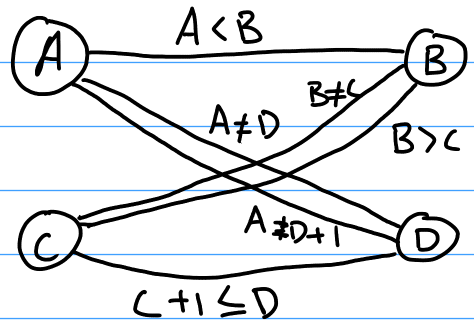
Ex. Sheet 2 2064363 F.B.

1. MRV – when selecting a variable, choose the one with the fewest remaining legal values.
Deg. Heur. – In case of MRV tiebreak, pick variable with the most constraints on remaining variables.
LCV – Pick the value which eliminates the least number of values for other variables.

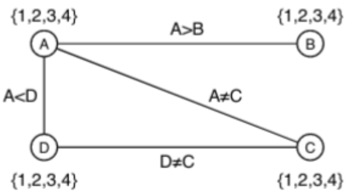
2. a)



Degrees:
A – 3 C – 3
B – 3 D – 3

b) By MRV – select C.
If C = 1: $D_A = \{1, 2, 3\}$ (-0)
$D_B = \{2, 3, 4\}$ (-1)
$D_D = \{2\}$ (-3) $\Rightarrow -4$
C = 2: $D_A =$ As above
$D_B = \{3, 4\}$ (-2)
$D_D = \{2, 3\}$ (-2) $\Rightarrow -4$
By LCV, either fine $\Rightarrow$ let C = 1.
By MRV, select D.
D = 2 (only choice)
$\therefore$ $D_A = \{1\}$ (-2)
$D_B = \{2, 3, 4\}$ (-0)
By MRV, select A.
A = 1
$\therefore$ B = {2, 3, 4} (-0)
$\therefore$ Solutions found:
A = 1, B = {2, 3, 4}, C = 1, D = 2.

3)



a) Arc $A > B$:

$A: \{\cancel{1}, 2, 3, 4\}$

$B: \{1, 2, 3, \cancel{4}\}$

Arc $A < D$:

$A: \{2, 3, \cancel{4}\}$

$D: \{\cancel{1}, \cancel{2}, 3, 4\}$

No singleton domains $\therefore$ cannot evaluate $A \neq C$ or $D \neq C$.

Arc-consistent domains:

$D_A = \{2, 3\}$, $D_B = \{1, 2, 3\}$, $D_C = \{1, 2, 3, 4\}$, $D_D = \{3, 4\}$.

b) By MRV & deg. heur.: Select D.

if $D = 3$: $D_A = \{2\}$ (−1)

$D_B = \{1, 2, 3\}$ (−0) $\Rightarrow -2$

$D_C = \{1, 2, 4\}$ (−1)

if $D = 4$: $D_A = \{2, 3\}$ (−0)

$D_B = \{1, 2, 3\}$ (−0) $\Rightarrow -1$

$D_C = \{1, 2, 3\}$ (−1) $\therefore$ select $D = 4$.

By MRV: select A.

if $A = 2$: $D_B = \{1\}$ (−2)

$D_C = \{1, 3\}$ (−1) $\Rightarrow -3$

if $A = 3$: $D_B = \{1, 2\}$ (−1)

$D_C = \{1, 2\}$ (−1) $\Rightarrow -2$ $\therefore$ select $A = 3$.

By MRV & deg. heur.: Select B.

if $B = 1$ OR $B = 2$: $D_C = \{1, 2\}$ (−0)

$\therefore$ Select $B = 1$ by default.

Finally, select C:

$C = \{1, 2\}$ $\therefore$ select $C = 1$ by default.

SOLUTION FOUND:

$A=3, B=1, C=1, D=4$.

4) New domains: $A = \{2\} / \{3\}$, $B = \{1, 2, 3\}$,
$C = \{1, 2, 3, 4\}$, $D = \{3, 4\}$

On 1st split:

$A = 2 \Rightarrow D_B = \{1\}$
$D_C = \{1, 3, 4\}$
$D_D = \{3, 4\}$

$B = 1 \Rightarrow D_C = \{1, 3, 4\}$
$D_D = \{3, 4\}$

Let $D = 3 \Rightarrow D_C = \{1, 4\}$

Let $D = 4 \Rightarrow D_C = \{1, 3\}$

$\therefore$ Solutions$_1$ = {[A=2, B=1, C=1, D=3], [A=2, B=1, C=4, D=3],
[A=2, B=1, C=1, D=4], [A=2, B=1, C=3, D=4]}

On 2nd split:

$A = 3 \Rightarrow D_B = \{1, 2\}$
$D_C = \{1, 2, 4\}$
$D_D = \{4\}$

$D = 4 \Rightarrow D_B = \{1, 2\}$
$D_C = \{1, 2\}$

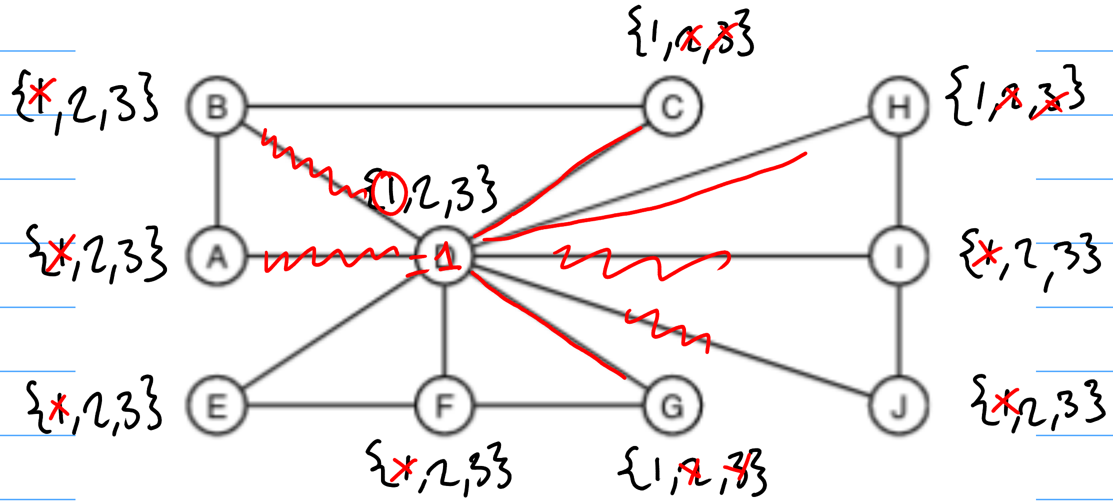
$B = 1 \Rightarrow D_C = \{1, 2\}$

$B = 2 \Rightarrow D_C = \{1, 2\}$

$\therefore$ Solutions$_2$ = {[A=3, B=1, C=1, D=4], [A=3, B=1, C=2, D=4],
[A=3, B=2, C=1, D=4], [A=3, B=2, C=2, D=4]}

$\therefore$ All solutions = solutions$_1$ $\cup$ solutions$_2$.

5) Cutset conditioning instansiates some nodes s.t. graph becomes a tree, reducing running time of backtrack alg.



UB of nodes expanded = 1? UB w/o = 6?

Yields tree: