Computational Modeling, Monitoring and Analysis of Economic Networks

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Abstract

This thesis proposes a novel formal framework for the analysis of economic phenomena through networks of interconnected agents, represented as directed graphs with quantified monetary flows. Based on the algebraic construction of the system, the formulation of individual intertemporal optimization programs and the implementation of an interactive real-time computational visualization, the framework models the money circulation from both a structural and dynamic perspective.
The proposed approach provides a general and flexible theoretical foundation applicable to diverse economic contexts. It offers a robust basis for empirical integration with real-world data, the incorporation of advanced techniques such as machine learning and the development of exploratory tools for economic analysis and monitoring.
A working interactive dashboard demonstrates the real-time behavior of the network and enables users to manipulate structural parameters on demand, opening new avenues for economic diagnostics and intervention.

Keywords: economic network, graph-based modeling, discrete dynamical system, inverse reinforcement learning, interactive real-time visualization.

Introduction

According to the first law of thermodynamics, *energy cannot be created or destroyed, only transformed.* While the principle originates in physics, it can be analogously applied to economics: money is neither created nor destroyed, but merely flows.
Although this analogy is not exact, as central banks can create money and individuals can burn bills, in general, one agent's income corresponds to another's expenditure.
This perspective enables us to reinterpret the economy as a dynamic system of interconnected agents.

Traditional macroeconomic models often overlook these explicit interactions, representing the economy in aggregate terms without detailing how individual decisions propagate through the underlying network of monetary transfers. This limits their ability to analyze contagion effects, imbalances, or structural changes.

This thesis proposes a formal framework grounded in economic networks, with nodes as economic agents and edges as monetary transfers.

The framework is built upon three pillars:
(1) Graph-based modeling.
(2) Behavioral inference from observed data.
(3) Design and implementation of an interactive real-time visualization interface.

Together, these components enable monitoring of real systems, behavioral extrapolation and scenario evaluation.

Literature Review

This contribution is situated at the intersection of seven key research areas in computational economics and macroeconomic modeling. The review synthesizes classical and recent developments within these domains, identifying structural limitations that motivate the formal and methodological framework proposed in this master's thesis.

Conceptual framework

Economic Network Theory

Foundational works by Jackson [Jackson, 2008] and subsequent contributions have shown how the structure of connections between agents influences phenomena such as information diffusion, market efficiency and financial shock propagation. However, most models in this strand are static, qualitative, or focused on specific topologies, lacking explicit monetary quantification and optimized decision-making. More recent studies like Bargigli et al. [Bargigli et al., 2015] and Battiston et al. [Battiston et al., 2016] incorporate empirical multilayer network data but still lack explicit microeconomic formalization.
Thus, a framework that dynamically integrates endogenous decisions within network structures remains missing.

Input-Output Modeling

Leontief's input-output framework [Leontief, 1936] has been central to structural analysis of sectoral interdependencies. Extensions such as Lenzen et al. [Lenzen et al., 2019] incorporate uncertainty and simulation but retain rigid technical coefficients and absence of autonomous agents. These constraints limit the ability to capture endogenous reconfigurations or strategic responses.
Therefore, a flexible input-output framework combining accounting structure with optimized behavior and microeconomic heterogeneity is still required.

Intertemporal Optimization

Ramsey's model [Ramsey, 1928] and later extensions established the tradition of intertemporal optimization in macroeconomics, typically centered on representative agents. While HANK models [Kaplan et al., 2018] introduce heterogeneity in income, wealth, or preferences, they lack explicit network interactions and monetary flows between agents. Traditional DSGE models [Kydland and Prescott, 1982], despite their analytical strengths, often rely on the assumption of unique equilibria and representative agents, which limits their ability to capture complex heterogeneity and emergent dynamics observed in real economies.
A decentralized model combining optimized intertemporal decisions with explicit economic network structures linking their accounting constraints remains undeveloped.

Discrete dynamical systems

Discrete dynamical systems provide a fundamental framework for analyzing time-based economic processes, especially through the Jacobian matrix at the steady-state [Elaydi, 2005]. The Perron–Frobenius theory [Perron, 1907, Frobenius, 1908] is crucial for extracting and interpreting eigenvalues of non-negative matrices, ensuring the existence of a unique, positive dominant eigenvalue that governs the long-term behavior, stability, and convergence of such systems.
Despite these analytical strengths, a comprehensive framework that integrates microeconomic foundations and explicit network structures within tractable discrete dynamical systems remains an open research frontier.

Observed Behavior Analysis

Economic behavior analysis from observable data has traditionally relied on revealed preference theory, such as Inverse Reinforcement Learning (IRL) [Ng and Russell, 2000]. But, while advanced methods like maximum-entropy IRL [Ziebart et al., 2008] have enhanced the capacity to infer utility functions from discrete action spaces, they face significant computational challenges when applied to continuous state and action spaces, where sophisticated integration techniques can lead to undetermined and intractable problems.

Agent-Based Simulation
Agent-based modeling (ABM) has allowed exploration of out-of-equilibrium dynamics, adaptive learning and emergent effects [Tesfatsion, 2006, Epstein and Axtell, 1996]. Nonetheless, many ABMs rely on heuristic rules, empirical replication, or ad hoc procedures without a formal algebraic framework ensuring consistency between individual decisions and aggregate outcomes. Assenza et al. [Assenza et al., 2020] highlight the need to link ABMs with optimization principles, but most current approaches remain fragmented and difficult to integrate.

Real-Time Visualization and Monitoring
Interactive visualizations for economic networks have advanced considerably in fields such as data science and graphic design [Kirk, 2016], yet their adoption in formal macroeconomics remains limited. Although institutions like the ECB or Federal Reserve publish macroeconomic dashboards, these tend to be aggregate, static and loosely connected to endogenous structures or decision models.
A visual platform integrating modeling, dynamic simulation and real-time monitoring is still absent.

Contributions

This contribution proposes a unified formal framework addressing structural gaps across six key domains in computational economics and macroeconomic modeling. The main contributions, structured around each research line, are:

Economic Network Theory:
Formalizes the economy as a complete directed graph where each node represents an economic agent and each edge a quantified monetary flow. This representation captures structured direct and indirect interactions among agents.

Input-Output Modeling:
Extends traditional input-output approaches with a dynamic matrix structure incorporating adaptive technical coefficients, optimized decisions and microeconomic heterogeneity. This enables simulation of endogenous structural reconfigurations in response to exogenous shocks.

Intertemporal Optimization:
Defines an individual intertemporal optimization program, where budget constraints emerge from network position. The model ensures coherence between microeconomic decisions and aggregate macroeconomic outcomes. It also introduces the concept of steady-state propensity as an analytical simplification for preference inference and equilibrium analysis.

Discrete dynamical systems:
Integrates established discrete-time modeling techniques and control theory within a network-based economic framework, enabling the analysis of system-wide dynamics and the stability of steady-states through local agent-level interactions and structural interdependencies, providing a foundation for policy-oriented diagnostics of systemic stability.

Observed Behavior Analysis:
Building upon the notion of bounded rationality [Simon, 1955] with cognitive constraints, introduces a novel methodology for applying IRL to environments with continuous state and action spaces.
The approach circumvents the intractability issues of traditional methods by using the likelihood function approximation which, setting the agents rationality, searchs for a utility expression that its expectation matches the rational elections made in the past.

This method measures the goodness-of-fit of a candidate utility function to observed behavior, providing a robust and computationally feasible alternative for analyzing preferences in complex economic behavior with noise.

Agent-Based Simulation:
Unlike heuristic ABM approaches, this framework provides a consistent algebraic basis enabling simulation of heterogeneous rational-agent networks, ensuring traceability from micro-foundations [Gabaix, 2011] to aggregate behavior.

Real-Time Visualization and Monitoring:
Designs an interactive interface based on normalized matrix representation that dynamically visualizes monetary flows. This tool facilitates diagnosis

of structural imbalances and early detection of anomalies in agents' preferences.

Unlike existing approaches, the proposed model offers an integrated framework linking mathematical formalization, revealed behavior analysis and real-time visualization. This theoretical foundation opens new pathways for developing institutional tools aimed at macroeconomic supervision, policy design and risk anticipation.

Benchmarking against existing models

Compared to standard modeling approaches:

Micro-level heterogeneity: Offers greater detail than DSGE and IO models, while remaining on par with ABM frameworks.

Explicit network interactions: Provides a fully formalized treatment, unlike DSGE and IO models, and captures network structure more systematically than typical ABMs.

Intertemporal optimization: Maintains rigorous optimization over time, which is often absent in ABMs and IO models.

