EFE ACER
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EEE 391 - Homework 2

Q1: $y[n] = x[n] + 3x[n-1] + 7x[n-2] + x[n-3]$

a: Linearity:

$$x_1[n] \mapsto y_1[n] = x_1[n] + 3x_1[n-1] + 7x_1[n-2] + x_1[n-3]$$

$$x_2[n] \mapsto y_2[n] = x_2[n] + 3x_2[n-1] + 7x_2[n-2] + x_2[n-3]$$

$$w[n] = \alpha y_1[n] + \beta y_2[n] = \alpha x_1[n] + \beta x_2[n] + 3\alpha x_1[n-1] + 3\beta x_2[n-1] + 7\alpha x_1[n-2] + 7\beta x_2[n-2] + \alpha x_1[n-3] + \beta x_2[n-3]$$

for $\forall \alpha, \beta, n$

$$x[n] = \alpha x_1[n] + \beta x_2[n] \mapsto y[n]$$

$$y[n] = (\alpha x_1[n] + \beta x_2[n]) + 3(\alpha x_1[n-1] + \beta x_2[n-1]) + 7(\alpha x_1[n-2] + \beta x_2[n-2]) + (\alpha x_1[n-3] + \beta x_2[n-3])$$

$$= \alpha x_1[n] + \beta x_2[n] + 3\alpha x_1[n-1] + 3\beta x_2[n-1] + 7\alpha x_1[n-2] + 7\beta x_2[n-2] + \alpha x_1[n-3] + \beta x_2[n-3]$$

$$\Downarrow$$

$$y[n] = w[n] \checkmark$$

$$\Updownarrow$$

$$\alpha x_1[n] + \beta x_2[n] \mapsto \alpha y_1[n] + \beta y_2[n]$$

Hence $y[n]$ is Linear

Time Invariance:

$$x[n] \mapsto y[n]$$

$$x[n-n_0] \mapsto w[n] = x[n-n_0] + 3x[(n-n_0)-1] + 7x[(n-n_0)-2] + x[(n-n_0)-3]$$

$$y[n-n_0] = x[n-n_0] + 3x[(n-n_0)-1] + 7x[(n-n_0)-2] + x[(n-n_0)-3] \quad \text{for } \forall n_0, n$$

$$\Downarrow$$

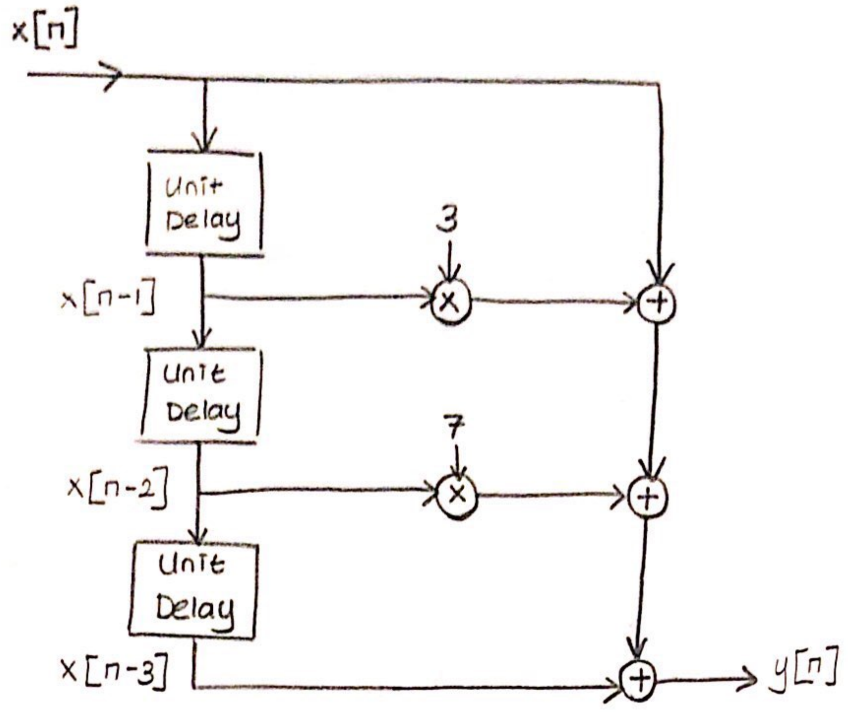
$$w[n] = y[n-n_0]$$

Hence $y[n]$ is Time Invariant

Thus, $y[n]$ is Linear Time Invariant (LTI).

EA

b: Signal Flow Diagram:



c:

$$x[n] = Ae^{J\phi}e^{J\hat{\omega}n} \longmapsto y[n] = Ae^{J\phi}e^{J\hat{\omega}n} + 3Ae^{J\phi}e^{J\hat{\omega}(n-1)} + 7Ae^{J\phi}e^{J\hat{\omega}(n-2)} + Ae^{J\phi}e^{J\hat{\omega}(n-3)}$$

$$y[n] = \underbrace{Ae^{J\phi}e^{J\hat{\omega}n}}_{x[n]} \cdot \Big(\underbrace{1 + 3e^{-J\hat{\omega}} + 7e^{-J2\hat{\omega}} + e^{-J3\hat{\omega}}}_{H(e^{J\hat{\omega}})} \Big)$$

$$H(e^{J\hat{\omega}}) = 1 + 3e^{-J\hat{\omega}} + 7e^{-J2\hat{\omega}} + e^{-J3\hat{\omega}} \;//$$

$$= 1 + 3\left[\cos(-\hat{\omega}) + J\sin(-\hat{\omega})\right] + 7\left[\cos(-2\hat{\omega}) + J\sin(-2\hat{\omega})\right] + \cos(-3\hat{\omega}) + J\sin(-3\hat{\omega})$$

converting to complex form

$$= 1 + 3\cos(\hat{\omega}) + 7\cos(2\hat{\omega}) + \cos(3\hat{\omega}) - 3J\sin(\hat{\omega}) - 7J\sin(2\hat{\omega}) - J\sin(3\hat{\omega})$$

$$\left(\begin{aligned}\cos(-\theta) &= \cos\theta \\ \sin(-\theta) &= -\sin\theta\end{aligned}\right)$$

In Complex form:

$$\boxed{H(e^{J\hat{\omega}}) = \left[1 + 3\cos(\hat{\omega}) + 7\cos(2\hat{\omega}) + \cos(3\hat{\omega})\right] + J\left[-3\sin(\hat{\omega}) - 7\sin(2\hat{\omega}) - \sin(3\hat{\omega})\right]}$$

EA

d: $y[n]$ is a FIR filter with $b_k = \{1, 3, 7, 1\}$,

hence the unit impulse response is:

$$h[n] = \begin{cases} 1 & n = 0, 3 \\ 3 & n = 1 \\ 7 & n = 2 \\ 0 & \text{otherwise} \end{cases} = \delta[n] + 3\delta[n-1] + 7\delta[n-2] + \delta[n-3]$$

Since $y[n]$ is LTI:

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \left(\delta[n] + \delta[n-1] + \delta[n-2]\right) * \left(\delta[n] + 3\delta[n-1] + 7\delta[n-2] + \delta[n-3]\right) \\ &= \delta[n] + 3\delta[n-1] + 7\delta[n-2] + \delta[n-3] \\ &\quad + \delta[n-1] + 3\delta[n-2] + 7\delta[n-3] + \delta[n-4] \\ &\quad + \delta[n-2] + 3\delta[n-3] + 7\delta[n-4] + \delta[n-5] \end{aligned}$$

$(x[n] * \delta[n-n_0] = x[n-n_0])$

$$\boxed{y[n] = \delta[n] + 4\delta[n-1] + 11\delta[n-2] + 11\delta[n-3] + 8\delta[n-4] + \delta[n-5]}$$

e: Again:

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \left(u[n] - u[n-3]\right) * \left(\delta[n] + 3\delta[n-1] + 7\delta[n-2] + \delta[n-3]\right) \\ &= u[n] + 3u[n-1] + 7u[n-2] + u[n-3] \\ &\quad - u[n-3] - 3u[n-4] - 7u[n-5] - u[n-6] \end{aligned}$$

$(x[n] * \delta[n-n_0] = x[n-n_0])$

$$\boxed{y[n] = u[n] + 3u[n-1] + 7u[n-2] - 3u[n-4] - 7u[n-5] - u[n-6]}$$

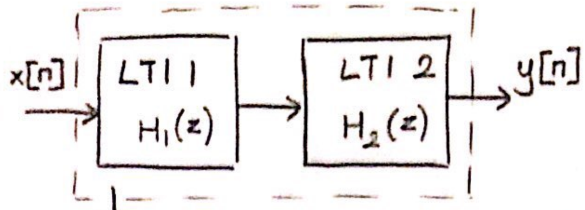
The inputs $x[n]$'s are the same for both part d and e:

$$\begin{aligned} u[n] &= \delta[n] + \delta[n-1] + \delta[n-2] + \cancel{\delta[n-3]} + \cancel{\delta[n-4]} + \dots \\ -u[n-3] &= \qquad\qquad\qquad\qquad\qquad - \cancel{\delta[n-3]} - \cancel{\delta[n-4]} + \dots \\ \hline & \delta[n] + \delta[n-1] + \delta[n-2] \end{aligned}$$

Hence, the outputs $y[n]$'s are also the same in parts d and e.

Q2:

a:



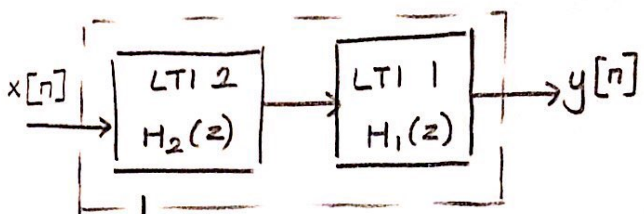
$$H(z) = H_1(z).H_2(z) = (1-2z^{-1})(1-z^{-2})$$
$$= \underbrace{1 - 2z^{-1} - z^{-2} + 2z^{-3}}$$

$$y[n] = x[n] - 2x[n-1] - x[n-2] + 2x[n-3]$$

$$(x[n] = u[n] - u[n-2]) \rightarrow y[n] = u[n] - u[n-2] - 2u[n-1] + 2u[n-3]$$
$$- u[n-2] + u[n-4] + 2u[n-3] - 2u[n-5]$$

$$\boxed{y[n] = u[n] - 2u[n-1] - 2u[n-2] + 4u[n-3] + u[n-4] - 2u[n-5]}$$

b:



$$H(z) = H_2(z).H_1(z) = (1-z^{-2})(1-2z^{-1})$$
$$= \underbrace{1 - 2z^{-1} - z^{-2} + 2z^{-3}}$$

$$y[n] = x[n] - 2x[n-1] - x[n-2] + 2x[n-3]$$

$$(x[n] = u[n] - u[n-2]) \downarrow$$

$$\boxed{y[n] = u[n] - 2u[n-1] - 2u[n-2] + 4u[n-3] + u[n-4] - 2u[n-5]}$$

c: The results in parts a and b are the same, which implies that the two LTI systems can be cascaded in any order.

d:

$$x[n] \longrightarrow \boxed{\begin{matrix}\text{LTI } 1 \\ H_1(z),\ h_1[n]\end{matrix}} \xrightarrow{w[n]} \boxed{\begin{matrix}\text{LTI } 2 \\ H_2(z),\ h_2[n]\end{matrix}} \longrightarrow y[n]$$

we know that in time domain;

