

# Economic Network Modeling: A Graph-Theoretical Framework Linking Micro-interactions to Macro-dynamics



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## Final Work Card

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**Abstract**

Traditional macroeconomic models often overlook the micro-level interactions through which shocks propagate and macroeconomic phenomena emerge. To address this limitation, this work introduces a novel graph-theoretical framework that formalizes the economy as a dynamical network of interconnected agents.

The methodology establishes an algebraic representation of the system, infers agent behavior from empirical data, and analyzes system convergence and shock diffusion within the network.

As a proof of concept, a computational simulation complemented by an interactive dashboard has been developed, enabling the conceptual exploration of hypothetical scenarios and policy interventions.

By moving beyond aggregate approaches, this framework offers a robust tool for macroeconomic analysis, monitoring, and stress testing, providing a network-based paradigm for understanding economic complexity.

**Keywords:** economic networks, behavioral inference, shock propagation, visual analytics.

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# Chapter 1

## Introduction

### 1.1 Context and justification of the work

According to the first law of thermodynamics, energy cannot be created or destroyed, only transferred. This physical principle provides a compelling analogy for economics, in which one agent's income corresponds to another's expenditure, illustrating monetary circulation. Although the analogy is not exact, since central banks can create money and individuals can burn bills, it still frames the economy as a dynamic system of interconnected agents whose actions are mutually interdependent.

Traditional macroeconomic models often simplify this complexity by aggregating flows, overlooking explicit interactions, thus limiting their capacity to analyze propagation effects. This work addresses these limitations by proposing a novel graph-theoretical framework that explicitly links micro-level interactions to macro-level dynamics, providing a richer basis for understanding how individual behaviors collectively shape macroeconomic phenomena.

Therefore, the fundamental motivation of this thesis is to develop an unified framework to capture and study economic complexity.

### 1.2 Work objectives

The primary objectives of this research are to propose a new framework for modeling economic environments as a discrete dynamic graph and to design an interactive dashboard for visual analytics as a proof of concept. This is achieved through the following specific objectives:

- Develop a graph-based model to represent monetary interactions and agent connectivity.
- Incorporate a novel behavioral inference method to calibrate agent interactions using empirical data.
- Formalize the system's evolution using discrete-time dynamical systems.
- Implement an interactive, real-time dashboard for scenario exploration and analysis.

Together, these objectives establish a cohesive framework that integrates theoretical modeling with practical tools for monitoring and simulating economic systems.

## 1.3 Impact on sustainability, ethics, and diversity

The proposed framework provides a substantial contribution to sustainable and inclusive economic analysis by explicitly modeling interactions among heterogeneous agents. This approach enables the identification of structural imbalances, contagion risks, and potential distributional inequalities, aligning with the objectives of the 2030 Agenda for Sustainable Development. Its impact spans four dimensions:

- **Economic stability and innovation (SDG 8 & SDG 9).** The framework facilitates the design of policies that foster stable and resilient economic growth while introducing methodological innovations that refine decision-making within complex economic systems.
- **Social inclusion (SDG 10).** By representing detailed interactions among agents, the model enables a precise analysis of inequality dynamics and resource flows, supporting interventions that reduce systemic risks and enhance social equity across the population.
- **Ethical governance and accountability (SDG 16).** Complementing the social and economic dimensions, elucidating assumptions and tracing the propagation of shocks reinforce transparency and accountability in economic modeling, thus strengthening confidence in policy-making processes.
- **Diversity and sustainable participation (SDG 5 & SDG 12).** Incorporating heterogeneous agents captures the complexity of real economies, allowing the assessment of differentiated impacts and fostering inclusive participation. This strengthens gender equity and aligns with the guiding principle of *leaving no one behind*.

## 1.4 Focus and method followed

The research adopts an Agile-inspired framework to support iterative development and conceptual validation. Rather than following a linear workflow, the project evolves through iterative cycles consisting of:

- **Objective definition.** Each cycle begins by specifying key goals, such as the algebraic modeling of economic networks or the dashboard design.
- **Implementation and validation.** This phase comprises module development and preliminary testing to ensure theoretical and computational consistency.
- **Refinement.** The cycle ends with the adjustment of intermediate outcomes based on analytical findings to improve the overall theoretical coherence.

This iterative and modular approach allowed each theoretical component to be developed independently, but cohesively, forming a unified framework.

In summary, the Agile-inspired methodology ensures both flexibility and rigor, facilitating the integration of theoretical modeling and computational experimentation.

## 1.5 Work schedule

Task	Start	End	Year	2025												2026					
				Month		09			10			11			12			01			
				Week		22	29	06	13	20	27	03	10	17	24	01	08	15	22	29	05
M1	25/09/25	12/10/25																			
Document structuring	25/09/25	26/09/25																			
Introduction writing	27/09/25	04/10/25																			
Bibliography review	05/10/25	12/10/25																			
M2	13/10/25	02/11/25																			
Conceptual framework	13/10/25	23/10/25																			
Literature review	24/10/25	29/10/25																			
Benchmarking analysis	30/10/25	02/11/25																			
M3	03/11/25	14/12/25																			
Graph-based modeling	03/11/25	11/11/25																			
Behavioral inference	12/11/25	19/11/25																			
Dynamical approach	20/11/25	30/11/25																			
Dashboard design	01/12/25	14/12/25																			
M4	15/12/25	06/01/26																			
Document draft	15/12/25	28/12/25																			
Audiovisual production	29/12/25	06/01/26																			
M5	07/01/26	30/01/26																			
Thesis submission	07/01/26	09/01/26																			
Defense rehearsal	10/01/26	30/01/26																			

Table 1.1: Gantt chart

## 1.6 Brief summary of products obtained

This work produces two main outcomes:

- An integrated graph-theoretical framework for economic modeling.
- A functional computational prototype for interactive, real-time visual analytics.

## 1.7 Brief description of document structure

The document is structured as follows.

**Chapter 2** reviews the related literature and presents the proposed graph-theoretical approach.

**Chapter 3** designs the computational prototype as a proof of concept.

**Chapter 4** summarizes key findings and outlines future research directions.

**Chapter 5** defines essential technical terms.

**Chapter 6** lists the references.

**Chapter 7** acknowledges the use of AI-assisted tools throughout the research process.

The **appendices** outline three extensions that address a simplified model execution, a hierarchical application, and an economic indicator for monetary crisis.

## 1.8 Risk assessment and mitigation strategies

The thesis may face certain risks that could affect its timely and successful completion. The most relevant are summarized below:

- **Technical implementation risk.** Compatibility issues between interdependent libraries may disrupt progress.  
The codebase is version-controlled and executed in an isolated virtual environment, with a `requirements.txt` file ensuring stability and reproducibility.
- **Time and schedule adherence risk.** Given the tight timeline defined in the work schedule, any delay could limit the revision time.  
Careful planning, weekly milestones, and early completion of key tasks will help ensure on-time delivery.
- **Conceptual and communicative risk.** Translating complex theoretical constructs into an interpretable model may challenge both coherence and accessibility.  
Iterative tutor feedback and visual aids will help maintain conceptual consistency while keeping the content accessible for a non-expert audience.

# Chapter 2

## Materials and Methods

### 2.1 State of the art

In recent years, macroeconomic modeling has diversified substantially, giving rise to a variety of approaches that differ in their assumptions and analytical objectives.

To situate this thesis, a structured review of traditional macroeconomic modeling is conducted.

#### 2.1.1 Conceptual framework

Macroeconomic models can be systematically analyzed along three key dimensions: **granularity**, **dynamics**, and **topology** (Jackson, 2008). Together, these dimensions define a conceptual space that enables coherent classification and comparison among modeling approaches.

Granularity denotes the degree of disaggregation in economic representation:

- **Aggregate.** The economy is treated as a whole.
- **Representative.** Groups of similar agents are aggregated.
- **Individual.** Each agent is modeled explicitly (Gabaix, 2011).

Temporal evolution describes how the system evolves over time:

- **Static.** Equilibrium without explicit temporal evolution.
- **Discrete.** Stepwise evolution capturing short-run adjustments.
- **Continuous.** Smooth evolution over time.

