega_t^a I_t^a \quad \forall a \in A,\ t \in \mathbb{N}$$

$$q_t^a = (1 - \Omega_t^a)\, I_t^a \quad \forall a \in A,\ t \in \mathbb{N}.$$

For practical implementation, the economic network is simulated alongside an interactive dashboard for real-time monitoring.

Design

The dashboard provides a minimal, interpretable interface for real-time monitoring and experimentation, offering:

- A normalized flow matrix to observe monetary circulation.
- Visual indicators of estimated exogenous propensity factors.
- Interactive controls for adjusting agents' preferences during simulation.

The dashboard enables systematic monitoring and analysis of monetary flows, facilitating the detection of shocks and dynamic network patterns.

Network normalization

To ensure consistent and interpretable visualizations, flow matrix are normalized as

$$\bar{S}_t = \frac{1}{Z_t} s_t^{i,j} \in \mathbb{R}_{\geq 0}^{a \times a} \quad \forall i, j \in A,\ t \in \mathbb{N},$$

where the normalization factor Z_t may be based on GDP for economic interpretation or on the aggregated network flow to bound all values within [0,1]. For automated, real-time monitoring, the latter approach is adopted:

$$Z_t = \sum_{i \in A} \sum_{j \in A} s_t^{i,j} \quad \forall t \in \mathbb{N}.$$

Limitations and extensions

Model limitations

Economic networks as a complete directed graph: The framework models the economy as a complete directed graph. In practice, most economic networks are sparse, since many agents do not interact directly. This is accommodated by assigning zero-valued flows to non-existent interactions. This approach preserves the formal completeness of the graph while simultaneously capturing its effective sparsity.

Exogenous and invariant roles: Agents' roles are assigned exogenously through a time-invariant function. While this simplification aids interpretation and clarity, it restricts the model's capacity to capture role transitions and limits its ability to represent adaptive economic dynamics.

Methodological limitations

Discretionary flows identification: The first step in the Observed Behavior Analysis section relies on identifying two discretionary flows, however, this process is context-dependent and subject to the researcher's judgment, with no universal protocol applicable across all cases or institutional settings.

Computational challenges

Available data: Behavioral analysis and dashboard monitoring require large volumes of granular streaming data. This, in turn, demands a dedicated data hub capable of collecting, preprocessing, and routing information to the application in real-time without interruptions.

Computational cost: The model's computational complexity grows as $\mathcal{O}(n^2)$ with the number of agents, posing a substantial challenge to scalability. Dimensionality reduction could be achieved through topological pruning based on observed economic structures and subnetwork encapsulation. Further performance gains may result from parallel computing or cloud-based simulations.

Future research directions

Building on the limitations discussed above, three promising avenues for extension can be identified:

- Incorporating endogenous, time-varying roles inferred from observed behavior, using stochastic modeling or machine learning techniques.
- Developing generalizable protocols to standardize the identification of discretionary flows beyond the two considered.
- Developing APIs and stream-processing pipelines to enable continuous ingestion of real-time economic data and seamless integration with the monitoring dashboard.

Closing remarks

Despite its limitations, the proposed framework establishes a robust foundation for modeling, monitoring, and analyzing economic networks. Its main contribution lies in uniting formal theoretical structures with interactive simulation and openness to automated learning methods.
The framework's consolidation will ultimately depend on data quality and availability, as well as on the development of hybrid methodologies that connect economic theory with empirical inference. By integrating formal theory, behavioral inference, and real-time monitoring, it sets the stage for a new generation of computational macroeconomic tools with both academic and institutional relevance.

Artificial Intelligence Usage

Generative Artificial Intelligence tools were employed to support the research process, enhancing efficiency and quality while preserving scientific integrity. Their application primarily encompassed:

- Literature review
- Mathematical formulation
- Academic writing
- Results verification

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Appendices

Appendix A: Intertemporal optimization

Following neoclassical microeconomics, each agent maximizes an intertemporal utility function $U^a(\Omega_t^a, \hat{s}_t^a)$, where $\Omega_t^a \in (0,1)$ represents exogenous propensity factors and $\hat{s}_t^a$ denotes discretionary monetary flows constrained by

$$S_t^a = \left\{ \hat{s}_t^a \in \mathbb{R}_{\geq 0}^n \mid R_t^a(\hat{s}_t^a) \right\}, \quad n = |\hat{s}_t^a|, \quad \forall a \in A, t \in \mathbb{N}.$$

Then, agent's optimal solutions are

$$s_t^a := \arg \max_{\hat{s}_t^a \in S_t^a} \sum_{t \in \mathbb{N}} (\Pi^a)^t U^a(\Omega_t^a, \hat{s}_t^a) \quad \forall a \in A, t \in \mathbb{N},$$

with discount factor $\Pi^a \in (0,1)$ and where non-determinism arises from the time-dependent exogenous factors Ω_t^a.