$w[n] = x[n] * h_1[n]$ and $y[n] = w[n] * h_2[n]$

which is the same as: $y[n] = x[n] * \underbrace{h_1[n] * h_2[n]}$

$= x[n] * h[n]$, where $h[n] = h_1[n] * h_2[n]$

In z-domain, we have:

$$y[n] = x[n] * h_1[n] * h_2[n] \overset{z}{\longleftrightarrow} Y(z) = X(z) . \underbrace{H_1(z) . H_2(z)}$$

$$= X(z) \cdot H(z), \text{ where } H(z) = H_1(z) . H_2(z)$$

Since multiplication is a commutative operator, we can write:

$$H(z) = H_1(z) . H_2(z) = H_2(z) . H_1(z)$$

$$\Downarrow$$

$$Y(z) = X(z)\, H_2(z) . H_1(z) \overset{z^{-1}}{\longleftrightarrow} y[n] = x[n] * h_2[n] * h_1[n]$$

Which implies $h[n] = h_1[n] * h_2[n] = h_2[n] * h_1[n]$, in other words convolution is also commutative and any two LTI systems can be cascaded in either order to obtain the same overall response.

The logic can be generalized to more than two LTI systems:

$$x[n] \rightarrow \boxed{\text{LTI } 1} \rightarrow \boxed{\text{LTI } 2} \cdots \rightarrow \boxed{- - -} \cdots \rightarrow \boxed{\text{LTI } N} \rightarrow y[n]$$

$$y[n] = x[n] * h_1[n] * \cdots * h_N[n] \overset{z}{\longleftrightarrow} Y(z) = X(z) . \underbrace{H_1(z) . \ldots . H_2(z)}$$

This multiplication will give the same result regardless of the H_is ordering

$\Downarrow$

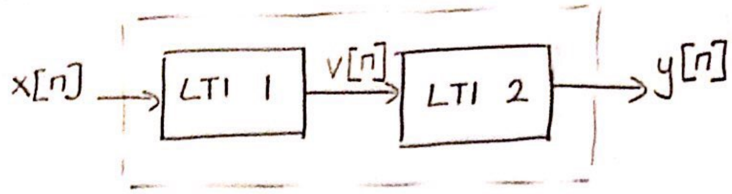
(Multiplication is associative)

$\Downarrow$

(Convolution is also associative)

Cascade order does not matter.

Q3:



a:

LTI 1 is a 5-point moving averager:

$$v[n] = \sum_{k=0}^{4} \frac{1}{5} x[n-k] \iff h_1[n] = \frac{1}{5}\delta[n] + \frac{1}{5}\delta[n-1] + \frac{1}{5}\delta[n-2] + \frac{1}{5}\delta[n-3] + \frac{1}{5}\delta[n-4]$$

LTI 2 is a first difference filter:

$$y[n] = v[n] - v[n-1] \iff h_2[n] = \delta[n] - \delta[n-1]$$

Then: $$y[n] = x[n] * \underbrace{h_1[n] * h_2[n]}_{h[n]} \overset{z}{\longleftrightarrow} Y(z) = X(z) \cdot \underbrace{H_1(z) \cdot H_2(z)}_{H(z)}$$

$$h_1[n] \overset{z}{\longleftrightarrow} \frac{1}{5}\left(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}\right) = H_1(z)$$

$$h_2[n] \overset{z}{\longleftrightarrow} 1 - z^{-1} = H_2(z)$$

$$H(z) = H_1(z) \cdot H_2(z) = \frac{1}{5}\left(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}\right)\left(1 - z^{-1}\right)$$

$$= \frac{1}{5}\left(1 + \cancel{z^{-1}} + \cancel{z^{-2}} + \cancel{z^{-3}} + \cancel{z^{-4}} - \cancel{z^{-1}} - \cancel{z^{-2}} - \cancel{z^{-3}} - \cancel{z^{-4}} - z^{-5}\right)$$

$$H(z) = \frac{1}{5} - \frac{1}{5} z^{-5} \overset{z^{-1}}{\longleftrightarrow} h[n] = \frac{1}{5}\delta[n] - \frac{1}{5}\delta[n-5]$$

The overall system is: $$y[n] = \frac{1}{5} x[n] - \frac{1}{5} x[n-5]$$

$$\Updownarrow$$

with $b_k = \{ 1/5, 0, 0, 0, 0, -1/5 \}$

Then, the frequency response is:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \frac{1}{5} - \frac{1}{5} e^{-j5\hat{\omega}} \; //$$

$H(e^{j\hat{\omega}})$ in complex form

$$= \frac{1}{5} - \frac{1}{5}\left[\cos(-5\hat{\omega}) + j\sin(-5\hat{\omega})\right] = \boxed{\left[\frac{1}{5} - \frac{1}{5}\cos(5\hat{\omega})\right] + j\left[-\sin(5\hat{\omega})\right]}$$

(convert to complex form)

$\left(\begin{array}{l}\cos(-\theta) = \cos\theta \\ \sin(-\theta) = -\sin\theta\end{array}\right)$

b: For the overall cascade system:

$$H(e^{j\hat{\omega}}) = \frac{1}{5} - \frac{1}{5}e^{-j5\hat{\omega}} = e^{-j\frac{5}{2}\hat{\omega}}\left(\frac{1}{5}e^{j\frac{5}{2}\hat{\omega}} - \frac{1}{5}e^{-j\frac{5}{2}\hat{\omega}}\right)$$

(factor out $e^{-j\frac{5}{2}\hat{\omega}}$, since b_k's are asymmetric)

$$= e^{-j\frac{5}{2}\hat{\omega}} \cdot \frac{1}{5}\left(\frac{e^{j\frac{5}{2}\hat{\omega}} - e^{-j\frac{5}{2}\hat{\omega}}}{2j}\right) \cdot 2j$$

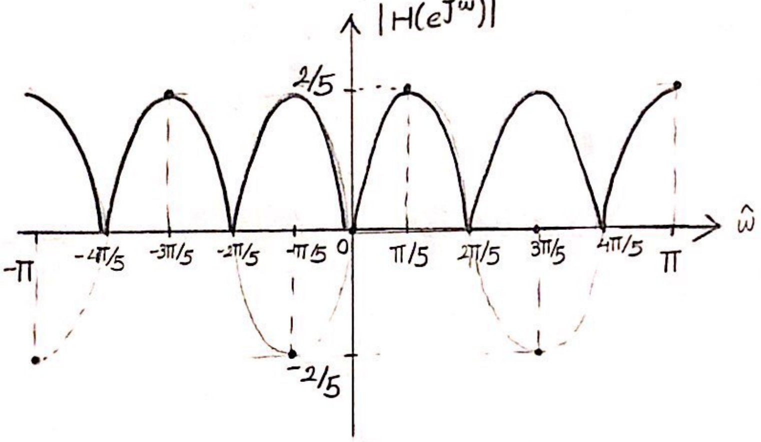
$$= e^{-j\frac{5}{2}\hat{\omega}} \cdot \frac{2}{5} \cdot e^{j\frac{\pi}{2}} \cdot \sin\left(\frac{5}{2}\hat{\omega}\right)$$

(inverse Euler's)

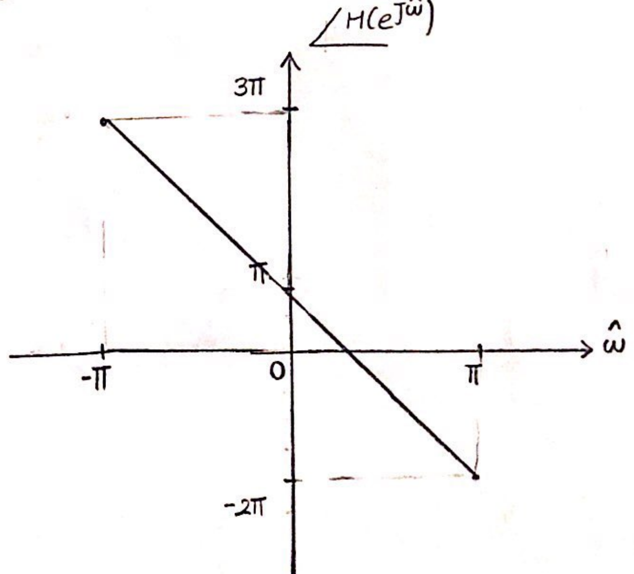
$$H(e^{j\hat{\omega}}) = \underbrace{\frac{2}{5}\sin\left(\frac{5}{2}\hat{\omega}\right)}_{|H(e^{j\hat{\omega}})| = \left|\frac{2}{5}\sin\left(\frac{5}{2}\hat{\omega}\right)\right|} \cdot e^{\overbrace{j\left(\frac{\pi}{2} - \frac{5}{2}\hat{\omega}\right)}}$$

$$\angle H(e^{j\hat{\omega}}) = \frac{\pi}{2} - \frac{5}{2}\hat{\omega}$$

Magnitude Response (Overall Cascade system):



Phase Response (Overall Cascade system):



For LTI system 1:

$$b_k = \{1/5, 1/5, 1/5, 1/5, 1/5\} \Rightarrow H(e^{j\hat{\omega}}) = \frac{1}{5}\left(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}\right)$$

$(M=4)$

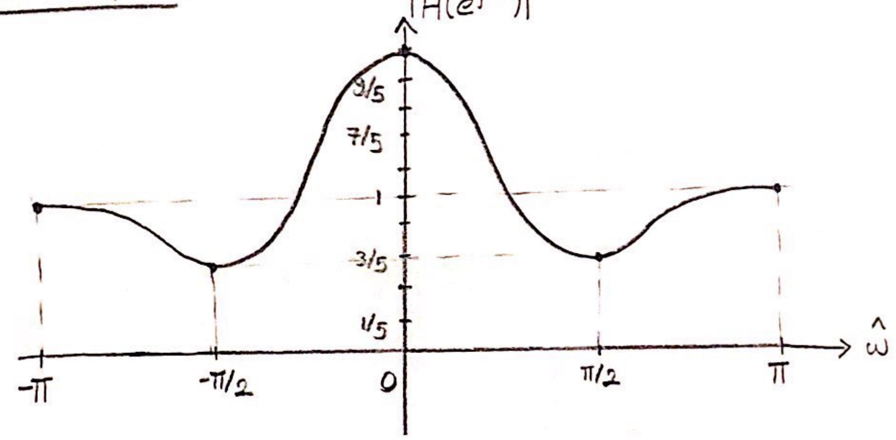
$$= \frac{1}{5} e^{-j2\hat{\omega}}\left(e^{j2\hat{\omega}} + e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right)$$

(factor out $e^{-j2\hat{\omega}}$, since b_k's are symmetric)