Topology captures the pattern of interactions among agents:

- **Isolated.** Agents act independently.
- **Connected.** Agents influence one another through interactions (Battiston et al., 2016).

Mapping macroeconomic models into this conceptual framework reveals their underlying methodological families, highlighting the mechanisms and scope of different modeling approaches.

## 2.1.2 Literature review

Building on this framework, the following overview maps canonical macroeconomic paradigms onto the conceptual space defined above, summarizing their core principles and methodological orientation.

**Vector Auto-Regression (VAR) Models.** VAR models capture empirical correlations between aggregate variables in discrete time. They lack explicit microfoundations or network structure, focusing instead on the statistical propagation of shocks (Sims, 1980).

**Computable General Equilibrium (CGE) Models.** CGE models simulate aggregated sectors linked by market-clearing conditions. They are typically static, used to assess comparative equilibria under external shocks through indirect interactions (Shoven and Whalley, 1984).

**Input-Output (IO) Models.** IO models depict a static network of production sectors connected by fixed technical coefficients (Leontief, 1936). They quantify proportional interdependencies and the immediate impact of sectoral shocks.

**Stock-Flow Consistent (SFC) Models.** SFC models describe aggregated institutional sectors linked by accounting identities in discrete time. Their interconnected balance sheets capture the propagation of monetary and real flows across the system (Godley and Lavoie, 2007).

**Overlapping Generations (OLG) Models.** OLG models represent successive cohorts of representative agents interacting through capital and labor markets. They evolve discretely, enabling the analysis of intergenerational and long-term policy dynamics (Diamond, 1965).

**Dynamic Stochastic General Equilibrium (DSGE) Models.** DSGE models build on the intertemporal optimization framework introduced by (Ramsey, 1928), evolving continuously through stochastic shocks with interactions mediated indirectly by market-clearing prices (Woodford, 2003).

**Agent-Based Models (ABM).** ABMs simulate heterogeneous agents interacting directly within network structures (Epstein and Axtell, 1996). They evolve discretely, generating aggregate outcomes from adaptive, bottom-up behaviors (Simon, 1955; Assenza et al., 2020), which often rely on ad-hoc rules and suffer from computational opacity.

Model	Granularity	Temporal evolution	Topology
VAR	Aggregate	Discrete	Isolated
CGE	Representative	Static	Connected
IO	Representative	Static	Connected
SFC	Representative	Discrete	Connected
OLG	Representative	Discrete	Connected
DSGE	Representative	Continuous	Connected
ABM	Individual	Discrete	Connected
<b>Proposed Framework</b>	Individual	Discrete	Connected

Table 2.1: Canonical macroeconomic models within the conceptual framework

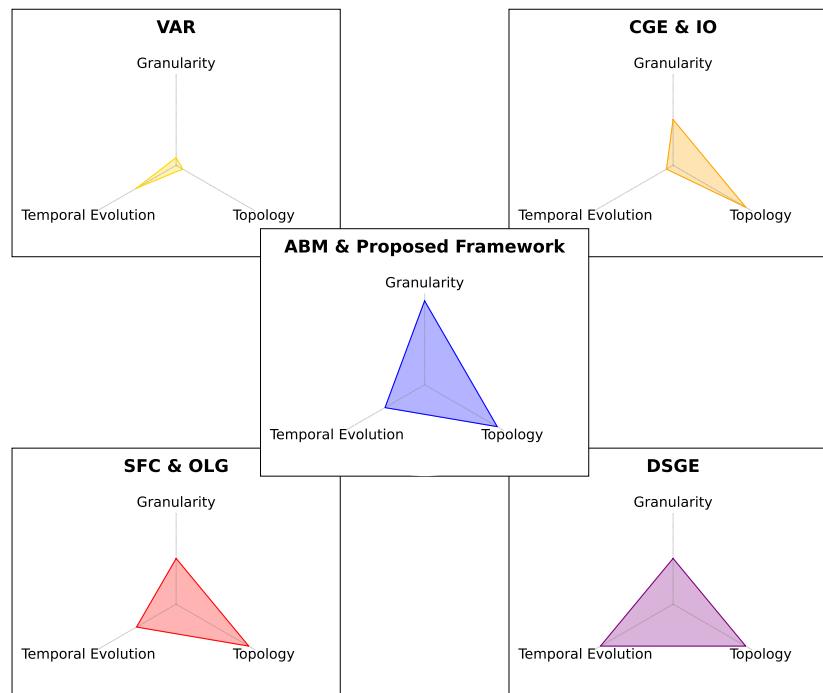


Figure 2.1: Conceptual mapping of macroeconomic canonical models

### 2.1.3 Proposed framework benchmarking against canonical models

The reviewed paradigms show key limitations, ranging from over-aggregation in traditional models to methodological opacity in canonical ABMs.

To highlight the native advantages of the proposed framework relative to canonical macroeconomic models, a benchmarking exercise is conducted across seven key dimensions.

**Micro-representation.** Modeling macro-dynamics in terms of low-level interactions between heterogeneous individual agents (Gabaix, 2011).

**Dynamical analysis.** Analyzing steady states, equilibrium convergence, and scenario simulations using dynamic systems (Elaydi, 2005).

**Behavioral inference.** Deriving agent behavior from observed empirical data, linking theoretical optimality with empirical measurability.

**Visual analytics.** Integrating a dynamic, real-time visualization interface for monitoring network topology and monetary flows (Munzner, 2014).

**Traceability.** Enabling full and consistent traceability that rigorously links aggregate outcomes back to explicit algebraic decision rules and observed behavioral parameters.

**Institutional applicability.** Designing the framework as an institutional analysis tool and support for decision-making by monetary and supervisory authorities (Battiston et al., 2016).

**Computational efficiency.** Balancing the complexity of the theoretical formulation with efficient practical implementation.

Dimension	VAR	CGE	IO	SFC	OLG	DSGE	ABM	Proposed Framework
Micro-representation	-	-	-	-	-	-	+	+
Dynamical analysis	+	-	-	+	+	+	+	+
Behavioral inference	-	-	-	-	-	-	-	+
Visual analytics	-	-	-	-	-	-	-	+
Traceability	-	+	+	+	+	+	-	+
Institutional applicability	+	+	+	+	+	+	-	+
Computational efficiency	+	+	+	+	+	+	-	-

Table 2.2: Benchmarking relative to canonical models

Legend: “-” := non-native application; “+” := native application.

Although the proposed framework is conceptually rooted in the ABM paradigm (Table 2.1), the benchmark analysis highlights critical departures from traditional ABM applications. Specifically, the thesis incorporates three key innovations to address traditional ABMs’ limitations.

**Behavioral inference.** Replacing the programming of ad hoc heuristic rules (Simon, 1955) with an inductive inference process grounded in observed empirical data.

**Traceability.** Solving the computational opacity with an algebraic formulation (Elaydi, 2005).

**Visual analytics.** Integrating a native visualization interface for real-time monitoring (Munzner, 2014).

Together, these innovations establish the framework as a transparent and auditable policy tool, enhancing its institutional applicability (Battiston et al., 2016).

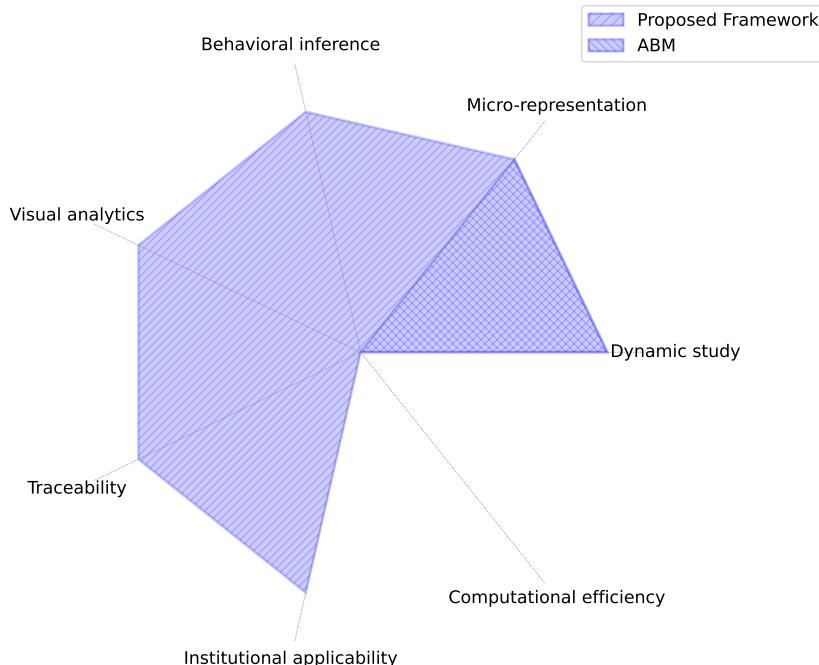


Figure 2.2: Proposed Framework innovations over ABMs

## 2.2 Theoretical framework

### 2.2.1 Graph-based modeling

#### Algebraic formulation

An economic network is modeled as a time-dependent, positive-weighted, reflexive, and complete directed graph  $G(A)$  where:

- $A := \{a_1, a_2, \dots, a_n\}$  is the set of  $n$  agents.
- $A \times A$  is the set of  $n^2$  possible directed edges.

