NETHERMIND RESEARCH

# Nova-style folding with STARK-friendly primes

Albert Garreta

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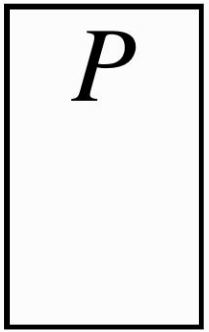


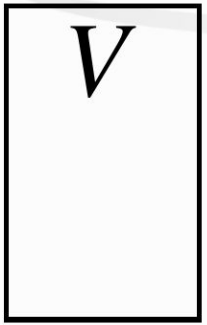




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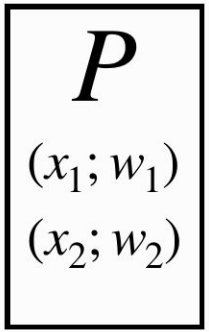


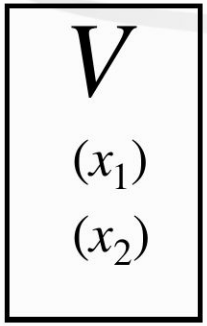




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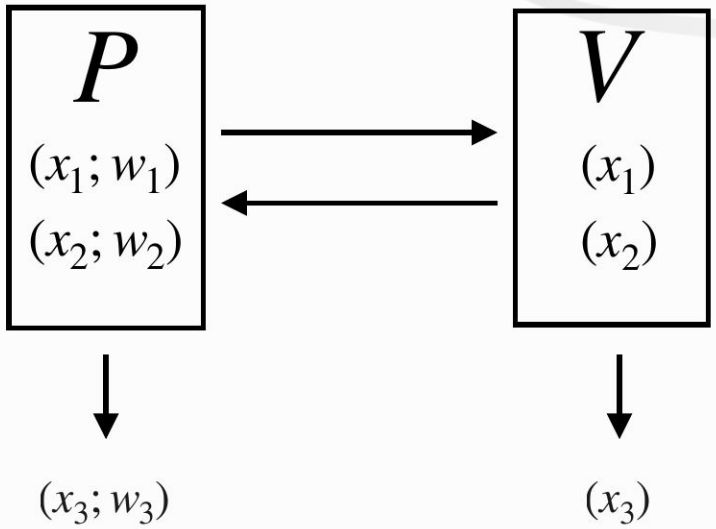




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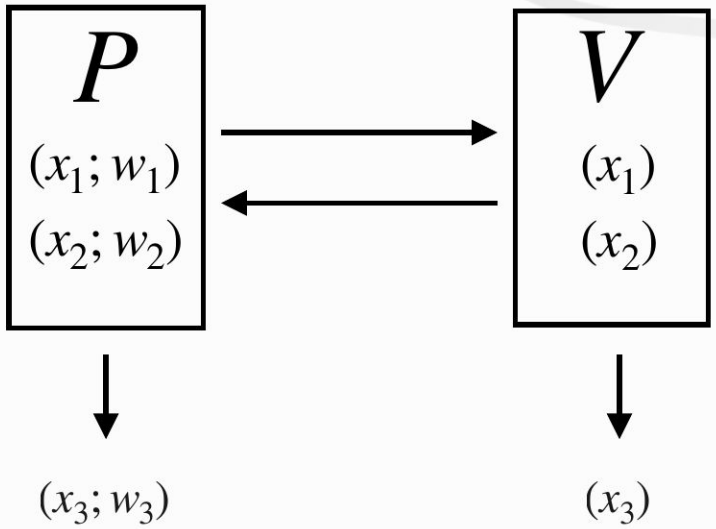




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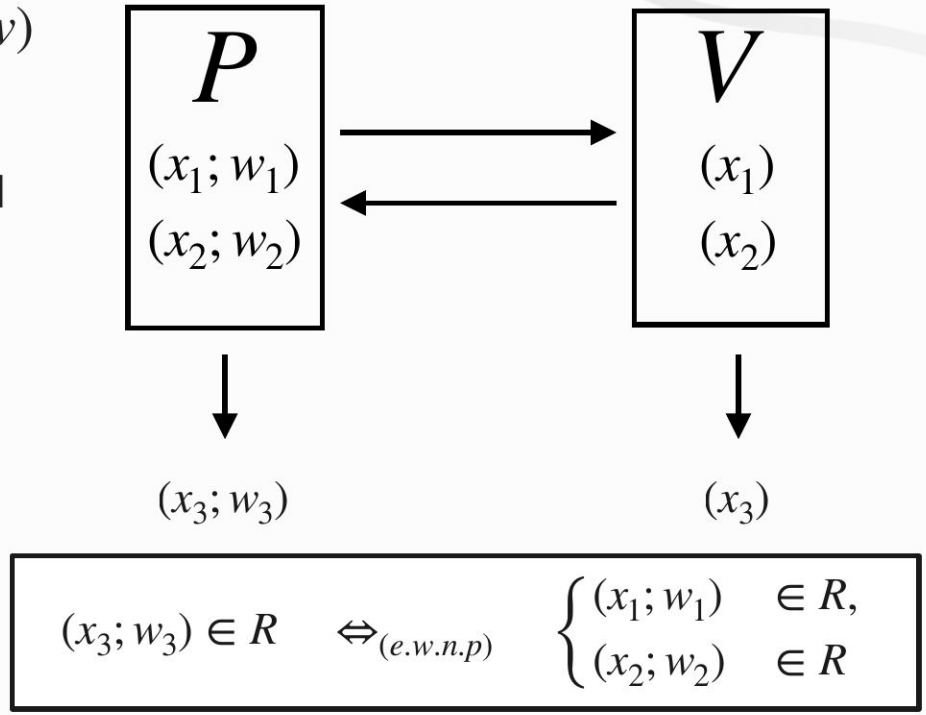




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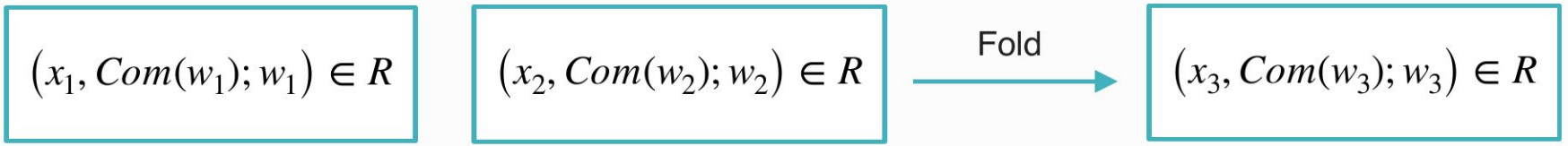




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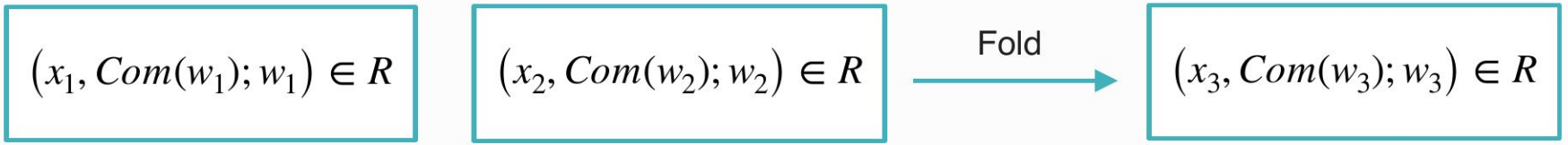


- Usually the commitment is homomorphic: $Com(w_1 + w_2) = Com(w_1) + Com(w_2)$



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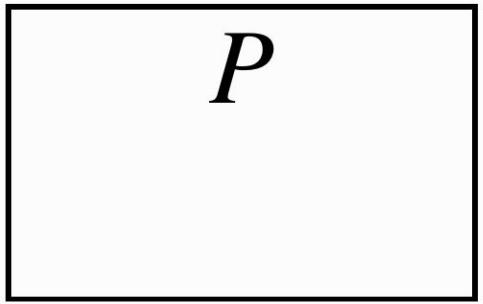


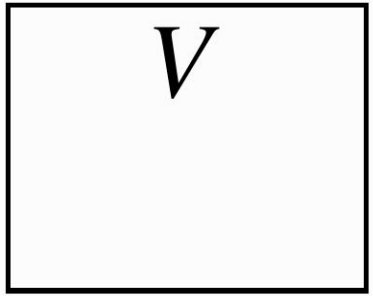

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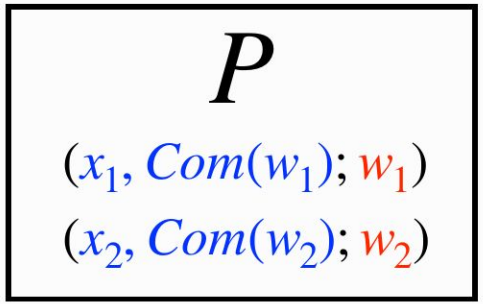


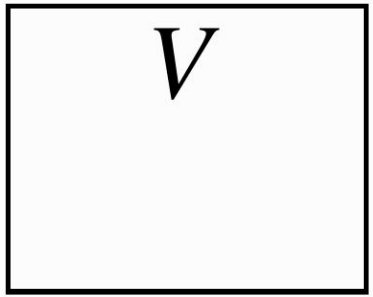




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| --- |
| $(x_1, Com(w_1); w_1)$ |
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| $V$ |
| --- |
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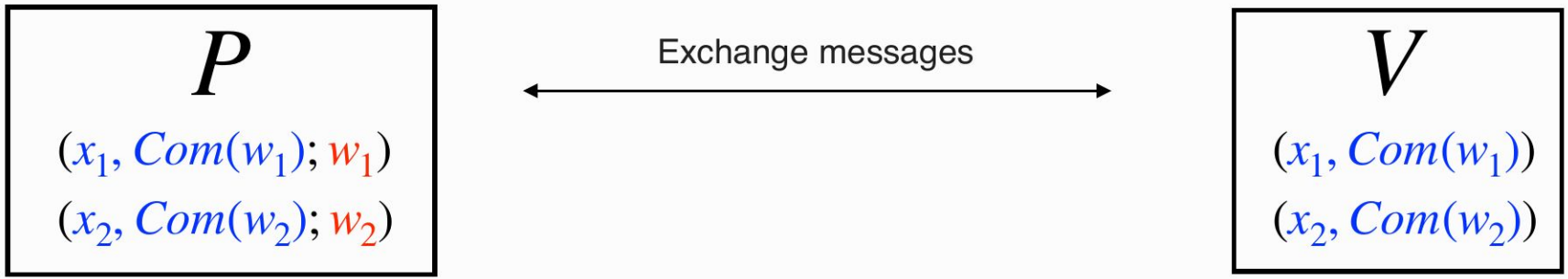




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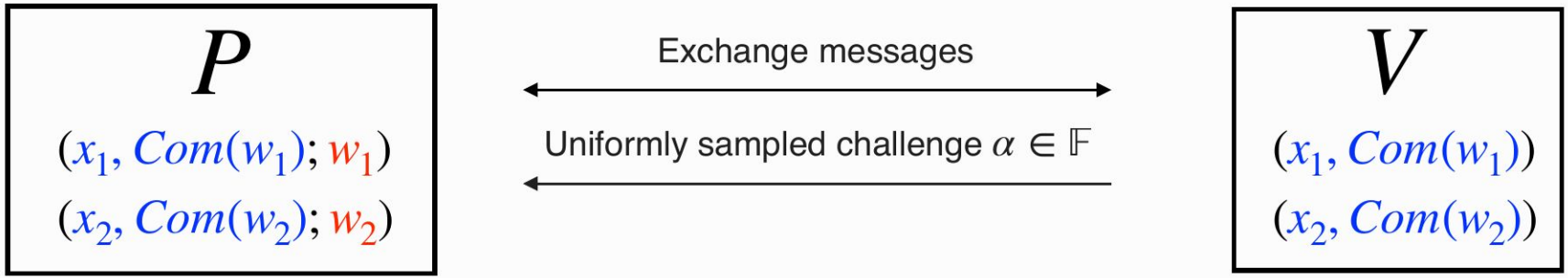




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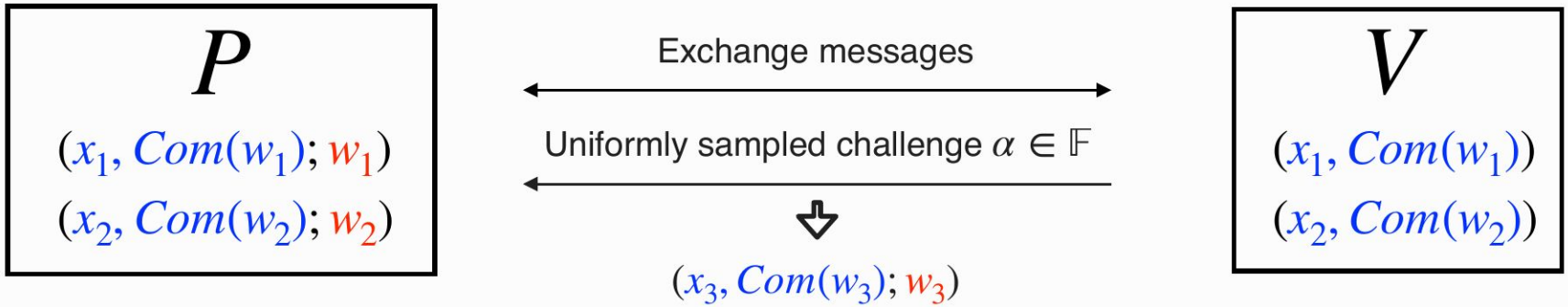




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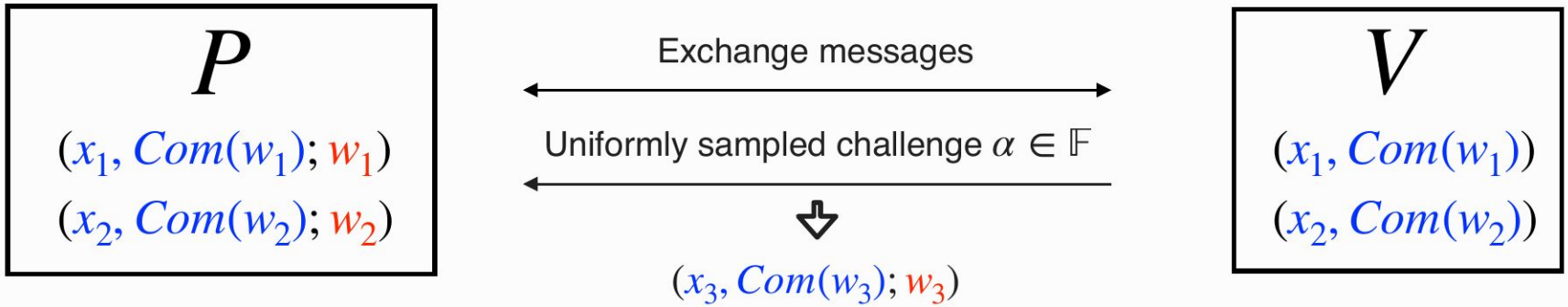


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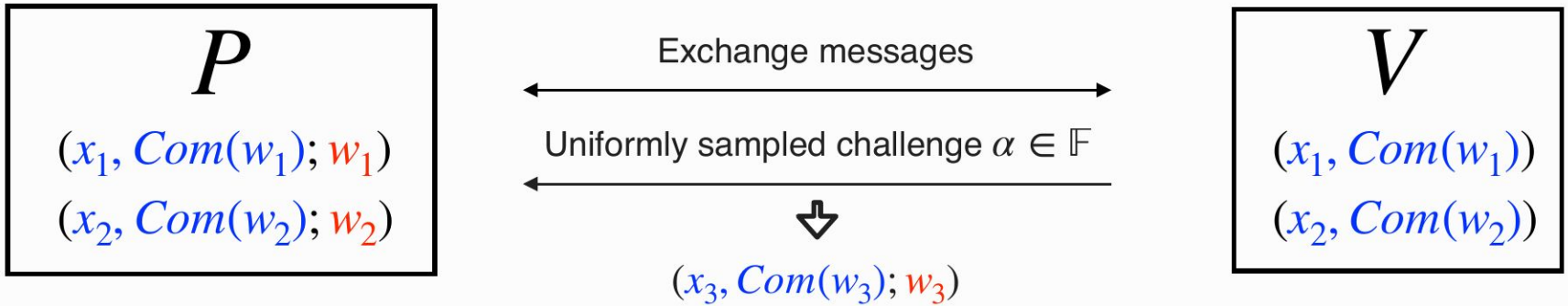


- Where $x_3 = x_1 + \alpha x_2 \qquad w_3 = w_1 + \alpha w_2$
- $V$ computes $Com(w_3)$ using that $Com(w_1) + \alpha Com(w_2) = Com(w_1 + \alpha w_2)$



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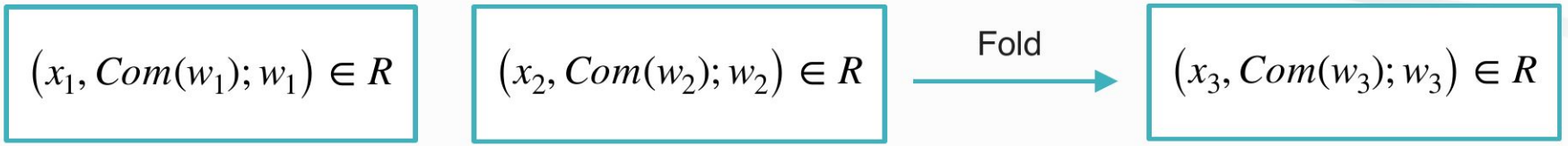


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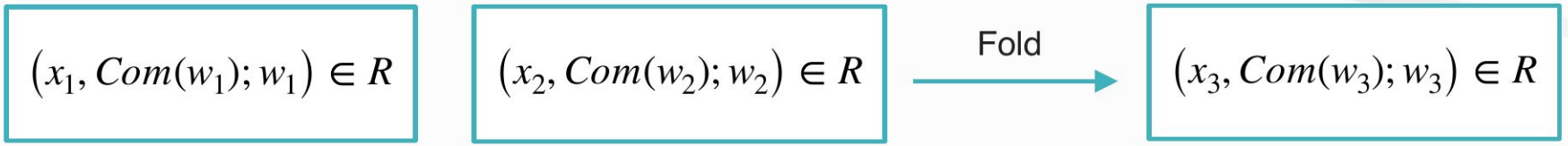


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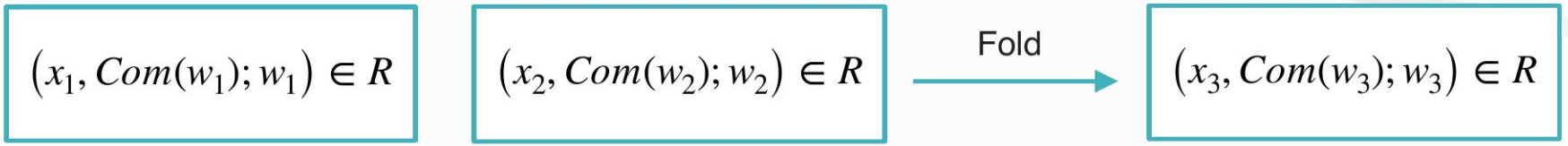


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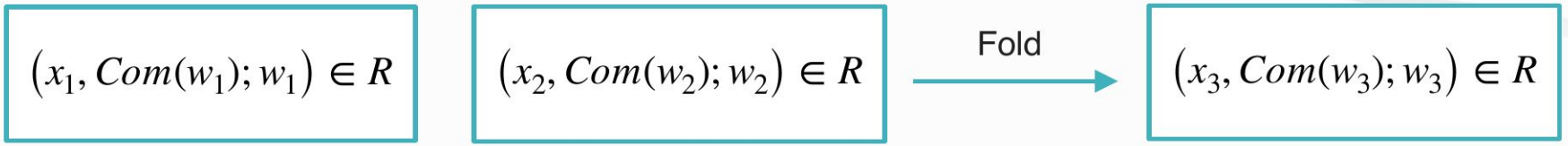


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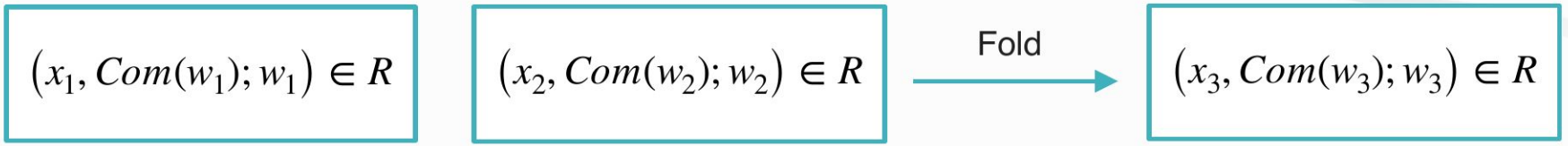


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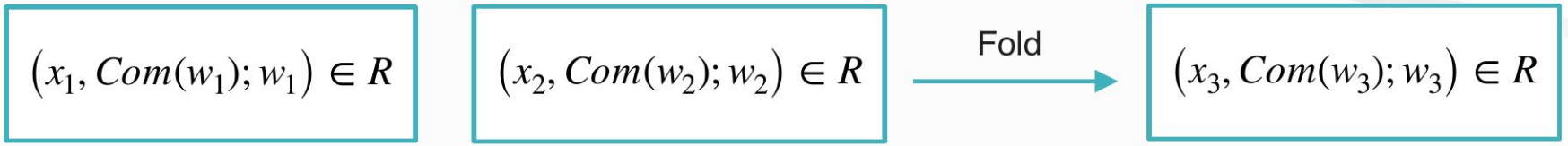


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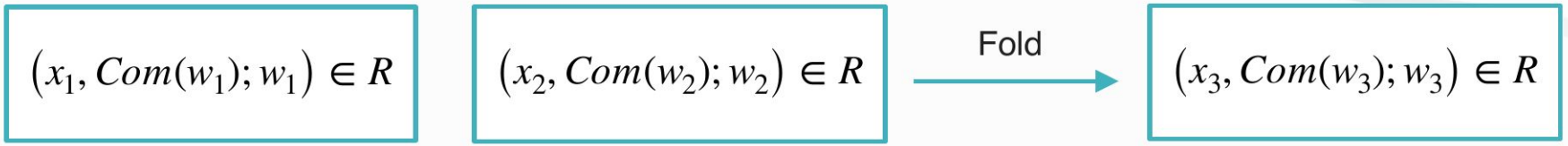


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- In principle, folding then can only be combined with KZG-based SNARKs

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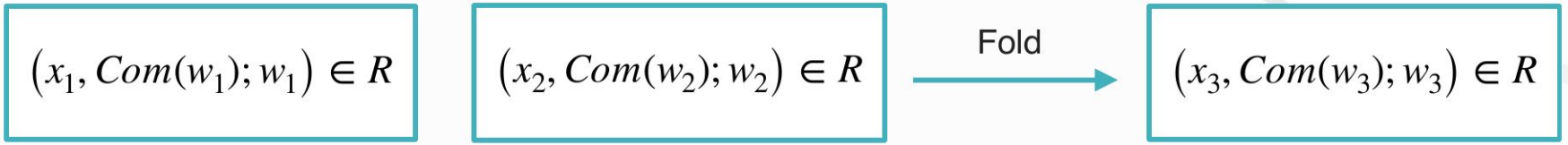


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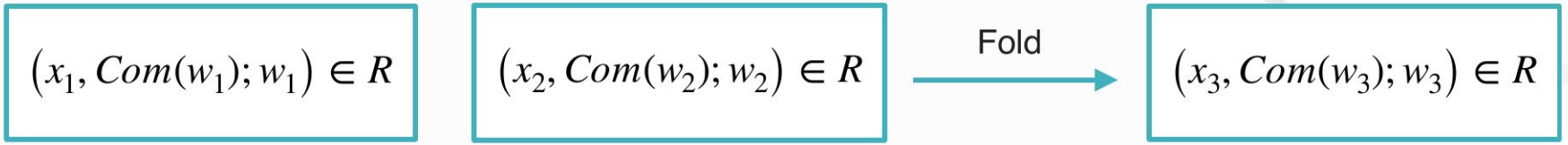


