EEE 482/582: Computational Neuroscience

Homework 2

(Due 19/03/2019, 17:00)

Instructions:

1. Prepare a report (including your answers/plots) to be uploaded on Moodle.
2. The report should be typeset (for lengthy derivations, the solution can be scanned and embedded into the report).
3. Show all the steps of your work clearly.
4. Unclear presentation of results will be penalized heavily.
5. No partial credits to unjustified answers.
6. Use Matlab or Python for computations.
7. Return all Matlab/Python code that you wrote in a single file.
8. Code should be commented, code for different HW questions should be clearly separated.
9. The code file should NOT return an error during runtime.
10. If the code returns an error at any point, the remaining part of your code will not be evaluated (i.e., 0 points).

Question	Points	Your Score
Q1	20	
Q2	30	
Q3	20	
Q4	30	
TOTAL	100	

Question 1. [20 points]

The subthreshold membrane-potential dynamics for the leaky integrate-and-fire neuron is determined by:

$$\tau_m \frac{dv}{dt} = -v + RI(t) \tag{1}$$

The resting potential is assumed to be zero. Spikes are emitted when the voltage reaches a threshold θ. After each spike emission, the potential is reset to 0. Answer the questions below. Be careful when picking the temporal step size in all parts, rough discretizations may lead to incorrect conclusions.

a) Find the analytical solution of $v(t)$ in Eq. 1 (assuming subthreshold activity), for a DC input current $I(t) = I_o$ and an initial value of $v(0^-) = 0$.

b) Confirm your answer to **part a** by numerically solving Eq. 1 for $\tau_m = 10$ msec, $R = 1\ k\Omega$, $I_o = 2$ mA. For this purpose, discretize the differential equation for the membrane potential with a sufficiently small temporal step size. Plot the solution $v(t)$ for $t \in [0\ 100]$ msec.

c) Numerically solve Eq. 1 for $\tau_m = 10$ msec, $R = 1\ k\Omega$, $I_o = 2$ mA, and $\theta = 1$ V. (Remember to set the potential to 0 after each spike emission.) Plot the solution $v(t)$ for $t \in [0\ 100]$ msec.

d) Compute and sketch the firing rate of the neuron characterized in **part c** as a function of the input current I_o, where $I_o \in [2\ 10]$ mA.

e) Simulate the solution to Eq. 1 for $\tau_m = 10$ msec, $R = 1\ k\Omega$, and $\theta = 1$ V. Assume that $I_o = 2$ mA + $n(t)$, where $n(t)$ is a stationary Gaussian process with mean of 0 and std of 4 mA. Simulate the affect of this noise on the firing rate curve plotted in **part d**.

Question 2. [30 points]

The responses of a cat LGN cell to two-dimensional visual images are contained in the file `c2p3.mat`, data are described in Kara et al., Neuron 30:803-817 (2000). In the file, `counts` is a vector containing the number of spikes in each 15.6 ms bin, and `stim` contains the 32767, 16x16 images that were presented at the corresponding times. Specifically, `stim(x,y,t)` is the stimulus presented at the coordinate `(x,y)` at time step `t`. Answer the questions below. (Note that `stim` is provided in integer format.)

a) Calculate the STA images for each of the 10 time steps before each spike and show them all. For display, use `imagesc` with a grayscale colormap and identical display windowing for all STA images. Based on the STA derived filter, describe what type of spatio-temporal stimulus this LGN cell is selective for.

b) Describe the changes in STA images across time. Sum the STA images over one of the spatial dimensions. You should obtain a matrix of 16 pixels by 10 time steps as a result of this process. Show this matrix (using `imagesc`). Based on the computed matrix, describe the temporal selectivity of the LGN cell. Is the matrix space-time separable?

c) Project the stimulus onto the STA image at a single time step prior to the spike. Obtain the projection for each time sample by computing the Frobenius inner product between the stimulus image and the STA image. Create a histogram from all stimulus projections, and another histogram from stimulus projections at time bins where a non-zero spike count was observed. Use identical binning for the two histograms, and normalize each histogram to a maximum of 1. Compare the histograms with a bar plot. Comment on whether STA significantly discriminates spike-eliciting stimuli.