The system is represented algebraically by the time-dependent flow matrix

$$S_t := [s_t^{a_i, a_j}]_{a_i, a_j \in A} \in \mathbb{R}_{\geq 0}^{n \times n} \quad \forall t \in \mathbb{N},$$

where  $s_t^{a_i, a_j}$  denotes the monetary flow from  $a_i$  to  $a_j$  at time  $t$  and diagonal entries  $s_t^{a_k, a_k}$  capture the intertemporal savings of  $a_k \in A$ .

#### Accounting constraints, roles, and net monetary flow

The accounting constraint is defined as the monetary restriction including both inter-agent and intertemporal transfers for each agent at time  $t$

$$R_t^{a_k} := \sum_{\substack{a_i \in A \\ a_i \neq a_k}} [s_t^{a_k, a_i} - s_t^{a_i, a_k}] + s_t^{a_k, a_k} - s_{t-1}^{a_k, a_k} \quad \forall a_k \in A, t \in \mathbb{N}.$$

To move beyond the pure conservation of the thermodynamic analogy and model real economic phenomena, agents are classified via the role function  $\text{role} : A \rightarrow \{\text{destroyer, neutral, creator}\}$

$$\text{role}(a_k) = \begin{cases} \text{destroyer} & \text{if } R_t^{a_k} < 0, \\ \text{neutral} & \text{if } R_t^{a_k} = 0, \\ \text{creator} & \text{if } R_t^{a_k} > 0, \end{cases}$$

embedding their capacity to create or destroy money into the economy.

Building on the agents' individual constraints, the system's net monetary flow is computed as the sum of all their restrictions

$$\Delta S_t := \sum_{a_k \in A} [R_t^{a_k}] \quad \forall t \in \mathbb{N},$$

quantifying the system's aggregate monetary imbalance according to the agents' roles at each time step.

## 2.2.2 Behavioral inference

Behavioral inference provides the microfoundations for the economic network by formalizing agent decision-making processes from empirical data.

Based on continuous flow observation, the analysis of a monitored environment enables the direct deduction of the underlying behavioral parameters and hence the dynamical properties of the system. The inference process is structured in two essential steps.

### Discretionary flows identification

Discretionary flows refer to monetary transfers that enter explicitly into the agent's utility function, reflecting decisions driven by individual preferences.

In the absence of a standardized identification procedure in the literature, discretionary flows are recognized through a combination of economic reasoning and institutional context.

The analysis is restricted to the simplified case where each agent has two discretionary flows

$$s_t^{a_k} := \{p_t^{a_k}, q_t^{a_k}\},$$

allowing Euler's equations (EE) to be reformulated as the marginal rate of substitution in terms of the agents' propensity factor.

### Relating via propensity factors

EE determine the optimal ratio between the discretionary flows of each agent

$$EE_t^{a_k} := \frac{p_t^{a_k}}{q_t^{a_k}}, \quad \forall a_k \in A, t \in \mathbb{N}.$$

By economic intuition, the propensity factor, which reflects agents' preferences, can be expressed relative to net income

$$\Omega_t^{a_k} := \frac{p_t^{a_k}}{I_t^{a_k}} \quad \forall a_k \in A, t \in \mathbb{N}.$$

From the accounting constraint and the thermodynamic principle of monetary circulation, each agent's net income becomes exactly the sum of its discretionary flows

$$I_t^{a_k} = p_t^{a_k} + q_t^{a_k} \quad \forall a_k \in A, t \in \mathbb{N}.$$

Hence, the propensity factor can be expressed as

$$\Omega_t^{a_k} = \left(1 + \frac{q_t^{a_k}}{p_t^{a_k}}\right)^{-1} \iff q_t^{a_k} = \left(\frac{1}{\Omega_t^{a_k}} - 1\right) p_t^{a_k} \quad \forall a_k \in A, t \in \mathbb{N},$$

characterizing each agent's behavior in terms of its exogenous but observable propensity factor.

## 2.2.3 Networks as dynamical systems

The dynamical systems approach formalizes the macro-dynamics of the economic network, revealing the patterns emerging from the propagation of micro-interactions.

Modeling the economy as a dynamical network provides a fundamental analytical basis for examining both the existence and convergence of steady states, as well as the propagation of exogenous shocks through propensity factors.

The propensity factors, inferred from individual agent behavior, are the sole parameters governing the network's evolution, naturally leading to its representation as a discrete-time linear system.

Let the network be represented as a discrete-time linear system

$$x_t = M(\Omega_t) x_{t-1} \quad \text{with} \quad M(\Omega_t) := V(\Omega_t) J(\Omega_t) V^{-1}(\Omega_t) \quad \forall t \in \mathbb{N},$$

where  $x_t$  denotes the state vector comprising  $2 \times n$  agents discretionary flows,  $V$  is the matrix of eigenvectors, and  $J$  is the Jordan canonical form.

### Steady-state existence and convergence

A steady-state is defined as a configuration that remains invariant over time in the absence of exogenous shocks

$$x^* := x_{t-1} = x_t \quad \forall t \in \mathbb{N}$$

and its stability properties are determined by the spectral radius of the system matrix

$$\rho(M(\Omega_t)) := \max_{\lambda \in \sigma(J)} |\lambda|.$$

From this perspective, different steady states correspond to different qualitative economic regimes.

Every linear dynamical system possesses at least the trivial steady-state ( $x^* = 0$ ), the economic collapse. However, if  $J$  has at least one diagonal element equal to one, the system also admits a non-trivial steady-state ( $x^* \neq 0$ ), a suitable economic stabilization.

After an exogenous shock, the economic convergence behavior depends critically on its spectral radius:

- If  $\rho(M(\Omega_t)) < 1$ , the economy is asymptotically convergent to its collapse  $x^* = 0$ .
- If  $\rho(M(\Omega_t)) = 1$  with associated Jordan blocks of size one, the economy is non-convergent and does not return to its previous steady-state. Instead, the system transitions towards a new equilibrium.
- Otherwise, the economy becomes unstable.

Therefore, the spectral radius  $\rho(M(\Omega_t))$  determines the economic network's resilience to shocks, serving as a threshold separating asymptotically contractive, adaptive, and unstable while simultaneously establishing the foundation for shock propagation analysis.

## Shock response

The propagation of exogenous shocks through the economic network is analyzed by examining their impact on the system's fundamental propensity factors. This analysis proceeds through the marginal sensitivity of the state vector, enabling the characterization of both local and global transmission mechanisms.

The sensitivity of the state vector to changes in agents' propensity factors is given by its derivative

$$\frac{\partial x_t}{\partial \Omega_t^{a_k}} = \frac{\partial M(\Omega_t)}{\partial \Omega_t^{a_k}} x_{t-1} \quad \forall a_k \in A, t \in \mathbb{N},$$

which yields a first-order approximation for shock propagation

$$x_t(\Omega_t + \Delta\Omega_t^{a_k}) \approx x_t(\Omega_t) + \frac{\partial x_t}{\partial \Omega_t^{a_k}} \Delta\Omega_t^{a_k} \quad \forall a_k \in A, t \in \mathbb{N}.$$

and captures the initial and most direct phase of shock diffusion before higher-order dynamic effects emerge.