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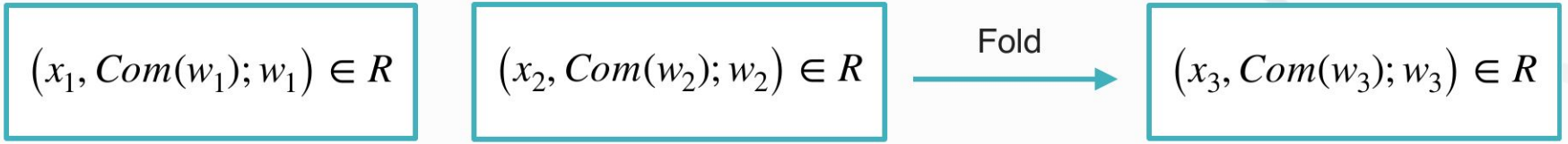


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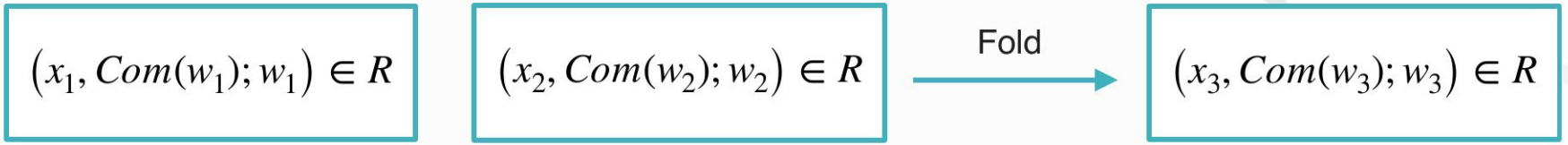


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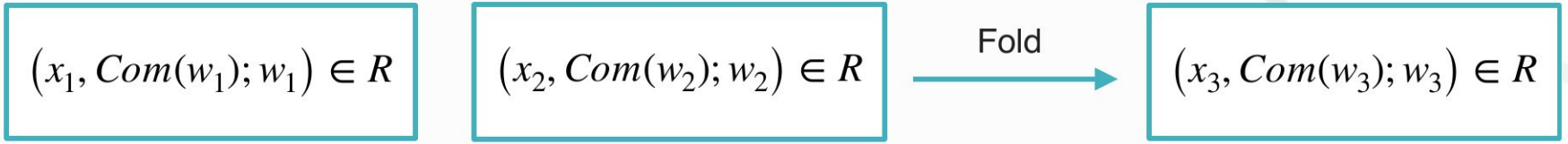


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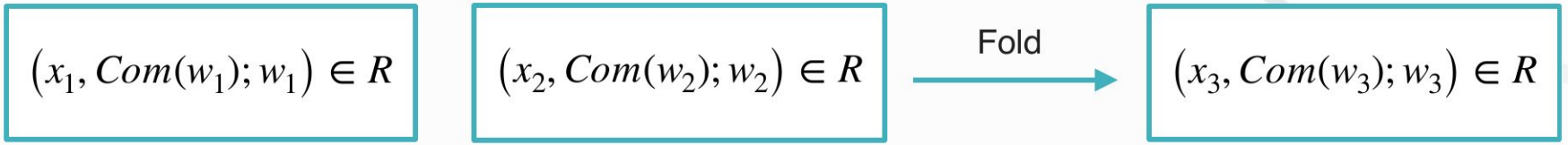


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    - Cheaper computations, since field elements are never very large.

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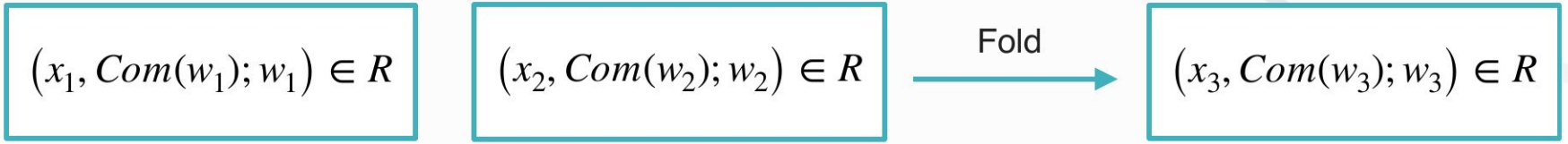


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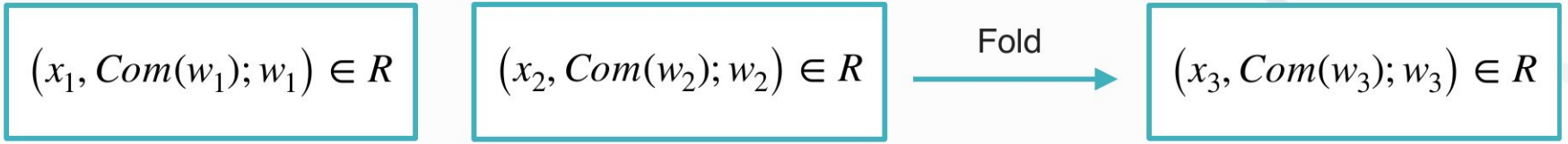


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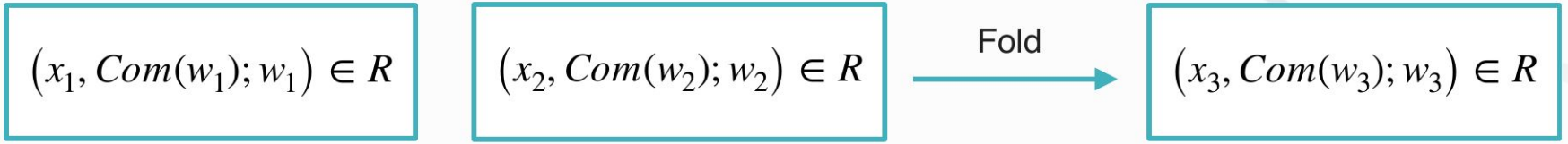


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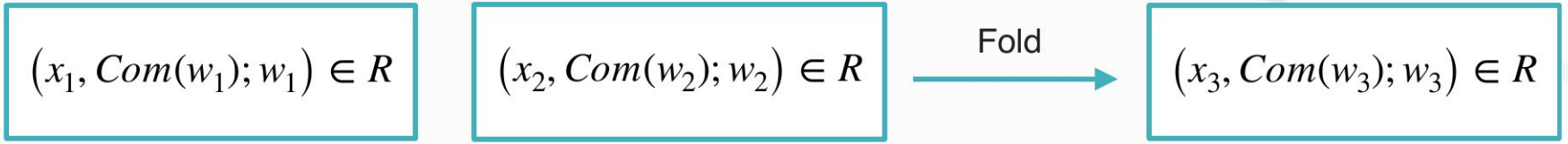


