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a) MoM and MLE Estimation

$$f(x) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

$X = \{0.3, 0.6, 0.8, 0.9\}$

MoM Estimation

$$\int_0^1 x.f(x)\,dx = m$$

mean of given sample is 0,65

$m = 0,65$

$$= \int_0^1 \theta . x^{\theta} = \theta \frac{x^{\theta+1}}{\theta+1}\Big|_0^1$$

$$= \frac{\theta}{\theta+1}$$

$\frac{\theta}{\theta+1} = 0,65$ $\quad \theta = 1,8571$

MLE Estimation

$$f(0,3) \times f(0,6) \times f(0,8) \times f(0,9)$$

$$= \theta^4 . (0,3 \times 0,6 \times 0,8 \times 0,9)^{\theta-1}$$

$$= \theta^4 . (0,1296)^{\theta-1}$$

$$\frac{d}{d\theta} \ln(\theta^4 . (0,1296)^{\theta-1}) = 0$$

$$\frac{\ln(0,1296)\theta + 4}{\theta} = 0$$

$$\theta = 1,9576$$

b) Population Generation

Inverse Transform Method

$$f(x) = \theta x^{\theta-1}$$

$$F(x) = \int \theta x^{\theta-1} = x^{\theta}$$

$$F^{-1}(x) = x^{\left(\frac{1}{\theta}\right)}$$

now we can generate random samples using this formula

c) Experiment Simulation

- Estimations got more accurate when I increase sample size (N).
- Mean and variance of estimations fell down.
- Distribution in histograms are more normal when N increase.

I would prefer MoM estimation. Its mean is more accurate. Faster and simpler to calculate. Only its variance bit higher than MLE.