Beyond local approximations, the full nonlinear response to a finite shock is captured by comparing the perturbed and baseline trajectories.

Building upon this local formulation, the analysis can be extended to evaluate the system's economic behavior across different time horizons.

The overall effect is quantified through two temporal perspectives:

- **Short-term analysis** with a finite horizon impact

$$\Delta x_t = \left( M(\tilde{\Omega})^t - M(\Omega)^t \right) x_0 \quad \forall t \in \mathbb{N}.$$

- **Long-term analysis** with an infinite horizon impact

$$\Delta x_\infty = \lim_{t \rightarrow \infty} \left[ M(\tilde{\Omega})^t - M(\Omega)^t \right] x_0.$$

As shown before, the eigenstructure of the economic network matrix  $M(\Omega_t)$  determines its temporal propagation patterns.

Short-term dynamics are governed by eigenvectors associated with  $|\lambda| > 0$ , while long-term outcomes are dominated by persistent modes ( $|\lambda| \geq 1$ ).

This decomposition reveals the economic architecture of shock propagation, distinguishing transient disturbances from enduring shifts.

From a policy perspective, this framework offers precise targeting. Short-term interventions should address flows associated with non-zero modes ( $|\lambda| > 0$ ) for immediate stabilization, while long-term reforms should adjust the persistent modes ( $|\lambda| \geq 1$ ) for resilience.

Collectively, this framework establishes an unified analytical basis for quantifying shock propagation, from initial local transmission to ultimate deep economic consequences.

# Chapter 3

## Results

This chapter presents the computational proof of concept developed under the MIT License and formulated from the theoretical framework introduced in Section 2.2.

The complete implementation is available in the public GitHub repository ([https://github.com/efarran0/Economic\\_Network/](https://github.com/efarran0/Economic_Network/)).

Its purpose is to demonstrate how the algebraic, behavioral, and dynamical components of the model manifest through economic networks.

### 3.1 Implementation overview

The prototype integrates two core components that together constitute the computational realization of the proposed economic network model:

- **Economic network simulation.** The simulation implements the algebraic formulation and dynamical system described in Appendix A.

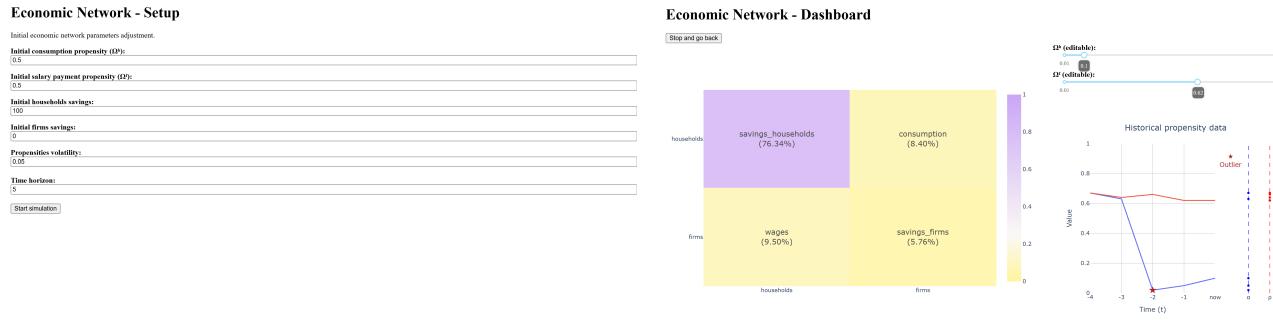
Agents' behavioral parameters are generated at each time step from a normally distributed process, emulating the temporal evolution of their preferences.

These values determine the agents' discretionary flows, which update the economy's flow matrix according to the model's dynamical rules.

This mechanism ensures that the simulated economy evolves coherently with the theoretical framework, preserving both accounting constraints and behavioral consistency.

- **Interactive dashboard.** A visual analytics interface renders the state of the simulated economy in real time and displays the normalized flow matrix, the historical evolution of propensity factors, and the system's response to user-defined parameter perturbations.

The dashboard constitutes the interpretability layer of the model, mapping formal mathematical structures into intuitive visual representations and enabling controlled interventions for experimentation.



### (a) Simulation setup

(b) Dashboard screen

Figure 3.1: Computational proof of concept

Together, these components form a complete implementation of the proposed framework. While the simulation enforces the algebraic logic and dynamical laws governing agent interactions, the dashboard reveals the emerging patterns and allows interactive exploration of the system's behavior under alternative scenarios.

### 3.2 Development setup

To run the computational prototype, the following setup is required:

- 1. Prerequisites.** The implementation is developed in Python within a standard scientific computing environment, designed to be compatible with major operating systems. Although optional, using an isolated virtual environment is recommended to ensure reproducibility and prevent version conflicts.
  - 2. Installation.** After cloning the repository, configure a Python environment and install all dependencies to ensure a stable and reproducible execution environment. Detailed instructions are provided in the repository's README.
  - 3. Requirements.** The project relies on widely used scientific computing libraries, each supporting a specific component of the framework. The main dependencies are:
    - **NumPy.** For numerical computation and vectorized operations.
    - **Pandas.** For data structures and time series handling.
    - **Statsmodels.** For time series decomposition and anomaly detection.
    - **Dash.** For constructing the interactive, web-based visual analytics dashboard.

The full specification is available in the requirements file.

4. **Execution environment.** Once installation is complete, the prototype can be executed via the project's main application module or through README hyperlinks. Execution triggers a local server hosting the dashboard, providing an unified environment for real-time visualization and stress testing.

### 3.3 Project architecture

A modular architecture is employed to separate the different components of the project, ensuring maintainability and facilitating future extensions. The project structure is organized as follows:

```

Economic_Network/
    assets/
        styles.css          (Visual layer styling)
    pdf/
        images/
            memory.pdf      (Figures documentation)
    src/
        __init__.py         (Package initialization)
        anomaly_detection.py (Behavioral inference - Section 2.2.2)
        app.py              (Main dash application)
        callbacks.py        (Interactive stress testing - Section 3.5.3)
        layout.py           (Dashboard UI components - Figure 3.3)
        sim.py              (Economic network simulation - Appendix A)
    requirements.txt      (Python dependencies)
    LICENSE              (Dual MIT & CC-BY-SA license)
    Procfile             (Process configuration)
    README.md            (Project documentation)
    render.yaml          (Deployment configuration),

```

implementing an adapted Model-View-Controller (MVC) pattern, where:

- **Model.** Implements the core economic simulation developed in Appendix A.
- **View.** Defines the visual interface of the dashboard.
- **Controller.** Handles user interaction, updates the model and, therefore, the dashboard.

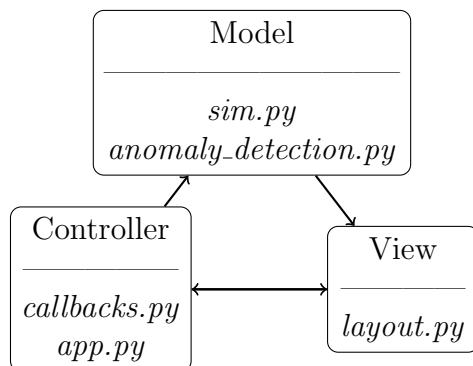


Figure 3.2: Model-View-Controller design.

The main file *app.py* acts as the orchestrator file, initializing these three layers and managing their interaction throughout the application workflow.

### 3.4 Application workflow

The simulation operationalizes the theoretical framework through a structured computational cycle, coordinated by the controller layer of the MVC architecture.

While the MVC architecture defines the responsibilities within the application, the workflow describes the temporal sequence through which these components operate during simulation

The execution comprises three sequential stages:

1. **Initialization.** The system is configured with a household and a firm via the setup screen (Figure 3.1.a).
2. **Iterative simulation loop.** While active, each time step executes the following phases:
  - (a) **Propensity factors generation.** Agents' behavioral parameters are either simulated or user-defined via the dashboard interface, emulating the temporal evolution of real economic preferences.
  - (b) **Flow matrix computation.** Economic flows are computed using the equations from Appendix A and the generated propensity factors.
  - (c) **Dashboard update.** The flow matrix is propagated to the dashboard interface, refreshing the visualizations.
3. **Termination.** The simulation runs until user termination.