Dynamic and qualitative analysis: Extends dynamic analysis beyond IO and ABM models by formalizing networks dynamics, while complementing DSGE frameworks through explicit heterogeneity.

Preference inference from data: Introduces inverse reinforcement learning, a feature not found in DSGE, ABM, or IO frameworks.

Shock adaptability: Exhibits greater flexibility than DSGE and IO models, comparable to ABMs but within a coherent and formal setup.

Real-time visualization: Integrates native real-time visualization capabilities, setting it apart from other models.

Computational scalability: Requires similar ABMs computational resources andmore than IO models, available for dimensionality reduction.

Institutional applicability: At an exploratory stage, but with promising potential for real-time monitoring and policy diagnostics.

Graph-Based Modeling

Algebraic formulation

An economic network is modeled as a complete directed graph $G = (A, I)$, where:

- $A = \{a_1, a_2, \dots, a_n\}$ is the set of agents, with cardinality n;
- $I = A \times A$ is the set of all directed interactions, of size n^2.

Each ordered pair $(a_i, a_j) \in I$ represents a monetary flow from agent a_i to agent a_j, with diagonal pairs (a_k, a_k) representing intertemporal savings of a_k.

The system is represented algebraically by the flow matrix

$$S_t = \left[s_t^{i,j}\right]_{i,j \in A} \in \mathbb{R}_{\geq 0}^{n \times n} \quad \forall t \in \mathbb{N},$$

where $s_t^{i,j}$ denotes the monetary flow from a_i to a_j at time t and diagonal entries $s_t^{k,k}$ capture the intertemporal savings of agent a_k.

Roles, constraints and net monetary flows

Agents are classified by an exogenous role function $k : A \to \{-1, 0, 1\}$, reflecting their capacity to create or destroy money.
The accounting constraint for each agent a at time t is defined as

$$R_t^a := \sum_{\substack{j \in A \\ j \neq a}} s_t^{a,j} + s_t^{a,a} - \sum_{\substack{i \in A \\ i \neq a}} s_t^{i,a} - s_{t-1}^{a,a}.$$

This satisfies

$$\begin{cases} R_t^a \leq 0 & \text{if } k(a) = -1 \quad \text{(destroyer)}, \\ R_t^a = 0 & \text{if } k(a) = 0 \quad \text{(neutral)}, \\ R_t^a \geq 0 & \text{if } k(a) = 1 \quad \text{(creator)}. \end{cases}$$

The total net monetary flow in the system is

$$D_t := \sum_{a \in A} R_t^a.$$

Optimization program

Each agent maximizes an intertemporal utility function $U^a(\Omega_t^a, \hat{s}_t^a)$, where $\Omega_t^a \in [0,1]$ captures exogenous propensity factors and $\hat{s}_t^a$ denotes optimized monetary flows constrained by

$$S_t^a = \left\{ \hat{s}_t^a \in \mathbb{R}_{\geq 0}^n \mid R_t^a(\hat{s}_t^a) \right\}, \quad \text{where} \quad n = |\hat{s}_t^a|.$$

The optimal decision for agent a is

$$s_t^{a^*} := \arg \max_{\hat{s}_t^a \in S_t^a} \sum_{t \in \mathbb{N}} (\Pi^a)^t U^a(\Omega_t^a, \hat{s}_t^a),$$

where $\Pi^a \in (0,1)$ is the discount factor.

Therefore, the network state is

$$s_t^* := (s_t^{a^*})_{a \in A} \quad \forall t \in \mathbb{N},$$

with non-determinism arising from the exogenous time-dependent propensity factors Ω_t^a.

Propensity to the steady-state

Due to the high degree of network interactivity and system complexity and depending on the sets of discretional flows—particularly when the utility function depends on savings—the optimal solutions to the maximization program may incorporate future expectations because of the temporal lag of savings in the agent's accounting constraint. Consequently, the propensity factor at time t cannot be analytically estimated.

A reasonable simplification that facilitates both the calculation of optimal flows and the estimation of agents' propensities is to assume steady-state propensity, whereby agents expect to select the same optimal flow across all time periods.

Under this assumption, it's considered that:

$$E(x_{t+1}) = x_t \quad \forall t \in \mathbb{N}$$

where x holds for either optimal flows or propensity factors. This hypothesis allows the model to be solved statically and enables inference on system behavior without requiring anticipation of complex dynamic expectations.

Qualitative system behavior on the steady-state

Steady-state conditions

To know whether the steady-state exists, the formulation of the extended vector with all the optimal flows $\mathcal{S}$ is required.
Thus, the condition can be written as:

$$\mathcal{S} = M\mathcal{S},$$

where non-trivial steady-state ($\mathcal{S} = 0$) exists while M has at least one eigenvalue equal to one.
If $\mathcal{S}$ includes at least one optimal extended flow, then non-trivial steady-state is guarantee.

Spectral analysis and economic policy

Once the steady-state is known, to study the network behavior after an external perturbation over the steady-state, it's necessary to compute the Jacobian of M evaluated at the steady-state $J_{\mathcal{S}=s^*}$ and get its associated spectral radius:

$$\rho(J_{\mathcal{S}=s^*}) := \max_{\lambda \in \sigma(J_{\mathcal{S}=s^*})} |\lambda|.$$

The steady-state is stable if $\rho(J_{\mathcal{S}=s^*}) < 1$ and unstable if $\rho(J_{\mathcal{S}=s^*}) \geq 1$.

The spectral properties of $J_{\mathcal{S}=s^*}$ govern the local dynamics and are fundamental for understanding how shocks propagate throughout the network.

From a policy perspective, stable modes ($|\lambda| < 1$) are self-correcting and typically require minimal intervention. Unstable modes ($|\lambda| > 1$) represent amplification channels through which shocks can propagate and destabilize the system, warranting early corrective measures to preserve systemic stability.

Note how external perturbations are modeled throw the propensity factors that guide the agents behavior and how they are considered inside the Jacobian.
That means that its spectral properties are time-dependent as well as propensity factors (Ω_t).

Otherwise, note also how the spectral characterization holds globally if the algebraic dynamical system is linear.

Revealed Behavior Analysis

In a monitored environment where the network state is continuously observed, it becomes crucial to infer the utility functions that rationalize the observed monetary flows. This requires, for each agent, identification of their discretionary flows, preference factors and the functional form of their utility.

Identification of discretionary flows

Discretionary flows $\hat{s}_t^a$ are determined via economic reasoning, based on the institutional context. For example, although savings are endogenously determined by agents, they are often excluded from the discretionary set as they result residually from other decisions.

Estimation of propensity factors

Two approaches can be employed to infer the agents' propensity factors Ω_t^a: analytical inversion and empirical estimation.

From the optimality condition derived in the decision-making framework, the discretionary flows can be expressed as a function of the exogenous propensity factors:

$$s_t^{a^*} = f^a(\Omega_t^a) \quad \Rightarrow \quad \Omega_t^a = (f^a)^{-1}(s_t^{a^*}).$$

Alternatively, an empirical approach uses observed net income, defined as:

$$I_t^a := \sum_{\substack{i \in A \\ s_t^{i,a} \notin \hat{s}_t^a}} s_t^{i,a} - \sum_{\substack{j \in A \\ s_t^{a,j} \notin \hat{s}_t^a}} s_t^{a,j}.$$

Agent preferences are then inferred via:

$$\Omega_t^a = \frac{s_t^{a^*}}{I_t^a} \quad \forall a \in A,\ t \in \mathbb{N}.$$

This empirical method circumvents assumptions on utility function forms, relying solely on observable flows.

Utility Function Inference

Once discretionary flows and propensities are identified, utility functions can be inferred using maximum entropy inverse reinforcement learning (IRL) combined with the Boltzmann distribution.