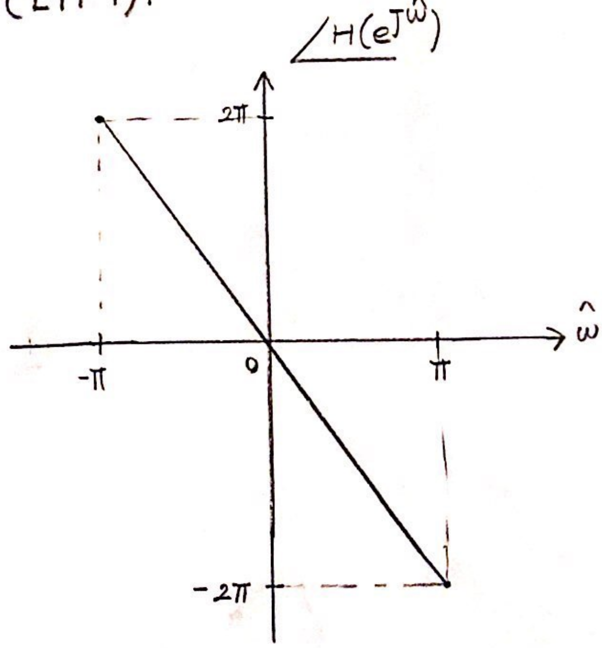
$$= \underbrace{e^{-j2\hat{\omega}}}_{\angle H(e^{j\hat{\omega}}) = -2\hat{\omega}}\left(\underbrace{\frac{2}{5}\cos(\hat{\omega}) + \frac{2}{5}\cos(2\hat{\omega}) + 1}_{|H(e^{j\hat{\omega}})| \text{ since always } \geq 0}\right)$$

(inv. Euler's)

Magnitude Response (LTI 1):



Phase Response (LTI 1):



For LTI System 2:

$$b_k = \{1, -1\} \implies H(e^{J\hat{\omega}}) = 1 - e^{-J\hat{\omega}}$$

$(M=1)$

$$= e^{-J\hat{\omega}/2}\left(e^{J\hat{\omega}/2} - e^{-J\hat{\omega}/2}\right)$$

(factor out $e^{-J\frac{\hat{\omega}}{2}}$, since b_k's are asymmetric)

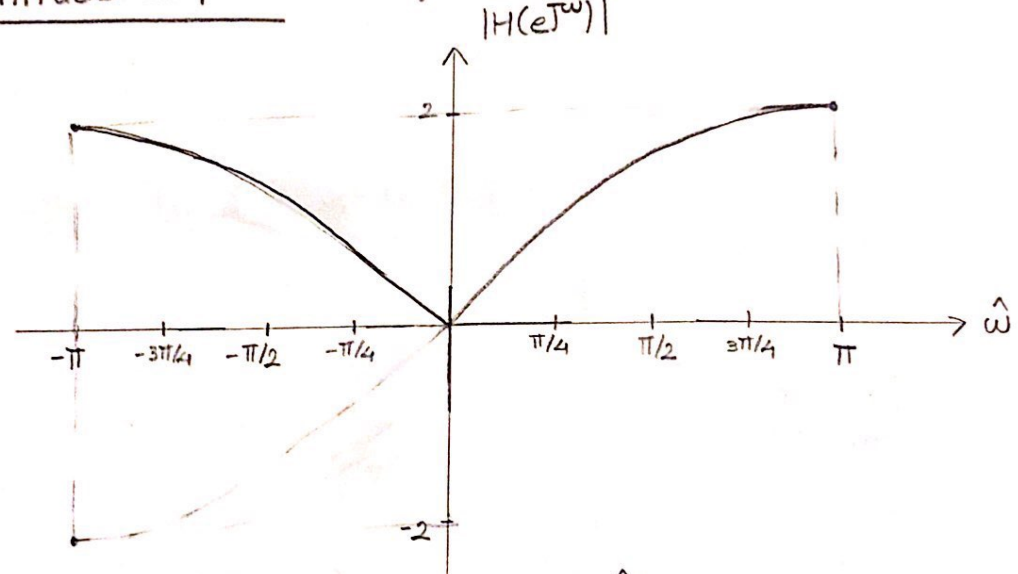
$$= e^{-J\hat{\omega}/2} \cdot 2J\left(\frac{e^{J\hat{\omega}/2} - e^{-J\hat{\omega}/2}}{2J}\right)$$

(inv. Euler's)

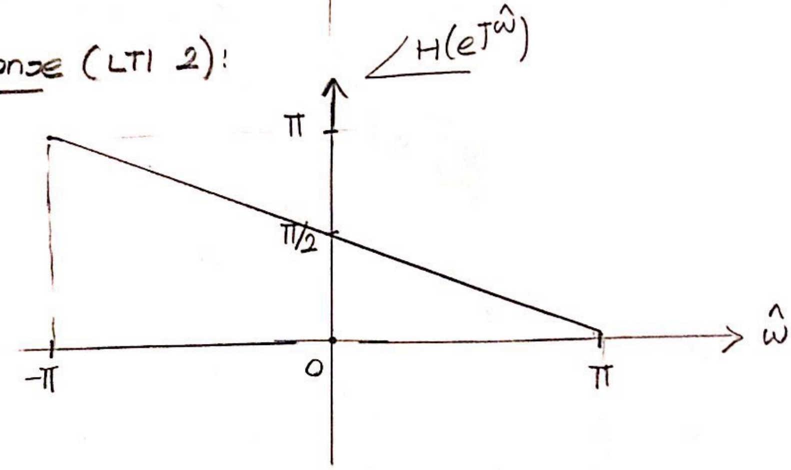
$$= e^{-J\hat{\omega}/2} \cdot e^{J\pi/2} \cdot 2\sin(\hat{\omega}/2)$$

$$= \underbrace{e^{J\left(\frac{\pi}{2} - \frac{\hat{\omega}}{2}\right)}}_{\angle H(e^{J\hat{\omega}}) = \frac{\pi}{2} - \frac{\hat{\omega}}{2}} \cdot \underbrace{2\sin(\hat{\omega}/2)}_{|H(e^{J\hat{\omega}})| = |2\sin(\hat{\omega}/2)|}$$

Magnitude Response (LTI 2):