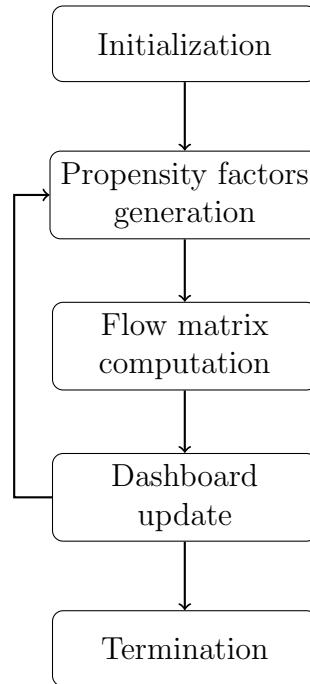


Figure 3.3: Application workflow pipeline

This workflow ensures that the theoretical components are faithfully translated into executable computational steps, maintaining the consistency while enabling continuous user interaction.

### 3.5 Dashboard components

The dashboard provides a minimal yet interpretable interface implementing the theoretical framework through three interconnected functional components:

- **Normalized flow visualization.** Direct rendering of matrix enabling pattern detection in monetary circulation flows.
- **Historical time series tracking.** Empirical monitoring of behavioral parameters derived via observational process.
- **Interactive stress testing.** Operationalization of shock propagation analysis through parameter perturbation tests.

This functional decomposition ensures each theoretical component from Chapter 2 has a direct computational counterpart, maintaining algebraic consistency while providing intuitive economic analytics.

#### Economic Network - Dashboard

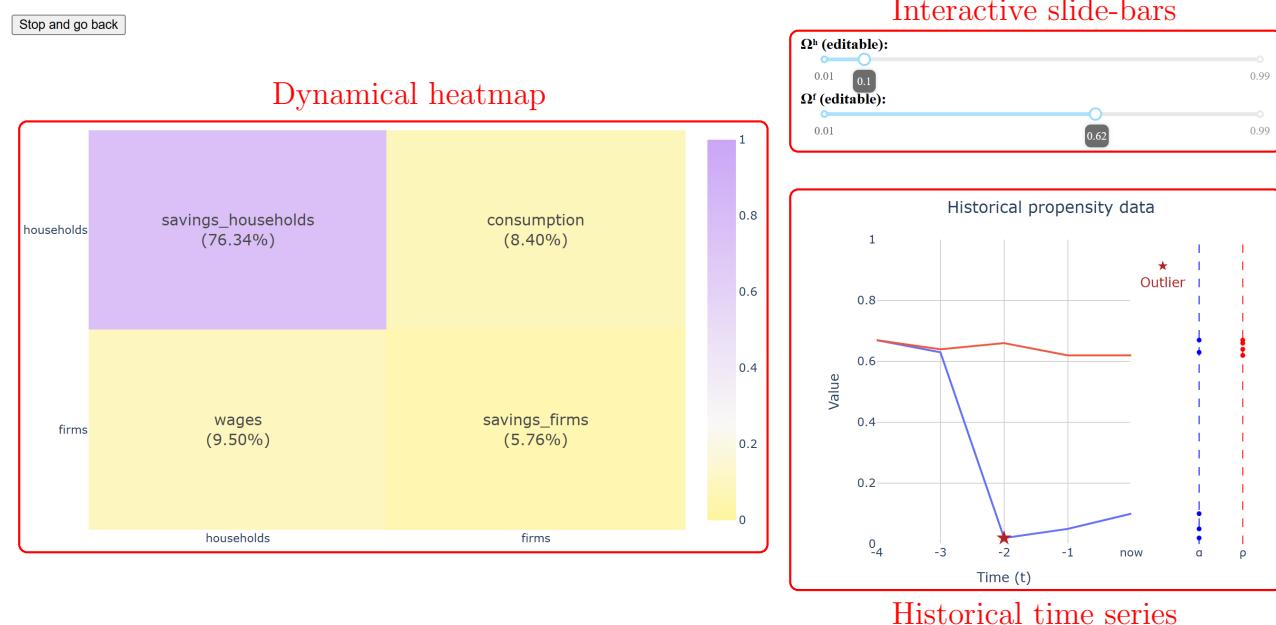


Figure 3.4: Dashboard components

### 3.5.1 Heatmap

For consistent and interpretable visualization, the flow matrix is normalized as

$$\bar{S}_t = \frac{1}{Z_t} S_t \quad \forall t \in \mathbb{N},$$

where the normalization factor

$$Z_t = \sum_{a_i \in A} \sum_{a_j \in A} [s_t^{a_i, a_j}]$$

bounds all values within  $[0, 1]$ , enabling automated real-time monitoring within a consistent scale.

The heatmap enables immediate identification of patterns within the economic network, where the chosen color scheme effectively visualizes two key economic behaviors and follow naturally from the constraint structure introduced in Section 2.2.1:

- **High-flow paths.** Characterized by a set of network connections carrying high monetary volumes.
- **Hoarder behavior.** A specific case of high-flow path, exhibiting persistent savings accumulation and thereby reducing monetary circulation in the network.

### 3.5.2 Historical time series with automated outlier detection

To support behavioral analysis and outlier detection, the dashboard incorporates historical time series visualization of propensity factors.

The outlier detection algorithm employs:

1. **Residuals extraction.** Propensity time series decomposition to isolate the residuals from trend and stationary components.
2. **Interquartile range test.** Performed over the residuals series to identify significant behavioral deviations.

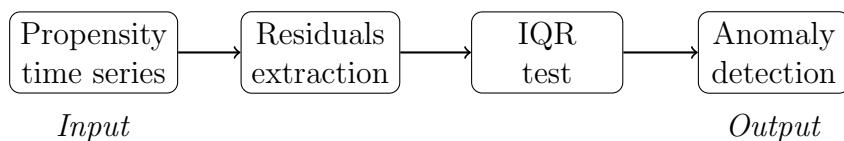


Figure 3.5: Automated outlier detection algorithm pipeline

This approach suits economic phenomena where behavioral patterns emerge from calendar events and periodic cycles, enabling the distinction between normal fluctuations and significant deviations, positioning the component as an early warning mechanism for potential recessions.

### 3.5.3 Interactive stress testing

Stress testing enables the interactive adjustment of agent propensity factors through sliders. This component provides an intuitive interface for exploring how micro-level perturbations propagate through the network, complementing the insights exposed in Section 2.2.3.

## 3.6 Summary of findings

This chapter presents a complete implementation of the Appendix A based on Section 2.2.

The adoption of a MVC architecture ensured a clean separation between the core economic simulation, the interactive visual interface, and the user-interaction logic, thereby reinforcing modularity and traceability across the framework.

The main outcomes demonstrate:

- **Theoretical-implementation coherence.** The prototype successfully translates algebraic formulations, behavioral inference mechanisms, and dynamical systems into functional code, maintaining mathematical consistency while achieving computational tractability.
- **Analytical utility.** The integrated simulation-dashboard system enables both real-time monitoring and interactive stress testing, revealing characteristic economic behaviors like shock propagation pathways, hoarding accumulation patterns, and monetary circulation dynamics.

All these macro-dynamics patterns emerging directly from micro-level agent interactions.

These findings substantiate the proposed framework as both theoretically sound and practically implementable, establishing a foundation for economic analysis that bridges micro-level interactions with macro-level phenomena.

## Chapter 4

# Conclusions and Future Work

This chapter synthesizes the main outcomes of the model by assessing the achievement of the stated objectives, outlining the framework's application contexts, and discussing its limitations and directions for future work.

Thereby, it substantiates the framework's core methodological contribution as a traceable and inference-ready evolution of the ABM paradigm.