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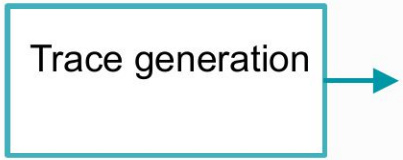




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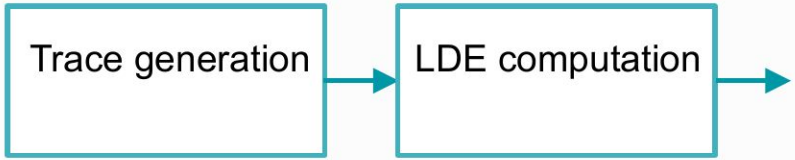




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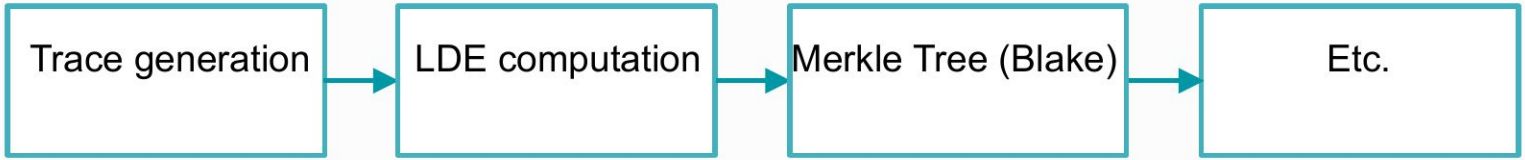




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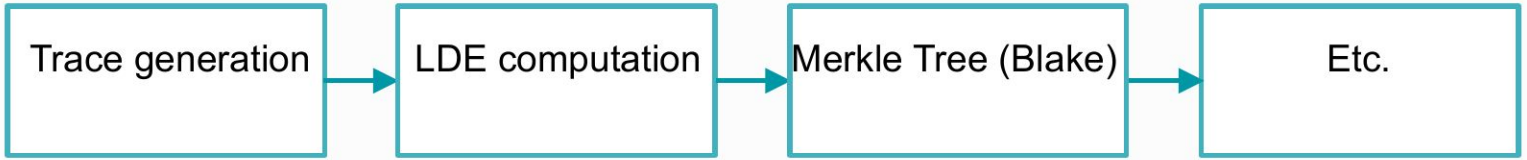




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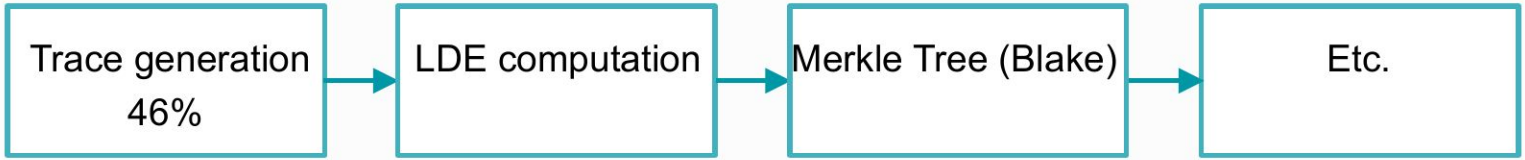




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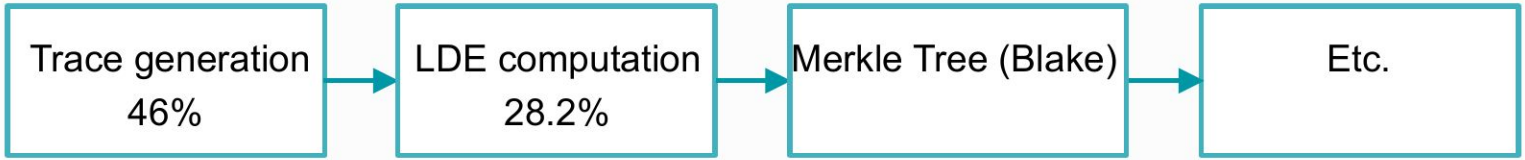




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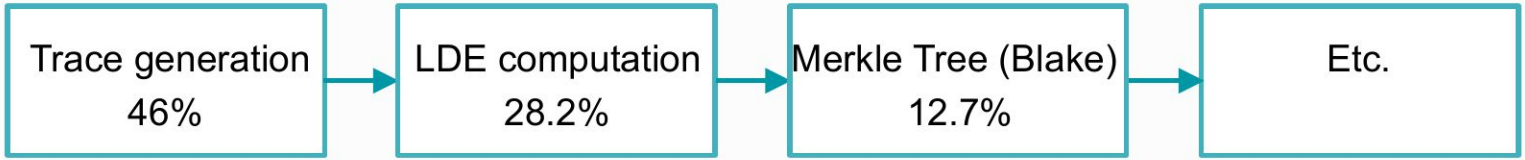




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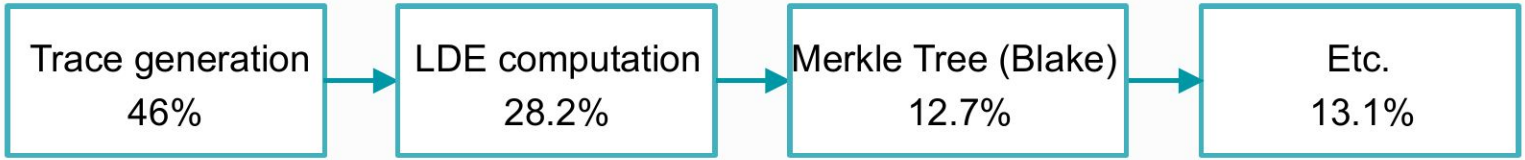




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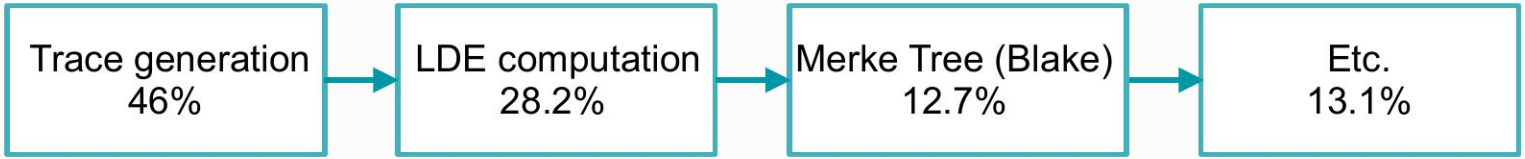