Phase Response (LTI 2):



c:

for $x[n] = 0.5^n u[n] - 0.1^n u[n-1]$:

$$\boxed{\begin{aligned} v[n] = &\frac{1}{5}\left(0.5^n u[n] + 0.5^{n-1} u[n-1] + 0.5^{n-2} u[n-2] + 0.5^{n-3} u[n-3] + 0.5^{n-4} u[n-4]\right) \\ &- \frac{1}{5}\left(0.1^n u[n-1] + 0.1^{n-1} u[n-2] + 0.1^{n-2} u[n-3] + 0.1^{n-3} u[n-4] + 0.1^{n-4} u[n-5]\right) \end{aligned}}$$

Since the cascaded system is LTI and $H(e^{j\hat{\omega}}) = \frac{1}{5} - \frac{1}{5} e^{-j5\hat{\omega}}$,

the impulse response of the overall system is $h[n] = \frac{1}{5}\delta[n] - \frac{1}{5}\delta[n-5]$,

which implies;

$$y[n] = \frac{1}{5} x[n] - \frac{1}{5} x[n-5]$$

$$\boxed{y[n] = \frac{1}{5} 0.5^n u[n] - \frac{1}{5} 0.1^n u[n-1] - \frac{1}{5} 0.5^{n-5} u[n-5] + \frac{1}{5} 0.1^{n-5} u[n-6]}$$

Q4:

a: i: $x[n] = c^n u[n] \xleftrightarrow{z} X(z) = \sum_{n=-\infty}^{\infty} c^n u[n] z^{-n} = \sum_{n=0}^{\infty} (cz^{-1})^n$

$$\boxed{X(z) = \frac{1}{1-cz^{-1}}} \quad \text{for } |cz^{-1}| < 1$$

$$\underbrace{|z| > |c|}_{\text{region of convergence}}$$

ii: $x[n] = 3\sin(0.5\pi n)u[n] \xleftrightarrow{z} X(z) = \sum_{n=-\infty}^{\infty} 3\sin(0.5\pi n)u[n] z^{-n}$

$$X(z) = 3\sum_{n=0}^{\infty} \sin(0.5\pi n) z^{-n}$$

$$= 3\sum_{n=0}^{\infty} \left(\frac{e^{j0.5\pi n} - e^{-j0.5\pi n}}{2j}\right) z^{-n}$$

$$= \frac{3}{2j}\left(\sum_{n=0}^{\infty}(e^{j0.5\pi}z^{-1})^n - \sum_{n=0}^{\infty}(e^{-j0.5\pi}z^{-1})^n\right)$$

$$= \frac{3}{2j}\left[\frac{z}{z-e^{j0.5\pi}} - \frac{z}{z-e^{-j0.5\pi}}\right] \quad \text{for } |e^{j0.5\pi}z^{-1}| < 1$$

$$|z| > |e^{j0.5\pi}|$$

and

$$= \frac{3}{2j}\left[\frac{\cancel{z^2} - ze^{-j0.5\pi} \cancel{- z^2} + ze^{j0.5\pi}}{z^2 - ze^{-j0.5\pi} - ze^{j0.5\pi} + 1}\right] \quad |e^{-j0.5\pi}z^{-1}| < 1$$

$$\underbrace{|z| > |e^{-j0.5\pi}|}_{\text{region of convergence}}$$

$$= 3\,\frac{z\sin(\cancel{0.5\pi})^{\to 1}}{z^2 - 2z\cos(\cancel{0.5\pi})^{\to 0} + 1}$$

$$\boxed{X(z) = \frac{3z}{z^2+1}}$$

b: i: $\dfrac{z}{z-a} = \dfrac{1}{1-az^{-1}} \xleftrightarrow{z^{-1}} \boxed{x[n] = a^n u[n]}$

ii: $\dfrac{z^{-2}}{1-0.9z^{-1}} \longleftrightarrow \boxed{x[n] = 0.9^n u[n-2]}$

- Using $x[n-n_0] \xleftrightarrow{z} z^{-n_0} X(z)$ and $a^n u[n] \longleftrightarrow \dfrac{1}{1-az^{-1}}$

c: i: $H_1(z) = 1 - 3z^{-1} + 3z^{-2} - z^{-3}$

$$= (1-z^{-1})(1-2z^{-1}+z^{-2}) = (1-z^{-1})^3$$

numerator: $(1-z^{-1})^3 = \left(\dfrac{z-1}{z}\right)^3 \longrightarrow$ $\boxed{\text{three zeros at } z=1, \text{ three poles at } z=0}$

$$H_2(z) = \frac{1+0.75z^{-1}}{1-0.25z^{-2}} = \frac{1+0.75z^{-1}}{(1+0.5z^{-1})(1-0.5z^{-1})}$$

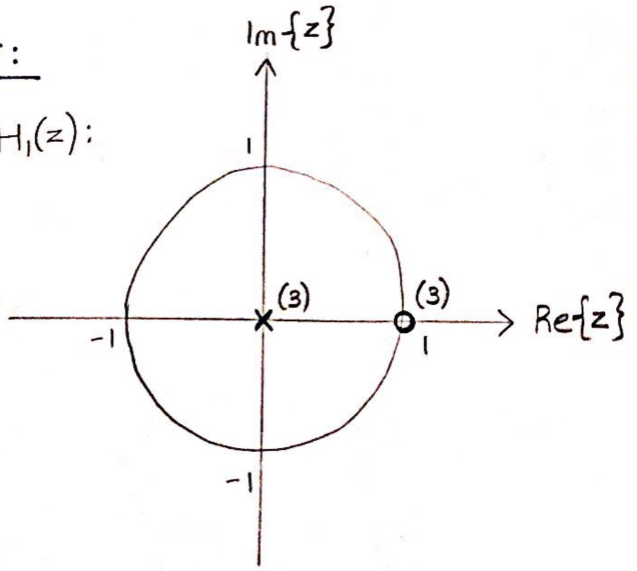
numerator: $1+0.75z^{-1} = \dfrac{z+0.75}{z} \longrightarrow$ zero at $z=-0.75$; ~~pole at $z=0$~~