## 4.1 Objectives achievement

In the development of this master's thesis, the objectives defined in Section 1.2 have been systematically addressed and fulfilled as follows:

- **Formulate a graph-theoretical economic representation.** The economy is modeled as a time-dependent, weighted, directed graph in which monetary interactions are encoded through an algebraic flow matrix subject to explicit accounting constraints.
- **Establish a novel behavioral inference mechanism using empirical data.** Agent behavior is inferred from observed monetary data through a propensity-based approach grounded in discretionary flows.
- **Characterize macroeconomic dynamics as dynamical systems.** The economic network is represented as a discrete-time linear dynamical system, enabling the analysis of steady-state properties, convergence, and shock propagation via spectral radius analysis.
- **Implement an interactive prototype as a proof of concept.** A computational prototype is developed, integrating simulation, network visualization, historical monitoring, and interactive stress testing.

## 4.2 Application contexts

Furthermore, the proposed framework provides differential value in several application contexts:

- **Institutional economic monitoring.** Enables transparent and auditable supervision of monetary flows and inferred behavioral dynamics across interconnected agents.
- **Stress testing and policy scenario analysis.** Supports controlled perturbations of behavioral parameters to evaluate shock propagation and systemic resilience.
- **Early warning and crisis monitoring.** Facilitates the detection of anomalous behavioral patterns through network-based indicators.
- **Exploratory macroeconomic analysis.** Provides a structured framework for analyzing the emergence of macroeconomic dynamics from micro-level interactions.

## 4.3 Limitations

While the framework successfully achieves its methodological objectives, its formulation is subject to several limitations that delimit its scope and simultaneously highlight opportunities for further refinement.

These limitations are naturally divided into methodological assumptions and computational challenges.

### 4.3.1 Methodological assumptions

A methodological assumption refers to a deliberate modeling choice introduced to ensure analytical tractability and internal consistency.

The assumptions introduced into the model are summarized as:

- **Complete-reflexive graph representation versus real world sparsity.** The framework represents the economy as a time-dependent, positive-weighted, reflexive, and complete directed graph. This assumption guarantees algebraic completeness and enables a matrix-based dynamical formulation, although real economic networks are typically sparse.

In practice, however, this discrepancy is mitigated by the fact that sparsity can be naturally encoded through zero-weight edges, preserving the framework's analytical structure without requiring a fully connected network.

- **Context-dependent identification of discretionary flows.** The behavioral inference mechanism requires selecting two discretionary flows for each agent, yet no universal standardized procedure exists for determining them.

As a result, this step necessarily depends on researcher judgment and contextual knowledge, which may hinder reproducibility and affect comparability across otherwise similar economic environments.

### 4.3.2 Computational challenges

Computational challenges capture the practical limitations associated with system deployment, data infrastructure, and computational complexity.

The challenges identified in this framework are characterized as follows:

- **Data requirements and infrastructure.** Dashboard operational deployment requires continuous, high-frequency, and granular data streams to support behavioral inference and real-time monitoring.

Such infrastructure is seldom available outside highly instrumented and digitally integrated environments, limiting the feasibility of its implementation in many contexts.

- **Computational scalability.** The framework relies on  $n \times n$  flow matrices, leading to a  $\mathcal{O}(n^2)$  computational cost. While this remains tractable for moderate-sized systems, it poses challenges for large-scale networks or real-time applications.

Potential mitigation strategies for large economies include hierarchical network representations, which reduce complexity while preserving the analytical structure of the model.

## 4.4 Future work

Building on these limitations, several directions emerge for extending the theoretical and practical scope of the framework:

- **Standardizing discretionary flow identification.** Develop formal and institutionally robust criteria for identifying discretionary flows to enhance reproducibility and support consistent application across similar economic environments.
- **Real-time economic monitoring.** Integrate the framework into real-time data infrastructures via APIs, stream-processing pipelines, and automated validation to test its explanatory power and strengthen its relevance for stress testing and policy evaluation.
- **Analytical extensions and reduced-form representations.** Future research should focus on the theoretical implications of economic networks. In particular, on their transient dynamics and on the reduced-form of the transition matrix to preserve essential dynamics while improving computational efficiency.

These avenues collectively promise to enhance both the theoretical understanding and practical applicability of the framework.

Its consolidation will ultimately depend on data quality and availability, as well as the development of hybrid methodologies that integrate formal economic theory, behavioral inference, and real-time monitoring, laying the groundwork for a new generation of computational macroeconomic tools with both academic and institutional relevance.

# Chapter 5

## Glossary

**Agent** ( $a_k \in A$ ) Economic entity that initiates and receives monetary flows.

**Agent accounting constraint** ( $R_t^{a_k}$ ) Condition for agent monetary balance at a given time.

**Agent behavioral inference** Agents' behavior parameters estimation.

**Agent discretionary flow** ( $s_t^{a_k}$ ) Monetary transfer freely chosen by an agent at a given time.

**Agent propensity factor** ( $\Omega_t^{a_k}$ ) Agent resource distribution preference at a given time.

**Agent role** ( $role(a_k)$ ) Agent classification by its net monetary contribution to the system.

**Dynamical system** ( $x_t = M(\Omega_t)x_{t-1}$ ) Agents' discretionary flows evolution over time.

**Economic network** ( $G(A)$ ) Graph of monetary flows between economic agents.

**Flow matrix** ( $S_t$ ) Monetary transfers matrix at a given time.

**Heatmap** ( $\bar{S}_t$ ) Normalized representation of the flow matrix at a given time.

**High-flow path** Connected sequence of edges with high monetary volume within the network.

**Hoarder behavior** Behavioral pattern of high savings allocation.

**Inter-agent flow** Monetary transfer between distinct agents within the economic network.

**Intertemporal flow** Self-directed monetary transfer representing savings.

**Monetary flow / transfer** ( $s_t^{a_i, a_j} \quad \forall a_i, a_j \in A$ ) Agents economic interaction at a given time.

**Net monetary flow** ( $\Delta S_t$ ) System-wide monetary balance measure at a given time.

**Shock** ( $\Delta\Omega_t$ ) Exogenous change on behavioral parameters at a given time.

**Shock propagation** Shock diffusion through the network.

**State vector** ( $x_t$ ) Agents' discretionary flows vector over the flow matrix at a given time.

**Steady-state** ( $x^*$ ) Invariant state vector without new shocks.

# Chapter 6

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## Chapter 7

# Acknowledgment of Artificial Intelligence Use

Generative Artificial Intelligence tools are employed to assist the research process, improving efficiency and consistency while preserving scientific integrity.

All AI-assisted outputs were critically reviewed, adapted, and validated.  
Their use primarily encompassed:

- Academic writing
- Glossary generator
- Literature review
- Mathematical formulation
- Results verification

## Appendix A

# An Economic Network Comprising a Household and a Firm

### A.1 Graph-based modeling

This appendix develops an explicit case of the general framework introduced in Section 2.2. Its purpose is to illustrate how the theoretical framework operates in a simple closed economy composed only of two neutral agents, providing an implementation for Chapter 3.

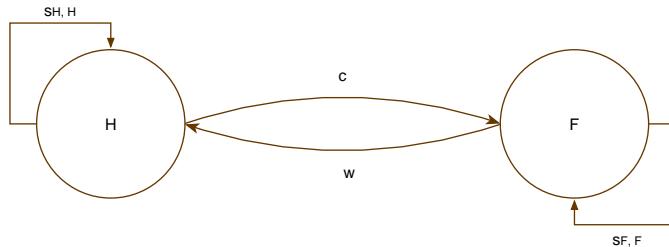


Figure A.1: Household and firm economy

#### A.1.1 Algebraic formulation

Consider a closed economy composed of a household and a firm, both neutral. According to Section 2.2.1, the economy can be modeled as a graph  $G(A)$  with

$$A = \left\{ \begin{array}{l} H : \text{household,} \\ F : \text{firm} \end{array} \right\}, \quad A \times A = \left\{ \begin{array}{l} s^{H,F} : \text{household consumption (c),} \\ s^{F,H} : \text{household wages (w),} \\ s^{H,H} : \text{household savings,} \\ s^{F,F} : \text{firm savings} \end{array} \right\}$$

and

$$S_t = \begin{bmatrix} s_t^{H,H} & c_t \\ w_t & s_t^{F,F} \end{bmatrix}.$$

### A.1.2 Accounting constraints, roles, and net monetary flow

Since agents are assumed to be neutral, their accounting restrictions are expressed in equality

$$\begin{aligned} R_t^H &:= c_t - w_t + s_t^{H,H} - s_{t-1}^{H,H} = 0 \quad \forall t \in \mathbb{N}, \\ R_t^F &:= w_t - c_t + s_t^{F,F} - s_{t-1}^{F,F} = 0 \quad \forall t \in \mathbb{N}, \end{aligned}$$

which leads by induction to a stationary net monetary flow where savings remain constant over time

$$s_t^{H,H} + s_t^{F,F} = s_0^{H,H} + s_0^{F,F} \quad \forall t \in \mathbb{N}.$$