Under bounded rationality, the probability of observing a particular discretionary flow is modeled by the continuous Boltzmann distribution, which captures both bounded rationality and external stochastic disturbances:

$$P\left(s_t^{a^*} \mid \hat{U}^a\right) = \frac{\exp\left(\hat{U}^a(\Omega_t^a, s_t^{a^*})\right)}{\int \exp\left(\hat{U}^a(\Omega_t^a, s)\right)\, ds} \quad \forall a \in A, t \in \mathbb{N}.$$

The corresponding likelihood function is

$$\mathcal{L}(\hat{U}^a) = \prod_{t\in\mathbb{N}} \left[P\left(s_t^{a^*} \mid \hat{U}^a\right)\right] \quad \forall a \in A$$

$$\ln\left(\mathcal{L}(\hat{U}^a)\right) = \sum_{t\in\mathbb{N}} \left[\hat{U}^a(\Omega_t^a, s_t^{a^*}) - \ln\left(\int \exp\left(\hat{U}^a(\Omega_t^a, s)\right)\, ds\right)\right].$$

The utility function estimate is obtained by maximizing this likelihood:

$$U^a := \arg\max_{\hat{U}^a} \left[ln\left(\mathcal{L}(\hat{U}^a)\right)\right] \quad \forall a \in A,$$

which leads to the first-order optimality condition:

$$\sum_{t\in\mathbb{N}} \left[\mathbf{1}_{\{s=s_t^{a^*}\}} - P(s \mid U^a)\right] = 0,$$

or equivalently,

$$E\left[P(s \mid U^a)\right] = \tau^a \quad \forall a \in A,$$

where $\tau^a \in [0, 1]$ quantifies the agent's rationality level with $\tau^a = 1$ holding for a perfect rational agent and $\tau^a = 0$ holding for a complete irrational agent.

Therefore, the real utility function U^a is that one that matches the empirical optimal frequency assumed (τ^a) of the observed flows.

Computational Architecture

Although some agents' optimal flows depend on future expectations, their temporal evolution can be simulated by fixing an arbitrary value for the propensity factor Ω_t^a.

Starting from the empirical derivation to estimate the propensity factor, the agent's optimal discretional flow can be expressed as:

$$\Omega_t^a = \frac{s_t^{a^*}}{I_t^a} \quad \Rightarrow \quad s_t^{a^*} = \Omega_t^a I_t^a \quad \forall a \in A,\ t \in \mathbb{N}$$

For practical implementation, it's proposed simulating an economic network alongside an interactive dashboard for real-time monitoring.

The dashboard includes:

- The **normalized flow matrix** to observe monetary circulation within the system in real time.
- **Visual indicators** displaying estimated exogenous propensity factors in a live setting.
- **Interactive interventions** enabling direct manipulation of propensity factors during the simulation.

This setup allows users to monitor and experiment with the network's dynamic state, providing an intuitive and operational representation of the theoretical framework. The tool demonstrates the model's practical applicability and aids in detecting and responding to imbalances, behavioral shifts, or risks within the economic system.

Real-time data can be used to track monetary flows among agents, aiming to design automated visualization that flags imbalances and changes in agent behavior.
The tool leverages the economy's matrix representation to produce a dynamic heatmap over time.

However, due to high variability and absence of an upper bound restricting flow volumes, the raw

visualization lacks consistency and cannot serve as a reliable study object.
To address this, graph normalization is proposed:

$$\bar{S}_t = \left[\frac{1}{Z_t} s_t^{i,j} \right] \in \mathbb{R}_{\geq 0}^{a \times a} \quad \forall i, j \in A,\ t \in \mathbb{N}.$$

Two options arise for determining the normalization factor Z_t:
On one hand, GDP allows economic interpretations of transaction evolution, such as consumption or wage variation relative to GDP over time. Yet in simplified networks, GDP calculation becomes exogenous and thus outside system scope.
On the other hand, the aggregated flow of the graph guarantees bounding all system flows within the interval $[0, 1]$.

Since the visualization aims for automated, real-time monitoring, it's preferable to use a normalization independent of nominal flows, even at the expense of some economic interpretability.

Therefore, at each time t:

$$Z_t = \sum_{i \in A} \sum_{j \in A} s_t^{i,j} \quad \forall t \in \mathbb{N}.$$

This thesis introduces a formal framework for modeling, monitoring and inferring behavior in economic networks, integrating decentralized decision-making, dynamic constraints and machine learning techniques within a unified architecture.
The key contributions are: (1) an algebraic representation of economic networks with explicit functional roles and accounting constraints; (2) an inverse reinforcement learning approach to infer agents' preferences from observational data; and (3) a real-time computational dashboard enabling interactive simulation and system-state manipulation.
This framework provides a foundation for a new class of tools for structural diagnostics, imbalance detection and the design of adaptive economic policies. Its broader applicability will depend on progress in data interface standardization, model scalability and extensions to large-scale systems.

Discussion

Model limitations

The proposed framework relies on an idealized theoretical structure that assumes perfect rationality and a fixed typology of agents defined by exogenous roles. While building a coherent computational architecture, these assumptions limit the model's ability to capture irrational behavior and role transitions. This functional rigidity also restricts the representation of complex and adaptive economic phenomena.

Computational challenges

The model's computational cost grows nonlinearly with the number of agents due to the nature of the economic graph. This imposes a significant barrier to system scalability, particularly in complex macroeconomic contexts. To mitigate this, future development may include topological pruning techniques based on observed economic topologies, as well as the encapsulation of subnetworks to reduce problem dimensionality. Moreover, additional performance gains could be achieved through parallel computing techniques or cloud-based simulations.

Directions for future research

Building on the limitations identified above, several relevant extensions are proposed:

- Endogenous functional roles by observed behavior, using stochastic models or machine learning.
- More advanced IRL techniques integration such as neural networks or maximum entropy approaches for continuous state spaces.
- Development of an API for continuous ingestion of real-time economic data and seamless integration with the monitoring dashboard.
- Generative adversarial networks (GANs) to create synthetic scenarios and evaluate economic policies under counterfactual conditions.

Closing remarks

Despite its limitations, the proposed framework provides a solid foundation for the modeling, monitoring and analysis of economic networks. Its value lies in combining rigorous theoretical formalization with interactive simulation capabilities and openness to automated learning methods. Its future consolidation will depend on the quality and availability of data, as well as the development of additional hybrid methodologies that bridge economic theory and empirical inference.
Taken together, this framework offers a novel perspective for exploring economic interactions, representing a significant extension of classical macroeconomic models. Not laying only in academic exploration but also in supporting institutional economic diagnostics and anticipatory policy design.

Artificial Intelligence Usage

For the development of this academic work, Generative Artificial Intelligence tools were employed to assist in various phases of the process, enhancing both efficiency and quality while maintaining full scientific integrity. Specifically, their application focused on:

- Literature review
- Mathematical formulation
- Academic writing
- Results verification

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Appendices

Appendix A: Mathematical proofs

Proof of the recursive flows by induction

The goal is to show that, given

$$s_t = s_{t-1} \quad \forall t \in \mathbb{N},$$

it holds that

$$s_t = s_0 \quad \forall t \in \mathbb{N}.$$

Base Case: For $t = 1$, it follows that

$$s_1 = s_0.$$

Inductive Step: Assume that

$$s_k = s_0 \quad \text{for some} \quad k \geq 1.$$

Then, by the recurrence relation,

$$s_{k+1} = s_k \Rightarrow s_{k+1} = s_0.$$

Conclusion: By the principle of mathematical induction,

$$s_t = s_0 \quad \forall t \in \mathbb{N}.$$

■

Proof of the uniqueness of the optimal solution under strict concavity of the utility function by contradiction

Let $S_t^a \subseteq \mathbb{R}^n$, where $n = |\hat{s}_t^a|$, denote the feasible set defined by the agent's budget constraint.
Although the constraints that define S_t^a are nonlinear, they are assumed to be continuous functions, which implies that S_t^a is a closed and bounded set. Therefore, S_t^a is compact.

Furthermore, in many economic contexts, any convex combination of two feasible decisions also respects the agent's budget constraint. This makes it reasonable to assume that S_t^a is convex.

Assume that the utility function

$$u : S_t^a \to \mathbb{R}$$

is strictly concave and differentiable on S_t^a.

Claim: Under these conditions, the utility maximization problem

$$\max_{x \in S_t^a} [u(x)]$$

has a unique optimal solution.

