Homework 4

(Due 07/05/2019, 17:00)

Instructions:

1. Prepare a report (including your answers/plots) to be uploaded on Moodle.

2. The report should be typeset (for lengthy derivations, the solution can be scanned and embedded into the report).

3. Show all the steps of your work clearly.

4. Unclear presentation of results will be penalized heavily.

5. No partial credits to unjustified answers.

6. Use Matlab or Python for computations.

7. Return all Matlab/Python code that you wrote in a single file.

8. Code should be commented, code for different HW questions should be clearly separated.

9. The code file should NOT return an error during runtime.

10. If the code returns an error at any point, the remaining part of your code will not be evaluated (i.e., 0 points).

Question	Points	Your Score
Q1	25	
Q2	25	
Q3	25	
Q4	25	
TOTAL	100	

Question 1. [25 points]

Stimuli consisting of face images have been used in a study on visual perception. Experimental stimuli are provided in the file `hw4_data1.mat`, which contains face images downsampled to a 32×32 square grid. The images are stored in a matrix $faces$, with 1000 rows (number of different images) and 1024 columns (number of image pixels). Answer the questions below.

a) The experimenter would like to fit encoding models between the stimuli (i.e., face images) and the measured neural responses. However, one first needs to define explanatory variables (i.e., regressors) that capture important variations in stimulus properties during the experiments. To accomplish this goal, perform PCA on the 1000 faces images. Plot the proportion of variance explained by each individual PC, for the first 100 PCs. Display the first 25 PCs using the function `dispImArray.m`.

b) The experimenter would like to know how many PCs are sufficient to obtain a reasonable representation of the stimuli. Reconstruct each image in the matrix $faces$ using their PC projections (i.e., reconstructed images are a linear weighted combination of stimulus PCs). Obtain separate reconstructions based on first 10, 25, and 50 PCs. Display the original images and the reconstructions using `dispImArray.m`, for the first 36 images. Find the mean-squared error (MSE) between the original and reconstructed images, and report the mean and standard deviation of MSE across 1000 images. Interpret the results.

c) Instead of PCA, find explanatory variables to capture stimulus properties using independent component analysis (ICA). Use the `FastICA` package available on Moodle, and use PCA-based reduction to 50 dimensions during the calls (i.e., by setting `lastEig`). Note that unlike PCA, ICA results are not deterministic. Recall `fastica.m` to return 10 ICs, 25 ICs, and 50 ICs. Display the obtained ICs using `dispImArray.m`. Reconstruct face images based on their IC projections. Display the original and reconstructed images based on 10, 25, and 50 ICs. Report the mean and std of MSE in each case. Compare your results with **part b**.

d) Finally, find explanatory variables to capture stimulus properties using non-negative matrix factorization (NNMF). NNMF requires its input to be strictly positive, so add a single scalar constant to all entries of the matrix $faces$ to satisfy this constraint (but only add the minimum amount required). Recall `nnmf.m` to return 10, 25, and 50 MFs. Display the obtained MFs using `dispImArray.m`. Reconstruct face images based on their MF projections. Display the original and reconstructed images based on 10, 25, and 50 MFs. Report the mean and std of MSE in each case. Compare your results with **parts b and c**.

Question 2. [25 points]

BOLD responses evoked by two distinct auditory stimulus categories, human speech versus nature sounds, have been recorded across human auditory cortex. The experimental data are provided in `hw4_data2.mat`. This file contains an array *stype*, identifying the stimulus category as 1 (speech) or 2 (nature sound), and a matrix *vresp*, representing the responses to each of 181 stimuli across 1626 voxels in auditory cortex. Answer the questions below.

a) Examine whether the two stimulus categories are discriminable based on the response patterns they elicit across auditory cortex. Using `pdist2.m`, find the similarity of the response patterns across 181 stimuli. Obtain separate confusion matrices (i.e., similarity estimates) based on Euclidean, Cosine and Correlation distance metrics. Show the confusion matrices for each distance metric with `imagesc`. Comment on the results. Which distance metric seems to discriminate best among the stimulus categories?

b) It is hard to visualize the stimulus categories in the space of response patterns, due to the sheer number of voxels. Obtain a low-dimensional visualization of the data, using multi-dimensional scaling (MDS, `cmdscale.m`). Perform MDS analysis with Euclidean, Cosine and Correlation distance metrics. Show scatter plots of the stimuli, where the x-axis represents projections onto the first MDS component, and the y-axis represents projections onto the second MDS component. Based on the information provided in the array *stype*, use different colors and symbols for two distinct stimulus categories. Are these categories distinguishable in the MDS space?

c) Since the experimenter used mutually-exclusive stimulus categories, you would like to separate the 181 stimulus instances into two groups based on their response patterns. Run a simple k-means algorithm to separate the stimuli into two clusters. Perform separate cluster analyses based on Euclidean, Cosine and Correlation distance metrics. Show scatter plots of the stimuli in the two-dimensional MDS space that was computed in **part b**. However this time, use different colors/symbols to label the data points for the two stimulus clusters returned by the k-means algorithm. Compare the results to those in **part b**. Are your results consistent with ground truth?