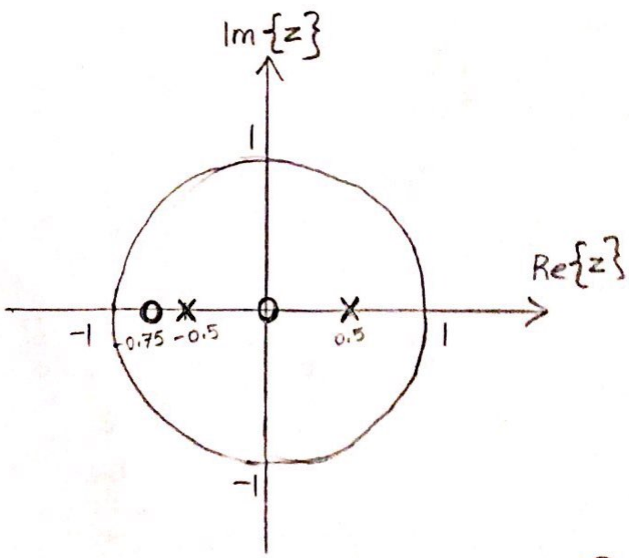
denominator: $1+0.5z^{-1} = \dfrac{z+0.5}{z} \longrightarrow$ pole at $z=-0.5$; ~~zero at $z=0$~~

$1-0.5z^{-1} = \dfrac{z-0.5}{z} \longrightarrow$ pole at $z=0.5$; zero at $z=0$

ii:

$H_1(z)$:





iii :

$$\Rightarrow Y(z) = H_1(z)\, X(z)$$

$$= H_1(z) = (1-z^{-1})^3 = 1 - 3z^{-1} + 3z^{-2} - z^{-3}$$

$(x[n] = \delta[n],$
$X(z) = 1)$

$$\Big\updownarrow z^{-1}$$

$$y[n] = h_1[n] * \delta[n] = \delta[n] - 3\delta[n-1] + 3\delta[n-2] - \delta[n-3]$$

$$\boxed{h_1[n] = \delta[n] - 3\delta[n-1] + 3\delta[n-2] - \delta[n-3]}$$

$$\Rightarrow Y(z) = H_2(z)\, X(z)$$

$$= H_2(z) = \frac{1 + 0.75z^{-1}}{(1+0.5z^{-1})(1-0.5z^{-1})} = \frac{A}{1+0.5z^{-1}} + \frac{B}{1-0.5z^{-1}}$$

$(x[n] = \delta[n],$
$X(z) = 1)$

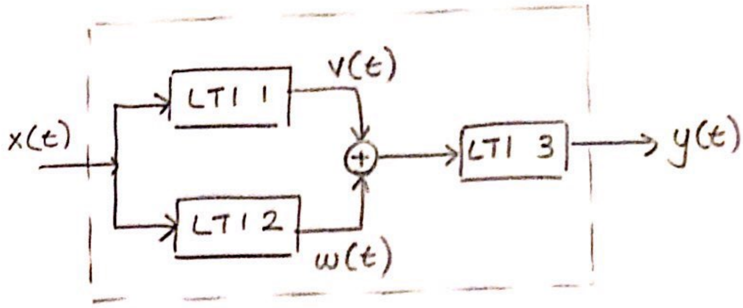
$$\left(\begin{aligned} A &= \left.\frac{1+0.75z^{-1}}{1+0.5z^{-1}}\right|_{z=0.5} = \frac{1+1.5}{2} = \frac{2.5}{2} = 5/4 \\ B &= \left.\frac{1+0.75z^{-1}}{1-0.5z^{-1}}\right|_{z=-0.5} = \frac{1-1.5}{2} = \frac{-0.5}{2} = -1/4 \end{aligned} \right)$$

$$H_2(z) = \frac{5/4}{1-(-0.5)z^{-1}} + \frac{1/4}{1-0.5z^{-1}}$$

$$\Big\updownarrow z^{-1}$$

$$\boxed{h_2[n] = \frac{5}{4}(-0.5)^n u[n] + \frac{1}{4}(0.5)^n u[n]}$$

Q5:



Overall impulse response:

$$h(t) = \left(h_1(t) + h_2(t)\right) * h_3(t)$$

a) $h(t) = \left(\delta(t+2) + \delta(t-2)\right) * u(t-2)$

$= \boxed{u(t) + u(t-4)}$ (*)

$\left(x(t) * \delta(t-t_0) = x(t-t_0) \right)$ (*)

Causal : No future value is used

Unstable: $\int_{-\infty}^{\infty} |h(t)|\,dt = \infty$

b) $h(t) = \left(\delta(t+2) + \delta(t-2)\right) * u(t)$

$= \boxed{u(t+2) + u(t-2)}$ (*)

Non-causal: A future value $(t+2)$ is used

Unstable: $\int_{-\infty}^{\infty} |h(t)|\,dt = \infty$

c) $h(t) = \left(u(t+2) + u(t-2)\right) * \delta(t-1)$

$= \boxed{u(t+1) + u(t-3)}$ (*)

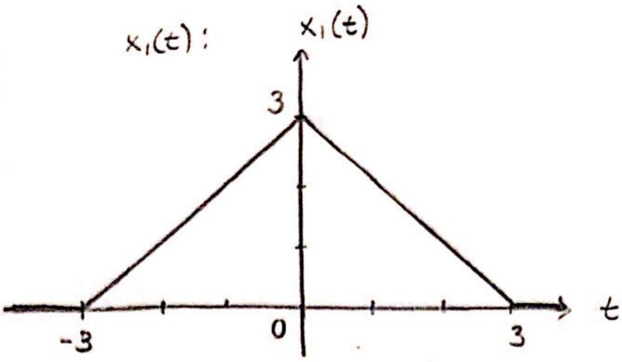
Non-causal: A future value $(t+1)$ is used