Proof: Suppose, for contradiction, that there exist two distinct optimal solutions

$$x_1, x_2 \in S_t^a, \quad \text{with} \quad x_1 \neq x_2,$$

such that

$$u(x_1) = u(x_2) = \max_{x \in S_t^a} u(x).$$

Since u is strictly concave and S_t^a is convex,

$$u(\lambda x_1 + (1-\lambda)x_2) > \lambda u(x_1) + (1-\lambda)u(x_2) = u(x_1)$$
$$\text{for any} \quad \lambda \in (0,1).$$

But this contradicts the optimality of x_1 and x_2, as the point $\lambda x_1 + (1-\lambda)x_2 \in S_t^a$ yields a strictly higher utility. Therefore, the optimal solution must be unique. ■

Appendix B: Closed economy network with households and firms

In a closed economy, that's, without money creation, consisting of households and firms, the system can be represented as a graph $G(2,4)$, with:

$$\text{A} = \{\text{Households (h), Firms (f)}\}$$

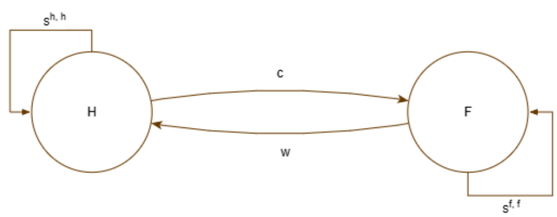

Figure 1: Households and firms

Matrix Representation

$$S_t = \begin{bmatrix} s_t^{h,h} & c_t \\ w_t & s_t^{f,f} \end{bmatrix}$$

Where:

$$c_t := s_t^{h,f} \quad \text{(consumption)} \qquad w_t := s_t^{f,h} \quad \text{(wage)}$$

Constraints

$$k(h) = 0 \qquad k(f) = 0$$

$$R_t^h := c_t + s_t^{h,h} - w_t - s_{t-1}^{h,h} = 0 \quad \forall t \in \mathbb{N}$$
$$R_t^f := w_t + s_t^{f,f} - c_t - s_{t-1}^{f,f} = 0 \quad \forall t \in \mathbb{N}$$

Net Monetary Flow

$$\textstyle\sum_{a \in A} [s_t^{a,a} - s_0^{a,a}] = 0 \quad \forall t \in \mathbb{N}$$

Discretional flows

$$\hat{s}_t^h := \{c_t, s_t^{h,h}\} \qquad \hat{s}_t^f := \{w_t, s_t^{f,f}\} \quad \forall t \in \mathbb{N}$$

Discount and Propensity Factors

$$\Pi^h = \{\beta\} \qquad \Pi^f = \{\delta\}$$
$$\Omega_t^h = \{\alpha_t\} \qquad \Omega_t^f = \{\rho_t\} \quad \forall t \in \mathbb{N}$$

Feasible Regions

$$S_t^h := \{\hat{s}_t^h \in \mathbb{R}_{\geq 0} \mid R_t^h(\hat{s}_t^h)\}$$
$$S_t^f := \{\hat{s}_t^f \in \mathbb{R}_{\geq 0} \mid R_t^f(\hat{s}_t^f)\}$$

Utility Functions

$$U^h(\Omega_t^h, \hat{s}_t^h) = \alpha_t \ln(c_t) + (1-\alpha_t)\ln(s_t^{h,h})$$
$$U^f(\Omega_t^f, \hat{s}_t^f) = \rho_t \ln(w_t) + (1-\rho_t)\ln(s_t^{f,f})$$

Optimal Flows

$$s_t^{h^*} := \arg\max_{\hat{s}_t^h \in S_t^h} \left[\textstyle\sum_{t \in \mathbb{N}} \left[\pi^h U^h \left(\Omega_t^h, \hat{s}_t^h\right)\right]\right]$$
$$s_t^{f^*} := \arg\max_{\hat{s}_t^f \in S_t^f} \left[\textstyle\sum_{t \in \mathbb{N}} \left[\pi^f U^f \left(\Omega_t^f, \hat{s}_t^f\right)\right]\right]$$
$$s_t^{h^*} = \left(c_t^*,\, s_t^{h,h^*}\right) \qquad s_t^{f^*} = \left(w_t^*,\, s_t^{f,f^*}\right)$$

Economic Network State at time t

$$s_t^* = \left(\hat{s}_t^{h^*}, \hat{s}_t^{f^*}\right) = \left((c_t^*, s_t^{h,h^*}), (w_t^*, s_t^{f,f^*})\right)$$
$$c_t^* = w_t^* + s_{t-1}^{h,h\,*} - s_t^{h,h^*}$$
$$w_t^* = c_t^* + s_{t-1}^{f,f\,*} - s_t^{f,f^*}$$
$$s_t^{h,h^*} = \left(\frac{1}{\alpha_t} - 1\right)\left(\frac{1}{c_t} - \beta\frac{\alpha_{t+1}}{\alpha_t}\frac{1}{c_{t+1}^*}\right)^{-1}$$
$$s_t^{f,f^*} = \left(\frac{1}{\rho_t} - 1\right)\left(\frac{1}{w_t} - \delta\frac{\rho_{t+1}}{\rho_t}\frac{1}{w_{t+1}^*}\right)^{-1}$$

In the steady-state:

$$c^* = w^*$$
$$s^{h,h^*} = \left(\frac{1}{\alpha_t} - 1\right)(1-\beta)^{-1} c^*$$
$$s^{f,f^*} = \left(\frac{1}{\rho_t} - 1\right)(1-\delta)^{-1} w^*$$

Propensity Factors Inference

Analytical inversion:

$$\alpha_t = \left(1 + \frac{s_t^{h,h}}{c_t}\right)^{-1} \left(1 + \beta\alpha_{t+1}\frac{s_t^{h,h}}{c_{t+1}}\right)$$

$$\rho_t = \left(1 + \frac{s_t^{f,f}}{w_t}\right)^{-1} \left(1 + \delta\rho_{t+1}\frac{s_t^{f,f}}{w_{t+1}}\right)$$

Empirical inference:

$$\alpha_t = \frac{c_t^*}{w_t^* + s_{t-1}^{h,h\,*}} \qquad \rho_t = \frac{w_t^*}{c_t^* + s_{t-1}^{f,f\,*}}$$

Economic behavior under steady-state

The network is holded by the extended vector

$$\mathcal{S} = (c_t, w_t, s_t^{h,h}, s_t^{f,f}, c_{t+1}, w_{t+1}, s_{t-1}^{h,h}, s_{t-1}^{f,f})^T,$$

so, the equilibrium is expressed as:

$$\mathcal{S} = M\mathcal{S}.$$

Since $\mathcal{S}$ considers at least one optimal extended flow, non-trivial steady-state exists.

In the steady-state, the Jacobian is:

$$\mathcal{S}^* = (c^*, w^*, s^{h,h^*}, s^{f,f^*})^T,$$

$$\mathcal{S}^* = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{\partial s^{h,h\,*}}{\partial c^*} & 0 & 0 & 0 \\ 0 & \frac{\partial s^{f,f\,*}}{\partial w^*} & 0 & 0 \end{bmatrix} \mathcal{S}^*$$

Therefore, the eigenvalues associated are:

$$\sigma(J_{\mathcal{S}=s^*}) = \{1, -1, 0, 0\}$$

which spectral radius equals to 1 so the equilibrium is unstable and the agents savings are self-corrective. That means that, in order to restore the steady-state in this network after a perturbation on propensity factors, it's imperative to keep the consumption and the wages invariants, and it's only possible if a new perturbation with opposite accumulative magnitude as the previous one is provoked.

Appendix C: Open national network economy

In a national economy, the graph is computed as $G(5, 25)$, where:

$$A = \left\{ \begin{array}{l} \text{Households (h),} \quad \text{Firms (f),} \quad \text{Government (g),} \\ \text{National Banks (nb),} \quad \text{Central Bank (cb)} \end{array} \right\}$$

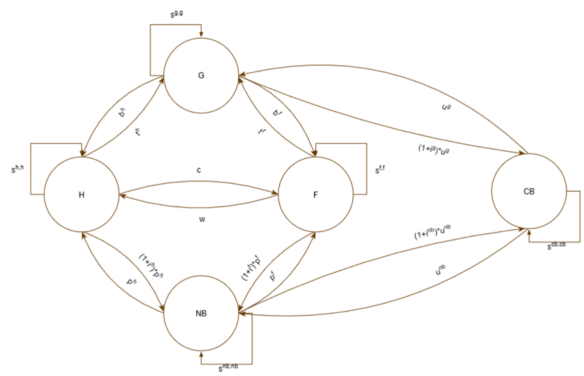

Figure 2: National Economy

Matrix Representation

$$S_t = \begin{bmatrix} s_t^{h,h} & c_t & t_t^h & (1+i_t^h)p_t^h & 0 \\ w_t & s_t^{f,f} & t_t^f & (1+i_t^f)p_t^f & 0 \\ b_t^h & b_t^f & s_t^{g,g} & 0 & (1+i_t^g)U^g \\ p_t^h & p_t^f & 0 & s_t^{nb,nb} & (1+i_t^{nb})U^{nb} \\ 0 & 0 & U^g & U^{nb} & s_t^{cb,cb} \end{bmatrix}$$

