EEE 391
Basics of Signals and Systems
Fall 2019–2020
Homework 2
due: 23 December 2019, Monday by 17:00 in the Homework Box

1) The difference equation of a system is given as:

$$y[n] = x[n] + 3x[n-1] + 7x[n-2] + x[n-3]$$

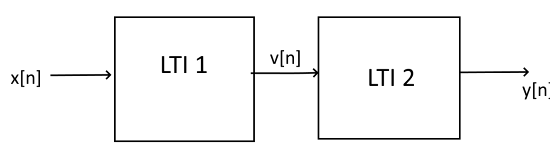
a) Show that this system is linear time invariant (LTI).
b) Make a complete signal flow diagram. Signal flow should be from left to right.
c) Obtain an expression for the frequency response function of the system in complex form.
d) Determine the output of the system when $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$.
e) Determine the output of the system when the input is $x[n] = u[n] - u[n-3]$. Compare your result with your result in c).

2) Suppose that we have two LTI systems, namely, System 1 and System 2, whose system functions are given as:

$$H_1(z) = 1 - 2z^{-1} \quad \text{and} \quad H_2(z) = 1 - z^{-2}$$

The sequence $x[n] = u[n] - u[n-2]$ is given as input to the cascaded arrangement.
a) If $x[n]$ first passes through System 1, followed by System 2, what will be the output?
b) If $x[n]$ first passes through System 2, followed by System 1, what will be the output?
c) Compare your results in parts a) and b).
d) Show that your conclusion in c) is true for any pair of LTI systems. (Please provide a rigorous mathematical proof). Is the result generalizable to more than two systems all of which are LTI?

3) In this question, you are given the following cascaded system:



a) If LTI-1 is a 5-point moving averager and LTI-2 is a first difference system, determine the frequency response function of the system in complex form.

b) Sketch the magnitude response and phase response functions of the individual systems and the overall cascade system for $-\pi \leq \hat{\omega} \leq \pi$.

c) Find $v[n]$ and $y[n]$ if $x[n] = 0.5^n u[n] - 0.1^n u[n-1]$ for $-\infty < n < \infty$.

4) *Forward and inverse z tranformation and the system function:*

a) Find the z-transform of the following functions:
i) $x[n] = c^n u[n]$, where c is an arbitrary constant.
ii) $x[n] = 3\sin(0.5\pi n)u[n]$ where $u[n]$ is the unit-step sequence.

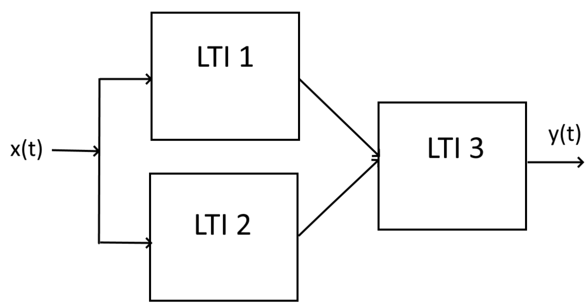
b) Find the inverse z-transform of the following functions:
i) $\dfrac{z}{z-a}$ ii) $\dfrac{z^{-2}}{1-0.9z^{-1}}$

c) You are given the following system functions:

$$H_1(z) = 1 - 3z^{-1} + 3z^{-2} - z^{-3} \qquad H_2(z) = \frac{1+0.75z^{-1}}{1-0.25z^{-2}}$$

i) Determine the poles and zeros of $H_1(z)$ and $H_2(z)$.
ii) Sketch the pole-zero diagrams of $H_1(z)$ and $H_2(z)$. Label the pole-zero locations and the axes clearly.
iii) Determine the impulse response functions ($h_1[n]$ and $h_2[n]$) of the corresponding systems.

5) In this question, you are given the following cascaded system:



In each of the three parts below, determine the impulse response of the overall system and answer the following: Is the overall system causal? Is it stable? Explain.

a) $h_1(t) = \delta(t+2)$, $h_2(t) = \delta(t-2)$, and $h_3(t) = u(t-2)$.
b) $h_1(t) = \delta(t+2)$, $h_2(t) = \delta(t-2)$, and $h_3(t) = u(t)$.
c) $h_1(t) = u(t+2)$, $h_2(t) = u(t-2)$, and $h_3(t) = \delta(t-1)$.

6) You are given the following two signals:

$$x_1(t) = \begin{cases} t+3 & \text{for} \quad t \in [-3, 0) \\ -t+3 & \text{for} \quad t \in [0, 3] \\ 0 & \text{otherwise} \end{cases} \qquad x_2(t) = \begin{cases} 1 & \text{for} \quad t \in [-3, 3] \\ 0 & \text{otherwise} \end{cases}$$

a) Sketch $x_1(t)$ and $x_2(t)$ over the interval $-3 \le t \le 3$.
b) Calculate $x_1(t) * x_2(t)$, $x_1(t) * x_1(t)$, and $x_2(t) * x_2(t-3)$.