EFE ACER
21602217

Q1)

(i) FCFS

Gantt Chart:

N	N - 1	...	2	1

$0 \quad N \quad 2N-1 \quad \sum_{i=3}^{N} i \quad \sum_{i=2}^{N} i$

Average Waiting Time:

$$\frac{1}{N} \sum_{i=1}^{N} \sum_{j=i+1}^{N} j = \frac{1}{N} \sum_{i=1}^{N} \left[\sum_{j=1}^{N} j - \sum_{j=1}^{i} j \right]$$

$$= \frac{1}{N} \sum_{i=1}^{N} \left[\frac{N(N+1)}{2} - \frac{i(i+1)}{2} \right] = \frac{1}{N} \sum_{i=1}^{N} \frac{N(N+1)}{2} - \frac{1}{N} \sum_{i=1}^{N} \frac{i(i+1)}{2}$$

$$= \frac{N(N+1)}{2} - \frac{1}{2N} \left[\sum_{i=1}^{N} i^2 + \sum_{i=1}^{N} i \right]$$

$$= \frac{N(N+1)}{2} - \frac{1}{2N} \left[\frac{N(N+1)(2N+1)}{6} + \frac{N(N+1)}{2} \right]$$

$$= \frac{N(N+1)}{2} \left[\underset{(6N)}{1} - \frac{2N+1}{6N} - \underset{(3)}{\frac{1}{2N}} \right] = \frac{\cancel{N}(N+1)}{2} \left[\frac{6N - 2N - 1 - 3}{6\cancel{N}} \right]$$

$$= \frac{(N+1)\,4\,(N-1)}{12} = \frac{(N+1)(N-1)}{3} = \boxed{\frac{N^2-1}{3}}$$

EFE ACER
21602217

(ii) SJF

Gantt Chart:

1	2		N - 1	N

$0 \quad 1 \quad 2 \quad \sum_{i=1}^{N-2} i \quad \sum_{i=1}^{N-1} i$

Average Waiting Time:

$$\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{i-1} j$$

$$= \frac{1}{N} \sum_{i=1}^{N} \frac{(i-1)\,i}{2} = \frac{1}{2N} \sum_{i=1}^{N} (i^2 - i)$$

$$= \frac{1}{2N} \left[\sum_{i=1}^{N} i^2 - \sum_{i=1}^{N} i \right]$$

$$= \frac{1}{2N} \left[\frac{N(N+1)(2N+1)}{6} - \frac{N(N+1)}{2} \right]$$

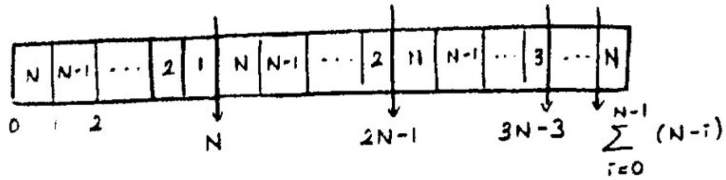
$$= \frac{N(N+1)}{2} \left[\frac{2N+1}{6N} - \underset{(3)}{\frac{1}{2N}} \right] = \frac{\cancel{N}(N+1)}{\cancel{2}} \cdot \frac{\cancel{2}(N-1)}{6\cancel{N}}$$

$$= \frac{(N+1)(N-1)}{6} = \boxed{\frac{N^2-1}{6}}$$

EFE ACER
21602217

(iii) RR (q = 1 time unit)

Gantt Chart :



Average waiting Time :

Process 1 waits for : $N-1$

Process 2 waits for : $N-2,\ N-1$

Process 3 waits for : $N-3,\ N-1,\ N-2$

⋮

Process N-1 waits for : $N-(N-1),\ N-1,\ N-2,\ \dots,\ N-(N-1)+1$

Process N waits for : $N-N,\ N-1,\ N-2,\ \dots,\ N-N+2,\ N-N+1$

Based on this observation :

$$AWT = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{i}(N-j)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{j=1}^{i}N - \sum_{j=1}^{i}j\right] = \frac{1}{N}\sum_{i=1}^{N}\left[Ni - \frac{i(i+1)}{2}\right]$$

$$= \frac{1}{N}\left[N\sum_{i=1}^{N}i - \frac{1}{2}\left(\sum_{i=1}^{N}i^2 + \sum_{i=1}^{N}i\right)\right]$$