$c_t := s^{(h,f)}$ Households consumption
$t^h := s^{(h,g)}$ Households tax
$p^h := s^{(nb,h)}$ Households credit
$w_t := s^{(f,h)}$ Wage
$t^f := s^{(f,g)}$ Firms tax
$p^f := s^{(nb,f)}$ Firms credit
$b^h := s^{(g,h)}$ Households subsidy
$b^f := s^{(g,f)}$ Firms subsidy
$u^g := s^{(cb,g)}$ Government debt
$u^{nb} := s^{(cb,nb)}$ National Banks debt

constraints

$$k(h)=0 \quad k(f)=0 \quad k(g)=0 \quad k(nb)=0 \quad k(cb)=1$$

$$R_t^h := c_t + t_t^h + (1+i_{t-1}^h)p_{t-1}^h + s_t^{h,h} - w_t - b_t^h - p_t^h - s_{t-1}^{h,h} = 0$$
$$R_t^f := w_t + t_t^f + (1+i_{t-1}^f)p_{t-1}^f + s_t^{f,f} - c_t - b_t^f - p_t^f - s_{t-1}^{f,f} = 0$$
$$R_t^g := b_t^h + b_t^f + (1+i_{t-1}^g)u_{t-1}^g + s_t^{g,g} - t_t^h - t_t^f - U^g - s_{t-1}^{g,g} = 0$$
$$R_t^{nb} := p_t^h + p_t^f + (1+i_{t-1}^{nb})u_{t-1}^{nb} + s_t^{nb,nb} - (1+i_{t-1}^h)p_{t-1}^h - (1+i_{t-1}^f)p_{t-1}^f - U^{nb} - s_{t-1}^{nb,nb} = 0$$
$$R_t^{cb} := U^g + U^{nb} + s_t^{cb,cb} - (1+i_{t-1}^g)u_{t-1}^g - (1+i_{t-1}^{nb})u_{t-1}^{nb} - s_{t-1}^{cb,cb} \geq 0$$

Net Monetary Flow

$$\textstyle\sum_{a\in A}\left[s_t^{a,a} - s_0^{a,a}\right] \geq 0 \quad \forall t \in \mathbb{N}$$

Discretional flows

$$\hat{s}_t^h = \left\{c_t, s_t^{h,h}\right\}, \qquad \hat{s}_t^f = \left\{w_t, s_t^{f,f}\right\},$$
$$\hat{s}_t^g = \left\{b_t^h, b_t^f, t_t^h, t_t^f\right\}, \quad \hat{s}_t^{nb} = \left\{i_t^h, i_t^f\right\},$$
$$\hat{s}_t^{cb} = \left\{i_t^g, i_t^{nb}\right\}$$

Discount and Propensity Factors

$$\Pi^h = \{\beta\} \qquad \Omega_t^h = \{\alpha_t\}$$
$$\Pi^f = \{\delta\} \qquad \Omega_t^f = \{\rho_t\}$$
$$\Pi^g = \{\sigma\} \qquad \Omega_t^g = \{\mu, \gamma_t\}$$
$$\Pi^{nb} = \{\theta\} \qquad \Omega_t^{nb} = \{\phi_t\}$$
$$\Pi^{cb} = \{\varpi\} \qquad \Omega_t^{cb} = \{\eta_t\}$$

Feasible Regions

$$S_t^h := \left\{\hat{s}_t^h \in \mathbb{R}_{\geq 0} \;\middle|\; R_t^h(\hat{s}_t^h)\right\}$$
$$S_t^f := \left\{\hat{s}_t^f \in \mathbb{R}_{\geq 0} \;\middle|\; R_t^f(\hat{s}_t^f)\right\}$$
$$S_t^g := \left\{\hat{s}_t^g \in \mathbb{R}_{\geq 0} \mid R_t^g(\hat{s}_t^g)\right\}$$
$$S_t^{nb} := \left\{\hat{s}_t^{nb} \in \mathbb{R}_{\geq 0} \;\middle|\; R_t^{nb}(\hat{s}_t^{nb})\right\}$$
$$S_t^{cb} := \left\{\hat{s}_t^{cb} \in \mathbb{R}_{\geq 0} \;\middle|\; R_t^{cb}(\hat{s}_t^{cb})\right\}$$

Utility Functions

$$U^h\left(\Omega_t^h, \hat{s}_t^h\right) = \alpha_t \ln(c_t) + (1-\alpha_t)\ln(s_t^{h,h})$$
$$U^f\left(\Omega_t^f, \hat{s}_t^f\right) = \rho_t \ln(w_t) + (1-\rho_t)\ln(s_t^{f,f})$$
$$U^g\left(\Omega_t^g, \hat{s}_t^g\right) = \mu \ln(b_t^h) + (1-\mu)\ln(b_t^f) + \gamma_t \ln(t_t^h) + (1-\gamma_t)\ln(t_t^f)$$
$$U^{nb}\left(\Omega_t^{nb}, \hat{s}_t^{nb}\right) = \phi_t \ln(i_t^h) + (1-\phi_t)\ln(i_t^f)$$
$$U^{cb}\left(\Omega_t^{cb}, \hat{s}_t^{cb}\right) = U^{cb}(\eta_t, i_t^g, i_t^{nb})$$

Optimal Flows

$$s_t^{h^*} := \arg\max_{\hat{s}_t^h \in S_t^h}\left[\textstyle\sum_{t\in\mathbb{N}}\left[\pi^h U^h\left(\Omega_t^h, \hat{s}_t^h\right)\right]\right]$$
$$s_t^{f^*} := \arg\max_{\hat{s}_t^f \in S_t^f}\left[\textstyle\sum_{t\in\mathbb{N}}\left[\pi^f U^f\left(\Omega_t^f, \hat{s}_t^f\right)\right]\right]$$
$$s_t^{g^*} := \arg\max_{\hat{s}_t^g \in S_t^g}\left[\textstyle\sum_{t\in\mathbb{N}}\left[\pi^g U^g\left(\Omega_t^g, \hat{s}_t^g\right)\right]\right]$$
$$s_t^{nb^*} := \arg\max_{\hat{s}_t^{nb} \in S_t^{nb}}\left[\textstyle\sum_{t\in\mathbb{N}}\left[\pi^{nb} U^{nb}\left(\Omega_t^{nb}, \hat{s}_t^{nb}\right)\right]\right]$$
$$s_t^{cb^*} := \arg\max_{\hat{s}_t^{cb} \in S_t^{cb}}\left[\textstyle\sum_{t\in\mathbb{N}}\left[\pi^{cb} U^{cb}\left(\Omega_t^{cb}, \hat{s}_t^{cb}\right)\right]\right]$$
$$s_t^{h^*} = \left(c_t^*, s_t^{h,h^*}\right), \qquad s_t^{f^*} = \left(w_t^*, s_t^{f,f^*}\right),$$
$$s_t^{g^*} = \left(b_t^{h^*}, b_t^{f^*}, t_t^{h^*}, t_t^{f^*}\right), \quad s_t^{nb^*} = \left(i_t^{h^*}, i_t^{f^*}\right),$$
$$s_t^{cb^*} = \left(i_t^{g^*}, i_t^{nb^*}\right)$$

Economyc Network State at time t

$$c_t^* = w_t^* + b_t^{h^*} + p_t^h + s_{t-1}^{h,h^*} - t_t^{h*} - (1+i_{t-1}^{h^*})p_{t-1}^h - s_t^{h,h^*} \quad \forall t \in \mathbb{N}$$
$$w_t^* = c_t^* + b_t^{f^*} + p_t^f + s_{t-1}^{f,f^*} - t_t^{f*} - (1+i_{t-1}^{f^*})p_{t-1}^f - s_t^{f,f^*} \quad \forall t \in \mathbb{N}$$
$$b_t^{h^*} + b_t^{f^*} - t_t^{h^*} - t_t^{f^*} = U^g + s_{t-1}^{g,g} - (1+i_{t-1}^{g^*})u_{t-1}^g - s_t^{g,g} \quad \forall t \in \mathbb{N}$$
$$i_{t-1}^{h^*} = \left(\frac{p_t^h + p_t^f + (1+i_{t-1}^{nb^*})u_{t-1}^{nb} + s_t^{nb,nb} - (1+i_{t-1}^{f^*})p_{t-1}^f - U^{nb} - s_{t-1}^{nb,nb}}{p_{t-1}^h} - 1\right) \quad \forall t \in \mathbb{N}$$
$$i_{t-1}^{g^*} \leq \left(\frac{U^g + U^{nb} + s_t^{cb,cb} - (1+i_{t-1}^{nb^*})u_{t-1}^{nb} - s_{t-1}^{cb,cb}}{u_{t-1}^g} - 1\right) \quad \forall t \in \mathbb{N}$$
$$s_t^{h,h^*} = \left(\frac{1}{\alpha_t} - 1\right)\left(\frac{1}{c_t} - \beta\frac{\alpha_{t+1}}{\alpha_t}\frac{1}{c_{t+1}^*}\right)^{-1}$$
$$s_t^{f,f^*} = \left(\frac{1}{\rho_t} - 1\right)\left(\frac{1}{w_t} - \delta\frac{\rho_{t+1}}{\rho_t}\frac{1}{w_{t+1}^*}\right)^{-1}$$
$$\mu\frac{1}{b_t^{h^*}} + (1-\mu)\frac{1}{b_t^{f^*}} = \gamma\frac{1}{t_t^{h^*}} + (1-\gamma)\frac{1}{t_t^{f^*}} \quad \forall t \in \mathbb{N}$$
$$i_t^{f^*} = \left(\frac{1}{\phi_t} - 1\right)\frac{p_t^h}{p_t^f} i_t^{h^*} \quad \forall t \in \mathbb{N}$$
$$i_t^{nb^*} = \ldots \quad \forall t \in \mathbb{N}$$