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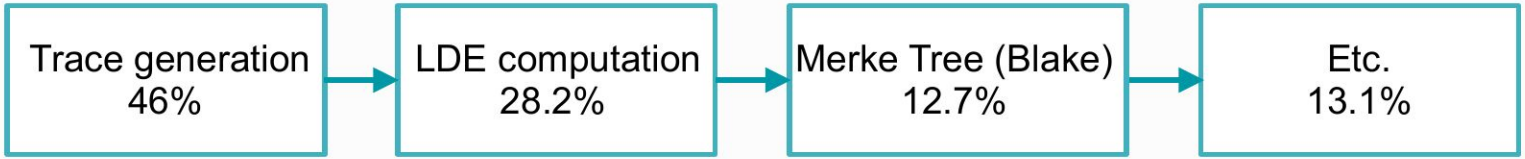


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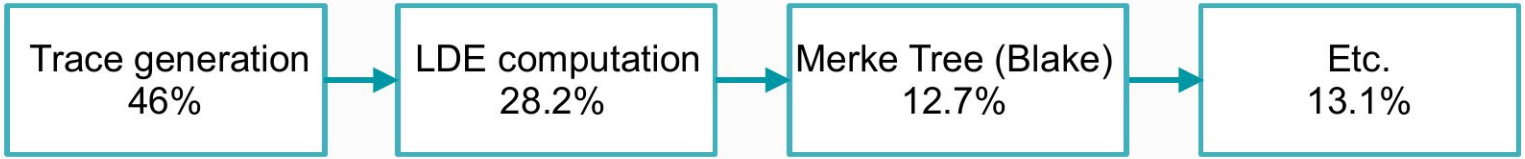


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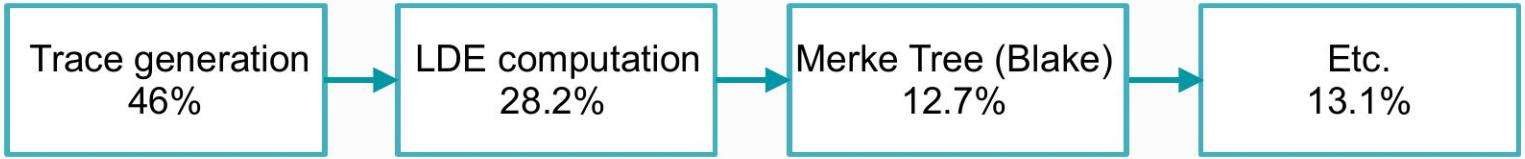


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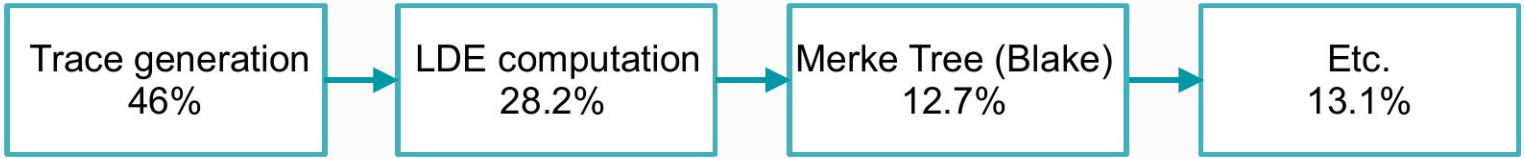


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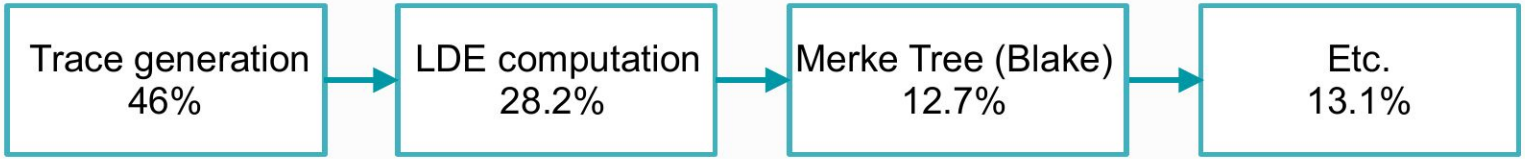


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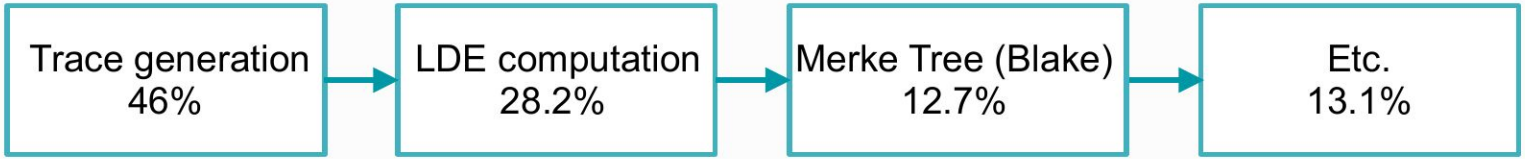


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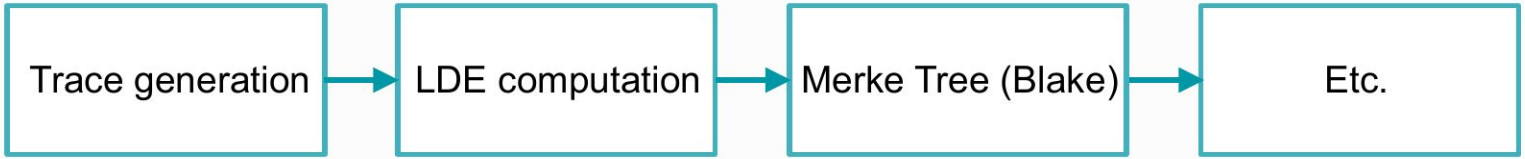