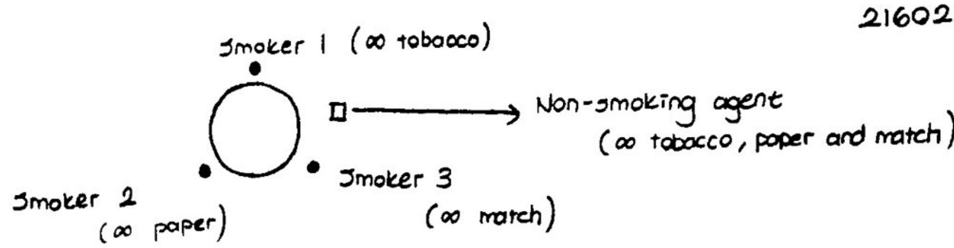
$$= \frac{1}{N}\left[\frac{N^2(N+1)}{2} - \frac{1}{2}\left(\frac{N(N+1)(2N+1)}{6} + \frac{N(N+1)}{2}\right)\right]$$

$$= \frac{1}{\cancel{N}}\left[\frac{\cancel{N}(N+1)}{2}\left(\underset{(6)}{N} - \frac{2N+1}{6} - \underset{(3)}{\frac{1}{2}}\right)\right] = \frac{N+1}{2}\left[\frac{6N-2N-1-3}{6}\right]$$

$$= \frac{(N+1)\,4(N-1)}{12} = \frac{(N+1)(N-1)}{3} = \boxed{\frac{N^2-1}{3}}$$

EFE ACER
21602217

Q2)



(The Code below is a pseudo-code)

```
// Shared Variables
Semaphore tableEmpty = 1; // binary semaphore for Agent
Semaphore S1 = 0; //counting semaphore for Smoker 1
Semaphore S2 = 0; //counting semaphore for Smoker 2
Semaphore S3 = 0; //counting semaphore for Smoker 3
```

Nonsmoking Agent Code

```
do {
    wait(tableEmpty);
    items = chooseTwoRandomItems(tobacco, paper, match);
    if (items are paper and match) {
        signal(S1); // there are available resources for Smoker 1
    } else if (items are tobacco and match) {
        signal(S2); // there are available resources for Smoker 2
    } else {
        signal(S3); // there are available resources for smoker 3
    }
} while(true);
```

Smoker 1's code:

```
do {
    wait(S1);
    signal(tableEmpty);
    smoke();
} while (true);
```

other smokers' codes are the same when S1 is changed to S2 and S3.

⇒ signal on Si means that the items on the table are retrieved by the ith smoker

⇒ wait on Si means that the one set of items in ith smoker's stock is used for smoking. If such a set does not exist, the smoker waits.

EFE ACER
21602217

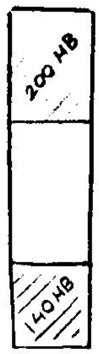
Q3)

Virtual Address :

p0	p1	p2	d (offset)
10	8	8	10

a) $2^d = 2^{10} = \boxed{1\text{ KB}}$

b)



$\frac{200 \cdot 2^{\cancel{20}\,10}}{2^{\cancel{10}}} = 200 \cdot 2^{10}$ third level page table entries needed

$\frac{140 \cdot 2^{\cancel{20}\,10}}{2^{\cancel{10}}} = 140 \cdot 2^{10}$ third level page table entries needed

In total : $340 \cdot 2^{10}$ third level page table entries needed

$\Rightarrow$ A third level page table can index at most $2^{p2} = 2^8$ entries (logical pages)

$\Downarrow$

$\frac{340 \cdot 2^{10}}{2^8} = 340 \cdot 2^2$ third level page tables needed

$\Rightarrow$ A second level page table can index at most $2^{p1} = 2^8$ entries (third level page tables)

$\left\lceil \frac{200 \cdot 2^{\cancel{10}}}{2^8 \cdot 2^{\cancel{8}\,6}} \right\rceil + \left\lceil \frac{140 \cdot 2^{\cancel{10}}}{2^8 \cdot 2^{\cancel{8}\,6}} \right\rceil = \left\lceil \frac{200}{64} \right\rceil + \left\lceil \frac{140}{64} \right\rceil = 4 + 3 = 7$ second level page tables needed