In the steady-state:

$$
\begin{aligned}
&c^* = w^* + b^{h^*} - t^{h*} - i^{h^*} p^h \\
&w^* = c^* + b^{f^*} - t^{f^*} - i^{f^*} p^f \\
&b^{h^*} + b^{f^*} + i^{g^*} u^g = t^{h^*} + t^{f^*} \\
&i^{h^*} = \frac{i^{nb^*} u^{nb} - i^{f^*} p^f}{p^h} \\
&i^{g^*} \leq -\frac{u^{nb}}{u^g} i^{nb^*} \\
&s_t^{h,h^*} = \left(\frac{1}{\alpha_t} - 1\right) (1-\beta)^{-1} c^* \\
&s_t^{f,f^*} = \left(\frac{1}{\rho_t} - 1\right) (1-\delta)^{-1} w^* \\
&\mu \frac{1}{b^{h^*}} + (1-\mu) \frac{1}{b^{f^*}} = \gamma \frac{1}{t^{h^*}} + (1-\gamma) \frac{1}{t^{f^*}} \\
&i^{f^*} = \left(\frac{1}{\phi} - 1\right) \frac{p^h}{p^f} i^{h^*} \\
&i^{nb^*} = ...
\end{aligned}
$$

Propensity Factors Inference

Analytical inversion:

$$\alpha_t = \left(1 + \left(c_t^* \left(\frac{1}{s_t^{h,h^*}} + \beta \alpha_{t+1} \frac{1}{c_{t+1}^*}\right)\right)^{-1}\right)^{-1} \quad \forall t \in \mathbb{N}$$

$$\rho_t = \left(1 + \left(w_t^* \left(\frac{1}{s_t^{f,f^*}} + \delta \rho_{t+1} \frac{1}{w_{t+1}^*}\right)\right)^{-1}\right)^{-1} \quad \forall t \in \mathbb{N}$$

$$\mu = \left(\frac{1}{b_t^f} + \frac{1}{t_t^f} - \gamma \left(\frac{1}{t_t^h} + \frac{1}{t_t^f}\right)\right) \left(\frac{1}{b_t^h} + \frac{1}{b_t^f}\right)^{-1} \quad \forall t \in \mathbb{N}$$

$$\phi_t = \left(1 + \frac{p_t^f}{p_t^h} \frac{i_t^{f*}}{i_t^{h*}}\right)^{-1} \quad \forall t \in \mathbb{N}$$

It's insightful to estimate the propensity factor of domestic banks, as its value can be directly inferred from the borrowing costs faced by households and firms. By algebraic manipulation, the expression can be rewritten as:

$$\phi_t = \left(1 + \frac{p_t^f}{p_t^h} \frac{i_t^{f*}}{i_t^{h*}}\right)^{-1} = \frac{p_t^h i_t^h}{p_t^h i_t^{h^*} + p_t^f i_t^{f^*}} \quad \forall t \in \mathbb{N}$$

From a practical standpoint, the propensity of domestic banks to increase the household interest rate i_t^h at time t can be deduced from the ratio of the domestic borrowing cost $\left(p_t^h i_t^h\right)$ to the total borrowing cost $\left(p_t^h i_t^h + p_t^f i_t^f\right)$.

Empirical inference:

$$\alpha_t = \frac{c_t^*}{w_t^* + b_t^{h^*} + p_t^h + s_{t-1}^{h,h^*} - t_t^{h^*} - (1 + i_{t-1}^{h^*}) p_{t-1}^h} \quad \forall t \in \mathbb{N}$$

$$\rho_t = \frac{w_t^*}{c_t^* + b_t^{f^*} + p_t^f + s_{t-1}^{f,f^*} - t_t^{f^*} - (1 + i_{t-1}^{f^*}) p_{t-1}^f} \quad \forall t \in \mathbb{N}$$

$$\mu = \frac{b_t^{h^*}}{t_t^{h^*} + t_t^{f^*} + U^g + s_{t-1}^{g,g} - (1 + i_{t-1}^{g^*}) u_{t-1}^g} \quad \forall t \in \mathbb{N}$$

$$\gamma_t = \frac{t_t^{h^*}}{U^g + s_{t-1}^{g,g} - b_t^{h^*} - b_t^{f^*} - (1 + i_{t-1}^{g^*}) u_{t-1}^g} \quad \forall t \in \mathbb{N}$$

Economic behavior under steady-state

The network is holded by the extended vector

$$\mathcal{S} = \begin{pmatrix} c_t, w_t, b_t^h, b_t^f, t_t^f, t_t^h, i_t^h, i_t^f, i_t^g, i_t^{nb}, s_t^{h,h}, s_t^{f,f}, \\ c_{t+1}, w_{t+1}, i_{t-1}^h, i_{t-1}^f, i_{t-1}^g, i_{t-1}^{nb}, s_{t-1}^{h,h}, s_{t-1}^{f,f} \end{pmatrix}^T$$

so, the equilibrium is expressed as:

$$\mathcal{S} = M\mathcal{S}.$$

Since $\mathcal{S}$ considers at least one optimal extended flow, non-trivial steady-state exists.

In the steady-state, the Jacobian is:

$$\mathcal{S}^* = (c^*, w^*, b^{h^*}, b^{f^*}, t^{h^*}, t^{f^*}, i^{h^*}, i^{f^*}, i^{g^*}, i^{nb^*}, s^{h,h^*}, s^{f,f^*})^T,$$

$$\mathcal{S}^* = \begin{bmatrix}
0 & 1 & 1 & 0 & -1 & 0 & \frac{\partial c^*}{\partial i^{h^*}} & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & -1 & 0 & \frac{\partial w^*}{\partial i^{f^*}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & \frac{\partial b^{h^*}}{\partial i^{g^*}} & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & \frac{\partial b^{f^*}}{\partial i^{g^*}} & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 & \frac{\partial t^{h^*}}{\partial i^{g^*}} & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & \frac{\partial t^{f^*}}{\partial i^{g^*}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial i^{h^*}}{\partial i^{f^*}} & 0 & \frac{\partial i^{h^*}}{\partial i^{nb^*}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial i^{f^*}}{\partial i^{h^*}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial i^{g^*}}{\partial i^{nb^*}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial i^{nb^*}}{\partial i^{g^*}} & 0 & 0 & 0 \\
\frac{\partial s^{h,h^*}}{\partial c^*} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\partial s^{f,f^*}}{\partial w^*} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \mathcal{S}^*$$

Therefore, the eigenvalues associated are:

$$\sigma(J_{\mathcal{S}=s^*}) = \begin{Bmatrix} -1, -3, 1, 1, 1, 1, \\ \sqrt{\frac{\partial i^{f^*}}{\partial i^{h^*}} * \frac{\partial i^{h^*}}{\partial i^{f^*}}}, -\sqrt{\frac{\partial i^{f^*}}{\partial i^{h^*}} * \frac{\partial i^{h^*}}{\partial i^{f^*}}}, \\ \sqrt{\frac{\partial i^{g^*}}{\partial i^{nb^*}} * \frac{\partial i^{nb^*}}{\partial i^{g^*}}}, -\sqrt{\frac{\partial i^{g^*}}{\partial i^{nb^*}} * \frac{\partial i^{nb^*}}{\partial i^{g^*}}}, \\ 0, 0 \end{Bmatrix}$$

which spectral radius equal or higher than 3 so the equilibrium is unstable and the agents savings are self-corrective.

That means that both consumption and wage are very sensitive and, in order to restore the steady-state in this network after a perturbation on propensity factors, it's imperative to keep it invariant, and it's only possible if a new perturbation with opposite accumulative magnitude as the previous one is provoked.