## A.2 Behavioral inference

### A.2.1 Discretionary flows identification

Consistent with Section 2.2.2, each agent is assumed to control two discretionary flows. The household chooses its consumption and savings, whereas the firm chooses wage payments and savings

$$s_t^H = \left\{ c_t, s_t^{H,H} \right\}, \quad s_t^F = \left\{ w_t, s_t^{F,F} \right\} \quad \forall t \in \mathbb{N}.$$

### A.2.2 Relating via propensity factors

With the discretionary flows sets, agents' propensity factors are measured as

$$\Omega_t^H = \left( 1 + \frac{s_t^{H,H}}{c_t} \right)^{-1}, \quad \Omega_t^F = \left( 1 + \frac{s_t^{F,F}}{w_t} \right)^{-1} \quad \forall t \in \mathbb{N}.$$

Solving for savings leads to an analytical expression linking the two discretionary flows of each agent

$$s_t^{H,H} = \left( \frac{1}{\Omega_t^H} - 1 \right) c_t, \quad s_t^{F,F} = \left( \frac{1}{\Omega_t^F} - 1 \right) w_t,$$

subject to the accounting constraints

$$\begin{aligned} c_t - w_t + s_t^{H,H} - s_{t-1}^{H,H} &= 0, \\ w_t - c_t + s_t^{F,F} - s_{t-1}^{F,F} &= 0. \end{aligned}$$

## A.3 Networks as dynamical systems

### A.3.1 Steady-state existence and convergence

Combining the behavioral relations with the accounting constraints yields the following closed-form expression for the system's evolution in terms of past savings

$$\begin{aligned} c_t &= \left( \frac{1}{\Omega_t^H \Omega_t^F} - 1 \right)^{-1} \left( \frac{1}{\Omega_t^F} s_{t-1}^{H,H} + s_{t-1}^{F,F} \right) \\ w_t &= \left( \frac{1}{\Omega_t^H \Omega_t^F} - 1 \right)^{-1} \left( s_{t-1}^{H,H} + \frac{1}{\Omega_t^H} s_{t-1}^{F,F} \right) \\ s_t^{H,H} &= \left( \frac{1}{\Omega_t^H \Omega_t^F} - 1 \right)^{-1} \left( \frac{1}{\Omega_t^H} - 1 \right) \left( \frac{1}{\Omega_t^F} s_{t-1}^{H,H} + s_{t-1}^{F,F} \right) \\ s_t^{F,F} &= \left( \frac{1}{\Omega_t^H \Omega_t^F} - 1 \right)^{-1} \left( \frac{1}{\Omega_t^F} - 1 \right) \left( s_{t-1}^{H,H} + \frac{1}{\Omega_t^H} s_{t-1}^{F,F} \right). \end{aligned}$$

Defining the state vector as

$$x = (c, w, s^{H,H}, s^{F,F})^T,$$

the dynamical system can be written as described in Section 2.2.3

$$x_t = M(\Omega_t) x_{t-1}$$

where

$$\begin{aligned} M(\Omega_t) &= V J V^{-1} = \left( \frac{1}{\Omega_t^H \Omega_t^F} - 1 \right)^{-1} \begin{bmatrix} 0 & 0 & \frac{1}{\Omega_t^F} & 1 \\ 0 & 0 & 1 & \frac{1}{\Omega_t^H} \\ 0 & 0 & \left( \frac{1}{\Omega_t^H} - 1 \right) \frac{1}{\Omega_t^F} & \left( \frac{1}{\Omega_t^H} - 1 \right) \\ 0 & 0 & \left( \frac{1}{\Omega_t^F} - 1 \right) & \left( \frac{1}{\Omega_t^F} - 1 \right) \frac{1}{\Omega_t^H} \end{bmatrix} \\ V &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & \left( \frac{1}{\Omega_t^F} - \frac{1}{\Omega_t^H} \right) \left( \frac{1}{\Omega_t^F} - 1 \right)^{-1} - 1 \\ 0 & 0 & \frac{1}{\Omega_t^H} - 1 & \frac{1}{\Omega_t^H} - 1 \\ 0 & 0 & \frac{1}{\Omega_t^F} - 1 & - \left( \frac{1}{\Omega_t^H} - 1 \right) \end{bmatrix}, \quad J = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & (\Omega_t^H + \Omega_t^F - 2) (\Omega_t^H \Omega_t^F - 1)^{-1} - 1 \end{bmatrix}. \end{aligned}$$

Since the Jordan canonical form has at least one diagonal term equal to 1 with a Jordan block of size one, the economy has a non-trivial steady-state

$$x^* = \left( c^*, c^*, \left( \frac{1}{\Omega_t^{H^*}} - 1 \right) c^*, \left( \frac{1}{\Omega_t^{F^*}} - 1 \right) c^* \right)^T$$

and, because

$$\left| (\Omega_t^H + \Omega_t^F - 2) (\Omega_t^H \Omega_t^F - 1)^{-1} - 1 \right| \leq 1,$$

the spectral radius of the economy (figure A.1) is equal to one, which implies that, after an exogenous shock on propensity factors and as long as  $x_0 \neq 0$ , the economy does not collapse nor diverge. Instead, the system always transitions towards a new steady-state.

### A.3.2 Shock response

Following Section 2.2.3, the sensitivity of the system to exogenous perturbations in the propensity factors is obtained by differentiating the state vector with respect to agents preferences.

For a shock in the household propensity:

$$\frac{\partial x_t}{\partial \Omega_t^H} = (1 - \Omega_t^H \Omega_t^F)^{-2} \begin{bmatrix} 0 & 0 & 1 & \Omega_t^F \\ 0 & 0 & \Omega_t^F & \Omega_t^{f^2} \\ 0 & 0 & -(1 - \Omega_t^F) & -(1 - \Omega_t^F) \Omega_t^F \\ 0 & 0 & 1 - \Omega_t^F & (1 - \Omega_t^F) \Omega_t^F \end{bmatrix} x_{t-1}$$

$\Omega_t^H$	$c_t$	$w_t$	$s_t^{H,H}$	$s_t^{F,F}$
$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$

Table A.1: Household propensity shock propagation

For a shock in the firm propensity:

$$\frac{\partial x_t}{\partial \Omega_t^F} = (1 - \Omega_t^H \Omega_t^F)^{-2} \begin{bmatrix} 0 & 0 & \Omega_t^{h^2} & \Omega_t^H \\ 0 & 0 & \Omega_t^H & 1 \\ 0 & 0 & (1 - \Omega_t^H) \Omega_t^H & (1 - \Omega_t^H) \\ 0 & 0 & -(1 - \Omega_t^H) \Omega_t^H & -(1 - \Omega_t^H) \end{bmatrix} x_{t-1}$$

$\Omega_t^F$	$c_t$	$w_t$	$s_t^{H,H}$	$s_t^{F,F}$
$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$

Table A.2: Firm propensity shock propagation

Note that a deviation in a propensity factor has the same qualitative impact on all economic transactions, except for the associated saving flow, which moves in the opposite direction of the other transfers.

## Appendix B

# Hierarchical Economic Networks

Up to this point, the graph-based modeling approach has been developed for networks of interconnected agents. However, the proposed methodology is also scalable to higher levels of aggregation. In the context of an economic network, it is possible to extend the model to a higher-level economic network  $G(A)$  composed of multiple subsystems where:

- $A = \{G_1, G_2, \dots, G_m\}$  denotes the set of subnetworks, where each node functions as a structural container encapsulating a complete lower-level economic network.
- $A \times A$  denotes the set of directed interactions between subeconomies.

At this level, the entire theoretical framework developed in the core sections of this contribution remains applicable, with the key distinction that the nodes of the higher-level graph no longer represent heterogeneous agents but instead encapsulate complete networks where each node functions as a structural container holding lower-level economic interactions.