Unstable: $\int_{-\infty}^{\infty} |h(t)|\,dt = \infty$

Q6:

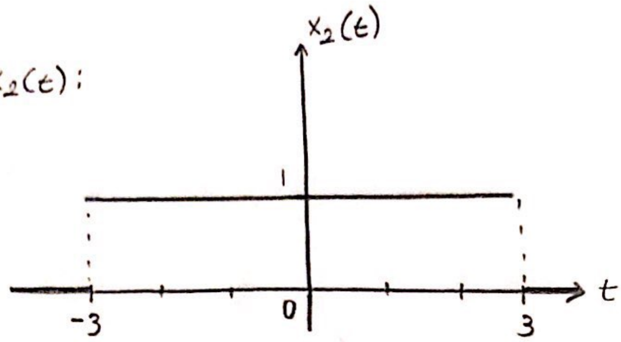
a:

$x_1(t)$:



$$x_1(t) = \begin{cases} t+3 & \text{for } t \in [-3,0) \\ -t+3 & \text{for } t \in [0,3] \\ 0 & \text{otherwise} \end{cases}$$

$x_2(t)$:



$$x_2(t) = \begin{cases} 1 & \text{for } t \in [-3,3] \\ 0 & \text{otherwise} \end{cases}$$

b:

$$\Rightarrow x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

CASE 1: $t+3 \geqslant -3$ and $t+3 < 0$: $-6 \leq t < -3$: $\displaystyle\int_{-3}^{t+3} (\tau+3)\, d\tau = \frac{\tau^2}{2} + 3\tau \Big|_{-3}^{t+3} = \frac{(t+6)^2}{2}$

CASE 2: $-3 \leqslant t < 0$: $\displaystyle\int_{-3}^{0} (\tau+3)\, d\tau + \int_{0}^{t+3} (-\tau+3)\, d\tau = 9 - \frac{t^2}{2}$

CASE 3: $0 \leqslant t \leqslant 3$: $\displaystyle\int_{t-3}^{0} (\tau+3)\, d\tau + \int_{0}^{3} (-\tau+3)\, d\tau = 9 - \frac{t^2}{2}$

CASE 4: $3 < t \leqslant 6$: $\displaystyle\int_{t-3}^{3} (-\tau+3)\, d\tau = \frac{1}{2}(t-6)^2$

$$\boxed{x_1(t) * x_2(t) = \begin{cases} \dfrac{(t+6)^2}{2} & -6 \leq t < -3 \\ 9 - t^2/2 & -3 \leq t \leq 3 \\ \dfrac{(t-6)^2}{2} & , 3 < t < 6 \\ 0 & , \text{otherwise} \end{cases}}$$

$$\Rightarrow \quad x_1(t) * x_1(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_1(t-\tau)\, d\tau$$

CASE 1: $-6 \le t < -3$: $\displaystyle\int_{-3}^{t+3} (\tau+3)(-\tau+t+3)\, d\tau = \frac{(t+6)^2}{6}$

CASE 2: $-3 \le t < 0$: $\displaystyle\int_{-3}^{t} (\tau+3)(\tau-t+3)\, d\tau + \int_{t}^{0} (\tau+3)(-\tau+t+3)\, d\tau$

$$+ \int_{0}^{t+3} (-\tau+3)(-\tau+t+3)\, d\tau = \frac{(-t^3-6t^2+36)}{2}$$

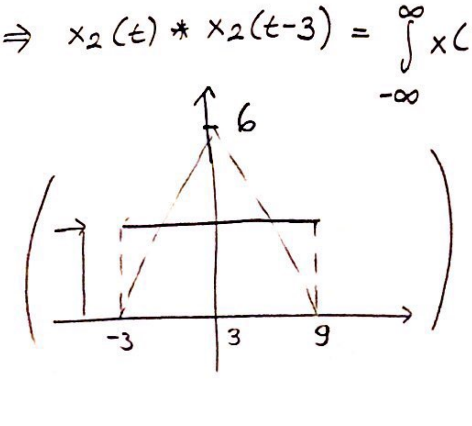
CASE 3: $0 \le t \le 3$: $\displaystyle\int_{-3}^{0} (\tau+3)(\tau-t+3)\, d\tau + \int_{0}^{t} (-\tau+3)(\tau-t+3)\, d\tau$

$$+ \int_{t}^{3} (-\tau+3)(-\tau+t+3)\, d\tau = \frac{(-t^3-6t^2+36)}{2}$$

CASE 4: $3 < t \le 6$: $\displaystyle\int_{t-3}^{3} (-\tau+3)(\tau-t+3)\, d\tau = \frac{(t-6)^2}{6}$

$$\boxed{x_1(t) * x_1(t) = \begin{cases} (t+6)^2/2 & , -6 \le t < -3 \\ (-t^3-6t^2+36)/2 & , -3 \le t \le 3 \\ (t-6)^2/2 & , 3 < t \le 6 \\ 0 & , \text{otherwise} \end{cases}}$$

$$\Rightarrow \quad x_2(t) * x_2(t-3) = \int_{-\infty}^{\infty} x(\tau)\, x_2(t-3-\tau)\, d\tau$$



$$\boxed{x_2(t) * x_2(t-3) = \begin{cases} t+3 & , -3 \le t < 3 \\ 9-t & , 3 \le t < 9 \\ 0 & , \text{otherwise} \end{cases}}$$