Appendix D: Bernanke and the Crises of 1929 and 2008

Ben Bernanke is an economist and policymaker who served as Chair of the United States Federal Reserve (FED) from 2006 to 2014. Both in academia and in policy circles, he became renowned for his research on macroeconomics and monetary policy, particularly for his studies on the Great Depression of 1929.

In his seminal article [Bernanke, 1983], Bernanke argued that the collapse of credit markets, beyond the mere contraction of the money supply, amplified and prolonged the crisis due to the breakdown of financial intermediation channels. Specifically, he demonstrated how banking dysfunction adversely affected credit allocation to solvent firms, distorted resource allocation and ultimately transformed a recession into a depression.

Within the methodological framework proposed in this work—using the practical case of a stationary-state national economy previously developed—the 1929 crisis can be represented as an exogenous shock to the propensity factor to tax household credit (φ), which, in the absence of intervention, results in:

$$\uparrow \varphi \Rightarrow \uparrow i_h^* \Rightarrow \downarrow c^* \Rightarrow \downarrow w^* \quad \text{(first causal channel)}$$

$$\uparrow \varphi \Rightarrow \downarrow i_f^* \Rightarrow \uparrow p_f \Rightarrow \uparrow w^* \Rightarrow \uparrow c^* \quad \text{(second causal channel)}$$

However, based on the equations derived for the national economy, the following inequalities hold:

$$\left|\frac{\partial i_h^*}{\partial \varphi}\right| > \left|\frac{\partial i_f^*}{\partial \varphi}\right|$$

$$\left(\frac{1}{\varphi} - 1\right)^{-1} > \left|\frac{p_h}{p_f} - \left(\frac{1}{\varphi} - 1\right)^{-1} \frac{i_h^*}{i_f^*}\right|, \quad \text{where } \frac{i_h^*}{i_f^*} \simeq 1$$

$$\text{and always that } \varphi \approx 1$$

This implies that the first causal channel dominates the second when starting from a high propensity to tax household credit, as identified by [Bernanke, 1983] in the historical data of the U.S. financial system during that period. In practical terms, without intervention, an increase in the propensity factor leads to lower consumption and reduced wage income, as observed during the 1929 stock market crash.

In his ex-post analysis, Bernanke identified two national-level policy measures with potential to mitigate the negative effects of the 1929 crisis:

- Liquidity injections into domestic banks to prevent banking sector collapse.
- Loan guarantee programs to restore credit channel functioning.

In a stationary-state national economy, the following identity holds:

$$i_{nb}u^{nb} = i_h^* p_h + i_f^* p_f$$

$$\uparrow u^{nb} \Rightarrow \uparrow (i_h^* p_h + i_f^* p_f)$$

In other words, liquidity injections into domestic banks, holding i^{nb} constant, worsen overall economic conditions by increasing the total borrowing cost.
In the context of an economic crisis, liquidity injections alone primarily boost demand for bank credit. As previously demonstrated, an increase in the propensity to tax credit reduces both wages and consumption. Consequently, due to liquidity injections, domestic banks respond by expanding credit supply to households and firms, thus temporarily alleviating the crisis's immediate impact.
Therefore, liquidity injections raise the medium-term borrowing cost and should only be interpreted as a temporary financial safety net.

The appropriate policy response for this type of crisis, as correctly highlighted by Bernanke, is to reduce the cost of bank debt:

$$\downarrow i_{nb}^* \Rightarrow \downarrow (i_h^* p_h + i_f^* p_f)$$

Under this scenario, domestic banks experience less pressure from their debt obligations to the central bank and, consequently, have reduced incentives to raise credit costs for households and firms. In the long run, credit interest rates decline irrespective of whether the outstanding debt of domestic banks to the central bank decreases or remains unchanged.
This final observation explains why the policy actions implemented by Bernanke during the 2008 financial crisis did not yield the expected results. Injecting liquidity at reduced cost had two opposing effects: on one hand, it lowered interest rates on bank credit, easing pressure on domestic credit markets in the long run; on the other hand, it temporarily expanded credit access for households and firms in a crisis context, thereby increasing medium-term pressure on aggregate credit demand.

In summary, the above analysis illustrates how Bernanke's policy measures produced simultaneous opposing effects, with the reduction in debt costs ultimately prevailing over liquidity injections, due to its more enduring long-term impact.

Appendix E: ECB and FED actions in 2020 pandemic

In early 2020, a novel strain of the well-known coronavirus family (SARS-CoV-2) was identified in Wuhan, China. Within just two months, it had spread globally and was declared a pandemic by the World Health Organization.

This event affected all aspects of human life, but with a focus on economics, it can be modeled as a significant negative shock to households' propensity to consume (α).

Applying the theoretical framework developed in this work, while keeping other flows constant and starting from the maximization program of a stationary-state national economy, the following relationship holds:

$$\downarrow \alpha \Rightarrow \downarrow c^* \Rightarrow \downarrow w^*$$

In other words, under a stationary-state equilibrium, a decrease in households' preference for consumption leads to a reduction in their wage income. Moreover, this reduction in wages further diminishes the propensity to consume, which in turn reduces consumption again, creating a recessionary spiral.

To analyze this dynamic, it's essential to determine under which conditions a decrease in α impacts wages more than it impacts corporate savings. Formally, this occurs when, in steady-state:

$$\left| \frac{\partial w^*}{\partial \alpha} \right| > \left| \frac{\partial s^{f,f*}}{\partial \alpha} \right|$$

Applying the chain rule yields:

$$\left| \frac{\partial w^*}{\partial c^*} \frac{\partial c^*}{\partial s^{h,h*}} \frac{\partial s^{h,h*}}{\partial \alpha} \right| > \left| \frac{\partial s^{f,f*}}{\partial w^*} \frac{\partial w^*}{\partial c^*} \frac{\partial c^*}{\partial s^{h,h*}} \frac{\partial s^{h,h*}}{\partial \alpha} \right|$$

Which simplifies to:

$$1 > \left| \frac{\partial s^{f,f*}}{\partial w^*} \right|$$

Or equivalently:

$$\rho > (2-\delta)^{-1} \quad \text{with } \delta \in (0,1)$$

This indicates that, in response to an exogenous shock lowering households' propensity to consume, firms transfer this adjustment primarily to labor compensation when $\rho > 0.5$ at least. In such cases, with absent policy intervention, the economy is driven into a self-reinforcing recessionary spiral due to the increased households preference for saving.

Christine Lagarde and Jerome Powell, then heads of the European Central Bank and the Federal Reserve, respectively, adopted different policy responses to mitigate the economic fallout of the pandemic.

In the United States, the response centered on aggressive fiscal stimulus directed at households, complemented by expansionary monetary policy: interest rates were cut to near zero and large-scale liquidity injections were implemented.

In Europe, policymakers focused on protecting the corporate sector through mechanisms such as short-time work schemes (e.g., ERTE in Spain), which covered wages and prevented mass layoffs. Simultaneously, the ECB applied expansionary monetary policy, reducing interest rates and increasing liquidity within the financial system.

From this, it's evident that while both the U.S. and the EU faced similar macroeconomic shocks, they adopted distinct policy approaches. The U.S. sought to directly counteract the drop in household consumption, whereas Europe prioritized labor market stability, expecting that preserving employment would help restore consumer confidence and economic activity.

Applying the national economy framework, the U.S. response is characterized by increases in b^h and u^{nb} and a reduction in i^{nb}, whereas the EU focused on increasing b^f and u^{nb}, alongside reductions in i^{nb}.

With respect to liquidity injections at reduced cost, the critique derived from the Bernanke case remains valid: the medium-term effect is neutral due to opposing mechanisms—simultaneous increases and decreases in credit costs. In cases where the objective is to indirectly influence the credit system, adjusting the debt cost of domestic banks is sufficient. To stimulate credit, lowering i^{nb} suffices; conversely, to restrict credit, increasing i^{nb} is effective.

Liquidity provision should be reserved for exceptional circumstances to prevent banking sector collapse, as it generally entails adverse side effects. In crisis conditions, providing liquidity incentivizes credit expansion, while withdrawing it raises the cost of new credit. In either case, the aggregate borrowing cost increases, leading to deteriorated credit conditions.

Regardless of whether government subsidies target households or firms, such measures ultimately aim to reactivate households' propensity to consume—directly in the U.S. via household transfers and indirectly in the EU via corporate subsidies preserving wage income.

In this context, governments should use their fiscal space to finance such measures at zero cost whenever possible. Where fiscal space is insufficient, the central bank should lower i^g to cover the remaining deficit.