Consider two national economies, each represented according to Figure B.1, composed of the complete set of institutional sectors in order to ensure full coverage of economic interactions: household, firm, government, national bank, and the central bank.

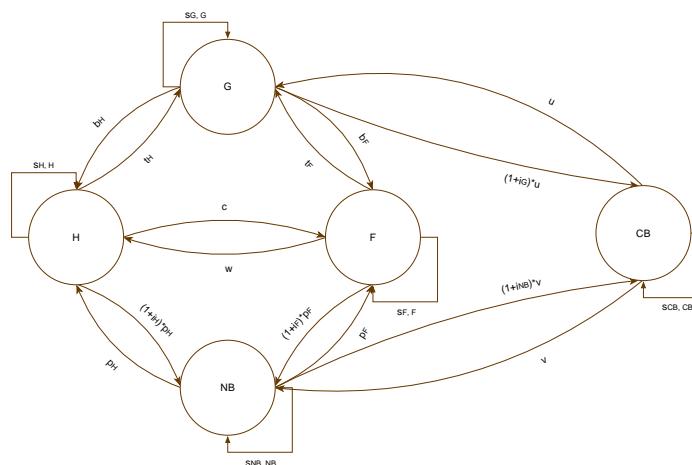


Figure B.1: National economy

To model an international economy, it suffices to define which agents trade out of their native network. For illustrative purposes, Figure B.2 considers interactions between central banks for currency exchange and between households and firms for international trade.

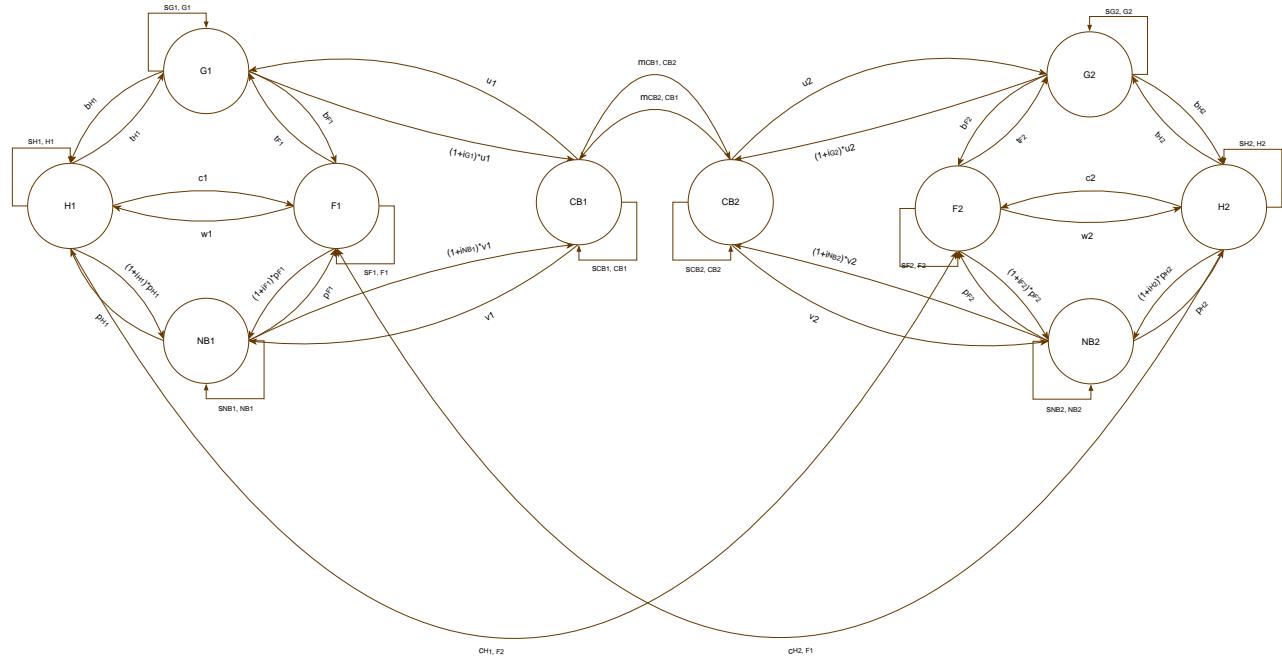


Figure B.2: International economy

It is important to note that the flow matrix associated with the extended graph grows rapidly in size. To facilitate its representation, each nation is treated as an encapsulated substructure while preserving only the relevant higher-level interactions, resulting in a hierarchical, second-order structure.

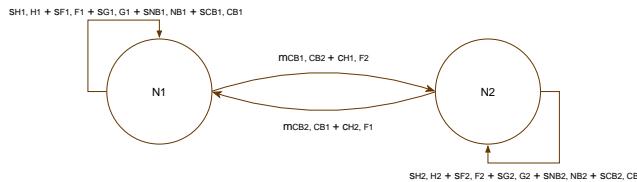


Figure B.3: Encapsulated international economy

It is worth emphasizing that this approach is inherently recursive: starting from national networks, one can model an international economy; with multiple planets, an interplanetary economy; and with stellar systems, an interstellar economy.

Similarly, the model also allows for a downward shift in the level of analysis to urban, sectoral, corporate, or even individual economic networks.

## Appendix C

# Monetary Hoarding Indicator for Crisis Monitoring

### C.1 Theoretical foundation

This appendix extends the graph-theoretical framework from Section 2.2 to develop a novel early warning indicator using the heatmap component of Section 3.5.1 to detect economic crisis.

### C.2 Economic interpretation of matrix structure

The structure of the normalized flow matrix introduced in Section 3.5.1 provides key insights into economic behavior by examining its diagonal and off-diagonal elements:

- **Inter-agent flows.** Represent active monetary circulation between distinct agents, facilitating economic exchange and promoting economic liquidity.
- **Intertemporal flows.** Capture savings behavior, representing liquidity withdrawn from current circulation for future use, reducing immediate economic liquidity.

### C.3 Monetary hoarding indicator

The Monetary Hoarding Indicator (MHI) is proposed as an indicator of economic performance. Given the normalized flow matrix in the dashboard, the MHI is defined as the proportion of total economic resources held as intertemporal savings

$$\phi_t := \text{tr} (\bar{S}_t) \in [0, 1],$$

while its complementary measures the active monetary circulation

$$1 - \phi_t = 1 - \text{tr} (\bar{S}_t).$$

## C.4 Economic interpretation and crisis detection

The MHI provides a clear, interpretable measure of economic health:

- **Low MHI values** indicate high monetary circulation, characteristic of healthy, active economies with efficient resource allocation and robust economic exchange.
- **High MHI values** indicate excessive liquidity hoarding, where economic agents tend to allocate resources to savings rather than to productive exchange. This behavior typically precedes economic contractions and serves as an early warning indicator of systemic dysfunction.

## C.5 Analytical properties

The MHI possesses several desirable properties for economic monitoring:

- **Computational efficiency.** Calculation requires only  $2n - 1 \in \mathcal{O}(n)$  operations:  $n$  accesses to diagonal entries of the normalized flow matrix followed by  $n - 1$  summations, with  $n = |A|$ .
- **Information sufficiency.** Monitoring aggregate savings behavior suffices for crisis detection, without requiring detailed knowledge of internal circulation.
- **Normalization invariance.** The indicator is scale-invariant and comparable across different economic scales and time periods.
- **Economic intuition.** The measure directly captures the fundamental trade-off between immediate circulation and future-oriented saving behavior.

## C.6 Future visual analytics implementation

Integrating the indicator into a future dashboard extension is straightforward.

1. **Real-time computation.** Where the MHI is computed at each time step from the normalized flow matrix.
2. **Threshold calibration.** With historical analysis establishing crisis thresholds  $\phi_{\text{critical}}$  based on empirical data.
3. **Alert mechanism.** Incorporating automated alerts triggered when  $\phi_t > \phi_{\text{critical}}$ .
4. **Visualization.** A time series visualization of the MHI, including historical comparisons and threshold markers.

This extension demonstrates that the graph-theoretical framework naturally yields practical monitoring tools with strong theoretical foundations and immediate policy relevance.