Question 3. [20 points] You are asked to characterize the response properties of 2 neurons whose functions are unknown. You conduct experiments, where you can measure neural responses (i.e., average firing rate) to an external stimulus. The compiled matlab functions that can be used to find the responses are `unknownNeuron1.p` and `unknownNeuron2.p`, respectively. During each experimental trial you can record response samples elicited by a stimulus vector of length $N = 50$ (column vector). Answer the questions below. Include plots of measured responses.

a) Assume the stimulus vector is an impulse in the first row of the column vector. Measure each neuron's responses to this stimulus. Check whether the impulse response is time-invariant by comparing the responses to impulses at any other row (assume circular time-invariance at the boundaries). Check whether the response to a sum of these impulses (at different locations in the stimulus vector) is equal to the sum of their individual responses.

b) Write a function that constructs the response profile of the first neuron as a function of stimulus temporal frequency. Assume that the stimulus vectors are cosines with unity amplitude and zero phase. Plot the response magnitude as a function of temporal frequencies in the range $[0\ 10\pi]/N$. What is the optimal stimulus for this neuron?

c) Write a function that constructs the response profile of the second neuron to stimulus intensity. Assume that each stimulus vector used in the experiment is constant across time (i.e., all elements must be the same). Plot the response magnitude as a function of stimulus intensity in the range [0 20]. Does this neuron respond linearly to stimulus intensity? What is the optimal stimulus for this neuron?

d) Modify the functions in **part b** and **part c**, such that the stimulus intensity during each trial is corrupted by additive noise, $n(t)$, due to spontaneous input signals from a population of neurons connected to the dentrites of the two neurons of interest. Assume that $n(t)$ is a stationary Gaussian process (0 mean, σ std). Measure the responses to each stimulus vector during 100 separate trials. When constructing the response profiles, plot the mean response and show the error bars (i.e., specifying the 68% confidence interval). Show your results for $\sigma = 1$, $\sigma = 2.5$, and $\sigma = 5$. Are your conclusions about the optimal stimulus the same for all noise levels?

Question 4. [30 points] Answer the questions below. Include plots whenever applicable.

a) Construct an on-center difference-of-gaussians (DOG) center-surround receptive field centered at 0:

$$D(x,y) = \frac{1}{2\pi\sigma_c^2} e^{-(x^2+y^2)/2\sigma_c^2} - \frac{1}{2\pi\sigma_s^2} e^{-(x^2+y^2)/2\sigma_s^2} \tag{1}$$

Sample this receptive field as a 21x21 matrix, with a central Gaussian width of $\sigma_c = 2$ pixels and a surround Gaussian width of $\sigma_s = 4$ pixels. Display the generated receptive field.

b) Neurons in lateral geniculate nuclei (LGN) have DOG receptive fields. Suppose that there is a separate LGN neuron with a receptive field centered on each pixel in the image. Compute the responses of each neuron to the image given in `hw2_image.bmp`. Place the neural responses topographically according to the centers of their receptive fields, and display the neural activity as an image (using `imagesc`). (Note: Be careful not to introduce artifacts at the image boundary.)

c) Build an edge detector by thresholding the neural activity image (i.e., setting all values above a certain threshold to 1 and the remainder to 0.) Tune the parameters of the DOG receptive fields and the threshold to optimize the edge detector's performance.

d) Construct a Gabor receptive field on the same 21x21 pixel grid:

$$D(\vec{x}) = \exp\left(-\left(\vec{k}(\theta)\cdot\vec{x}\right)^2/2\sigma_l^2 - \left(\vec{k}_\perp(\theta)\cdot\vec{x}\right)^2/2\sigma_w^2\right)\cos\left(2\pi\frac{\vec{k}_\perp(\theta)\cdot\vec{x}}{\lambda} + \phi\right) \tag{2}$$

Here, $\vec{k}(\theta)$ is a unit vector with the orientation θ, $\vec{k}_\perp(\theta)$ is a unit vector orthogonal to $\vec{k}(\theta)$, and θ, σ_l, σ_w, λ and ϕ are parameters the comprise the Gabor filter. Start with assumption that $\theta = \pi/2$, $\sigma_l = \sigma_w = 3$ pixels, $\lambda = 6$ pixels, and $\phi = 0$. Display the generated receptive field.

e) Simple cells in V1 have Gabor receptive fields. Suppose that there is a separate V1 neuron with a receptive field centered on each pixel in the image. Compute the responses of each neuron to the image given in `hw2_image.bmp`. Place the neural responses topographically according to the centers of their receptive fields, and display the neural activity as an image (using `imagesc`). What is the function of this Gabor filter?

f) Construct 4 Gabors with $\theta = 0, \pi/6, \pi/3, \pi/2$. Compute combined neural responses to the image `hw2_image.bmp`, by summing the outputs of the individual receptive fields (for different θ). Does the edge detection performance look better in this case? What can you do with these 4 Gabors to further improve the performance?