$\Rightarrow$ A first level page table can index at most $2^{p0} = 2^{10}$ entries (second level page tables)

$\Downarrow$

1 first level page table suffices

Thus :

$\boxed{7 \text{ second level page tables are used}}$

$340 \cdot 2^2 = \boxed{1360 \text{ third level page tables are used}}$

EFE ACER
21602217

Q4)

For all parts, "x" indicates a page fault and "✓" indicates a hit.

a) Algorithm: FIFO

Reference String:

x	x	x	✓	✓	x	x	x	✓	x	x	x	x	✓	✓	x	x	✓	x
3	5	4	3	5	6	2	5	2	3	4	2	5	4	2	7	4	7	3

Frames:

0	~~3~~	~~6~~ ~~3~~ ~~5~~ 3
1	~~5~~	~~2~~ ~~4~~ 7
2	~~4~~	~~5~~ ~~2~~ 4

Number of Page Faults: $\boxed{13}$

b) Algorithm: LRU

Reference String:

x	x	x	✓	✓	x	x	✓	✓	x	x	✓	x	✓	✓	x	✓	✓	x
3	5	4	3	5	6	2	5	2	3	4	2	5	4	2	7	4	7	3

Frames:

0	~~3~~	~~2~~ 3
1	~~5~~	4
2	~~4~~	~~6~~ ~~3~~ ~~5~~ 7

Number of Page Faults: $\boxed{10}$

c) Algorithm: OPT

Reference String:

x	x	x	✓	✓	x	x	✓	✓	✓	x	✓	✓	✓	✓	x	✓	✓	x
3	5	4	3	5	6	2	5	2	3	4	2	5	4	2	7	4	7	3

Frames:

0	~~3~~	~~4~~ 3
1	5	
2	~~4~~	~~6~~ ~~2~~ 7

(smallest page number is removed in case of tie)

Number of Page Faults: $\boxed{8}$

EFE ACER
21602217

d) **Algorithm:** R bit

x x x ✓ | ✓ x x ✓ | ✓ x x ✓ | x ✓ x x | ✓ ✓ x

Reference String: 3, 5, 4, 3, | 5, 6, 2, 5, | 2, 3, 4, 2, | 5, 4, 2, 7 | 4, 7, 3

Frames:

0	~~3~~	~~6~~ 4
1	~~5~~	~~3~~ ~~2~~ 7
2	~~4~~	~~2~~ ~~5~~ 3

Number of Page Faults: $\boxed{11}$

e) **Algorithm:** Second - chance

x x x ✓ ✓ x | x x ✓ x x x | x ✓ ✓ x x ✓ x

Reference String: 3, 5, 4, 3, 5, 6, | 2, 5, 2, 3, 4, 2, | 5, 4, 2, 7, 4, 7, 3

Frames:

0	~~3~~	~~6~~ ~~3~~ ~~5~~ 3
1	~~5~~	~~2~~ ~~4~~ 7
2	~~4~~	~~5~~ ~~2~~ 4

FIFO list:
Head (oldest)
↓
$3^1 - 5^1 - 4^1$
$5^0 - 4^0 - 6^1$

$4^0 - 6^0 - 2^1$
$6^0 - 2^1 - 5^1$
$2^1 - 5^1 - 3^1$
$5^0 - 3^0 - 4^1$
$3^0 - 4^1 - 2^1$

$4^0 - 2^0 - 5^1$
$2^0 - 5^1 - 7^1$
$5^1 - 7^1 - 4^1$

$7^0 - 4^0 - 3^1$

Number of Page Faults: $\boxed{13}$

f) **Algorithm:** LFU

x x x ✓ ✓ x x ✓ ✓ ✓ x x ✓ x x x x x x

Reference String: 3, 5, 4, 3, 5, 6, 2, 5, 2, 3, 4, 2, 5, 4, 2, 7, 4, 7, 3

Frames:

0	~~3~~	~~2~~ ~~7~~ 3
1	5	
2	~~4~~	~~6~~ ~~2~~ ~~4~~ ~~2~~ ~~4~~ ~~7~~ 4

Page Number	Reference Count
2	||||
3	||||
4	||||
5	||||
6	|
7	||

Number of Page Faults: $\boxed{13}$

(smallest page number is removed in case of tie)

EFE ACER
21602217

Q5)

a) $\begin{pmatrix}\text{Number of}\\ \text{disk blocks}\end{pmatrix} = \dfrac{(\text{Disk size})}{(\text{Block size})} = \dfrac{32\text{ GB}}{4\text{ KB}} = \dfrac{2^{35}}{2^{12}} = \boxed{2^{23}\text{ disk blocks}}$

b) Bit vector associates 1 bit per disk block

$\Downarrow$

$\begin{pmatrix}\text{Bit vector}\\ \text{size}\end{pmatrix} = 2^{23}\text{ bits} = \dfrac{2^{\cancel{23}\,20}}{2^{3}}\text{ Bytes} = \dfrac{2^{\cancel{20}\,8}}{2^{12}}\text{ disk blocks} = \boxed{2^{8} = 256\text{ disk blocks}}$

c) 1 FCB is associated with each file (an FCB is also associated with the root directory but it is disregarded for simpler calculations)

$\Downarrow$

$\begin{pmatrix}\text{Size of}\\ \text{FCBs}\end{pmatrix} = \begin{pmatrix}\text{Number of}\\ \text{Files}\end{pmatrix}\begin{pmatrix}\text{FCB}\\ \text{size}\end{pmatrix} = (200\ 000)\,2^{8}\text{ Bytes} = \dfrac{(200\ 000)\,\cancel{2^{8}}}{2^{\cancel{12}\,4}}\text{ disk blocks}$

$= \dfrac{200\ 000}{16}\text{ disk blocks} = \boxed{12\ 500\text{ disk blocks}}$

d) Each file corresponds to 1 (root) directory entry

$\Downarrow$

$\begin{pmatrix}\text{Number of}\\ \text{directory entries}\end{pmatrix} = \begin{pmatrix}\text{Number of}\\ \text{Files}\end{pmatrix} = 200\ 000$

$\Downarrow$

$\begin{pmatrix}\text{Size of the}\\ \text{directory}\\ \text{information}\end{pmatrix} = \begin{pmatrix}\text{Number of}\\ \text{directory entries}\end{pmatrix}\cdot\begin{pmatrix}\text{Directory}\\ \text{Entry size}\end{pmatrix} = (200\ 000)\cdot 2^{8}\text{ Bytes}$

$= \dfrac{(200\ 000)\,2^{8}}{2^{\cancel{12}\,4}}\text{ disk blocks} = \boxed{12\ 500\text{ disk blocks}}$

e) $\begin{pmatrix}\text{Free disk}\\ \text{space}\end{pmatrix} = \begin{pmatrix}\text{Disk}\\ \text{size}\end{pmatrix} - \left[\underbrace{\begin{pmatrix}\text{Bitvector}\\ \text{size}\end{pmatrix}}_{\text{part b}} + \underbrace{\begin{pmatrix}\text{Size of}\\ \text{FCBs}\end{pmatrix}}_{\text{part c}} + \underbrace{\begin{pmatrix}\text{Size of the}\\ \text{directory}\\ \text{information}\end{pmatrix}}_{\text{part d}} + \underbrace{\begin{pmatrix}\text{Size of}\\ \text{Files}\end{pmatrix}}_{\approx\begin{pmatrix}\text{Number}\\ \text{of files}\end{pmatrix}\begin{pmatrix}\text{Average}\\ \text{File size}\end{pmatrix} = 200\ 000\cdot 3 = 600\ 000\text{ disk blocks}}\right]$

$\approx 2^{23} - \left[256 + 12\ 500 + 12\ 500 + 600\ 000\right]\text{ disk blocks}$

$= 2^{23} - 625\ 256 = \boxed{7\ 763\ 352\text{ disk blocks}}$

and $(7\ 763\ 352)\,2^{12}\text{ Bytes} \approx 2^{4.88}\text{ GB} \approx \boxed{29.5\text{ GB}}$