Appendix F: Composite graph modeling

Up to this point, the graph modelling approach has been developed for a network of interconnected agents. However, the proposed methodology is also scalable to higher levels of aggregation. Referring back to Appendix C, where a national economic network was designed, it's possible to extend this approach to an international economic network $G(A, I)$ composed of multiple national systems.

- $A = \{G_1, G_2, \ldots, G_n\}$ denotes the set of subgraphs, where each G_i represents a national economy.

- $I \subseteq A \times A$ denotes the set of directed interactions between nations.

At this level, the entire theoretical framework developed in the core sections of this contribution remains applicable, with the key distinction that the nodes of the supergraph no longer represent heterogeneous agents but instead encapsulate complete networks. Each node functions as a structural container holding a lower-level economic graph.

Consider two national economies, each represented according to the following figure.

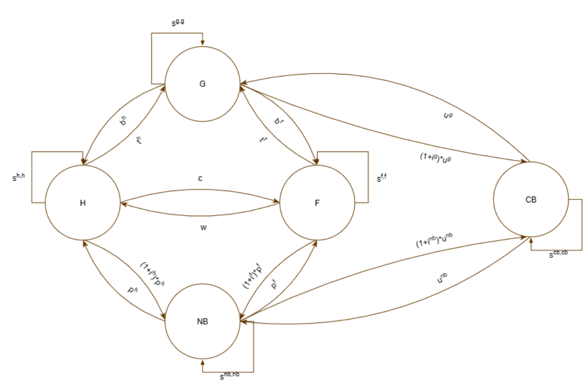

Figure 3: National economy

To model an international economy, it suffices to define which agents trade out of their native network. For illustrative purposes, it's considered interactions between central banks for currency exchange and households and firms for international trade.

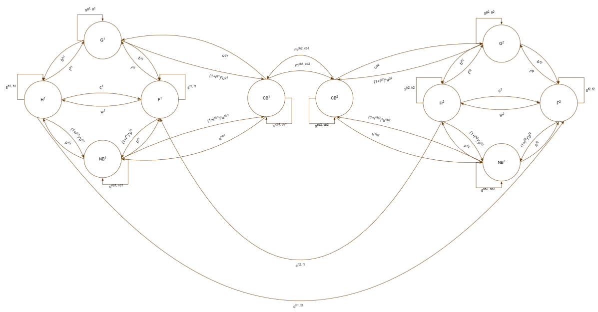

Figure 4: International economy with two nations

It's important to note that the adjacency matrix associated with the composite graph expands rapidly in size. To facilitate its representation, each nation is treated as an encapsulated substructure, while preserving the relevant higher-level interactions.

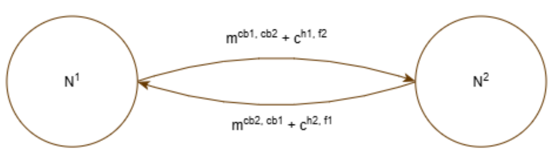

Figure 5: Encapsulated international economy

The result is a hierarchical second-order structure.

Finally, it's worth emphasizing that this approach is inherently recursive: starting from national networks, one can model an international system; with multiple planets, an interplanetary economy; with stellar systems, an interstellar economy. Similarly, the model allows for descending the level of analysis to urban, sectoral, corporate, or even individual economic networks.

Appendig G: Maximum entropy IRL - Exercise

Consider a close economy with households and firms where

$$\left(s_0^{h,h^*}, s_0^{f,f^*}\right) = (100, 0)$$

$$\beta = \sigma = 0.5$$

and agents decision-making holds from its respective unobserved utility functions:

$$U^h(\alpha_t, c_t, s_t^{h,h}) = \alpha_t ln(c_t) + (1-\alpha_t) ln(s_t^{h,h})$$

$$U^f(\rho_t, w_t, s_t^{f,f}) = \rho_t (w_t)^{1/2} + (1-\rho_t)(s_t^{f,f})^{1/2}.$$

Solving the associated optimal decision-making program and setting random values at agents propensity factors over time:

t	c_t^*	s_t^{h,h^*}	w_t^*	s_t^{f,f^*}	α_t	ρ_t
0		100		0		
1	29.52	72.17	1.69	27.83	0.45	0.33
2	33.12	52.04	12.99	47.96	0.56	0.51
3	29.99	53.20	31.15	46.80	0.53	0.62
4	28.43	50.43	25.66	49.57	0.53	0.59
5	25.33	50.65	25.55	49.35	0.50	0.59

Table 1: Observed behavior

Note how the empirical estimation for propensities doesn't work, that's because the propensity steady-state assumption which is followed by households and firms.

Now let's apply the IRL method to figure out the mathematical expression of the two agents utilities. Three utility expressions are proposed:

$$\text{log } \left(U_l^h\right):\ \alpha_t ln(c_t) + (1-\alpha_t) ln(s_t^{h,h})$$

$$\text{root } \left(U_r^h\right):\ \alpha_t (c_t)^{1/2} + (1-\alpha_t)(s_t^{h,h})^{1/2}$$

$$\text{mix } \left(U_m^h\right):\ \alpha_t ln(c_t) + (1-\alpha_t)(s_t^{h,h})^{1/2}$$

In order to test the predicted flows associated with each utility candidate, the observed data must respect the Euler optimal equations. Assuming perfect rationality $\tau^a = 1$ for both agents, the predicted flows must match all the time with the real ones.

Discount factors β and δ are derived from de Euler optimal equations and the behavior observed.
Since $\pi^a \in (0,1)$ and is not time-dependent, the correct one must be between the (0, 1) range and be constant all the time.

For each utility candidate, impatience factors are:

t	β			δ		
	U_l^h	U_r^h	U_m^h	U_l^f	U_r^f	U_m^f
0						
1	0.50	0.22	1.00	0.88	0.50	1.00
2	0.50	0.37	1.00	0.74	0.50	1.00
3	0.50	0.33	1.00	0.59	0.50	1.00
4	0.50	0.33	1.00	0.64	0.50	1.00
5	0.50	0.29	1.00	0.64	0.50	1.00

Table 2: Discount factors inference

Holding for consumption and wage, the associated savings are:

t	s^{h,h^*}			s^{f,f^*}		
	U_l^h	U_r^h	U_m^h	U_l^f	U_r^f	U_m^f
0	100	100	100	0	0	0
1	72.17	176.41	1301.97	6.85	27.83	11.75
2	52.04	81.78	677.01	24.96	47.96	155.74
3	53.20	94.35	707.43	38.18	46.80	364.48
4	50.43	89.44	635.72	35.67	49.57	318.06
5	50.65	101.30	641.36	35.51	49.35	315.20

Table 3: Savings inference

Note how discount inference already tells which utility expression holds for each actor.
Since discount inference is easier for utility IRL, savings inference can be used as a validation test when first comes ambiguous.

In conclusion, households savings matches perfectly with the log utility expression while firms savings matches perfectly with the root utility expression. Thereby, log utility for households and root utility for firms are assumed.

Appendix F: Simulation and dashboard

As a practical exercise, a simulation for a closed economy consisting of households and firms has been developed to illustrate the design of an interactive dashboard for scenario tracking and simulation.

The dashboard consists of two main views: the first for configuring the simulation parameters, and the second for monitoring the simulation and enabling user intervention.
In the interactive dashboard, the user can interact with normalized values via a heatmap, monitor the time series of propensity factors, and modify these factors using the sliders located in the upper-right section of the interface.

This panel serves as a functional proof of concept for the proposed theoretical model and demonstrates its feasibility for practical applications in real-time economic monitoring.
Visit the online repository for access to the visualization.

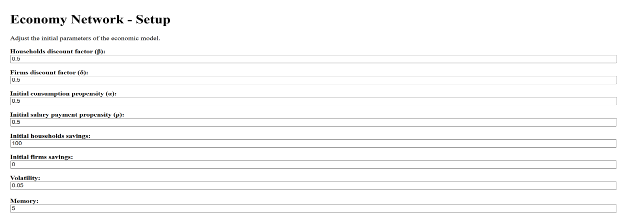

Figure 6: Simulation — Setting

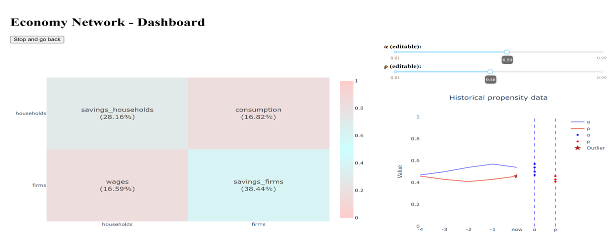

Figure 7: Simulation — Dashboard