Stationary expectation

The compleixity of the economic network often causes the optimal solutions to incorporate forward-looking expectations.
That's because lagged savings appear in the intertemporal accounting constraint, implying that current decisions are conditioned by anticipated future states. As a result, Euler's equation cannot be derived analytically in terms of its parameters.

A tractable simplification consists in adopting the *stationary expectation hypothesis*, according to which agents are assumed to expect identical optimal flows across all time periods.

$$\mathbb{E}[x_{t+1}] = x_t \quad \forall\, t \in \mathbb{N},$$

where x denotes either optimal flows or propensity factors.

Appendix B: Closed economy network with households and firms

In a closed economy consisting of households and firms, without money creation, the system can be represented as a graph $G(2,4)$, with:

$$A = \{\text{Households (h), Firms (f)}\}$$

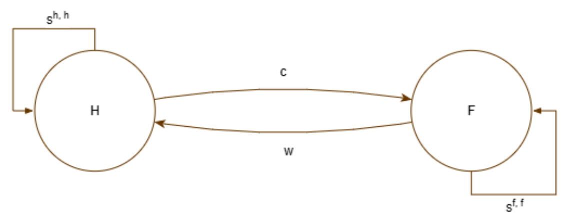

Figure 1: Households and firms economy

Matrix Representation

$$S_t = \begin{bmatrix} s_t^{h,h} & c_t \\ w_t & s_t^{f,f} \end{bmatrix}$$

Where:

$$c_t := s_t^{h,f} \quad \text{(consumption)}, \qquad w_t := s_t^{f,h} \quad \text{(wage)}$$

Constraints

$$k(h) = 0 \quad (\textit{neutral}) \qquad k(f) = 0 \quad (\textit{neutral})$$

$$R_t^h := c_t + s_t^{h,h} - w_t - s_{t-1}^{h,h} = 0 \quad \forall t \in \mathbb{N}$$
$$R_t^f := w_t + s_t^{f,f} - c_t - s_{t-1}^{f,f} = 0 \quad \forall t \in \mathbb{N}$$

Net monetary flows

$$\textstyle\sum_{a \in A} [s_t^{a,a} - s_0^{a,a}] = 0 \quad \forall t \in \mathbb{N}$$

Observed behavior analysis

Assume the following discretionary flows pairs by each agent:

$$\hat{s}_t^h := \left\{c_t, s_t^{h,h}\right\}, \qquad \hat{s}_t^f := \left\{w_t, s_t^{f,f}\right\} \quad \forall t \in \mathbb{N}$$

Then agents' propensity factors can be written as

$$\Omega_t^h := \left(1 + \frac{s_t^h}{c_t}\right)^{-1} \qquad \Omega_t^f := \left(1 + \frac{s_t^f}{w_t}\right)^{-1} \quad \forall t \in \mathbb{N},$$

and marginal rate of substitution becomes

$$s_t^h = \left(\frac{1}{\Omega_t^h} - 1\right) c_t, \quad s_t^f = \left(\frac{1}{\Omega_t^f} - 1\right) w_t$$

subject to:

$$c_t = w_t + s_{t-1}^{h,h} - s_t^{h,h}$$
$$w_t = c_t + s_{t-1}^{f,f} - s_t^{f,f}$$