EFE ACER
21602217

f) $$\begin{pmatrix}\text{portion of}\\\text{metadata in}\\\text{the used disk}\\\text{space}\end{pmatrix} = \frac{\begin{pmatrix}\text{Bitvector}\\\text{size}\end{pmatrix} + \begin{pmatrix}\text{size of}\\\text{FCBs}\end{pmatrix} + \begin{pmatrix}\text{size of the}\\\text{directory}\\\text{information}\end{pmatrix}}{(\text{used disk space})}$$

$$\underset{(\text{part e})}{\overset{=}{\downarrow}} \frac{25\,256}{625\,256} \approx \boxed{4.04\,\%}$$

EFE ACER
21602217

Q6)

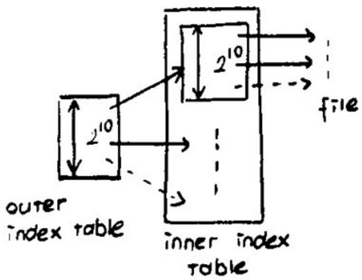
a) FAT stores a pointer for each disk block

$$\begin{pmatrix}\text{FAT}\\ \text{size}\end{pmatrix} = \underbrace{\begin{pmatrix}\text{Number of}\\ \text{Disk Blocks}\end{pmatrix}}_{(\text{Disk size})/(\text{Block size})} \cdot \begin{pmatrix}\text{Entry}\\ \text{size}\end{pmatrix} = \frac{64\text{ GB}}{4\text{ KB}} \cdot 4\text{ Bytes} = \frac{2^{\cancel{38}\,26}}{2^{\cancel{14}}}\text{ Bytes}$$

$$= \frac{2^{26}}{2^{12}}\text{ disk blocks} = \boxed{2^{14}\text{ disk blocks}}$$

b) An index block can map $\dfrac{(\text{Block size})}{(\text{Pointer size})} = \dfrac{4\text{KB}}{4\text{ Bytes}} = 2^{10}$ entries

Two-level index structure looks like:



$$\begin{pmatrix}\text{Maximum}\\ \text{File size}\end{pmatrix} = \underbrace{2^{10} \cdot 2^{10}}_{\left(\substack{\text{Number of}\\ \text{blocks that can}\\ \text{be mapped}}\right)} \cdot \underbrace{4\text{ KB}}_{\left(\substack{\text{Block}\\ \text{size}}\right)} = \boxed{4\text{ GB}}$$

Index blocks required for:

(i) File A of size 1 MB :

$$\underbrace{\begin{pmatrix}\text{Number of inner}\\ \text{index tables required}\end{pmatrix}}_{\left\lceil \frac{(\text{File size})}{(\text{Block size})(\text{Number of Entries})} \right\rceil} + \left\lceil \frac{(\text{Number of inner index tables})}{(\text{Number of Entries})} \right\rceil$$

$$+ \begin{pmatrix}\text{Number of outer}\\ \text{index tables required}\end{pmatrix}$$

$$= \left\lceil \frac{1\text{ MB} \Rightarrow 2^{20}}{\underbrace{4\text{ KB}}_{2^{12}} \cdot 2^{10}} \right\rceil \to 1 + \left\lceil \frac{1}{2^{10}} \right\rceil \to 1 = \underset{\text{inner}}{1} + \underset{\text{outer}}{1} = \boxed{2\text{ index blocks}}$$

Similarly:

(ii) File B of size 10 MB :

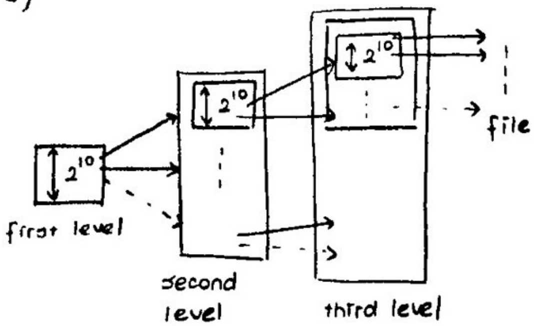
$$\left\lceil \frac{10\text{ MB} \Rightarrow 10 \cdot 2^{20}}{\underbrace{4\text{ KB} \cdot 2^{10}}_{2^{22}}} \right\rceil \to 3 + \left\lceil \frac{3}{2^{10}} \right\rceil \to 1 = \underset{\text{inner}}{3} + \underset{\text{outer}}{1} = \boxed{4\text{ index blocks}}$$