Question 3. [25 points]

Consider a population of 21 independent neurons with Gaussian-shaped tuning curves:

$$f_i(x) = A \cdot e^{-(x-\mu_i)^2/(2\sigma_i^2)} \tag{1}$$

The tuning curves have an amplitude of 1 and a standard deviation of $\sigma_i = 1$, with centers μ_i evenly spaced between -10 and 10 along the x-axis. Answer the questions below.

a) Plot all tuning curves in the population on the same axis. Simulate the population response to the stimulus $x = -1$, and plot the population response as a function each neuron's preferred stimulus value.

b) Perform a simulated experiment with 200 trials. In each trial, sample a stimulus intensity uniformly from the interval $[-5\ 5]$, simulate the 21-long vector of population response $\vec{r}$. Assume that each neuron's response is corrupted by an additive Gaussian noise with zero mean and $\sigma/20$ standard deviation. Implement a winner-take-all decoder, and calculate the stimulus estimate x_{WTA} for each trial. Plot the actual and estimated stimulus on the same graph. Compute the mean and standard deviation of error in stimulus estimation across 200 trials.

c) For the experimental trials simulated in **part b**, implement a maximum-likelihood decoder, and calculate the stimulus estimate x_{ML} for each trial. Plot the actual and estimated stimulus on the same graph. Compute the mean and standard deviation of error in stimulus estimation across 200 trials. (Hint: To find x_{ML}, you can calculate the log-likelihood for the entire stimulus range. Note that the tuning curve gives you the expected value (i.e., mean) of the neural response, and there is additional variability due to additive noise that you need to consider.)

d) For the experimental trials simulated in **part b**, implement a maximum-a-posteriori decoder, and calculate the stimulus estimate x_{MAP} for each trial. Assume that the prior of the stimulus value x follows a Gaussian distribution with a mean of 0 and a standard deviation of 2.5. Plot the actual and estimated stimulus on the same graph. Compute the mean and standard deviation of error in stimulus estimation across 200 trials. Interpret your results.

e) Perform an experiment with 200 trials of stimulus intensity. In each trial, sample a stimulus intensity from the interval $[-5\ 5]$. For the resulting stimulus vector (of length 200), separately simulate the population response vectors ($\vec{r}$) for $\sigma_i = 0.1$, $\sigma_i = 0.2$, $\sigma_i = 0.5$, $\sigma_i = 1$, $\sigma_i = 2$, and $\sigma_i = 5$. In each case, assume additive Gaussian noise with zero mean and $1/20$ standard deviation. Calculate MLE estimates of the stimulus x_{ML} based on each population response separately. Compare the mean and standard deviation of error in stimulus estimation for various σ values. Is it better to have narrow or wide tuning curves?

Question 4. [25 points]

Response patterns elicited by 'face' and 'building' stimuli have been recorded across 1626 voxels in ventral-temporal cortex. The experimental data are provided in `hw4_data3.mat`. This file contains an array *stype*, identifying the category of each stimulus 1 (face) or 2 (building), and a matrix *vresp*, representing the responses to 181 stimuli across 1626 voxels. Answer the questions below.

a) Perform multi-dimensional scaling analysis on *vresp* with a Euclidean distance metric. Build an LDA-based classifier using a two-dimensional MDS representation of the data. Measure classification accuracy using leave-one-out cross validation. Show a scatter plot of *vresp* in MDS space, labeling responses for the two stimulus categories with different colors. On the same figure, plot the decision boundary of the classifier with a line.

b) Perform multi-dimensional scaling analysis on *vresp* with a correlation distance metric. Build an LDA-based classifier using a two-dimensional MDS representation of the data. Measure classification accuracy using leave-one-out cross validation. Show a scatter plot of *vresp* in MDS space, labeling responses for the two stimulus categories with different colors. On the same figure, plot the decision boundary of the classifier with a line. Compare your results to **part a**.

c) Perform multi-dimensional scaling analysis on *vresp* with a correlation distance metric. Build separate LDA-based classifiers using MDS representations in [1:1:5] dimensions. Measure classification accuracy using leave-one-out cross validation. Show a bar plot of classification accuracy as a function of number of MDS dimensions. During display, zoom in on the appropriate range of classification accuracies to clearly show differences between classifiers (e.g., [95 100]). How many dimensions are sufficient?