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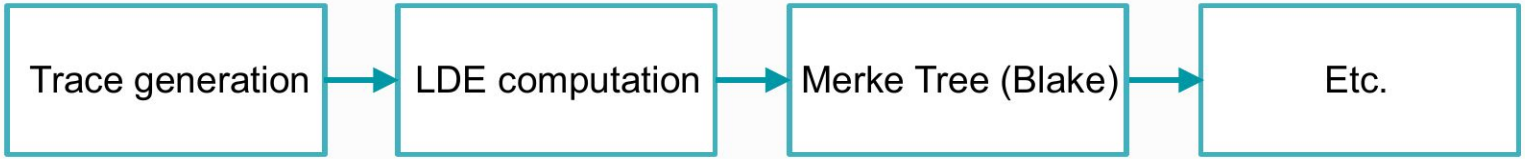


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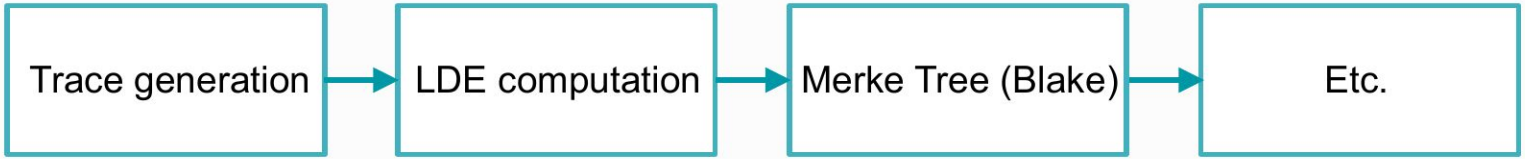


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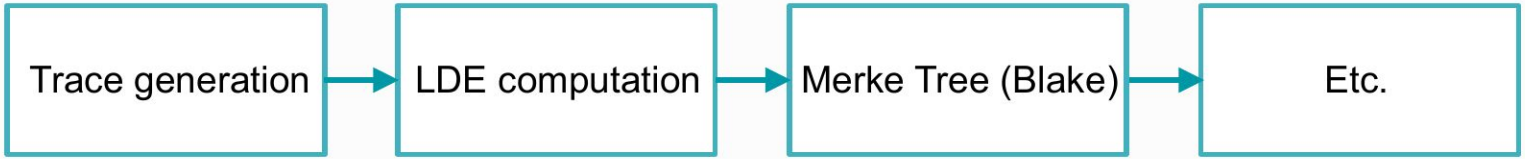


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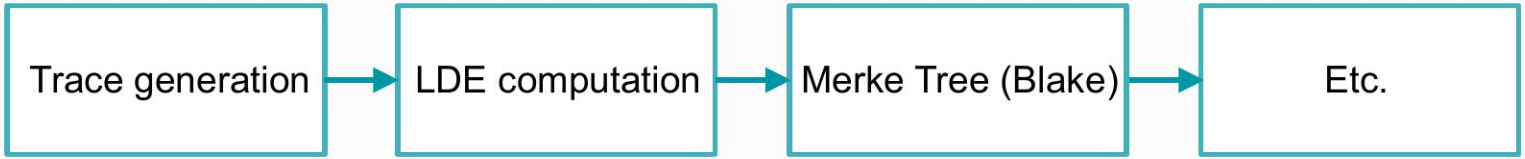


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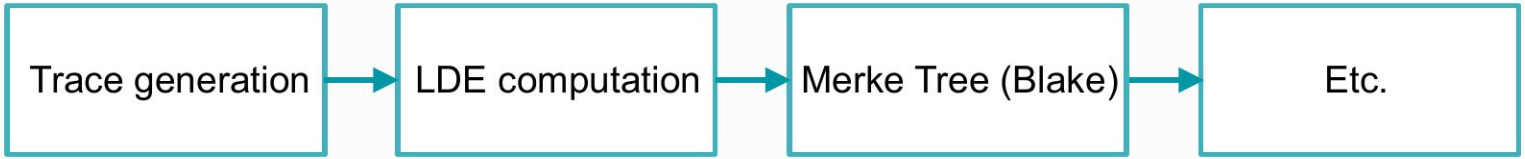


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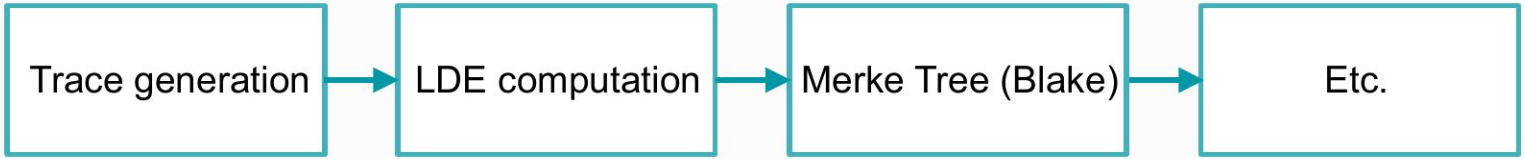


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# Framework for folding STARKs

Let $R$ be the AIR or Plonkish relation. Let $(x_1; w_1) \in R$, $(x_2; w_2) \in R$.

Framework:

1. P commits to the trace (i.e. the witness) with a homomorphic scheme $Com$:

$$(x_1, Com(w_1); w_1) \in R, \qquad (x_2, Com(w_2); w_2) \in R$$

2. Fold (somehow) in a way that the folded instance belongs to $R$ (or similar):

$$(x_3, Com(w_3); w_3) \in R$$

3. When proving a folded instance, say $(x_3, Com(w_3); w_3)$,
    1. Compute the LDE of $w_3$ and commit to it with a Merkle tree: $Mer(LDE(w_3)$
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    3. Finish the proof for $(x_3; w_3) \in R$ with your favorite STARK

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Our main result is designing such.

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| | \|v\| | Field |
|---|---|---|
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| **Merkle (Blake)** | 2^16 | 14ms |
| **Merkle (Blake)** | 2^19 | 50ms |
| **Merkle (Blake)** | 2^20 | 100ms |
| **Merkle (Poseidon)** | 2^16 | 2.5s |



Ajtai: AMD Ryzen 7 3700X 8-Core - 64 Gb RAM

Merkles: i7 12th 12700F 8-core - 32Gb RAM

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- We follow Mova, a variation of Nova where $E$ is not committed.

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- We express the property (*) as an R1CS, and fold it along with the original R1CS

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# Thanks