(iii) File C of size 100 MB :

$$\left\lceil \frac{\cancel{100}^{\,25} \cdot \cancel{2^{20}}}{\cancel{4} \cdot \cancel{2^{20}}} \right\rceil \to 25 + \left\lceil \frac{25}{2^{10}} \right\rceil \to 1 = \underset{\text{inner}}{25} + \underset{\text{outer}}{1} = \boxed{26\text{ index blocks}}$$

EFE ACER
21602217

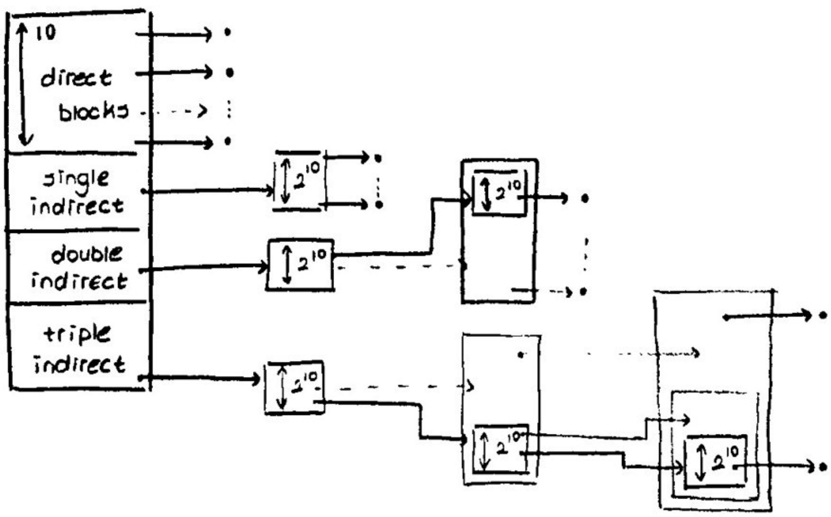
c) Three-level index structure looks like:



$$\begin{pmatrix}\text{Maximum}\\ \text{File size}\end{pmatrix} = \underbrace{2^{10}\cdot 2^{10}\cdot 2^{10}}_{\begin{pmatrix}\text{Number of blocks}\\ \text{that can be}\\ \text{mapped}\end{pmatrix}} \cdot \underbrace{4KB}_{(\text{Block size})}$$

$$= \boxed{4\ TB}$$

d) Mixed index structure looks like:



$$\begin{pmatrix}\text{Maximum}\\ \text{File size}\end{pmatrix} = \underbrace{10}_{\text{mapping of direct blocks}} \cdot 4KB + \underbrace{2^{10}}_{\text{mapping of single indirect}} \cdot 4KB + \underbrace{2^{10}\cdot 2^{10}}_{\text{mapping of double indirect}} \cdot 4KB + \underbrace{2^{10}\cdot 2^{10}\cdot 2^{10}}_{\text{mapping of triple indirect}} \cdot 4KB$$

$$= \boxed{40\ KB + 4\ MB + 4\ GB + 4TB \approx 4\ TB}$$

Disk accesses required to access a byte at:

i) offset 0: $\underbrace{1}_{\text{pointer retrieval from direct blocks}} + \underbrace{1}_{\text{data retrieval}} = \boxed{2 \text{ disk accesses}}$

ii) offset $\underbrace{1\ 000\ 000}$:
Corresponds to single indirect
($10^6 \approx 1\ MB$)

$\underbrace{1}_{\text{index block address}} + \underbrace{1}_{\text{pointer}} + \underbrace{1}_{\text{data}} = \boxed{3 \text{ disk accesses}}$

iii) offset $\underbrace{100\ 000\ 000}$:
Corresponds to double indirect
($10^8 \approx 0.1\ GB$)

$\underbrace{1}_{\text{outer index block address}} + \underbrace{1}_{\text{inner index block address}} + \underbrace{1}_{\text{pointer}} + \underbrace{1}_{\text{data}} = \boxed{4 \text{ disk accesses}}$