Network dynamical properties

Solving the system it holds that

$$x = \left(c, w, s^{h,h}, s^{f,f}\right)^T$$

where

$$x_t = VJV^{-1}x_{t-1}$$

$$VJV^{-1} = \left(\frac{1}{\Omega_t^h\Omega_t^f} - 1\right)^{-1} \begin{bmatrix} 0 & 0 & \frac{1}{\Omega_t^f} & 1 \\ 0 & 0 & 1 & \frac{1}{\Omega_t^h} \\ 0 & 0 & \left(\frac{1}{\Omega_t^h} - 1\right)\frac{1}{\Omega_t^f} & \left(\frac{1}{\Omega_t^h} - 1\right) \\ 0 & 0 & \left(\frac{1}{\Omega_t^f} - 1\right) & \left(\frac{1}{\Omega_t^f} - 1\right)\frac{1}{\Omega_t^h} \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \left(\Omega_t^h + \Omega_t^f - 2\right)\left(\Omega_t^h\Omega_t^f - 1\right)^{-1} - 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & \left(\frac{1}{\Omega_t^f} - \frac{1}{\Omega_t^h}\right)\left(\frac{1}{\Omega_t^f} - 1\right)^{-1} - 1 \\ 0 & 0 & \frac{1}{\Omega_t^h} - 1 & \frac{1}{\Omega_t^h} - 1 \\ 0 & 0 & \frac{1}{\Omega_t^f} - 1 & -\left(\frac{1}{\Omega_t^h} - 1\right) \end{bmatrix}$$

Shock propagation

$$\frac{\partial x_t}{\partial \Omega_t^h} = \left(\Omega_t^h\Omega_t^f - 1\right)^{-2} \begin{bmatrix} 0 & 0 & 1 & \Omega_t^f \\ 0 & 0 & \Omega_t^f & \Omega_t^{f^2} \\ 0 & 0 & -\left(1 - \Omega_t^f\right) & -\left(1 - \Omega_t^f\right)\Omega_t^f \\ 0 & 0 & 1 - \Omega_t^f & \left(1 - \Omega_t^f\right)\Omega_t^f \end{bmatrix} x_{t-1}$$

$$\frac{\partial x_t}{\partial \Omega_t^f} = \left(\Omega_t^h\Omega_t^f - 1\right)^{-2} \begin{bmatrix} 0 & 0 & \Omega_t^{h^2} & \Omega_t^h \\ 0 & 0 & \Omega_t^h & 1 \\ 0 & 0 & \left(1 - \Omega_t^h\right)\Omega_t^h & \left(1 - \Omega_t^h\right) \\ 0 & 0 & -\left(1 - \Omega_t^h\right)\Omega_t^h & -\left(1 - \Omega_t^h\right) \end{bmatrix} x_{t-1}$$

Ω_t^h	c_t	w_t	$s_t^{h,h}$	$s_t^{f,f}$
↑	↑	↑	↓	↑
↓	↓	↓	↑	↓

Table 1: Households propensity shock propagation

Ω_t^f	c_t	w_t	$s_t^{h,h}$	$s_t^{f,f}$
↑	↑	↑	↑	↓
↓	↓	↓	↓	↑

Table 2: Firms propensity shock propagation

Shock response

Assume $\left\{\Omega_{t-1}^h = 0.40, \Omega_{t-1}^f = 0.35, s_{t-1}^{h,h} = 2, s_{t-1}^{f,f} = 2\right\}$..

Let's measure a two-steps forward $(t+1)$ shock of 0.25 on Ω^h.

$$\Delta x_{t+1} = x_{t+1}(0.50, 0.35) - x_{t+1}(0.40, 0.35)$$

$$\Delta x_{t+1} = \begin{bmatrix} 1.49 \\ 1.35 \\ 1.49 \\ 2.51 \end{bmatrix} - \begin{bmatrix} 1.22 \\ 1.17 \\ 1.83 \\ 2.17 \end{bmatrix} = \begin{bmatrix} 0.27 \\ 0.18 \\ -0.34 \\ 0.34 \end{bmatrix}$$

Now lets measure the shock response in infinity.

$$\Delta x_\infty = x_\infty(0.50, 0.35) - x_\infty(0.40, 0.35)$$

$$\Delta x_\infty = \begin{bmatrix} 1.4 \\ 1.4 \\ 1.4 \\ 2.6 \end{bmatrix} - \begin{bmatrix} 1.19 \\ 1.19 \\ 1.79 \\ 2.21 \end{bmatrix} = \begin{bmatrix} 0.21 \\ 0.21 \\ -0.39 \\ 0.39 \end{bmatrix}$$

Appendix C: Hierarchical economic networks

Up to this point, the graph modeling approach has been developed for a network of interconnected agents. However, the proposed methodology is also scalable to higher levels of aggregation. Referring to a national economic network, it's possible to extend its model to an international economic network $G(A, I)$ composed of multiple national systems.

- $A = \{G_1, G_2, \ldots, G_n\}$ denotes the set of subgraphs, where each G_i represents a national economy.
- $I \subseteq A \times A$ denotes the set of directed interactions between nations.

At this level, the entire theoretical framework developed in the core sections of this contribution remains applicable, with the key distinction that the nodes of the supergraph no longer represent heterogeneous agents but instead encapsulate complete networks. Each node functions as a structural container holding a lower-level economic interactions.
Consider two national economies, each represented according to the following figure.

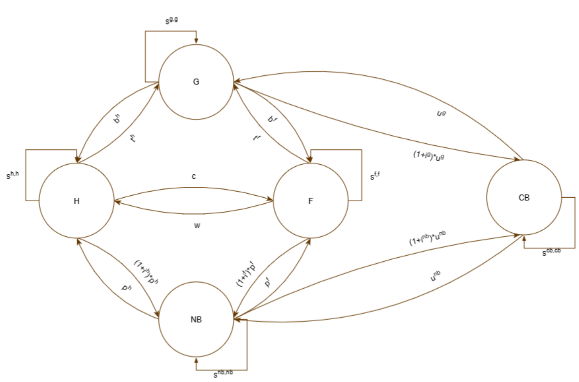

Figure 2: National economy

To model an international economy, it suffices to define which agents trade out of their native network. For illustrative purposes, it's considered interactions between central banks for currency exchange and households and firms for international trade.

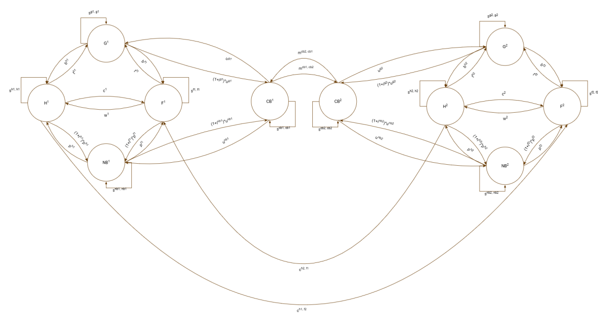

Figure 3: International economy with two nations

It's important to note that the adjacency matrix associated with the extended graph expands rapidly in size. To facilitate its representation, each nation is treated as an encapsulated substructure, while preserving the relevant higher-level interactions.

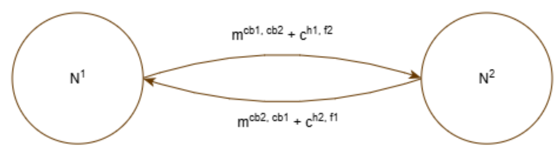

Figure 4: Encapsulated international economy

The result is a hierarchical second-order structure.

Finally, it's worth emphasizing that this approach is inherently recursive: starting from national networks, one can model an international economy; with multiple planets, an interplanetary economy; with stellar systems, an interstellar economy. Similarly, the model allows for descending the level of analysis to urban, sectoral, corporate, or even individual economic networks.

Appendix D: Simulation and dashboard

As a practical exercise, a simulation for a closed economy consisting of households and firms has been developed to illustrate the design of an interactive dashboard for scenario tracking and simulation.

The dashboard consists of two main views: the first for configuring the simulation parameters, and the second for monitoring the simulation and enabling user intervention.
In the interactive dashboard, the user can interact with normalized values via a heatmap, monitor the time series of propensity factors, and modify these factors using the sliders located in the upper-right section of the interface.

This panel serves as a functional proof of concept for the proposed theoretical model and demonstrates its feasibility for practical applications in real-time economic monitoring.
Visit the online repository for access to the visualization.

Figure 5: Simulation — Setting

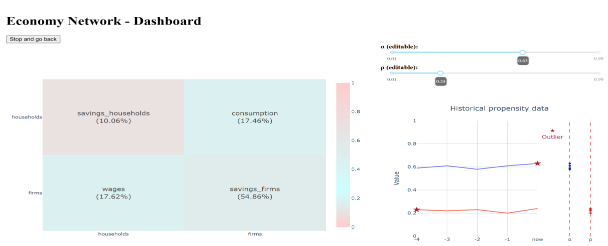

Figure 6: Simulation — Dashboard